

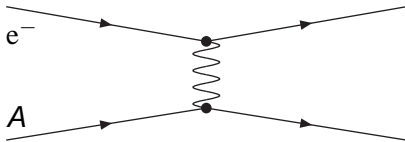
# Particle Physics: The Standard Model

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- QCD works, but are quarks real or mathematical constructions?
- use  $eA \rightarrow eA'$
- electrons or muons pointlike probe possibly non-pointlike objects
- Fixed target experiments:
  - SLAC:  $1\text{GeV} < E < 30\text{GeV}$
  - FERMILAB/CERN:  $E \sim 300\text{GeV}$
- Colliding beams experiment:
  - DESY (HERA):  $E_e = 20\text{GeV}$ ,  $E_p = 800\text{GeV}$



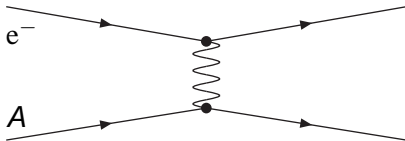
## Process

- incoming electron  $\mathbf{k}$
- outgoing electron  $\mathbf{k}'$
- emission of photon  
 $\mathbf{q} = \mathbf{k} - \mathbf{k}'$
- incoming nucleon  $A$   $\mathbf{p}$
- outgoing nucleon  $A'$   $\mathbf{p}'$

## Measurements

- $M$ : mass of the nucleon  $A$
- $E$ : energy of the incoming electron (ref:lab)
- $E'$ : energy of the outgoing electron (ref:lab)
- $\theta$ : scattering angle in lab frame

$k$  is known,  $k'$  is reconstructed from  $E'$  and  $\theta$



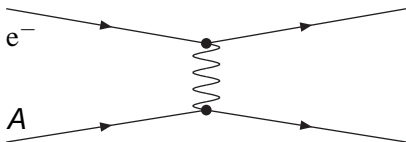
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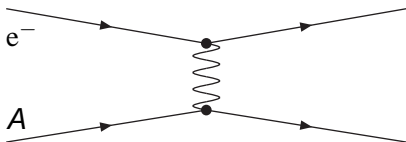
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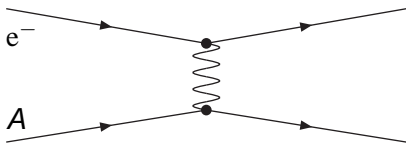
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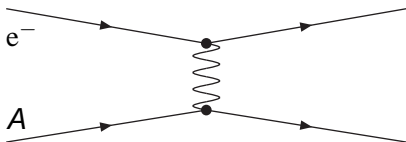
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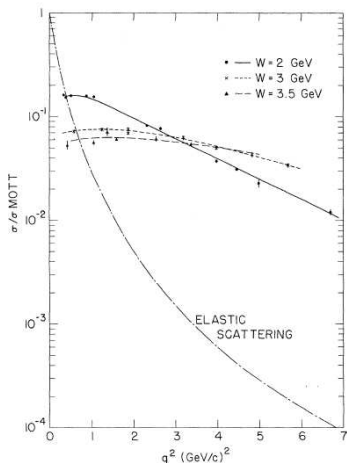
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Scattering of an electron on a point-like object (proton):  
Mott

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{4E^2} \frac{\cos^2 \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}}$$

- Low  $Q^2$
- $E = 7\text{GeV}$  and  $E = 17\text{GeV}$
- angles:  $6^\circ, 10^\circ$
- **not compatible with Mott**





## Dirac equation for adjoint spinor

$$\begin{aligned}
 i\gamma^\mu \partial_\mu \psi - m\psi &= 0 \\
 -i(\gamma^\mu)^* \partial_\mu \psi^* - m\psi^* &= 0 \\
 -i(\partial_\mu \psi^\dagger)(\gamma^\mu)^\dagger - m\psi^\dagger &= 0 \\
 -i(\partial_\mu \psi^\dagger)\gamma^0 \gamma^\circ (\gamma^\mu)^\dagger \gamma^0 - m\psi^\dagger \gamma^0 &= 0 \\
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 i(\partial_\mu \bar{\psi})\gamma^\mu + m\bar{\psi} &= 0
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 \partial_\mu j^\mu &= \partial_\mu [-e\bar{\psi}\gamma^\mu\psi] \\
 &= -e(\partial_\mu \bar{\psi})\gamma^\mu\psi - e\bar{\psi}\gamma^\mu\partial_\mu\psi && \text{Dirac} \\
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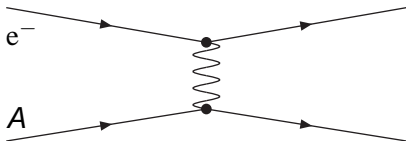
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(In)-Elastic scattering



$$2M\nu \geq Q^2$$

$A'$ : mass of target plus photon

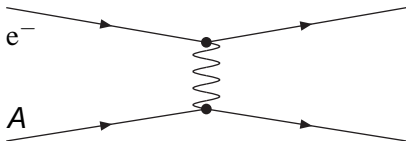
$$\begin{aligned} (\mathbf{p} + \mathbf{q})^2 &= \mathbf{p}^2 + 2 \cdot \mathbf{p} \cdot \mathbf{q} + \mathbf{q}^2 \\ &= M^2 + 2 \cdot M \cdot \nu - Q^2 \\ M^2 &\leq M^2 + 2 \cdot M \cdot \nu - Q^2 \end{aligned}$$

$$\begin{aligned} T_{fi} &= 4\pi\alpha \bar{u}(\mathbf{k}') \gamma^\mu u(\mathbf{k}) \frac{1}{q^2} W_\mu \\ d\sigma &\sim L^{\mu\nu} W_{\nu\mu}^{eN} \\ L^{\mu\nu} &= k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} (\mathbf{k} \cdot \mathbf{k}') \end{aligned}$$

- hadronic tensor:  
 $W_{\nu\mu}^{eN} = f(W_\mu, W_\nu) = f(\mathbf{p}, \mathbf{q})$
- EM current:  $p^\mu W_{\nu\mu} = (p' - p)^\nu W_{\nu\mu} = q^\nu W_{\nu\mu} = 0$

## (In)-Elastic scattering

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$A'$ : mass of target plus  
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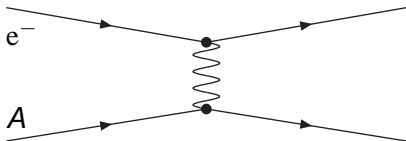
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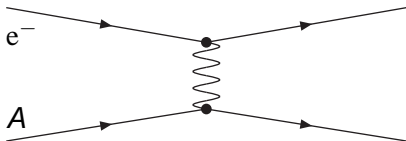
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$$2M\nu \geq Q^2$$



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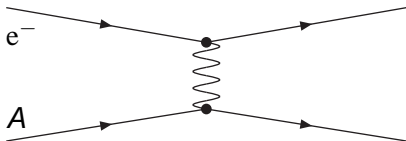
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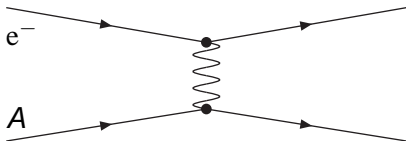
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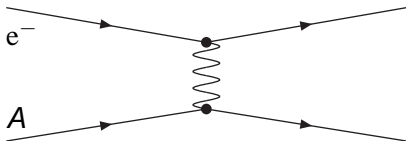
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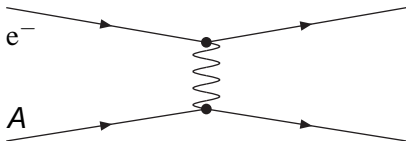
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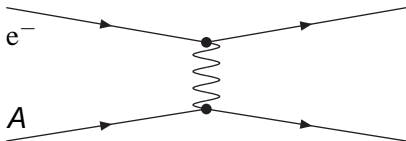
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Bjorken: a nucleon is a beam of partons where  $x$  describes the fraction of the nucleon momentum carried by the partons

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 2MW_1(\nu, Q^2) &= F_1(x) + \mathcal{O}\left(\frac{1}{Q^2}\right) && \text{Magnetic SF} \\
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The functions  $F_1(x)$  and  $F_2(x)$  are related to the probability to find a parton (= quark):

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$Q_i$  charge of the quark,  $s$  scalars (no magnetism!)

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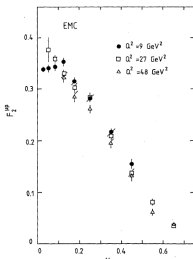
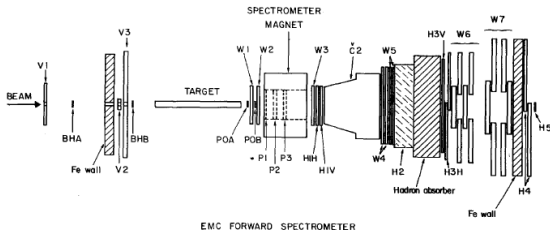
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- boost (300GeV on target at rest) leads to forward detection (problem solving)
- to first order  $F_2$  is independent of  $Q^2$ , i.e. scale invariant
- QPM works to first order

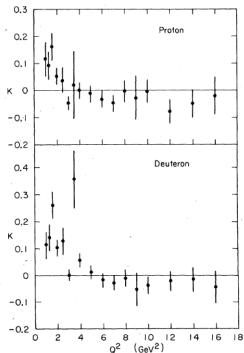
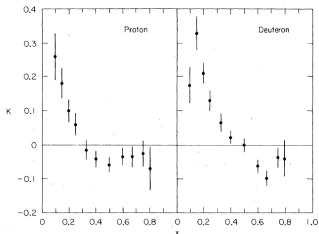
## Callan-Gross 1969

$$R(x) = \frac{F_2(x) - xF_1(x)}{F_2(x)}$$

= 0 for Spin- $\frac{1}{2}$

= 1 for Spin-0

1983:  $R = -0.01 \pm 0.11$

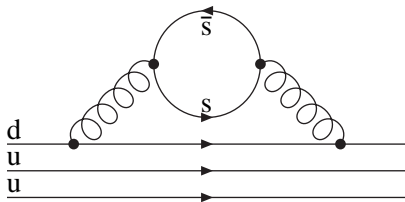


(SLAC) small  $x$  and small  $Q^2$

## Description of a Proton



- BREIT frame: ( $E = E'$ )
- partons without transverse momentum
- $|uud\rangle$

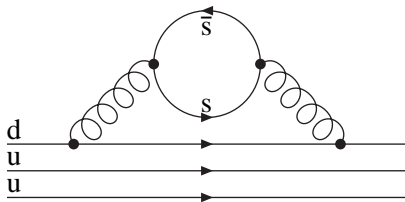


- $|uud + u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c} + \dots\rangle$
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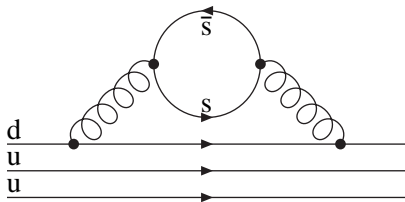


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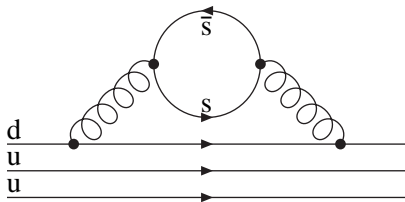


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$f_u(x)$  etc are the **parton** distribution function (PDF). Not a probability because of the normalization:

$$\int f_u^p(x) + f_{\bar{u}}^p(x) dx = 2$$

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$F_2^n$  of the neutron is then:

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$U(\pi)|u\rangle = |d\rangle$  The probability to find a  $u$  in a proton

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$$f_u^p S = f_u^p S = f_d^p S \dots$$

$$\begin{aligned} F_2^p - F_2^n &= \frac{x}{3}(f_u^p V - f_d^p V) \\ &= \frac{x}{3}(f_u^p V - \frac{1}{2}f_u^p V) \\ &= \frac{x}{6}f_u^p V \end{aligned}$$

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$U(\pi)|u\rangle = |d\rangle$  The probability to find a  $u$  in a proton

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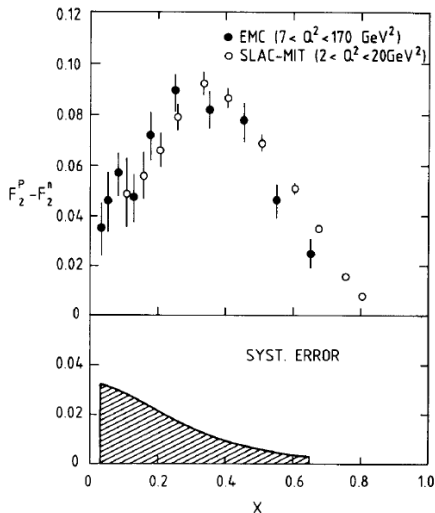
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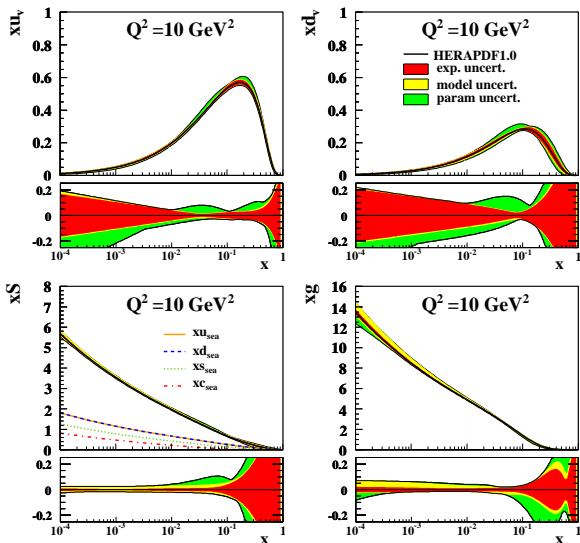
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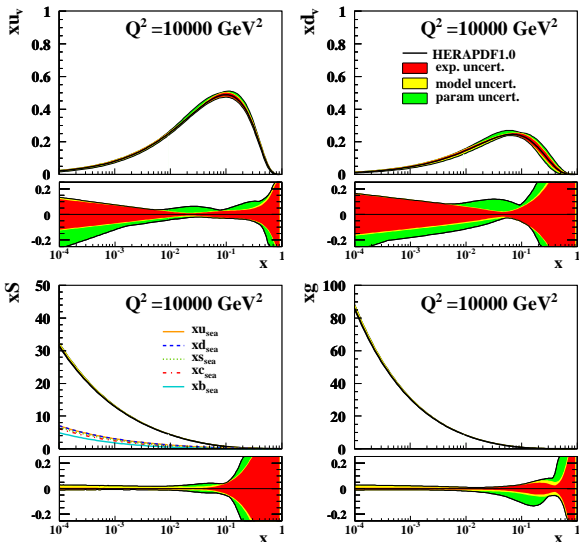


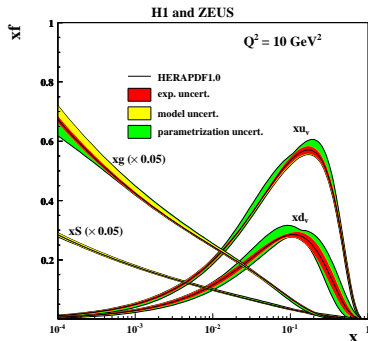
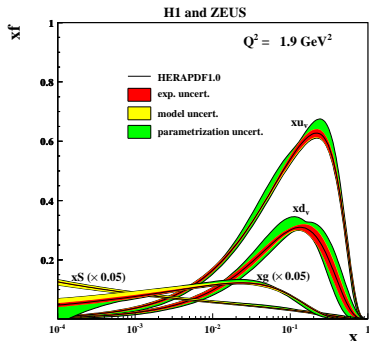
- $F_2^p$  from  $\mu + H$
- $F_2^n$  from  $\mu + D$
- **correct and subtract.....**
- distribution peaks at 0.3
- essentially independent of  $Q^2$  (scaling)

## H1 and ZEUS



## H1 and ZEUS





## PDF Summary

- PDFs measured over a large range of  $x$  and  $Q^2$
- small  $x$  strong increase of  $xg(x)$

## Parton Model Sum Rules

Baryonnumber:

$$\int_0^1 \sum_q \frac{1}{3} (f_q - f_{\bar{q}}) dx = 1$$

The baryonnumber is 1.

The hadron charge:

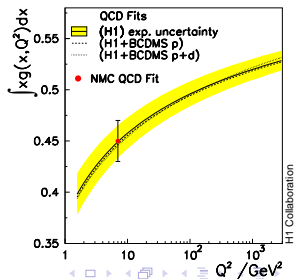
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The momentum:

$$\int_0^1 x \sum_j f_j dx = 1 - \epsilon$$

$\epsilon$  gluon momentum  
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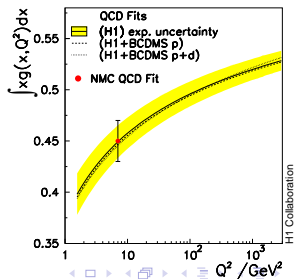
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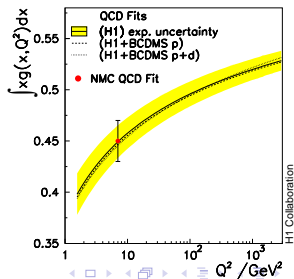
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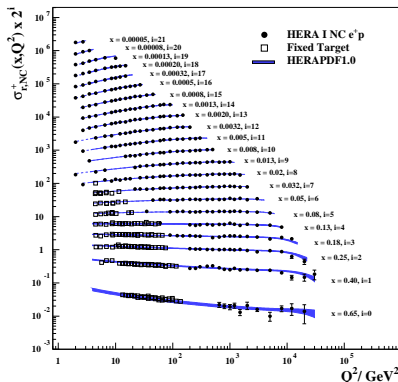




## QPM $\rightarrow$ QCD

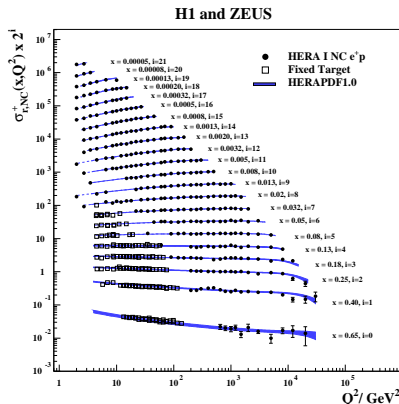
- $f_i(x) \rightarrow f_i(x, Q^2)$
- increase  $Q$  improves parton resolution
- small  $Q$  parton  $p$
- increase  $Q$   
 $p = p_1 + p_2$
- small  $x$  more particles  $\rightarrow f_i \uparrow$
- large  $x$  less particles  $\rightarrow f_i \downarrow$
- Attempt to describe evolution with  $Q^2$

H1 and ZEUS



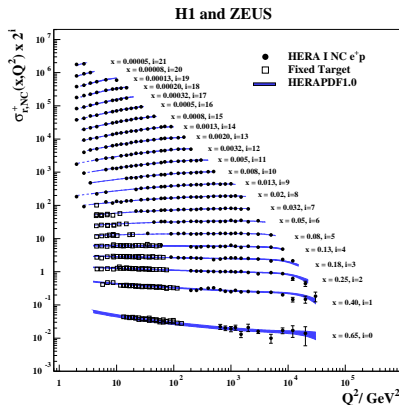
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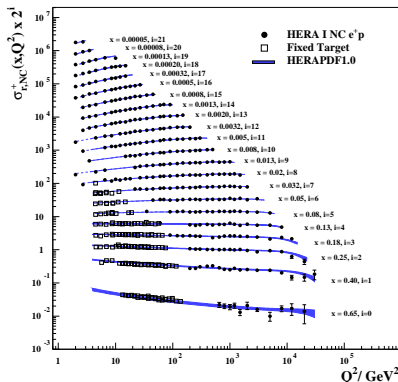
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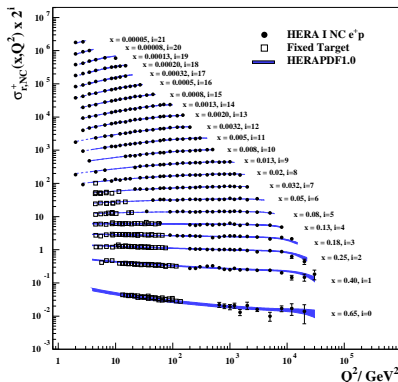
H1 and ZEUS



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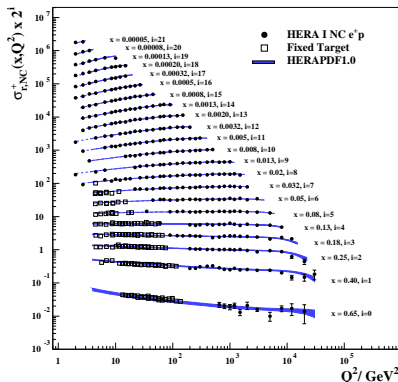
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H1 and ZEUS



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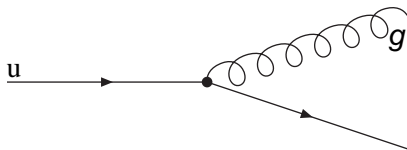
$$\frac{\partial f_i(x, Q^2)}{\partial \log Q^2} = \frac{g_S^2}{8\pi^2} \sum_j \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}\right) f_j(y, Q^2)$$

- proportional to  $g_S^2$
- daughter momentum  $x$  smaller than mother momentum  $y$   
 $\rightarrow \int_x^1$
- only depends on the relative longitudinal momenta  $\frac{x}{y}$

Example:

$$P_{Gq}(x) = \frac{4}{3} \frac{1 + (1-x)^2}{x}$$

$\frac{1}{x}$  typical for radiation  
 Derivation of functions  $\rightarrow$  QCD



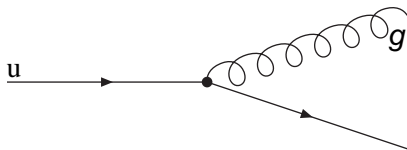
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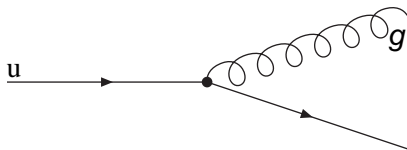
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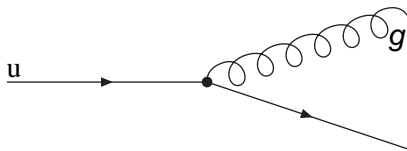
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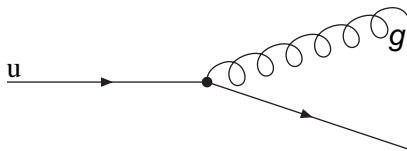
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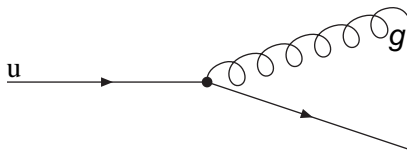
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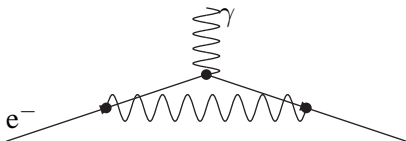
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## Asymptotic freedom



Electron in (out):  $\mathbf{p}$  ( $\mathbf{p}'$ )

Photon:  $\mathbf{l}$

inner Electron in (out):  $\mathbf{p} - \mathbf{l}$ ,  
 ( $\mathbf{p}' - \mathbf{l}$ )

$$\int d^4 l \frac{1}{l^2} \frac{\not{p}' - \not{l} + m}{(p-l)^2 - m^2} \frac{\not{p}' - \not{l} + m}{(p'-l)^2 - m^2}$$

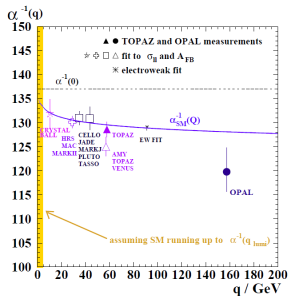
$$\int d^4 l \frac{1}{l^2} \frac{\not{p}' - \not{l} + m}{p^2 - m^2 - 2pl + l^2} \frac{\not{p}' - \not{l} + m}{p'^2 - m^2 - 2p'l + l^2}$$

$$\int d^4 l \frac{1}{l^2} \frac{\not{p}' - \not{l} + m}{-2pl + l^2} \frac{\not{p}' - \not{l} + m}{-2p'l + l^2}$$

- $p, p'$  are fixed!
- infrared catastrophe for  $l \rightarrow 0$ :  $\frac{1}{l^2} \frac{1}{pl} \frac{1}{p'l}$
- cured by compensation real/virtual diagrams
- ultraviolet catastrophe:  $l \rightarrow \infty$
- logarithmic divergence
- renormalization and regularisation

## QED

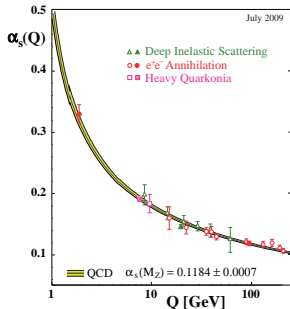
$$\alpha_{EM}(Q^2) = \frac{\alpha_{EM}}{1 - \sum_f \Pi_f(Q^2)}$$

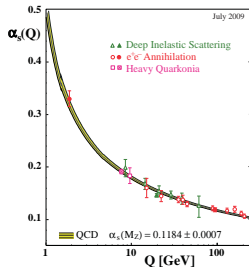
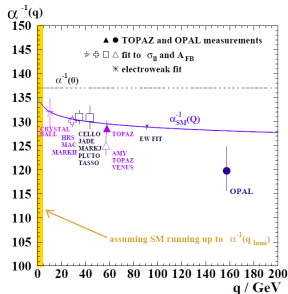


## QCD

$$\alpha_S(Q^2) = \frac{12\pi}{(33 - 2N_f) \log(Q^2/\Lambda^2)}$$

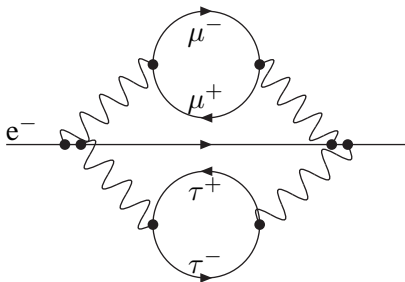
with  $\Lambda \sim 200\text{MeV}$ ,  $N_f$  number of active flavors (max 6 quarks)



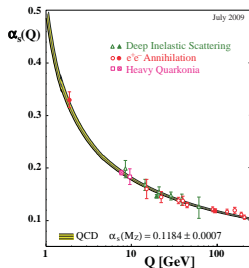


- energy  $\uparrow \rightarrow$  coupling  $\uparrow$
- finite (1/137) at 0
- shielding like a di-electric medium

- energy  $\uparrow \rightarrow$  coupling  $\downarrow$
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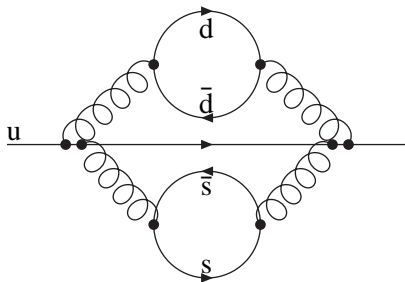
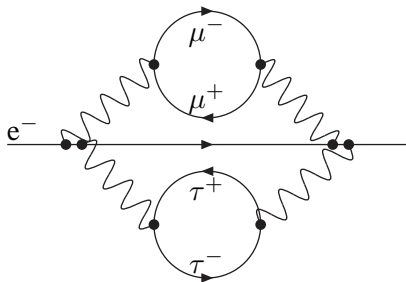


- energy  $\uparrow \rightarrow$  coupling  $\uparrow$
- finite ( $1/137$ ) at 0
- shielding like a di-electric medium
- $Q^2 \uparrow$  resolves bare charge



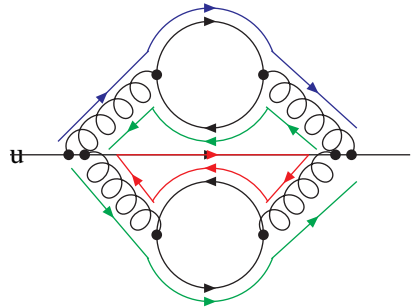
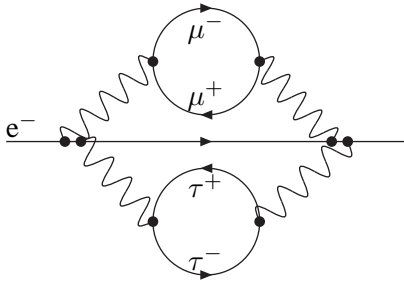
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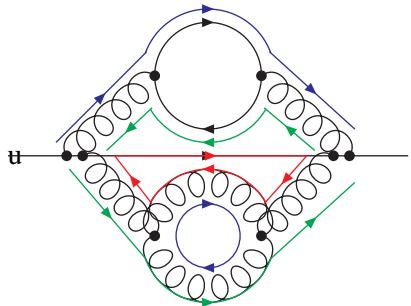
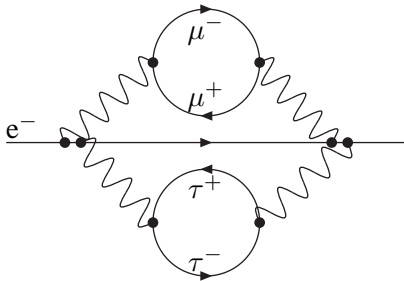
- energy  $\uparrow \rightarrow$  coupling  $\uparrow$
- finite ( $1/137$ ) at 0
- shielding like a di-electric medium
- $Q^2 \uparrow$  resolves bare charge

- **Color charge!**
- shielding as in QED
- TGV changes the shielding to anti-shielding



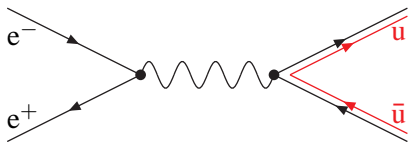
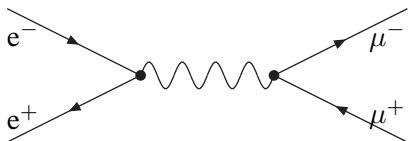
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- hadronization does not disturb measurement

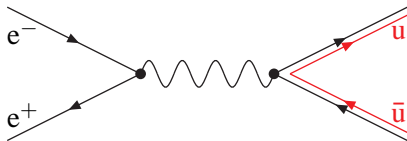
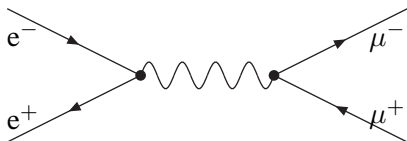
remember: quark-photon vertex  $\sim q$

$$R = \frac{\sum_i \sigma(e^-e^- \rightarrow q\bar{q})_i}{\sigma(e^-e^- \rightarrow \mu^+\mu^-)}$$

$$= \sum_i q_i^2 N_C$$

The ratio is sensitive to the number of colors!

- $\sqrt{s} > 10\text{GeV}$   $R = 3 \times (3 \times \frac{1}{3}^2 + 2 \times \frac{2}{3}^2) = \frac{11}{3}$
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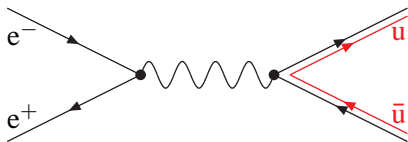
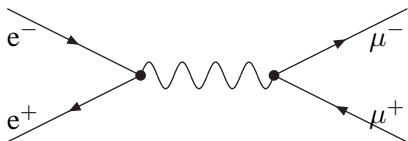
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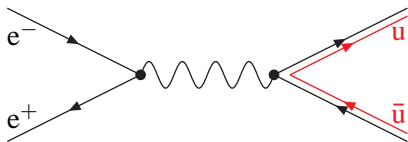
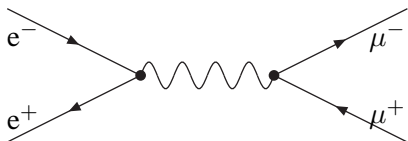
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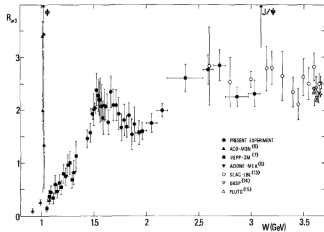
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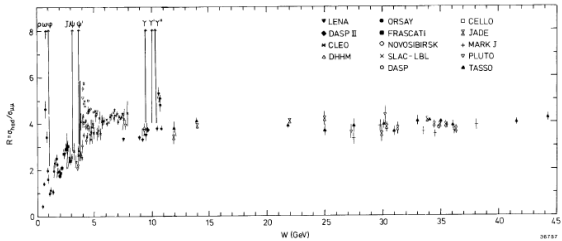
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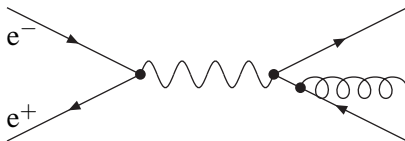
## Experiments

- ADONE (QED)
- PETRA (data)
- in agreement with quark counting
- resonances spoil

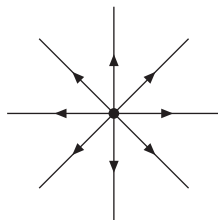




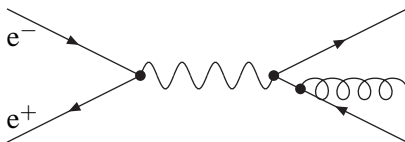
- Quarks and gluons are bound in hadrons
- Color is compatible with 3
- the gluon carries  $\mathcal{O}(50\%)$  or the proton momentum
- $SU(3)_C$  the gluon is a spin-1 particle
- $e^+e^- \rightarrow gg$  **not possible**
- $e^+e^- \rightarrow q\bar{q}$
- LAB frame is CM frame
- Problem: Hadronization
- reconstruct jets



Start with two jets at threshold:



- Quarks and gluons are bound in hadrons
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- $e^+e^- \rightarrow q\bar{q}g$
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Two jets above threshold:



Transverse momentum  
 $\sim 200\text{MeV}$

Boost collimates

Remember infrared catastrophe:

$$\sigma(e^+e^- \rightarrow q\bar{q}g) = \infty$$

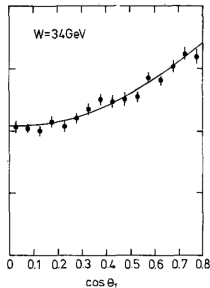
Reason:

$$\mathbf{P}_q = \mathbf{P}'_q + \mathbf{K}_g$$

cannot distinguish left from right.

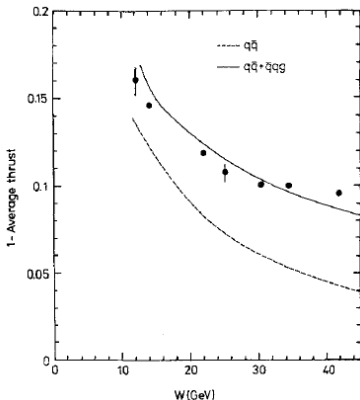
Solution: define infrared-safe observables like thrust:

$$T = \max\left(\frac{\sum_i |\mathbf{p}_i \cdot \hat{n}|}{\sum_i |\mathbf{p}_i|}\right)$$



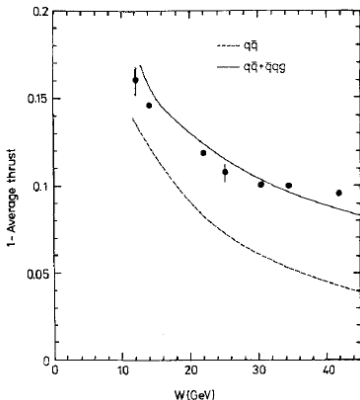
- thrust memorizes the quark spin
- compatible with Spin- $\frac{1}{2}$

- collimated 2jets:  $T = 1$   
 $\rightarrow 1 - T = 0$
- $q\bar{q}$ :  $1 - T$  should decrease to 0 as function of energy
- gluon should increase  $1 - T$  (more isotropical)
- distribution compatible with  $q\bar{q}g$



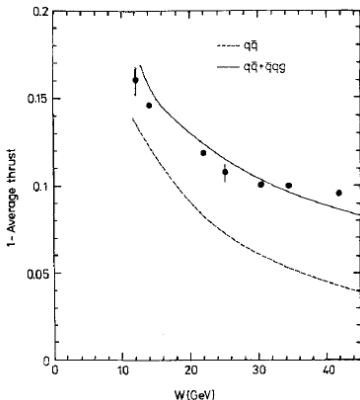
More Jet Algorithms in Problem Solving

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