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Double pole in the neutral Higgs sector

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Motivation

- Degeneracy of the heavy neutral Higgses will be manifested for H_2^0 and H_3^0 as coherent states.

  $\rightarrow$ mass degeneracy will be revealed as an exceptional point in the complex parameter space.

- Proper treatment of Non-Hermitian systems will ensure correct theoretical predictions in the study of possible sources of CP violation

  $\rightarrow$ $\not{C}\not{P}$ phases in the Higgs line-shape, may have an unexpected behavior

  at the exact degeneracy of a coherent system

Necessary conditions for a double pole in the Higgs sector

① **multi-Higgses** $\rightarrow$ as 2HDM and MSSM

② **very close in mass at tree level** $\rightarrow$ as in the MSSM $m_H^{(0)} = m_A^{(0)}$ for $m_A \gg m_Z$ and $\tan\beta \gg 1$.

③ **masses larger than the EW scale** $\rightarrow$ including radiative corrections $m_h < m_H, m_A$

④ **system manifestly non-Hermitian** $\rightarrow$ CP non-invariant Higgs sector: h, H, A CP defined $\rightarrow H_1, H_2, H_3$ CP-mixed

Example: Degeneracy of non-coherent two single poles.

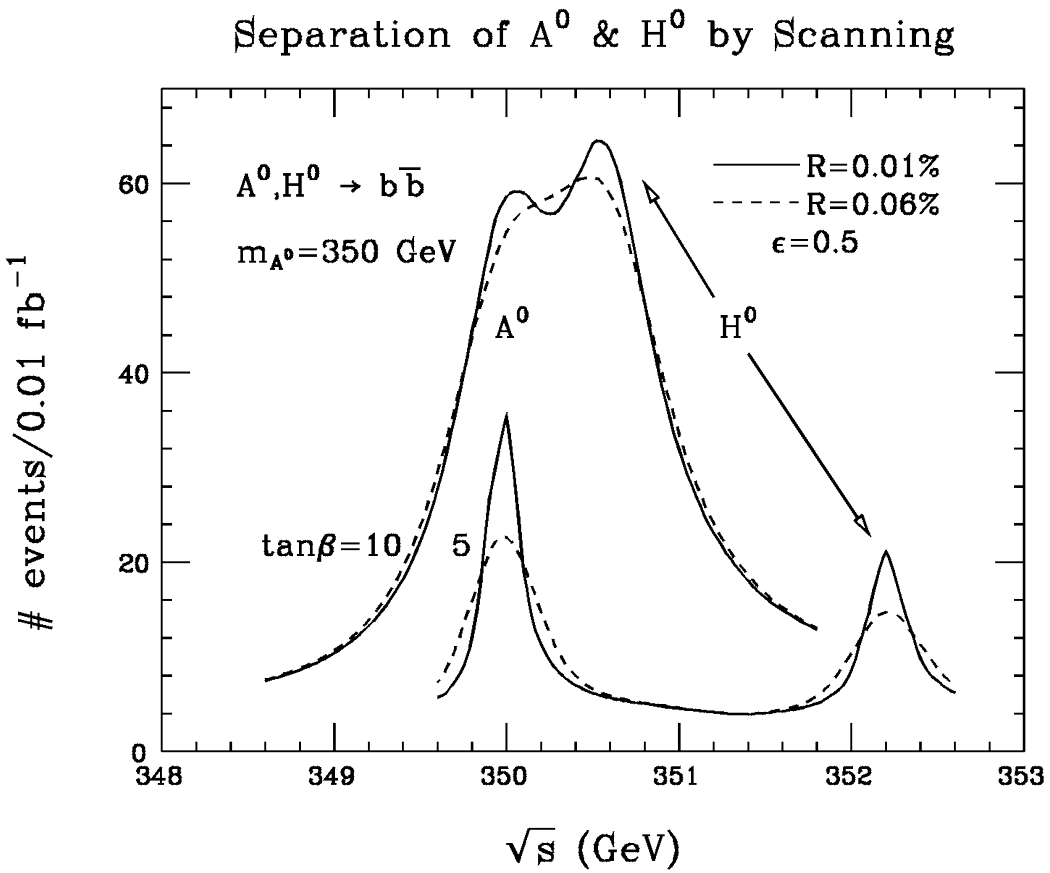

Figure 1: two non-coherent states, [Barger,Berger,Gunion,Han 96]

[Bernabeu, Binosi, Papavassiliou, 06]

Example: Degeneracy and coherent states.

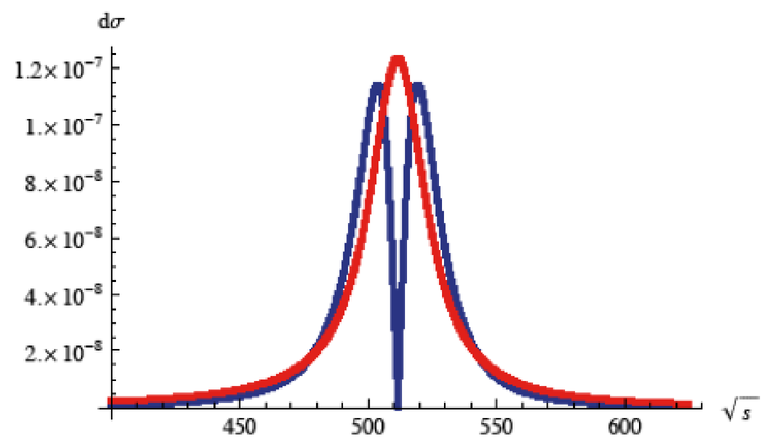

Figure 2: two coherent states may have destructive interference.

For general model see [Cacciapaglia,Deandrea,De Curtis 09]

Higgs masses as the poles of the propagator

Neutral heavy Higgs bosons as s-channel resonances

- Masses and mixings of the H-A system may be detected as two closely spaced or even overlapping resonances in the s-channel reaction, *i.e.*

$$\mu^+\mu^- \to A^*/H^* \to f\bar{f} \tag{1}$$

[Pilaftsis 97],[Bernabeu, Binosi, Papavasiliu 06]

- The form of the line shape of this process would indicate the presence (or absence) of CPV in the heavy Higgs system.
- In the resonant region, the t-channel amplitude is relatively small and may be ignored
- Then, in the electroweak basis, the transition amplitude matrix between states with CP-violation via resonant Higgs exchange is

$$\mathcal{T}^{res}(s) = V^P \hat{\Delta}^{-1}_{H_2-H_3}(s) V^D \tag{2}$$

where we identify the propagator as

$$\hat{\Delta}^{-1}_{H_2-H_3}(s) = s\mathbf{1}_{2\times 2} - \mathcal{M}^2_{H_2-H_3}(s) = \begin{pmatrix} s-(M_H^2-\hat{\Pi}_{HH}(s)) & \hat{\Pi}_{HA}(s) \\ \hat{\Pi}_{HA}(s) & s-(M_A^2-\hat{\Pi}_{AA}(s)) \end{pmatrix} \tag{3}$$

Physical masses as the poles of the propagator

The physical masses of the neutral heavy Higgs bosons are identified with the poles of the propagator matrix $\hat{\Delta}_{H_2-H_3}(s)$.

Hence, the masses of the neutral, heavy Higgs bosons are defined as the solutions of the implicit equation

$$det\left[\hat{\Delta}_{H_2-H_3}(s^\star)\right] = det\left[(s^\star)\mathbf{1}_{2\times 2} - \mathcal{M}^2_{H_2-H_3}(s^\star)\right] = 0. \tag{4}$$

In the physical basis, $\mathcal{M}^2_{H_2-H_3}(s)$ is diagonal,

$$M^2_{H_i}(s^\star) - iM_i\Gamma_i(s^\star) := \mu^2_{H_i}(s^\star_i). \tag{5}$$

then eq. (4) becomes

$$(s_2 - \mu^2_{H_2}(s_2))(s_3 - \mu^2_{H_3}(s_3)) = 0\,, \tag{6}$$

Degeneracy of neutral heavy CP non-invariant Higgs bosons

Considering the full s-dependance, the true physical masses are identified with the poles of the propagator. Therefore, the heavy Higgs bosons masses should be defined by the solutions of the implicit equations [Stuart 95],[Bohm,Kaldass and Wickramasekara 02]

$$\mu^2_{H_i}(s^*_{H_i}) - s^*_{H_i} = 0; \qquad i = 2, 3 \tag{7}$$

We say that the two heavy neutral Higgs bosons are mass degenerate if there exist an s^* such that

$$\begin{array}{l} s^* - \mu^2_{H_2}(s^*) = 0 \\ \\ s^* - \mu^2_{H_3}(s^*) = 0 \end{array} \Rightarrow \mu^2_{H_2}(s^*) = \mu^2_{H_3}(s^*),$$

In order to avoid a clumsy notation, we will write the effective squared mass matrix $\mathcal{M}^2_{H_2H_3}(s)$ in the basis of the Pauli spin matrices as

$$\mathcal{M}^2_{H_2H_3}(s) = \frac{1}{2}T\mathbf{1}_{2\times2} + (\vec{R} - i\vec{\Gamma}) \cdot \vec{\sigma}. \tag{8}$$

where T is the trace of $\mathcal{M}^2_{H_2H_3}$

Degeneracy conditions

And we may write the degeneracy condition on the eigenvalues as

$$\frac{1}{2}\left[\mu^2_{H_3}(s^*) - \mu^2_{H_2}(s^*)\right] = \sqrt{(\vec{R}_d - i\vec{\Gamma}_d)^2} = 0, \tag{9}$$

where we have defined the vectors

$\vec{R} = \left(\frac{1}{2}(M_H^2 - M_A^2), 0, Re\Delta^2_{HA}\right)$ and $\vec{\Gamma} = \left(\frac{1}{2}(M_H\Gamma_H - M_A\Gamma_A), 0, Im\Delta^2_{HA}\right)$,

The eq.(9) implies $R_d^2(s^\star) = \Gamma_d^2(s^\star)$ and $\vec{R}_d(s^\star)\cdot\vec{\Gamma}_d(s^\star) = 0$ for $\vec{R}_d, \vec{\Gamma}_d \neq 0$.

From the above conditions we found that the traceless term of the propagator, at degeneracy, will be given by

$$(\vec{R}_d - i\vec{\Gamma}_d)\cdot\vec{\sigma} = M_d\Gamma_d\begin{pmatrix}1 & i\\ i & -1\end{pmatrix}, \tag{10}$$

s-channel transition matrix at degeneracy

The propagator can be written as

$$\Delta^{(d)}_{H_2-H_3}(s) = \begin{pmatrix} \frac{1}{(s-M_d^2+iM_d\Gamma_d)} & \frac{iM_d\Gamma_d}{(s-M_d^2+iM_d\Gamma_d)^2} \\ 0 & \frac{1}{(s-M_d^2+iM_d\Gamma_d)} \end{pmatrix} \tag{11}$$

We then can write the resonant transition matrix in the mass representation as

$$\begin{aligned} \mathcal{T}^{res(d)}(s) &= (\tilde{V}_H^P, \tilde{V}_A^P)\Delta^{(d)}_{H_2,H_3}(s) \begin{pmatrix} \tilde{V}_H^D \\ \tilde{V}_A^D \end{pmatrix} \\ &= \tilde{V}_H^P \frac{1}{s-\mu_d^2(s)} \tilde{V}_H^D + \tilde{V}_A^P \frac{1}{s-\mu_d^2(s)} \tilde{V}_A^D + \tilde{V}_H^P \frac{iM_d\Gamma_d}{(s-\mu_d^2(s))^2} \tilde{V}_A^D \end{aligned} \tag{12}$$

with

$$\begin{pmatrix} \tilde{V}_H^{P,D} \\ \tilde{V}_A^{P,D} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & +i \\ 1 & -i \end{pmatrix} \begin{pmatrix} V_H^{P,D} \\ V_A^{P,D} \end{pmatrix} \tag{13}$$

Application for the 2HDM

Can be shown that only two of the neutral Higgs states are degenerated obtaining a 2×2 mass matrix.

In the decoupling limit, defined by the inequality [Gunion, Haber 03]

$$m_A^2 >> |\lambda_i| v^2, \tag{14}$$

the, mixing between the light state, $H_1(\rightarrow\ h^0)$, and the heavy states, H_2 and H_3, is small, compared with the mixing of the nearly degenerate heavy Higgs states H_2 and H_3.

[Félix-Beltrán,Gómez-Bock,Hernández, Mondragón,Mondragón 2009]

$$\mathcal{M}^2_{H_2-H_3}(s) = \begin{pmatrix} M_H^2(s) - iM_H\Gamma_H(s) & \Delta^2_{HA}(s) \\ \Delta^2_{HA}(s) & M_A^2(s) - i\Gamma_A M_A(s) \end{pmatrix} \tag{15}$$

[Pilaftsis 98],[Demir 99]

The matrix elements are expressed as functions of the model parameters. In the decoupling limit $M_A^2 >> |\lambda_i| v^2$, we may find a simplifying approach for the relations of the mass matrix elements as [Choi,Kalinowski,Liao and Zerwas 05]

$$
\begin{aligned}
M_H^2 - M_A^2 &\approx \lambda v^2 \cos\phi && (16)\\
32\pi [M_H \Gamma_H - M_A \Gamma_A] &\approx [\Delta_t + 9\lambda^2 v^2 \cos 2\phi] && (17)\\
Re\Delta_{HA}^2 &\approx -\frac{1}{2}\lambda v^2 \sin\phi && (18)\\
32\pi \ Im\Delta_{HA}^2 &\approx -\frac{9}{2}\lambda^2 v^2 \sin 2\phi && (19)
\end{aligned}
$$

We have taken the magnitudes of all λ_i as same order, and ϕ is the CP violating common phase of the complex couplings. And

$$\Delta_t = -12 M_{H/A}^2 (m_t/v)^2 (1-\beta_t^2)\beta_t, \qquad (20)$$

is the one loop contribution of the top quark.

Exceptional point in the mass complex surfaces

We are able to write explicity the masses of the heavy neutral Higgs bosons as functions of the parameters λ and ϕ and if further more we neglect the weak $\mathbf{s}$ dependence of the elements of $\mathcal{M}^2_{HA}$, we found an approximation for pole position mass. The term under the square root admits a Puiseux expansion series around the exceptional point, [Hernández,Jáuregui and Mondragón 06].

$$\mu^2_{2,3}(\lambda,\phi) = \frac{1}{2}\sqrt{c_1^{(1)}(\lambda-\lambda^\star) + c_2^{(1)}(\phi-\phi^\star) + \ldots} \qquad (21)$$

where the degeneracy conditions we get the exceptional point as: $\lambda^\star = 0.1075,\ \ \phi^\star = \pi/2$ and $c_k^{(1)}$ are the derivatives of $\mu^2_{2,3}$ with respect to the parameters λ and ϕ.

$$\Re e\mu^2_{2,3} = \pm\frac{1}{2\sqrt{2}}|\vec{\zeta}|^{1/2}\left[\sqrt{(\vec{\mathcal{R}}\cdot\hat{\zeta})^2 + (\vec{\mathcal{I}}\cdot\hat{\zeta})^2} + (\vec{\mathcal{R}}\cdot\hat{\zeta})\right]^{1/2} \qquad (22)$$

$$\Im m\mu^2_{2,3} = \pm\frac{1}{2\sqrt{2}}|\vec{\zeta}|^{1/2}\left[\sqrt{(\vec{\mathcal{R}}\cdot\hat{\zeta})^2 + (\vec{\mathcal{I}}\cdot\hat{\zeta})^2} - (\vec{\mathcal{R}}\cdot\hat{\zeta})\right]^{1/2} \qquad (23)$$

with

$$\vec{\mathcal{R}} = \left(\Re e\, c_1^{(1)}, \Re e\, c_2^{(1)}\right), \quad \vec{\mathcal{I}} = \left(\Im m\, c_1^{(1)}, \Im m\, c_2^{(1)}\right), \quad \vec{\zeta} = \begin{pmatrix} \lambda-\lambda^\star \\ \phi-\phi^\star \end{pmatrix}. \qquad (24)$$

Unfolding of the exceptional point

The figures show the mass hypersurface representing the imaginary parts of $\mu^2_{2,3}$ as function of the Lagrangian parameters in the neighbourhood of the exceptional point

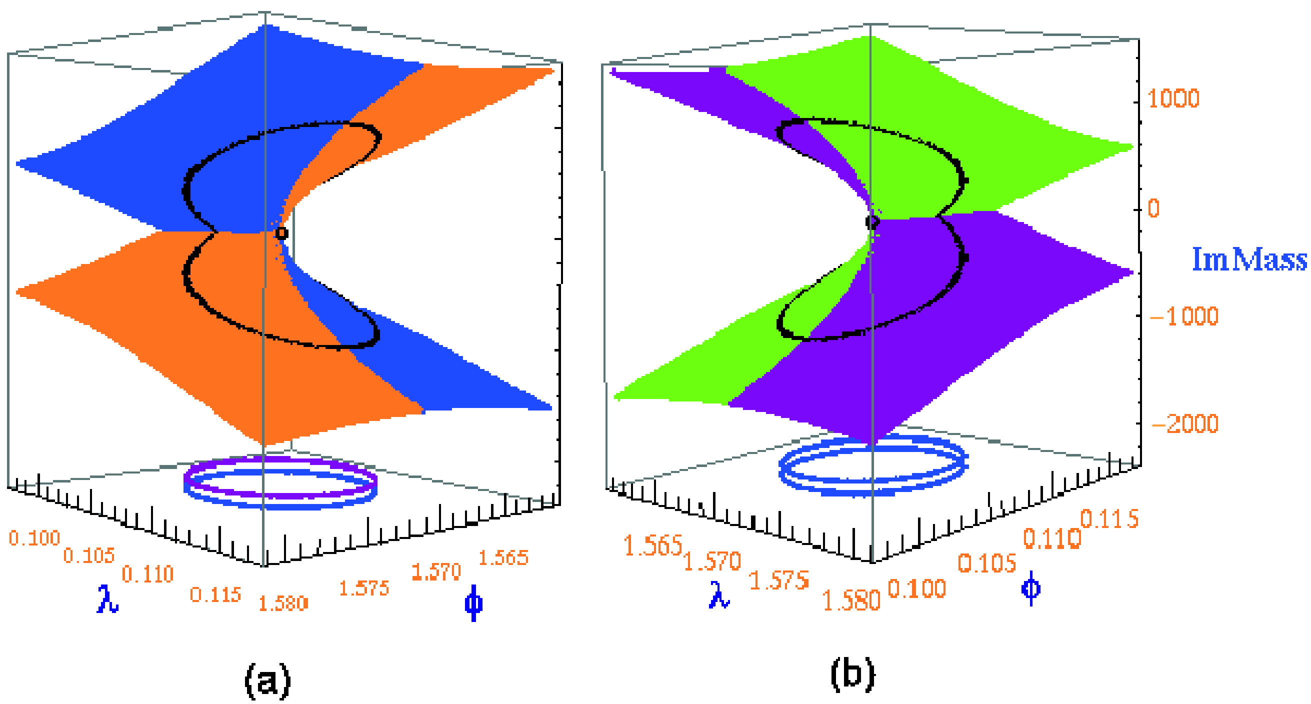

Figure 3: Real and imaginary mass surfaces around the exceptional point

Conclusions

- CP-violating complex couplings allow for the possibility of mixing of $H, \ A$ and degeneracy of the $H_2, \ H_3$ physical states.

- At exact mass degeneracy, the propagator of the system has a combination of one double and two single poles in the complex energy s-plane.

- In parameter space the mass surfaces have one branch point of rank one where exact degeneracy occurs.

- At degeneracy, the identification of the two particles will depend strongly on the values of the parameters where the mass surfaces are displayed.

- It is imperative to consider the main s-dependent 1-loop diagrams for the Higgs self-energy in order to find the exceptional point in terms of the MSSM parameters.

- These features would make a CP-violating Higgs sector of the MSSM easily discernible from a CP-preserving one.

thank you