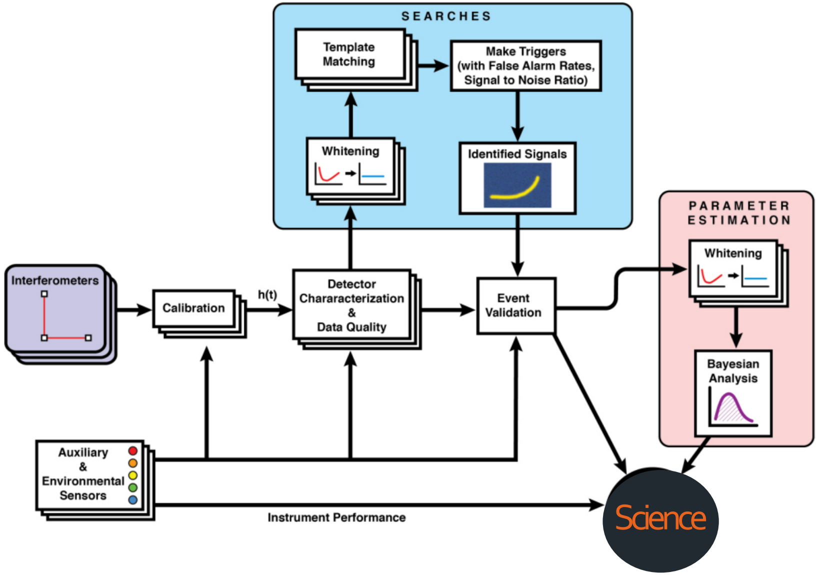
Gravitational wave astronomy:
from interferometric strain to astrophysics

Dr Greg Ashton

ROYAL HOLLOWAY UNIVERSITY OF LONDON

Summary of the talk

- Introduction and overview
- Searching for signals
- Parameter estimation
- Observational results



Davis & Walker (2022)

Summary of the talk

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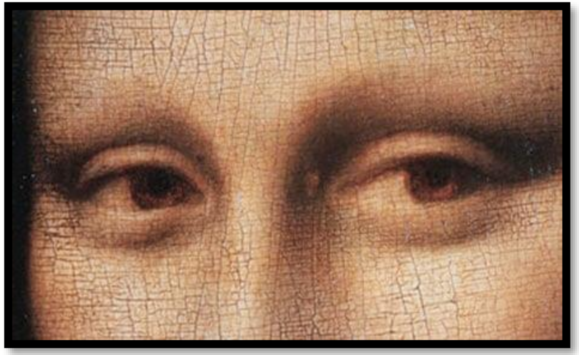
The cartoon overview

The view from the back of the room

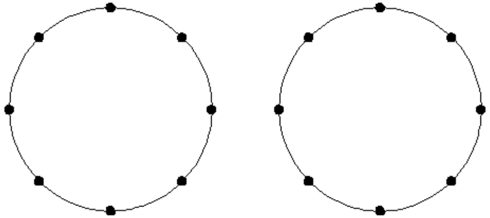
The detailed view of some of the field



Introduction and overview

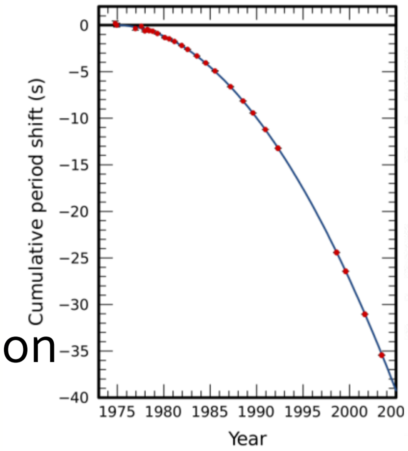
Gravitational waves

- Einstein (1918) "On gravitational waves"
  - Plane wave solutions
  - Travelling at c
  - Two polarisations

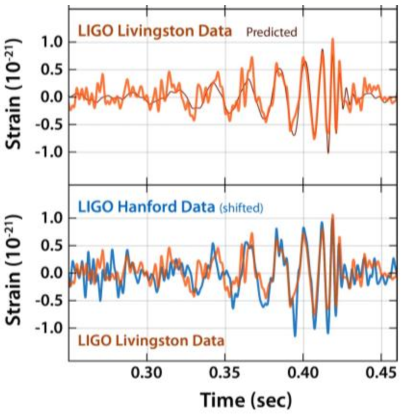


Credit: V. Cardosa & M Cavaglia

- Thorne (1980): "*gravitational waves will become a powerful tool for astronomy*"
- Taylor & Weisberg (1982): First unambiguous evidence for energy loss
- Abbott et al. (2016): First direct observation using the LIGO interferometers



Credit: Weisberg & Taylor (2005)



Credit: LVK

The gravitational wave spectrum

Cosmic fluctuations in the early Universe

Merging supermassive black holes

Rotating asymmetric neutron stars

Merging stellar-mass black holes

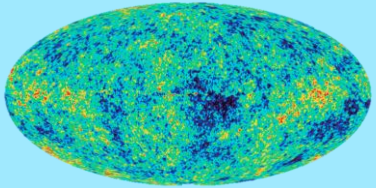
Merging neutron stars

Supernovae

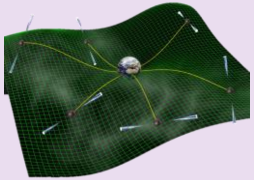
????

Frequency [Hz]

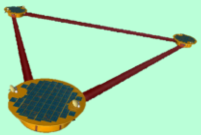
10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4}



Cosmic Microwave Background polarisation



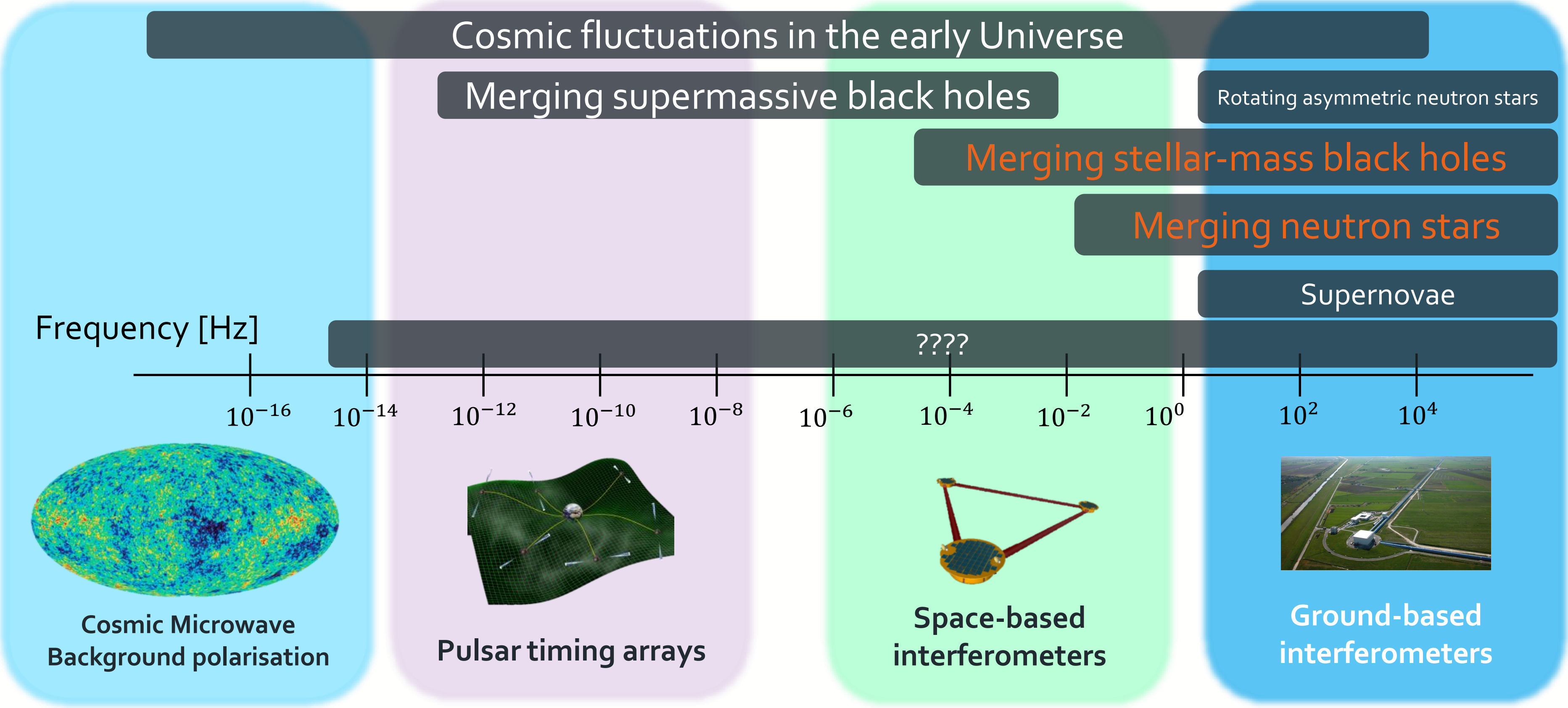
Pulsar timing arrays



Space-based interferometers

Ground-based interferometers

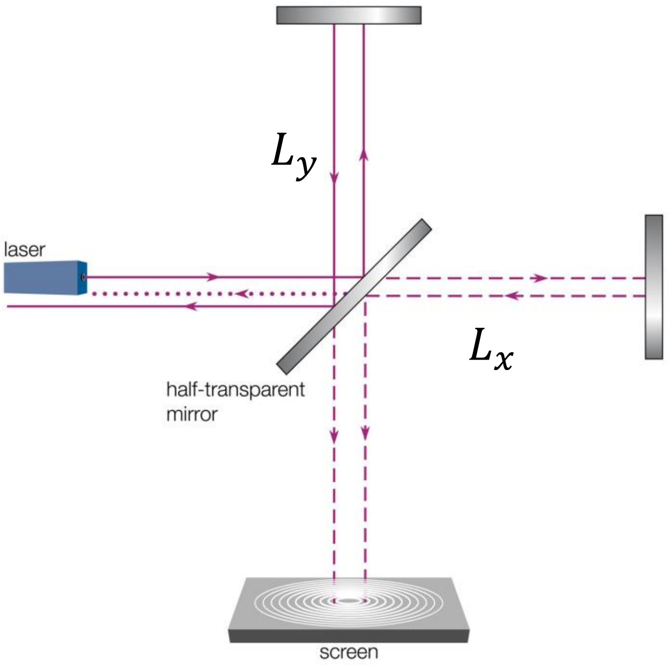
The gravitational wave spectrum: this talk



Ground-based interferometric detectors

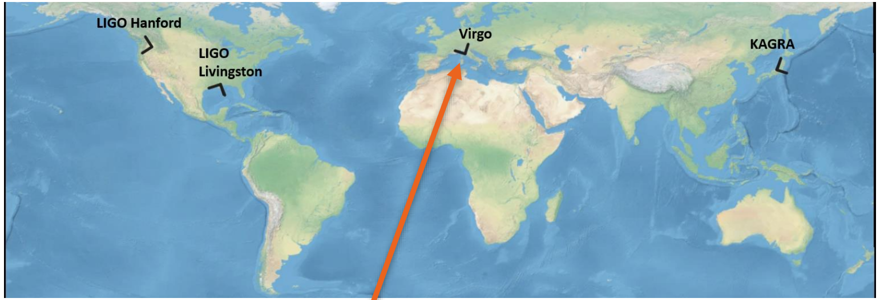
- Basic principle: Michelson Morley interferometer
- Device to convert **relative arm-length differences** into a **changing interferometer pattern**
- Define the **strain**:

$$h = \frac{L_x - L_y}{L}$$



An international network of detectors

- LIGO, Virgo, and KAGRA (LVK) are **kilometre-scale interferometers**
- Operate in tandem to perform **gravitational wave astronomy**




The Virgo detector

Compact binary mergers (CBC): **binary black holes**

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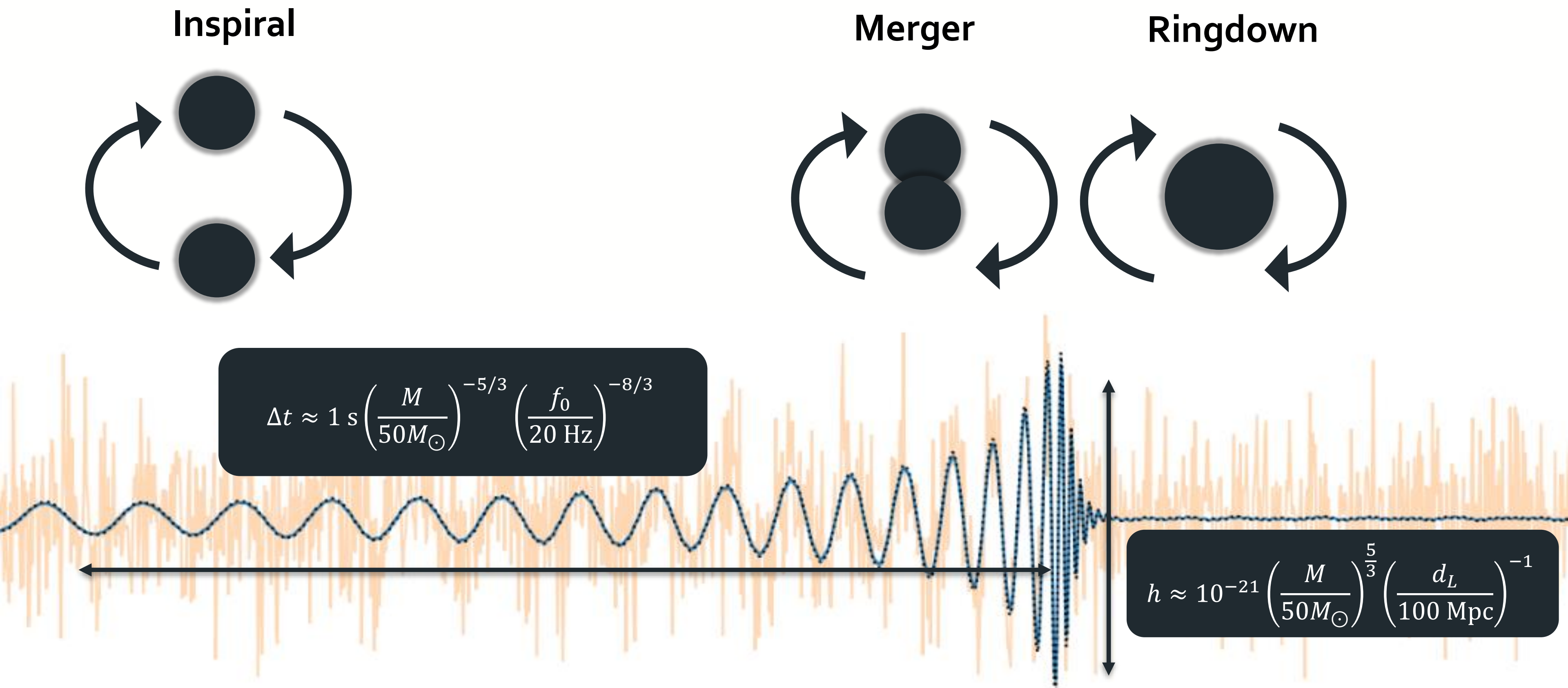
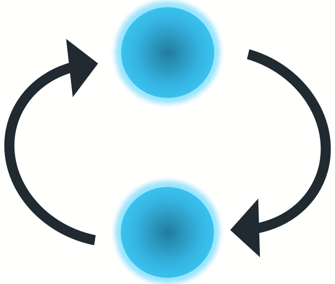


$$\Delta t \approx 1\ \mathrm{s} \left(\frac{M}{50 M_\odot}\right)^{-5/3} \left(\frac{f_0}{20\ \mathrm{Hz}}\right)^{-8/3}$$

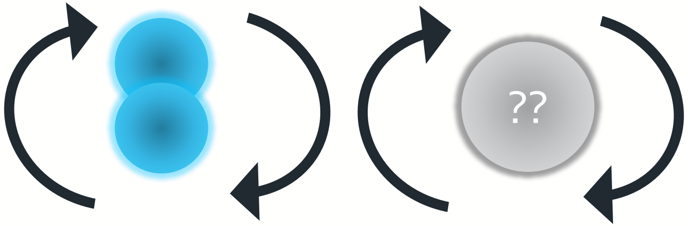
$$h \approx 10^{-21} \left(\frac{M}{50 M_\odot}\right)^{\frac{5}{3}} \left(\frac{d_L}{100\ \mathrm{Mpc}}\right)^{-1}$$

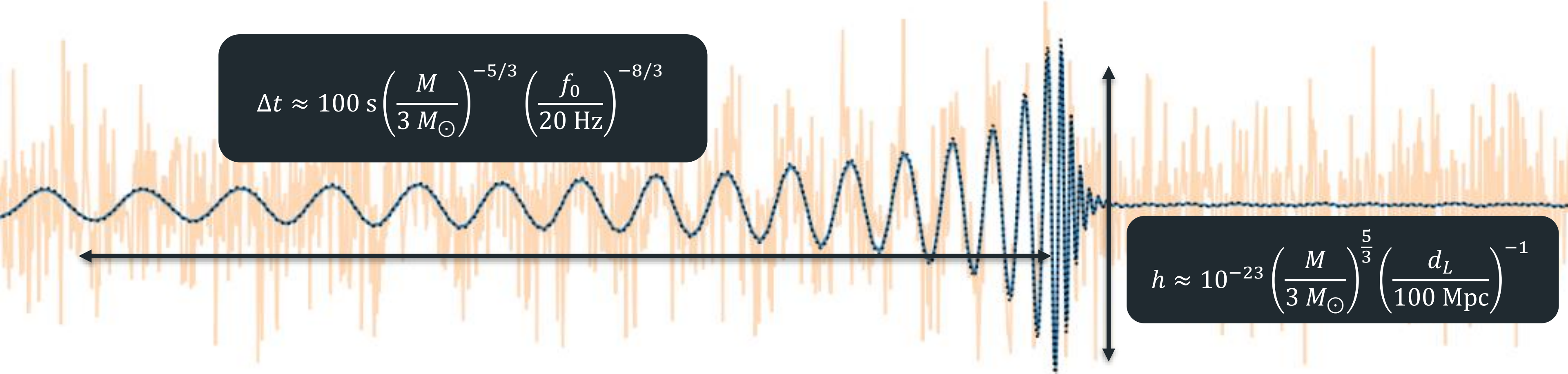
Compact binary mergers (CBC): binary neutron stars

Inspiral



Merger **Ringdown**





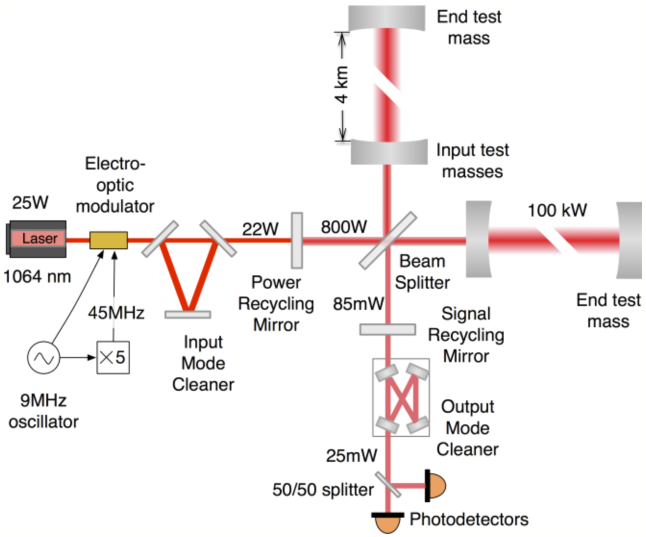
$$\Delta t \approx 100\ \mathrm{s}\left(\frac{M}{3\ M_\odot}\right)^{-5/3}\left(\frac{f_0}{20\ \mathrm{Hz}}\right)^{-8/3}$$

$$h \approx 10^{-23}\left(\frac{M}{3\ M_\odot}\right)^{\frac{5}{3}}\left(\frac{d_L}{100\ \mathrm{Mpc}}\right)^{-1}$$

A more realistic estimate

- Detectors are more complicated than simple interferometers
- Taken at face value, $h = 10^{-21}$ suggests we measure the arm-length to better than the width a proton
- We measure **power output**, not mirror position
- A more realistic calculation (Saulson 1994) propagating uncertainties from the phase measurement:

$$\sigma_h = 1.6 \times 10^{-23} \left(\frac{1000\text{ km}}{L}\right)\left(\frac{\lambda}{1064\text{ nm}}\right)^{1/2}\left(\frac{1\text{ kw}}{P_{\text{in}}}\right)^{1/2}\left(\frac{10\text{ ms}}{\tau}\right)^{1/2}$$



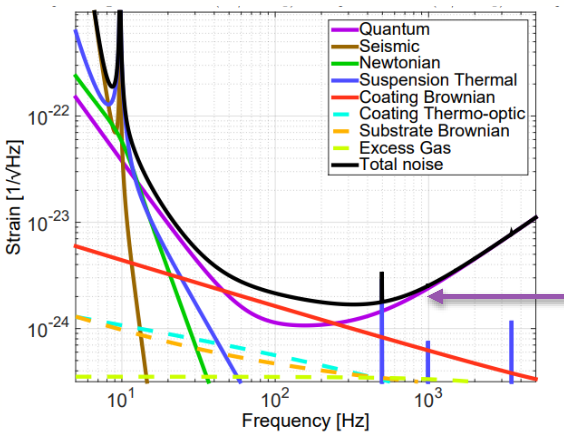
Real detector data

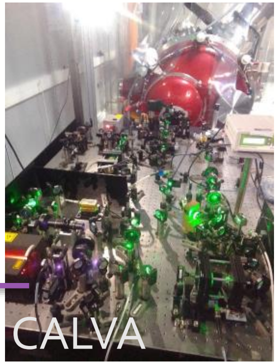
Barsotti et al.

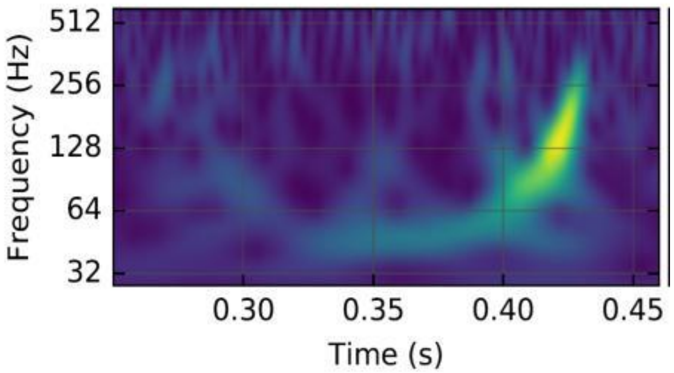
- In practise, noise is frequency-dependent: can be characterised by a Power Spectral Density (PSD)
- Ideally the data consists of

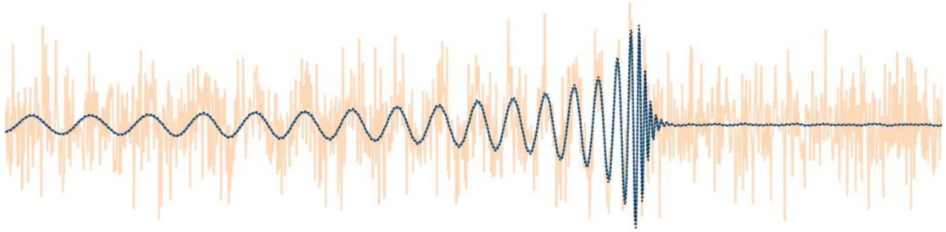
$$h = \text{signal} + \text{colored Gaussian noise}$$

- To "see" the signal, either whiten or filter









Real detector data is full of glitches

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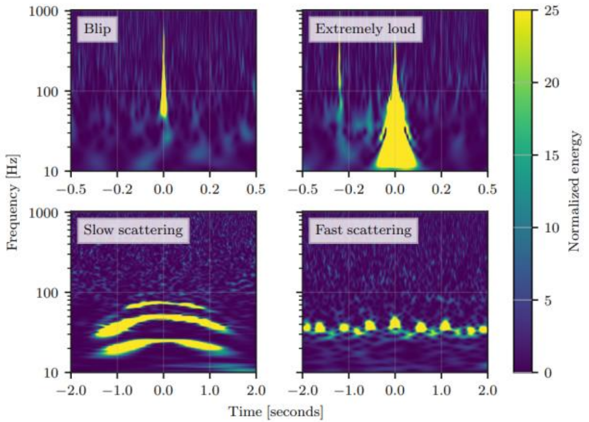
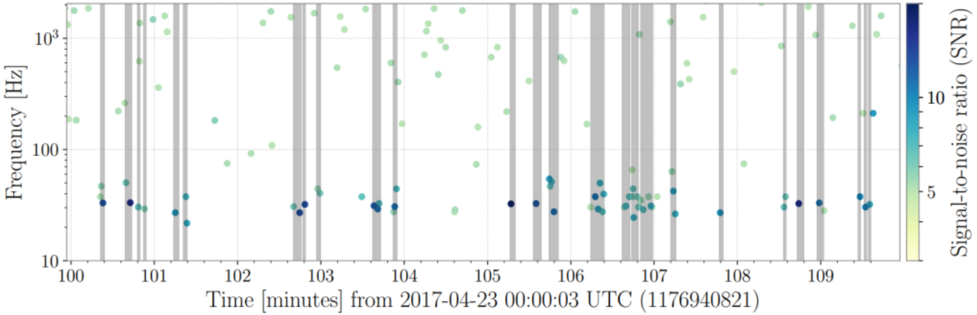
- **Glitches**: transient non-Gaussian noise
- One every few minutes
- Impact:
  - Reduce search sensitivity
  - Contaminate observed signals





Davis & Walker (2022)

Gravitational-wave data analysis

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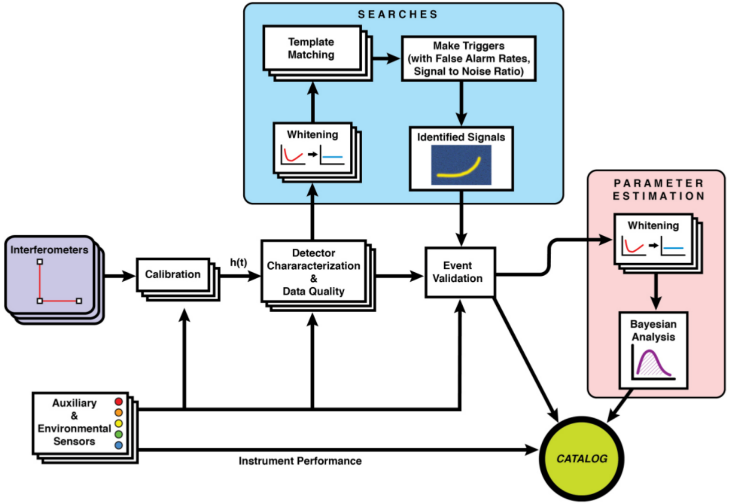
- Finn (1992):
  - Search: decide if the data contains a signal
  - Parameter Estimation: assume the presence of a signal and measure its parameters
- LIGO-Virgo-KAGRA:
  - Calibration, Detector Characterisation
  - Search + Parameter Estimation
  - Population studies, Tests of General Relativity, Cosmology, Lensing, …



Davis & Walker (2022)

Searching for signals

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Searching for signals

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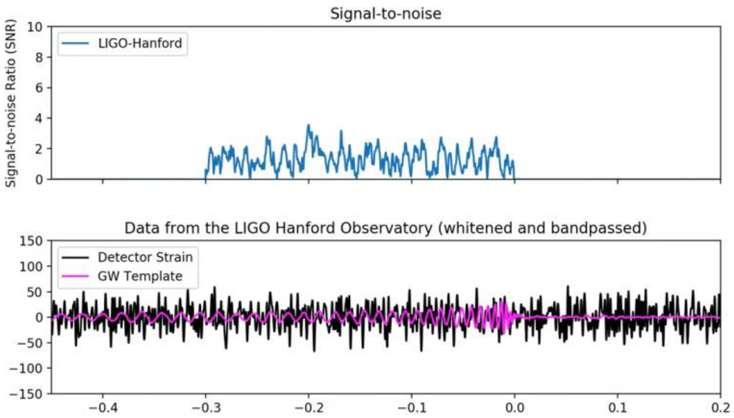
Given data d and "template" waveform μ, construct the signal-to-noise ratio (SNR):

$$\rho = \frac{\langle d|\mu\rangle}{\sqrt{\langle \mu|\mu\rangle}}$$

where the noise-weighted inner product:

$$\langle x|y\rangle = \frac{4}{T}\sum_j \Re\left(\frac{x_j y_j^*}{P_j}\right)$$

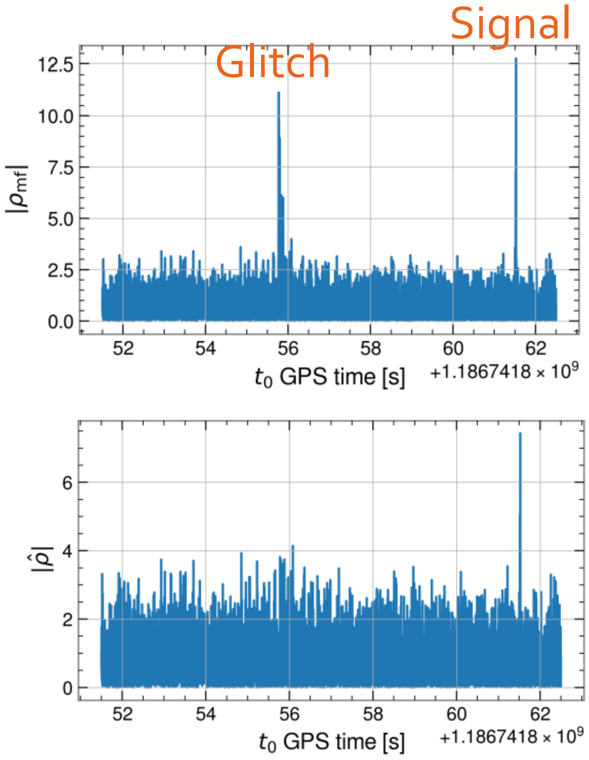
T is the duration while P is the PSD.



Credit: A. Nitz

Searching for signals

- Without glitches
  - Background is known analytically
  - Can construct an **optimal** detection statistic
  - Standard statistical decision problem
- With glitches
  - Background must be empirically estimated
  - Optimal statistic unknown
  - Need to determine a modified detection statistic $\hat{\rho}$ (see example using χ^2 approach Allen (2005))



False alarm rates

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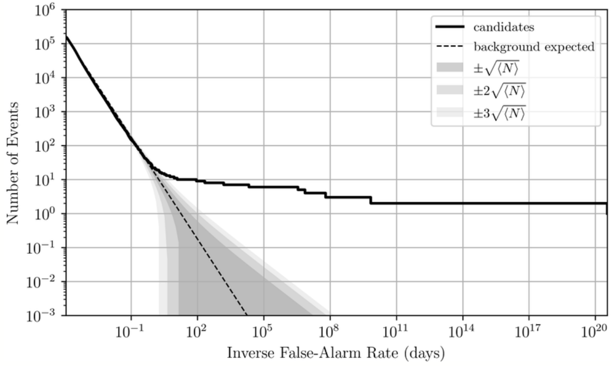
- Construct an empirical background
  - E.g., using "time slides"
  - Estimate of $P(\hat{\rho}\ |H_0)$ where H_0 is the null hypothesis
- Calculate a one-sided empirical p-value scaled by the search duration called the "False Alarm Rate":

$$\mathrm{FAR} = \frac{1}{T} P(\hat{\rho} > \hat{\rho}'|H_0)$$

- The FAR is then used to determine significance

Credit: Ewing et al (2023)



Pipelines

- The LVK runs a set of search "pipelines":
  - Modelled
  - Unmodelled
- Run in "online" and "offline" modes
- Identify events and measure significance:
  - FAR: Frequentist and **fundamental for detection**
  - p_{astro}: Bayesian modelled probability **for routine observations**

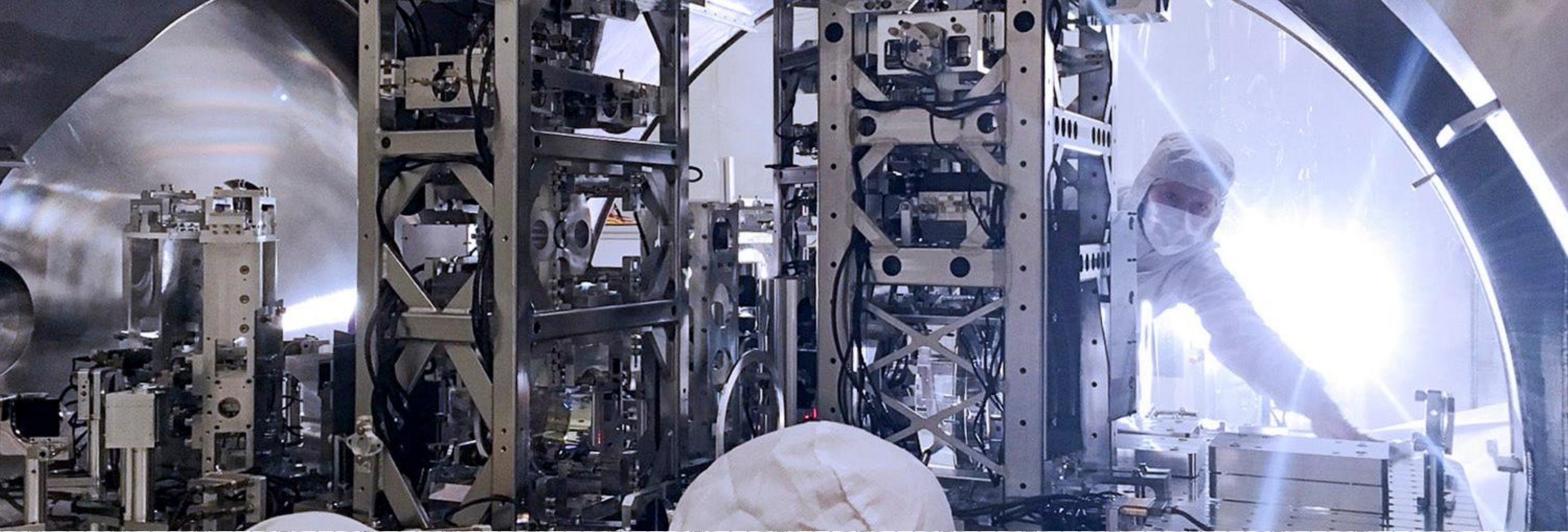
PyCBC

gstLAL

MBTA

SPIIR

cWB



Parameter Estimation

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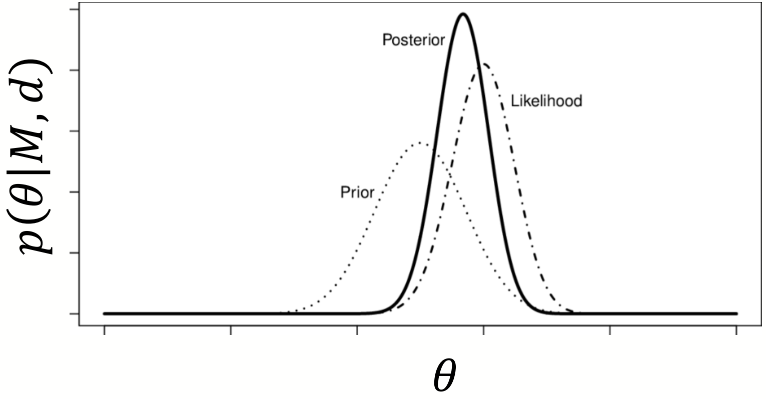
Bayesian inference

For data d and model M with parameters θ:

$$p(\theta|M,d) = \frac{\mathcal{L}(d|\theta,M)\pi(\theta|M)}{\mathcal{Z}(d|M)}$$



with

$$\mathcal{Z}(d|M) = \int \mathcal{L}(d|\theta,M)\pi(\theta|M)\,\mathrm{d}\theta$$

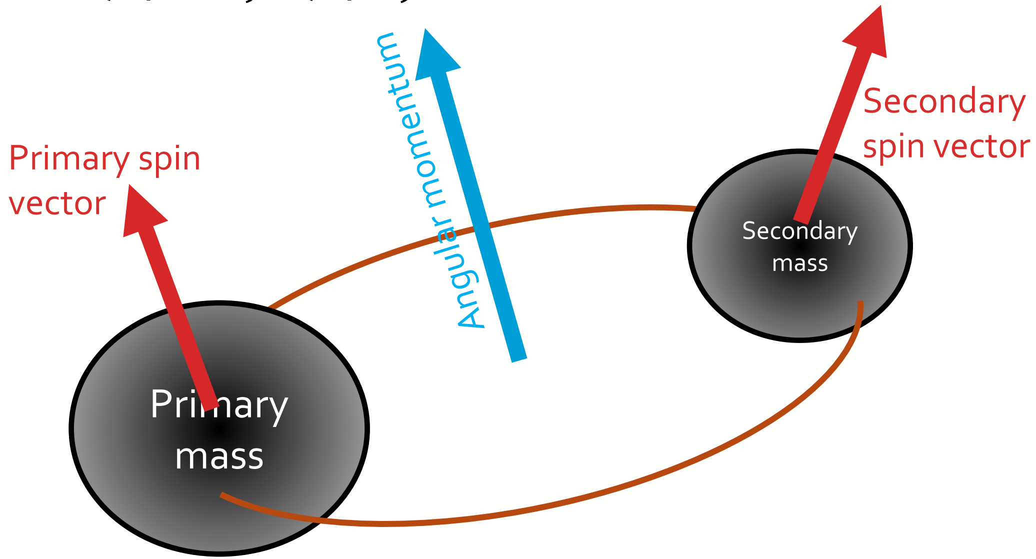
Generally, θ is a vector of parameters

Parameter estimation

- In gravitational-wave astronomy, we are primarily interested in **parameter estimation:**

$$p(\theta|M, d) \propto \mathcal{L}(d|\theta, M)\pi(\theta|M)$$

- We generally split θ into
  - Intrinsic parameters
  - Extrinsic parameters



Parameter estimation

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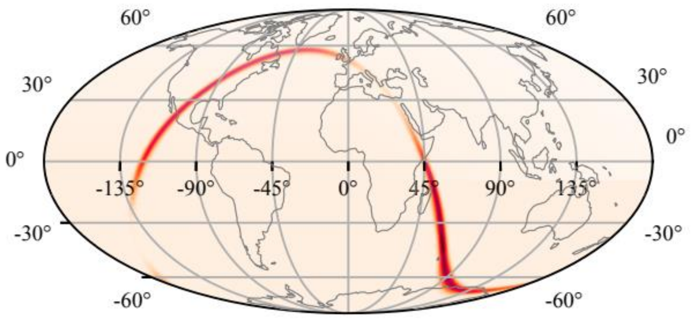
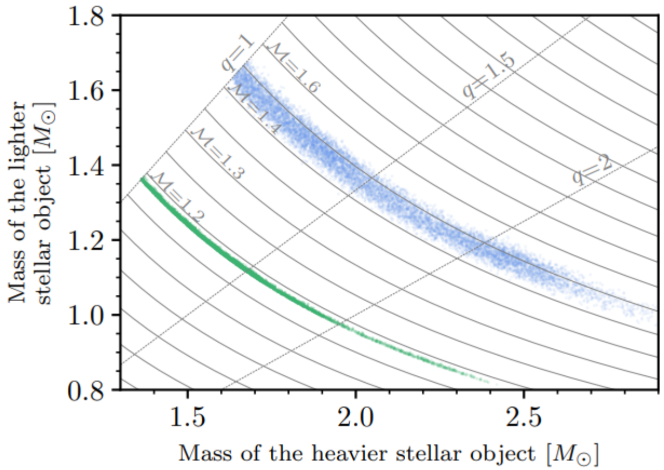
- Upwards of 15 parameters
- Strong correlations and curving degeneracies

Ashton et al. (2022)



The posterior skymap

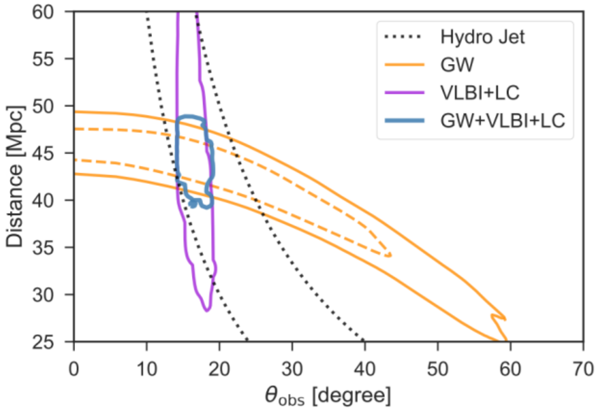


The posterior masses

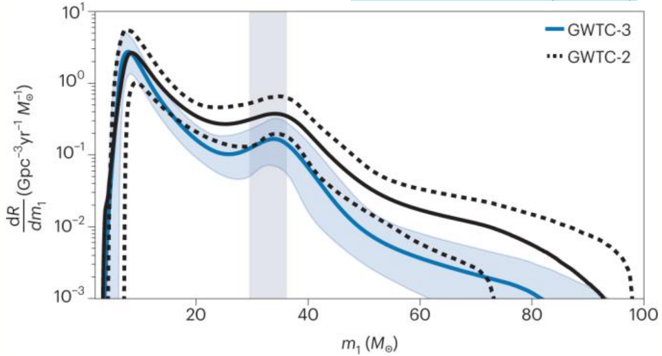
Why do we use Bayesian inference?

- Framework to probe model validity
- Combine data sets in a probabilistic manner
- Natural connection with hierarchical Bayesian methods to infer the population

Hotokezaka et al. (2018)

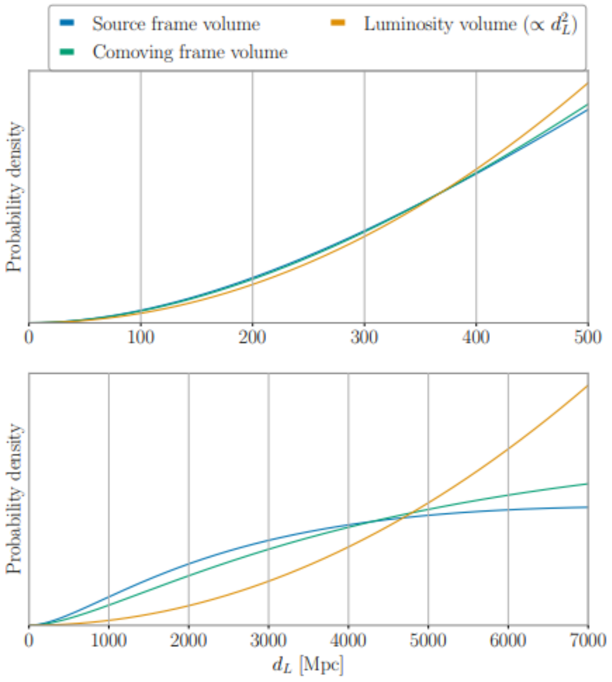


Abbott et al. (2022)



What is the role of the prior?

- Provides a framework for assigning prior knowledge
- We **always** have some prior:
  - Can be constrained by astrophysical knowledge
  - Can be constrained by model validity ⇒ take care if the posterior is prior-informed
- For example: cosmological priors
  - Uniform in a Euclidean universe: $\pi(d_L) \propto d_L^2$
  - Uniform source-frame: $\pi(z) \propto \frac{1}{1+z}\frac{dV_c}{dz}$
- Population-weighted priors



An introduction to MCMC

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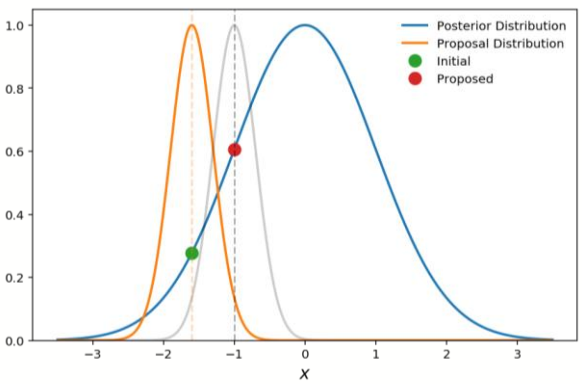
- Computational Bayesian inference is required to estimate the posterior
- Let's look at the MCMC algorithm

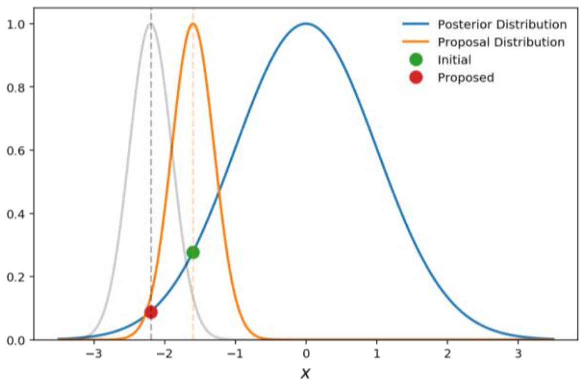
Algorithm 5 The Metropolis MCMC algorithm to draw samples from a target $f(\theta)$ given an initialization point θ_0.

$c \leftarrow []$ ▷ Initialise an empty Markov chain
$c_0 \leftarrow [\theta_0]$ ▷ Set the first element of the to an initial value θ_0
for i in range(1, N_{steps}) **do** ▷ Repeat the loop N_{steps} times
  $\theta' \sim Q(\theta' | c_{i-1})$ ▷ Draw a proposed point θ' from the proposal distribution
  $u \sim U(0, 1)$ ▷ Draw a uniform random number u
  $\alpha \leftarrow f(\theta')/f(\theta)$ ▷ Calculate the acceptance ratio α
  if $u \leq \alpha$ **then**
    $c_i \leftarrow \theta'$ ▷ Accept the proposed point and append it to the chain
  else
    $c_i \leftarrow \theta$ ▷ Reject the proposed point and append the existing point to the chain
  end if
end for

A jump "up": always accepted



A jump "down": accept in proportion



Stochastic sampling: MCMC

Set the target distribution to

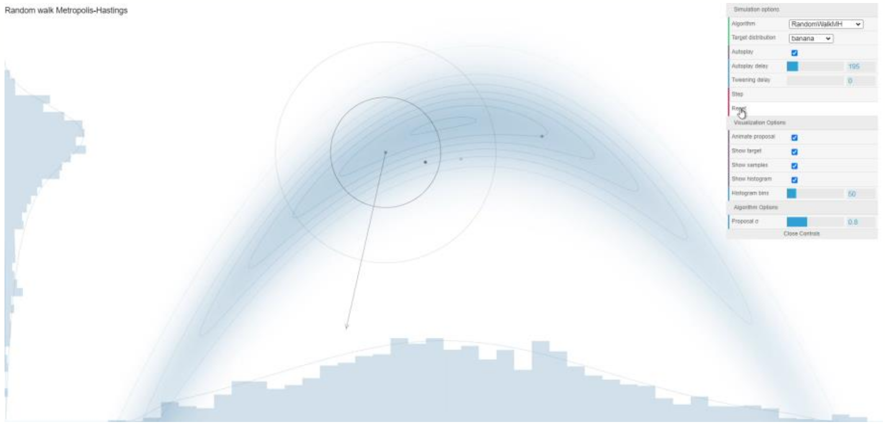
$$f(\theta) = \mathcal{L}(d|\theta, M)\pi(\theta|M)$$

Run the algorithm

Result: a set of samples from the posterior

$$p(\theta) \sim [\theta_0, \theta_1, \theta_2, \ldots]$$

- Able to handle many dimensions
- Able to handle arbitrary posterior distributions

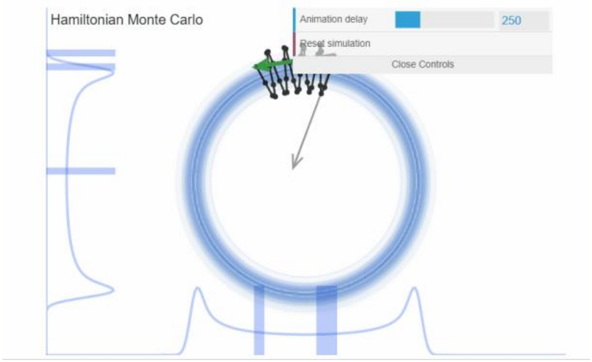


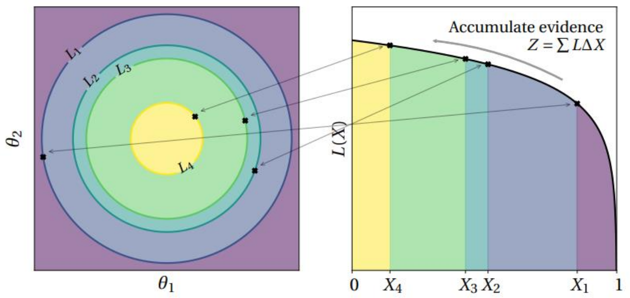
Credit: github.com/chi-feng/mcmc-demo

Modern stochastic sampling

Two primary algorithms used to date:

- **MCMC**
  - Goal: estimate the posterior distribution $p(\theta|M, d)$
  - Evidence estimates possible
  - Tuned proposals needed for multi-modal and correlated posterior
- **Nested Sampling**
  - Goal: estimate the evidence $\mathcal{Z}(d|M) = \int \mathcal{L}(d|\theta, M)\pi(\theta|M)\ \mathrm{d}\theta$
  - Posterior distributions obtained from weighted samples
  - Multi-modal by design





Stochastic sampling is slow

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To analyse a typical transient gravitational-wave signal, it takes at least a few hours:

$$T \approx 5\,\mathrm{hrs}\left(\frac{n^{\mathrm{eff}}_{\mathrm{samples}}}{1000}\right)\left(\frac{t_\ell}{10\mathrm{ms}}\right)\left(\frac{\epsilon}{0.01\%}\right)^{-1}\left(\frac{m}{0.75}\right)^{-1}\left(\frac{n_{\mathrm{cores}}}{8}\right)^{-1}.$$

But can take many weeks

How can we make it faster?

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- Increase efficiency:
  - Choose better parameterizations
  - Analytically marginalize over subsets of the parameters
  - Use a better sampler
- Replace the likelihood (reduce t_ℓ)
  - Reduced Order Quadrature
  - Heterodyning (AKA "relative binning")
- Computational parallelization:
  - HPC cluster: Nested Sampling (pbilby + dynesty)
  - Large-core-count CPUs (e.g. 128)
  - HTC cluster: run multiple MCMC chains and combine

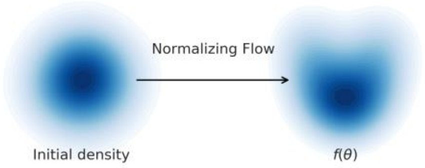
$$T \approx 5\,\mathrm{hrs}\left(\frac{n^{\mathrm{eff}}_{\mathrm{samples}}}{1000}\right)\left(\frac{t_\ell}{10\mathrm{ms}}\right)\left(\frac{\epsilon}{0.01\%}\right)^{-1}\left(\frac{m}{0.75}\right)^{-1}\left(\frac{n_{\mathrm{cores}}}{8}\right)^{-1}.$$

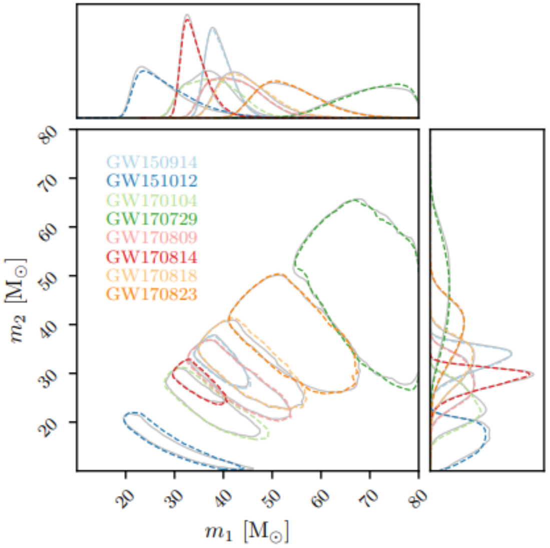
Simulation-based inference

Credit: astroautomata.com/blog/simulation-based-inference/

Neural posterior density estimation:

- "Learn" a mapping from the posterior to a latent space, invert to generate posterior samples
- Dax et al. (2023) reproduce stochastic-sampling with two orders of magnitude improvement
- Most interesting feature: "likelihood-free"





Observational results

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The international gravitational-wave detector network

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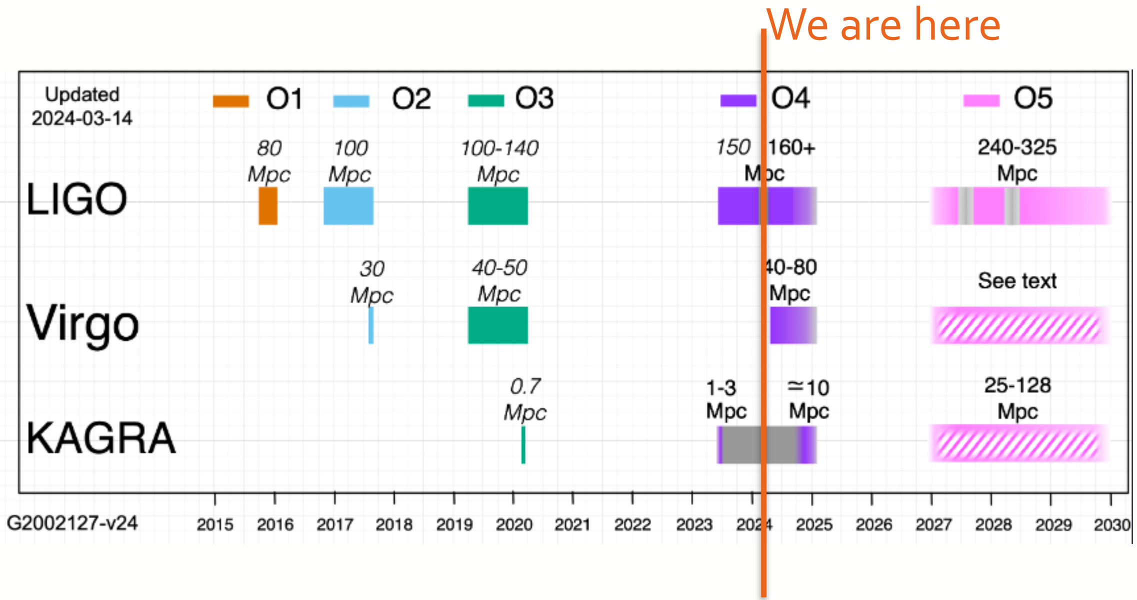
O4b will start on the 10th of April



observing.docs.ligo.org/plan/

Observations to date

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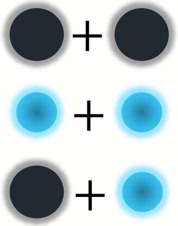
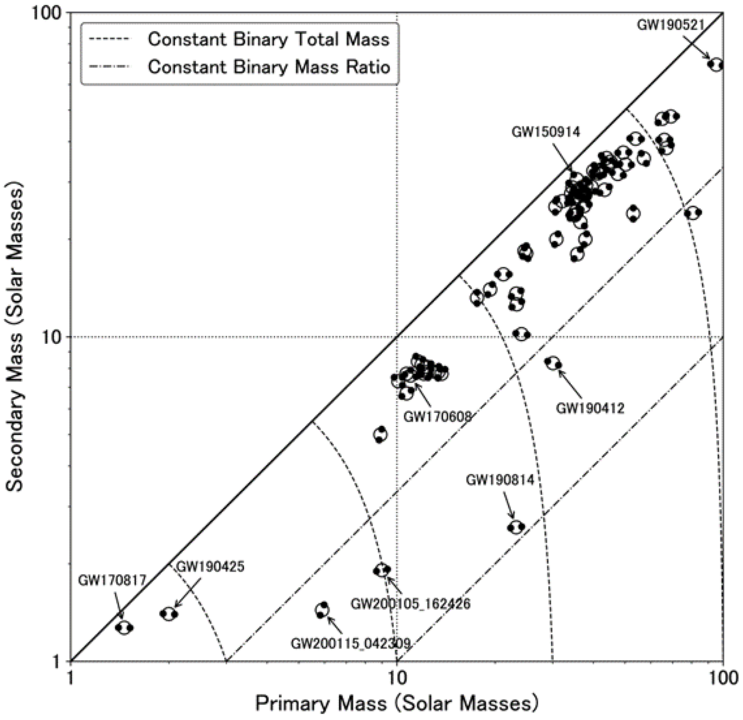
- O1-O3 produced nearly 100 observations
- All signals arise from CBC:
  - Binary black hole collisions
  - Binary neutron star collisions
  - Neutron star – black holes



- Binary black holes:
  - Single events enable precise tests of General Relativity
  - Populations enable inferences of stellar evolution
  - + much more

Credit: LIGO-Virgo-KAGRA Collaboration / IGFAE / Thomas Dent



Confident events containing a neutron star

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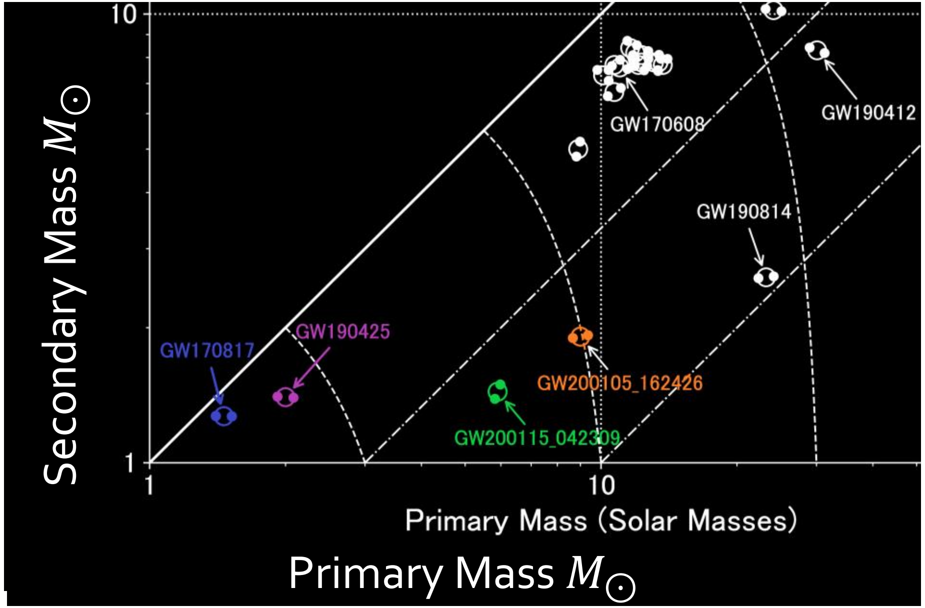
Neutron star + neutron star

- GW170817
- GW190425

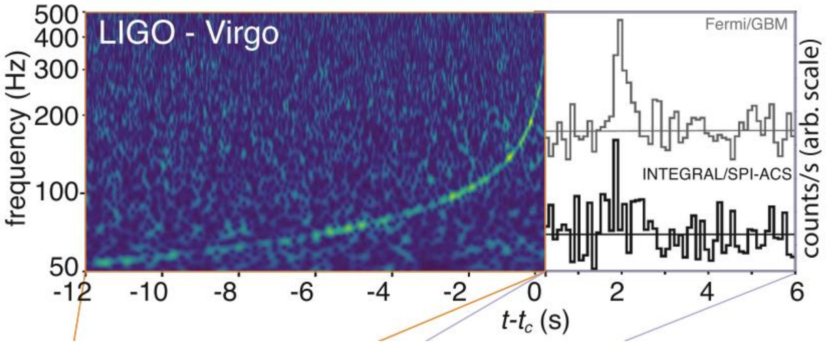
Black hole + neutron star

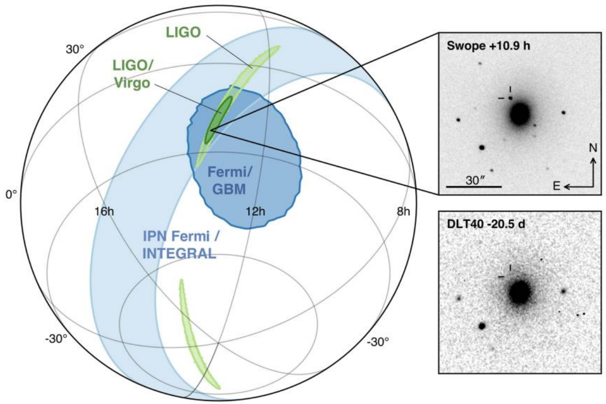
- GW200115
- GW200105



GW170817

- A multi-messenger event:
  - Gravitational-waves
  - Gamma-ray Burst
  - Kilonova
- Enabled new probes of:
  - The NS equation of state
  - Cosmology
  - + more more





Observations to date

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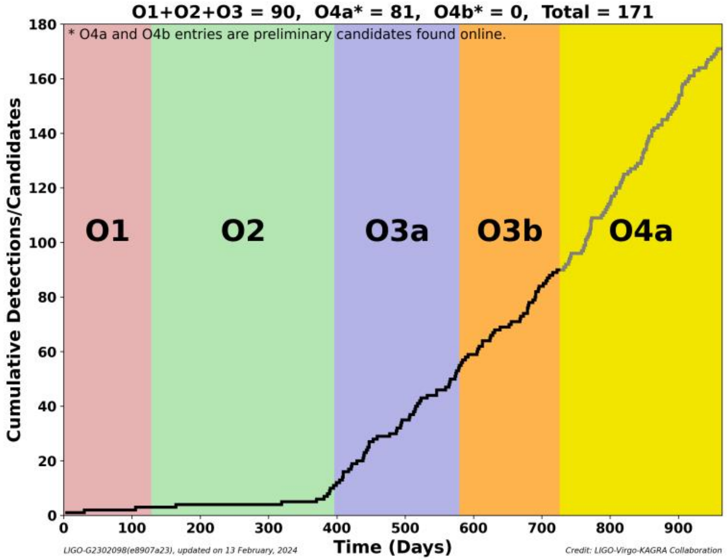
- Known events and public alerts (gracedb.ligo.org/latest/)
- O4a nearly doubled the number of events
- Watch out for new results in the next 24hrs..
- Virgo/KAGRA not online in O4a

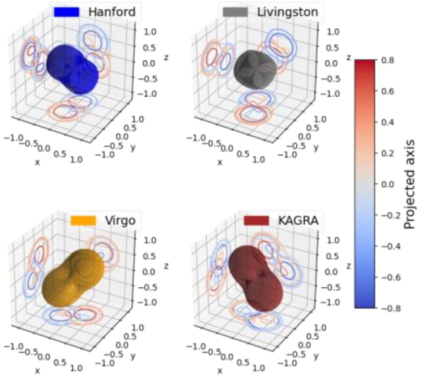


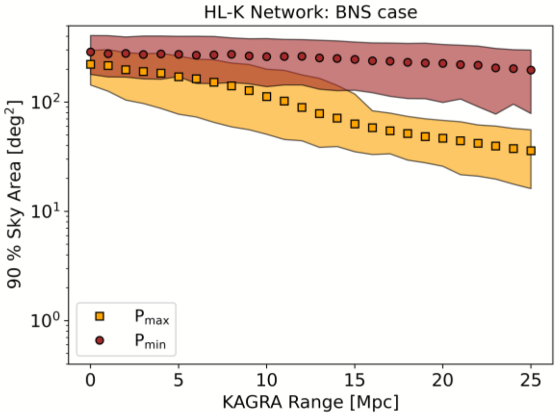
Asymmetric detector sensitivities

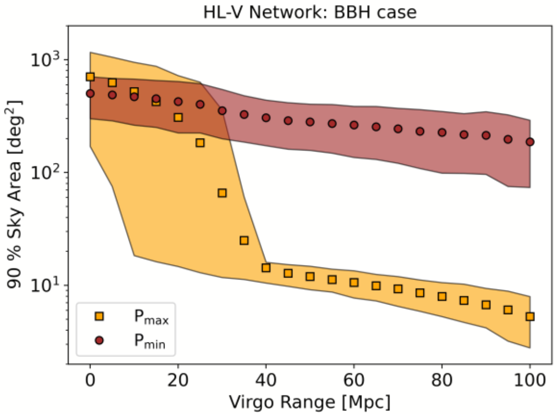
Addition of Virgo/KAGRA in O4b:

- Improve sky localisation
- Improve overall duty cycle
- Enable more precise source parameter measurements

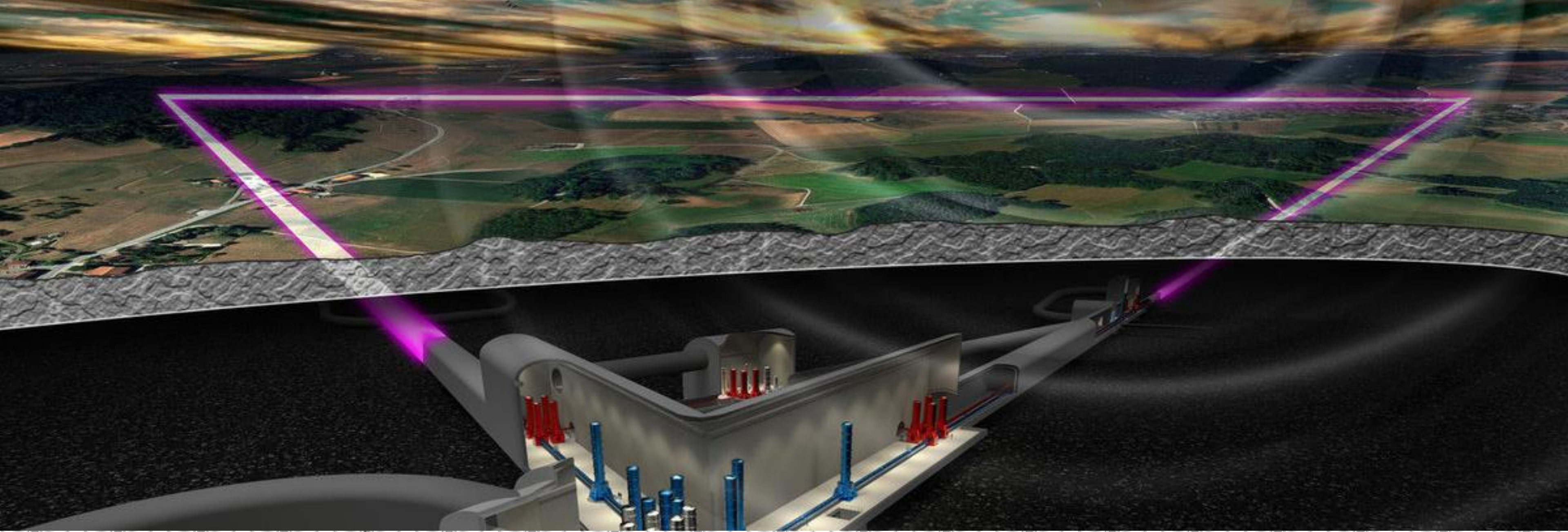
Detector	Horizon
LIGO	160 Mpc
Virgo	55 Mpc
KAGRA	10 Mpc







Emma, M (in prep)



The future

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Near-term future

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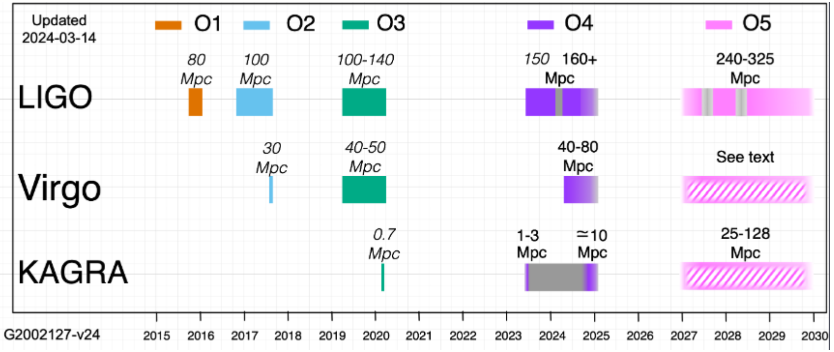
- LIGO, Virgo, and KAGRA will observe **thousands of CBC signals**
- Transition from discovery to population era
- Start to probe redshifts above 1 and the star-formation rate
- New classes of sources:
  - Stochastic gravitational-wave background
  - Isolated rotating neutron stars
  - Supernovae
  - ???



The longer-term future

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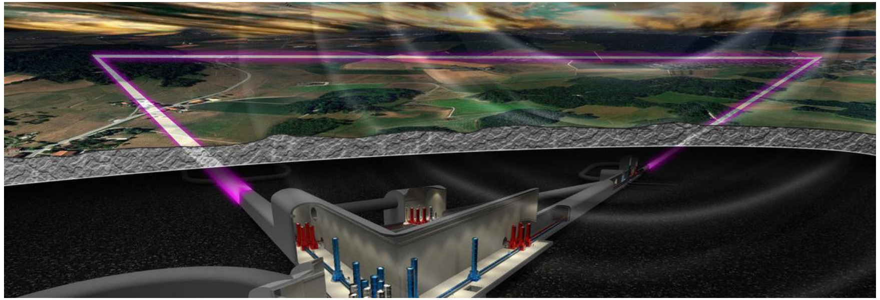
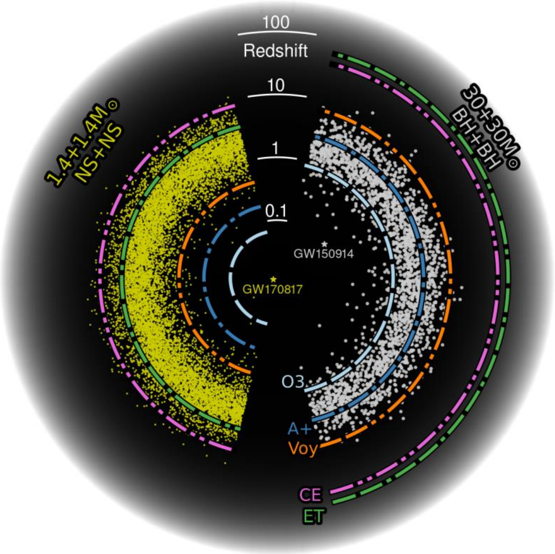
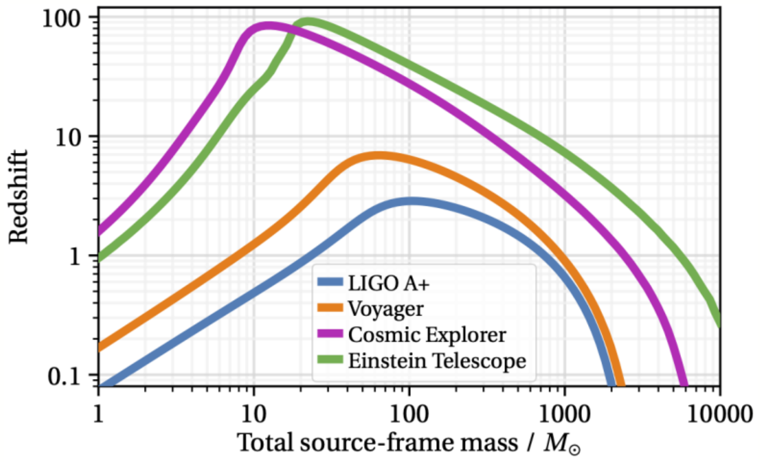
- Einstein telescope (EU)
- Cosmic Explorer (US)







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Thank you for listening!