CHIRAL EXTRAPOLATION OF BARYON PROPERTIES

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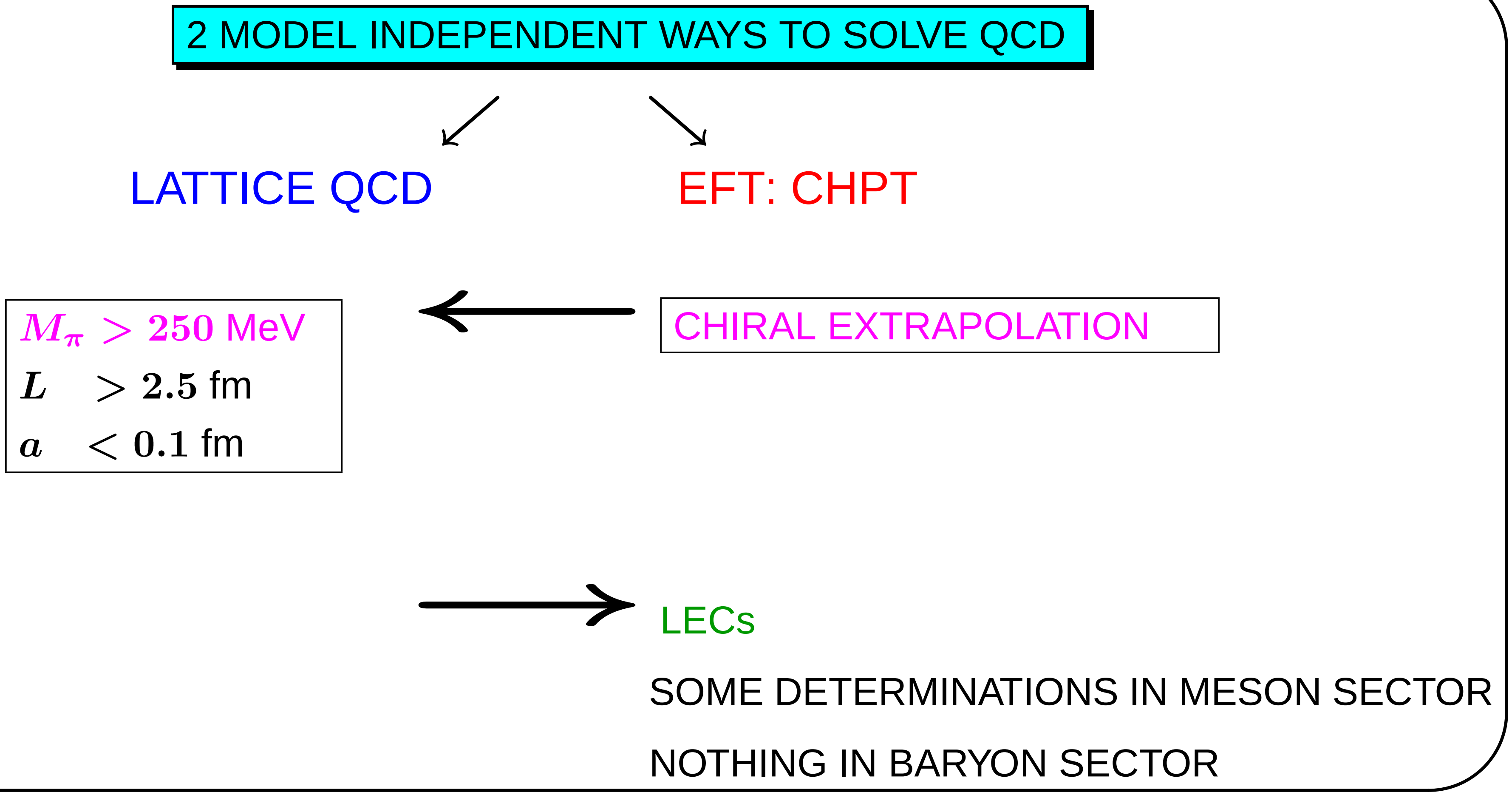
QCD AT LOW ENERGY

– NON PERTURBATIVE

– IMPORTANT PROPERTY: CHIRAL SYMMETRY in the limit of massless quarks



2 MODEL INDEPENDENT WAYS TO SOLVE QCD

LATTICE QCD

EFT: CHPT

$M_\pi > 250$ MeV
$L \quad > 2.5$ fm
$a \quad < 0.1$ fm

CHIRAL EXTRAPOLATION

LECs

SOME DETERMINATIONS IN MESON SECTOR

NOTHING IN BARYON SECTOR

SU(2)

- **CHPT**
  - Effective Lagrangian
  - Low energy constants
  - Regularizations
- **Magnetic moments**
- **Nucleon mass**
- **Δ mass**
  - Infinite volume
  - Finite volume
- **Axial vector coupling**

SU(3)

- **octet masses**

CONCLUSION

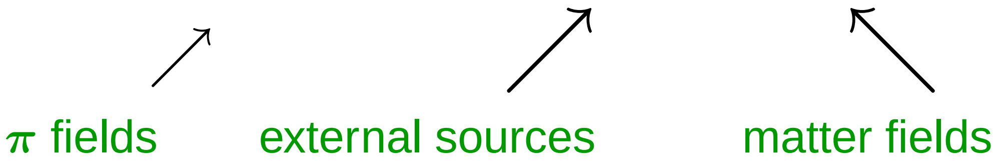
CHIRAL PERTURBATION THEORY

EFT OF THE STANDARD MODEL

PROPERTIES

- MOST GENERAL EFFECTIVE LAGRANGIAN in agreement with SYMMETRIES OF QCD

$$\mathcal{L}_{EFF}[U, \partial_\mu U, \cdots, \underbrace{\mathcal{M}, v_\mu, a_\mu,} \cdots, N]$$



π fields external sources matter fields

- EXPANSION
  - in external momenta p: interaction between GB is weak
  - in quark masses m_q: small compared to $\Lambda_\chi \simeq 1$ GeV

$$\mathcal{L}_{EFF} = \sum_i \mathcal{L}_{\pi N}^{(i)} + \sum_j \mathcal{L}_{\pi\pi}^{(j)}$$

i, j power in small parameter $q = \{p, m_q\}$

EFFECTIVE LAGRANGIAN AT TWO LOOP ORDER

$$\boxed{\begin{aligned}
&\mathcal{L}_{\text{eff}} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \mathcal{L}^{(5)}_{\pi N} + \mathcal{L}^{(6)}_{\pi N} + \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \cdots \\
&\mathcal{L}^{(1)}_{\pi N} = \bar{\psi}(i \not{D} - m_0 + \tfrac{1}{2} g_0 \not{u} \gamma_5)\psi \\
&\mathcal{L}^{(2)}_{\pi N} = \sum_{i=1}^{7} c_i \bar{\psi} \mathcal{O}_i^{(2)} \psi = \bar{\psi}_N \Big[c_1 \langle \chi_+ \rangle - c_2 \frac{1}{4m_0^2} \{ \langle u_\mu u_\nu \rangle D^\mu D^\nu + \text{h.c.} \} + c_3 \frac{1}{2} \langle u^2 \rangle \\
&+ c_4 \frac{i}{4} [u_\mu, u_\nu] \sigma^{\mu\nu} + \ldots \Big] \psi_N \\
&\mathcal{L}^{(3)}_{\pi N} = \sum_{i=1}^{23} d_i \bar{\psi} \mathcal{O}_i^{(3)} \psi \qquad \mathcal{L}^{(4)}_{\pi N} = \sum_{i=1}^{118} e_i \bar{\psi} \mathcal{O}_i^{(4)} \psi \quad , \qquad \ldots
\end{aligned}}$$

Here: include eom terms in $\mathcal{L}_{\text{eff}}$: Fettes, Meißner, Moijzis & Steininger: Ann. Phys. 283 (2000) 273

Eliminate eom terms in $\mathcal{L}_{\text{eff}}$: Ecker & Mojzis, Phys. Lett. B365 (1996) 312

$$\longrightarrow \quad \boxed{\text{LECs: } F, l_i; m_0, g_0, c_i, d_i, e_i}$$

χ limit value of: pion decay constant ⇝ F: $F_\pi = F(1 + \mathcal{O}(m_q))$

nucleon mass ⇝ m_0

axial vector coupling ⇝ g_0

DETERMINATION OF THE LECs

- not constrained by symmetries
- in general scale dependent (renormalization) $\rightsquigarrow$ d_i, e_i : $b_i = b_i^r(\lambda) + \kappa_i \lambda^{d-4}(1/(d-4) + \cdots)$
- describe influence of "heavy" degrees of freedom not contained explicitely in χ Lagrangians
- RELATE MANY OBSERVABLES
- naturalness: $c_i \sim g_A/\Lambda_\chi \sim 1$ GeV^{-1}

TWO CLASSES

- dynamical LECs: ∂_μ^n

govern momentum dependence ⟶ accessible EXPERIMENTALLY

- symmetry breaking LECS: $\sim m_q^n, \;\; m_q^n \partial_\mu^m$

specify quark mass dependence of amplitudes ⟶ more difficult to extract from experiments

ADDITIONAL INPUT FROM THEORY

large-N_c method
lattice QCD
$\Big\}$ RESULTS IN MESON SECTOR BUT NOT FOR BARYONS

c_i

- πN scattering inside the Mandelstam triangle: best convergence, relies on dispersive analysis,

Büttiker & Meißner, Nucl. Phys. A668 (2000) 97

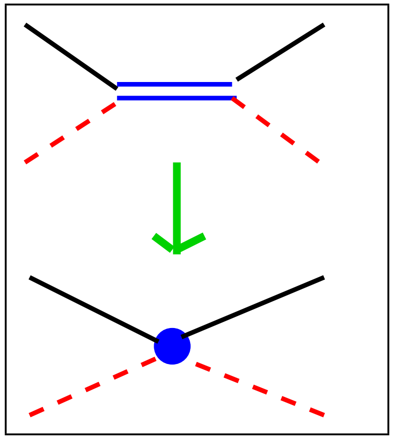
- πN scattering in the threshold region: large data basis, Fettes, Meißner & Steininger, Nucl. Phys. A640 (1998) 119
- peripheral phases in NN scattering: large data basis, Epelbaum, Glöckle & Meißner, Eur. Phys. J. A19 (2004) 125

Resulting values in $\mathbf{GeV}^{-1}$

$$\boxed{c_1 = -0.9^{+0.2}_{-0.5}, \quad c_2 = 3.3 \pm 0.2, \quad c_3 = -4.7^{+1.2}_{-1.0}, \quad c_4 = 3.5^{+0.5}_{-0.2}}$$

RESONANCE SATURATION:

- Effective Lagrangian with resonances chirally coupled to nucleons and pions
- Let the resonance masses become very large



$$c_3 = c_3^{\Delta} + c_3^{S} + c_3^{R} = -3.83 - 1.40 - 0.06 = -5.29$$

d_i

- d_{16}: $\pi N \longrightarrow \pi\pi N$: not very well determined, Fettes, B & Meißner, Nucl. Phys. A669 (2000) 269
- d_{18}: Goldberger Treiman discrepancy and $g_{\pi NN} = 13.18$

$$g_{\pi N} = \frac{g_A m}{F_\pi}\left(1 - \frac{2M_\pi^2 \bar{d}_{18}}{g_A}\right) + \mathcal{O}(M_\pi^4)$$

Resulting values in GeV^{-2}

$$\boxed{\bar{d}_{16} = -3.4 \cdots - 0.92, \quad \bar{d}_{18} = -0.72 \pm 0.27, \quad \bar{d}_{28} = 0}$$

e_1

only badly determined from πN scattering, negative, of natural size

l_3

$$M_\pi = M\left(1 - \frac{M^2}{32\pi^2 F^2}\bar{l}_3 + \mathcal{O}(M^4)\right)$$

l_4

$$F_\pi = F\left(1 + \frac{M^2}{16\pi^2 F^2}\bar{l}_4 + \mathcal{O}(M^4)\right), \qquad \bar{l}_4 = 16\pi^2 F^2 l_4^r(\lambda) - 2\ln(M_\pi/\lambda)$$

$$\boxed{\bar{l}_4 = 4.33 \longrightarrow F = 87\mathrm{MeV}}$$

	$\bar{l}_3$	$\bar{l}_4$
sources	$\pi\pi$ scat.	$\langle r_\pi^S \rangle$
standard CHPT	2.9 ± 2.4 [1]	4.4 ± 0.2 [2]
lattice [3]	3.65 ± 0.12	4.52 ± 0.06
[4]	$3.5 \pm 0.5 \pm 0.1$	
[5]	0.6 ± 1.2	3.9 ± 0.5

[1] Gasser and Leutyler

[2] Gasser, Leutwyler and Colangelo

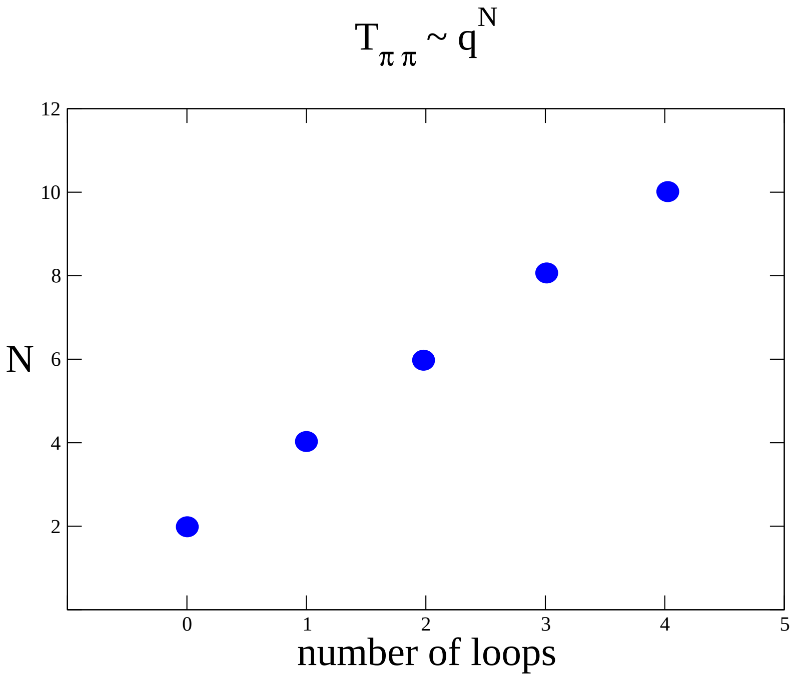
[3] ETM Collaboration 2007

[4] Del Debbio et al. 2006

[5] MILC COllaboration 2006

- POWER COUNTING SCHEME: Weinberg
  - MESON SECTOR

ONE TO ONE CORRESPONDANCE BETWEEN LOOP EXPANSION AND POWERS IN q

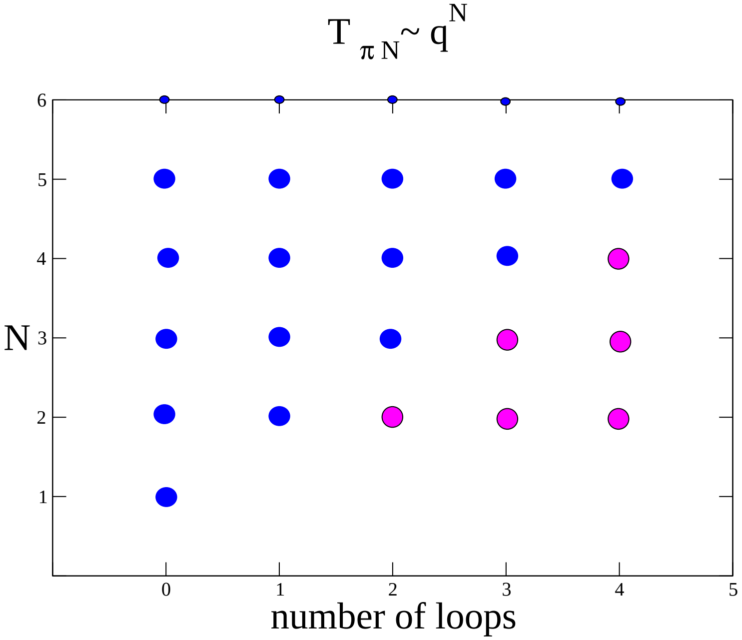


– BARYON SECTOR

PROBLEM: $m_N \sim \Lambda_\chi$ does not vanish in χ limit

$\rightsquigarrow$ EXTRA SCALE

EXPANSION in q/Λ_χ & q/m_N



SOLUTIONS:

⋆ HBCHPT: E. Jenkins & A. Manohar

m_N: PROPAGATOR $\longrightarrow$ VERTEX

HOW:

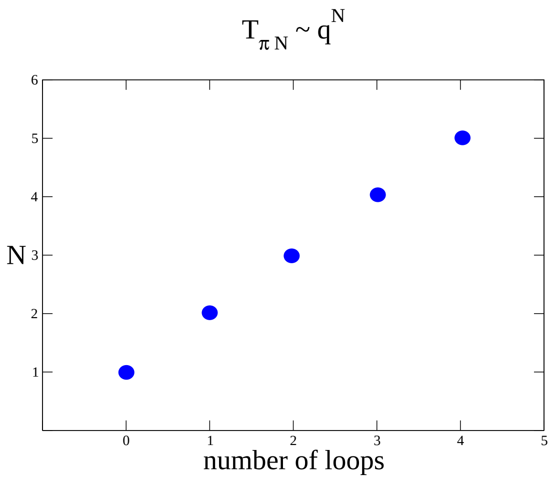
Ψ_N = H + h

$\nearrow$ $\nearrow$

" LIGHT" KEPT "HEAVY" INTEGRATED OUT

$\longrightarrow \mathcal{L}_{eff}(H)$: NO MASS TERM to leading order
Next orders $1/m$ SUPPRESSED

- STRICT EXPANSION OF SERIES

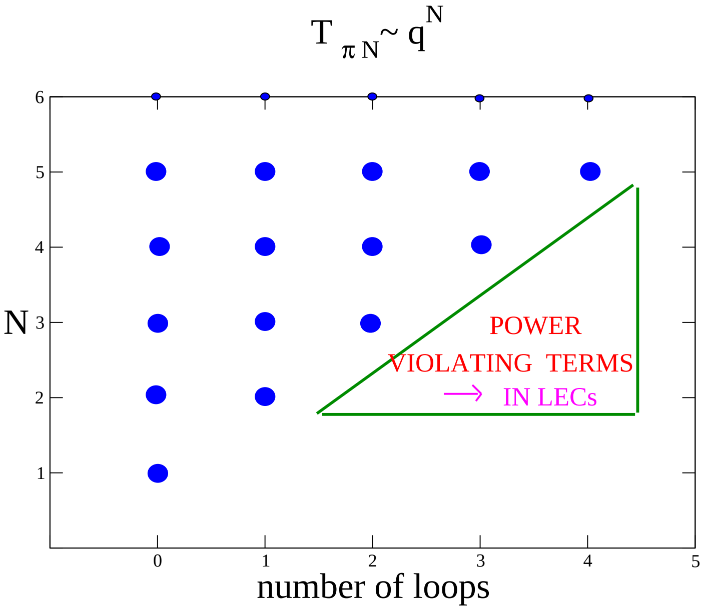


- MANY SUCCESSES

  BUT

- INCORRECT THRESHOLDS AND SINGULARITIES IN SOME CASES

⋆ Covariant CHPT



- Tang and Ellis: POWER VIOLATING TERMS ARE POLYNOMS IN M_π AND EXT. MOM

⟶ CAN BE ABSORBED IN LECs of $\mathcal{L}_{eff}$

↗

most general Lagrangian

- Infinite series in 1/m

– IR regularization; T. Becher & H. Leutwyler

RESUMS THE KINETIC 1/m terms

but UNPHYSICAL CUTS →DIVERGING quantities when M_π becomes large

– EOMS scheme: S. Scherer et al.

SUBTRACT POWER COUNTING VIOLATING TERMS

Loop functions DECREASE as M_π becomes large

⋆ CUT-OFF

- ALL THESE REGULARIZATIONS DIFFER BY HIGHER ORDER TERMS
- ALL NON-ANALYTICAL TERMS ARE THE SAME
- ANALYTICAL TERMS CAN BE DIFFERENT: DIFFERENCE ABSORBED IN THE LECs

$\longrightarrow$ VALUES OF LECs DEPEND IN PRINCIPLE ON REGULARIZATION

FOR χ EXTRAPOL. NO DIFFERENCE
RESULTS MUST BE INDEPENDENT OF REGULARIZATION SCHEME

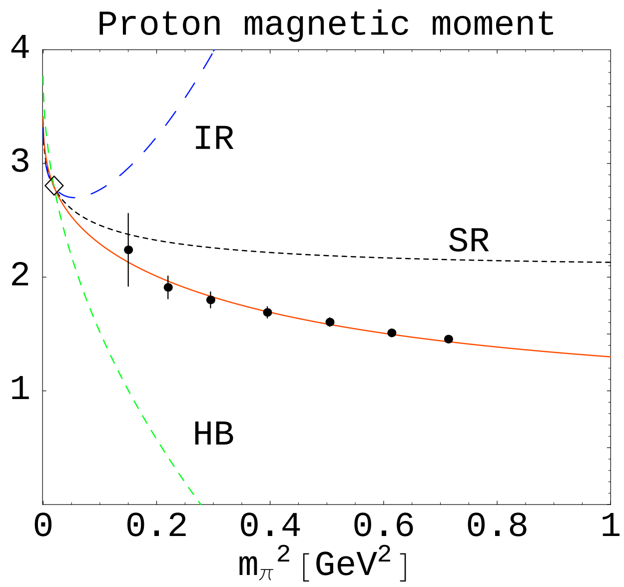
QUARK MASS EXPANSION OF MAGNETIC MOMENTS in SU(2):

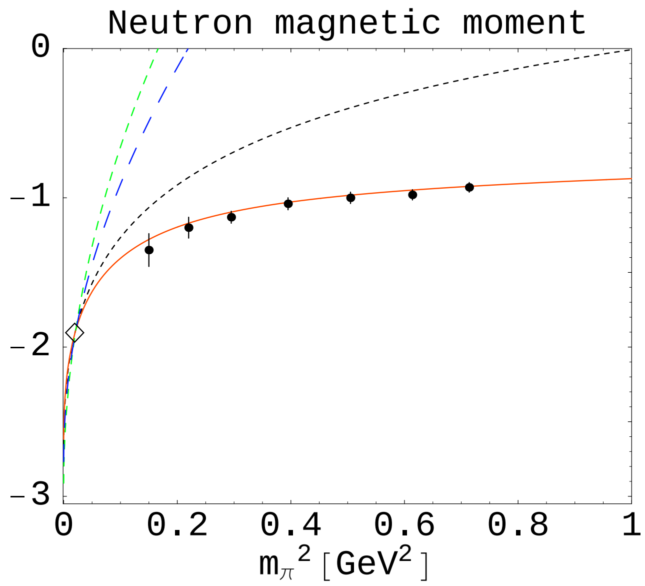
B. Kubis and U.-G. Meißner, Nucl. Phys. A 679 (2001) 698, B. R. Holstein, V. Pascalutsa and M. Vanderhaegen, Phys. Rev. D 72 (2005) 094014

HB: $\kappa_p = \kappa_0 + \frac{g^2 m^2}{(4\pi F_\pi)^2} \left\{ -2\pi\mu - 2\,(1 + 5\ln\mu)\,\mu^2 + \frac{21\pi}{4}\,\mu^3 + O(\mu^4) \right\}$ $\qquad \mu = M_\pi/m$

Rel: $\kappa_p = \kappa_0 + \frac{g_A^2 m^2}{(4\pi F_\pi)^2} \left\{ \boxed{1} - \frac{\mu\,(4 - 11\mu^2 + 3\mu^4)}{\sqrt{1-\mu^2/4}} \arccos\frac{\mu}{2} - 6\mu^2 + 2\mu^2\,(-5 + 3\,\mu^2)\ln\mu \right\}$

IR: $\kappa_p = \kappa_0 + \frac{g_A^2 m^2}{(4\pi F_\pi)^2} \left\{ -\frac{\mu\,(4 - 11\mu^2 + 3\mu^4)}{\sqrt{1-\mu^2/4}} \arccos\left(-\frac{\mu}{2}\right) - \frac{3}{2}\mu^4 + 2\mu^2\,(-5 + 3\,\mu^2)\ln\mu \right\}$





CHIRAL EXTRAPOLATION VALID UP TO $M_\pi \sim 350$MeV

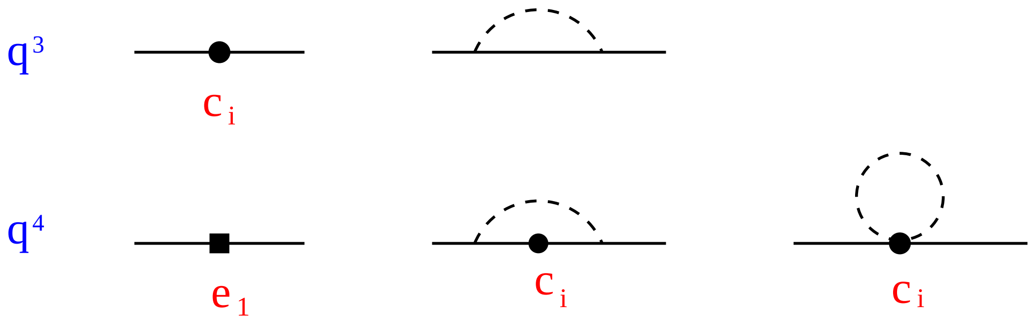
BARYON OBSERVABLE TO TWO LOOPS: $\mathcal{O}(\underbrace{M, F, l_i}_{\text{meson sector}}; m_0, g_0, c_i, d_i, e_i, \cdots)$

CALCULATION TO ONE LOOP $\mathcal{O}(q^4)$

- all TREE graphs with insertion from $\mathcal{L}_{\pi N}^{(i)}$, i=1, 2, 3, 4
- all LOOP graphs with insertions at most from $\mathcal{L}_{\pi N}^{(2)}$

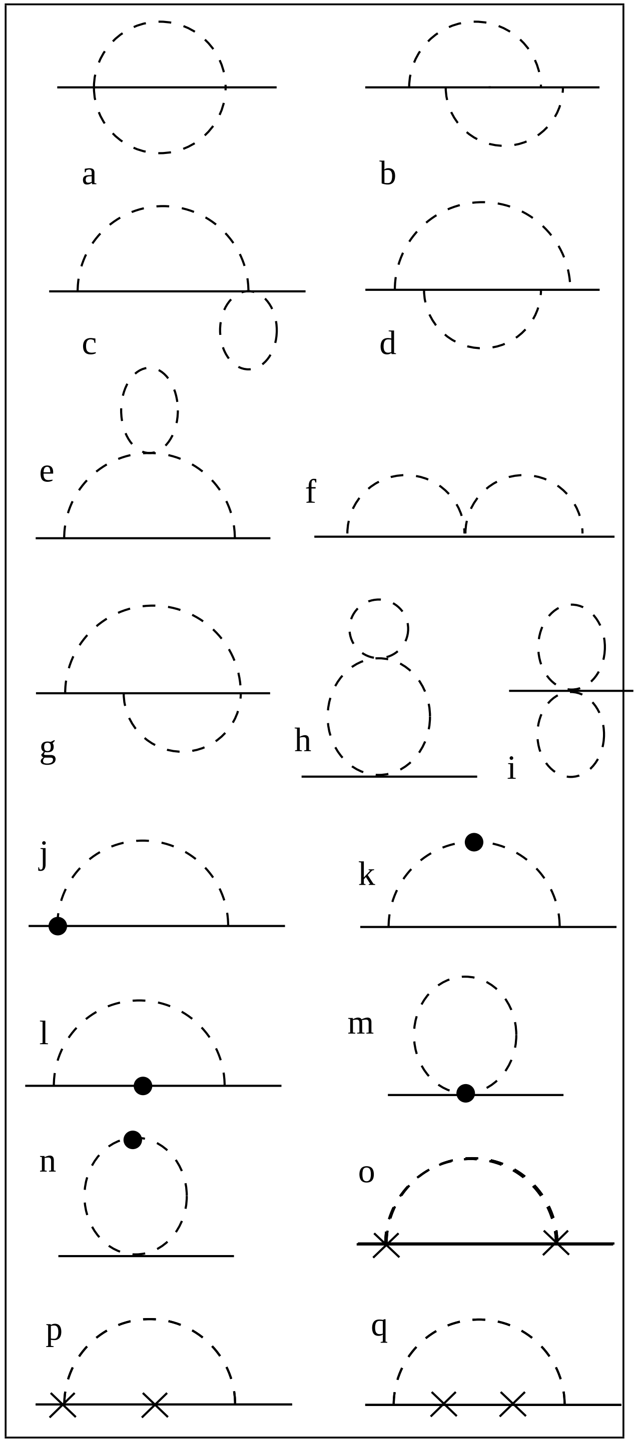
→ CONTRIBUTION OF LECs up to $\mathcal{O}(q^4)$

EXAMPLE: NUCLEON MASS



CALCULATION TO TWO LOOPs $\mathcal{O}(q^6)$

- TREE graphs with insertion from $\mathcal{L}_{\pi N}^{(i)}$, i=5,6
- LOOP graphs with insertions at most from $\mathcal{L}_{\pi N}^{(4)}$
- TWO LOOP graphs with insertions at most from $\mathcal{L}_{\pi N}^{(2)}$



EXAMPLE: NUCLEON MASS

QUARK MASS EXPANSION OF m_N in SU(2)

- Fifth order calculation: McGovern & Birse, Phys. Rev. D74 (2006) 097501

$$\boxed{m_N = m_0 - 4c_1 M^2 - \frac{3g_0^2 M^3}{32\pi F^2} + k_1 M^4 \ln\frac{M}{m_N} + k_2 M^4 + k_3 M^5 \ln\frac{M}{m_N} + k_4 M^5 + \mathcal{O}(M^6)}$$

$$k_1 = -(3/32\pi^2 F^2)(-8c_1 + c_2 + 4c_3 + g_0{}^2/m_0)$$

$$k_2 = -4e_1 + (3/128\pi^2 F^2)(c_2 - 2g_0{}^2/m_0)$$

$$k_3 = (3/1024\pi^3 F^4)(16g_0{}^2 - 3)$$

$$k_4 = (3/32\pi F^2)((2l_4^r - 3l_3^r) - 4(2d_{16}^r - d_{18})/g_0 + 16\, d_{28}^r + g_0{}^2/(8\pi^2 F^2) + 1/8m_0^2)$$

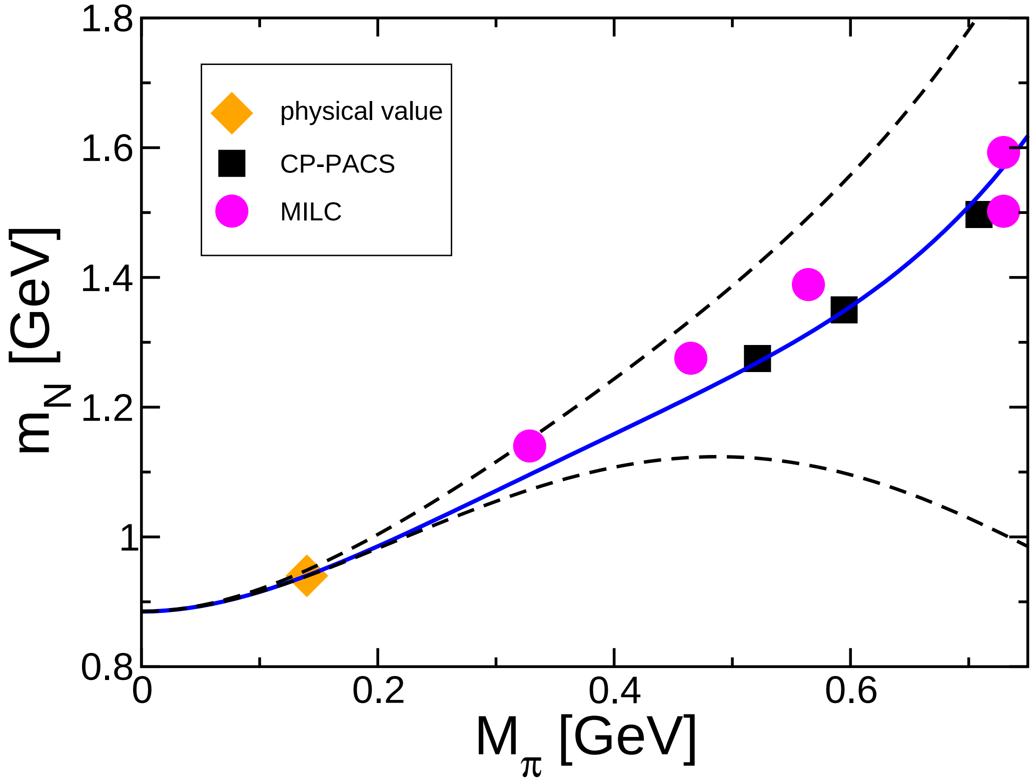
- $M \longrightarrow M_\pi$: EXTRA l_3 CONTRIBUTION extremely small in the considered range

$\nwarrow$

Pion mass to leading order

- COEFF. OF $M^5 \ln M$ TERM FIXED BY CHIRAL SYMMETRY
- ONLY $\mathcal{O}(q^3)$ COUNTERTERMS IN M_π^5 TERM $\longrightarrow$ COEFFICIENTS RATHER WELL KNOWN
- FITS TO THE DATA WITH CONSTRAINT FROM PHYSICAL VALUE

- $\boxed{\mathcal{O}(q^4)}$: B., Hemmert & Meißner, Nucl. Phys. A732 (2004) 149 and similar work by Hemmert, Procura & Weise



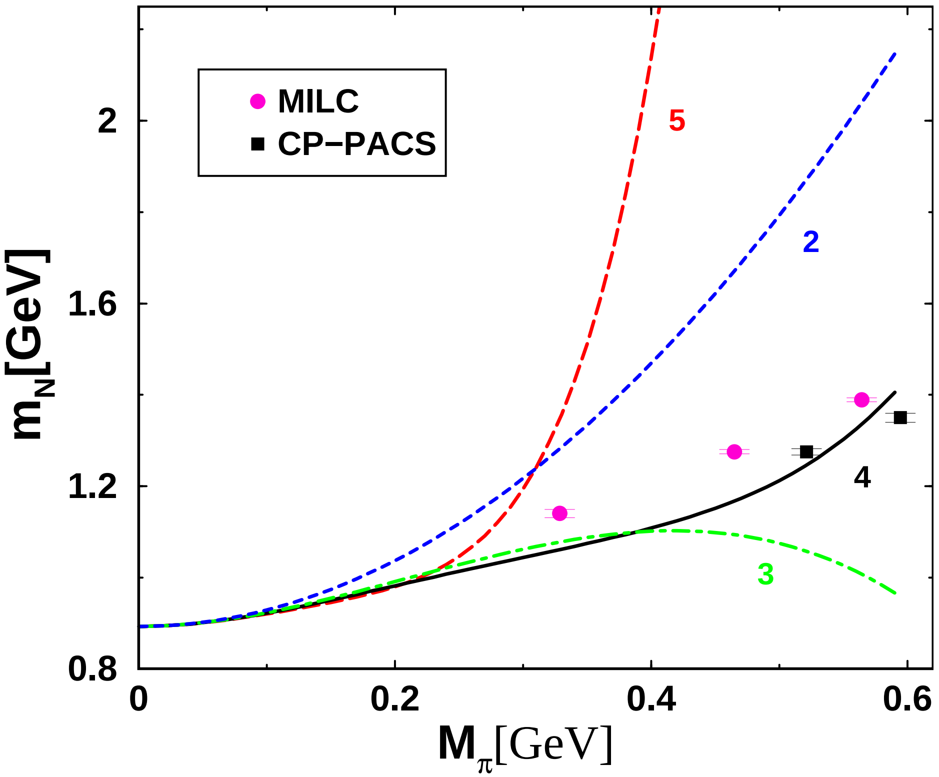
- LECs close to empirical values: $c_1 = -0.9, c_2 = 3.2, c_3 = -3.5, e_1 = -1$ (in appropriate units)
- Chiral SU(2) value of nucleon mass: $m_0 \sim 0.88 GeV$
- Moderate/Large theoretical uncertainty for M_π above 400/500 MeV

- CONVERGENCE OF THE SERIES

less known (e_1) FIXED BY χ SYM.

$$m_N = 0.89 + 3.6M^2 - 5.7M^3 - 20.0M^4 \ln\frac{M}{m_N} + 8.6M^4 + 56.0M^5 \ln\frac{M}{m_N} + 213.4M^5 + \mathcal{O}(M^6)$$

M_π physical $\longrightarrow m_N = 0.893(1 + 0.078 - [0.017 + 0.015] + 0.006 + \cdots)$

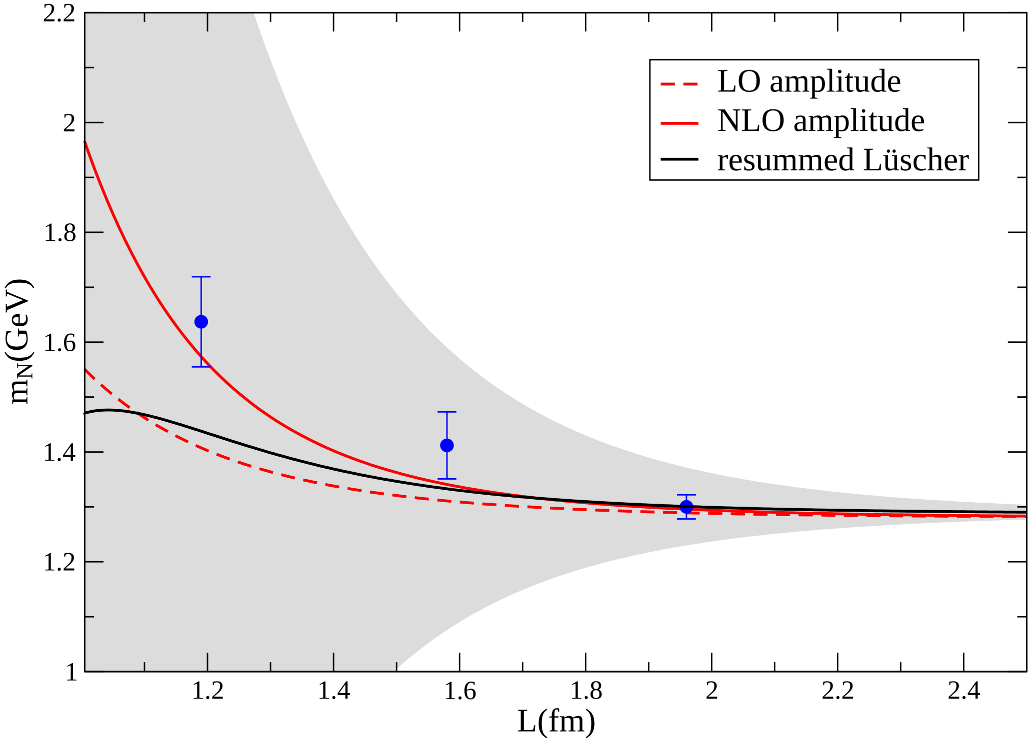


ONLY ONE FREE PARAMETER e_1

USE d_{16} & g_0 from g_A

CHIRAL EXTRAPOLATION VALID UP TO $M_\pi \sim 350$MeV

- VOLUME DEPENDENCE: Ali Khan et al. Nucl. Phys. B689 (2004) 175; Colangelo et al. Nucl.Phys.Proc.Suppl. 153 (2006) 41



- different combination of c_i from finite volume correction ⟶ FURTHER CONSTRAINS

RESULT CONSISTENT WITH INFINITE VOLUME

M. Procura et al., Phys. Rev. D73 (2006) 114510

- Problem in nucleon sector: CHIRAL SYMMETRY RESTRICTS THE πN INTERACTION LESS SEVERELY THAN $\pi\pi$.

Δ (1232):

$\star$ $m_\Delta - m_N$ SMALL

$\star$ COUPLES STRONGLY TO PIONS, NUCLEONS & PHOTONS

$\longrightarrow$ COVARIANT EFT w/ EXPLICIT Δ(1232) (spin 3/2) DOFs

Hemmert, Holstein, Kambor, Phys. Lett. B 395 (1997) 89

- $\Delta \equiv (m_\Delta - m_N)$ as additional small parameter: does not vanish in the χ limit

$\longrightarrow$ $\boxed{\text{EXPANSION IN } \epsilon \equiv \{p, m_q, \Delta\}}$

- Should fulfill DECOUPLING THEOREM: Gasser & Zepeda, Nucl. Phys. B174 (1980)445

Leading chiral non-analytic terms stem from pion (Goldstone boson) one-loop graphs coupled to pions or nucleons (ground state baryons)

$\longrightarrow$ severe constraints

- more operator structures → more LECs

QUARK MASS EXPANSION OF m_Δ in SU(2):

–INFINITE VOLUME V. B., Hemmert & Meißner, Phys. Lett. B622 (2005) 141

•work to fourth order in the ϵ-expansion within IR regularization

$$\boxed{m_\Delta = m_0^\Delta - 4a_1 M_\pi^2 - 4e_1^\Delta M_\pi^4 + m_\Delta^{N-\mathrm{loop}} + m_\Delta^{\Delta-\mathrm{loop}}}$$

11 combinations of parameters

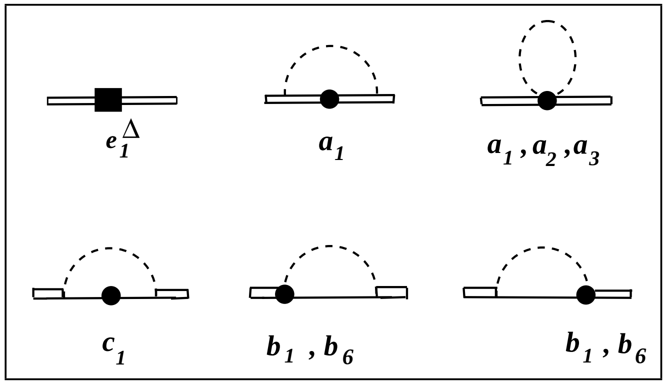
- $\longrightarrow$ 3 from nucleon sectors

  use resonance saturation

- $\longrightarrow$ from complex Δ pole:

  $c_A = 1.1$ (axial $N\Delta$ coupling)

  $\Delta_0 \equiv m_\Delta - m_N = 0.33$ GeV

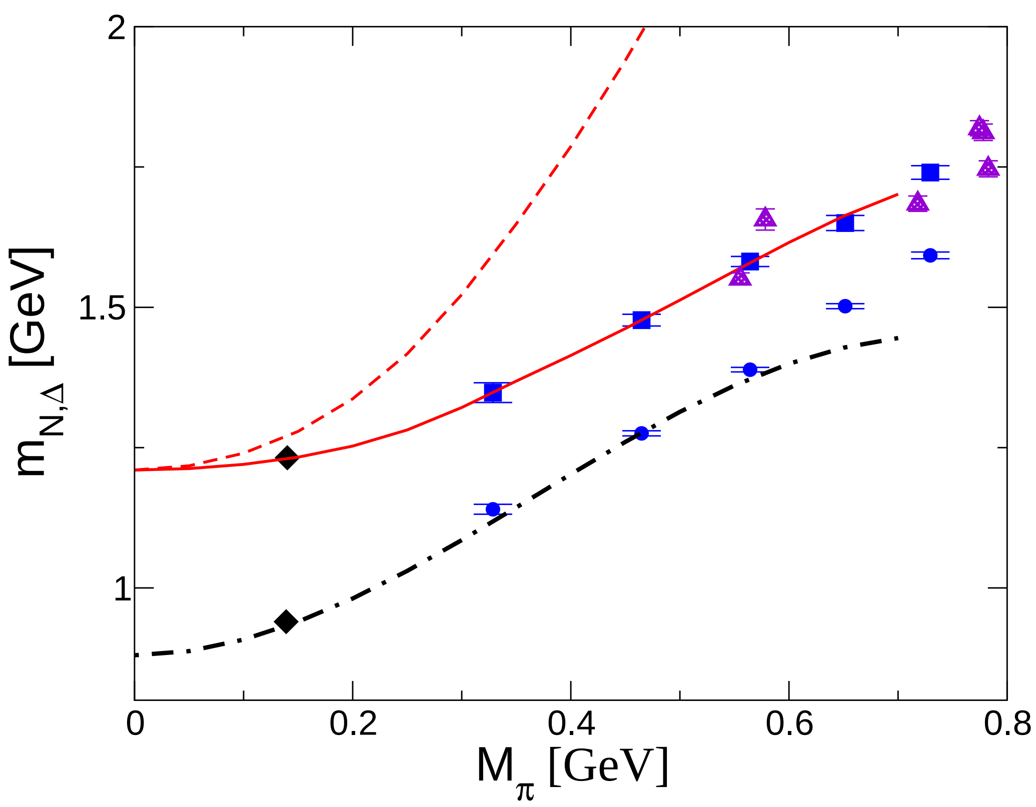


FITS TO LATTICE DATA + CONSTRAINT FROM PHYSICAL VALUE

6 combinations of LEC's to fit

▲ & ● MILC coll., Phys. Rev. D64 (2001) 054506

▲ QCDSF, G. Schierholz, priv. comm.



- ALL LECs OF NATURAL SIZE
- AXIAL $\Delta\Delta$ COUPLING $g_1 = 2$ NOT FAR FROM $9g_A/5 = 2.28$ as in SU(6)
- LEADING SYMMETRY BREAKER $a_1 = -0.3\mathrm{GeV}^{-1}$ FAR FROM SU(6)

  $\longrightarrow$ FURTHER STUDY
- sigma terms $\sigma_{\pi N}(0) = 48.9$ MeV, $\sigma_{\pi\Delta}(0) = 20.6$ MeV
- FIT FOR NUCLEON MASS SIMILAR TO HBCHPT

–FINITE VOLUME:

- $M_\pi \gtrsim \mathbf{290}$ MeV: Δ IS A STABLE PARTICLE

  $\longrightarrow$ CALC. AS NUCLEON

$$\boxed{E_1 - m_\Delta \propto exp(-M_\pi L)}$$

- $M_\pi \lesssim \mathbf{290}$ MeV: Δ IS UNSTABLE

  INVERSE POWER RATHER THAN EXPONENTIAL LAW

CAN ONE DETERMINE THE Δ MASS AND WIDTH ON THE LATTICE WHEN IT DECAYS?

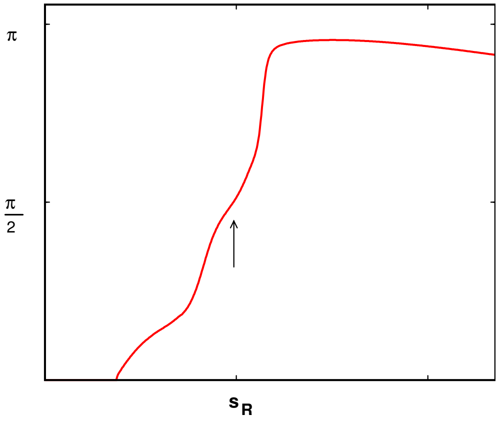
RELATED CALCULATION:

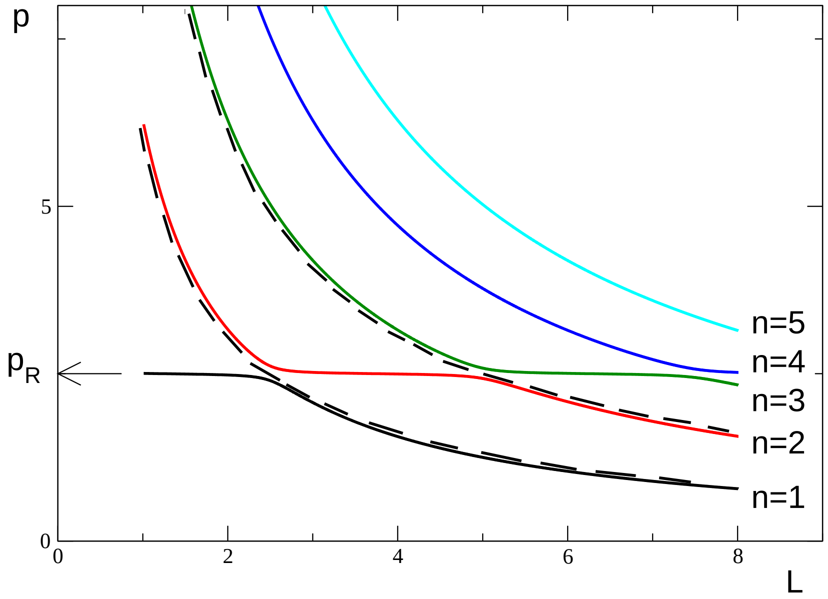
TWO-BODY SCATTERING PHASE SHIFTS FROM THE ENERGY LEVELS IN A FINITE BOX:

M. Lüscher, Lectures at Les Houches (1988), NPB 364 (1991) 237, Wiese

1 Dimension case:

- free: $pL = 2\pi n$
- interacting: $pL + 2\delta(p) = 2\pi n$





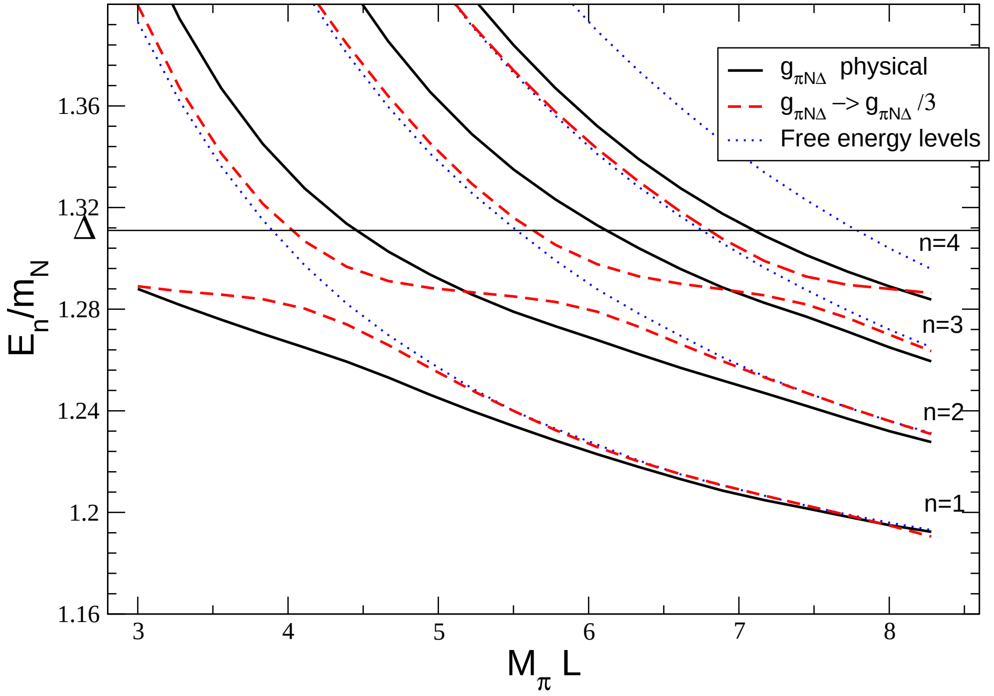
AVOIDED LEVEL CROSSING NEAR THE RESONANCE ENERGY

- HORIZONTAL PLATEAUS $\longrightarrow$ POSITION OF THE RESONANCE
- DISTANCE BETWEEN ENERGY LEVELS NEAR AVOIDED CROSSING $\longrightarrow$ DECAY WIDTH OF THE RESONANCE

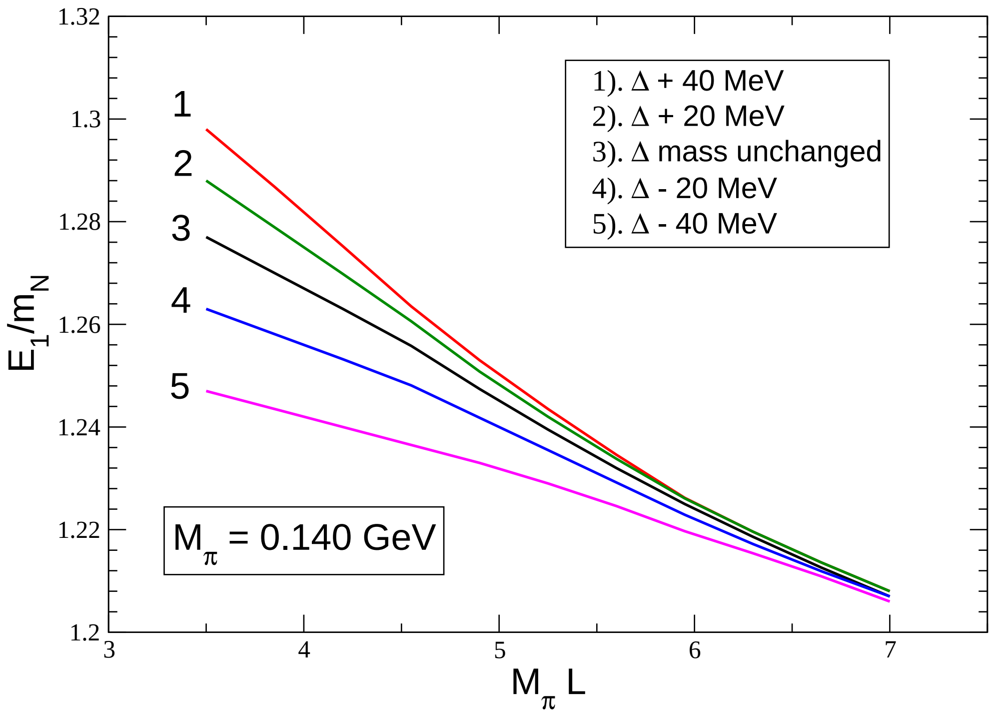
ENERGY LEVELS IN SSE: $\mathcal{O}(\epsilon^3)$ V. B., U.-G. Meißner and A. Rusetsky

Δ self-energy at a finite volume ⟶ POLES OF THE PROPAGATOR

$E_n = f(m_\Delta, g_{\pi N\Delta})$: $\pi\Delta$ Loops and Tadpole exponentially suppressed

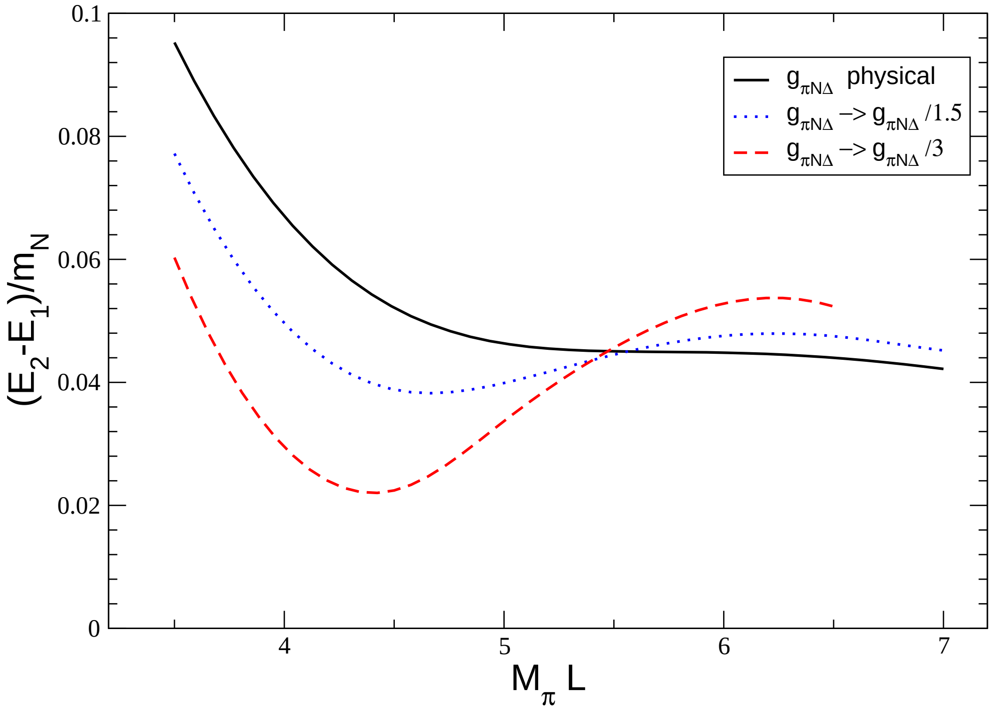


- AVOIDED LEVEL CROSSING FOR $g_{\pi N\Delta}$ SMALL
- WIDTH OF **Δ** TOO LARGE FOR $g_{\pi N\Delta}$ PHYSICAL, WASHED OUT



SENSITIVE TO THE INPUT VALUES OF $\boldsymbol{m_\Delta}$ for $M_\pi^{\rm phys} L \leq 5$

INCREASE SENSITIVITY CLOSER TO $\boldsymbol{N} + \boldsymbol{\pi}$ THRESHOLD



- $E_2 - E_1$ SENSITIVE TO $\boldsymbol{g_{\pi N \Delta}}$

- CONCLUSION

check the convergence $\boldsymbol{\mathcal{O}(\epsilon^4)}$

FITTING THE MASS AND WIDTH OF THE $\boldsymbol{\Delta}$ IS FEASIBLE DESPITE THE FACT THAT THE AVOIDED LEVEL CROSSING IS WASHED OUT

AXIAL-VECTOR COUPLING CONSTANT

$$\begin{aligned} g_A &= g_0 \left\{ 1 + \left(\frac{\alpha_2}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_2 \right) M_\pi^2 + \alpha_3 \, M_\pi^3 \right. \\ &\quad \left. + \left(\frac{\alpha_4}{(4\pi F)^4} \ln^2 \frac{M_\pi}{\lambda} + \frac{\gamma_4}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_4 \right) M_\pi^4 + \alpha_5 \, M_\pi^5 \right\} + \mathcal{O}(M_\pi^6) \, , \\ &= g_0 \left\{ 1 + \underbrace{\Delta^{(2)} + \Delta^{(3)}}_{\text{1-loop}} + \underbrace{\Delta^{(4)} + \Delta^{(5)}}_{\text{2-loop}} \right\} + \mathcal{O}(M_\pi^6) \end{aligned}$$

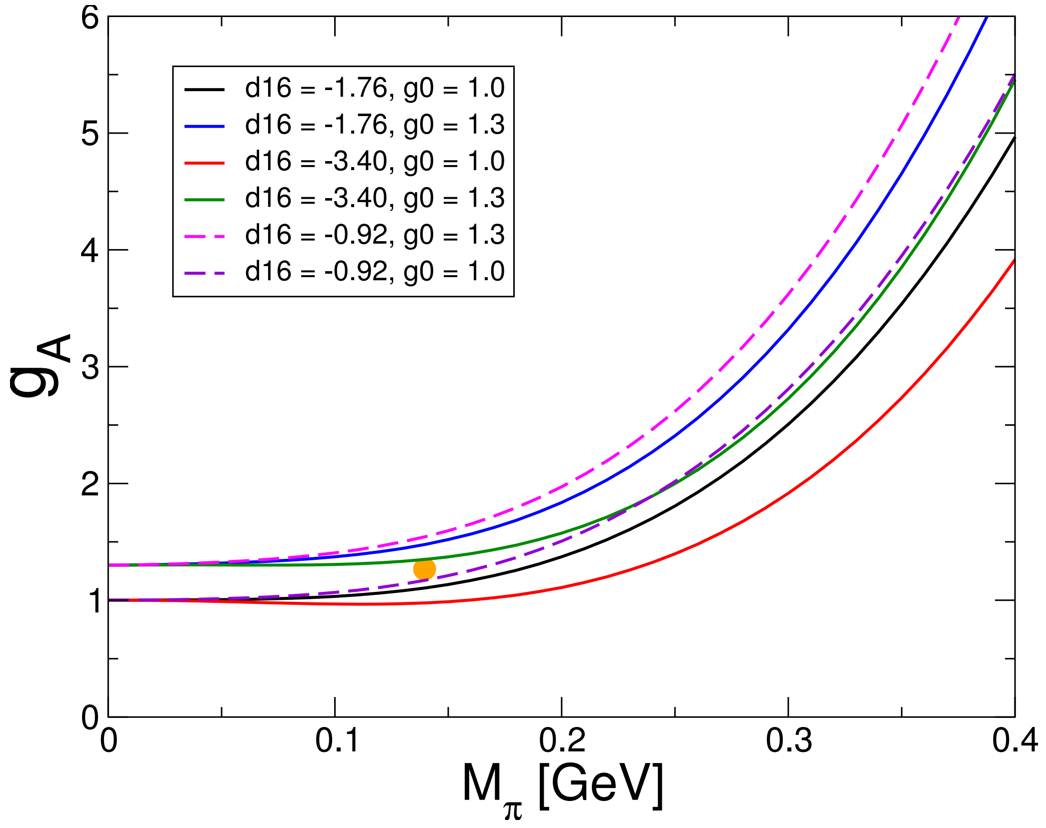
- in principle also $M_\pi^5 \ln M_\pi$ term ⟶ put it in uncertainty of α_5
- 1-loop result

$$\begin{aligned} &\alpha_2 = -2 - 4g_0^2 \, , \;\; \beta_2 = \frac{4}{g_0} \left(d_{16}^r(\lambda) - 2 d_{28}^r(\lambda) \right) - g_0^2/(4\pi F)^2 \\ &\alpha_3 = \left(3 + 3g_0^2 - 4m_0 c_3 + 8 m_0 c_4\right) / (24\pi F^2 m_0) \end{aligned}$$

At physical pion mass: $g_A = g_0(1 - 0.15 + 0.26 + \ldots)$

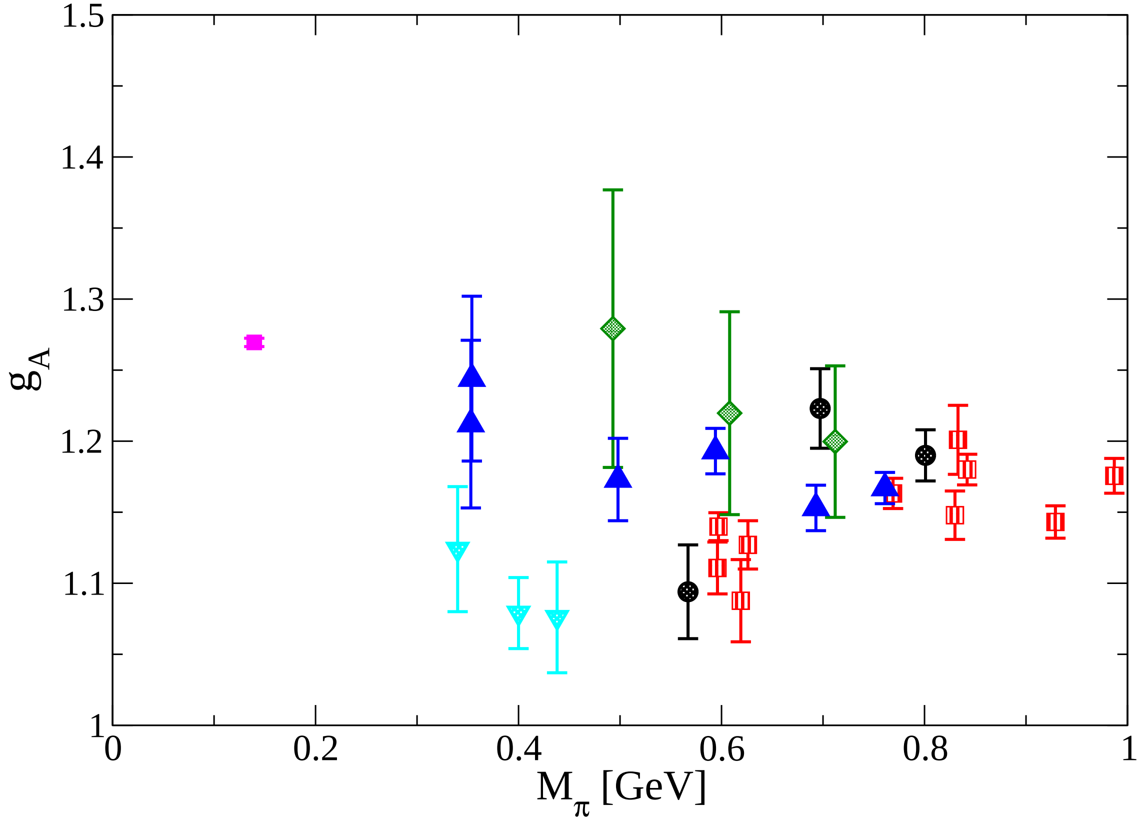
⋆ natural correction at $\mathcal{O}(M_\pi^2)$ ($\bar{d}_{16} = -1.76$ GeV^{-2})

⋆unnaturally large correction at $\mathcal{O}(M_\pi^3)$ for central values of c_i due to large values of these LECs



- input parameters: c's fixed at central values
- SHARP RISE BEYOND $M_\pi \simeq 300$ MeV → dominance of the M_π^3 term

LATTICE DATA



▲ LPHC/MILC

R. G. Edwards et al, Phys. Rev. Lett. 96 (2006) 052001

■ QCDSF/UKQCD

A. A. Khan et al, Nucl. Phys. Proc. Suppl. 140 (2005) 408

◆ RBC

S. Sasaki et al, Nucl. Phys. Proc. Suppl. 106 (2002) 302

● LHPC/SESAM

D. Dolgov et al., Phys. Rev. D 66 (2002) 034506

▼ QCDSF preliminary

RATHER FLAT DEPENDENCE IN M_π IN CONTRADICTION WITH ONE LOOP RESULT

- INCLUDING THE $\boldsymbol{\Delta}$: T. H.Hemmert, M. Procura & W. Weise, Phys. Rev. D 68 (2003) 075009

-COULD BE IMPORTANT $\longrightarrow$ Adler-Weisberger sum rule

-IMPROVE THE CHIRAL EXPANSION BY SHUFFLING TO LOWEST ORDER TERMS

WHICH WOULD APPEAR LATER IN THE SERIE WITH NO Δ

$\mathcal{O}(\epsilon^3)$ calculation

$$\boxed{\begin{array}{l} g_A^{SSE}(M_\pi^2) = g^0(1+(\alpha_2 \ln \frac{M_\pi}{\lambda}+\beta_2)M_\pi^2+\gamma_2 \frac{M_\pi^3}{\Delta_0} + \delta_2 M_\pi^2 \ln R + \epsilon_2 \Delta_0^2 \ln R + \mathcal{O}(\epsilon^4)) \\ R = \frac{\Delta_0}{m_\pi} + \sqrt{\frac{\Delta_0^2}{m_\pi^2}} - 1; \qquad \Delta_0 \equiv m_\Delta - m_N \end{array}}$$

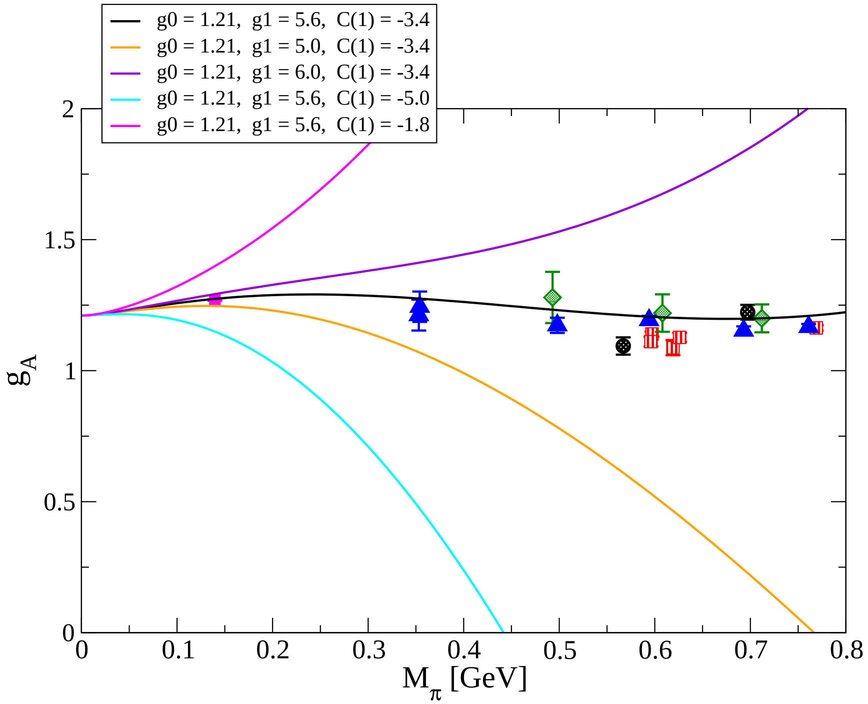
PARAMETERS: $\overbrace{c_A,\ \Delta_0}^{\text{taken at }\phi\text{sical values}},\ g_1,\ C(\lambda)$

↖

Combination of LECs: not known: MATCHING WITH HBCHPT AT $\lambda = 2\Delta_0$

Result insensitive for λ between 0.4 & 0.8 MeV

C(1GeV)=(-3.4 ± 1.2) GeV



- g_1 LARGER THAN SU(6) VALUE: NOT CONSISTENT WITH m_Δ RESULT ??
- Ali Khan et al. hep-lat/0603028 SIMILAR FIT BUT c_A LARGER $\longrightarrow g_1 \sim 3$
- Large theoretical uncertainties

- TWO LOOP CALCULATION V. B. & Meißner, Phys. Lett. B 639 (2006) 278

$$\begin{aligned} g_A &= g_0 \left\{ 1 + \left(\frac{\alpha_2}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_2 \right) M_\pi^2 + \alpha_3 \, M_\pi^3 \right. \\ &\left. + \left(\frac{\alpha_4}{(4\pi F)^4} \ln^2 \frac{M_\pi}{\lambda} + \frac{\gamma_4}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_4 \right) M_\pi^4 + \alpha_5 \, M_\pi^5 \right\} + \mathcal{O}(M_\pi^6) \, , \\ &= g_0 \left\{ 1 + \underbrace{\Delta^{(2)} + \Delta^{(3)}}_{\text{1-loop}} + \underbrace{\Delta^{(4)} + \Delta^{(5)}}_{\text{2-loop}} \right\} + \mathcal{O}(M_\pi^6) \end{aligned}$$

- CALCULATIONAL STRATEGY:
  - calculate α_4 EXACTLY using RGE TECHNIQUE
  - calculate DOMINANT contribution to $\gamma_4, \beta_4, \alpha_5$ +NATURALNESS

$\boxed{\alpha_4}$

COEFF OF THE DOUBLE LOG $\sim \ln^2 M_\pi$ CAN BE ENTIRELY EXPRESSED IN TERMS OF THE COUPLING CONSTANTS OF THE ONE-LOOP FUNCTIONAL generic form of two loop divergences:

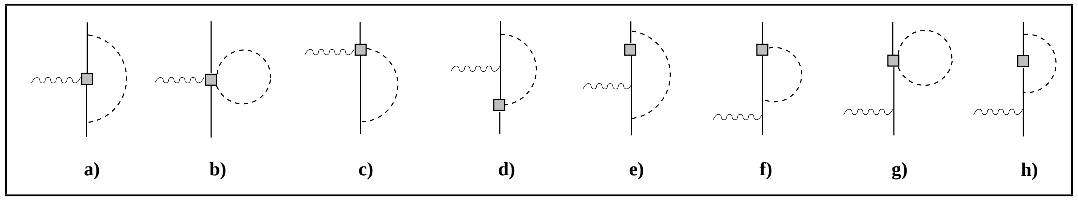
$$\boxed{k(d)\,\frac{\lambda^{2\epsilon}}{(4\pi)^4}\left[\frac{1}{\epsilon^2}+\frac{2}{\epsilon}\ln\frac{M_\pi}{\lambda}+\ln^2\frac{M_\pi}{\lambda}+\ldots\right]}$$

generic form of one loop graph with insertions from dimension three πN Lagrangian:

$$\boxed{-\frac{h_i(d)}{2}\,\frac{\lambda^{2\epsilon}}{(4\pi)^4}\left[\frac{\kappa_i}{\epsilon^2}+\frac{\kappa_i}{\epsilon}\ln\frac{M_\pi}{\lambda}+\frac{(4\pi)^2 d_i^r(\lambda)}{\epsilon}+(4\pi)^2\,d_i^r(\lambda)\ln\frac{M_\pi}{\lambda}+\ldots\right]}$$

RG condition $\boxed{k_0 = h_i^0 k_i}$

- Topologies of one-loop graphs that generate the coeff of the double log:



Altogether: 8 LECs ($d_{16}\cdots$)without eom terms + 7 LECS from eom terms

β functions are known for both type of Lagrangian ⟶ good check of the calc.

$$\boxed{\alpha_4 = -\frac{16}{3} - \frac{11}{3}g_0^2 + 16 g_0^4}$$

$\gamma_4, \beta_4, \alpha_5$

- From previous one loop graph one gets contributions to γ_4

$$\gamma_4^{d_{16}} = -12\, d_{16}^r(\lambda)\left(\tfrac{5}{3} + g_0^2\right)$$

- Relativistic propagator generates the $1/m_N$ corrections

$$\gamma_4^{c_i} = \frac{4(c_4 - c_3)}{m_0}\,, \quad \beta_4^{c_i} = \frac{c_4}{m_0}\,\frac{1}{4\pi^2 F^2}\,, \quad \alpha_5^{c_i} = \frac{c_3}{m_0^2}\,\frac{1}{16\pi F^2}$$

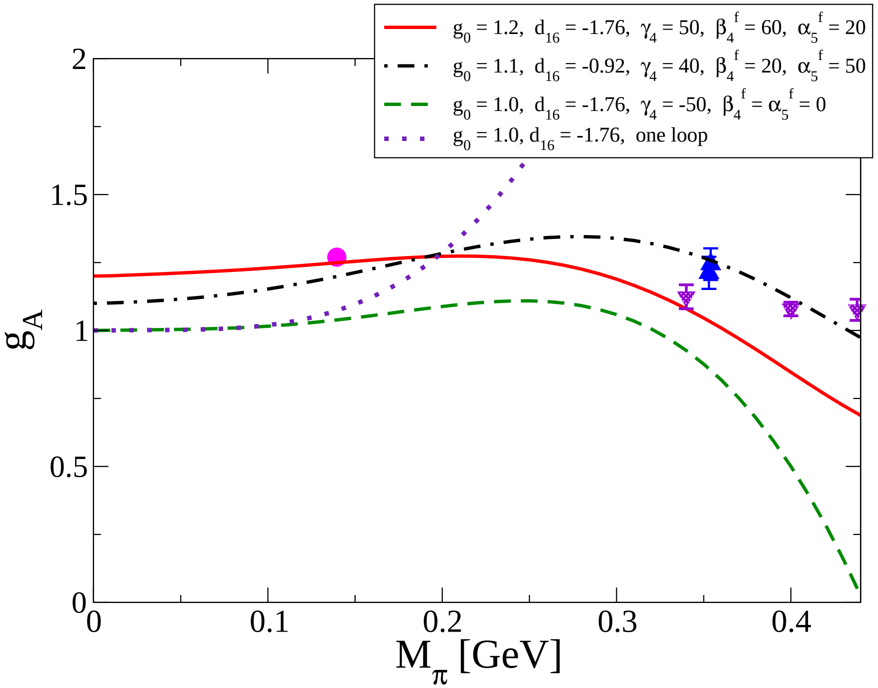
- Further corrections from the quark mass expansion of F_π

$$\tilde{\alpha}_4 = 4\,\alpha_2\,, \tilde{\gamma}_4 = -\frac{2}{F^2}\,\alpha_2\, l_4^r(\lambda)\,, \tilde{\beta}_4 = \frac{2g_0^2}{(4\pi F)^2 F^2} l_4^r(\lambda)\,, \tilde{\alpha}_5 = -\frac{2\alpha_3}{(4\pi F)^2}\left(l_4^r(\lambda) - \frac{2}{16\pi^2}\ln\frac{M_\pi}{\lambda}\right)$$

- Further contributions can only be estimated assuming naturalness
- At physical pion mass

$$\boxed{g_A = g_0(1 - 0.15 + 0.26 - 0.06 - 0.01 + \ldots)}$$

→ CONVERGENCE



Theoretical uncertainty small for $M_\pi \leq 300$MeV

SU(3)

- More fields and operator structures $\Longrightarrow$ MORE LECs

  nucleon doublet & pion triplet $\longrightarrow$ baryon octet & Goldstone boson octet

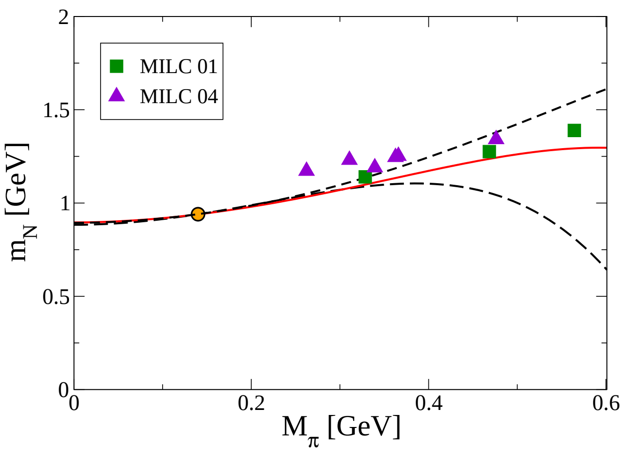
  meson-baryon coupling: $$g_a \bar{\psi}_N u_\mu \gamma_\mu \gamma_5 \psi_N \;\longrightarrow\; D\,\langle \bar{B}\,\{\not{u}\gamma_5, B\}\,\rangle + F\,\langle \bar{B}\,[\not{u}\gamma_5, B]\,\rangle$$

  symmetry breaking: $$c_1 \bar{\psi}_N \langle \chi_+ \rangle \psi_N \;\longrightarrow\; b_0 \langle \bar{B} B \rangle + b_D \langle \bar{B} \{\chi_+, B\} \rangle + b_F \langle \bar{B} [\chi_+, B] \rangle$$

- MATCHING CONDITIONS $\longrightarrow$ IMPORTANT CONSTRAINTS M. Frink and U.-G. Meißner, JHEP 0407 (2004) 028

  leading symmetry breakers $$c_1 = b_0 + \tfrac{1}{2}(b_D + b_F) + \mathcal{O}(\sqrt{m_s})$$

NUCLEON MASS: Frink, Scheller and U.-G. Meißner, Eur. Phys. J. A24 (2005) 395



FIT TO MILC 01

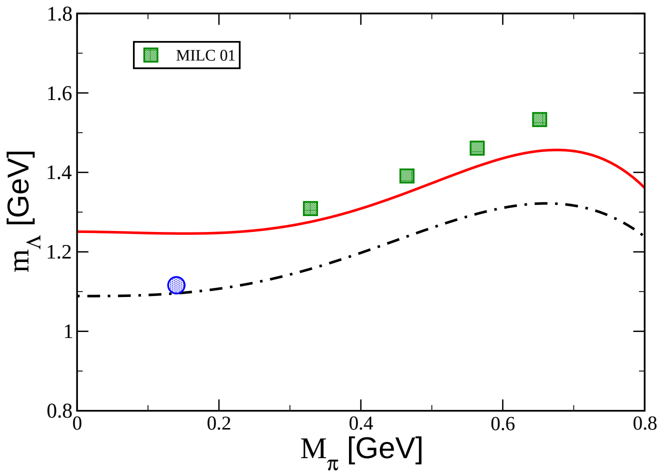
○ m_N in χ limit : $\boxed{710\text{MeV} \lesssim m_0 \lesssim 1070\text{MeV}}$

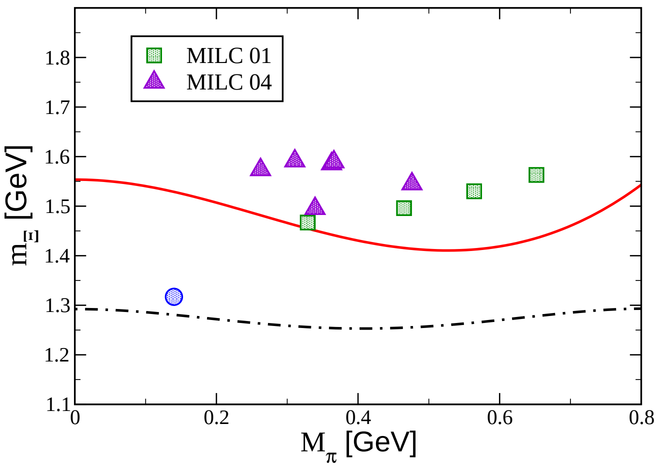
○ πN sigma term: $\boxed{39.5\text{MeV} \lesssim \sigma_{\pi N} \lesssim 46.7\text{MeV}}$

○ Strangeness fraction: $\boxed{0.07 \lesssim y \lesssim 0.22}$

SIMILAR FIT AS IN SU(2)

PION MASS EXPANSION IN THE BARYON OCTET:





— d_i from nucleon mass

- - d_i from octet mass

- WHAT ABOUT THE CASCADE?

SHORT DIGRESSION ON $F_\pi \& F_K/F_\pi$

GENUINE QCD quantities so far unknown experimentally

$$\boxed{\mathcal{L} = \tilde{g}(l_\mu + \frac{1}{2}\bar{U}(\mathcal{V}_{eff}\gamma_\mu + \mathcal{A}_{eff}\gamma_\mu\gamma_5)D)W^\mu + h.c}$$

- $\boxed{F_\pi}$

$\pi \to \mu\nu \longrightarrow |F_\pi \mathcal{A}^{ud}_{eff}|$

SM input: NO RH currents $\longrightarrow \mathcal{A}^{ud}_{eff} = \mathcal{V}^{ud}_{eff} = V_{CKM}$

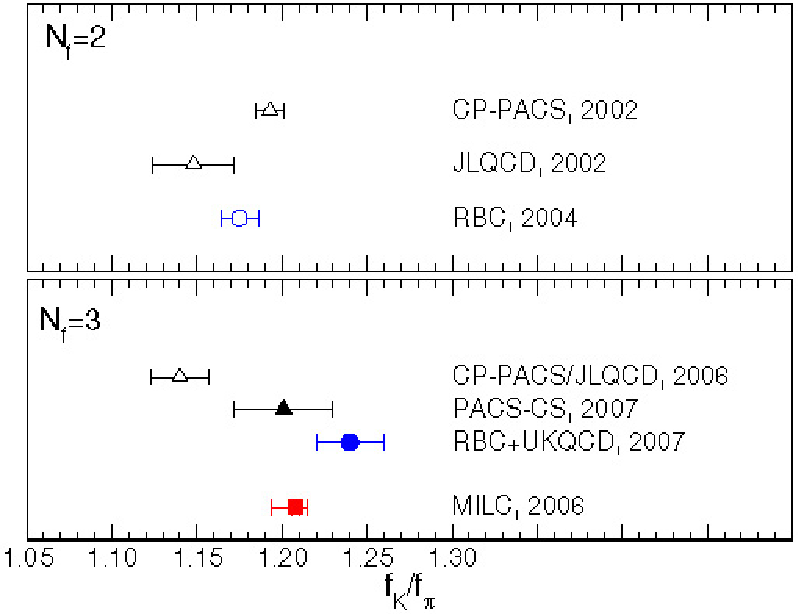
$\mathcal{V}^{ud}_{eff} = 0.97377(26)$ from $0^+ \to 0^+$ Towner & Hardy updated by Marciano & Sirlin '05

$$\boxed{F_\pi|_{SM} = (92.4 \pm 0.2)\mathrm{MeV}}$$

- $\boxed{F_K/F_\pi}$

$\frac{\Gamma(K\to\mu\nu)}{\Gamma(\pi\to\mu\nu)} \to |\frac{F_K \mathcal{A}^{us}_{eff}}{F_\pi \mathcal{A}^{ud}_{eff}}|^2 = 0.07602(23)$ (Marciano hep-ph/0402299)

$$\boxed{F_K/F_\pi|_{SM} = (1.180 \pm 0.006)}$$



- from T. Kaneko, KAON 2007
- simulations at heavy m_{sea}

  underestimate $F_K\,/F_\pi$
- PACS-CS 2007 very preliminary

SU(3) RECENT LATTICE DATA BARELY COMPATIBLE WITH SM???

CONCLUSION

- NOT IMPORTANT TO FIT LATTICE DATA FOR ONE QUANTITY UP TO LARGE M_π

BUT

- IMPORTANCE OF PERFORMING SIMULTANEOUS SYSTEMATIC EXTRAPOLATION WITH CONSISTENT SET OF LEC's

- IMPORTANCE OF CAREFULLY EVALUATING THE THEORETICAL UNCERTAINTIES

- CHIRAL EXTRAPOLATION VALID UP TO $M_\pi \sim \mathbf{350}$ MeV