Towards analytic “bottom up” approach in reconstruction of basic SUSY parameters

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1. Introduction, Motivation

2. Tree Level analytic inverted diagonalization

- Ino sector: μ, M_2, M_1 from Chargino and/or Neutralino masses

search for minimal input, different scenarios/strategies are possible if:

-gauge unification or not

-assuming LHC or ILC data

- Higgs sector inversion: scenario$M_{\tilde{t}}$, M_h input

- Squarks/sleptons sector inversion

3. More realistic: incorporate rad. corr. (different approximation levels)

4. Renormalization Group ”bottom up” evolution

5. A case study: mSUGRA SPS1a point

6. NO Conclusion, yet..

CAUTION: very preliminary study: not complete, NOT reliable numerical illustrations!!

At the moment we only sketch the main steps of a plausible procedure, trying to identify the difficulties

Introduction / Motivations

Best of all SUSY world: all sparticles +Higgses found at LHC; fit mSUGRA model; find something like 'SPS1a'

But nobody believes that, no?...

- Direct "top-down" approach:

GUT scale Lagrangian $\rightarrow$ RG evolution $\rightarrow$ Electroweak Symmetry Breaking (low scale) $\rightarrow$ Spectrum determination (diagonalization+ rad. corr.)

Standard Fitting procedure: Fit model parameters (e.g mSUGRA) to data set (masses, cross-sections, etc)

Works well only if $\#$ data $\gg$ $\#$ fitted parameters.

-rather time consuming (in χ^2 fits, SUSY spectrum calculator called thousand of times...)

+ Pb if too much parameters: hardly fitting general MSSM (22 parameters) even if (optimistically) all sparticle masses, cross-sections known!!

-Even in mSUGRA, a standard fit (probably?) not very good if only a few (4,5) sparticles discovered...

- Alternative: "Un-diagonalization": from physical masses to basic (Lagrangian) parameters (at EWSB scale)

(then RG evolution up to high (GUT) scale)

• Analytical, if possible

-At tree-level such inversion works (Moultaka, JLK '98)

extended by Kalinowski et al, P. Zerwas et al, many others '98-01

-Transparent, fast and useful guide to "blind fit" analysis

(exhibit e.g. "mass sum rules" $\rightarrow$ cross-check)

Aim: extend tree-level inversion to more realistic reconstruction, including rad. corr., semi-realistic input scenario choice (LHC, or LHC+LC), etc

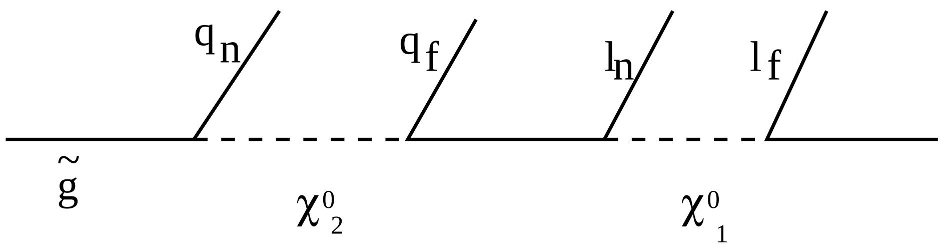
Part of ”SPA” project (”global analysis program” cf. SFITTER, FITTINO), but:

-less ambitious: theoretical exercise to begin (masses only input, true events/data not used)

-more ambitious: As much as possible analytical expressions.

(in contrast FITTINO uses some tree-level inversions, but only as starting point to “guide” standard fitting procedure)

Possible strategy: at LHC, can determine quite accurately some masses from "kinematical endpoints" analysis of (2-body) cascade decays



→quite precise $m_{\tilde{g}}, m_{N_2}, m_{N_1}, m_{\tilde{q}_L}, m_{\tilde{l}_R}, m_{b_1}$

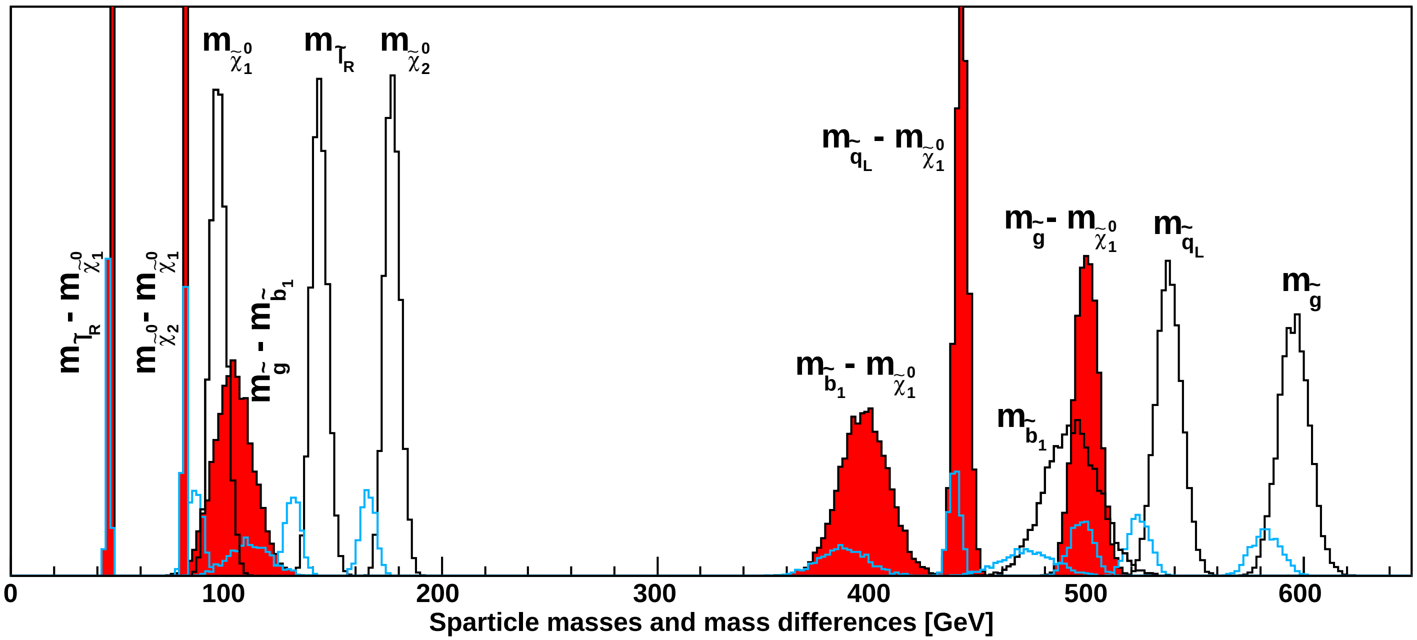
(Allanach et al '01, Gjelsen, Miller, Osland '05)

+M_h, +$\chi^{\pm}$,...

At ILC , access to chargino, neutralino masses and coupling via pair production

But we also like to consider minimal (pessimistic) scenario..

say, only M_h, m_{N_2}, m_{N_1}, $m_{\tilde{q}}$.. suffi cient for mSUGRA??..



[B.K. Gjelsten, D.J. Miller, P. Osland, hep-ph/0501033]

2. Inversion in Gaugino sector

-Scenario S1: input $M_{\chi_1^+}$, $M_{\chi_2^+}$, M_{N_2} - Chargino mass matrix:

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

-Inversion gives:

$$\mu^2 = \frac{1}{2}\Big(M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2m_W^2 \pm \big[(M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2m_W^2)^2 - 4(m_W^2 \sin 2\beta \pm M_{\chi_1^+} M_{\chi_2^+})^2\big]^{1/2}\Big)$$

$$M_2 = [M^2_{\chi_1^+} + M^2_{\chi_2^+} - 2m_W^2 - \mu^2]^{1/2}$$

-Pb: needs both $M_{\chi_1^+}, M_{\chi_2^+}$ (may be difficult at LHC..)

+$\tan\beta$ (assumed known from another sector: e.g. Higgs?)

+Difficulties: $M_2 \leftrightarrow \mu$ symmetric! $\rightarrow$ has to consider $\mu < M_2$ **and** $M_2 > \mu$ (cf mSUGRA) + quadratic μ Eq. $\rightarrow$ 4-fold ambiguities..

- Neutralino mass matrix:

$$M_N = \begin{pmatrix} M_1 & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M_2 & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -\mu \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

Trick: use the 4 invariants (under diagonalization):

$$Tr M_N, \quad \frac{(Tr M_N)^2}{2} - \frac{Tr(M_N^2)}{2}$$

$$\frac{(Tr M_N)^3}{6} - \frac{Tr M \;\; Tr(M_N^2)}{2} + \frac{Tr(M_N^3)}{3}, \quad Det M_N$$

-Gives M_1 (unique solution) as:

$$M_1 = -\frac{P_{2i}^2 + P_{2i}(\mu^2 + M_Z^2 + M_2 S_{2i} - S_{2i}^2) + \mu M_Z^2 M_2 s_w^2 \sin 2\beta}{P_{2i}(S_{2i} - M_2) + \mu(c_w^2 M_Z^2 \sin 2\beta - \mu M_2)}$$

- $S_{2i} \equiv \tilde{M}_{N_2} + \tilde{M}_{N_i}$ $\quad$ $P_{2i} \equiv \tilde{M}_{N_2}\tilde{M}_{N_i}$
- $\tilde{M}_{N_i} \equiv$ ANY of the remaining neutralinos

-Scenario S2: input $M_{\chi_1^+}$, M_{N_1}, M_{N_2}

previous ambiguities partially solved

(But needs iteration on e.g. M_2)

Same basic equations but different input/output

-If gaugino unifi cation $M_1 = M_2 = M_3$ (GUT scale):

determines e.g. $\tan\beta$, μ from $M_{\tilde{g}}$, M_{N_1}, M_{N_2}

Incorporating Radiative Corrections

To very good approximation, keeps tree-level form

$$\mathcal{M}_C = \begin{pmatrix} M_2 + \Delta M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu + \Delta\mu \end{pmatrix}$$

Similarly for Neutralino matrix

$\rightarrow$ preserves analytic form of inversion

(But of course $\Delta\mu$, ΔM_1, ΔM_2 depend on other sector: squarks, sleptons, ..)

Higgs sector

$$\begin{pmatrix} m_Z^2\ \cos^2\beta + m_A^2\ \sin^2\beta - \Pi_{11} + \frac{t_1}{v_1} & -(m_Z^2 + m_A^2)\ \sin\beta\cos\beta - \Pi_{12} \\ -(m_Z^2 + m_A^2)\ \sin\beta\cos\beta - \Pi_{12} & m_Z^2\ \sin^2\beta + {}_r m_A^2\ \cos^2\beta - \Pi_{22} + \frac{t_2}{v_2} \end{pmatrix}$$

Inversion, including Rad. Corr., gives

$$\bar{M}_A^2 = \frac{\bar{m}_h^2(m_Z^2 - \bar{m}_h^2)}{m_Z^2\cos^2 2\beta - \bar{m}_h^2} + R.C.$$

NB:where (leading RC)

$$\bar{m}_h^2 = M_{h,pole}^2 - 3g^2\frac{m_t^4}{8\pi^2 m_W^2\sin^2\beta}(\ln\frac{m_{\tilde{t}_1}m_{\tilde{t}_2}}{m_t} + ..)$$

But full 1(2)-loop Higgs R.C. preserve *linear* $\bar{M}_A^2$ solution!!

Next:

$$m_{H_u}^2 = \frac{\bar{M}_A^2 - (\mu^2 + m_Z^2/2)(\tan^2\beta - 1)}{1 + \tan^2\beta} \quad m_{H_d}^2 = M_A^2 - M_{H_u}^2 - 2\mu^2$$

Squarks/sleptons sector

$$\begin{pmatrix} M_Q^2 + m_t^2 + (\frac{2}{3}m_W^2 - \frac{1}{6}m_Z^2)\cos 2\beta + RC & m_t\,(A_t - \mu/\tan\beta) + RC \\ m_t(A_t - \mu/\tan\beta) + RC & m_{t_R}^2 + m_t^2 - \frac{2}{3}(m_W^2 - m_Z^2)\cos 2\beta + RC \end{pmatrix}$$

$$A_t = \frac{\mu}{\tan\beta} + (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)\frac{\sin 2\theta_{\tilde{t}}}{2\,m_t}$$

$$M_Q^2 = m_{\tilde{t}_1}^2 \cos^2\theta_{\tilde{t}} + m_{\tilde{t}_2}^2 \sin^2\theta_{\tilde{t}} - m_t^2 - \cos(2\beta)\,(4m_W^2 - m_Z^2)/6 \quad (1)$$

$$M_R^2 = m_{\tilde{t}_1}^2 \sin^2\theta_{\tilde{t}} + m_{\tilde{t}_2}^2 \cos^2\theta_{\tilde{t}} - m_t^2 + \frac{2}{3}\cos(2\beta)\,(m_W^2 - m_Z^2)$$

NB alternative (or cross-check) determination of $\tan\beta$ if several squark masses known (squark mass sum rules):

$$m_W^2 \cos 2\beta = m_b^2 - m_t^2 + m_{\tilde{t}_1}^2 c_t^2 + m_{\tilde{t}_2}^2 s_t^2 - m_{\tilde{b}_1}^2 c_b^2 - m_{\tilde{b}_2}^2 s_b^2$$

Renormalization Group "bottom-up" evolution

The RGE are evidently invertible, but to evolve MSSM parameters from EWSB scale UP to GUT scale, while matching low-energy (gauge+yukawa) data is a bit involved.

RGE bottom-up option installed in SuSpect 2.3
(already used in e.g. SFITTER)

A case study: mSUGRA SPS1a point

$m_{1/2} = 250$ GeV, $m_0 = -A_0 = 100$ GeV, $\tan\beta = 10$, $\mu > 0$

Preliminary results (no RC in chargino, stops yet!!)

```
BLOCK Au Q=  2.52600000E+16  # The trilinear coupli
  1  1     -9.54515109E+01   # A_u(Q) DRbar
  3  3     -9.17357973E+01   # A_t(Q) DRbar
  1  1     -9.97765857E+01   # A_d(Q) DRbar
  3  3     -9.85197420E+01   # A_b(Q) DRbar
  1  1     -9.97545874E+01   # A_e(Q) DRbar
  3  3     -9.96269348E+01   # A_tau(Q) DRbar
BLOCK MSOFT Q=  2.52600000E+16  # soft SUSY breakin
         1      2.49126619E+02   # M_1
         2      2.50519425E+02   # M_2
```

```
 3      2.50231883E+02   # M_3
21      1.00450023E+04   # M^2_Hd
22      5.71579396E+02   # M^2_Hu
31      1.01976186E+02   # M_eL
33      1.00085423E+02   # M_tauL
34      9.85548280E+01   # M_eR
36      9.86746478E+01   # M_tauR
41      9.60221905E+01   # M_q1L
43      8.05690580E+01   # M_q3L
44      1.00730369E+02   # M_uR
46      6.02183953E+01   # M_tR
47      9.86457209E+01   # M_dR
49      9.44621807E+01   # M_bR
```

Work under construction...

Plan to study propagation of exp + th errors on masses, etc.

comparison with standard top-down fit results (for same numbers of input masses) Hope to have more numerical illustration for Barcelone GDR meeting!!..