

# Model-independent spin and coupling determination of Higgs-like resonances

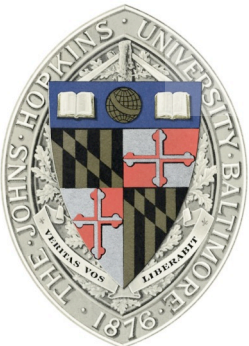


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Higgs Hunting 2010

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# What if a resonance is found?

- Resonances could be sign of Higgs...or something else!

- How can we distinguish?

- Mass and width

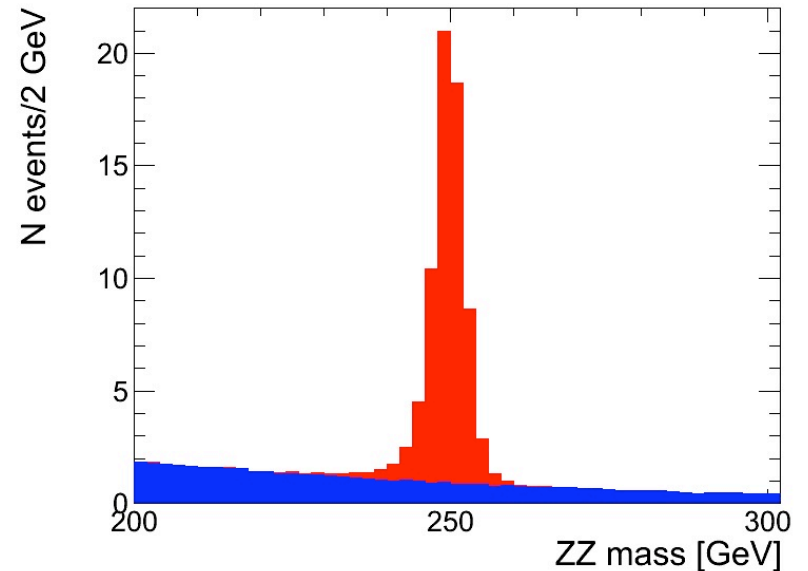
- Cross-section and branching fractions

- Angular distributions and spin correlations

past contributions countless, most recent advances to be discussed

Gao, Gritsan, Guo, Melnikov, Schulze, N.T. 2010 [arXiv:1001.3396] PRD81,075022(2010)

De Rujula, Lykken, Pierini, Spiropulu, Rogan 2010 [arXiv:1001.5300]

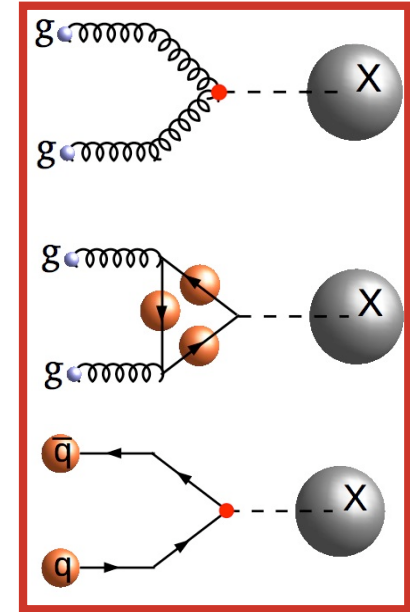


Techniques and analysis tools for determining the spin, parity, and interactions with SM fields of a resonance by analyzing the angular distributions of its decay products.

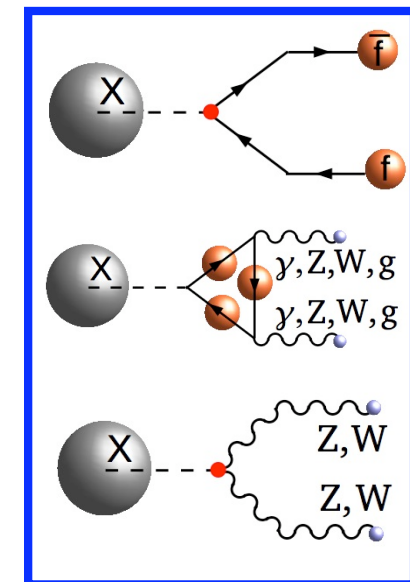


# Some motivated examples

- **Spin-zero**
  - **SM Higgs**,  $J^P = 0^+$ , or other non-SM scalar
  - Pseudoscalar  $J^P = 0^-$ , multi-Higgs case
- **Spin-one**
  - Heavy photon
  - Kaluza-Klein gluon
- **Spin-two**
  - **RS Graviton**,  $J^P = 2^+$ : classic model
    - SM fields localized to TeV brane
  - Non-classic RS Graviton model
    - SM fields in the bulk
- Hidden valley models
  - "Hidden glueballs" of various spin/CP



Production

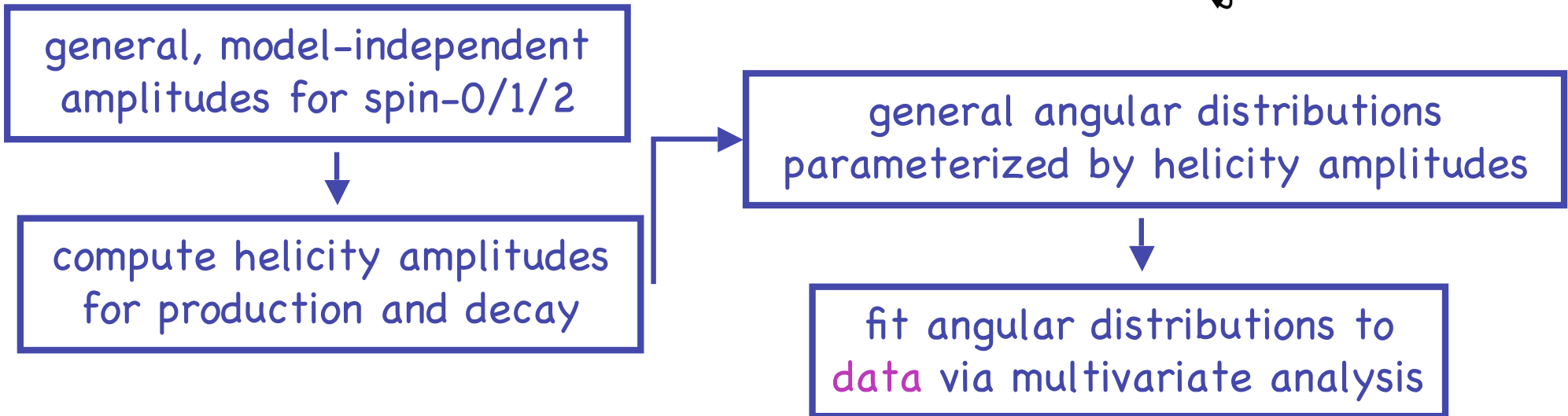
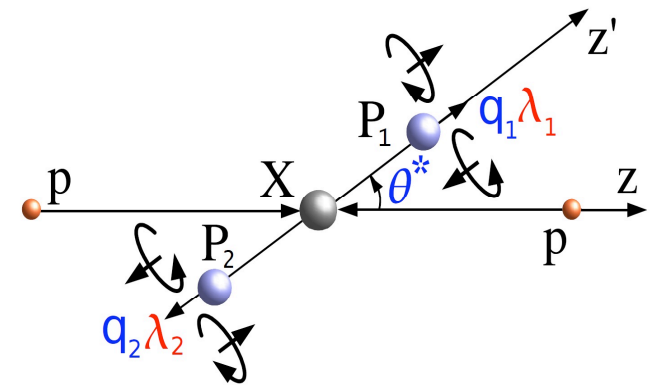


Decay



# Program

- A model-independent approach: **choose most general couplings** of a **spin-zero, -one, -two** particle to SM fields
- Analysis **applicable to many cases** such as  $ZZ$ ,  $W^+W^-$ ,  $\gamma\gamma$ ,  $gg$ ,  $l^+l^-$ :  
 $2 \rightarrow 2$  analysis via production angle,  $\cos \theta^*$
- Focus on the  $X \rightarrow ZZ \rightarrow 4l$  decay channel
  - **Final state fully reconstructed accurately**
  - **More information in four-body final state**
  - **$ZZ$  decay can be large or even dominant**



\*data = MC generator based on amplitudes



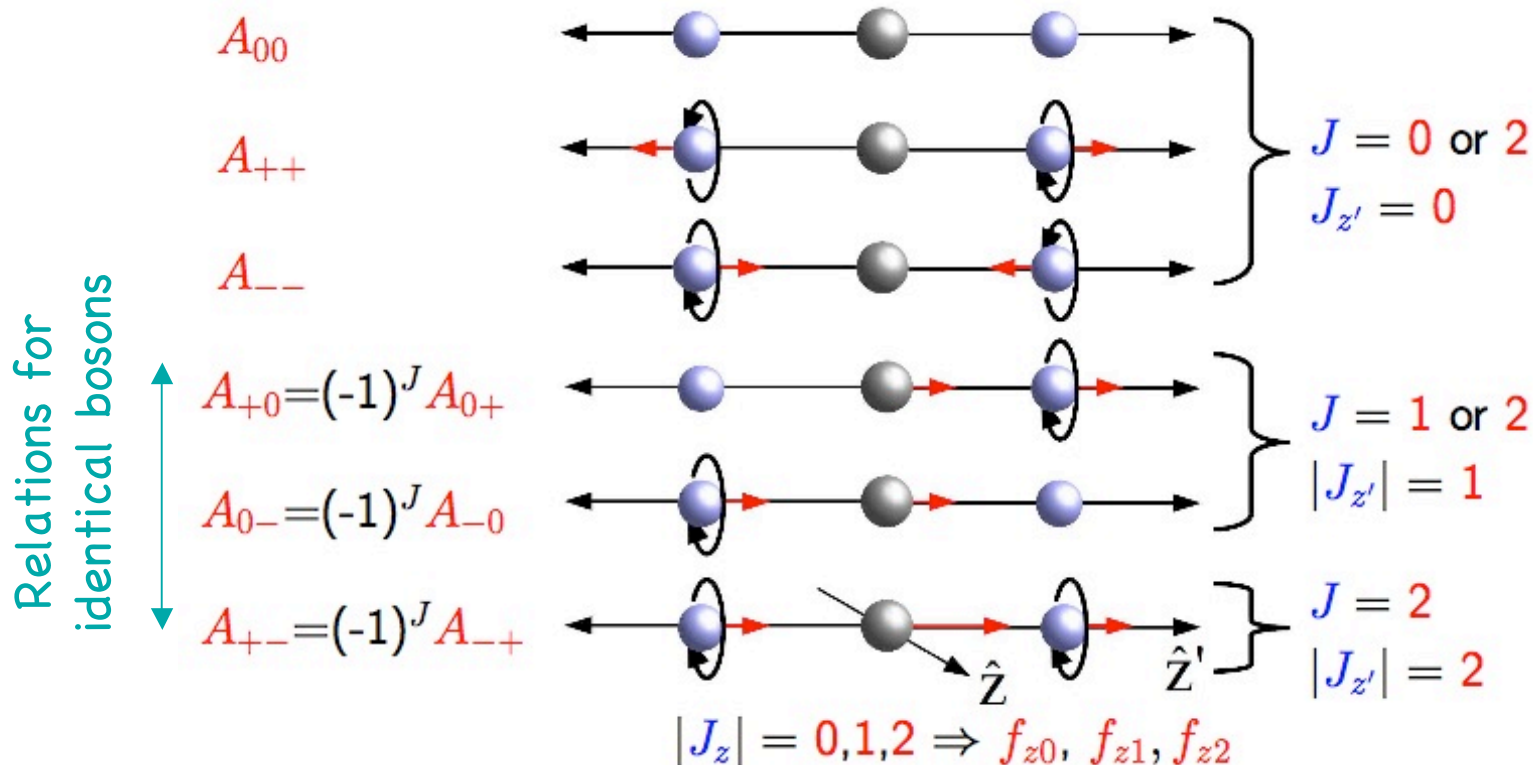
# Helicity amplitude formalism

Helicity amplitudes: contributions to the total amplitude from the different daughter helicities

Determined by theory, measured by experiment

Example:

Massive gauge bosons (W,Z) have  $J_z = 0, \pm 1$  possible helicity states;  
9 total amplitudes,  $A_{kl}$





# Theory to experiment:

General amplitudes to helicity amplitudes

Interactions of **spin-zero X** to two gauge bosons:

$$A(X \rightarrow VV) = v^{-1} \epsilon_1^{*\mu} \epsilon_2^{*\nu} \left( a_1 g_{\mu\nu} M_X^2 + a_2 q_\mu q_\nu + a_3 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \right)$$

Dimensionless *complex* coupling constants  
Gauge boson polarization vectors

e.g. For SM Higgs:  $a_1 \rightarrow$  tree level,  $a_2 \rightarrow$  radiative corrections  $O(\%)$ ,  
 $a_3 \rightarrow$  3-loop CP-violating  $O(10^{-11})$

By applying gauge boson polarization vectors to the general amplitudes, we can read off the helicity amplitudes

$$A_{00} = -\frac{m_X^4}{4vm_V^2} (a_1(1 + \beta^2) + a_2\beta^2);$$
$$A_{++} = \frac{m_X^2}{v} \left( a_1 + \frac{ia_3\beta}{2} \right); \quad A_{--} = \frac{m_X^2}{v} \left( a_1 - \frac{ia_3\beta}{2} \right)$$

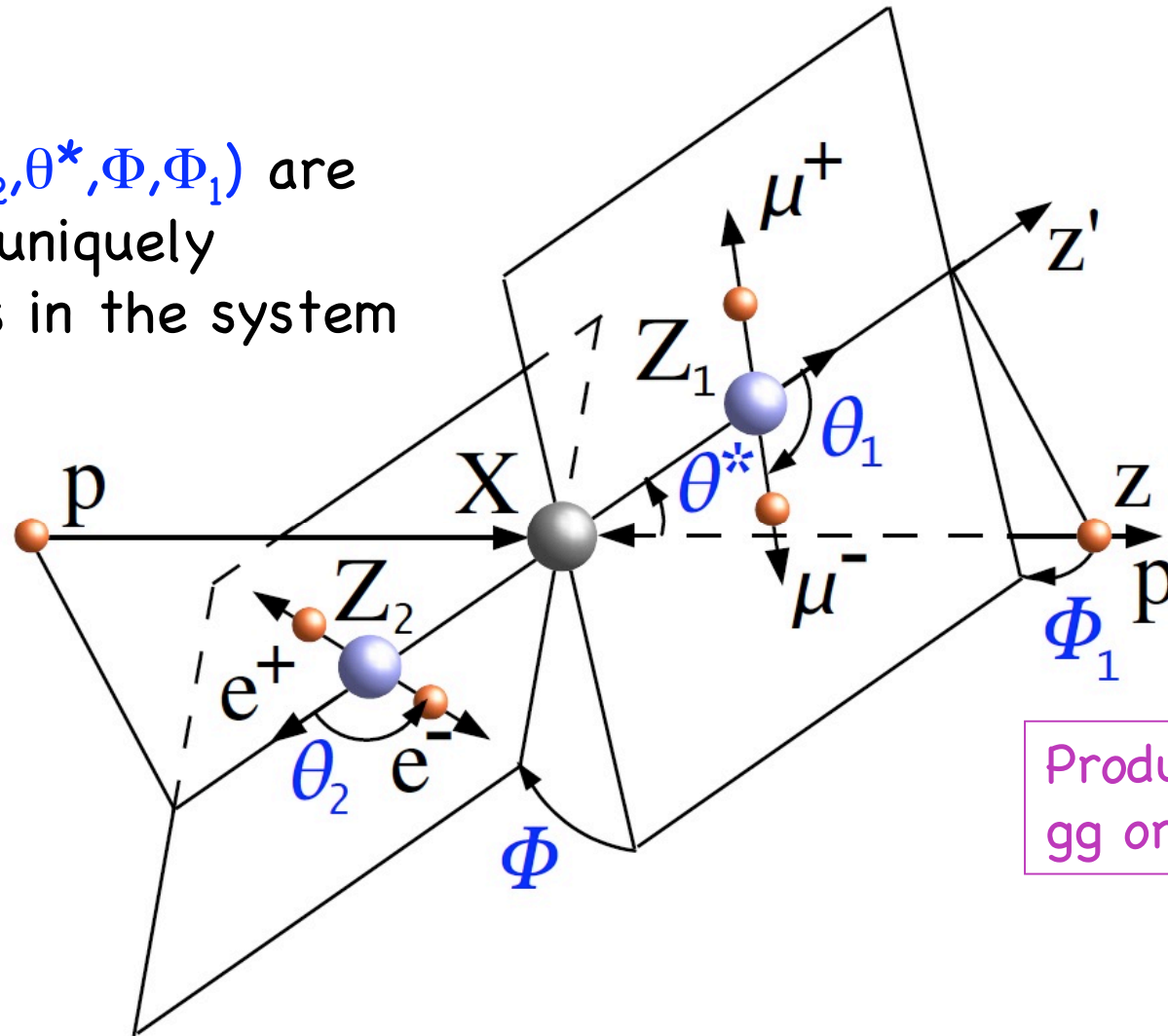
We do the same thing for spin-one and spin-two X



# Definition of the system

$X \rightarrow ZZ \rightarrow 4l$ :

5 angles ( $\theta_1, \theta_2, \theta^*, \Phi, \Phi_1$ ) are the maximal, uniquely defined angles in the system



Production via  
gg or qqbar

$\theta^*, \Phi_1$ : production angles

$\theta_1, \theta_2, \Phi$ : helicity angles, independent of production



# Angular distributions

## General spin- $J$ angular distribution

$$F_{00}^J(\theta^*) \times \left\{ 4 f_{00} \sin^2 \theta_1 \sin^2 \theta_2 + (f_{++} + f_{--}) \left( (1 + \cos^2 \theta_1)(1 + \cos^2 \theta_2) + 4R_1 R_2 \cos \theta_1 \cos \theta_2 \right) \right. \\ \left. - 2(f_{++} - f_{--}) \left( R_1 \cos \theta_1 (1 + \cos^2 \theta_2) + R_2 (1 + \cos^2 \theta_1) \cos \theta_2 \right) \right. \\ \left. + 4\sqrt{f_{++} f_{00}} (R_1 - \cos \theta_1) \sin \theta_1 (R_2 - \cos \theta_2) \sin \theta_2 \cos(\Phi + \phi_{++}) \right. \\ \left. + 4\sqrt{f_{--} f_{00}} (R_1 + \cos \theta_1) \sin \theta_1 (R_2 + \cos \theta_2) \sin \theta_2 \cos(\Phi - \phi_{--}) \right. \\ \left. + 2\sqrt{f_{++} f_{--}} \sin^2 \theta_1 \sin^2 \theta_2 \cos(2\Phi + \phi_{++} - \phi_{--}) \right\}$$

$$+ 4F_{11}^J(\theta^*) \times \left\{ (f_{+0} + f_{0-})(1 - \cos^2 \theta_1 \cos^2 \theta_2) - (f_{+0} - f_{0-})(R_1 \cos \theta_1 \sin^2 \theta_2 + R_2 \sin^2 \theta_1 \cos \theta_2) \right. \\ \left. + 2\sqrt{f_{+0} f_{0-}} \sin \theta_1 \sin \theta_2 (R_1 R_2 - \cos \theta_1 \cos \theta_2) \cos(\Phi + \phi_{+0} - \phi_{0-}) \right\} \\ + (-1)^J \times 4F_{-11}^J(\theta^*) \times \left\{ (f_{+0} + f_{0-})(R_1 R_2 + \cos \theta_1 \cos \theta_2) - (f_{+0} - f_{0-})(R_1 \cos \theta_2 + R_2 \cos \theta_1) \right. \\ \left. + 2\sqrt{f_{+0} f_{0-}} \sin \theta_1 \sin \theta_2 \cos(\Phi + \phi_{+0} - \phi_{0-}) \right\} \sin \theta_1 \sin \theta_2 \cos(2\Psi)$$

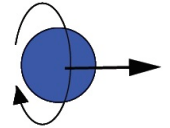
$$+ 2F_{22}^J(\theta^*) \times f_{+-} \left\{ (1 + \cos^2 \theta_1)(1 + \cos^2 \theta_2) - 4R_1 R_2 \cos \theta_1 \cos \theta_2 \right\} \\ + (-1)^J \times 2F_{-22}^J(\theta^*) \times f_{+-} \sin^2 \theta_1 \sin^2 \theta_2 \cos(4\Psi)$$

+ interference terms

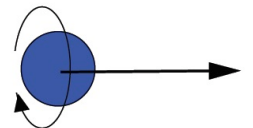
$$J_Z = 0$$



$$J_Z = \pm 1$$



$$J_Z = \pm 2$$



- Spin-zero  $X$ : only  $J_Z = 0$  part contributes
- Spin-one  $X$ : only  $J_Z = \pm 1$  part contributes
- Spin-two  $X$ : all contributions exist  $J_Z = 0, \pm 1, \pm 2$

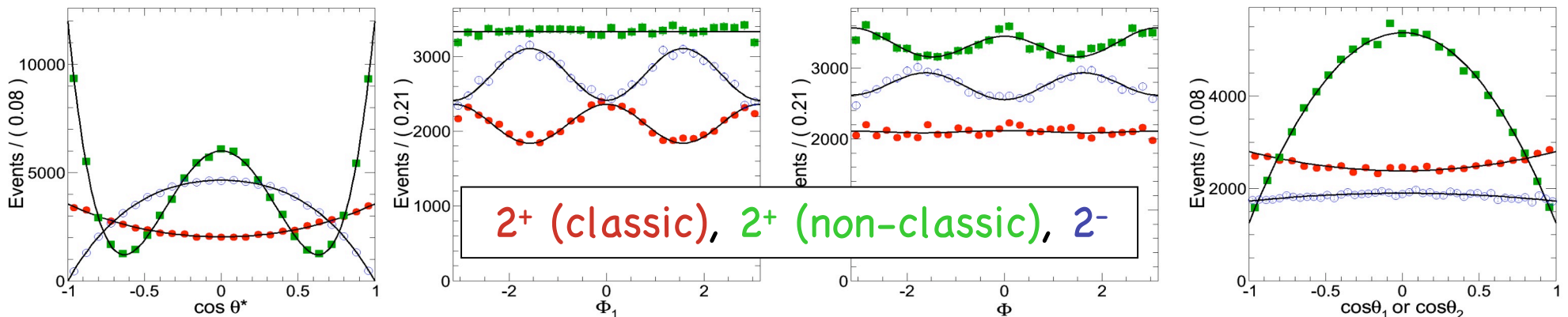




# MC Simulation

- A **MC program developed** to simulate production and decay of  $X$  with spin-zero, -one, or -two
  - Includes all spin correlations and all general couplings
  - Inputs are general dimensionless couplings - calculates matrix elements
  - Both  $gg$  and  $q\bar{q}$  production
  - Contains both final states for  $ZZ \rightarrow 4l$  and  $ZZ \rightarrow 2l2j$
  - Output in LHE format; can interface to Pythia
  - All code publicly available: [www.pha.jhu.edu/spin](http://www.pha.jhu.edu/spin)

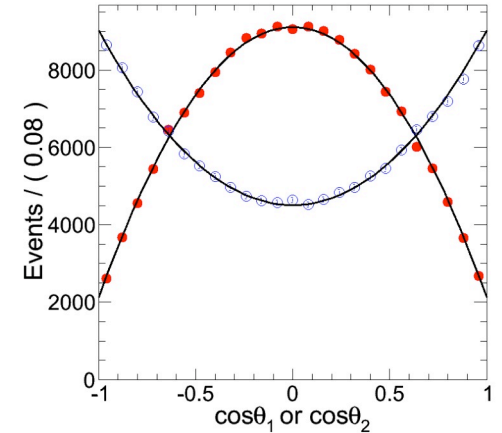
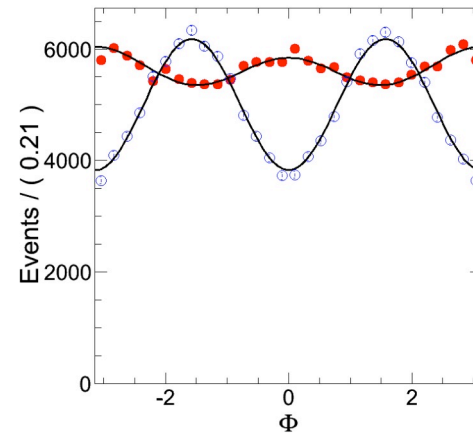
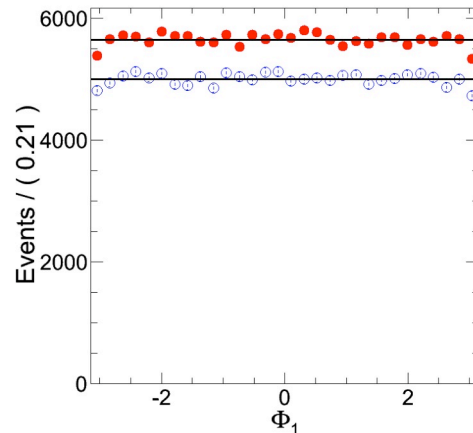
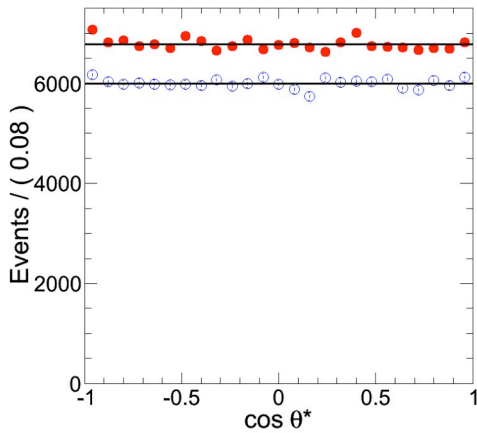
Example of agreement for MC (points) and angular distributions (lines)



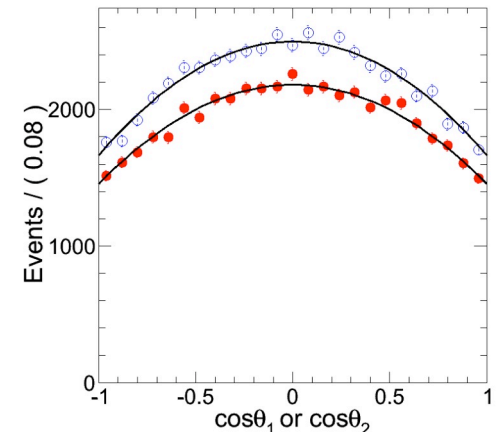
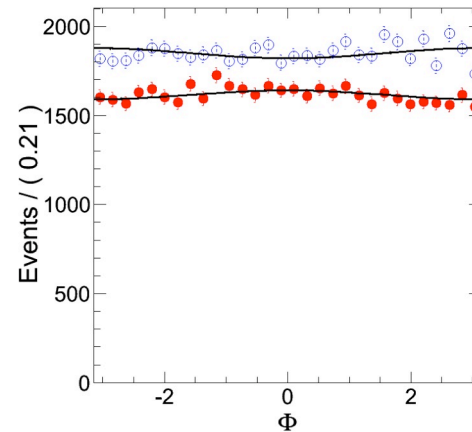
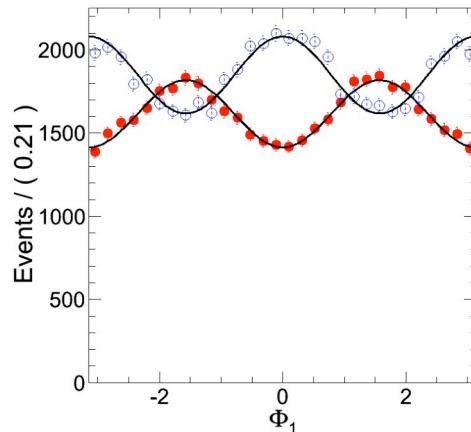
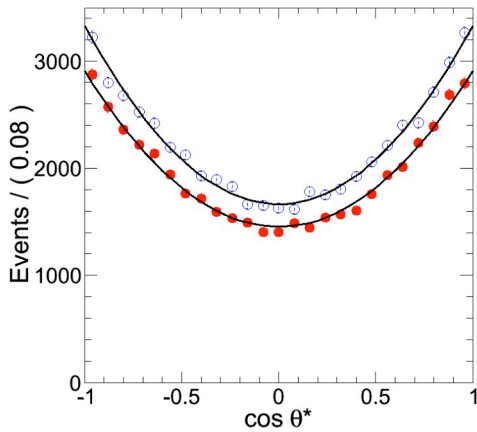


# MC Simulation

Spin Zero:  $\mathcal{J}^P = 0^+, 0^-$



Spin One:  $\mathcal{J}^P = 1^+, 1^-$



N.B. 1D projections of angles for illustration,  
statistical power comes from 5D *angular correlations*



# What we do in practice...

- To determine the helicity amplitudes, we need
  - Data: our MC generator
  - Angular distributions
  - Detector: approximate model with acceptance and smearing
  - Fit: multivariate likelihood method
- Fit used for
  - “Hypothesis separation” study: lower statistics, how much separation between different signal hypotheses achieved?
  - “Parameter fitting” study: higher statistics, how well can we determine the parameters of a certain hypothesis?

Example:

Hypothesis separation of signal scenarios near time of discovery

We can already make a statement about spin/CP!

	$0^-$	$1^+$	$1^-$	$2_m^+$	$2_L^+$	$2^-$
$0^+$	4.1	2.3	2.6	2.8	2.6	3.3
$0^-$	–	3.1	3.0	2.4	4.8	2.9
$1^+$	–	–	2.2	2.6	3.6	2.9
$1^-$	–	–	–	1.8	3.8	3.4
$2_m^+$	–	–	–	–	3.8	3.2
$2_L^+$	–	–	–	–	–	4.3



# Conclusion and outlook

- A program is developed to determine the spin of a resonance in a model-independent way
- A MC generator is introduced which simulates production and decay of spin-zero, -one, -two resonance including all spin correlations
- Data analysis is performed using multivariate likelihood method for both hypothesis separation and parameter fitting
- **We need to be ready for anything!**
  - Should not be limited to certain models; consider most general cases
- **Use all information available!**
  - Full 5D formalism provides the best separation and background suppression
  - At time of discovery, can already constrain spin/CP



# Backup



# Helicity Amplitudes

In general, 9 complex amplitudes,  $A_{kl}$ , where  $k, l = 0, \pm 1$

$$\mathcal{J}_X = 0$$

Production:  $gg^{\wedge}$

Allowed spin projection:  
0

Helicity Amplitudes:

$$A_{00}, A_{++}, A_{--}$$

4 [free parameters]

$$\mathcal{J}_X = 1$$

Production:  $qqbar^*$

Allowed spin projection:  
 $\pm 1^{\wedge}$

Helicity Amplitudes:

$$A_{+0} = -A_{0+}, A_{0-} = -A_{-0}$$

2

$$\mathcal{J}_X = 2$$

Production:  $gg$  or  $qqbar$

Allowed spin projection:  
0,  $\pm 1$ ,  $\pm 2$

Helicity Amplitudes:

$$A_{00}, A_{++}, A_{--}, A_{+0} = A_{0+}, A_{0-} = A_{-0}, A_{+-} = A_{-+}$$

10

\* $gg$  fusion forbidden due to Landau-Yang theorem

$\wedge$  assume chirality a good quantum number for massless quarks

For identical vector bosons:  $A_{kl} = (-1)^J A_{lk}$

For definite CP states:  $A_{kl} = \eta_P (-1)^J A_{-k-l}$



# Theory to experiment:

## General amplitudes to helicity amplitudes

Interactions of **spin-two X** to two gauge bosons:

$$A(X \rightarrow ZZ) = \Lambda^{-1} e_1^{*\mu} e_2^{*\nu} \left[ c_1 (q_1 q_2) t_{\mu\nu} + c_2 g_{\mu\nu} t_{\alpha\beta} \tilde{q}^\alpha \tilde{q}^\beta + c_3 \frac{q_{2\mu} q_{1\nu}}{M_X^2} t_{\alpha\beta} \tilde{q}^\alpha \tilde{q}^\beta + 2c_4 (q_{1\nu} q_2^\alpha t_{\mu\alpha} + q_{2\mu} q_1^\alpha t_{\nu\alpha}) + c_5 t_{\alpha\beta} \frac{\tilde{q}^\alpha \tilde{q}^\beta}{M_X^2} \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma + c_6 t^{\alpha\beta} \tilde{q}_\beta \epsilon_{\mu\nu\alpha\rho} q^\rho + \frac{c_7 t^{\alpha\beta} \tilde{q}^\beta}{M_X^2} (\epsilon_{\alpha\mu\rho\sigma} q^\rho \tilde{q}^\sigma q_\nu + \epsilon_{\alpha\nu\rho\sigma} q^\rho \tilde{q}^\sigma q_\mu) \right]$$

Dimensionless *complex* coupling constants

Gauge boson polarization vectors

By applying gauge boson polarization vectors to the general amplitudes, we can read off the helicity amplitudes

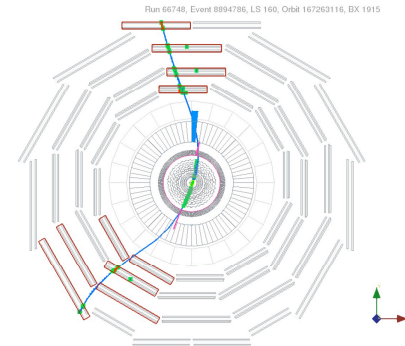
For massive gauge boson, can have 9  $A_{k,l}$  where  $k, l = 0, \pm 1$

$$\begin{aligned} A_{+-} = A_{-+} &= \frac{m_X^2}{4\Lambda} c_1 (1 + \beta^2), & A_{+0} = A_{0+} &= \frac{m_X^3}{m_V \sqrt{2}\Lambda} \left[ \frac{c_1}{8} (1 + \beta^2) + \frac{c_4}{2} \beta^2 - \frac{c_6 + c_7 \beta^2}{2} i\beta \right], \\ A_{++} &= \frac{m_X^2}{\sqrt{6}\Lambda} \left[ \frac{c_1}{4} (1 + \beta^2) + 2c_2 \beta^2 + i\beta(c_5 \beta^2 - 2c_6) \right], & A_{-0} = A_{0-} &= \frac{m_X^3}{m_V \sqrt{2}\Lambda} \left[ \frac{c_1}{8} (1 + \beta^2) + \frac{c_4}{2} \beta^2 + \frac{c_6 + c_7 \beta^2}{2} i\beta \right], \\ A_{--} &= \frac{m_X^2}{\sqrt{6}\Lambda} \left[ \frac{c_1}{4} (1 + \beta^2) + 2c_2 \beta^2 - i\beta(c_5 \beta^2 - 2c_6) \right], & A_{00} &= \frac{m_X^4}{m_V^2 \sqrt{6}\Lambda} \left[ (1 + \beta^2) \left( \frac{c_1}{8} - \frac{c_2}{2} \beta^2 \right) - \beta^2 \left( \frac{c_3}{2} \beta^2 - c_4 \right) \right]. \end{aligned}$$

We do the same thing for spin-zero and spin-one X

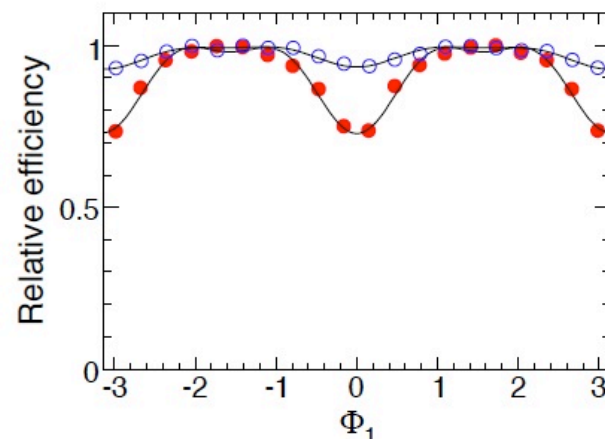
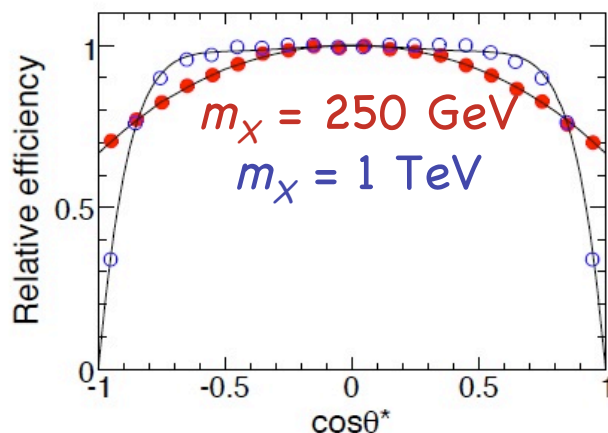


# Detector Effects



- Experimental effects addressed in standalone ROOT
  - **Parameter resolution:** we smear four-momenta of decay products in pT and angular resolution by values determined from CMS cosmic ray studies (*JINST*)
    - Angular resolution very good; on the order of 0.01 radians
  - **Geometric acceptance:** assume hermetic detector out to  $\eta = 2.5$ 
    - Helicity angles weakly dependent on detector acceptance
    - Production angles most directly affected
    - Parameterize acceptance in PDF by:

$$\mathcal{P}(\text{angles}) = \mathcal{P}_{\text{ideal}}(\text{angles}) * G_{\text{acc}}(\text{angles})$$







# Multivariate Techniques

- Using RooFit: unbinned maximum-likelihood fit
- Joint fit to combine all 3 channels:  $4\mu$ ,  $4e$ ,  $2e2\mu$

$$\mathcal{L} = \exp\left(-\sum_{J=1}^3 n_J - n_{\text{bkg}}\right) \prod_i^N \left( \sum_{J=1}^3 n_J \times \mathcal{P}_J(\mathbf{x}_i; \zeta_J; \xi) + n_{\text{bkg}} \times \mathcal{P}_{\text{bkg}}(\mathbf{x}_i; \xi) \right)$$

$$\begin{aligned} \mathbf{x}_i &= \{m_{ZZ}, \theta_1, \theta_2, \Phi, \theta^*, \Phi_1\}_i \\ \zeta_J &= \{f_{kb}, \phi_{kb}, f_m\} \\ \xi &= \text{other parameters} \end{aligned}$$

- Use the multivariate likelihoods for:
  - Distinguishing between different signal hypotheses
  - Improving background suppression - both in case of signal or no signal
  - Parameter determination for a certain hypothesis



# Hypothesis Separation

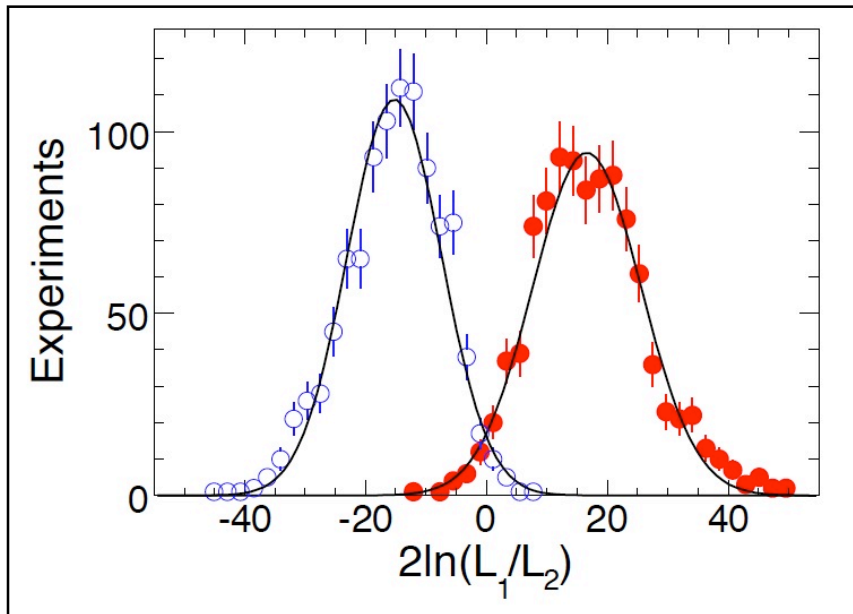
Neyman-Pearson hypothesis testing:

Run 1000 toy experiments...

Determine likelihood ratio estimator [ $S = 2 \cdot \ln(L_A/L_B)$ ] for data samples "A" and "B". Quote effective separation of Gaussian peaks.

Probability Density Function constructed of  $m_{ZZ}$  + angular distributions

Example case of  $0^+$  vs  $0^-$  at 250 GeV



Separation of:

- Signal scenarios (left)
- Signal vs. Background  
 $L_A$  (S+B) and  $L_B$  (B only)

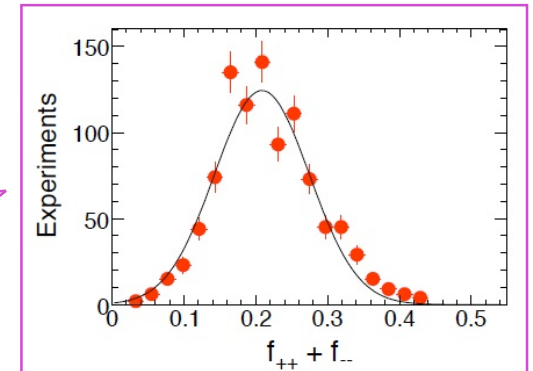
e.g. SM Higgs: can achieve  $5.7\sigma$  using kinematic variables only. We can improve by  $\sim 16\%$  if we include angular variables



# Parameter fitting

- Fit w/ and w/out detector effects for 2 mass points, **compare with generated parameters**
- As an example, we take  $0^+$  and  $0^-$  cases

$0^+$	generated	$m_X = 250 \text{ GeV}$		generated	$m_X = 1 \text{ TeV}$	
		without detector	fitted with detector		without detector	fitted with detector
$n_{\text{sig}}$	150	$150 \pm 13$	$153 \pm 15$	150	$150 \pm 12$	$152 \pm 12$
$(f_{++} + f_{--})$	0.208	$0.21 \pm 0.07$	$0.23 \pm 0.08$	0.000	$0.00 \pm 0.03$	$0.00 \pm 0.03$
$(f_{++} - f_{--})$	0.000	$0.01 \pm 0.13$	$0.01 \pm 0.14$	0.000	$0.00 \pm 0.02$	$0.00 \pm 0.02$
$(\phi_{++} + \phi_{--})$	$2\pi$	$6.30 \pm 1.46$	$6.39 \pm 1.54$	$2\pi$	free	free
$(\phi_{++} - \phi_{--})$	0	$0.00 \pm 1.06$	$0.01 \pm 1.09$	0	free	free



$0^-$	generated	$m_X = 250 \text{ GeV}$		generated	$m_X = 1 \text{ TeV}$	
		without detector	fitted with detector		without detector	fitted with detector
$n_{\text{sig}}$	150	$150 \pm 13$	$151 \pm 15$	150	$151 \pm 12$	$150 \pm 13$
$(f_{++} + f_{--})$	1.000	$1.00 \pm 0.05$	$1.00 \pm 0.06$	1.000	$1.00 \pm 0.05$	$1.00 \pm 0.06$
$(f_{++} - f_{--})$	0.000	$0.00 \pm 0.35$	$0.00 \pm 0.40$	0.000	$0.00 \pm 0.31$	$-0.01 \pm 0.32$
$(\phi_{++} + \phi_{--})$	N/A	free	free	N/A	free	free
$(\phi_{++} - \phi_{--})$	$\pi$	$3.15 \pm 0.31$	$3.14 \pm 0.41$	$\pi$	$3.15 \pm 0.31$	$3.14 \pm 0.33$

$0^+$ :  $f_{++} + f_{--} = 0.23 \pm 0.08$

$0^-$ :  $f_{++} + f_{--} = 1.00 \pm 0.06$

A naive separation  
between  $0^+/0^-$  of  $\sim 10\sigma$