Forward-backward and CP-violating asymmetries in rare $B_{d,s} \to (V, \gamma)\, \ell^+\ell^-$ decays

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    (a) Form factors model
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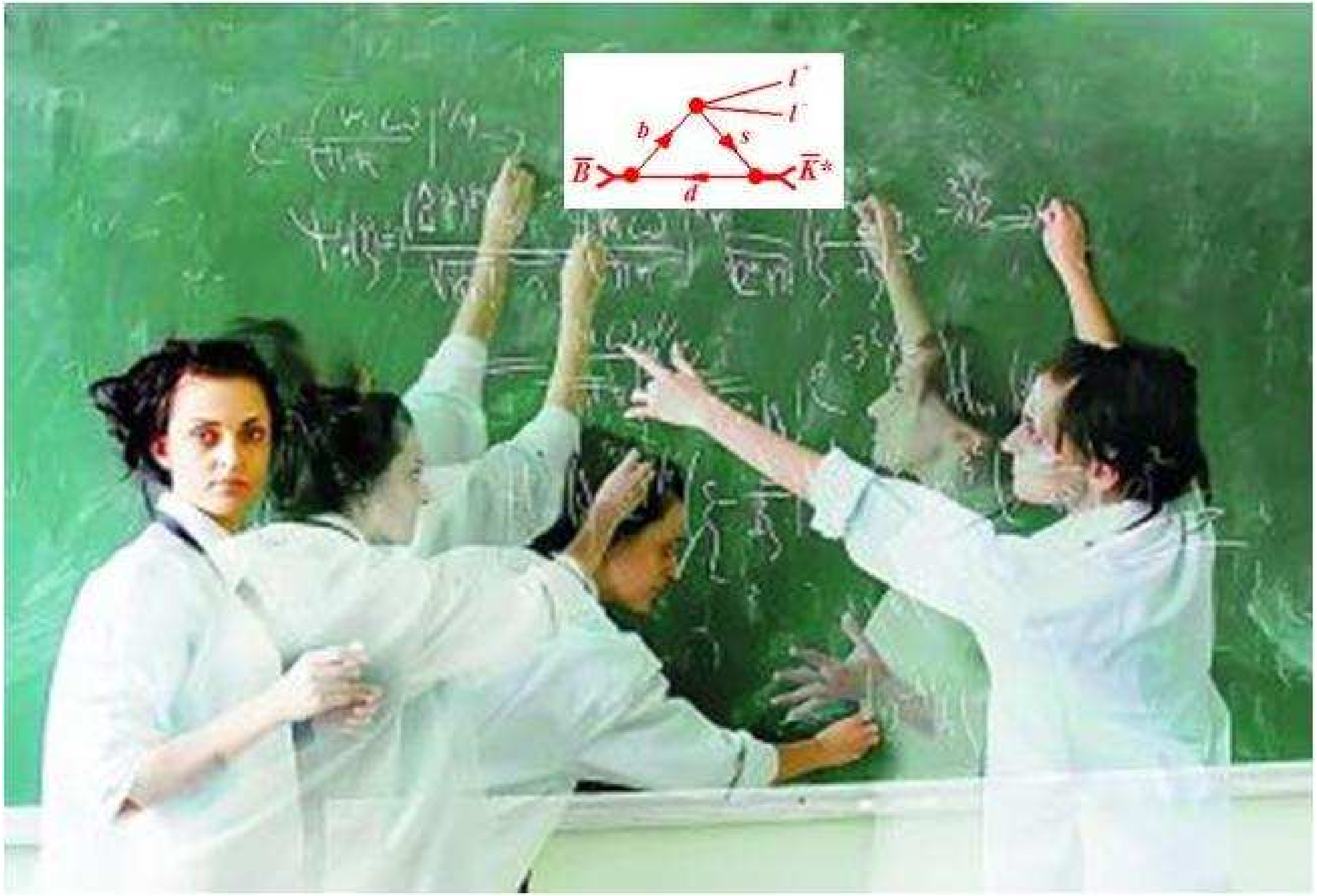
    (c) CP-violating asymmetry

Introduction



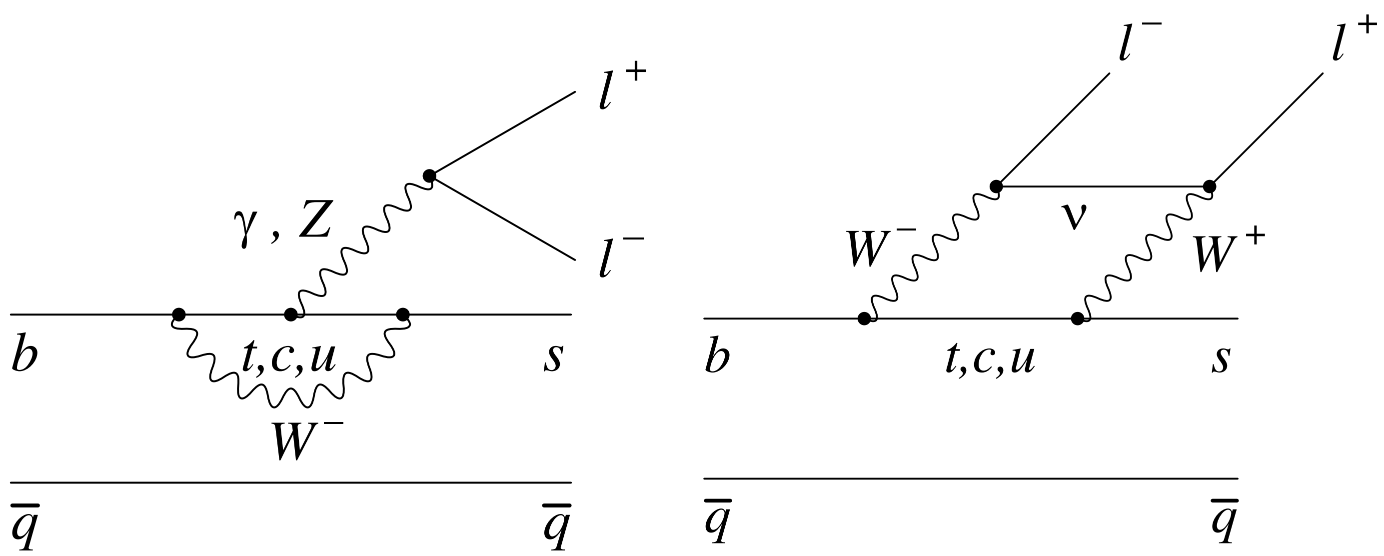
Introduction

During my previous visit in LAL I made a talk about some differential distributions (generally about Forward-backward asymmetry A_{FB}) in rare semileptonic and radiative leptonic decays. This talk is a logical continuation of the previous one. CP-violating effects in rare semileptonic and leptonic radiative decays are going to be concidered.

Rare semileptonic $\bar{B} \to (\bar{P}, \bar{V})\, \ell^+\ell^-$ and rare radiative leptonic $\bar{B} \to \gamma\, \ell^+\ell^-$ decays are induced by $b \to d,\, s$ transitions (so-called Flavor Changing Neutral Currents). Such currents are forbidden at the tree level in the framework of the Standard Model (SM) and occur starting from the lowest order only through the one-loop "penguin" and "box" diagrams.

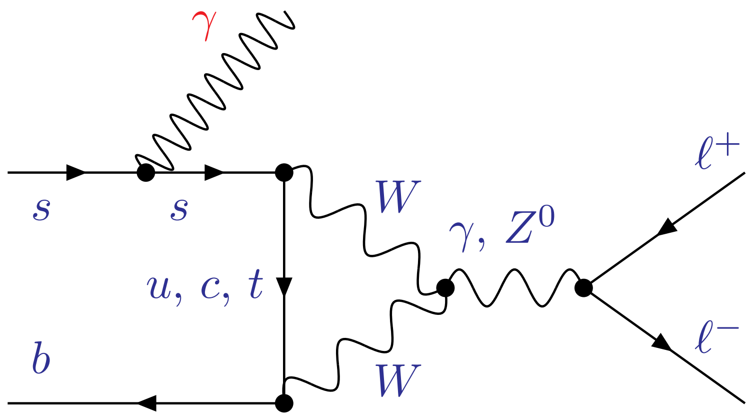
Introduction
Some examples of one-loop diagrams – I

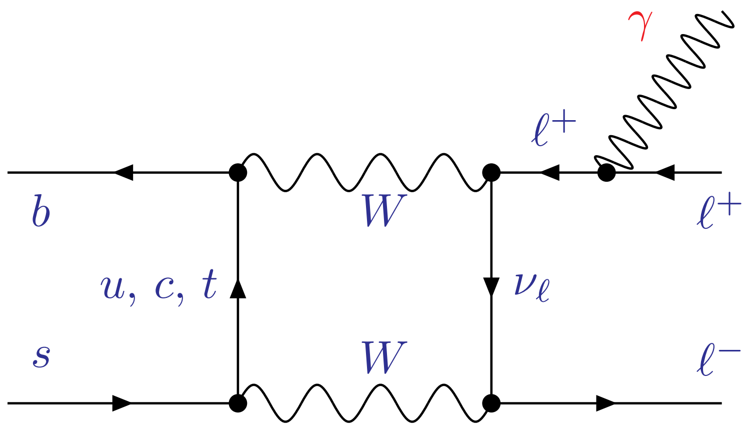
Lowest order Standard Model contributions to the $\bar{B}_q^0 \to \bar{K}^{0\,*}\ell^+\ell^-$ **decays.**



Introduction
Some examples of one-loop diagrams – II

Lowest order Standard Model contributions to the $B_s^0 \to \gamma \ell^+ \ell^-$ decays.





Introduction. Theoretical predictions for the branching ratios of rare semileptonic decays in SM

The branching ratios of these decays are very small: from $\sim 4 \times 10^{-5}$ **for rare radiative decay** $B_d^0 \to K^*\gamma$ **(discovered by CLEO at 1993). Here the branching**

Decay channel	Branching ratio in SM	Reference
$\mathbf{B \to K^* \mu^+\mu^-}$	$\mathbf{1.3 \times 10^{-6}}$	
$B \to K \mu^+\mu^-$	5.3×10^{-7}	D.Melikhov, N.Nikitin, S.Simula, PRD 57, 6814, 1998
$\mathbf{B_s^0 \to \phi \mu^+\mu^-}$	$\mathbf{1.0 \times 10^{-6}}$	
$\mathbf{B \to \rho \mu^+\mu^-}$	$\mathbf{3.2 \times 10^{-7}}$	D.Melikhov, B.Stech, PRD 62, 014006, 2000
$B \to \pi \mu^+\mu^-$	2.0×10^{-8}	
$\mathbf{B_d^0 \to \gamma \mu^+\mu^-}$	$\mathbf{4.0 \times 10^{-10}}$	D.Melikhov, N.Nikitin, PRD 70, 114028, 2004
$\mathbf{B_s^0 \to \gamma \mu^+\mu^-}$	$\mathbf{1.9 \times 10^{-8}}$	F.Kruger, D.Melikhov, PRD 67,034002, 2003

ratios for rare $B_{d,s}^0$ **and** $B^\pm$**-mesons decays are recalculated using** $|V_{ts}^* V_{tb}|^2 = 2.2 \times 10^{-3}$ **and** $|V_{td}^* V_{tb}|^2 = 6.9 \times 10^{-5}$**.**

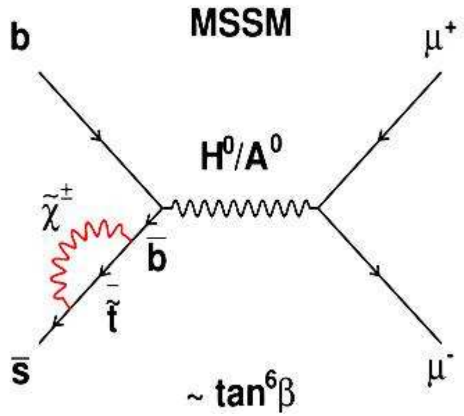
Introduction. Experimental results for BR from Belle, BaBar and Tevatron Collaborations

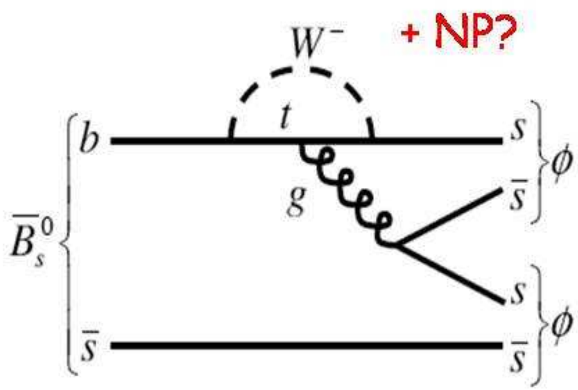
Decay channel	Experimental BR	Ref.
$B \to K^* \mu^+ \mu^-$	$(8.2 \pm 3.1 \pm 1.0) \times 10^{-7}$	CDF, PRD 79, 011104(R), 2009
	$(10.7^{+1.1}_{-1.0} \pm 0.9) \times 10^{-7}$	Belle, **arXiv:0904.0770** [hep-ex], 2009
	$(11.1^{+1.9}_{-1.8} \pm 0.7) \times 10^{-7}$	BaBar, **arXiv:0906.1306** [hep-ex], 2009
$B^+ \to K^+ \mu^+ \mu^-$	$(3.9 \pm 0.07 \pm 0.2) \times 10^{-7}$	BaBar, **arXiv:0906.1306** [hep-ex], 2009
	$(4.8^{+0.5}_{-0.4} \pm 0.3) \times 10^{-7}$	Belle, **arXiv:0904.0770** [hep-ex], 2009
	$(5.9 \pm 1.5 \pm 0.4) \times 10^{-7}$	CDF, PRD 79, 011104(R), 2009
$B_s^0 \to \phi \mu^+ \mu^-$	$< 3.2 \times 10^{-6}$ at 90% CL	D0, PRD 74, 031107, 2006
	$< Br(B_s^0 \to J/\psi \phi) \times$ $\times 2.3 \times 10^{-3}$ at 90% CL	CDF, PRD 79, 011104(R), 2009

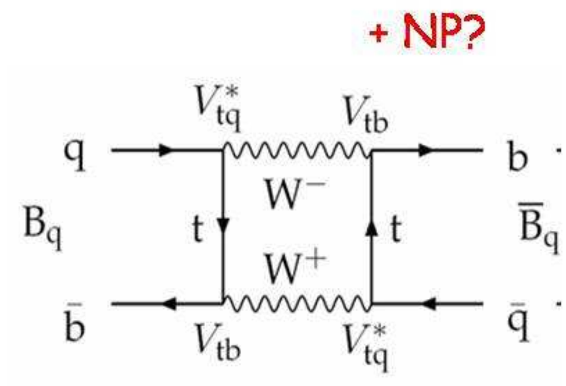
Introduction
Scientific motivation

The main motivation: INDIRECT search of "New Physics" (NP) on LHC and (Super)B-factories.

Heavy virtual non-standard particles may provide a contribution into amplitudes of rare decays. Absolute value of such a contribution may be measured by studying the invariant dimuon mass distribution and A_{FB}. Phase – by the studies of some CP-violation effects corresponding to an interference between weak and strong phases in a matrix element.







Introduction
References

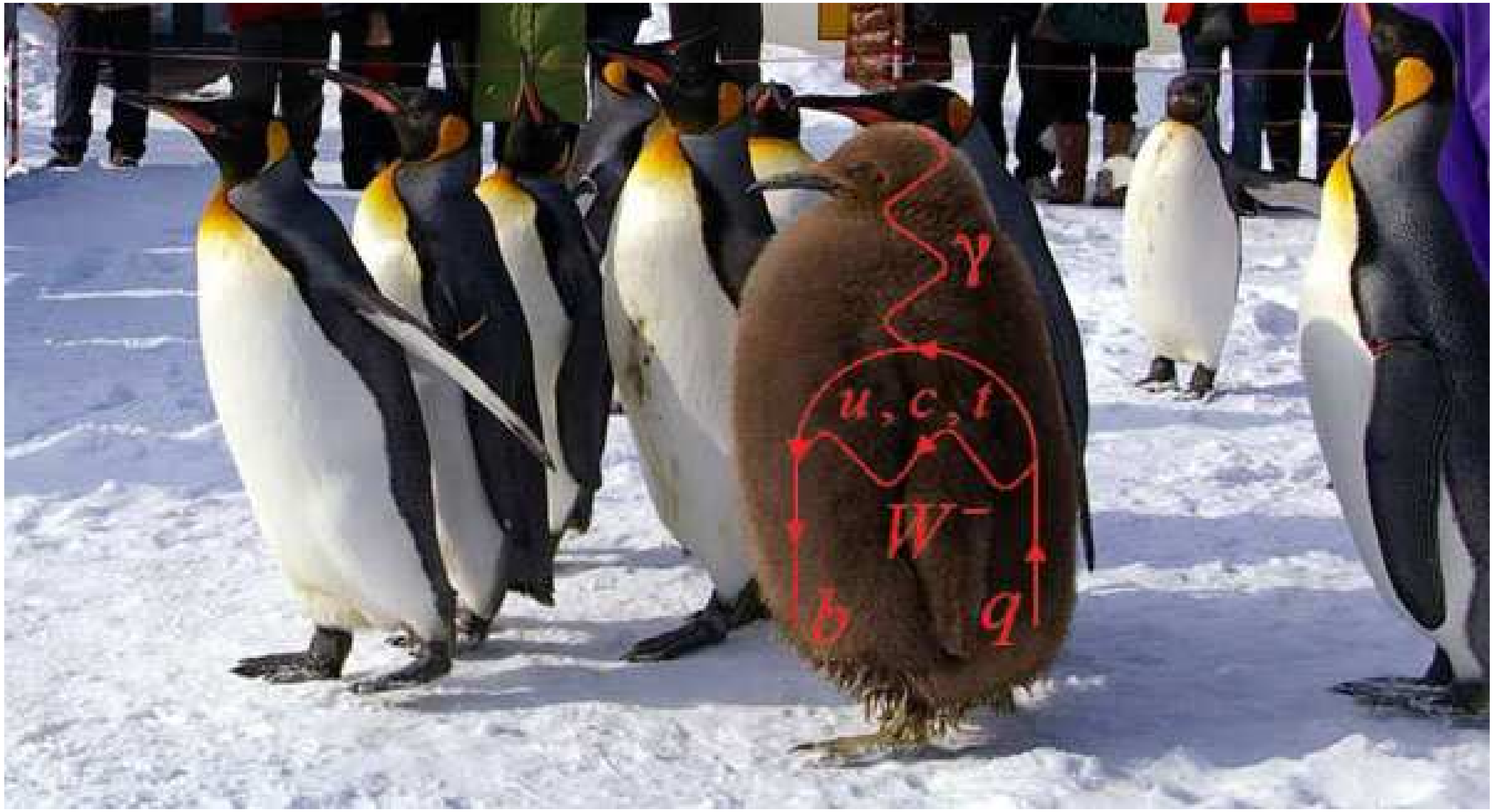
This report is based on following works:

1) I. Balakireva, D. Melikhov, N. Nikitin, D. Tlisov, "Forward-backward and CP-violating asymmetries in rare $B_{d,s} \to (\phi, \gamma) l^+ l^-$ decays", e-Print: arXiv:0911.0605 [hep-ph] (is accepted to the publication in PRD);

2) D. Melikhov, N. Nikitin, K. Toms, "Rare radiative leptonic decays $B_{(d,s)} \to \ell^+ \ell^- \gamma$", Phys.Rev. D70, 114028 (2004); Phys.Atom.Nucl. 68, 1842 (2005);

3) F. Kruger, D. Melikhov, "Gauge invariance and form-factors for the decay $B \to \gamma \ell^+ \ell^-$", Phys.Rev. D67, 034002 (2003).

EFFECTIVE HAMILTONIANS FOR RARE DECAYS



Effective Hamiltonian for rare decays
Common theoretical framework

From the theoretical point of view the $b(\bar{b}) \to q(\bar{q})$ transitions $q = \{d, s\}$ are considered using the effective Hamiltonian

$$H_{\rm eff}^{b(\bar{b})\to q(\bar{q})} = \frac{G_{\rm F}}{\sqrt{2}} V_{tb}V_{tq}^* \Big[\left(1 + \lambda_u^{(q)}\right) \left(C_1(\mu) O_1^{(c)}(\mu) + C_2(\mu) O_2^{(c)}(\mu) \right) - $$
$$- \lambda_u^{(q)} \left(C_1(\mu) O_1^{(u)}(\mu) + C_2(\mu) O_2^{(u)}(\mu) \right) + \sum_{i=3}^{\dots} C_i(\mu) O_i(\mu) \Big] + (\bar{b} \to \bar{q}) \, ,$$

in the form of Wilson expansion, where G_F is Fermi constant, V_{tq} and V_{tb} are the CKM matrix elements, $\lambda_Q^{(q)} = V_{Qb}V_{Qq}^*/V_{tb}V_{tq}^*$ where $Q = \{u, c, t\}$. The set of Wilson coefficients $C_i(\mu)$ depends on the current model. The scale parameter μ separates the perturbative and nonperturbative contributions of the strong interactions. The value of the μ is approximately equal to the mass of b-quark that is $\mu \sim 5$ GeV.

Effective Hamiltonian for rare decays
Set of the basic operators in SM

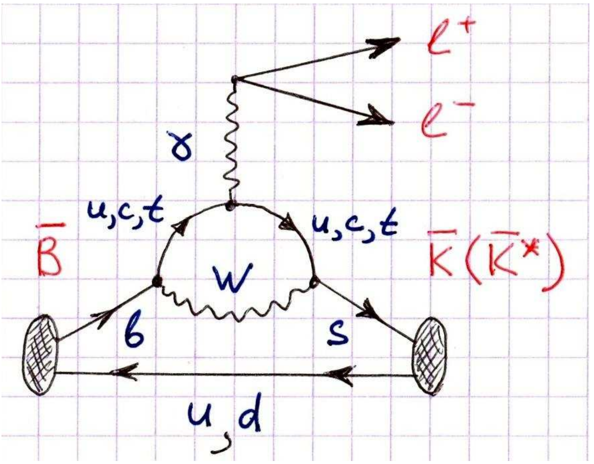
$O_i(\mu)$ **is the set of the basic operators (specific for each model like the set of the Wilson coefficients). The following set of the basic operators provides the main contribution in the matrix elements of the rare leptonic, radiative leptonic and semileptonic** B**-decays:**

$$\begin{aligned}
O_1^{(Q)} &= (\bar{q}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha) \left(\bar{Q}_\beta \gamma_\mu (1-\gamma_5) Q_\beta\right), \\
O_2^{(Q)} &= (\bar{q}_\alpha \gamma^\mu (1-\gamma_5) b_\beta) \left(\bar{Q}_\beta \gamma_\mu (1-\gamma_5) Q_\alpha\right), \\
O_{7\gamma} &= \frac{e}{8\pi^2} \bar{q}_\alpha \sigma_{\mu\nu} [m_b(1+\gamma_5) + m_s(1-\gamma_5)] b_\alpha \; F^{\mu\nu}, \\
O_{9V} &= \frac{e^2}{8\pi^2} (\bar{q}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha) \bar{\ell} \gamma_\mu \ell, \\
O_{10A} &= \frac{e^2}{8\pi^2} (\bar{q}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha) \bar{\ell} \gamma_\mu \gamma_5 \ell,
\end{aligned}$$

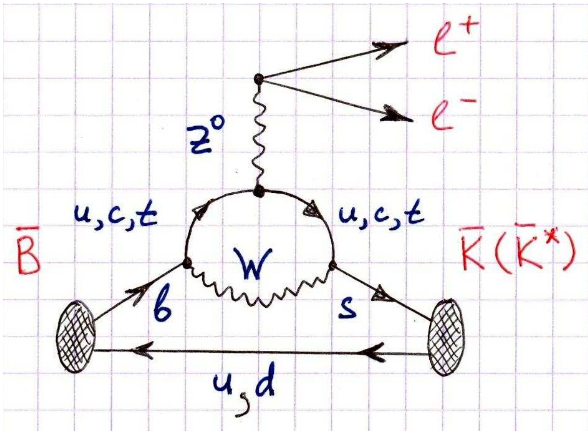
where $F^{\mu\nu}$ – the electromagnetic field tensor, $F^{\mu\nu\,\dagger} = F^{\mu\nu}$ and

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu], \quad e = \sqrt{4\pi\alpha_{\rm em}} > 0.$$

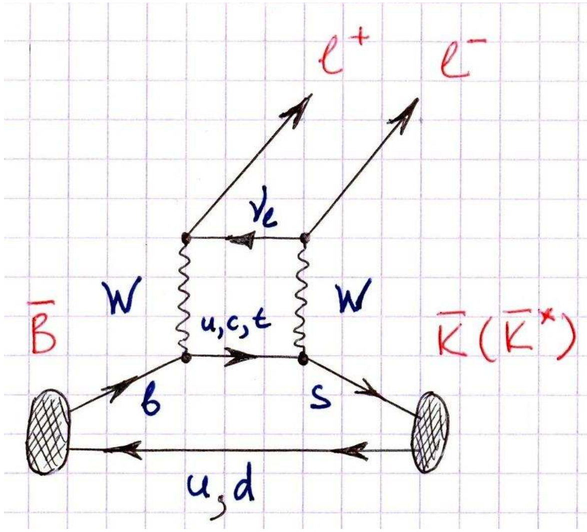
Operator $O_{7\gamma}$ is determined by the contribution from the "penguin" diagram with the photon emission. The "penguin" diagram with a Z^0-boson emission and the "box"-diagram contribute into the O_{9V} and O_{10A} operators.



$C_{7\gamma}(\mu)$



$C_{9V}(\mu) + C_{10A}(\mu)$



$C_{9V}(\mu) + C_{10A}(\mu)$

Effective Hamiltonian for rare decays
The Matrix Elements

The nonperturbative contributions of the strong interactions are contained in the matrix elements of these operators

$$\langle\, \textbf{final states}\ |O_i(\mu)|\ \textbf{initial states}\,\rangle\,.$$

These matrix elements can be described in terms of Lorentz-invariant form factors and structures constructed using 4-momenta of initial and final particles, tensors $g^{\mu\nu}$ **and** $\varepsilon_{abcd} \equiv \varepsilon_{\alpha\beta\mu\nu} a^\alpha b^\beta c^\mu d^\nu$, **where** $\varepsilon^{0123} = -1$.

For the decays $\bar{B} \to (\bar{V}, \gamma)\ell^+\ell^-$ **main basic operators have the form** $Q_i(\mu) = H_i^{\cdots} L_{i\,\ldots}$. **Therefore, the matrix elements have the form:**

$$\sum_i \left\langle \bar{V}(p_2,\, M_2,\, \varepsilon)\ \textbf{or}\, \gamma(k,\, \epsilon)\, |H_i^{\cdots}|\ \bar{B}(p_1,\, M_1)\,\right\rangle\, \bar{\ell}(k_2,\, m) L_{i\,\ldots} \ell(-k_1,\, m).$$

The Effective Hamiltonians for $b\,(\bar b) \to q\,(\bar q)\,\ell^+\,\ell^-$ transitions in SM

$$H_{\rm eff}^{\rm SM} =$$
$$= \frac{G_F}{\sqrt{2}}\,\frac{\alpha_{em}}{2\pi}\,V_{tb}V_{tq}^*\,\Bigg[-2\,\frac{C_{7\gamma}(\mu)}{s}\,\left(\bar q\, i\sigma_{\mu\nu}\left\{m_b\,\left(1+\gamma^5\right)+m_q\,\left(1-\gamma^5\right)\right\}q^\nu b\right)\left(\bar\ell\gamma^\mu\ell\right)+$$
$$+C_{9V}^{{\rm eff}\,(q)}(\mu,\,s)\left(\bar q\gamma_\mu\left(1-\gamma^5\right)b\right)\left(\bar\ell\gamma^\mu\ell\right)+C_{10A}(\mu)\left(\bar q\gamma_\mu\left(1-\gamma^5\right)b\right)\left(\bar\ell\gamma^\mu\gamma^5\ell\right)\Bigg]+$$
$$+\frac{G_F}{\sqrt{2}}\,\frac{\alpha_{em}}{2\pi}\,V_{tb}^*V_{tq}\,\Bigg[2\,\frac{C_{7\gamma}^*(\mu)}{s}\,\left(\bar b\, i\sigma_{\mu\nu}\left\{m_b\,\left(1-\gamma^5\right)+m_q\,\left(1+\gamma^5\right)\right\}q^\nu q\right)\left(\bar\ell\gamma^\mu\ell\right)+$$
$$+C_{9V}^{{\rm eff}\,(\bar q)}(\mu,\,s)\left(\bar b\gamma_\mu\left(1-\gamma^5\right)q\right)\left(\bar\ell\gamma^\mu\ell\right)+C_{10A}^*(\mu)\left(\bar b\gamma_\mu\left(1-\gamma^5\right)q\right)\left(\bar\ell\gamma^\mu\gamma^5\ell\right)\Bigg].$$

The μ dependence of the Wilson coefficients $C_{7\gamma}$, C_{9V} and C_{10A} is calculated using RGE. See A.Buras, M.Munz, PRD52, p.182, 1995.

Numericaly: if $C_2(M_W) = -1$, then at NLO approach $C_{7\gamma}(5\,\textbf{GeV}) \approx 0.312$, $\tilde C_{9V}^{NDR}(5\,\textbf{GeV}) \approx -4.21$ and $C_{10A}(5\,\textbf{GeV}) \approx 4.64$.

Effective Hamiltonian for rare decays
The effective coefficients $C_{9V}^{\text{eff}\,(q)}(\mu,\, s)$ and $C_{9V}^{\text{eff}\,(\bar q)}(\mu,\, s)$.

The Wilson coefficients $C_{9V}^{\text{eff}\,(q)}(\mu,\, s)$ and $C_{9V}^{\text{eff}\,(\bar q)}(\mu,\, s)$ contain the contributions from $u\bar u$- and $c\bar c$-pairs, ρ^0-, ω-, J/ψ-, ψ'-, $\psi(3770)$-, $\psi(4040)$-, $\psi(4160)$- and $\psi(4415)$-resonances in the $s = q^2$ - channel.

We obtain the following expression for the effective coefficients:

$$C_{9V}^{\text{eff}\,(q)}(\mu,\, s) = C_{9V}(\mu,\, s) + C^{(1)}_{\mathbf{res}}(\mu,\, s) + \lambda_u^{(q)}\, C^{(2)}_{\mathbf{res}}(\mu,\, s),$$
$$C_{9V}^{\text{eff}\,(\bar q)}(\mu,\, s) = C_{9V}(\mu,\, s) + C^{(1)}_{\mathbf{res}}(\mu,\, s) + \lambda_u^{(q)\,*}\, C^{(2)}_{\mathbf{res}}(\mu,\, s),$$

where in the SM

$$C_{9V}(\mu,\, s) = \left(1 + \frac{\alpha_s(\mu)}{\pi}\,\omega(s/m_b^2)\right)\tilde C_9^{NDR}(\mu) + \frac{2}{9}\,\left(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)\right).$$

The functions $\tilde C_9^{NDR}(\mu)$, $\omega(s/m_b^2)$ and $h(m_q/m_b, s/m_b^2)$ are calculated in (A.Buras, M.Munz, PRD52, p.182, 1995).

$$C^{(1)}_{\textbf{res}} = (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6)\; H\left(\frac{m_c}{m_b}, \frac{s}{m_b^2}\right) - \\ - \frac{1}{2}(4C_3 + 4C_4 + 3C_5 + C_6)\; h\left(1, \frac{s}{m_b^2}\right) - \frac{1}{2}(C_3 + 3C_4)\; h\left(\frac{m_d}{m_b}, \frac{s}{m_b^2}\right);$$

$$C^{(2)}_{\textbf{res}} = (3C_1 + C_2)\left[H\left(\frac{m_c}{m_b}, \frac{s}{m_b^2}\right) - H\left(\frac{m_u}{m_b}, \frac{s}{m_b^2}\right)\right].$$

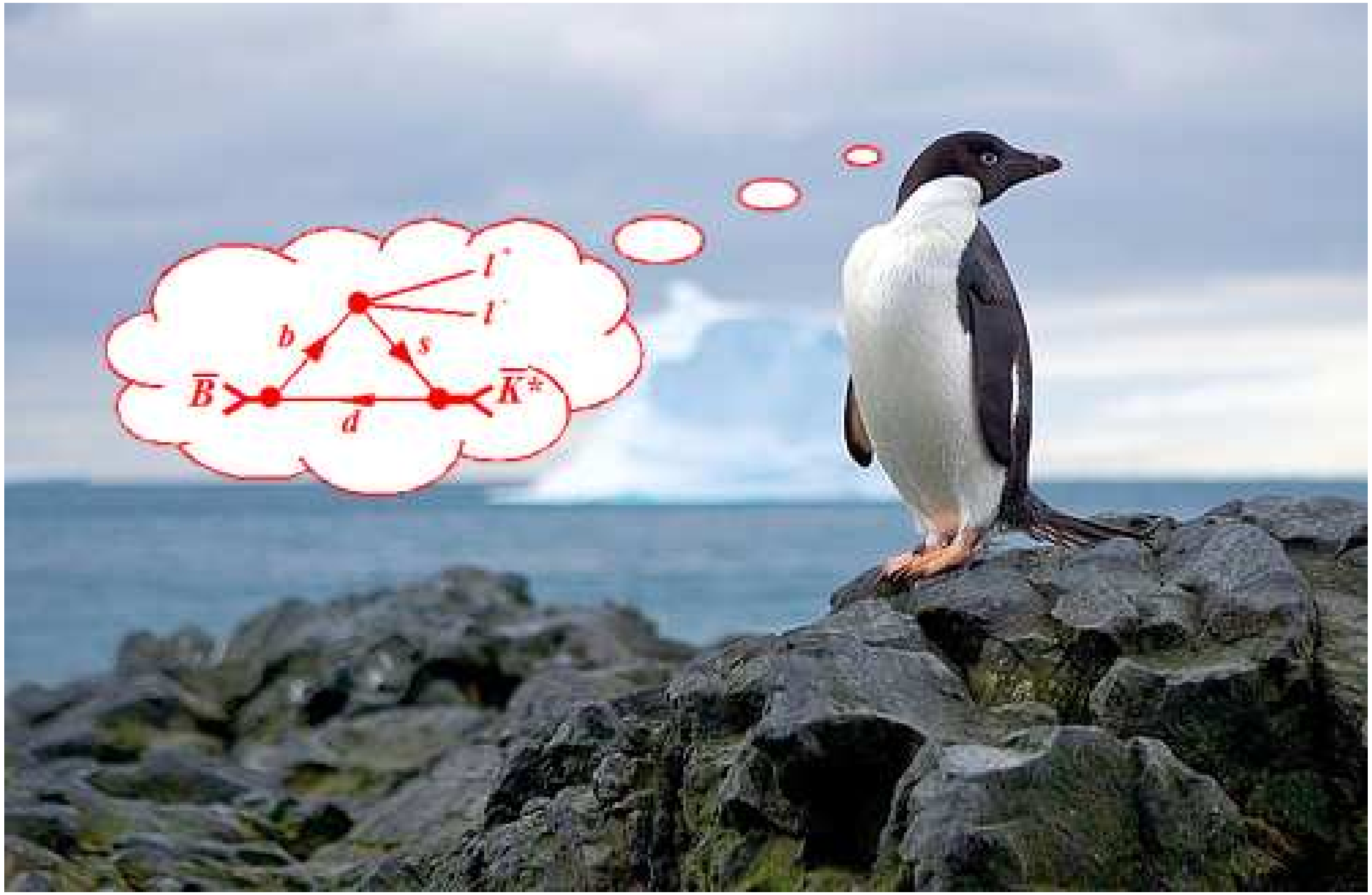
The functions $H\left(m_Q/m_b, s/m_b^2\right)$ include a contribution from $Q\bar{Q}$-pairs, as well as a contribution from the vector resonances with the corresponding quark structure:

$$H\left(\frac{m_c}{m_b}, \frac{s}{m_b^2}\right) = h\left(\frac{m_c}{m_b}, \frac{s}{m_b^2}\right) - \frac{3}{3C_1 + C_2}\,\frac{\pi}{\alpha_{\text{em}}^2}\sum_{V=J/\psi}^{\psi(4415)}\frac{s}{M_V}\,\frac{\Gamma(V\to\ell^+\ell^-)}{M_V^2 - s - iM_V\Gamma_V};$$

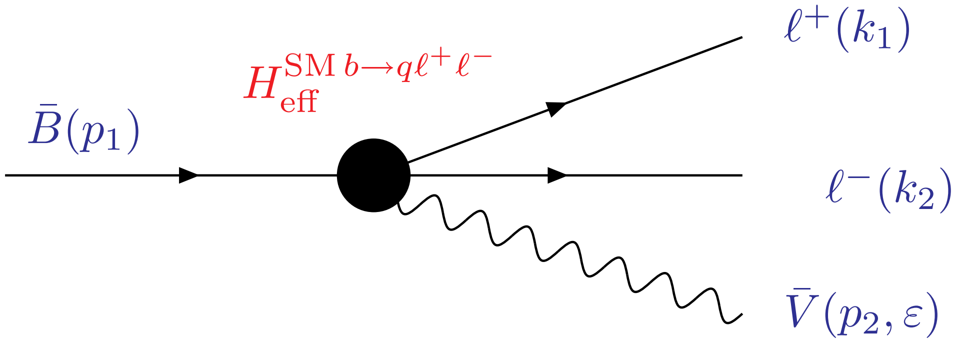
$$H\left(\frac{m_u}{m_b}, \frac{s}{m_b^2}\right) = h\left(\frac{m_u}{m_b}, \frac{s}{m_b^2}\right) - \frac{3}{3C_1 + C_2}\,\frac{\pi}{\sqrt{2}\,\alpha_{\text{em}}^2}\sum_{V=\rho}^{\omega}\frac{s}{M_V}\,\frac{\Gamma(V\to\ell^+\ell^-)}{M_V^2 - s - iM_V\Gamma_V}.$$

The additional factor $1/\sqrt{2}$ in the resonant contribution for the function $H\left(m_u/m_b, s/m_b^2\right)$ takes into account the quark structure of ρ^0- and ω-mesons.

RARE SEMILEPTONIC B-DECAYS



Rare semileptonic decays
Kinematics



$$p_1 = k_1 + k_2 + p_2, = q + p_2, \quad p_1^2 = M_1^2, \quad p_2^2 = M_2^2, \quad k_1^2 = k_2^2 = m^2.$$

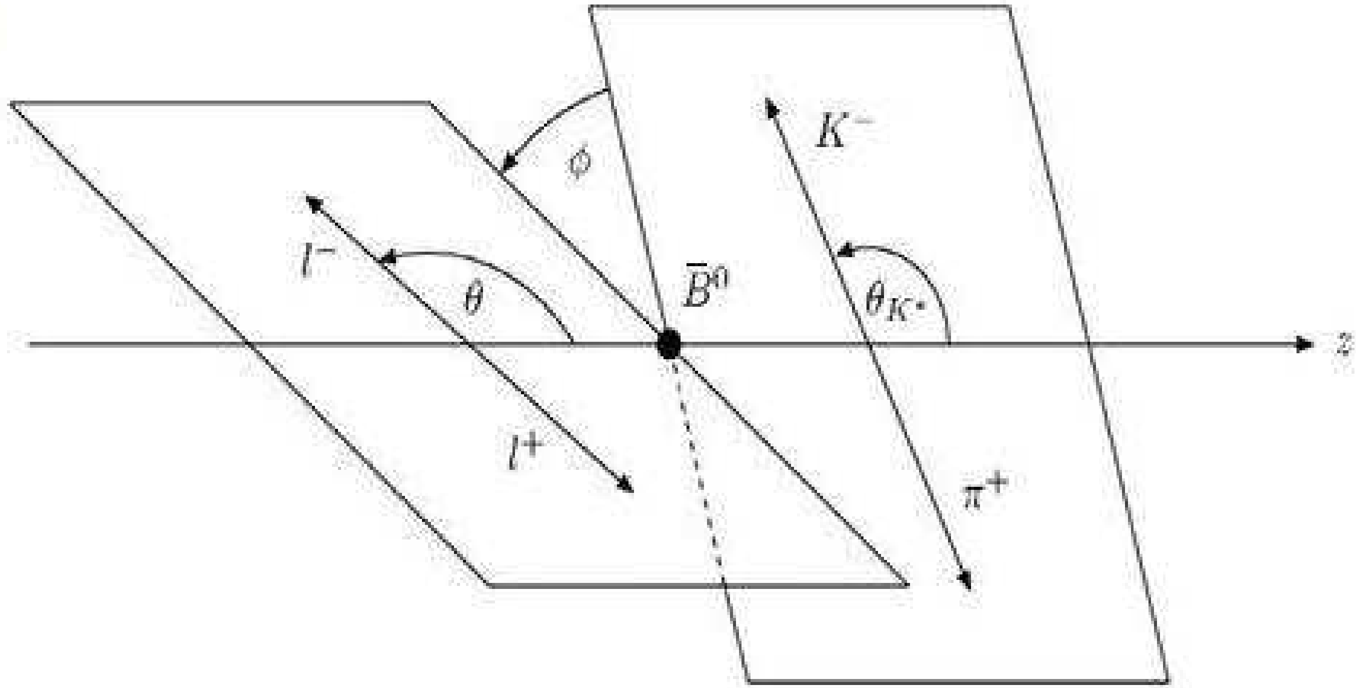
The kinematics of three-body decays can be described in terms of two independent variables. The first independent variable:

$$4\,m^2 \le (s \equiv q^2) \le (M_1 - M_2)^2.$$

In the rest frame of the leptonic pair we define the second independent variable: the angle $\theta \equiv \theta_-$ between ℓ^- and final $\bar{V}$ meson (initial $\bar{B}$) directions.

Alternatively, we can define the angle θ_+ between ℓ^+ and $\bar{V}$ meson (initial $\bar{B}$).

Rare semileptonic decays Kinematics for decay $\bar{B}_d^0 \to (\bar{K}^{*\,0} \to K^-\pi^+)\ell^+\ell^-$



θ_{K^*}: **the angle between** K^- **and** $\bar{B}_d^0$ **in the** $\bar{K}^{*\,0}$ **rest frame;**

ϕ: **the angle between** $\bar{K}^{*\,0} \to K^-\pi^+$ **and** $\ell^+\ell^-$ **decay planes.**

Rare semileptonic decays Form factors for $\bar{B} \to \bar{V}$ transitions

For $\bar{B}(p_1, M_1) \to \bar{V}(p_2, M_2, \varepsilon^*)$ **transitions we define the following form factors:**

$$\langle \bar{V}(p_2, M_2, \varepsilon) \,|\bar{q}\, \gamma_\mu \, b|\, \bar{B}(p_1, M_1)\rangle = \frac{2\, V(q^2)}{M_1 + M_2}\, \epsilon_{\mu\nu\alpha\beta}\, \varepsilon^{*\,\nu}\, p_1^\alpha\, p_2^\beta;$$

$$\langle \bar{V}(p_2, M_2, \varepsilon) \left|\bar{q}\, \gamma_\mu \gamma^5 \, b\right| \bar{B}(p_1, M_1)\rangle = i\, \varepsilon^*_\mu (M_1 + M_2)\, A_1(q^2) -$$

$$- i\, (\varepsilon^*\, p_1)\, (p_1 + p_2)_\mu \frac{A_2(q^2)}{M_1 + M_2} - i\, (\varepsilon^*\, p_1)\, q_\mu \frac{2\, M_2}{q^2} \left(A_3(q^2) - A_0(q^2)\right);$$

$$\langle \bar{V}(p_2, M_2, \varepsilon) \,|\bar{q}\, \sigma_{\mu\nu}\, q^\nu \, b|\, \bar{B}(p_1, M_1)\rangle = 2\, i\, T_1(q^2)\, \epsilon_{\mu\nu\alpha\beta}\, \varepsilon^{*\,\nu}\, p_1^\alpha\, p_2^\beta;$$

$$\langle \bar{V}(p_2, M_2, \varepsilon) \left|\bar{q}\, \sigma_{\mu\nu} \gamma^5\, q^\nu \, b\right| \bar{B}(p_1, M_1)\rangle = T_2(q^2) \left(\varepsilon^*_\mu\, (M_1^2 - M_2^2) - (\varepsilon^*\, p_1)\, (p_1 + p_2)_\mu\right) +$$

$$+ T_3(q^2)\, (\varepsilon^*\, p_1) \left(q_\mu - \frac{q^2}{M_1^2 - M_2^2}\, (p_1 + p_2)_\mu\right).$$

The form factors satisfy the following conditions

$$A_3(q^2) = \frac{M_1 + M_2}{2M_2} A_1(q^2) - \frac{M_1 - M_2}{2M_2} A_2(q^2), \qquad A_0(0) = A_3(0), \qquad T_1(0) = T_2(0).$$

Charge conjugation and form factors for $B \to V$ transitions

We start from the following definitions:

$$\hat{C}\left|\bar{B}_q^0\right\rangle = e^{-i\varphi_B}\left|B_q^0\right\rangle, \quad \hat{C}\left|\bar{V}\right\rangle = e^{-i\varphi_V}\left|V\right\rangle \quad \textbf{and} \quad e^{-i\varphi_h} = e^{-i(\varphi_V - \varphi_B)}.$$

According to these definitions, $B \to V$ transitions form factors can be obtained from the $\bar{B} \to \bar{V}$ form factors using the following replacements:

$$A_0(q^2) \to A_0(q^2)\, e^{-i\varphi_h}; \quad A_1(q^2) \to A_1(q^2)\, e^{-i\varphi_h};$$
$$A_2(q^2) \to A_2(q^2)\, e^{-i\varphi_h}; \quad A_3(q^2) \to A_3(q^2)\, e^{-i\varphi_h};$$
$$V(q^2) \to -V(q^2)\, e^{-i\varphi_h}; \quad T_1(q^2) \to T_1(q^2)\, e^{-i\varphi_h};$$
$$T_2(q^2) \to T_2(q^2)\, e^{-i\varphi_h}; \quad T_3(q^2) \to T_3(q^2)\, e^{-i\varphi_h};$$

Rare semileptonic decays
Amplitudes for $\bar{B} \to \bar{V}\, \ell^+ \ell^-$ transitions

are described by the relations:

$$\bar{A}^{(q)} = \langle \bar{V}(p_2,\, M_2,\, \varepsilon),\, \ell^+(k_1),\, \ell^-(k_2) \left| H_{\rm eff}^{\rm SM} \right| \bar{B}(p_1,\, M_1) \rangle = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi} V_{tb} V_{tq}^*$$

$$\left[\frac{a(\mu,\, s)}{M_1} \epsilon_{\mu \varepsilon^* p_1 p_2} \left(\bar{\ell}(k_2) \gamma^\mu \ell(-k_1) \right) - i\, b(\mu,\, s)\, M_1\, \varepsilon^*_\mu \left(\bar{\ell}(k_2) \gamma^\mu \ell(-k_1) \right) + \right.$$

$$i\, \frac{c(\mu,\, s)}{M_1} P_\mu\, (\varepsilon^* p_1) \left(\bar{\ell}(k_2) \gamma^\mu \ell(-k_1) \right) + i\, \frac{d(\mu,\, s)}{M_1} q_\mu\, (\varepsilon^* p_1) \left(\bar{\ell}(k_2) \gamma^\mu \ell(-k_1) \right) +$$

$$\frac{e(\mu,\, s)}{M_1} \epsilon_{\mu \varepsilon^* p_1 p_2} \left(\bar{\ell}(k_2) \gamma^\mu \gamma^5 \ell(-k_1) \right) - i\, f(\mu,\, s)\, M_1\, \varepsilon^*_\mu \left(\bar{\ell}(k_2) \gamma^\mu \gamma^5 \ell(-k_1) \right) +$$

$$\left. i\, \frac{g(\mu,\, s)}{M_1} P_\mu\, (\varepsilon^* p_1) \left(\bar{\ell}(k_2) \gamma^\mu \gamma^5 \ell(-k_1) \right) + i\, \frac{h(\mu,\, s)}{M_1} q_\mu\, (\varepsilon^* p_1) \left(\bar{\ell}(k_2) \gamma^\mu \gamma^5 \ell(-k_1) \right) \right],$$

where $a(\mu,\, s)$, ..., $h(\mu, s)$ are functions depending on the Wilson coefficients and form factors in the current model (SM, MSSM, LR, 2HD etc).

Rare semileptonic decays
Amplitudes for $B \to V\,\ell^+\ell^-$ transitions

Considering the expression for the effective Hamiltonian, the rules of the form factors replacement and the fact that all form factors are the real functions of s, one receives the matrix elements of $B \to V\ell^+\ell^-$:

$$A^{(q)} = \langle V(p_2,\, M_2,\, \varepsilon),\, \ell^+(k_1),\, \ell^-(k_2)\left|H_{\rm eff}^{\rm SM}\right| B(p_1,\, M_1)\rangle = \frac{G_F}{\sqrt{2}}\,\frac{\alpha_{em}}{2\pi}\,V_{tb}^*V_{tq}$$

$$\Bigg[-\frac{\tilde a(\mu,\, s)}{M_1}\,\epsilon_{\mu\varepsilon^* p_1 p_2}\,\big(\bar\ell(k_2)\gamma^\mu\ell(-k_1)\big) - i\,\tilde b(\mu,\, s)\,M_1\,\varepsilon^*_\mu\,\big(\bar\ell(k_2)\gamma^\mu\ell(-k_1)\big) +$$

$$i\,\frac{\tilde c(\mu,\, s)}{M_1}\,P_\mu\,(\varepsilon^* p_1)\,\big(\bar\ell(k_2)\gamma^\mu\ell(-k_1)\big) + i\,\frac{\tilde d(\mu,\, s)}{M_1}\,q_\mu\,(\varepsilon^* p_1)\,\big(\bar\ell(k_2)\gamma^\mu\ell(-k_1)\big) -$$

$$-\frac{e^*(\mu,\, s)}{M_1}\,\epsilon_{\mu\varepsilon^* p_1 p_2}\,\big(\bar\ell(k_2)\gamma^\mu\gamma^5\ell(-k_1)\big) - i\,f^*(\mu,\, s)\,M_1\,\varepsilon^*_\mu\,\big(\bar\ell(k_2)\gamma^\mu\gamma^5\ell(-k_1)\big) +$$

$$i\,\frac{g^*(\mu,\, s)}{M_1}P_\mu\,(\varepsilon^* p_1)\,\big(\bar\ell(k_2)\gamma^\mu\gamma^5\ell(-k_1)\big) + i\,\frac{h^*(\mu,\, s)}{M_1}q_\mu\,(\varepsilon^* p_1)\,\big(\bar\ell(k_2)\gamma^\mu\gamma^5\ell(-k_1)\big)\Bigg].$$

Explicit form for coefficients $a(\mu,\, s)$, ..., $h(\mu,\, s)$ in SM

$$a(\mu,\, s) \;=\; 4\, C_{7\gamma}(\mu)\, \frac{(\hat m_b + \hat m_q)}{\hat s}\, T_1(s) \;+\; 2\, C_{9V}^{\mathrm{eff}\,(q)}(\mu,\, s)\, \frac{V(s)}{1+\hat M_2},$$

$$b(\mu,\, s) \;=\; \left(1+\hat M_2\right)\left(2\, C_{7\gamma}(\mu)\, \frac{(\hat m_b - \hat m_q)}{\hat s}\, \left(1-\hat M_2\right) T_2(s) \;+\; C_{9V}^{\mathrm{eff}\,(q)}(\mu,\, s)\, A_1(s),\right)$$

$$c(\mu,\, s) \;=\; \frac{1}{1-\hat M_2^2}\left(2\, C_{7\gamma}(\mu)\, \frac{(\hat m_b - \hat m_q)}{\hat s}\, \left(1-\hat M_2^2\right) T_2(s) \;+\; 2\, C_{7\gamma}(\mu)\, (\hat m_b - \hat m_q)\, T_3(s) \;+\; C_{9V}^{\mathrm{eff}\,(q)}(\mu,\, s)\, \left(1-\hat M_2\right)\, A_2(s)\right),$$

$$e(\mu,\, s) \;=\; 2\, C_{10A}(\mu)\, \frac{V(s)}{1+\hat M_2},$$

$$f(\mu,\, s) \;=\; C_{10A}(\mu)\, \left(1+\hat M_2\right)\, A_1(s), \qquad g(\mu,\, s) \;=\; C_{10A}(\mu)\, \frac{A_2(s)}{1+\hat M_2},$$

$$h(\mu,\, s) \;=\; \frac{C_{10A}(\mu)}{\hat s}\left(\left(1+\hat M_2\right)\, A_1(s) \;-\; \left(1-\hat M_2\right)\, A_2(s) \;-\; 2\, \hat M_2\, A_0(s)\right).$$

Explicit form for coefficients $\tilde{a}(\mu,\, s)$, ..., $\tilde{c}(\mu,\, s)$ in SM

If one assumes that all form factors are the real functions of the variable s and BSM Wilson coefficients contain only new weak phases for coefficients $\tilde{a}(\mu,\, s)$, ..., $\tilde{c}(\mu,\, s)$it is possible to write the following expressions:

$$\tilde{a}(\mu,\, s) \;=\; 4\, C^{*}_{7\gamma}(\mu)\, \frac{(\hat{m}_b + \hat{m}_q)}{\hat{s}}\, T_1(s) \,+\, 2\, C^{\mathrm{eff}\,(\bar{q})}_{9V}(\mu,\, s)\, \frac{V(s)}{1+\hat{M}_2},$$

$$\tilde{b}(\mu,\, s) \;=\; \left(1+\hat{M}_2\right) \left(2\, C^{*}_{7\gamma}(\mu)\, \frac{(\hat{m}_b - \hat{m}_q)}{\hat{s}} \left(1-\hat{M}_2\right) T_2(s) \,+\, C^{\mathrm{eff}\,(\bar{q})}_{9V}(\mu,\, s)\, A_1(s)\right),$$

$$\tilde{c}(\mu,\, s) \;=\; \frac{1}{1-\hat{M}_2^2} \left(2\, C^{*}_{7\gamma}(\mu)\, \frac{(\hat{m}_b - \hat{m}_q)}{\hat{s}} \left(1-\hat{M}_2^2\right) T_2(s) \,+\right.$$
$$\left. +\, 2\, C^{*}_{7\gamma}(\mu)\, (\hat{m}_b - \hat{m}_q)\, T_3(s) \,+\, C^{\mathrm{eff}\,(\bar{q})}_{9V}(\mu,\, s) \left(1-\hat{M}_2\right) A_2(s)\right).$$

For coefficients $e(\mu,\, s)$, ..., $h(\mu,\, s)$ there is a following rule:

$$\{e(\mu,\, s),\, \ldots\, h(\mu,\, s)\} \;\to\; \{e^{*}(\mu,\, s),\, \ldots\, h^{*}(\mu,\, s)\}.$$

Rare semileptonic decays Forward-Backward Asymmetry definition

The Forward-Backward Asymmetry A_{FB} definition shows great variation in the literature. Therefore we present here the most explicit definition.

$$A_{FB}(s) = \frac{\displaystyle\int_0^1 d\cos\theta\, \frac{d^2\Gamma(\bar{B}\to\bar{V}\ell^+\ell^-)}{ds\,d\cos\theta} - \int_{-1}^0 d\cos\theta\, \frac{d^2\Gamma(\bar{B}\to\bar{V}\ell^+\ell^-)}{ds\,d\cos\theta}}{\displaystyle\frac{d\Gamma(\bar{B}\to\bar{V}\ell^+\ell^-)}{d\,s}}.$$

The equivalent definition for A_{FB} is

$$A_{FB}(s) = \frac{\displaystyle\int_0^1 d\cos\theta_+\, \frac{d^2\Gamma(B\to V\ell^+\ell^-)}{ds\,d\cos\theta_+} - \int_{-1}^0 d\cos\theta_+\, \frac{d^2\Gamma(B\to V\ell^+\ell^-)}{ds\,d\cos\theta_+}}{\displaystyle\frac{d\Gamma(B\to V\ell^+\ell^-)}{d\,s}}.$$

In the terms of $a(\mu,\, s)$, ..., $h(\mu,\, s)$ functions the $A_{FB}(s)$ is given by

$$A_{FB}(s) \;=\; \frac{G_F^2}{M_1^3}\,\frac{\alpha_{em}^2}{2^{10}\,\pi^5}\,\left|V_{tq}^{*}V_{tb}\right|^2\,\frac{s\left(1-\frac{4m^2}{s}\right)\,\lambda(s,M_1^2,M_2^2)}{d\Gamma(\{\bar{B},B\}\to\{\bar{V},V\}\ell^+\ell^-)/\,d\,s}\,Re\big(a\,f^{*}+\,b\,e^{*}\big).$$

The alternative definition for the Forward-Backward Asymmetry has the following form:

$$A_{FB}^{(\mathrm{alt})}(s) \;=\; \frac{\int\limits_0^1 d\cos\theta_+\,\frac{d^2\Gamma(\bar{B}\to\bar{V}\ell^+\ell^-)}{ds\,d\cos\theta_+} - \int\limits_{-1}^0 d\cos\theta_+\,\frac{d^2\Gamma(\bar{B}\to\bar{V}\ell^+\ell^-)}{ds\,d\cos\theta_+}}{\frac{d\Gamma(\bar{B}\to\bar{V}\ell^+\ell^-)}{ds}}.$$

It is obvious that

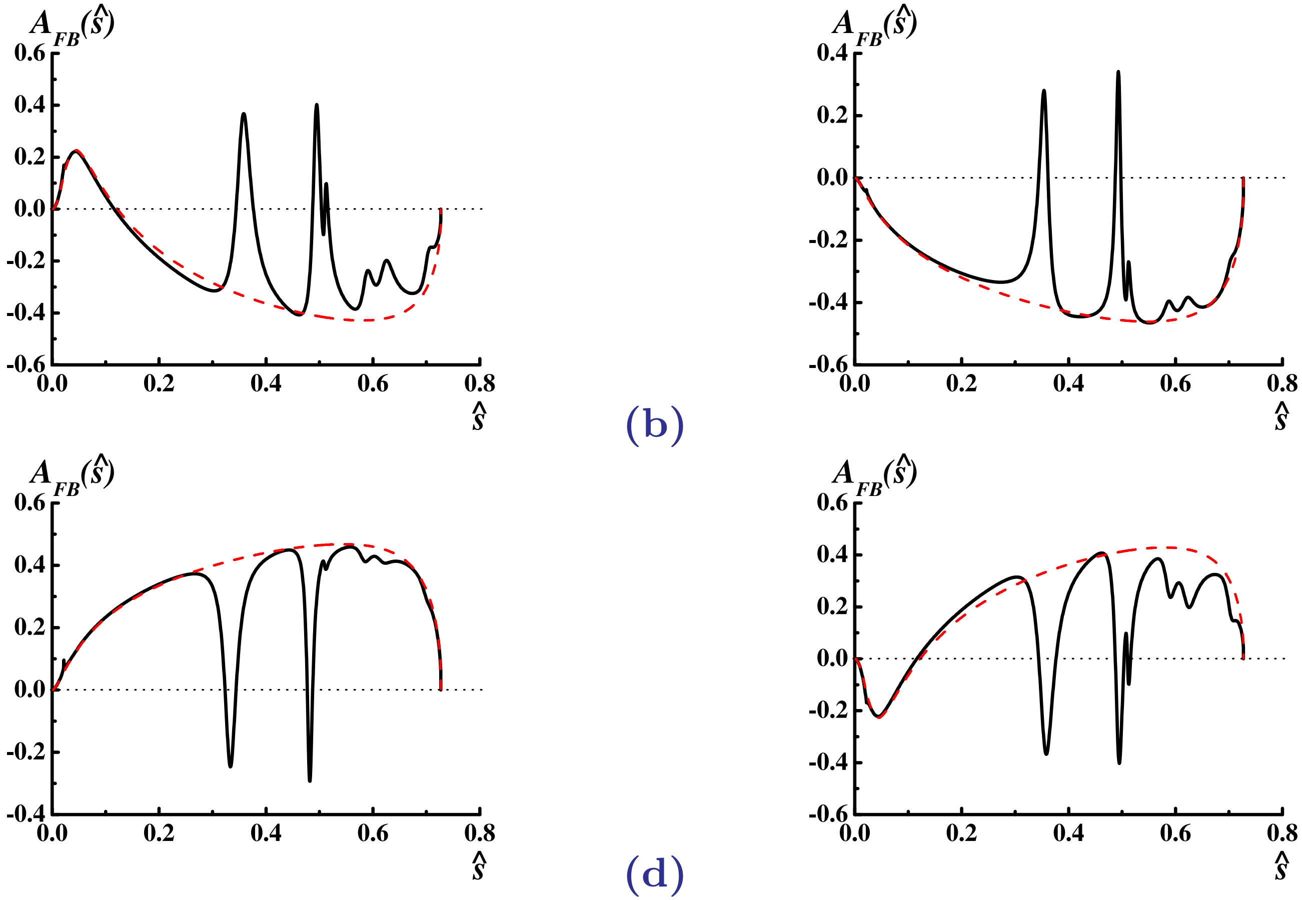
$$A_{FB}^{(\mathrm{alt})}(s) \;=\; -\,A_{FB}(s).$$

Thus we can measure A_{FB} in experiment for the decays $B \to (K^* \to K\pi)\ell^+\ell^-$ where the final state fix the initial flavor of B–meson.

For the decays like $(\bar{B}_d^0, B_d^0) \to (\rho^0, \gamma)\ell^+\ell^-$ or $(\bar{B}_s^0, B_s^0) \to (\phi, \gamma)\ell^+\ell^-$ where the final state does not fix the flavor of the initial B–meson, the mean $A_{FB}(s)$ integrated on time is equal to zero when not taking into account any of small CP–violation effects.

That's why the very effective procedure of the initial B–meson flavor tagging is needed for the A_{FB} measurements. This leads to larger statistics requirements.

Example for the Forward-Backward Asymmetry



(a) (b) (c) (d)

A_{FB} decays $\bar{B}_d^0 \to \rho^0 \mu^+ \mu^-$: (a) in the SM; (b) For $C_{7\gamma} = -C_{7\gamma}^{\rm SM}$; (c) For $C_{9V} = -C_{9V}^{\rm SM}$; (d) For $C_{10A} = -C_{10A}^{\rm SM}$. Solid line (black): the full asymmetry which takes into account the J/ψ, ψ', etc contributions. Dashed line (red): the non-resonant asymmetry.

Rare semileptonic decays Zero point of A_{FB}

As follows from the previos slide, for the SM in the lower s-region $A_{FB} > 0$, but in case of large s this asymmetry < 0. Consequently, there does exist such value $s = s_0$, where $A_{FB}(s_0) = 0$, the so-called zero point.

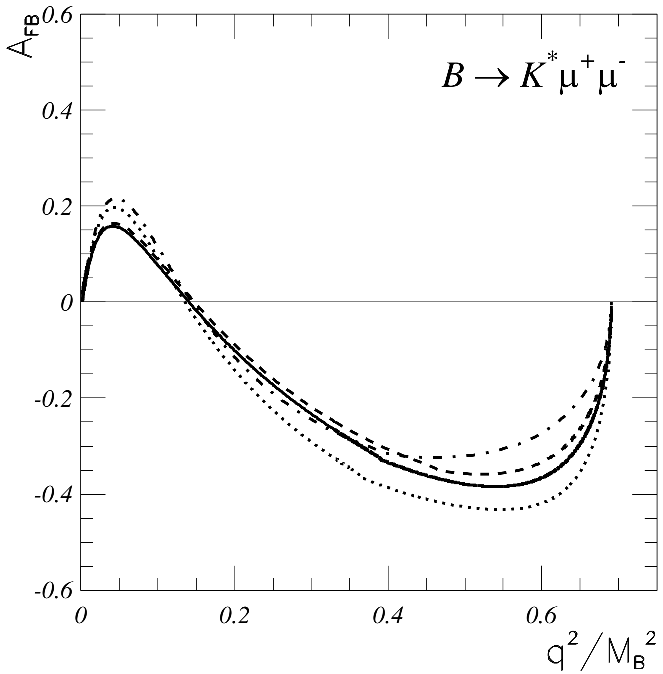
Zero point condition: $\mathbf{Re}\big(a\, f^* + b\, e^*\big) = 0$.

One may roughly accept $\hat{m}_q \approx 0$, $\hat{m}_b \approx 1$, $T_1(s) \approx T_2(s)$ and $A_1(s) \approx V(s)$. Within these assumptions

$$s_0 \approx \frac{2\, C_{7\gamma}}{C_{9V}}\, M_1^2 \approx 4.1\, \mathbf{GeV}^2$$

and in the current model depends only on the Wilson coefficients, but not on the hadronic form factors.

Rare semileptonic decays Zero point Example



Here we present the $A_{FB}(s/M_1^2)$ for the decay $B_d^0 \to K^{*\,0}\mu^+\mu^-$ with four form factors sets without resonant contribution. From this picture it's quite obvious, that zero point $s_0/M_1^2 \approx 0.15$ position is weakly dependent on the form factors sets (see G.Burdman, PRD 57, p.4254, 1998).

Rare semileptonic decays
A_{FB} definition in the experimental papers

In the last paper of Belle Collaboration (arXiv:0904.0770 [hep-ex], 2009; angle $\theta_{B_\ell} \equiv \theta_+$) the alternative definition of Forward-Backward Asymmetry is used:

$$A_{FB}^{\mathbf{Belle}}(s) = \frac{\displaystyle\int_0^1 d\cos\theta_+ \frac{d^2\Gamma(\bar{B} \to \bar{V}\ell^+\ell^-)}{ds\, d\cos\theta_+} - \int_{-1}^0 d\cos\theta_+ \frac{d^2\Gamma(\bar{B} \to \bar{V}\ell^+\ell^-)}{ds\, d\cos\theta_+}}{\displaystyle\frac{d\Gamma(\bar{B} \to \bar{V}\ell^+\ell^-)}{d\, s}}.$$

It is obvious, that in this case $A_{FB}^{\mathbf{Belle}}(s) = -A_{FB}(s)$.

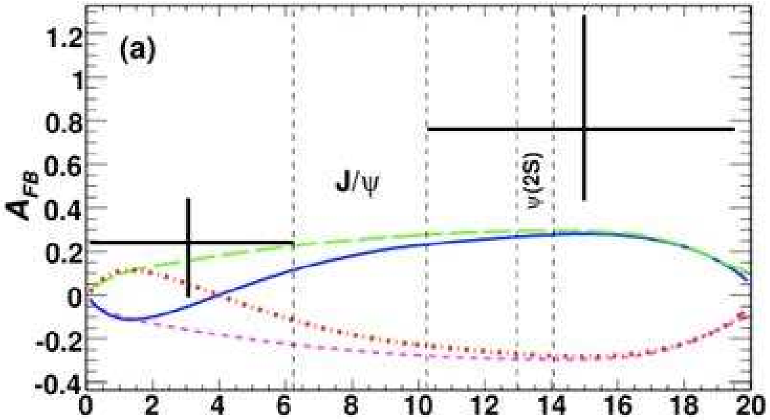
In the paper of BaBar Collaboration (PRD 79, 031102(R), 2009; angle $\theta_\ell \equiv \theta$) the choice of parametrization (2) gives $A_{FB}^{\mathbf{BaBar}}(s) = -A_{FB}(s)$.

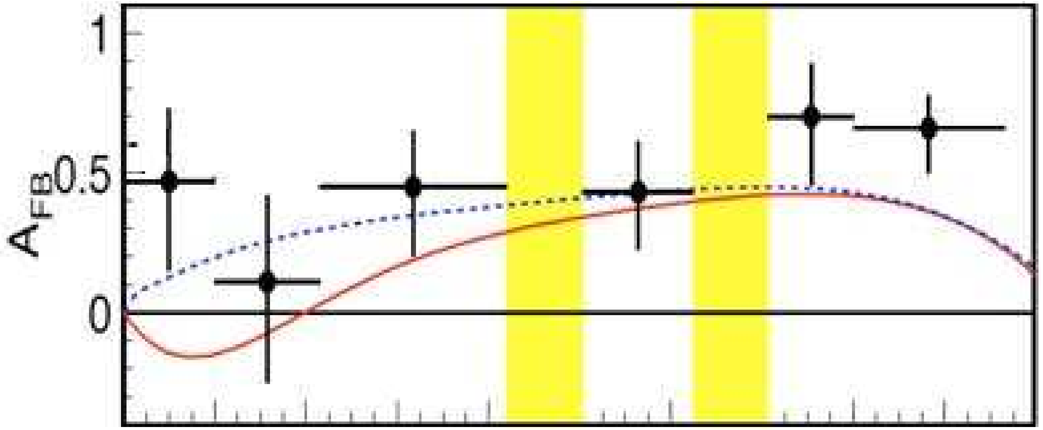
The measure of A_{FB} at Belle and BaBar

The $\cos\theta$-fit of the angular distribution (see the useful conventions in **G.Buchalla et al., EPJ C57, p.309, 2008**, equation (91))

$$\frac{1}{d\Gamma/ds}\,\frac{d^2\Gamma}{ds\,d\cos\theta_+} = \frac{3}{4}F_L(1-\cos^2\theta_+) + \frac{3}{8}(1-F_L)(1+\cos^2\theta_+) + \cos\theta_+ A_{FB}^{\text{Belle}\atop\text{BaBar}}$$

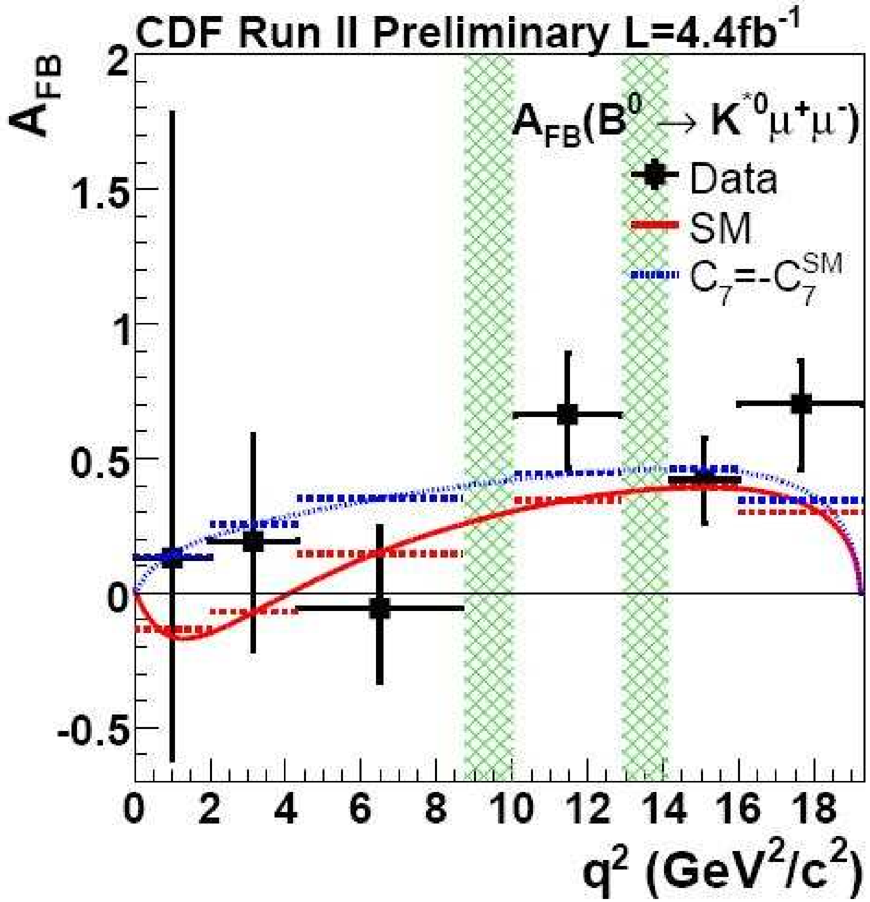
gives the value of A_{FB}. We assume that $\Gamma \equiv \Gamma(\bar{B}\to\bar{K}^*\ell^+\ell^-)$.





BaBar (left) and Belle (right) $A_{FB}(s)$ fits. The solid lines corresponds to SM, the green long dashed line (BaBar) and the blue dotted line (Belle) corresponds to the model with $C_{7\gamma} = -C_{7\gamma}^{\mathbf{SM}}$.
BaBar has ~ 100 events and Belle has ~ 250 events.

The measurement of A_{FB} at CDF



A_{FB} **for** $B_d^0 \to K^{*0}\mu^+\mu^-$ **at CDF (see The CDF Collaboration, Public Note 10047 09-11-12, 2009).**

Rare semileptonic decays CP-violating asymmetries

1) Experimental study of CP-violating observables requires greater samples of beauty hadrons and effective procedure of the flavour tagging. Only LHCb and Super-B-factories can solve this problem.

2) The time-independent CP-asymmetries in rare semileptonic decays were considered first for the process $b \to d\ell^+\ell^-$ in F. Kruger and L. M. Sehgal, PRD55, 2799 (1997); PRD 56, 5452 (1997). An asymmetry of the order of a few percent has been predicted. For the $b \to s\ell^+\ell^-$-transitions the asymmetry is expected to be much smaller.

3) The first attempt of the time-dependent CP-asymmetries consideration for rare semileptonic decays contains in the paper C. Bobeth, G. Hiller, and G. Piranishvili, JHEP 0807, 014017106 (2008).

Rare semileptonic decays
Time-dependent CP-asymmetry

Time-dependent CP-violating asymmetry is defined in the B-meson rest frame as follows

$$A_{CP}^{B_q\to f}(\tau) = \frac{\dfrac{d\Gamma(\bar{B}_q^0 \to f)}{d\tau} - \dfrac{d\Gamma(B_q^0 \to f)}{d\tau}}{\dfrac{d\Gamma(\bar{B}_q^0 \to f)}{d\tau} + \dfrac{d\Gamma(B_q^0 \to f)}{d\tau}},$$

where f is the common final state for B_q^0 and $\bar{B}_q^0$ decays. In this case a pronounced CP violation is expected in interference between the oscillation and decay amplitudes.

For instance, for semileptonic decays of B_d^0- and $\bar{B}_d^0$-mesons this final state is $f \equiv \rho^0 \ell^+ \ell^-$.

Taking into account meson oscillations

$$\frac{d\Gamma(B_q^0 \to f)}{d\tau} = \frac{e^{-\Gamma\tau}}{2}\Big[A\,\mathbf{ch}\,(y\,\Gamma\tau) + B\,\mathbf{cos}\,(x\,\Gamma\tau) - $$
$$- 2\,C\,\mathbf{sh}\,(y\,\Gamma\tau) - 2\,D\,\mathbf{sin}\,(x\,\Gamma\tau)\Big],$$
$$\frac{d\Gamma(\bar{B}_q^0 \to f)}{d\tau} = \frac{e^{-\Gamma\tau}}{2}\Big[A\,\mathbf{ch}\,(y\,\Gamma\tau) - B\,\mathbf{cos}\,(x\,\Gamma\tau) - $$
$$- 2\,C\,\mathbf{sh}\,(y\,\Gamma\tau) + 2\,D\,\mathbf{sin}\,(x\,\Gamma\tau)\Big],$$

we obtain the following expression for time-dependent asymmetry:

$$A_{CP}^{B_q \to f}(\tau) = \frac{2\,D\,\mathbf{sin}\,(x\,\Gamma\tau) - B\,\mathbf{cos}\,(x\,\Gamma\tau)}{A\,\mathbf{ch}\,(y\,\Gamma\tau) - 2\,C\,\mathbf{sh}\,(y\,\Gamma\tau)},$$

where

$$\Delta m = M_h - M_\ell,\ \Gamma = (\Gamma_\ell + \Gamma_h)/2,\ \Delta\Gamma = \Gamma_\ell - \Gamma_h,\ x = \Delta m/\Gamma,\ y = \Delta\Gamma/\Gamma.$$

Rare semileptonic decays
Definition of the coefficients A, ..., D

The coefficients A, B, C, and D may be expressed via the helicity amplitudes as follows:

$$A = \int d\hat{s}\, \tilde{A}(\hat{s}) = \int \frac{d\Phi_3}{2\, M_1} \sum_{\lambda_i\, \lambda_1\, \lambda_2} \left(\left| A^{(q)}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \right|^2 + \left| \bar{A}^{(q)}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \right|^2 \right),$$

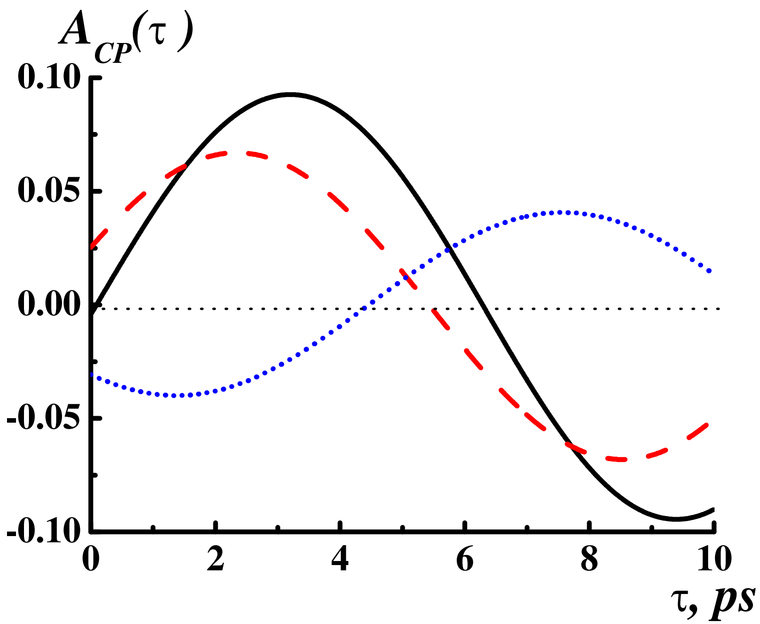
$$B = \int d\hat{s}\, \tilde{B}(\hat{s}) = \int \frac{d\Phi_3}{2\, M_1} \sum_{\lambda_i\, \lambda_1\, \lambda_2} \left(\left| A^{(q)}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \right|^2 - \left| \bar{A}^{(q)}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \right|^2 \right),$$

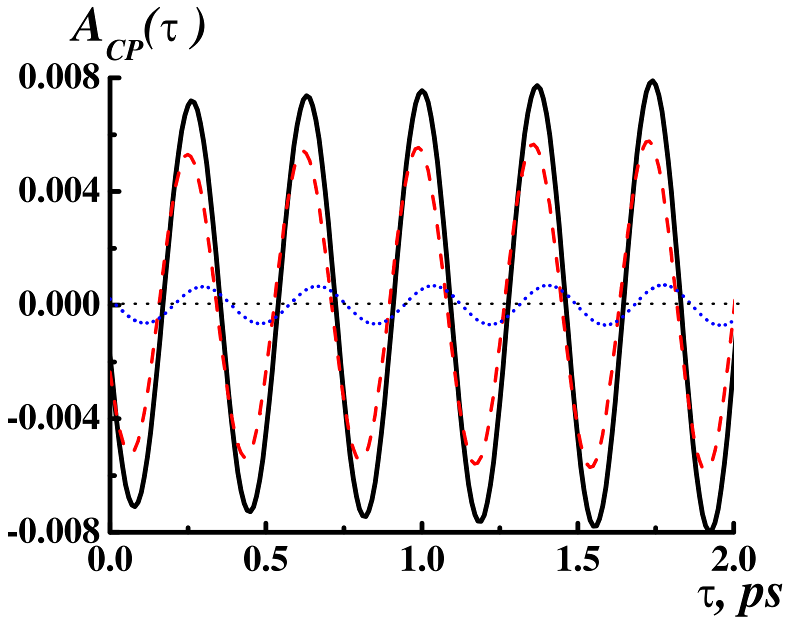
$$C = \int d\hat{s}\, \tilde{C}(\hat{s}) = \int \frac{d\Phi_3}{2\, M_1} \sum_{\lambda_i\, \lambda_1\, \lambda_2} \mathbf{Re} \left(e^{-2i\phi_{\rm ckm}} A^{(q)\,*}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \bar{A}^{(q)}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \right),$$

$$D = \int d\hat{s}\, \tilde{D}(\hat{s}) = \int \frac{d\Phi_3}{2\, M_1} \sum_{\lambda_i\, \lambda_1\, \lambda_2} \mathbf{Im} \left(e^{-2i\phi_{\rm ckm}} A^{(q)\,*}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \bar{A}^{(q)}_{\lambda_i\, \lambda_1\, \lambda_2}(\hat{s},\, \cos\theta) \right),$$

where $V_{tb}^* V_{tq} = |V_{tb}^* V_{tq}| e^{-i\phi_{\rm ckm}}$.

Rare semileptonic decays Time-dependent CP-asymmetry (figures)





Time-dependent asymmetry for $B_d^0 \to \rho^0\mu^+\mu^-$ (left) and $B_s^0 \to \phi\mu^+\mu^-$ (right) decays. Solid line (black): SM. Dashed line (red): $C_{7\gamma} = -C_{7\gamma}^{\rm SM}$. Dotted line (blue): $C_{9V} = -C_{9V}^{\rm SM}$. The region around the J/ψ and ψ' resonances was excluded from the integration.

Rare semileptonic decays
Time-independent CP-asymmetry

Time-independent CP-asymmetry may be represented via $\tilde{A}(\hat{s})$, ..., $\tilde{D}(\hat{s})$ as follows:

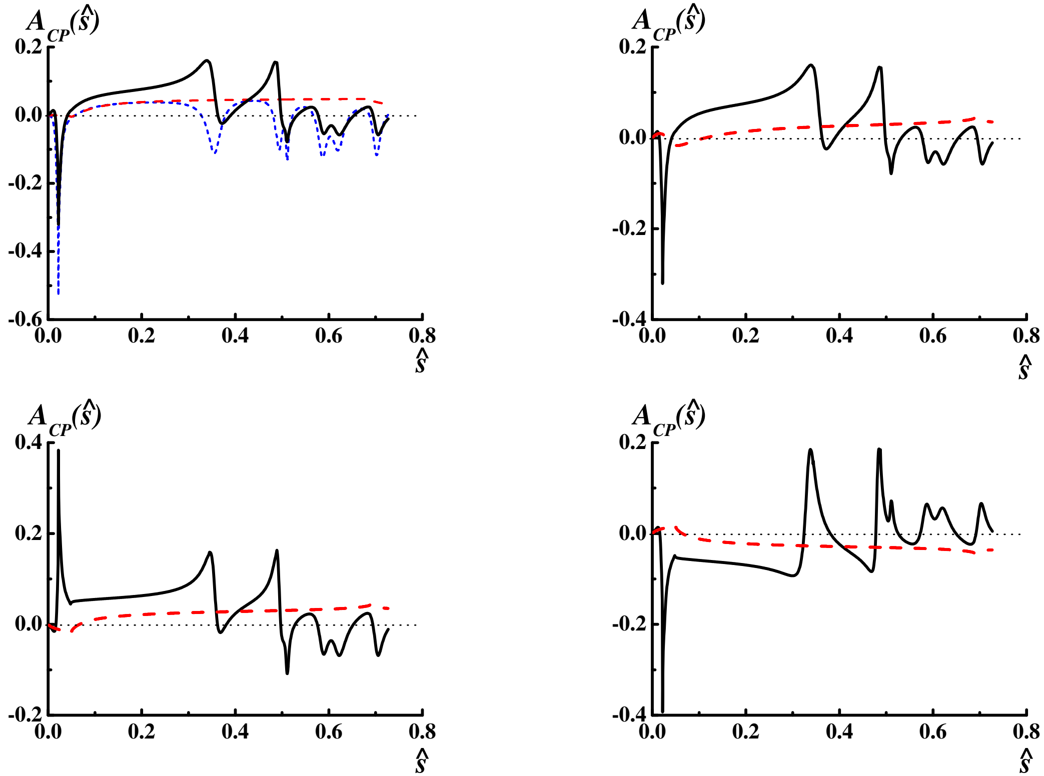
$$A_{CP}^{B_q\to f}(\hat{s}) = \frac{\dfrac{d\Gamma(\bar{B}_q \to f)}{d\hat{s}} - \dfrac{d\Gamma(B_q \to f)}{d\hat{s}}}{\dfrac{d\Gamma(\bar{B}_q \to f)}{d\hat{s}} + \dfrac{d\Gamma(B_q \to f)}{d\hat{s}}} = $$

$$= - \left(\frac{1-y^2}{1+x^2}\right) \frac{\tilde{B}(\hat{s}) - 2x\tilde{D}(\hat{s})}{\tilde{A}(\hat{s}) - 2y\tilde{C}(\hat{s})}.$$

For B_d^0-mesons: $x \approx 0.8$ and $y \approx 10^{-2}$.

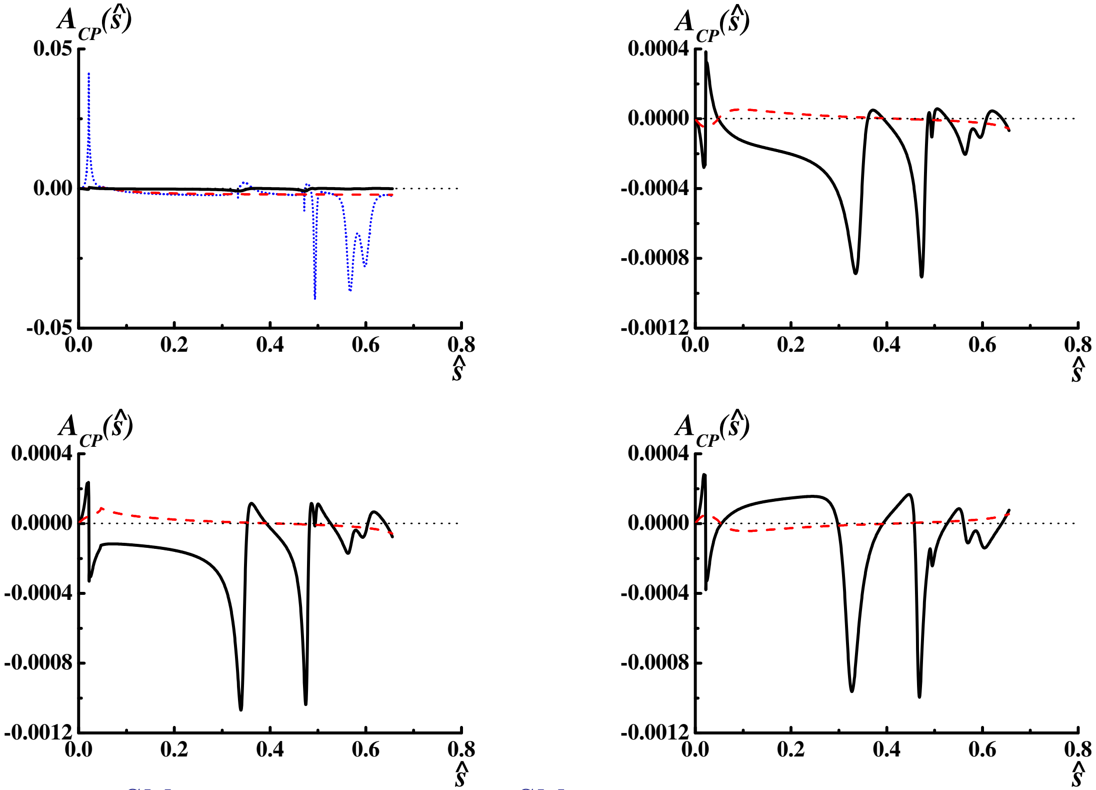
For B_s^0-mesons: $x \approx 25$ and $y \approx 0.5$.

Rare semileptonic decays
Time-independent CP-asymmetry for $\{\bar{B}_d^0, B_d^0\} \to \rho^0 \mu^+ \mu^-$



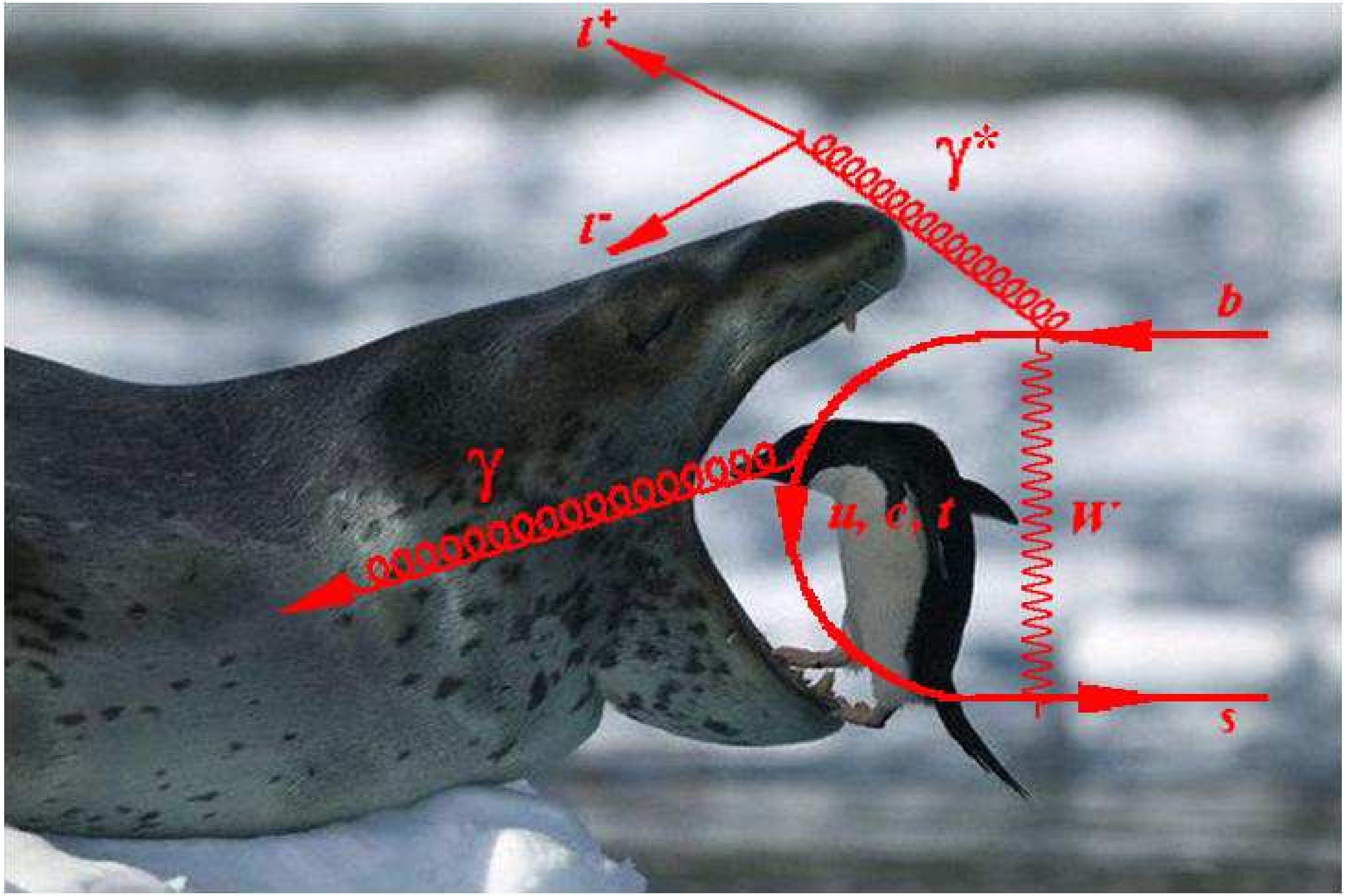
(b) SM; (c) $C_{7\gamma} = -C_{7\gamma}^{\rm SM}$; (d) $C_{9V} = -C_{9V}^{\rm SM}$. Flavor oscillations have been taken into account. Solid line (black) line: full asymmetry. Dashed (red) line: nonresonant asymmetry. (a) SM. Dotted (blue) line shows the asymmetry if flavor oscillations are not taken into account.

Rare semileptonic decays
Time-independent CP-asymmetry for $\{\bar{B}_s^0, B_s^0\} \to \phi\mu^+\mu^-$



(b) SM; (c) $C_{7\gamma} = -C_{7\gamma}^{\rm SM}$; (d) $C_{9V} = -C_{9V}^{\rm SM}$. Flavor oscillations have been taken into account. Solid line (black) line: full asymmetry. Dashed (red) line: nonresonant asymmetry. (a) SM. Dotted (blue) line shows the asymmetry if flavor oscillations are not taken into account.

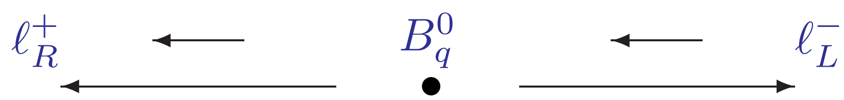
RARE RADIATIVE LEPTONIC DECAYS



Rare radiative leptonic decays Naive estimate of the ratio $Br(B_q^0 \to \ell^+\ell^-\gamma)/Br(B_q^0 \to \ell^+\ell^-)$

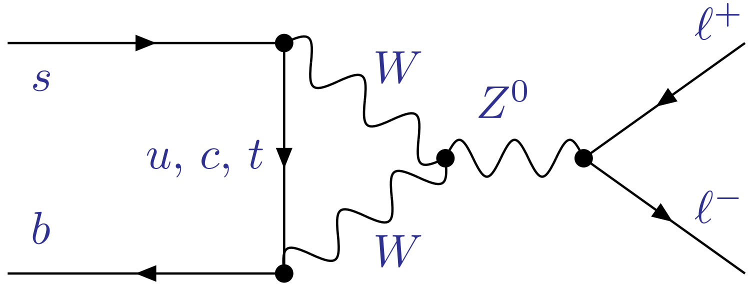
$$\frac{Br(B_q^0 \to \ell^+\ell^-\gamma)}{Br(B_q^0 \to \ell^+\ell^-)} \sim \frac{M^2_{B_q^0}}{m^2_\ell}\,\frac{\alpha_{em}}{4\pi}.$$

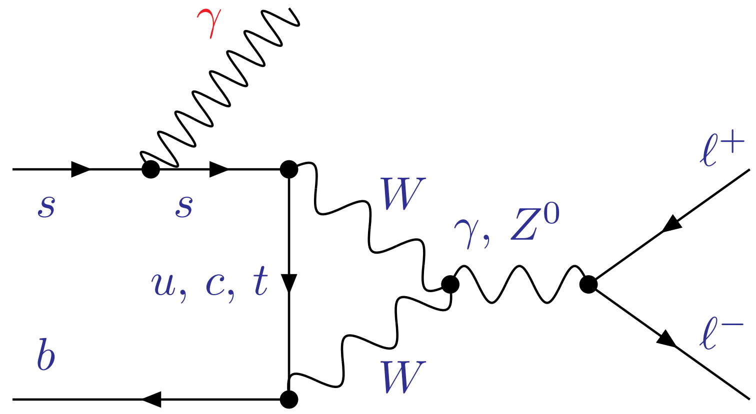
1. **The ratio $m_\ell^2/M^2_{B_q^0}$ corresponds to the helicity-suppressed factor in the decay $B_q^0 \to \ell^+\ell^-$ (like in the decay $\pi \to \mu\nu_\mu$).**



Decays $B_q^0 \to \ell^+\ell^-\gamma$ are not helicity-suppressed, because in the final state we have a photon.

2. The constant α_{em} corresponds to the additional γ-emission.





3. The factor 4π corresponds to the ratio of the two-body ($\ell^+\ell^-$) and three-body ($\gamma\ell^+\ell^-$) phase spaces.

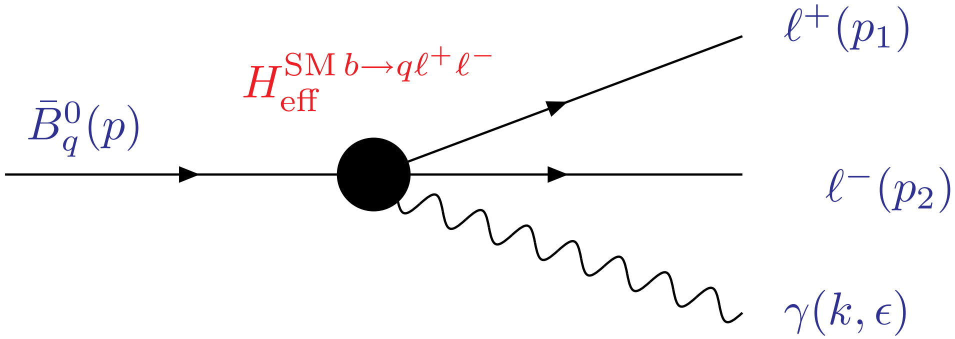
Numericaly:

1. $Br(B_q^0 \to e^+e^-) \ll Br(B_q^0 \to e^+e^-\gamma)$, **because** $M^2_{B_q^0}/m_e^2 \sim 10^8 \gg 4\,\pi/\alpha_{em} \sim 10^3$;

2. $Br(B_q^0 \to \mu^+\mu^-) \sim Br(B_q^0 \to \mu^+\mu^-\gamma) \approx Br(B_q^0 \to e^+e^-\gamma)$, **because** $M^2_{B_q^0}/m_\mu^2 \sim 2.5 \times 10^3 \sim 4\,\pi/\alpha_{em}$

3. **and** $Br(B_q^0 \to \tau^+\tau^-\gamma) \sim \alpha_{em}\, Br(B_q^0 \to \tau^+\tau^-)$.

Rare radiative leptonic decays
Kinematics



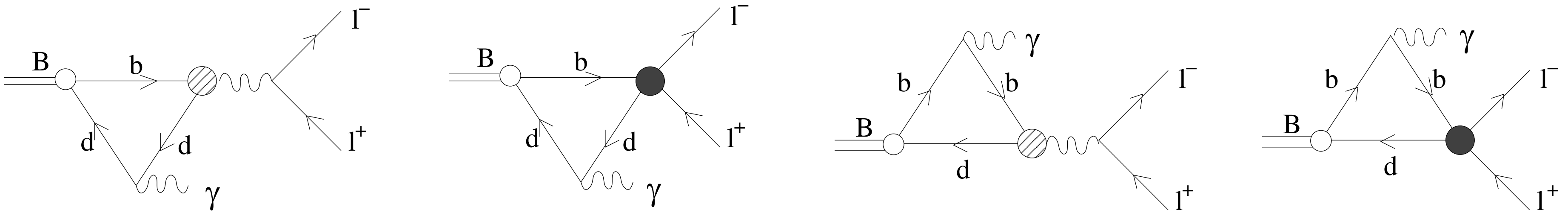
$$p = p_1 + p_2 + k = q + k,\ p^2 = M_B^2 \equiv M_1^2,\ k^2 = 0,\ p_1^2 = p_2^2 = m_\ell^2.$$

$$m_\ell^2 \le q^2 \le M_B^2.$$

In the rest frame of the $\bar{B}_q^0$ meson the photon energy is:

$$E_\gamma = \frac{M_B}{2}\left(1 - \frac{q^2}{M_B^2}\right) = \frac{M_B}{2}\left(1 - \hat{s}\right) = \frac{M_B}{2}\,x.$$

Decay Amplitude Contributions
Emission of the real photon from valence quarks



In these diagrams the real photon is directly emitted from the valence b or q quarks. Dashed circles denote the virtual photonic penguin contribution. Solid circles denote the Z-penguin and box contributions from $H_{\rm eff}^{{\rm SM}\, b\to q\ell^+\ell^-}$ effective Hamiltonian.

For description of this photon emission we use FOUR form factors: $F_V(q^2)$, $F_A(q^2)$, $F_{TV}(q^2,\,0)$ and $F_{TA}(q^2,\,0)$.

Rare radiative leptonic decays F_V, F_A, F_{TV} and F_{TA} formfactors

For the transition to a real photon, matrix elements of the vector, axial-vector, tensor and pseudotensor currents are given by the formulas

$$\begin{aligned}
\langle \gamma(k,\,\epsilon)|\bar{q}\gamma_\mu b|\bar{B}^0_q(p)\rangle &= e\,\epsilon^{*\,\alpha}\,\varepsilon_{\mu\alpha\xi\eta}p^\xi k^\eta\,\frac{F_V(q^2)}{M_B}, \\
\langle \gamma(k)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}_q(p)\rangle &= ie\epsilon^{*\alpha}[g_{\mu\alpha}(pk)-p_\alpha k_\mu]\,\frac{F_A(q^2)}{M_B}, \\
\langle \gamma(k,\,\epsilon)|\bar{q}\sigma_{\mu\nu}b|\bar{B}^0_q(p)\rangle\,(p-k)^\nu &= i\,e\,\epsilon^{*\,\alpha}\varepsilon_{\mu\alpha\xi\eta}p^\xi k^\eta\,F_{TV}(q^2,0), \\
\langle \gamma(k)|\bar{q}\sigma_{\mu\nu}\gamma_5 b|\bar{B}_q(p)\rangle(p-k)^\nu &= e\epsilon^{*\alpha}\,[g_{\mu\alpha}(pk)-p_\alpha k_\mu]\,F_{TA}(q^2,\,0).
\end{aligned}$$

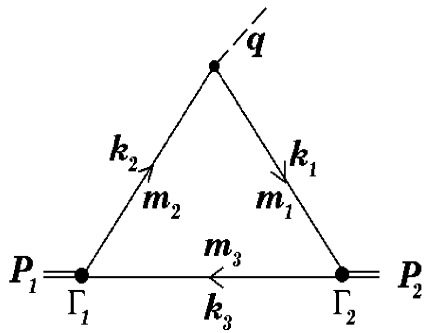
We treat the penguin form factors $F_{TV}(q_1^2, q_2^2)$ and $F_{TA}(q_1^2, q_2^2)$ as functions of two variables: q_1 is the momentum of the photon emitted from the $b \to q$ vertex, and q_2 is the momentum of the photon emitted from the valence quark of the $\bar{B}^0_q$ meson.

Rare radiative leptonic decays Form factors model

We proposed a simple parametrization for the form factors:

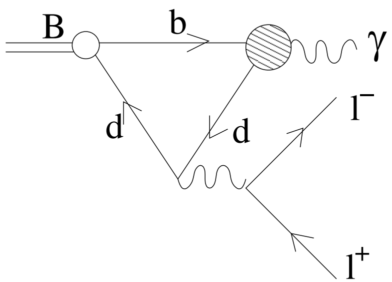
$$F_i(E_\gamma) = \beta_i \frac{M_B f_B}{\Delta_i + E_\gamma}, \qquad i = A, V, TA, TV.$$

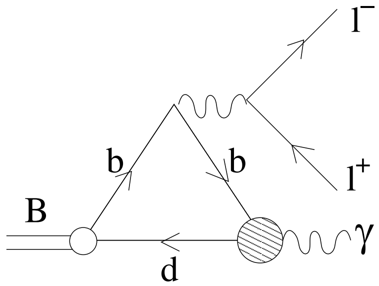
The numerical parameters were calculated using the dispersion approach of the Quark Model:



D.Melikhov, N.Nikitin, PRD 70, 114028, 2004,
F.Kruger, D.Melikhov, PRD 67,034002, 2003

Decay Amplitude Contributions
Emission of the virtual photon from valence quarks





In these diagrams the valence quarks directly emit the virtual photon which then goes into the final $\ell^+\ell^-$ pair. Dashed circles denote the $b \to q\gamma$ operator from $H_{\rm eff}^{{\rm SM}\, b\to q\gamma}$ effective Hamiltonian.

The corresponding amplitude has the same structure as the photonic penguin amplitude in the previous page with $F_{TA,TV}(q^2,0)$ replaced by $F_{TA,TV}(0,q^2)$.

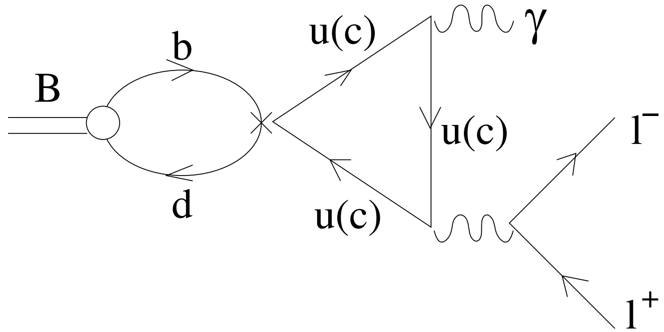
The form factors $F_{TA,TV}(0,q^2)$ for the necessary timelike momentum transfers are not known. The difficulty with these form factors comes from neutral light vector mesons, ρ^0 and ω for B_d decay and ϕ for B_s decay, which appear in the physical $B \to \gamma\ell^+\ell^-$ decay region. These resonances emerge in the amplitude of the subprocess when the photon is emitted from the light valence d or s quark.

We obtain the form factors $F_{TA,TV}(0,q^2)$ for $q^2 > 0$ using gauge-invariant version of the vector meson dominance (see D.Melikhov, O.Nachtmann, V.Nikonov, T.Paulus, EPJ C34, 345 (2004)). This allows us to express the form factors $F_{TA,TV}(0,q^2)$ in terms of the $B \to V$ transition form factors at zero momentum transfer and leptonic constants f_V of vector mesons:

$$F_{TV,TA}(0,q^2) = F_{TV,TA}(0,0) - \sum_V 2\, f_V g_+^{B\to V}(0) \frac{q^2/M_V}{q^2 - M_V^2 + iM_V\Gamma_V},$$

where M_V and Γ_V are the mass and width of the vector meson resonance.

Decay Amplitude Contributions
Weak annihilation



The weak annihilation amplitude is given by a triangle diagrams when the u and c quarks are in the loop, but here it is suppressed by a power of a heavy quark mass compared to the previous contributions of the real and virtual photon emission from valence quarks.

Rare radiative leptonic decays
Final formulas for tensor and pseudotensor form factors

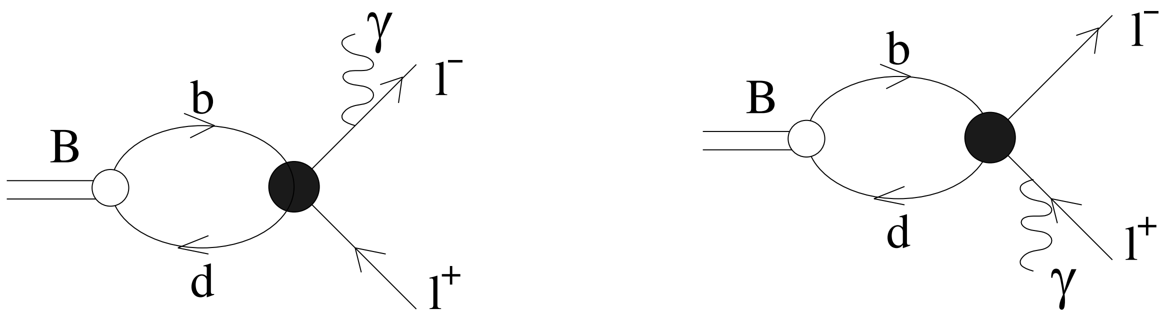
If we take into account the weak annihilation contribution and nonzero mass of the light quark q, we can write following final expressions for tensor and pseudotensor form factors in the form:

$$F_{TV}^{b\to q}(q^2) = \left(1+\frac{m_q}{m_b}\right)\left(F_{TV}(q^2,0)+F_{TV}(0,q^2)\right) - \frac{16}{3}\left(\frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*}+\frac{V_{cb}V_{cq}^*}{V_{tb}V_{tq}^*}\right)\frac{a_1}{C_{7\gamma}}\frac{f_{B_q}}{m_b},$$
$$F_{TA}^{b\to q}(q^2) = \left(1-\frac{m_q}{m_b}\right)\left(F_{TA}(q^2,0)+F_{TA}(0,q^2)\right).$$

and

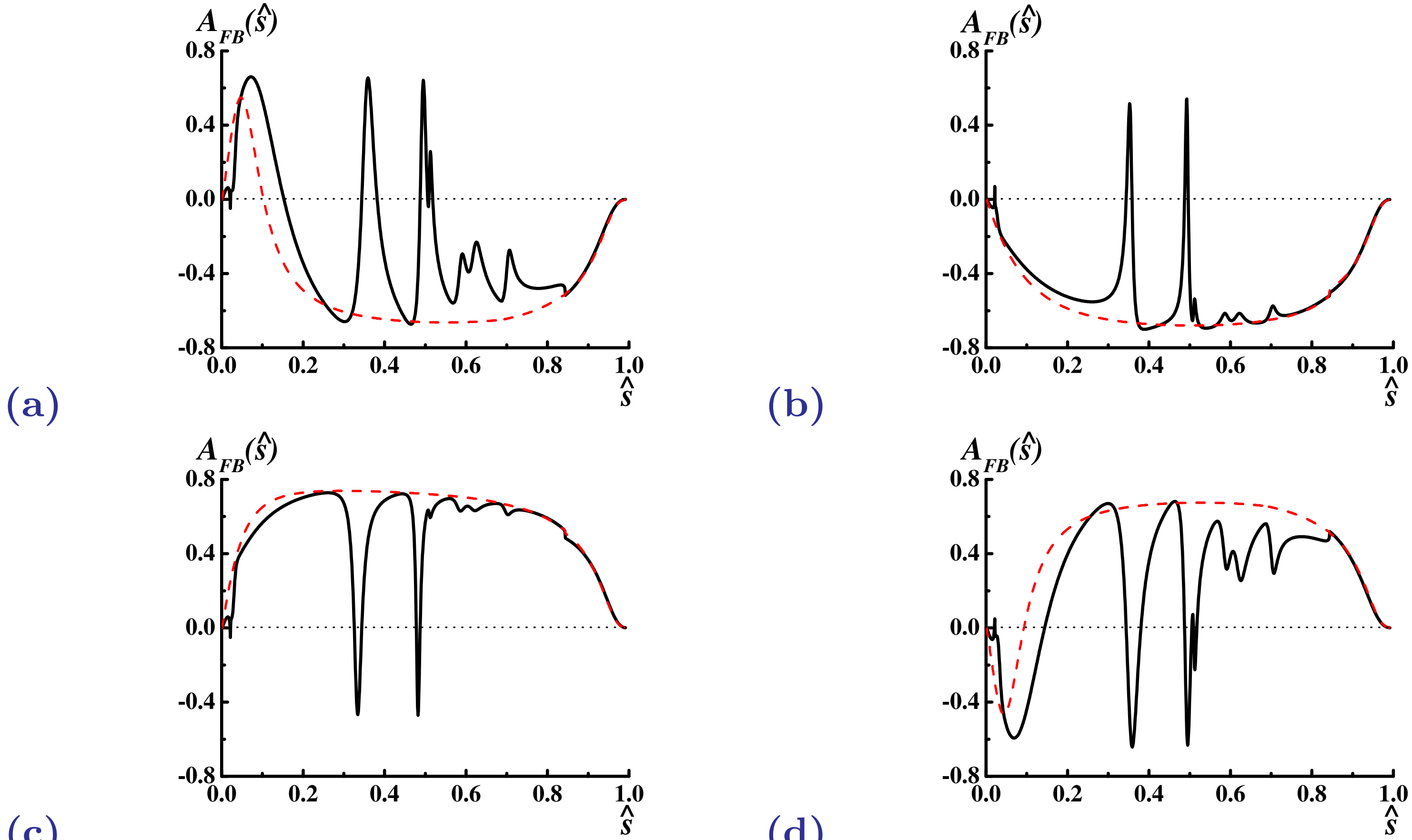
$$F_{TV}^{\bar b\to \bar q}(q^2) = \left(1+\frac{m_q}{m_b}\right)\left(F_{TV}(q^2,0)+F_{TV}(0,q^2)\right) + \frac{16}{3}\left(\frac{V_{ub}^*V_{uq}}{V_{tb}^*V_{tq}}+\frac{V_{cb}^*V_{cq}}{V_{tb}^*V_{tq}}\right)\frac{a_1}{C_{7\gamma}}\frac{f_{B_q}}{m_b},$$
$$F_{TA}^{\bar b\to \bar q}(q^2) = \left(1-\frac{m_q}{m_b}\right)\left(F_{TA}(q^2,0)+F_{TA}(0,q^2)\right).$$

Decay Amplitude Contributions
Bremsstrahlung



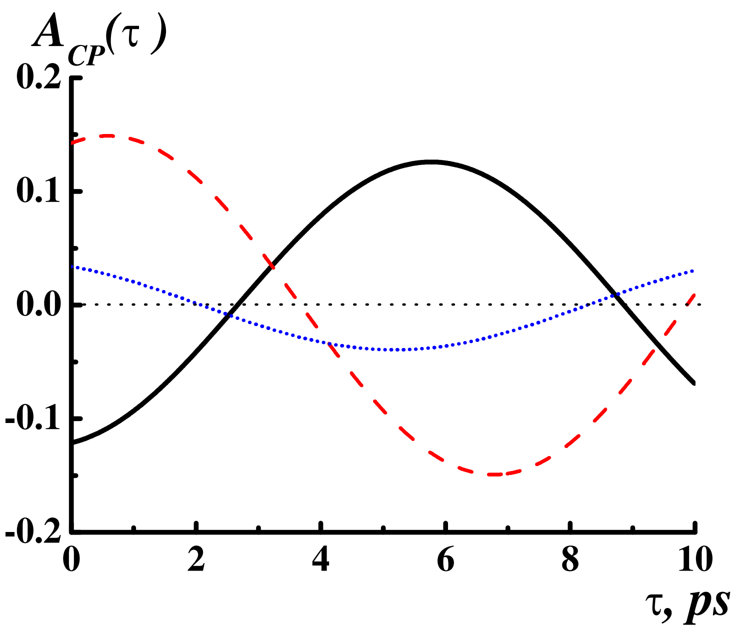
Solid circles denote the Z**-penguin and box contributions from the effective Hamiltonian** $H_{\text{eff}}^{\text{SM}\, b\to q\ell^+\ell^-}$.

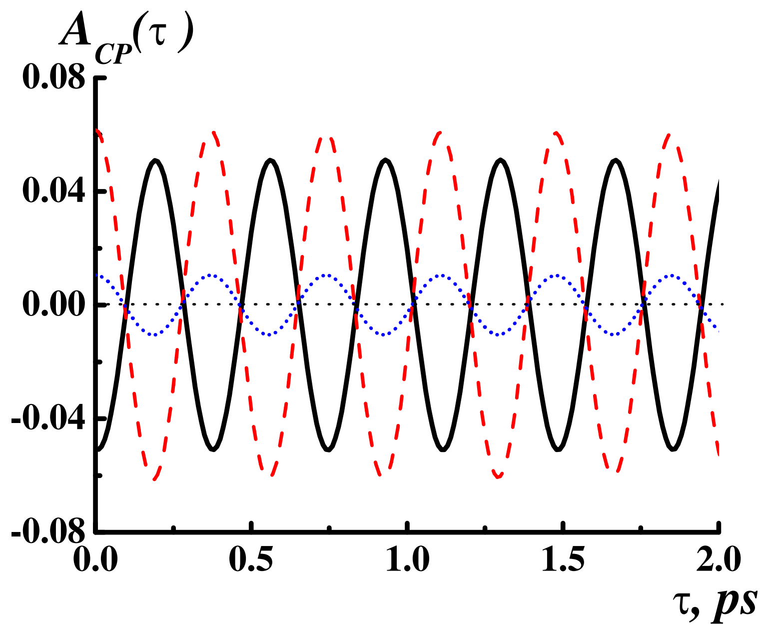
Forward-Backward Asymmetry for the decay $\bar{B}_d^0 \to \gamma\mu^+\mu^-$



A_{FB} **decays** $\bar{B}_d^0 \to \gamma\mu^+\mu^-$**: (a) in the SM; (b) For** $C_{7\gamma} = -C_{7\gamma}^{\rm SM}$**; (c) For** $C_{9V} = -C_{9V}^{\rm SM}$**; (d) For** $C_{10A} = -C_{10A}^{\rm SM}$**. Solid line (black): the full asymmetry which takes into account the** J/ψ**,** ψ'**, etc contributions. Dashed line (red): the non-resonant asymmetry.**

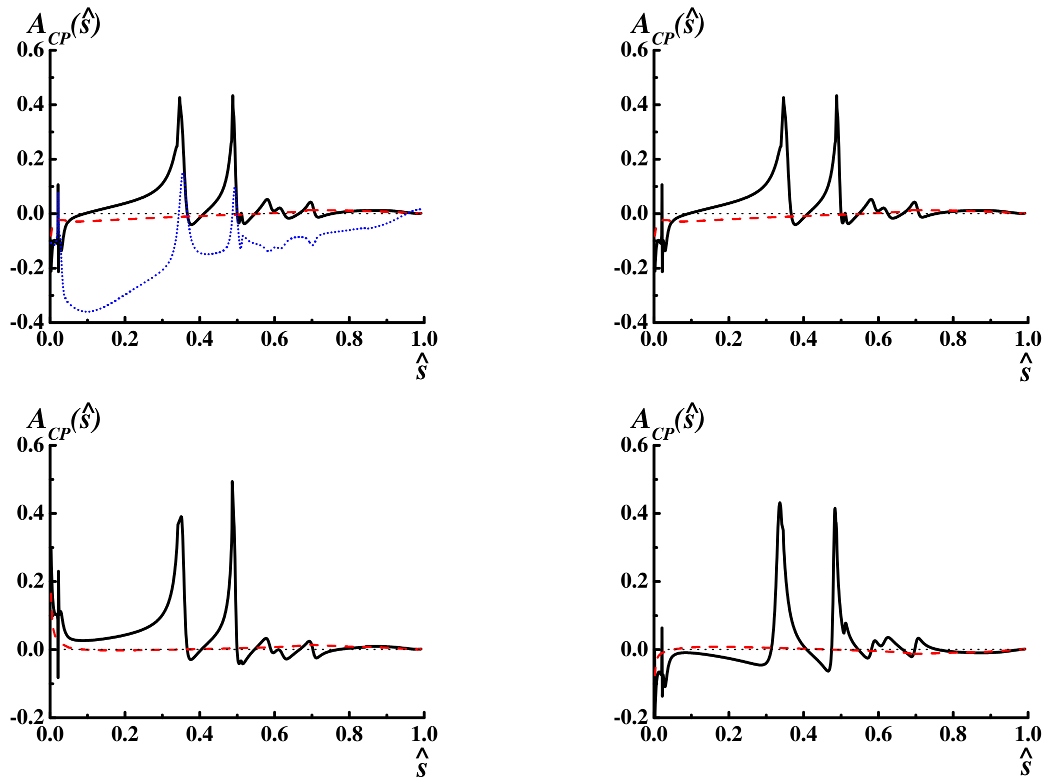
Rare radiative leptonic decays
Time-dependent CP-asymmetry (figures)





Time-dependent asymmetry for $B_d^0 \to \gamma\mu^+\mu^-$ (left) and $B_s^0 \to \gamma\mu^+\mu^-$ (right) decays. Solid line (black): SM. Dashed line (red): $C_{7\gamma} = -C_{7\gamma}^{\mathrm{SM}}$. Dotted line (blue): $C_{9V} = -C_{9V}^{\mathrm{SM}}$. The region around the J/ψ and ψ' resonances was excluded from the integration.

Rare semileptonic decays
Time-independent CP-asymmetry for $\{\bar{B}^0_d, B^0_d\} \to \gamma\mu^+\mu^-$



(b) $C_{10A} = -C_{10A}^{\rm SM}$; **(c)** $C_{7\gamma} = -C_{7\gamma}^{\rm SM}$; **(d)** $C_{9V} = -C_{9V}^{\rm SM}$. **Flavor oscillations have been taken into account. Solid line (black) line: full asymmetry. Dashed (red) line: nonresonant asymmetry. (a) SM. Dotted (blue) line shows the asymmetry if flavor oscillations are not taken into account.**

Conclusion

We presented the analysis of the forward-backward and the CP-violating asymmetries in rare semileptonic and radiative leptonic B-decays. Our results may be summarized as follows:

1. **We obtained the analytic results for the time-dependent and time-independent CP-asymmetries in rare semileptonic and rare radiative leptonic B-decays.**
2. **We studied the forward-backward asymmetry in $B_{d,s} \to \gamma\ell^+\ell^-$ decays taking into account the vector resonance contributions, the Bremsstrahlung, and the weak annihilation effects.**

   We noticed that the light neutral vector resonances strongly distort the shape of the asymmetry at small values of the dilepton invariant mass. In particular, in the SM these resonances lead to a sizeable shift of the zero point of the full asymmetry compared to the zero-point of the non-resonant asymmetry.

3. We analysed the CP-violating asymmetries (both time-dependent and time-independent) in $B_d \to \rho\mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$, and $B_{s,d} \to \gamma\mu^+\mu^-$ decays.

The asymmetries in B_s decays are found to be very small and therefore to be of no practical interest.

The asymmetries in B_d decays reach measurable values and thus might provide additional tests of the SM and its extentions.