

# New Physics models giving rise to LFV

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# Outline

- LFV in minimally extended SM
- What to expect from observation of LFV
- LFV in BSM models and its connections to other phenomena

(LFV= lepton flavor violation)

(cLFV= charged lepton flavor violation)


# Standard Model (SM)


Q  
U  
A  
R  
K  
S


<b>UP</b> mass $2,3 \text{ MeV}/c^2$ charge $\frac{2}{3}$ spin $\frac{1}{2}$ 	<b>CHARM</b> mass $1,275 \text{ GeV}/c^2$ charge $\frac{2}{3}$ spin $\frac{1}{2}$ 	<b>TOP</b> mass $173,07 \text{ GeV}/c^2$ charge $\frac{2}{3}$ spin $\frac{1}{2}$ 
<b>DOWN</b> mass $4,8 \text{ MeV}/c^2$ charge $-\frac{1}{3}$ spin $\frac{1}{2}$ 	<b>STRANGE</b> mass $95 \text{ MeV}/c^2$ charge $-\frac{1}{3}$ spin $\frac{1}{2}$ 	<b>BOTTOM</b> mass $4,18 \text{ GeV}/c^2$ charge $-\frac{1}{3}$ spin $\frac{1}{2}$ 

L  
E  
P  
T  
O  
N  
S


<b>ELECTRON</b> mass $0,511 \text{ MeV}/c^2$ charge $-1$ spin $\frac{1}{2}$ 	<b>MUON</b> mass $105,7 \text{ MeV}/c^2$ charge $-1$ spin $\frac{1}{2}$ 	<b>TAU</b> mass $1,777 \text{ GeV}/c^2$ charge $-1$ spin $\frac{1}{2}$ 
<b>ELECTRON NEUTRINO</b> mass $<2,2 \text{ eV}/c^2$ charge $0$ spin $\frac{1}{2}$ 	<b>MUON NEUTRINO</b> mass $<0,17 \text{ MeV}/c^2$ charge $0$ spin $\frac{1}{2}$ 	<b>TAU NEUTRINO</b> mass $<15,5 \text{ MeV}/c^2$ charge $0$ spin $\frac{1}{2}$ 


<b>GLUON</b> mass $0$ charge $0$ spin $1$ 
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<b>HIGGS BOSON</b> mass $126 \text{ GeV}/c^2$ charge $0$ spin $0$ 
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<b>PHOTON</b> mass $0$ charge $0$ spin $1$ 
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G  
A  
U  
G  
E  
B  
O  
S  
O  
N  
S

<b>Z BOSON</b> mass $91,2 \text{ GeV}/c^2$ charge $0$ spin $1$ 
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<b>W BOSON</b> mass $80,4 \text{ GeV}/c^2$ charge $\pm 1$ spin $1$ 
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# Global Symmetries in the SM

$$\mathcal{G}_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$Q_{L_i} \sim (3, 2)_{+1/6}, \quad u_{R_i} \sim (3, 1)_{+2/3}, \quad d_{R_i} \sim (3, 1)_{-1/3}$$

$$L_{L_i} \sim (1, 2)_{-1/2}, \quad \ell_{R_i} \sim (1, 1)_{-1}$$

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{Hig}}$$

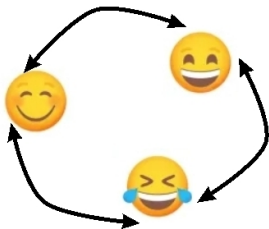
$$\mathcal{L}_{\text{Yuk}} \rightarrow 0 : \quad \mathcal{G}_{\text{flavor}} = U(3)_q^3 \times U(3)_{\text{lep}}^2$$

$$m_{\text{fermions}}^{\text{charged}} \neq 0 : \quad \mathcal{G}_{\text{flavor}}^{(\text{global})} \rightarrow U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- individual lepton-flavor numbers are conserved by the SM Lagrangian

$B + L$  broken at quantum level; 't Hooft 1976

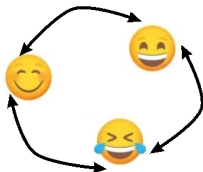
# Discovery of Neutrino Oscillations: $m_\nu \neq 0$



$$\begin{aligned}
 Y_\ell \not\propto 1 : U(3)_L \times U(3)_\ell &\rightarrow U(1)_e \times U(1)_\mu \times U(1)_\tau \\
 Y_u \not\propto 1 : U(3)_Q \times U(3)_u &\rightarrow U(1)_u \times U(1)_c \times U(1)_t \\
 Y_d \not\propto 1 : U(3)_Q \times U(3)_d &\rightarrow U(1)_d \times U(1)_s \times U(1)_b \\
 [Y_u, Y_d] \neq 0 : U(1)_q^6 &\rightarrow U(1)_B \quad (V_{\text{CKM}}) \\
 [Y_\ell, Y_\nu] \neq 0 : U(1)_\ell^3 &\rightarrow U(1)_L \quad (V_{\text{PMNS}})
 \end{aligned}$$

# Neutrino oscillations: Consequences

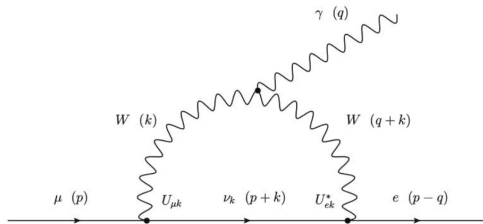
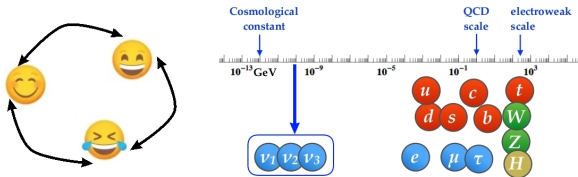
- Individual lepton flavor no longer conserved  
(Transition between flavors)
- Direct consequence  $\rightarrow$  cLFV
- Total  $L$  could still be conserved



$$m_{\text{Dirac}}^{\nu} \neq 0 : \mathcal{G}_{\text{flavor}}^{(\text{global})} \rightarrow U(1)_B \times U(1)_L$$

$$m_{\text{Majorana}}^{\nu} \neq 0 : \mathcal{G}_{\text{flavor}}^{(\text{global})} \rightarrow U(1)_B$$

# cLFV with SM Particles



$$\text{Br}(\mu \rightarrow e \gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U_{\mu i}^* U_{ei} \frac{\Delta m_{i1}^2}{M_W^2} \right|^2 < 10^{-54},$$

•  $\mathcal{G}_{\text{flavor}}^{(\text{global})} \supset \sim U(1)_e \times U(1)_\mu \times U(1)_\tau$

# Neutrino mass & cLFV

- $m_\nu \neq 0 \Rightarrow \nu$ FV processes
- $\nu$ FV  $\Rightarrow$  cLFV (at some order in perturbation theory)
- $m_\nu$  mechanism depends on new physics (NP) scenario
- cLFV depends completely on NP model
- NP can lead to cLFV  $\gg$  the ones in the SM (with  $m_\nu \neq 0$ )
- observation of cLFV  $\Rightarrow$  direct implication of NP
- Standard convention: processes with  $\Delta L = 0$
- Two  $U(1)$  factors:  
$$U(1)_e \times U(1)_\mu \times U(1)_\tau \rightarrow U(1)_{\mu-\tau} \times U(1)_{\mu+\tau-2e}$$



# cLFV Grouping

- Model independent expectations from observations?

Grouping	Process	Current bound	Future sensitivity
$\Delta(L_e - L_\mu) = 2$	$\mu \rightarrow e\gamma$	$4.2 \times 10^{-13}$	$4 \times 10^{-14}$
	$\mu \rightarrow e\bar{e}e$	$1.0 \times 10^{-12}$	$10^{-16}$
	$\mu \rightarrow e \text{ conv.}$	$\mathcal{O}(10^{-12})$	$10^{-17}$
	$h \rightarrow e\bar{\mu}$	$3.5 \times 10^{-4}$	$2 \times 10^{-4}$
	$Z \rightarrow e\bar{\mu}$	$7.5 \times 10^{-7}$	–
	$had \rightarrow e\bar{\mu}(had)$	$4.7 \times 10^{-12}$	$10^{-12}$
$\Delta(L_e - L_\tau) = 2$	$\tau \rightarrow e\gamma$	$3.3 \times 10^{-8}$	$10^{-9}$
	$\tau \rightarrow e\bar{e}e$	$2.7 \times 10^{-8}$	$10^{-9}$
	$\tau \rightarrow e\bar{\mu}\mu$	$2.7 \times 10^{-8}$	$10^{-9}$
	$\tau \rightarrow e \text{ had}$	$\mathcal{O}(10^{-8})$	$10^{-9}$
	$h \rightarrow e\bar{\tau}$	$6.9 \times 10^{-3}$	$5 \times 10^{-3}$
	$Z \rightarrow e\bar{\tau}$	$9.8 \times 10^{-6}$	–
	$had \rightarrow e\bar{\tau}(had)$	$\mathcal{O}(10^{-6})$	–
$\Delta(L_\mu - L_\tau) = 2$	$\tau \rightarrow \mu\gamma$	$4.4 \times 10^{-8}$	$10^{-9}$
	$\tau \rightarrow \mu\bar{e}e$	$1.8 \times 10^{-8}$	$10^{-9}$
	$\tau \rightarrow \mu\bar{\mu}\mu$	$2.1 \times 10^{-8}$	$10^{-9}$
	$\tau \rightarrow \mu \text{ had}$	$\mathcal{O}(10^{-8})$	$10^{-9}$
	$h \rightarrow \mu\bar{\tau}$	$1.2 \times 10^{-2}$	$5 \times 10^{-3}$
	$Z \rightarrow \mu\bar{\tau}$	$1.2 \times 10^{-5}$	–
	$had \rightarrow \mu\bar{\tau}(had)$	$\mathcal{O}(10^{-6})$	–
$\Delta(L_\mu + L_\tau - 2L_e) = 6$	$\tau \rightarrow ee\bar{\mu}$	$1.5 \times 10^{-8}$	$10^{-9}$
$\Delta(L_\tau + L_e - 2L_\mu) = 6$	$\tau \rightarrow \mu\mu\bar{e}$	$1.7 \times 10^{-8}$	$10^{-9}$
$\Delta(L_e + L_\mu - 2L_\tau) = 6$	$\mu e \rightarrow \tau\tau$	–	–

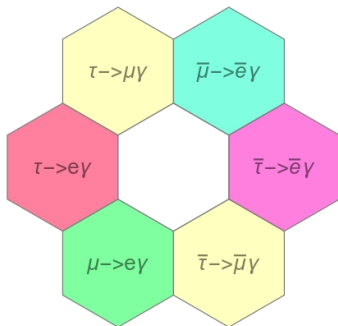
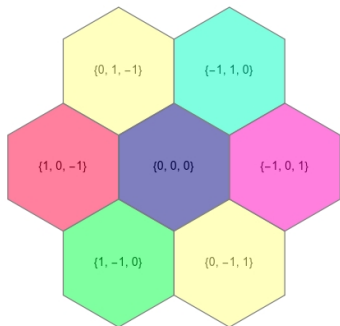
# cLFV grouping: $U(1)_e \times U(1)_\mu \times U(1)_\tau$

- observation of 1 process (1 linear combination is violated)  $\Rightarrow$  all other processes within that group (rates: model dependent)
- it does not violate the other groups
- two unequal vectors [quantum number] need to be observed to know that all cLFV (typically)

Representative	$\Delta(e, \mu, \tau)$	Vector ( $e + \mu + \tau = 0$ )
$\mu \rightarrow e\gamma$	(1,-1,0)	minimal
$\tau \rightarrow e\gamma$	(1,0,-1)	minimal
$\tau \rightarrow \mu\gamma$	(0,1,-1)	minimal
$\tau \rightarrow ee\bar{\mu}$	(2,-1,-1)	next to minimal
$\tau \rightarrow \mu\mu\bar{e}$	(-1,2,-1)	next to minimal
$\mu e \rightarrow \tau\tau$	(-1,-1,2)	next to minimal

# cLFV on the Hexagonal Grid

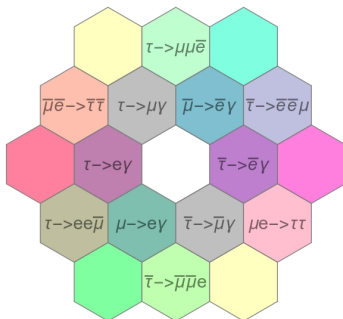
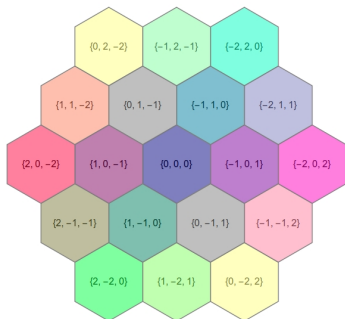
Representative	$\Delta(e, \mu, \tau)$	Vector ( $e + \mu + \tau = 0$ )
$\mu \rightarrow e\gamma$	$(1, -1, 0)$	minimal
$\tau \rightarrow e\gamma$	$(1, 0, -1)$	minimal
$\tau \rightarrow \mu\gamma$	$(0, 1, -1)$	minimal



Each Hexagon preserves  $L$  number (cube coordinates)

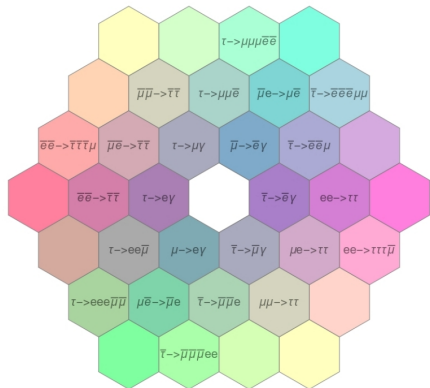
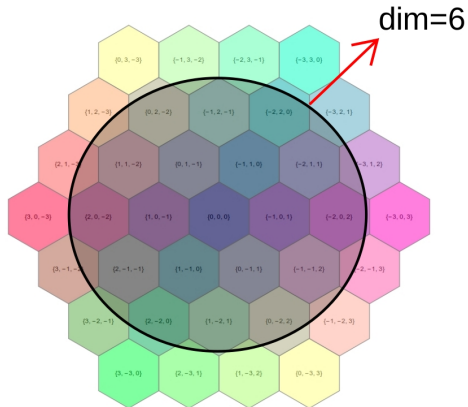
# cLFV on the Hexagonal Grid

Representative	$\Delta(e, \mu, \tau)$	Vector ( $e + \mu + \tau = 0$ )
$\tau \rightarrow ee\bar{\mu}$	(2,-1,-1)	next to minimal
$\tau \rightarrow \mu\mu\bar{e}$	(-1,2,-1)	next to minimal
$\mu e \rightarrow \tau\tau$	(-1,-1,2)	next to minimal



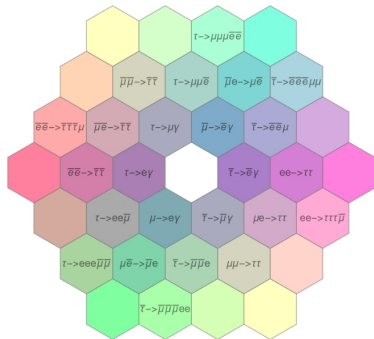
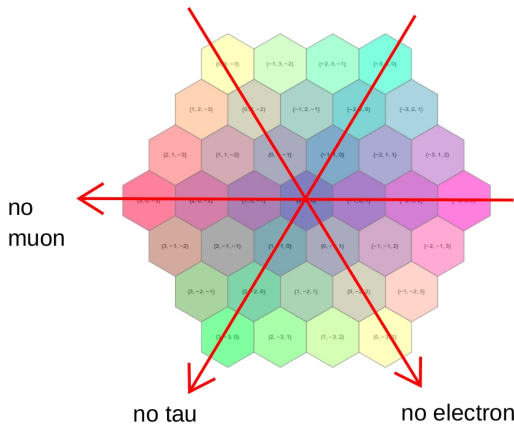
# cLFV on the Hexagonal Grid

## Beyond Next to Minimal Vectors

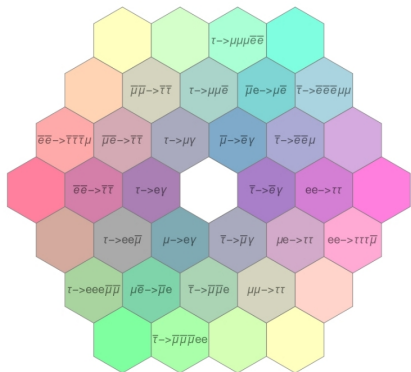
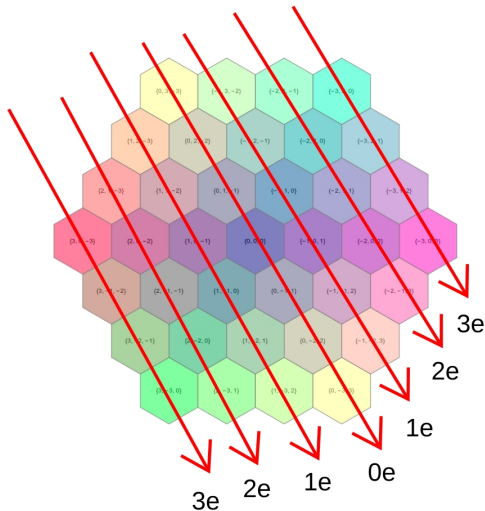


examples:  $(1, -1, 0): \sqrt{2}$ ;  $(2, -1, -1): \sqrt{6}$ ;  $(2, -2, 0): 2\sqrt{2}$ ;  $(3, -2, -1): \sqrt{14}$ ;

# cLFV on the Hexagonal Grid



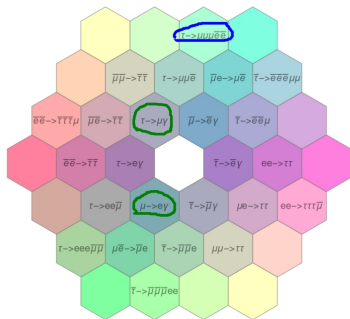
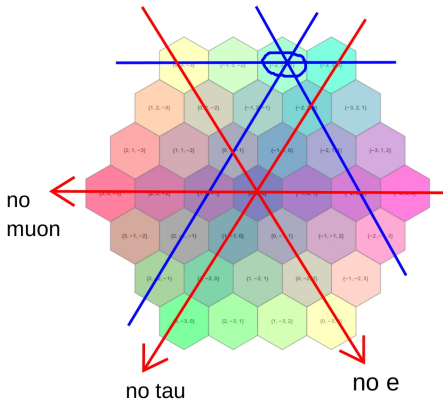
# cLFV on the Hexagonal Grid



(similar for muon, tau)

# cLFV on the Hexagonal Grid

- If  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  are observed: all other cLFV must occur
- Any other cLFV =  $x(1, -1, 0) + y(0, 1, -1)$ ;  $x, y \in \mathbb{Z}$
- $x = -2, y = 1 \Rightarrow cLFV = (-2, 3, -1) \Rightarrow \tau \rightarrow \mu\mu\mu\bar{e}\bar{e}$





# cLFV: Orthogonal

Representative	$\Delta(e, \mu, \tau)$	Vector ( $e + \mu + \tau = 0$ )
$\mu \rightarrow e\gamma$	(1,-1,0)	minimal
$\tau \rightarrow e\gamma$	(1,0,-1)	minimal
$\tau \rightarrow \mu\gamma$	(0,1,-1)	minimal
$\tau \rightarrow ee\bar{\mu}$	(2,-1,-1)	next to minimal
$\tau \rightarrow \mu\mu\bar{e}$	(-1,2,-1)	next to minimal
$\mu e \rightarrow \tau\tau$	(-1,-1,2)	next to minimal

$\vec{a} \cdot \vec{b} = 0$  :

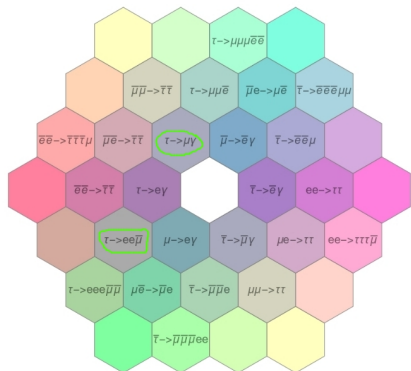
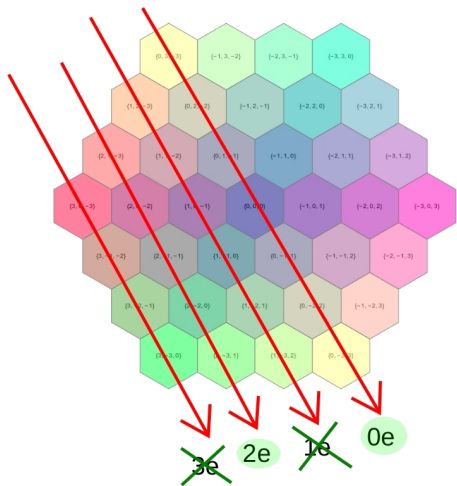
$$(\mu \rightarrow e\gamma) \perp (\mu e \rightarrow \tau\tau)$$

$$(\tau \rightarrow e\gamma) \perp (\tau \rightarrow \mu\mu\bar{e})$$

$$(\tau \rightarrow \mu\gamma) \perp (\tau \rightarrow ee\bar{\mu})$$

# cLFV on the Hexagonal Grid

$$(\tau \rightarrow \mu\gamma) \perp (\tau \rightarrow ee\bar{\mu})$$

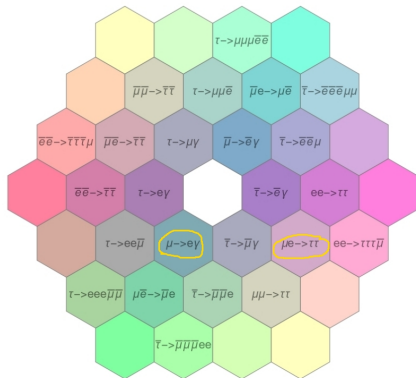
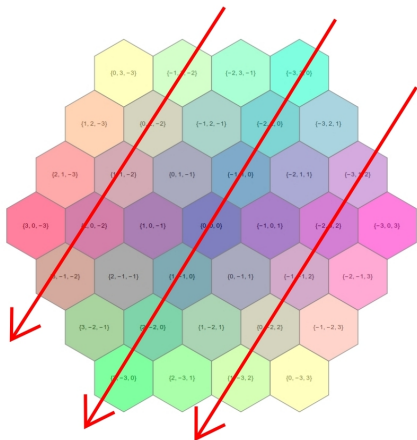


Leftover  $\mathbb{Z}_2^e$  (further observation required ...)

(no solution with  $x, y \in \mathbb{Z}$ )

# cLFV on the Hexagonal Grid

$$(\mu \rightarrow e\gamma) \perp (\mu e \rightarrow \tau\tau)$$



even taus

Leftover  $\mathcal{Z}_2^T$  (further observation required ...)

# New Physics Models for Neutrino Mass

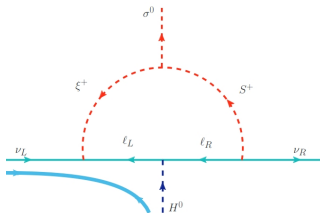
# cLFV in BSM: Simplest Dirac Scenario

- NP states: singlets  $\nu_{R_i}$ ,  $L = +1$
- $\mathcal{L} \supset Y_\nu \bar{L} H \nu_R$
- $m_\nu \Rightarrow Y_\nu \sim 10^{-12}$
- cLFV highly suppressed

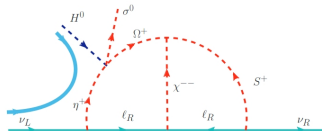
In the presence of other BSM states, one can achieve **unsuppressed cLFV** with **Dirac neutrinos** (next page)

# Radiative Dirac schemes

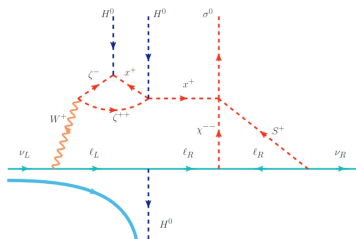
Topology	$SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
T-I-F-i	$\sigma^0(1, 0, 3)$
	$S^+(1, 1, 5)$
	$\xi^+(1, 1, 2)$



Topology	$SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
T-II-S-i	$\sigma^0(1, 0, 3)$
	$S^+(1, 1, 5)$
	$\chi^{++}(1, 2, 2)$
	$\eta(2, \frac{1}{2}, 0)$
	$\Omega^+(1, 1, -3)$



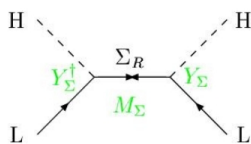
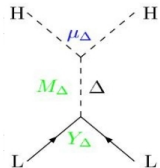
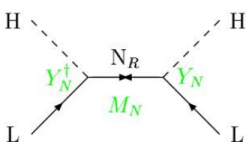
Topology	$SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
T-III-F-i	$\sigma^0(1, 0, 3)$
	$S^+(1, 1, 5)$
	$\chi^{++}(1, 2, 2)$
	$\zeta(2, -\frac{3}{2}, 0)$
	$x^+(1, 1, 0)$



# Tree Level Majorana Scheme

- $|\Delta L| = 2$
- $\mathcal{L} \supset \frac{c_5}{\Lambda} LLHH \rightarrow m_\nu \sim \frac{c_5 v^2}{\Lambda}$

Weinberg 1979



$$N_R \sim (1, 1, 0)_{+1}^F,$$

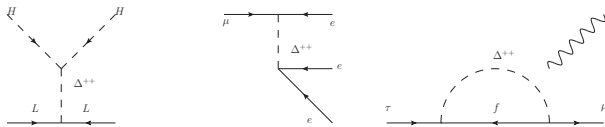
$$\Delta \sim (1, 3, 1)_{-2}^S,$$

$$\Sigma_R \sim (1, 3, 0)_{+1}^F$$

Minkowski 1977; ...

(see talk by Enrique Fernandez Martinez)

# cLFV in Type-II Seesaw



- $\Delta \sim (1, 3, 1)_{-2}^S \supset (\Delta^{++}, \Delta^+, \Delta^0)$
- tree-level  $\mu \rightarrow eee, \dots$
- loop-level  $\mu \rightarrow e\gamma, \dots$

$$\mathcal{L} \supset \mu \tilde{H}^T \epsilon \Delta \tilde{H} + \bar{L}^c \epsilon Y_\Delta \Delta L$$

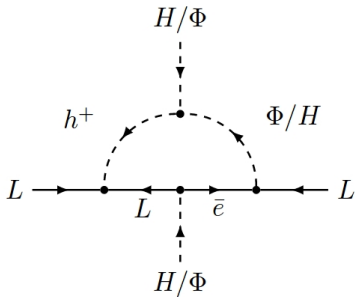
$$\mathcal{L}_5 \rightarrow m_\nu \sim Y_\Delta \mu \frac{v^2}{m_\Delta^2} \sim 0.1 \text{eV} \rightarrow Y_\Delta \sim 10^{-2.5} \Rightarrow \frac{\mu}{m_\Delta (\text{TeV})} \sim 10^{-9}$$

$$\mathcal{L}_6 \supset \frac{Y_\Delta^\dagger Y_\Delta}{2m_\Delta^2} (\bar{L} \gamma_\mu L) (\bar{L} \gamma^\mu L) \rightarrow \text{Br}(\mu \rightarrow eee) \sim \frac{Y_\Delta^4}{m_\Delta^4 G_F^2} \rightarrow \text{saturates}_{\text{expt}}$$

Abada et. al. 2007; Dinh et. al. 2012; UV completion– Antusch, Hinze, Saad 2023

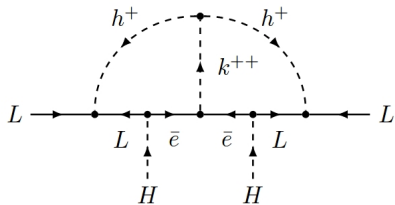


# Radiative Majorana Scheme



2HDM +  $h^+(1, 1, 1)$

Zee 1980

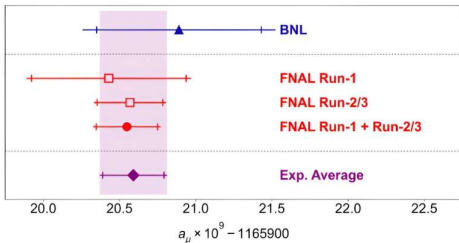


$h^+(1, 1, 1) + k^{++}(1, 1, 2)$

Babu 1988

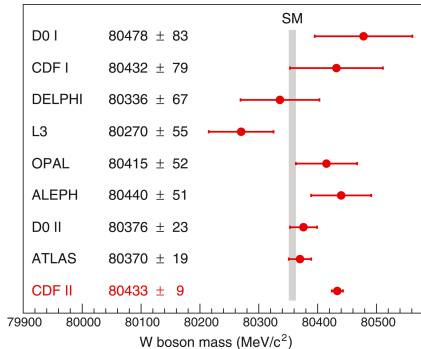
## cLFV and its connection to other phenomena

# $m_W, (g - 2)_\mu, \text{cLFV}$ in the Zee model



$\sim 5\sigma$

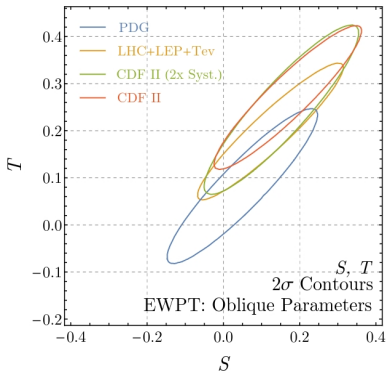
Phys. Rev. Lett. 131, 161802



$\sim 7\sigma$

Science 376 no. 6589, (2022) 170–176

# $m_W, (g - 2)_\mu, \text{cLFV in the Zee model}$



Asadi et. al. 2022

$$M_W^2 = M_{W,SM}^2 \left[ 1 + \frac{\alpha_{em} \left( c_W^2 T - \frac{1}{2} S + \frac{c_W^2 - s_W^2}{4s_W^2} U \right)}{c_W^2 - s_W^2} \right]$$

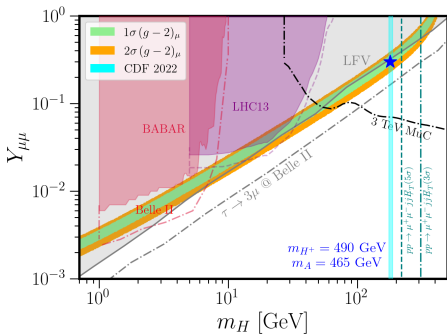
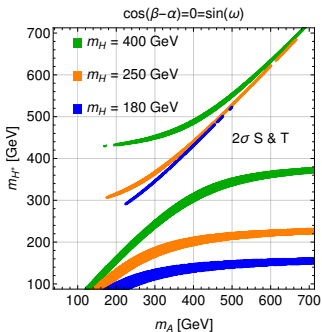
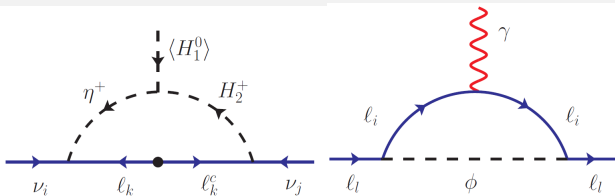
Grimus et. al. 2008

Coupling of the 2nd Higgs Doublet:

$$\begin{pmatrix} 0 & y_{e\mu} & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}, \quad \begin{pmatrix} * & 0 & * \\ 0 & y_{\mu\mu} & 0 \\ 0 & * & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & * & 0 \\ * & 0 & y_{\mu\tau} \\ 0 & y_{\tau\mu} & * \end{pmatrix}$$

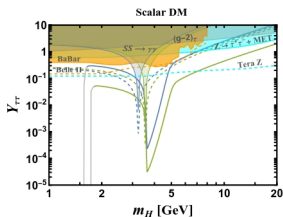
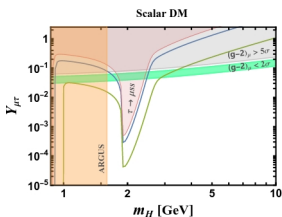
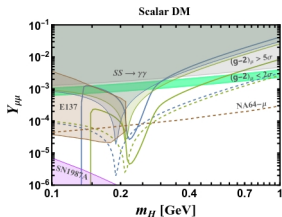
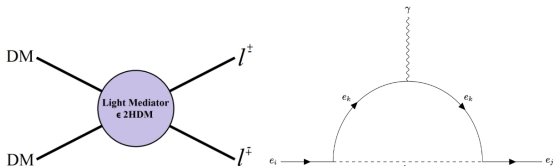
Heeck, Saad et. al. 2022

# $m_W, (g-2)_\mu, \text{cLFV in the Zee model}$



# cLFV in Zee Model + sub-GeV Singlet DM

- Light mediator from two-Higgs-doublet model
- DM mass  $\sim$  lepton mass
- Natural resolution to  $(g-2)_\mu$
- MEG-II will fully test  $\mu \rightarrow e\gamma$



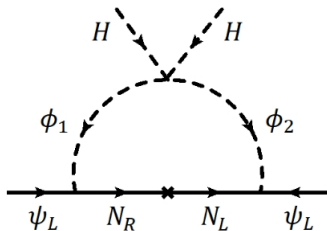
# sub-GeV DM & cLFV in Scotogenic Framework

$$SU(2)_L \times U(1)_Y \times Z_3$$

$$N_{R,L} \sim (1, 0; \omega),$$

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \sim (2, \frac{1}{2}; \omega),$$

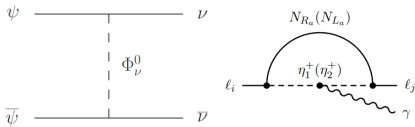
$$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \sim (2, \frac{1}{2}; \omega^2),$$



$$(M_N, m_{S_1}) = (20, 100) \times 10^{-3} \text{ GeV},$$

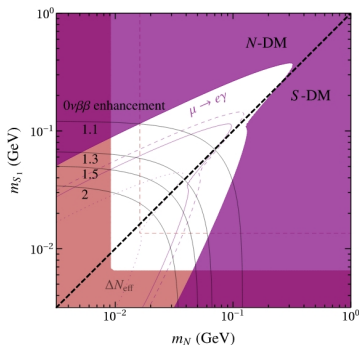
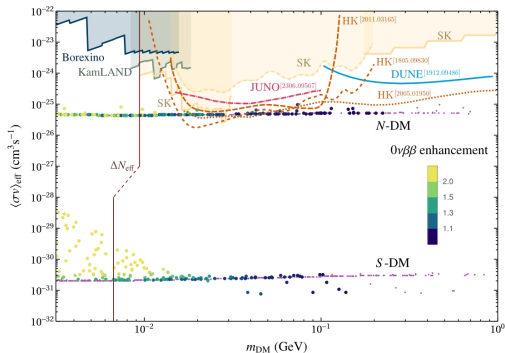
$$(m_{S_2}, m_{\phi_1^+}, m_{\phi_2^+}) = (110, 110, 110) \text{ GeV}.$$

$$Y_1 = 10^{-2} \begin{pmatrix} -0.24402 \\ -1.76064 \\ 3 \end{pmatrix}, \quad Y_2 = 10^{-7} \begin{pmatrix} -3.31953 \\ 8.39950 \\ -3.22219 \end{pmatrix}$$



Hermes, Jana, Vishnu, Saad 2023

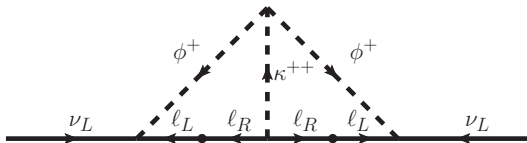
# sub-GeV DM & cLFV in Scotogenic Framework



Herns, Jana, Vishnu, Saad 2023

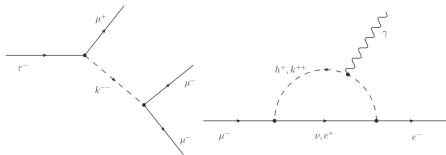


# Zee-Babu States at the Muon Collider



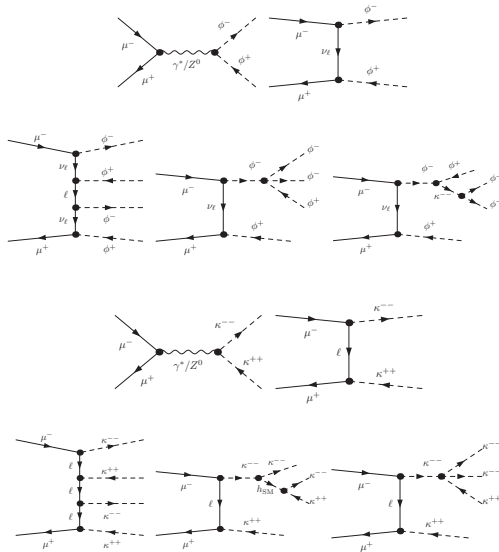
Benchmark point	BP1	BP2	BP3	BP4	BP5
<i>Parameters</i>					
$m_\kappa$ (GeV)	1250	1250	2500	1250	3750
$m_\phi$ (GeV)	1250	2500	1250	3750	1250
$\mu$ (GeV)	1903.01	1957.01	1994.75	1730.09	2067.06
$f_{e\mu}$	-0.03809	-0.06687	-0.02157	-0.1026	-0.03558
$f_{e\tau}$	0.02037	0.03577	0.02918	0.05487	0.01925
$f_{\mu\tau}$	0.06297	0.11052	0.05291	0.16973	0.05893
$g_{ee}$	-0.19669	-0.02474	-0.02499	-0.01731	-0.00269
$g_{e\mu}$	$9.89 \times 10^{-6}$	$-5.05 \times 10^{-4}$	-0.00237	-0.00160	0.00132
$g_{e\tau}$	0.00462	0.00289	0.04409	0.02005	-0.00699
$g_{\mu\mu}$	0.48	0.487	0.99	0.488	1.0
$g_{\mu\tau}$	0.02542	0.02579	0.05029	0.02582	0.05270
$g_{\tau\tau}$	0.00222	0.00225	0.00420	0.00225	0.00457

# Unsuppressed cLFV

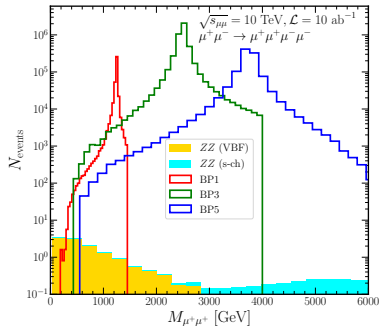
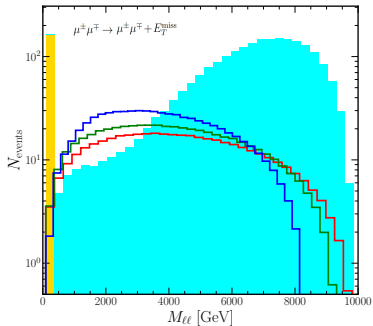


Benchmark point	BP1	BP2	BP3	BP4	BP5
$BR(\ell_i \rightarrow \ell_j \gamma)$					
$BR(\mu \rightarrow e \gamma)$	$2.93 \times 10^{-13}$	$2.15 \times 10^{-13}$	$3.73 \times 10^{-13}$	$3.06 \times 10^{-13}$	$2.29 \times 10^{-13}$
$BR(\tau \rightarrow e \gamma)$	$5.19 \times 10^{-13}$	$9.77 \times 10^{-14}$	$6.62 \times 10^{-14}$	$1.56 \times 10^{-13}$	$1.22 \times 10^{-13}$
$BR(\tau \rightarrow \mu \gamma)$	$6.51 \times 10^{-11}$	$6.90 \times 10^{-11}$	$6.74 \times 10^{-11}$	$6.91 \times 10^{-11}$	$1.50 \times 10^{-11}$
$BR(\ell_i \rightarrow \ell_j \ell_k \ell_k)$					
$BR(\mu^- \rightarrow e^+ e^- e^-)$	$2.85 \times 10^{-15}$	$1.17 \times 10^{-13}$	$1.65 \times 10^{-13}$	$5.78 \times 10^{-13}$	$1.18 \times 10^{-16}$
$BR(\tau^- \rightarrow e^+ e^- e^-)$	$1.11 \times 10^{-10}$	$6.88 \times 10^{-13}$	$1.02 \times 10^{-11}$	$1.62 \times 10^{-11}$	$5.87 \times 10^{-16}$
$BR(\tau^- \rightarrow e^+ e^- \mu^-)$	$5.06 \times 10^{-19}$	$5.74 \times 10^{-16}$	$1.84 \times 10^{-13}$	$2.67 \times 10^{-13}$	$2.84 \times 10^{-16}$
$BR(\tau^- \rightarrow e^+ \mu^- \mu^-)$	$6.59 \times 10^{-10}$	$2.66 \times 10^{-10}$	$1.59 \times 10^{-8}$	$1.28 \times 10^{-8}$	$8.11 \times 10^{-11}$
$BR(\tau^- \rightarrow \mu^+ e^- e^-)$	$3.26 \times 10^{-9}$	$5.32 \times 10^{-11}$	$1.29 \times 10^{-11}$	$2.61 \times 10^{-11}$	$3.24 \times 10^{-14}$
$BR(\tau^- \rightarrow \mu^+ e^- \mu^-)$	$1.65 \times 10^{-17}$	$4.44 \times 10^{-14}$	$2.32 \times 10^{-13}$	$4.46 \times 10^{-13}$	$1.57 \times 10^{-14}$
$BR(\tau^- \rightarrow \mu^+ \mu^- \mu^-)$	$1.94 \times 10^{-8}$	$2.06 \times 10^{-8}$	$2.02 \times 10^{-8}$	$2.07 \times 10^{-8}$	$4.48 \times 10^{-9}$

# Muon Collider Probes



# Muon Collider Probes

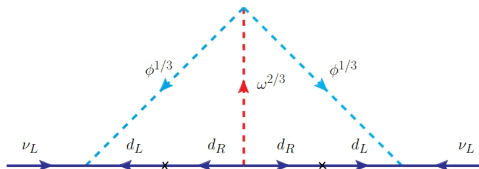


Saad et. al. 2023

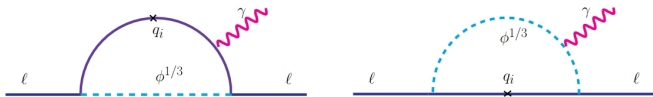
# Zee-Babu Model: Colored version

Leptoquark:  $S_1 \sim (\bar{3}, 1, 1/3)$

Di-quark:  $\omega \sim (\bar{6}, 1, 2/3)$



- Cannot escape to high energies: reach collider prospect
- Expected large rates for cLFV
- Flavor anomalies ...



Babu, Leung 2001; Khoda et. al. 2012; Saad 2020

(see talk by Nejc Košnik)

# SU(5) grand Unification

## Georgi-Glashow Model

- Fermions

$$\bar{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}, \quad \mathbf{10}_F = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_2^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}.$$

- Scalars

$$\mathbf{24}_H : SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathbf{5}_H : SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em}$$

Georgi, Glashow 1974

# Limitations & Resolutions

Georgi, Glashow 1974

- $\bar{5}_F^i + 10_F^i + 5_H + 24_H$
- ✗  $M_d = M_e^T$
- ✗  $M_\nu = 0$
- ✗ Gauge coupling unification

Georgi, Jarlskog 1979

- $\bar{5}_F^i + 10_F^i + 5_H + 24_H + 45_H$
- ✓  $M_d \neq M_e^T$
- ✗  $M_\nu = 0$
- ✓ Gauge coupling unification
- ✓ Proton decay (safe)

Dorsner, Perez 2006

# Neutrino Mass: Leptoquark option

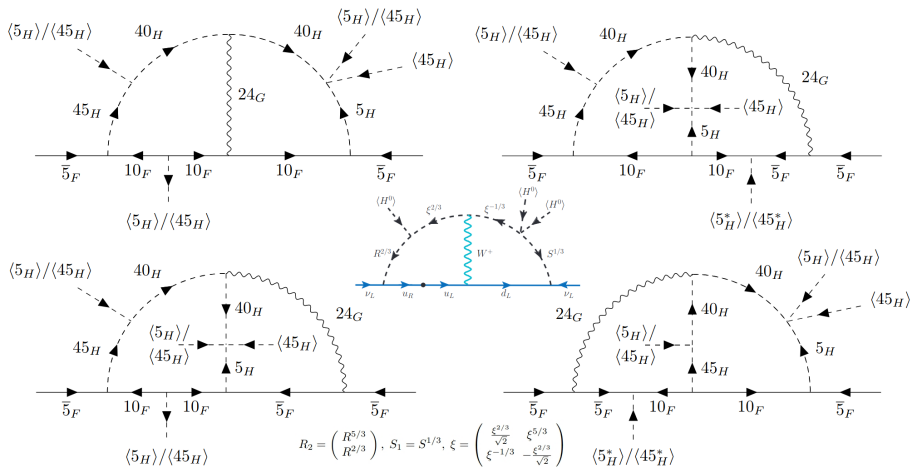
Hinze, Saad 2024

$$\begin{aligned} \mathbf{5}_H &\equiv \phi = \phi_1(1, 2, \frac{1}{2}) \oplus \phi_2(3, 1, -\frac{1}{3}), \\ \mathbf{45}_H &\equiv \Sigma = \Sigma_1(1, 2, \frac{1}{2}) \oplus \Sigma_2(3, 1, -\frac{1}{3}) \oplus \Sigma_3(\bar{3}, 1, \frac{4}{3}) \\ &\quad \oplus \Sigma_4(\bar{3}, 2, -\frac{7}{6}) \oplus \Sigma_5(3, 3, -\frac{1}{3}) \\ &\quad \oplus \Sigma_6(\bar{6}, 1, -\frac{1}{3}) \oplus \Sigma_7(8, 2, \frac{1}{2}). \\ \mathbf{40}_H &\equiv \eta = \eta_1(1, 2, -\frac{3}{2}) \oplus \eta_2(\bar{3}, 1, -\frac{2}{3}) \oplus \eta_3(3, 2, \frac{1}{6}) \\ &\quad \oplus \eta_4(\bar{3}, 3, -\frac{2}{3}) \oplus \eta_5(\bar{6}, 2, \frac{1}{6}) \oplus \eta_6(8, 1, 1). \end{aligned}$$

cf. Saad 2019; Ilja, Saad 2019; Ilja, Saad 2021; Julio, Saad, Thapa 2022



# Neutrino Mass: Leptoquark option



## Fermion masses

$$-\mathcal{L}_Y = Y_A 10_F \bar{5}_F 5_H^* + Y_B 10_F 10_F 5_H \\ + Y_C 10_F \bar{5}_F 45_H^* + Y_D 10_F 10_F 45_H .$$

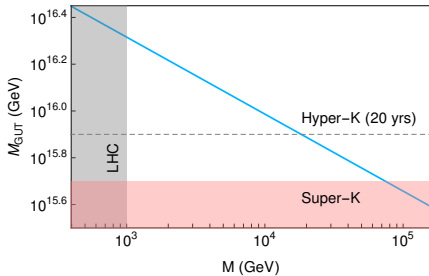
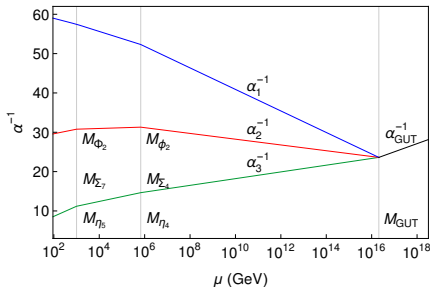
$$M_D = \frac{v_5}{2} Y_A - \frac{v_{45}}{2\sqrt{6}} Y_C ,$$

$$M_E = \frac{v_5}{2} Y_A^T + \frac{\sqrt{3}v_{45}}{2\sqrt{2}} Y_C^T ,$$

$$M_U = \sqrt{2}v_5 (Y_B + Y_B^T) + \frac{v_{45}}{\sqrt{3}} (Y_D - Y_D^T) ,$$

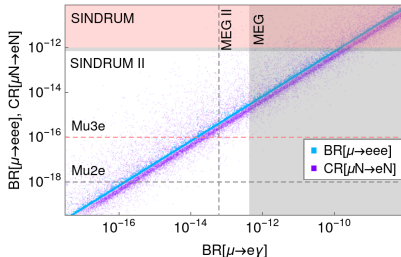
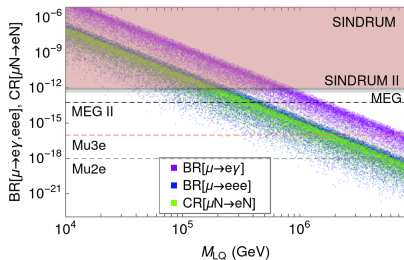
$$M_N = -\frac{3g^2}{\sqrt{2}(16\pi^2)^2} \left\{ 2Y_L^T M_U^{\text{diag}} F_L + M_E^{\text{diag}} Y_R^\dagger F_L \right. \\ \left. + M_E^{\text{diag}} Y_L^T F_R^* \right\} J_0 + (\text{transpose}) .$$

# Neutrino Mass in SU(5)



- $m_\nu$ : masses of the leptoquarks  $\phi_2, \Sigma_4, \eta_4 \Rightarrow M_{LQ} \lesssim 10^8 \text{ GeV}$
- Proton decay:  $\phi_2 \Rightarrow M_{LQ} \gtrsim 10^{12} \text{ GeV}$
- Suppression:  $(U_L^\dagger (Y_B + Y_B^T) D_L^*)_{1\beta} = (D_R^\dagger Y_A^\dagger U_R^*)_{\beta 1} = 0$  for  $\beta = 1, 2$

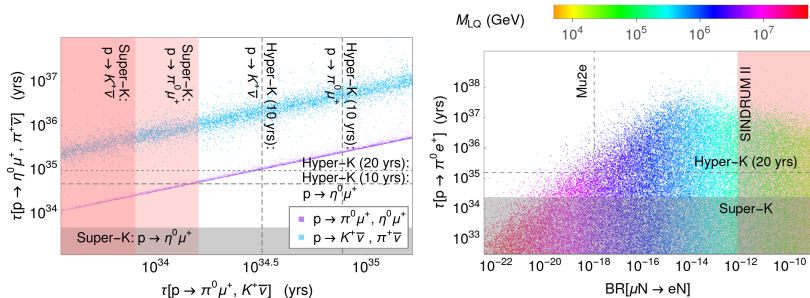
# Neutrino Mass in SU(5)



- Fermion mass+PD suppression determine the couplings
- LFV: typically  $M_{LQ} \gtrsim 10^5$  GeV

Hinze, Saad 2024

# Neutrino Mass in SU(5)



Hinze, Saad 2024

# Summary

- ❄ Neutrino oscillations  $\implies$  cLFV
- ❄ Origin of neutrino mass  $\implies$  New Physics
- ❄ New Physics  $\implies$  unsuppressed cLFV
- ❄ Observation of cLFV  $\implies$  Direct signature of New Physics
- ❄ cLFV rates  $\implies$  Model dependent
- ❄ NP models: cLFV  $\iff$  Complementary probes

THANK YOU!