Theory of double charmonium production

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Outline

- Introduction / Motivation
- Old and new measurements and possible contributions
- More in detail about SPS CS LO calculation
  - MC and predictions for LHCb
  - Signatures within LHCb acceptance
- DPS studies
- If there is time left – tetraquarks
- Conclusions

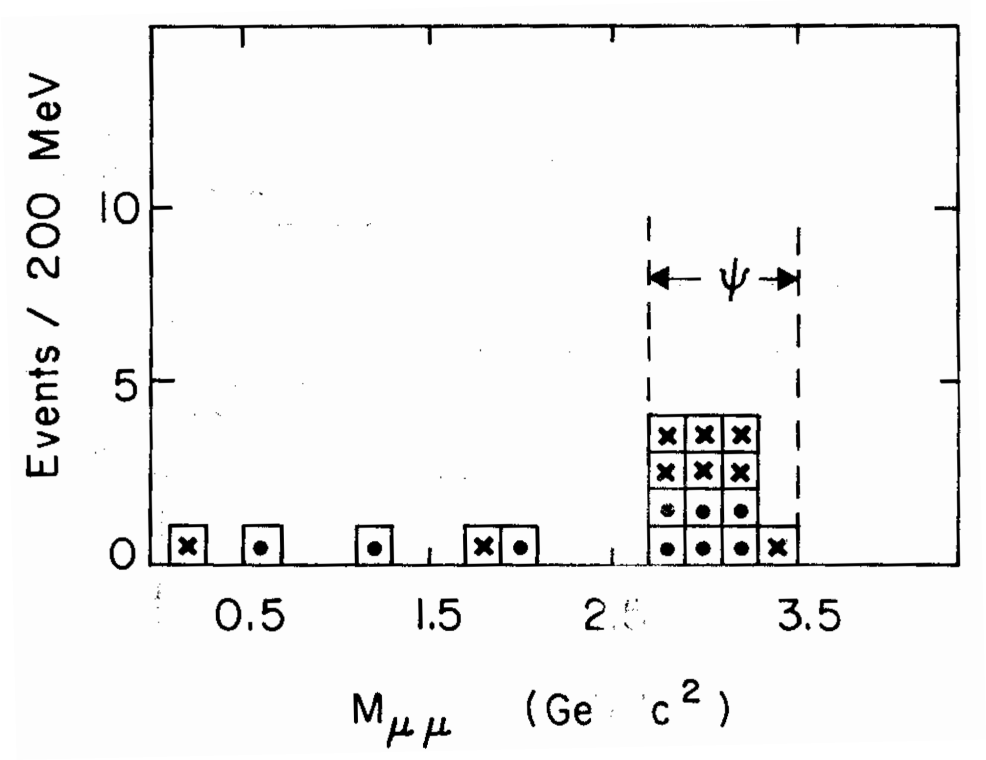
Introduction

- Why is charmonia production unclear?
  - First attempt to explain prompt J/ψ production at Tevatron on the basis of Color-Singlet (CS) model led to ~order of magnitude discrepancy
  - Color-Octet (CO) model allowed to improve the description and lead to the prediction that J/ψ are transversely-polarized
  - Subsequent experimental studies didn't confirm transverse polarization
  - Large NLO-corrections gave strong enhancement to the CSM contribution and allowed the longitudinal polarization of J/ψ
- Additional studies are needed to clarify the interplay between CS and CO models
- Why is double charmonium production interesting?
  - New test of CS and CO approaches in quarkonium production
  - Is there new popular phenomenon - DPS contributing double charmonium production?

First Experiment

- First observation of *J*/ψ pair production in hadronic experiment: by NA3, Phys.Lett. B114 (1982) 457, 150 and 280 GeV *πp*-collisions

- 13 events observed, $\sigma(J/\psi J/\psi) / \sigma(J/\psi) \sim 3 \cdot 10^{-4}$



New Experiment

- We already have a measured number:

$$\sigma(2 \times J/\psi)^{\rm LHCb}_{\rm exp.} = 5.1 \pm 1.0_{stat.} \pm 1.1_{syst.}\ {\rm nb}$$

  - $2 < y^{J/\psi} < 4.5$ and $p_T^{J/\psi} < 10$ GeV.
  - No cut on minimal p_T – great advantage of LHCb as p_T-distributions are most model-dependent in all theoretical predictions

- What phenomena do contribute to this cross section?

Single parton scattering

- First, there must be a contribution from single parton scattering (SPS) processes, mainly gluon fusion.
- The LO prediction was obtained:

$$\sigma(2\times J/\psi)^{\mathrm{LHCb}}_{\mathrm{SPS}} = 4.1 \pm 1.2\ \mathrm{nb}$$

  - This accounts CS contribution only, which dominates in fiducial region.
  - Theoretical uncertainties are due to the scale dependence mainly. Remember, it is an $\alpha(\mu)^4$-process.
- $q\bar{q} \to J/\psi\, J/\psi$ — V.G. Kartvelishvili and Sh.M. Esakiya. Yad.Fiz., 38:722–726, 1983.
  $gg \to J/\psi\, J/\psi$ — B. Humpert and P. Mery. Z.Phys., C20:83, 1983.

Double parton scattering

- It is also double parton scattering (DPS), which can contribute to the *J*/ψ pair production.
- The idea is simple: relatively large charm production cross section at LHC leads to occasional production of two charm particles in one *pp*-collision but in two partonic (gluon fusion mainly) interactions.
- The model is also simple (Phys.Rev., D56:3811–3832, 1997; Phys.Rev., D81:052012, 2010)

$$\sigma_{DPS}^{AB} = \frac{i}{2}\frac{\sigma_A\sigma_B}{\sigma_{eff}}, \sigma_{eff} \approx 15\text{mb}$$

and leads to

$$\sigma(2\times J/\psi)_{\text{DPS}}^{\text{LHCb}} \approx 4\ \text{nb}$$

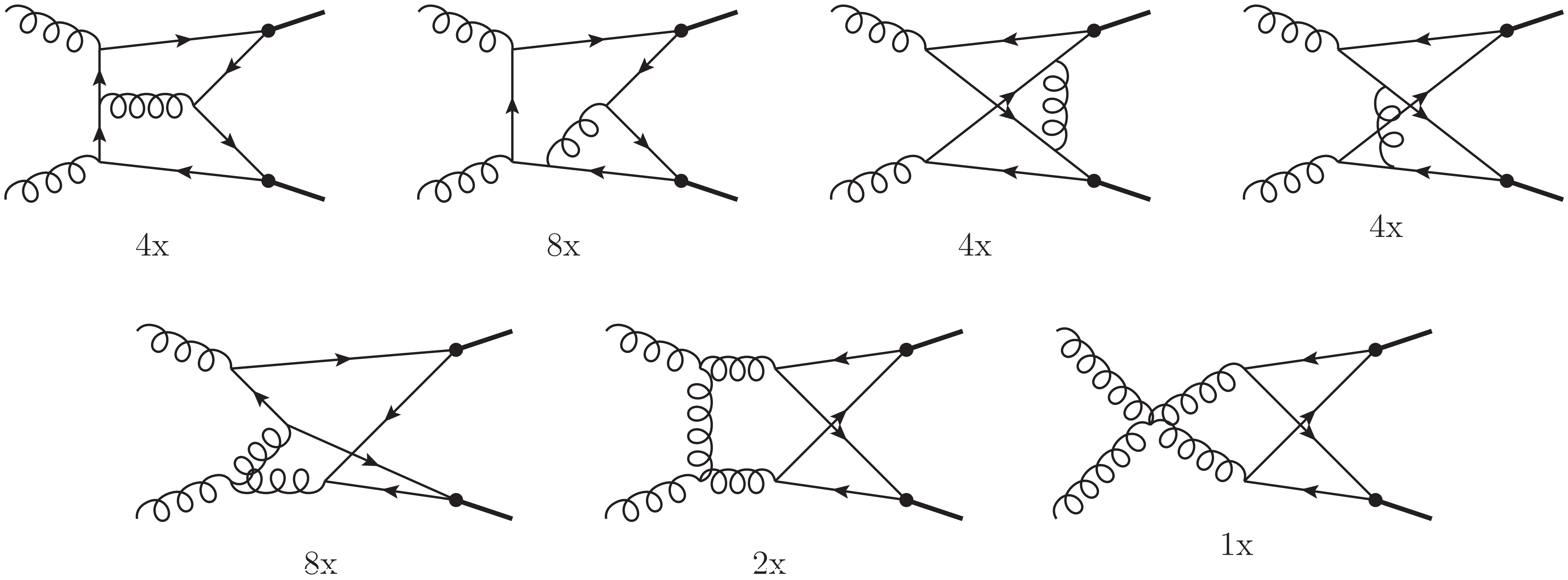
  - The uncertainties are not well known

What interplay is there between SPS an DPS?

- The question is if
  - $\text{Exp}_{(5.1\text{nb})} = \text{SPS}_{(4.1\text{nb})}$ or
  - $\text{Exp}_{(5.1\text{nb})} = \text{SPS}_{(4.1\text{nb})} + \text{DPS}_{(4\text{nb})}$ or
  - $\text{Exp}_{(5.1\text{nb})} = \text{SPS}_{(4.1\text{nb})}$ + NLO corrections, parameter variance, etc?
- Theoretical and experimental uncertainties allow all these possibilities.
- What signatures are expected for these contributions?

LO CS model

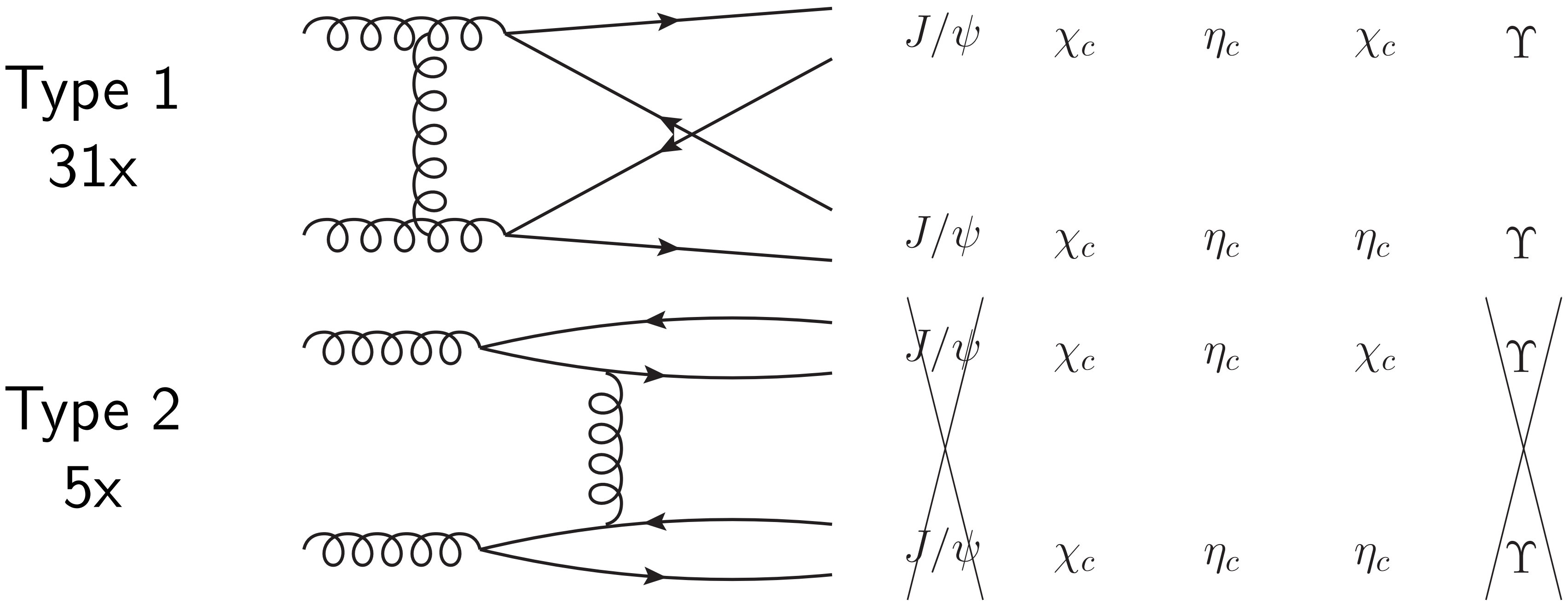
- Formation of two $c\bar{c}$ pair in color-singlet (1C) configuration with definite quantum numbers (1^{--} for J/ψ).
- Restriction on quantum numbers of quarkonia produced: CS selection rules.



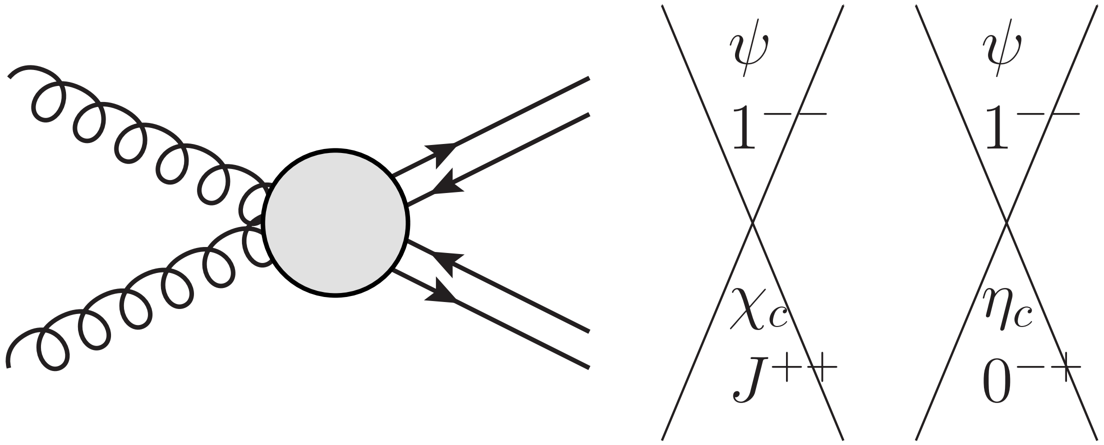
All types of LO diagrams for $gg \to J/\psi\, J/\psi$

LO CS model – selection rules

- There are 36 Feynman diagrams of 2 types:

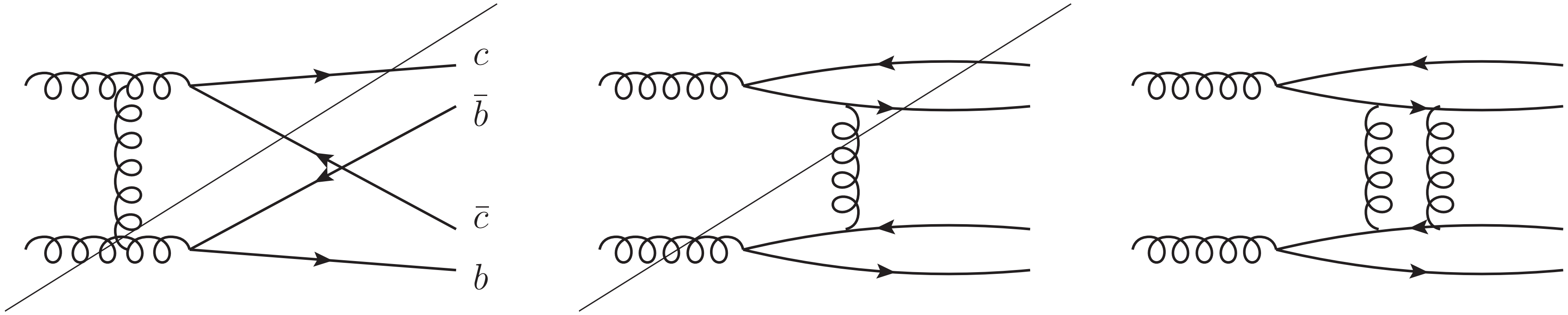


- The C-parity conservation forbids $\psi\chi_c$ and $\psi\eta_c$ exclusive production in gluon fusion:



LO CS model – selection rules

- There are no LO diagrams contributing CS *J*/ψϒ production:



- This leads to “unnatural” ordering of the cross section values:

$$\sigma_{\rm SPS}^{J\psi J\psi} > \sigma_{\rm SPS}^{\Upsilon\Upsilon} > \sigma_{\rm SPS}^{J/\psi\Upsilon}$$

- For DPS (or NLO SPS) an “expected” ordering take place:

$$\sigma_{\rm DPS}^{J\psi J\psi} > \sigma_{\rm DPS}^{J/\psi\Upsilon} > \sigma_{\rm DPS}^{\Upsilon\Upsilon}$$

- Predicted numerical values for the quantities involved are:

$$\sigma_{\rm SPS}^{J\psi J\psi} = 4{\rm nb} \qquad \sigma_{\rm SPS}^{\Upsilon\Upsilon} = 10{\rm pb} \qquad \sigma_{{\rm SPS}\ (8_{\rm C})}^{J/\psi\Upsilon} = 3{\rm pb}$$

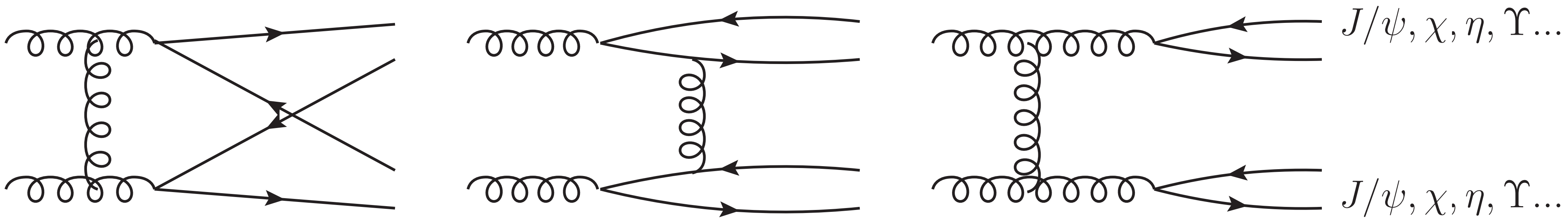
$$\sigma_{\rm DPS}^{J\psi J\psi} = 4{\rm nb} \qquad \sigma_{\rm DPS}^{\Upsilon\Upsilon} = 1{\rm pb} \qquad \sigma_{\rm DPS}^{J/\psi\Upsilon} = 100{\rm pb}$$

All these values are for LHCb acceptance.

Color-octet *J*/ψϒ prediction from arXiv:1007.3095 [hep-ph] (P. Ko, Jungil Lee and Chaehyun Yua)

LO CS vs. CO model

- CO model allow to produce whatever quarkonia in whatever combination in whatever diagram:



Type 1 ($\sim 1/\hat{p}_T^8$) Type 2 ($\sim 1/\hat{p}_T^6$) Type 3 ($\sim 1/\hat{p}_T^4$) $\leftarrow d\hat{\sigma}/d\hat{p}_T^2$

- Small (~$10^{-2..-3}$) color-octet contribution to the quarkonia wave function
- But may dominate
  - In high-p_T region, especially because of gluon fragmentation (type 3)
  - For final states disallowed in the CS mechanism

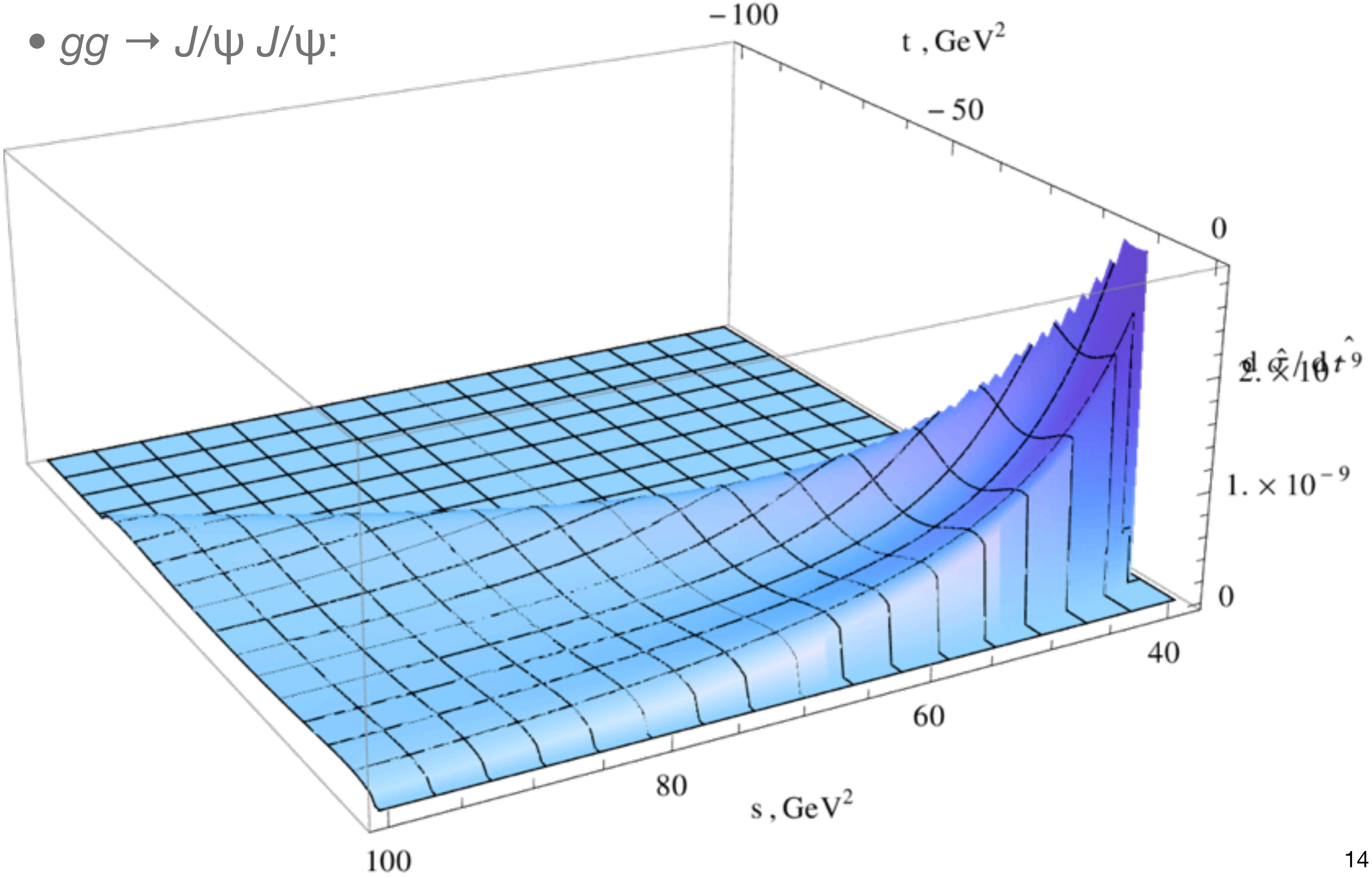
LO CS model – Matrix Element

- δ-approximation: $p_c = p_{\bar{c}} = p_{J/\psi}/2$. Also $m_c = m_{J/\psi}/2$.
- For $gg \to J/\psi\, J/\psi$:

$$
\begin{aligned}
|M|^2 \quad = \quad & \frac{8\pi^4\alpha_s^4\psi(0)^4}{81m^2s^6\left(m^2-t\right)^4\left(m^2-u\right)^4}\left(335s^{12}+274s^{11}(t+u)+\right.\\
+ \quad & s^{10}\left(-3393t^2+6782tu-3393u^2\right)+\\
+ \quad & s^9\left(-7124t^3+7204t^2u+7204tu^2-7124u^3\right)+\\
+ \quad & s^8\left(3290t^4-24440t^3u+42684t^2u^2-24440tu^3+3290u^4\right)+\\
+ \quad & 4s^7(t-u)^2\left(5459t^3-4631t^2u-4631tu^2+5459u^3\right)+\\
+ \quad & 2s^6(t-u)^2\left(7771t^4-1016t^3u-13126t^2u^2-1016tu^3+7771u^4\right)-\\
- \quad & 16s^5(t-u)^4\left(778t^3-821t^2u-821tu^2+778u^3\right)-\\
- \quad & s^4(t-u)^4\left(20405t^4-18628t^3u-3938t^2u^2-18628tu^3+20405u^4\right)-\\
- \quad & 2s^3(t-u)^6\left(2647t^3-1375t^2u-1375tu^2+2647u^3\right)+\\
+ \quad & 21s^2(t-u)^8\left(223t^2+398tu+223u^2\right)+\\
+ \quad & \left.3780s(t-u)^{10}(t+u)+972(t-u)^{12}\right)
\end{aligned}
$$

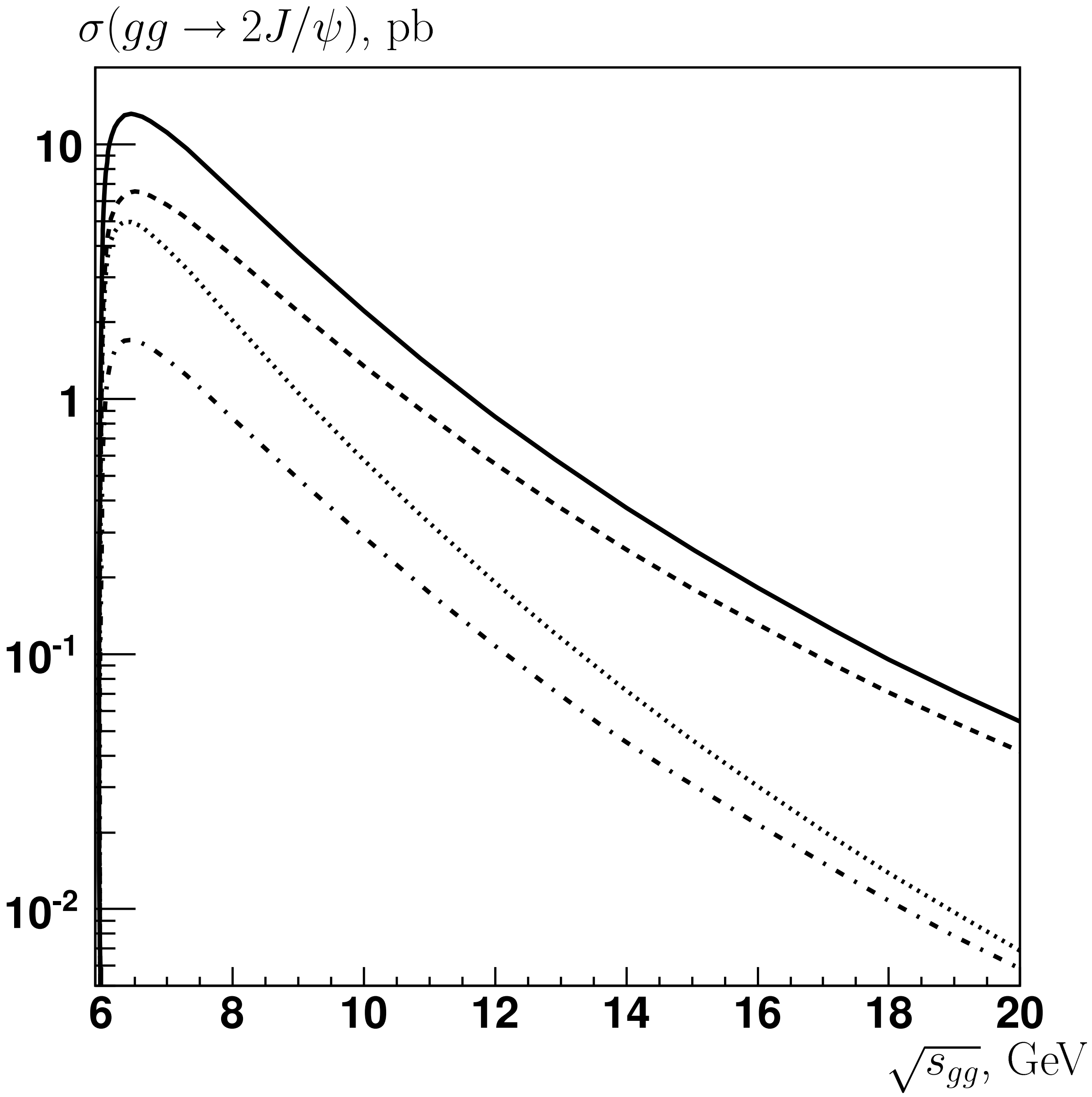
LO CS model – Matrix Element

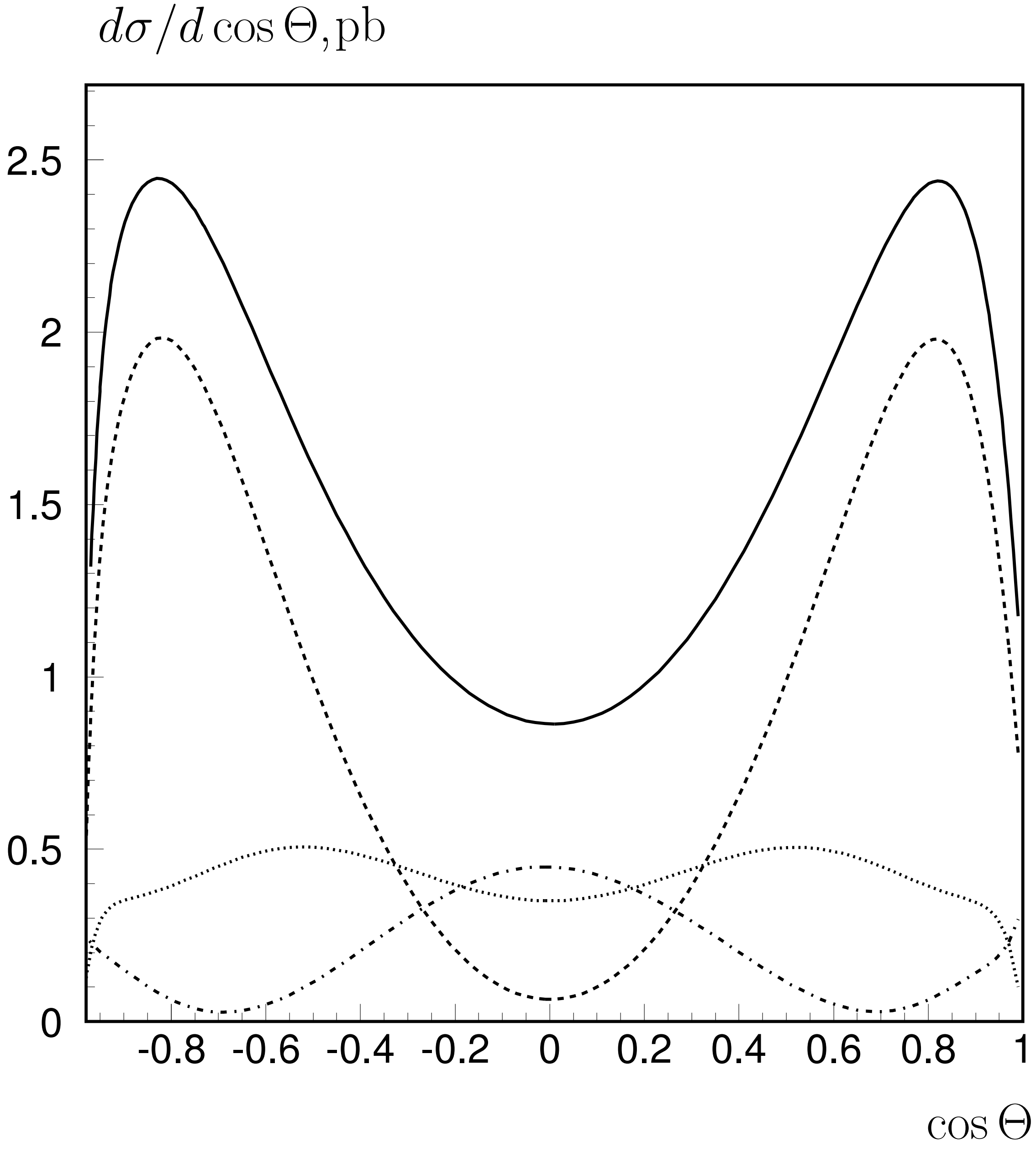
- *gg* → *J*/ψ *J*/ψ:



LO CS model – Polarization

- Solid, dashed, dotted and dot-dashed curves correspond to the total cross section, TT , TL and LL polarizations, respectively.
- Angular distribution is for $\sqrt{s_{gg}}$ = 10 GeV

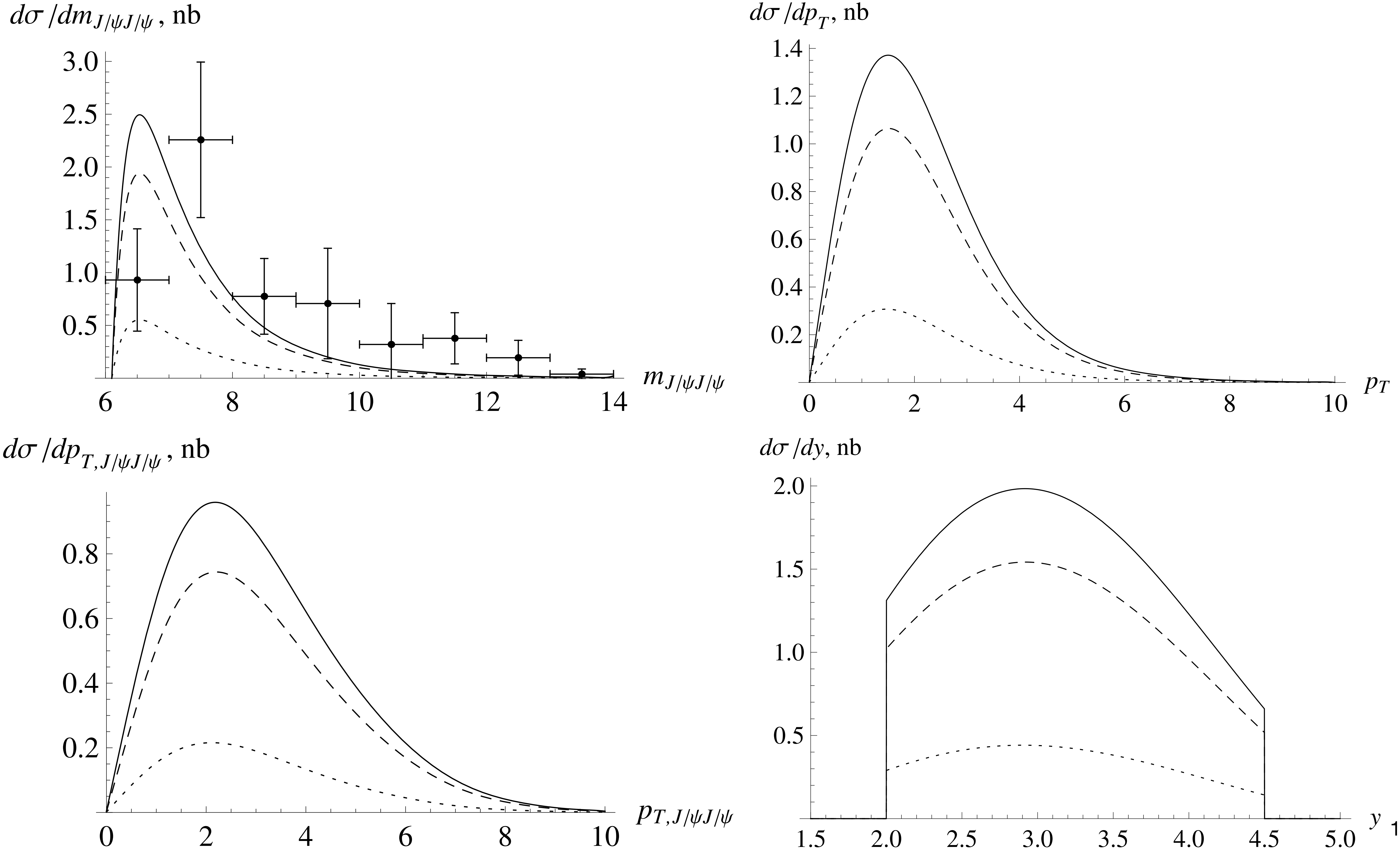




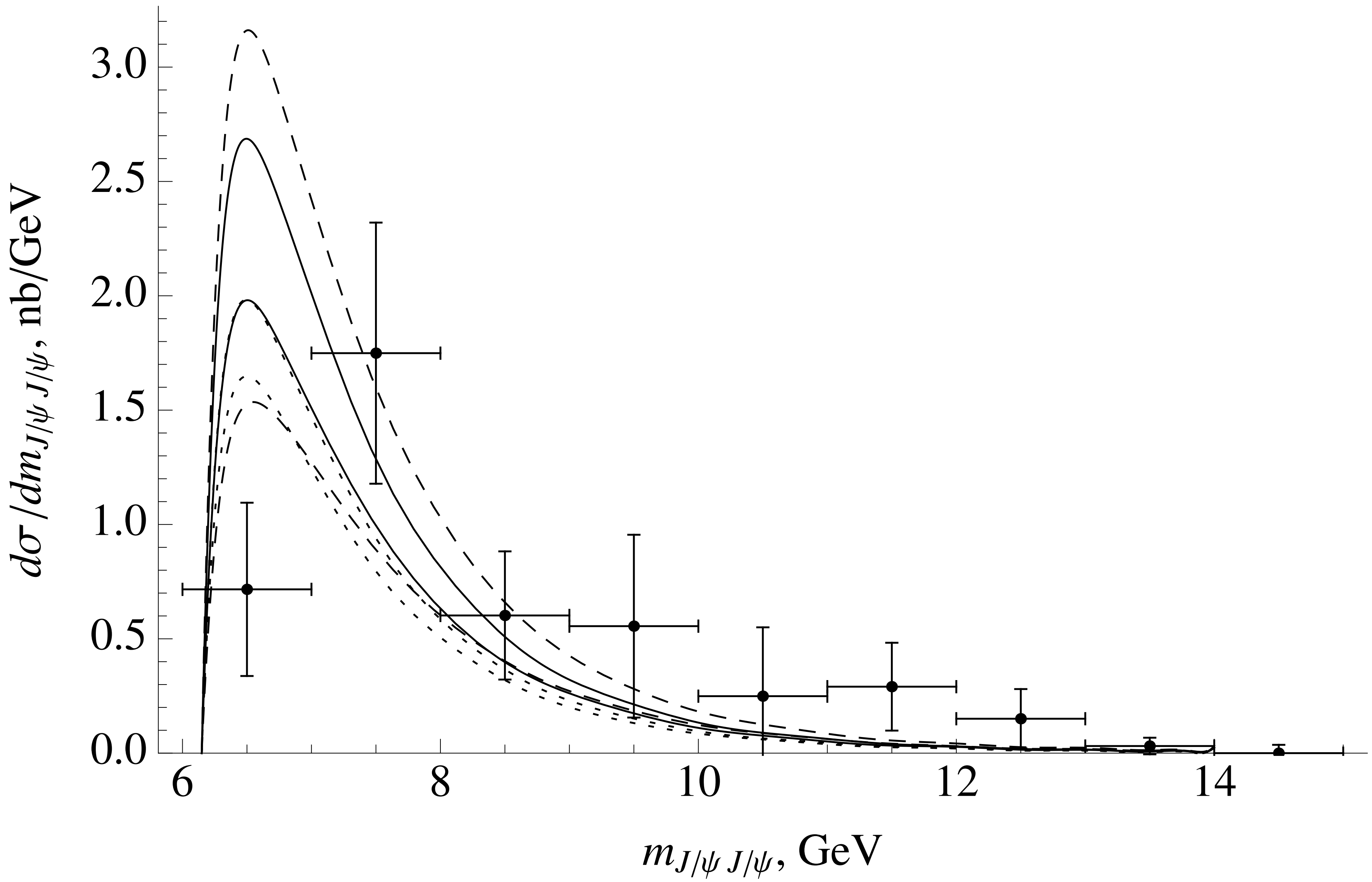
LO CS model – MC calculation for LHC, LHCb

- Calculation for LHC conditions – implemented as an external process for Pythia 6.4. Works standalone on within LHCb software. We also have a working code for Pythia 8.
- With ordinary collinear PDFs one can obtain realistic cross section for such experiment as LHCb (with no cut on minimal p_T).
- But the p_T-distribution of final particles (like $d\sigma/d(p_T^2) \sim p_T^{-8}$) has nothing in common with reality if the k_T of initial gluons is neglected.
- So Pythia is also used to give initial k_T to the gluons. This adds additional model-dependence to distributions over the p_T of single J/ψ and J/ψ pairs. But no significant influence on invariant mass and rapidity spectra.
- We also use Pythia (or EvtGen) to treat with $\psi(2S)$ decays to $J/\psi + X$. $J/\psi + \psi(2S)$ and $2\times\psi(2S)$ is simulated analogously to $2\times J/\psi$.
- A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, A.A.N., Phys.Rev, **D**84 (2011), 094023, arXiv:1101.5881
- Alternative – k_T-factorization, S.P. Baranov, Phys.Rev. D84 (2011) 054012, consistent results.

LO CS model – MC calculation for LHC, LHCb



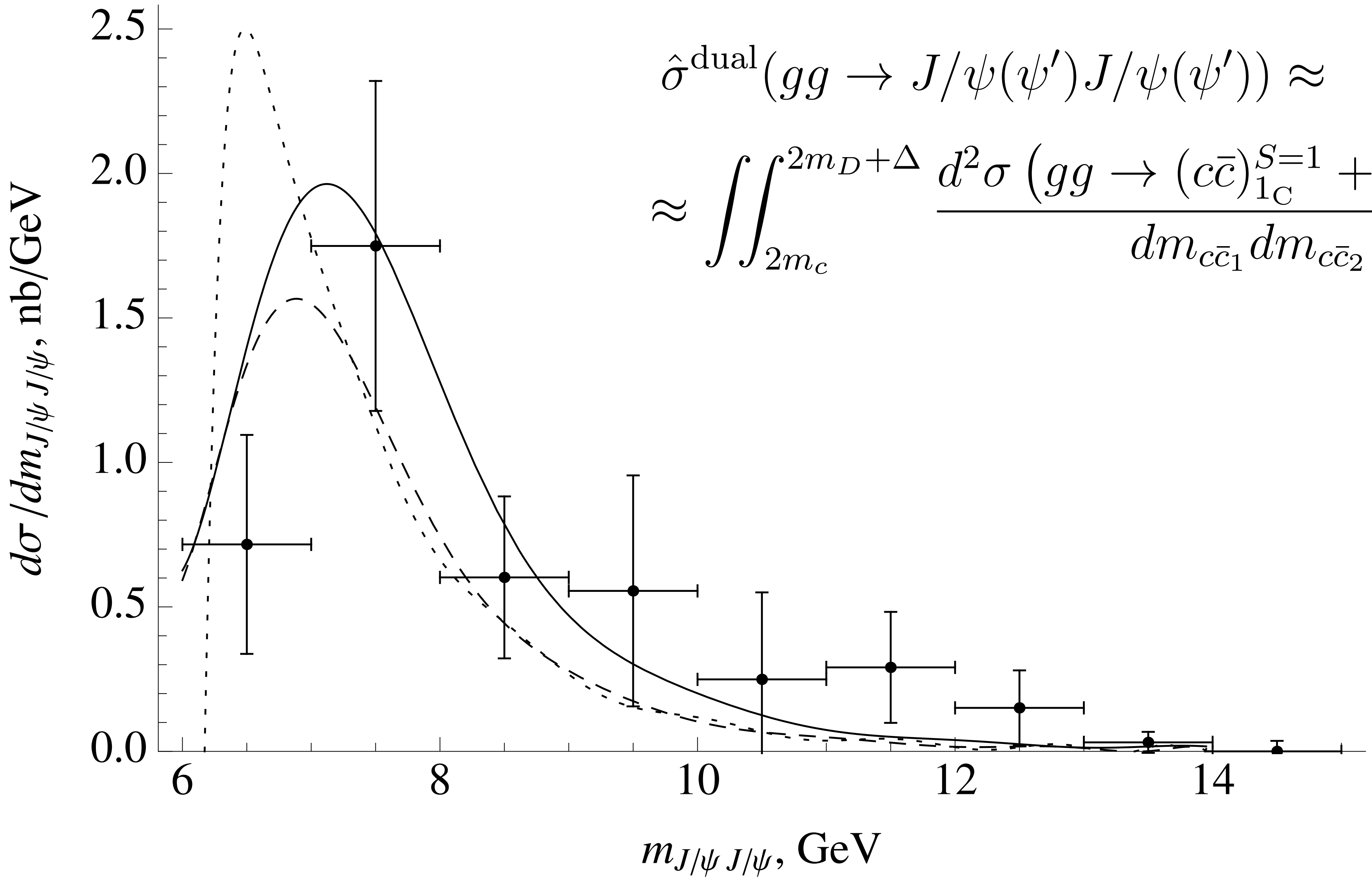
LO CS model – MC calculation for LHC, LHCb



Solid curves were obtained with $m_T^{J/\psi}$ as the hard scale, dashed — $0.5 \cdot m_T^{J/\psi}$ and dotted — $2 \cdot m_T^{J/\psi}$. For every scale choice upper curve corresponds to the CTEQ5L, lower — to the CTEQ6LL *pdf* used.

LO CS model – relative quark motion

- Distribution over the invariant mass of the *J*/ψ-meson pairs in the “duality” approach compared with the LHCb measurement. Solid curve was obtained with Δ = 0.5 GeV, dashed — with Δ = 0.3 GeV and dotted — in the δ-approximation.

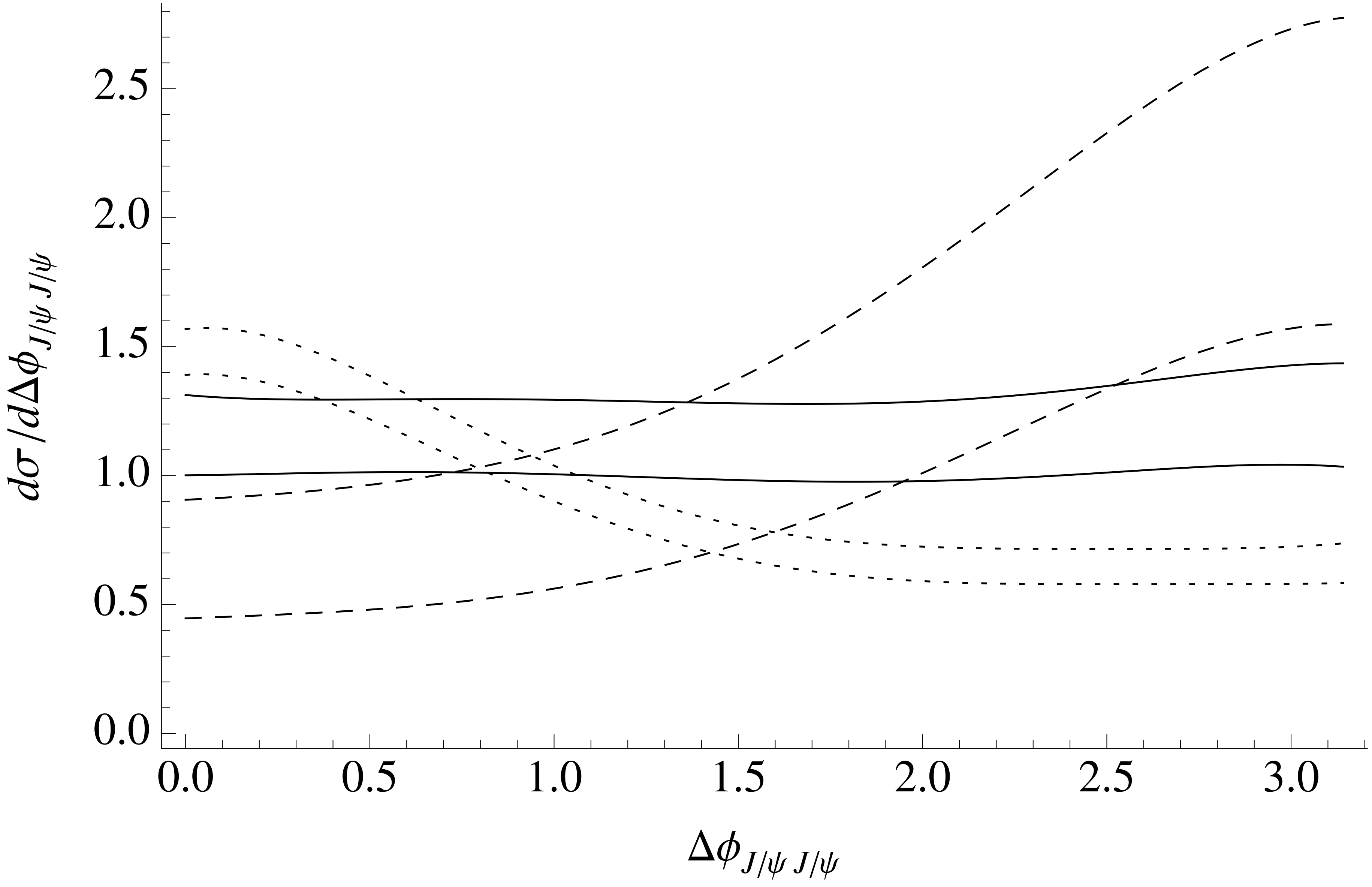


$$\hat{\sigma}^{\text{dual}}(gg \to J/\psi(\psi')J/\psi(\psi')) \approx$$

$$\approx \iint_{2m_c}^{2m_D+\Delta} \frac{d^2\sigma\left(gg \to (c\bar{c})_{1_\mathrm{C}}^{S=1} + (c\bar{c})_{1_\mathrm{C}}^{S=1}\right)}{dm_{c\bar{c}_1}\,dm_{c\bar{c}_2}}\,dm_{c\bar{c}_1}\,dm_{c\bar{c}_2}$$

LO CS model – signatures in LHCb

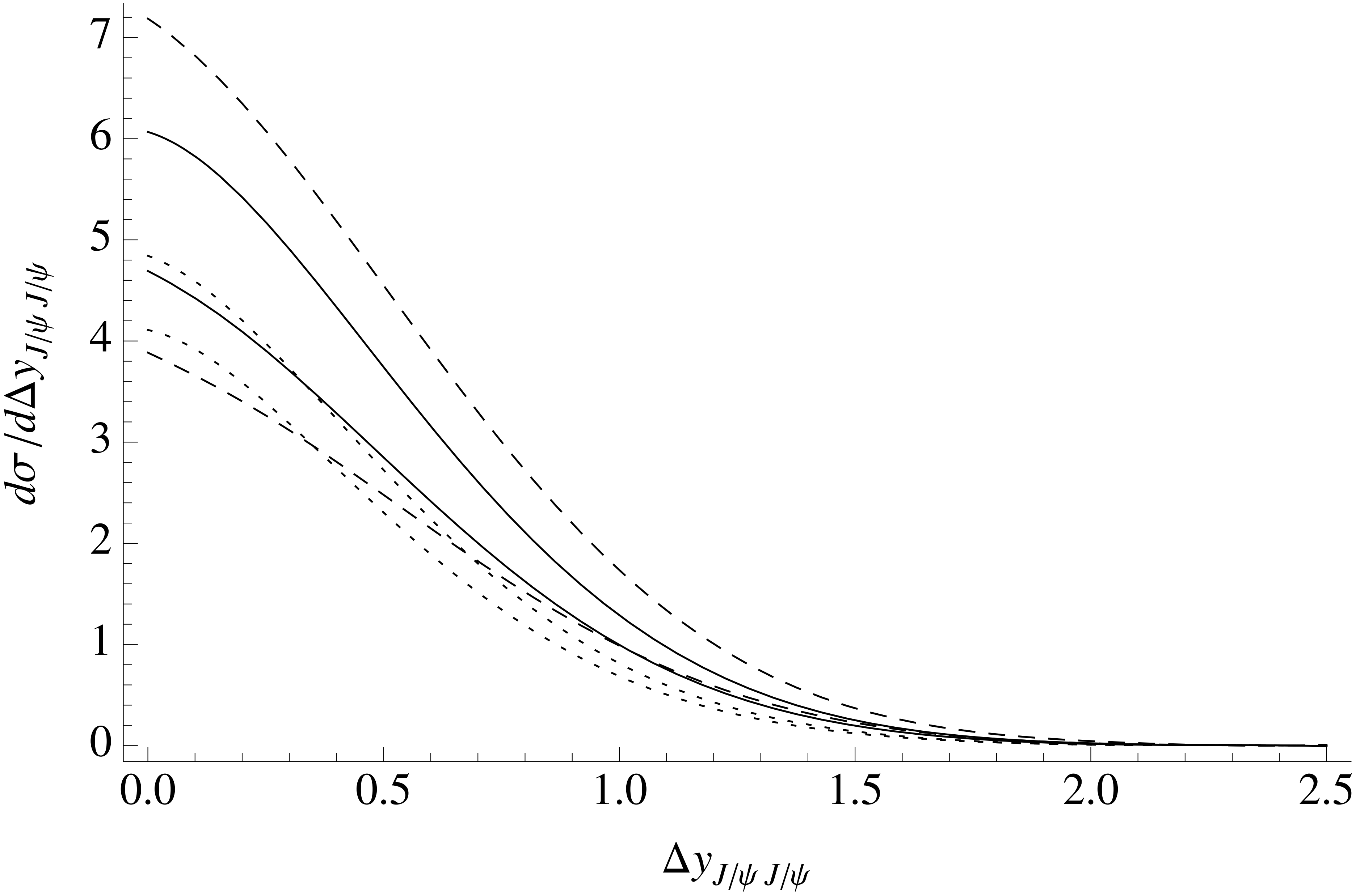
- (Absence of) *J*/ψ azimuthal angle correlation



- Same conclusion in the work by the k_T-factorization adepts
  S.P. Baranov, A.M. Snigirev, N.P. Zotov, A. Szczurek, W. Schäfer, arXiv:1210.1806

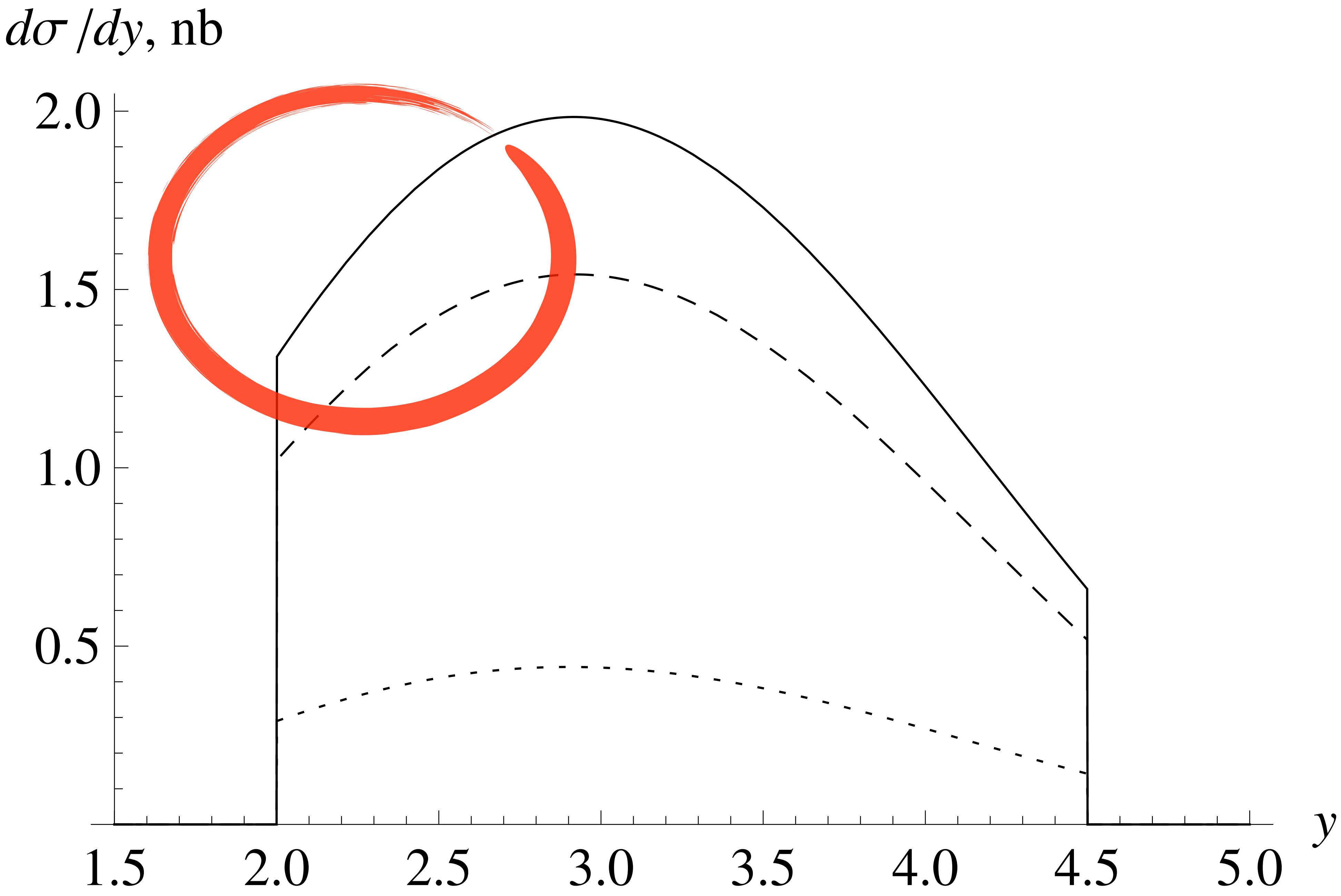
LO CS model – signatures in LHCb

- Correlation in rapidity – also visible in distribution over *y* of single *J*/ψ from a pair.



LO CS model – signatures in LHCb

- Just to recall:



Turn to DPS now

- For double *J*/ψ production hadronic cross section is 10^3 larger than partonic one ⇒ enhancement due to the high low-*x* gluon luminosity.
- Double parton scattering:

$$d\sigma^{AB}_{\mathrm{DPS}} = \frac{i}{2\sigma_{eff}} F_{gg}(x_1, x_2, Q_1^2, Q_2^2) F_{gg}(x_1', x_2', Q_1^2, Q_2^2) d\hat{\sigma}^A(x_1, x_1', Q_1^2) d\hat{\sigma}^B(x_2, x_2', Q_2^2)$$

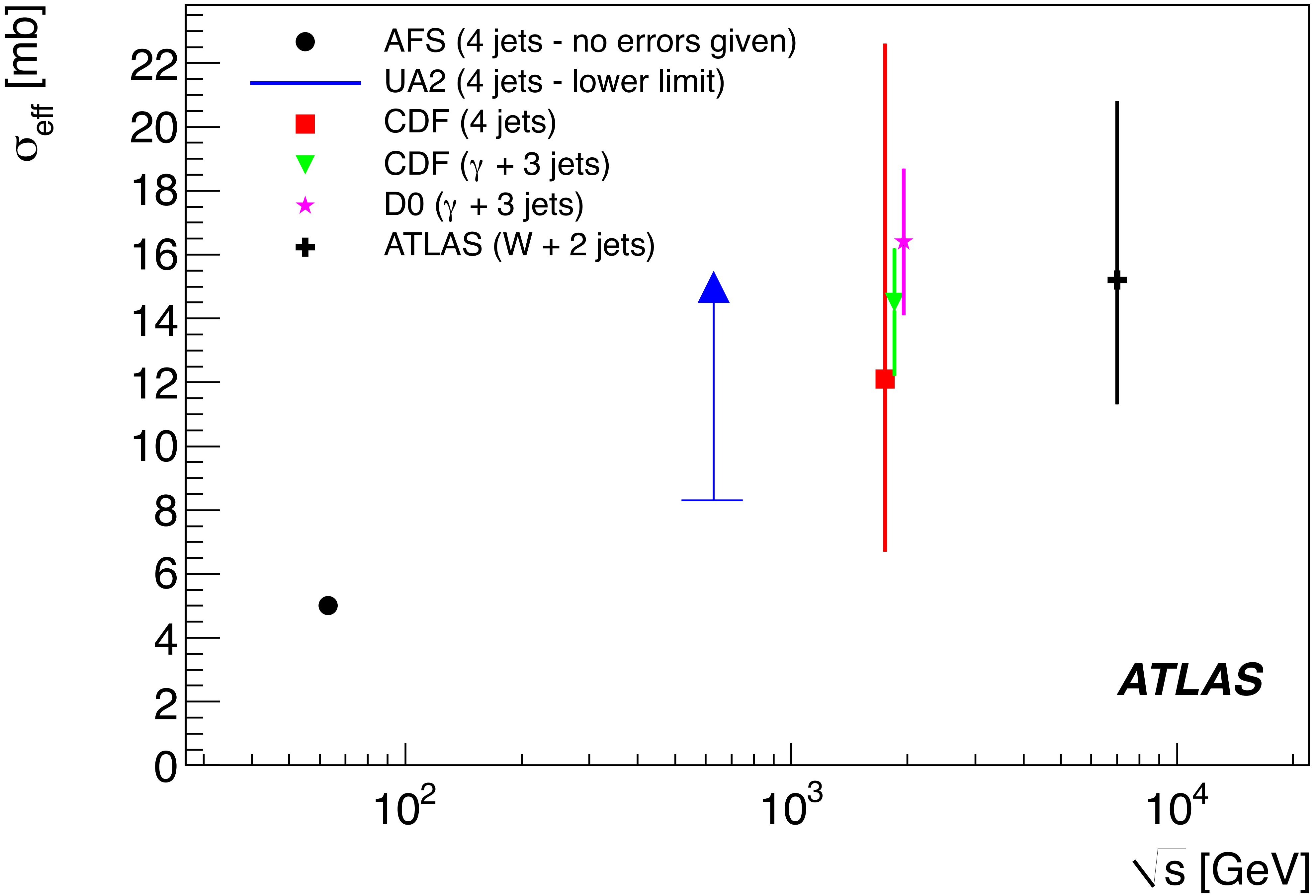
  - here *i* = 1 for identical subprocesses *A* and *B*, *i* = 2 – for different.
- $F_{gg}(x_1, x_2, Q_1^2, Q_2^2)$ are (unknown) generalized *pdf*s.
- Cross sections can be estimated by a simple expression:

$$\sigma^{AB}_{DPS} = \frac{i}{2}\frac{\sigma_A \sigma_B}{\sigma_{eff}}, \sigma_{eff} \approx 15\mathrm{mb}$$

- Phys.Rev., D56:3811–3832, 1997; Phys.Rev., D81:052012, 2010

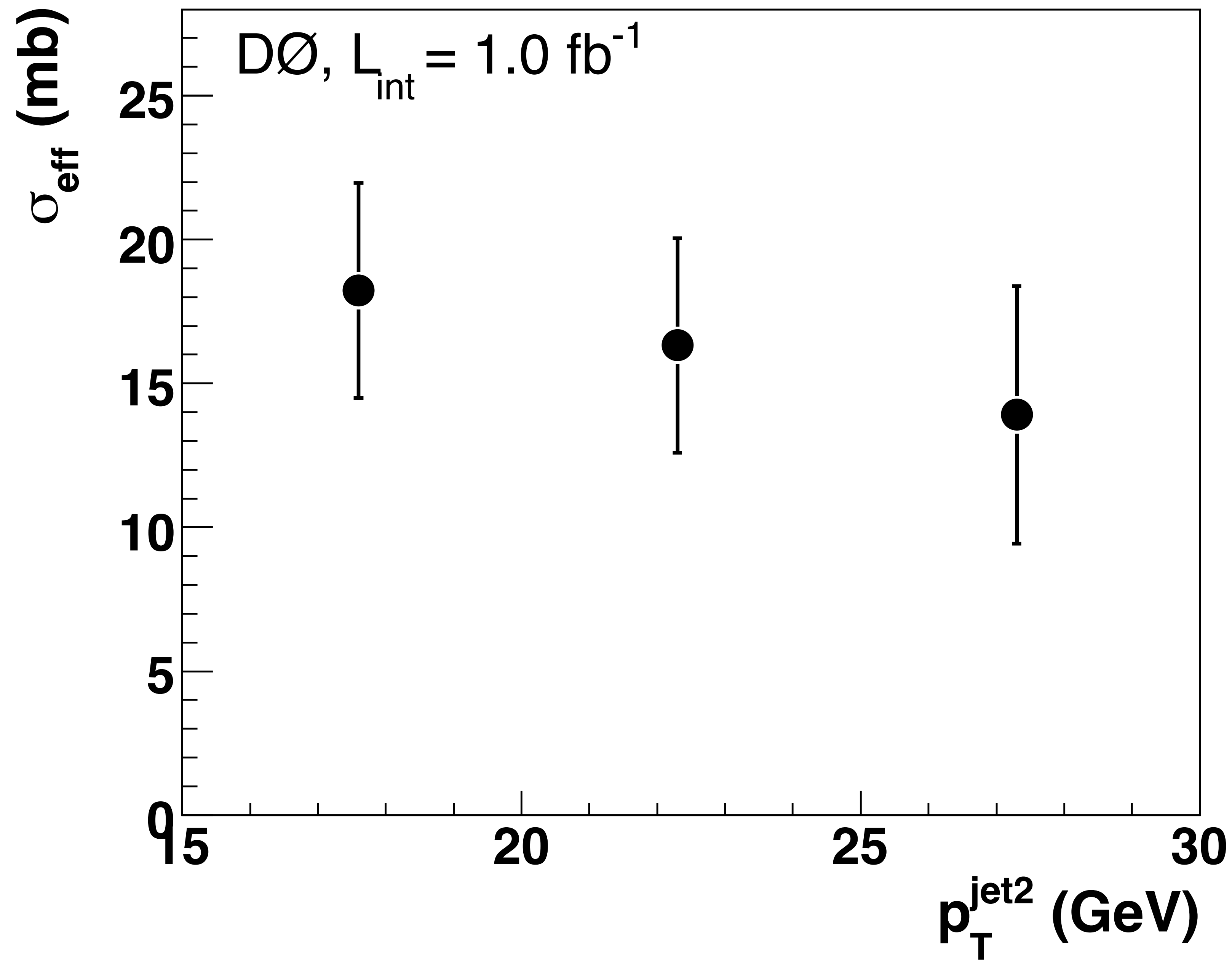
$\sigma_{\text{eff.}}$ measurements

- ATLAS Collaboration, CERN-PH-EP-2012-355, arXiv:1301.6872 [hep-ex]



$\sigma_{\text{eff.}}$ measurements

- D0 Collaboration, Phys.Rev.D81:052012,2010, arXiv:0912.5104 [hep-ex]



4*c*-tetraquarks

- Mesons built from two quarks and two antiquarks, $T = [q_1 q_2][\bar{q}_3 \bar{q}_4]$.
- If one assumes it is a bound state of two almost pointlike diquarks, non-relativistic quantum mechanics can be used to calculate these states.
- Color structure:

$$(Q_1 Q_2 \bar{Q}_3 \bar{Q}_4) = [Q_1 Q_2]_{\bar{3}_c}[\bar{Q}_3 \bar{Q}_4]_{3c} + [Q_1 Q_2]_{6c}[\bar{Q}_3 \bar{Q}_4]_{6c}$$

  - $3_c \otimes \bar{3}_c - \mathrm{attraction}$
  - $6_c \otimes 6_c - \mathrm{repulsion}$
  - Kunihiko Terasaki, Prog.Theor.Phys.125:199-204,2011
- Schrodinger equations:
  - $\left[-\frac{1}{2m_{\boldsymbol{Q}}}\nabla^2 + \frac{1}{2}V(r)\right]\Psi_{diq}(r) = \delta E_{diq}\Psi_{diq}(r)$
  - $\left[-\frac{1}{2m_{\boldsymbol{diq}}}\nabla^2 + V(r)\right]\Psi_T(r) = \delta E_T \Psi_T(r)$

4*c*-tetraquarks

- Tetraquark states can be characterized by total and diquark spins

$$\left|J; S_{1,2}, S_{3,4}\right\rangle$$

- or total spin and P-, C -parities

$$\left|J^{PC}\right\rangle$$

- $$\begin{aligned}
&|0^{++}\rangle = |0;0,0\rangle \\
&|0^{++}‘\rangle = |0;1,1\rangle \\
&|1^{+\pm}\rangle = (|1;0,1\rangle \pm |1;1,0\rangle)/\sqrt{2} \\
&|1^{+-}‘\rangle = |1;1,1\rangle \\
&|2^{++}\rangle = |2;1,1\rangle
\end{aligned}$$

4*c*-tetraquarks

- Spin-spin interaction Hamiltonian

$$H = M_0 + 2\sum_{i<j} \kappa_{ij} \left(\mathbf{S}_i \mathbf{S}_j\right)$$

- κ_{ij} can be determined from wave function at origin R(0)
    - $[QQ]$

$$\kappa_{ij} = \frac{1}{2}\frac{4}{9 m_{Q_1} m_{Q_2}} \alpha_s \left|R_{[ij]}(0)\right|^2$$

$R_{[QQ]}(0)$ from Schroedinger equation

-
    - $(Q\overline{Q})$

$$\kappa_{ij} = \frac{1}{2}\frac{8}{9 m_{Q_1} m_{Q_2}} \alpha_s \left|R_{(ij)}(0)\right|^2$$

$R_{(Q\bar{Q})}(0)$ from $\Gamma(V \to ee)$

4*c*-tetraquarks

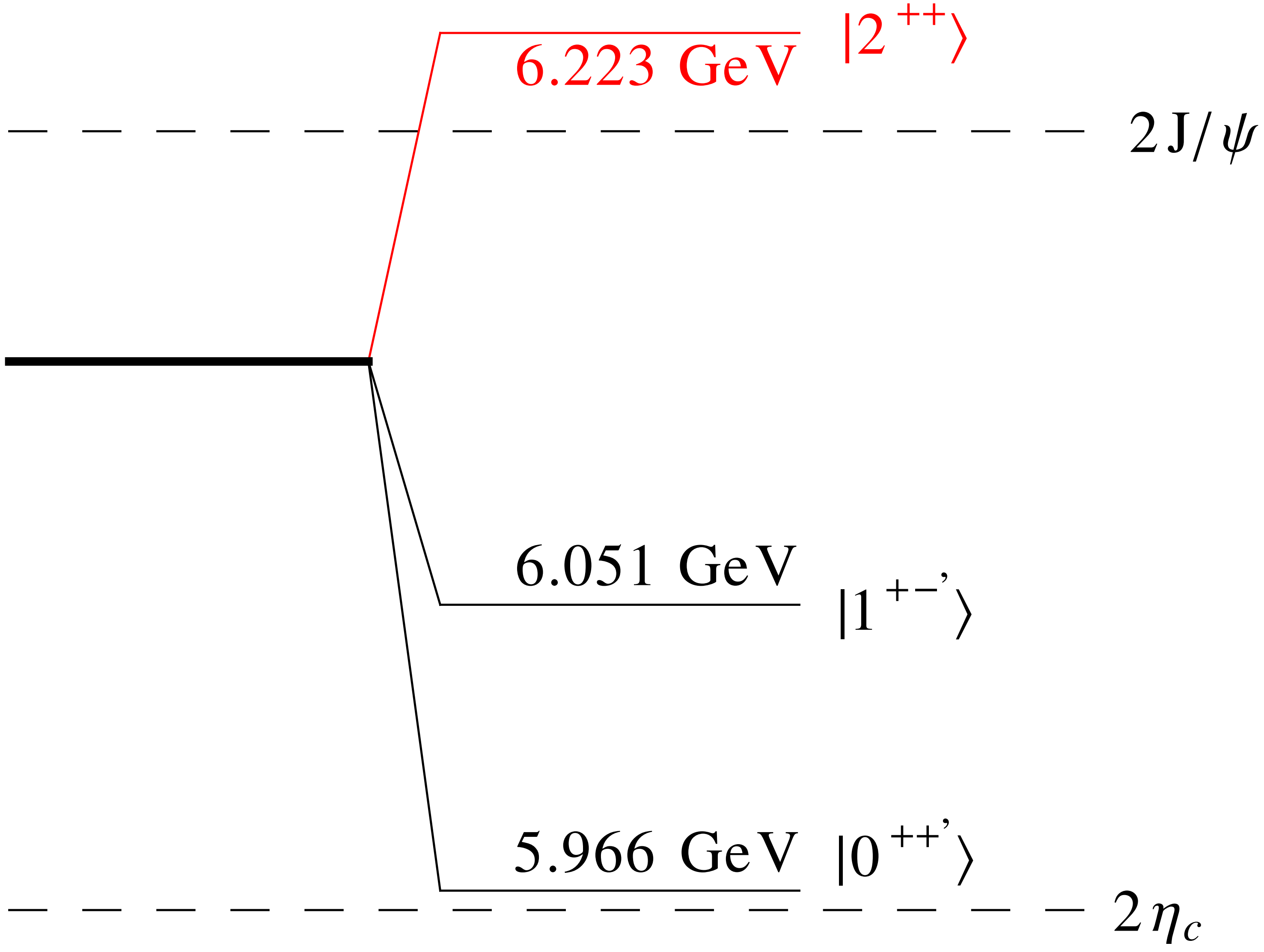
- In the case of identical quarks only $S_{diq} = 1$ is possible and

$$M(0^{++\prime}) = M_0 + \kappa_{12} - 2\kappa_+$$

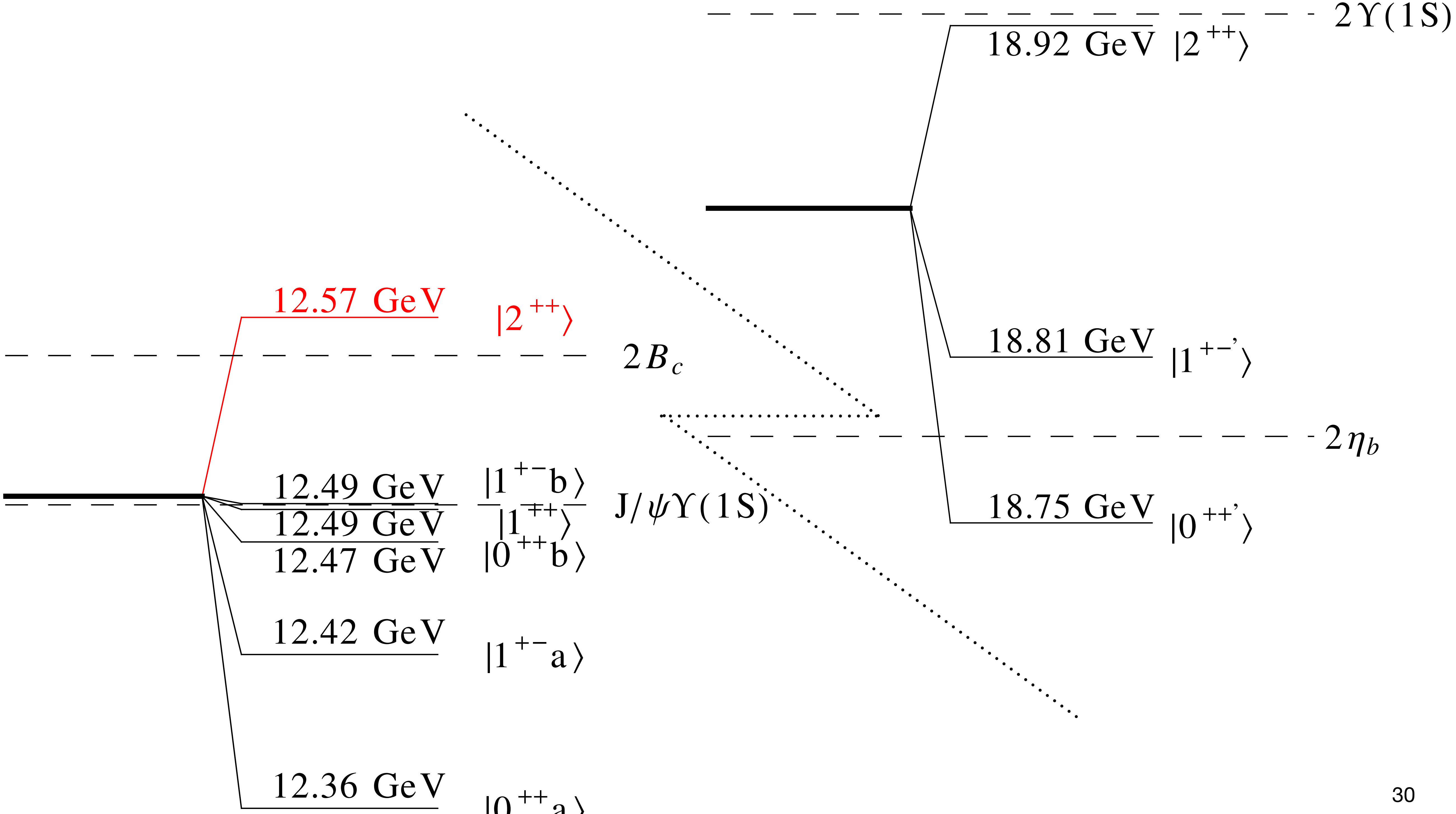
$$M(1^{+-\prime}) = M_0 + \kappa_{12} - \kappa_+$$

$$M(2^{++}) = M_0 + \kappa_{12} + \kappa_+$$

where $\kappa_+ = 2\kappa_{(Q\bar{Q})}$



2[*bc*]- and 4*b*-tetraquarks



4*c*-tetraquarks – production

- $gg \to [QQ][\bar{Q}\bar{Q}]$
- Below 2×Ξ_{QQ} production threshold diquarks can hadronize into
  - Light mesons
  - Quarkonia pair
  - Tetraquark (with subsequent decay into 2 quarkonia)
- Duality relation:

$$S_T = \int\limits_{2M_{\mathcal{Q}}}^{2M_{\Xi_{QQ}}} dm_{gg}\hat{\sigma}[gg \to T \to 2\mathcal{Q}] = \epsilon \int\limits_{2M_{\mathcal{Q}}}^{2M_{\Xi_{QQ}}} dm_{gg}\hat{\sigma}(gg \to [Q_1 Q_2] + [\bar{Q}_3 \bar{Q}_4])$$

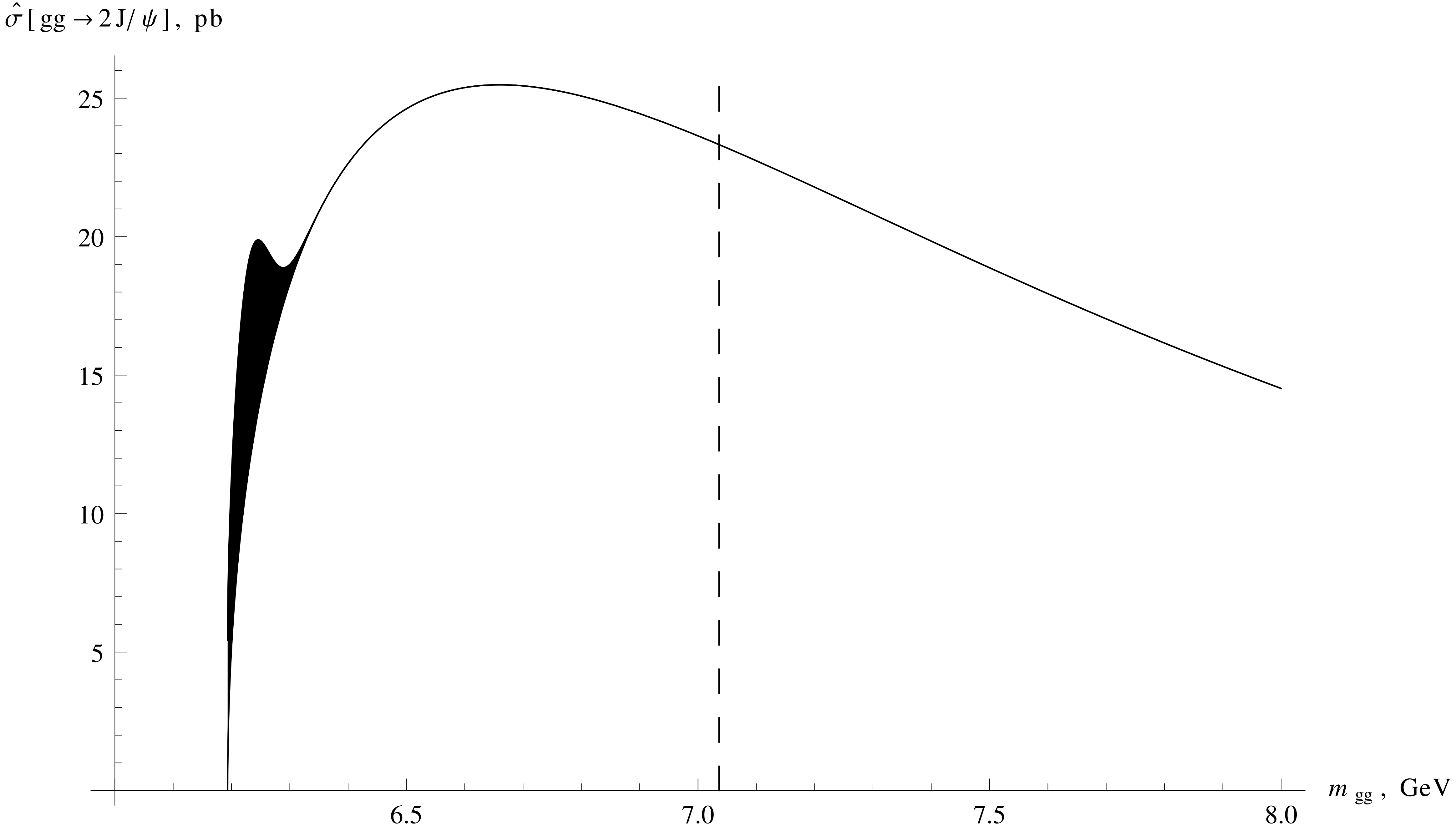
with $\varepsilon < 1$

- Width of tetraquarks are small compared to the detector resolution $\Delta \approx 50$MeV:

$$\hat{\sigma}(gg \to T \to 2\mathcal{Q}) = \frac{S_T}{\sqrt{\pi}\Delta} \exp\left\{ -\frac{(m_{gg} - M_T)^2}{\Delta^2} \right\}$$

$gg \to T_{4c} \to 2\times J/\psi$

- For instance, with $\varepsilon = 0.1$:



Conclusion

- The SPS CS LO calculation gives reliable predictions for the J/ψ pair production. As was already mentioned yesterday, no free parameters, but irreducible uncertainties due to the scale choice.
- Selection rules suppress some feeddown channels – less mess than in inclusive production. Uncertainties from feeddown not exceed those from the scale choice.
- DPS estimation leads to comparable cross-section. Though SPS has validatable signatures and even LHCb rapidity window (2–4.5) is sufficient for rapidity correlation studies.
- It is worth looking for other quarkonia pairs, like $\Upsilon\Upsilon$, $J/\psi\Upsilon$ or $J/\psi\chi_c$. First of them can be more accessible for ATLAS.
- Two J/ψ channel is interesting for looking for tetraquarks. Though, only upper limit on their production can be estimated.

Thank you for attention!

Thanks to LAL and organizers for this charming workshop :)