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Orsay, 9-12 January '07

Beyond the Standard Model

Neutrino Masses & Mixings

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In the last decade data on ν oscillations have added some (badly needed) fresh experimental input in particle physics

ν masses are not all vanishing but they are very small

ν mixing angles follow a different pattern from quark mixings

For ν masses and mixings we do not have so far a "Standard Model": many possibilities are still open.

In fact, this is also the case for quarks and charged leptons; we do not have a theory of flavour that explains the observed spectrum, mixings and CP violation.

ν's are interesting because they can provide new clues on this important problem

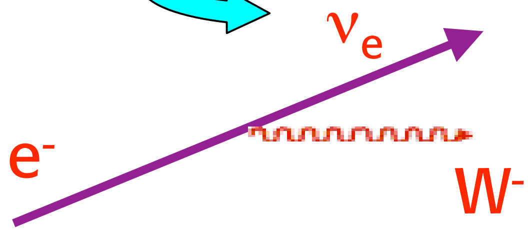
ν Oscillations Imply Different ν Masses

ν_e: same weak isospin doublet as e^-

flavour mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

$$\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$$
$$\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2$$

e.g 2 flav.

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$$\nu_a(x) = e^{ip_a x}\ \nu_a \qquad p_a^2 = E^2 - m_a^2$$

$$P(\nu_e <-> \nu_\mu) = |< \nu_\mu(L)|\ \nu_e>|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

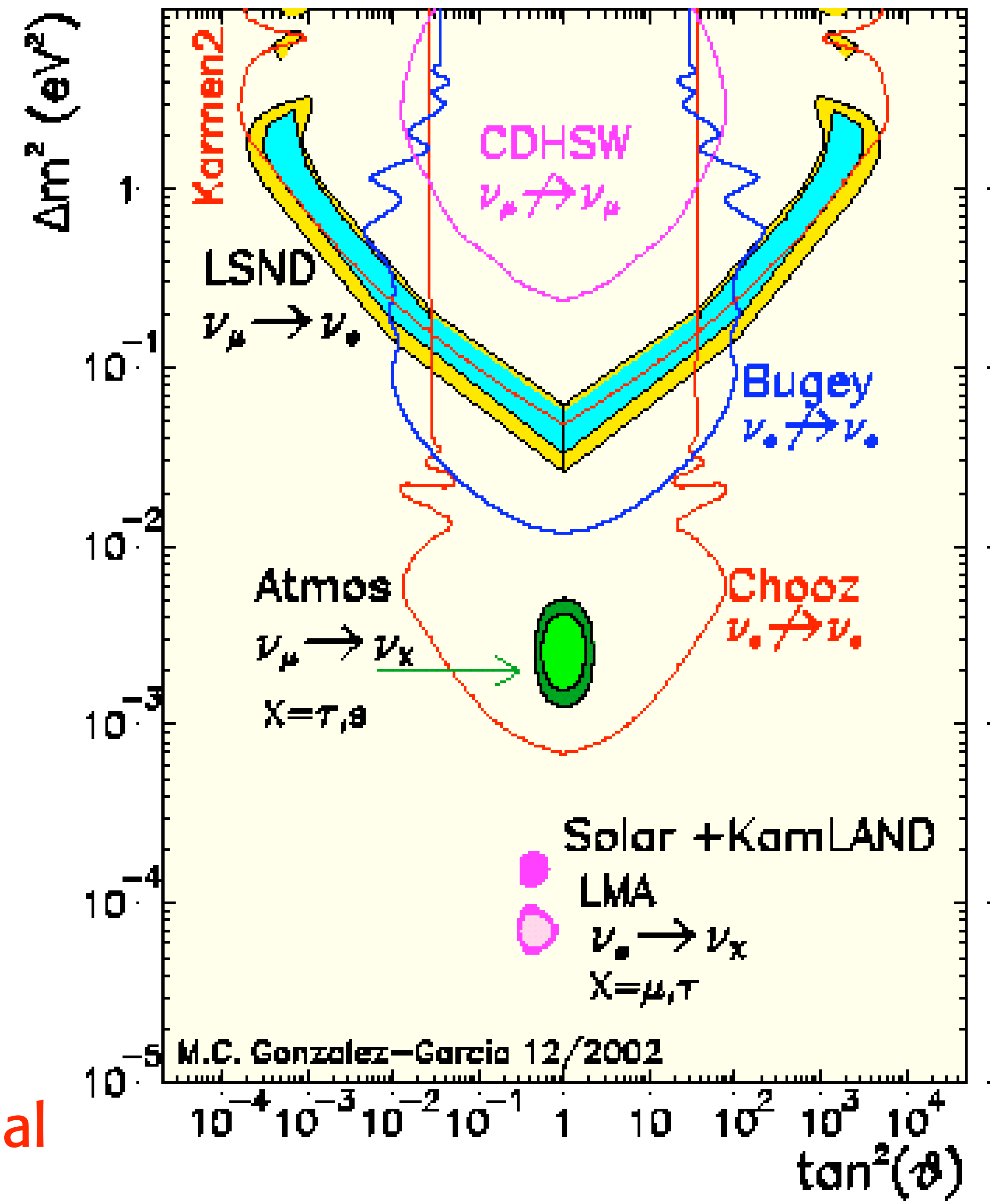
At a distance L, ν_μ from μ^- decay can produce e^- via charged weak interact's

Solid evidence for solar and atmosph. ν oscillations (+LSND unclear)

Δm^2 values fixed:
$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,
$\Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$
($\Delta m^2_{LSND} \sim 1\ eV^2$)

Also confirmed in laboratory exp.ts KamLAND, K2K

mixing angles:
θ_{12} (solar) large
θ_{23} (atm) large,~maximal
θ_{13} (CHOOZ) small



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV   End-point tritium β decay (Mainz, Troitsk)

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.3 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal?   Klapdor-Kleingrothaus

- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\, /94$eV   ($h^2 \sim 1/2$)

$\Sigma_i m_i < 0.17$-0.68-2.1 eV (dep. on data&priors)

$\Longrightarrow$ Any ν mass < 0.06-0.23-0.7 eV

WMAP,
2dFGRS,
Ly-α

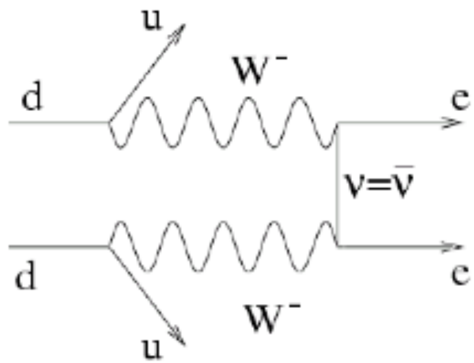
$0\nu\beta\beta$ experiments

$$\langle m_\nu \rangle^2 = \frac{1}{G(Q,Z)\,|M_{nucl}|^2\,\tau}$$

phase space

matrix elmnt
large uncrtnts

dd -> uuee



Pavan

Experiment	Isotope	$\tau_{1/2}^{0\nu}$ > [y]	range $\langle m_\nu \rangle$ [eV]
Heidelberg Moscow 2001	^{76}Ge	$1.9\ 10^{25}$	0.3-2.5
IGEX 2002	^{76}Ge	$1.57\ 10^{25}$	0.3-2.5
Cuoricino 2005	^{130}Te	$2\ 10^{24}$	0.3-0.7
NEMO 2005	^{100}Mo	$4.6\ 10^{23}$	0.6-1.0

claimed evidence only by a part of the collaboration

started in 2003

$$m_{ee} = \langle m_\nu \rangle = |\Sigma\ U_{ej}^2\ m_j\ e^{i\alpha j}|$$

Future: a factor ~ 10 improvement in next decade

ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2$

- Direct limits

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Evidence of signal? Klapdor-Kleingrothaus

- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i / 94 eV$ $(h^2 \sim 1/2)$

$\Sigma_i m_i < 0.17\text{-}0.68\text{-}2.1$ eV (dep. on data&priors)

Any ν mass $< 0.06\text{-}0.23\text{-}0.7$eV

WMAP, SDSS, 2dFGRS, Ly-α

By itself CMB (eg WMAP) is only mildly sensitive to $\Sigma_i m_i$

Only in combination with Large Scale Structure (2dFGRS, SDSS) the limit becomes stronger.

And even stronger by adding the Lyman alpha forest data (but some tension among the data).

95%cl

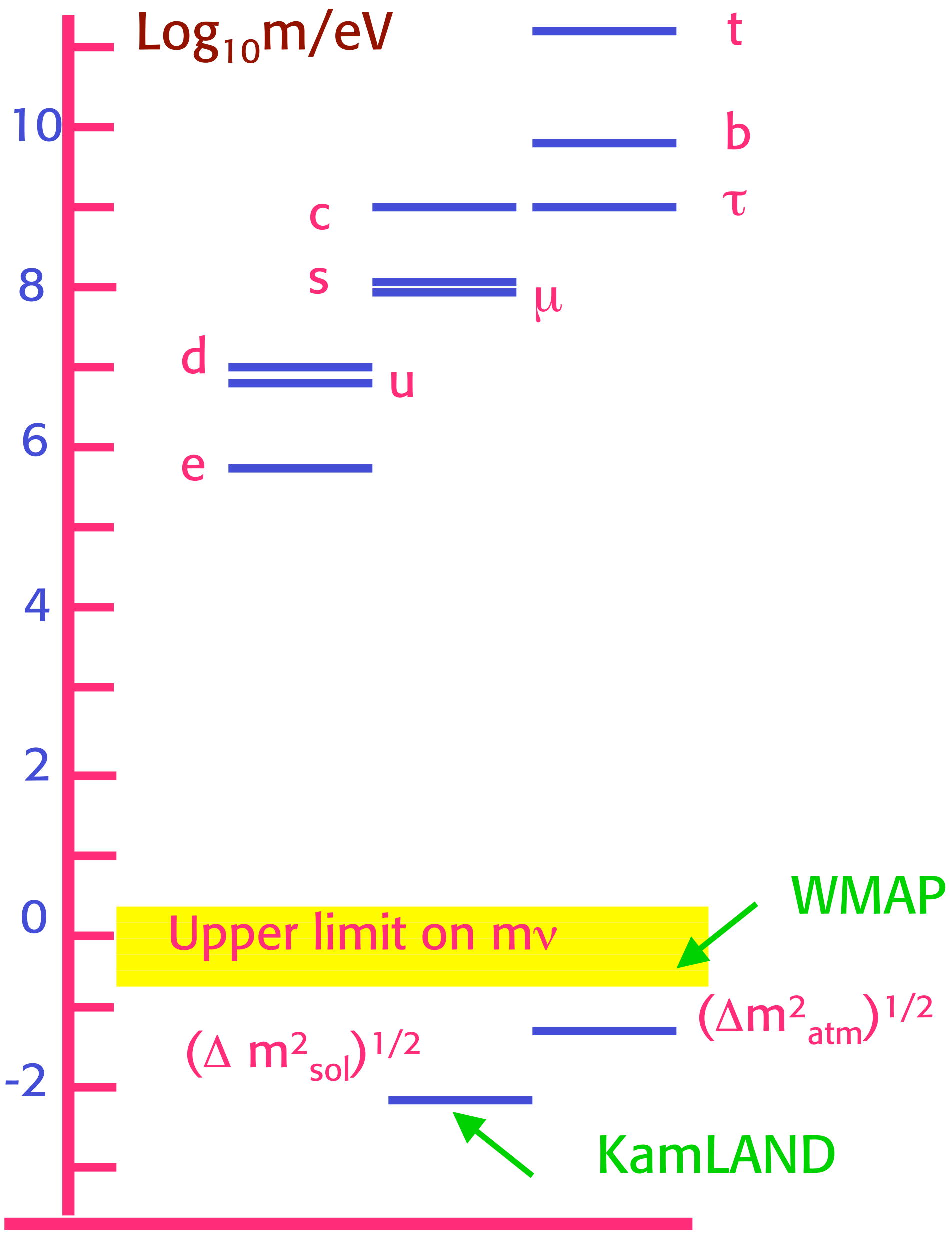
CMB only	(CMB +) LSS	+ Ly-α
$\sum m_{\nu_i} < 2.11$ eV	$\sum m_{\nu_i} < 0.68$ eV	$\sum m_{\nu_i} < 0.17$ eV

Seljac et al '06

Note: for degenerate ν's the mass of each would be $\Sigma m_{\nu i}/3$.

eg 0.68/3 ~ 0.23 eV to be compared with

$(\Delta m^2_{atm})^{1/2} \sim 0.05$ eV



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

How to guarantee a massless neutrino?

1) ν_R does not exist

→ No Dirac mass

$$\bar{\nu}_L \nu_R + \nu_R \bar{\nu}_L$$

and

2) Lepton Number is conserved

→ No Majorana mass

$$\bar{\nu}^c \nu \rightarrow \nu^T{}_R C \nu_R \text{ or } \nu^T{}_L C \nu_L$$

$C = i\gamma^0\gamma^2$

ν masses:

Dirac mass: $\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$

(needs ν_R)

ν's have no electric charge. Their only charge is lepton number L.

IF L is not conserved (not a good quantum number)

ν and $\bar{\nu}$ are not really different

TCP, "Lorentz"

$$|\nu, h = -1/2\rangle \longrightarrow |\bar{\nu}, h = +1/2\rangle$$

Majorana mass: $\nu^T_R \nu_R$ or $\nu^T_L \nu_L$

(we omit the charge conj. matrix C)

Violates L, B-L by $|\Delta L| = 2$

Weak isospin I

$\nu_L \Rightarrow I = 1/2, I_3 = 1/2$

$\nu_R \Rightarrow I = 0, I_3 = 0$

Dirac Mass:

$\overline{\nu}_L \nu_R + \overline{\nu}_R \nu_L$ $\quad |\Delta I|=1/2$

Can be obtained from Higgs doublets: $\nu_L \overline{\nu}_R H$

Majorana Mass:

- $\nu^T{}_L \nu_L$ $\quad |\Delta I|=1$

Non ren., dim. 5 operator: $\nu^T{}_L\ \nu_L HH$

- $\nu^T{}_R \nu_R$ $\quad |\Delta I|=0$

Directly compatible with SU(2)xU(1)!

See-Saw Mechanism

Minkowski; Glashow;
Yanagida; Gell-Mann, Ramond , Slansky;
Mohapatra, Senjanovic…..

$M\nu^T{}_R\nu_R$ allowed by SU(2)xU(1)

Large Majorana mass M (as large as the cut-off)

$m_D\bar{\nu}_L\nu_R$

Dirac mass m_D from
Higgs doublet(s)

$$\begin{array}{cc} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \end{array}$$

$M \gg m_D$

Eigenvalues

$$\nu_{light} = \frac{-m_D{}^2}{M} , \quad \nu_{heavy} = M$$

sign conventional
for fermions

In general ν mass terms are:

$$\mathcal{L}_\nu = \bar{L} h \nu_R H + \text{h.c.} + \nu_R^T M_R \nu_R + \nu_L^T \frac{\lambda}{M_L} \nu_L H H$$

Maiorana

Dirac

$m_D = hv$

$v = \langle 0|H|0\rangle$

$$m = \frac{\lambda v^2}{M_L}$$

More general see-saw mechanism:

$$\begin{array}{ccc} & \nu_L & \nu_R \\ \begin{matrix}\nu_L \\ \nu_R\end{matrix} & \multicolumn{2}{c}{\begin{bmatrix} \lambda v^2/M_L & m_D \\ m_D & M_R \end{bmatrix}} \end{array}$$

$$m_{light} \sim \frac{m_D{}^2}{M_R} \quad \text{and/or} \quad \frac{\lambda v^2}{M_L}$$

$m_{heavy} \sim M_R$ $\qquad$ $m_{eff} = \nu^T{}_L m_{light} \nu_L$

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale M ~ M_{GUT}

$$m_\nu \sim \frac{m^2}{M}$$

m:$\leq$ m_t ~ v ~ 200 GeV
M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV
m ~ v ~ 200 GeV

$M \sim 10^{14} - 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

The current experimental situation is still unclear

- LSND: true or false? -> MiniBooNE soon will tell
- what is the absolute scale of ν masses?
- no detection of $0\nu\beta\beta$ (proof that ν's are Majorana) ••••••

Different classes of models are still possible:

If LSND true

sterile ν(s)??

CPT violat'n??

- "3-1" or "3-n"

$m^2 \sim 1\text{-}2\ eV^2$

LSND

$\nu_{sterile}$

If LSND false → 3 light ν's are OK

We assume this case here

- Degenerate ($m^2 >> \Delta m^2$) $m^2 < o(1) eV^2$
- Inverse hierarchy

sol $m^2 \sim 10^{-3}\ eV^2$

atm

- Normal hierarchy

$m^2 \sim 10^{-3}\ eV^2$

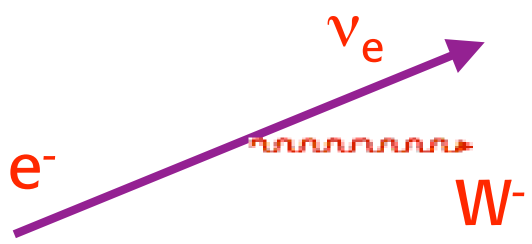
atm

sol

3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^{+} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour   mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| <\sim 0.2$

$$\sim \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & c_{13}s_{23} \\ & & c_{13}c_{23} \end{bmatrix}$$

atm.: ~ max

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(some signs are conventional)

In general: $U = U^{+}_{e} U_{\nu}$

$$m_\nu \sim U^* \begin{bmatrix} e^{i\alpha_1} m_1 & 0 & 0 \\ 0 & e^{i\alpha_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^+$$

$L^T m_\nu L$

In general 9 parameters:
3 masses, 3 angles,
3 phases

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{bmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2)cs/\sqrt{2} & (m_1 - m_2)cs/\sqrt{2} \\ \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 & (m_1 s^2 + m_2 c^2 - m_3)/2 \\ \dots & \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 \end{bmatrix}$$

Note:

- m_ν is symmetric
- phases included in m_i

Relation between masses and frequencies:

$P(\nu_e \text{<->} \nu_\mu) = P(\nu_e \text{<->} \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$
$P(\nu_\mu \text{<->} \nu_\tau) = \sin^2 \Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0

Defining:

$$\Delta m^2_{atm} = m_3^2 - m_2^2 > \text{or} < 0$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 > 0$$

one has:

$$m_3^2 = \overline{m^2} + \frac{2}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_2^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_1^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} - \frac{2}{3}\Delta m^2_{sol}$$

and

$\overline{m^2} >> \left|\Delta m^2_{atm}\right| > \Delta m^2_{sol}$ degenerate

$\Delta m^2_{atm} < 0$ inverse hierarchy

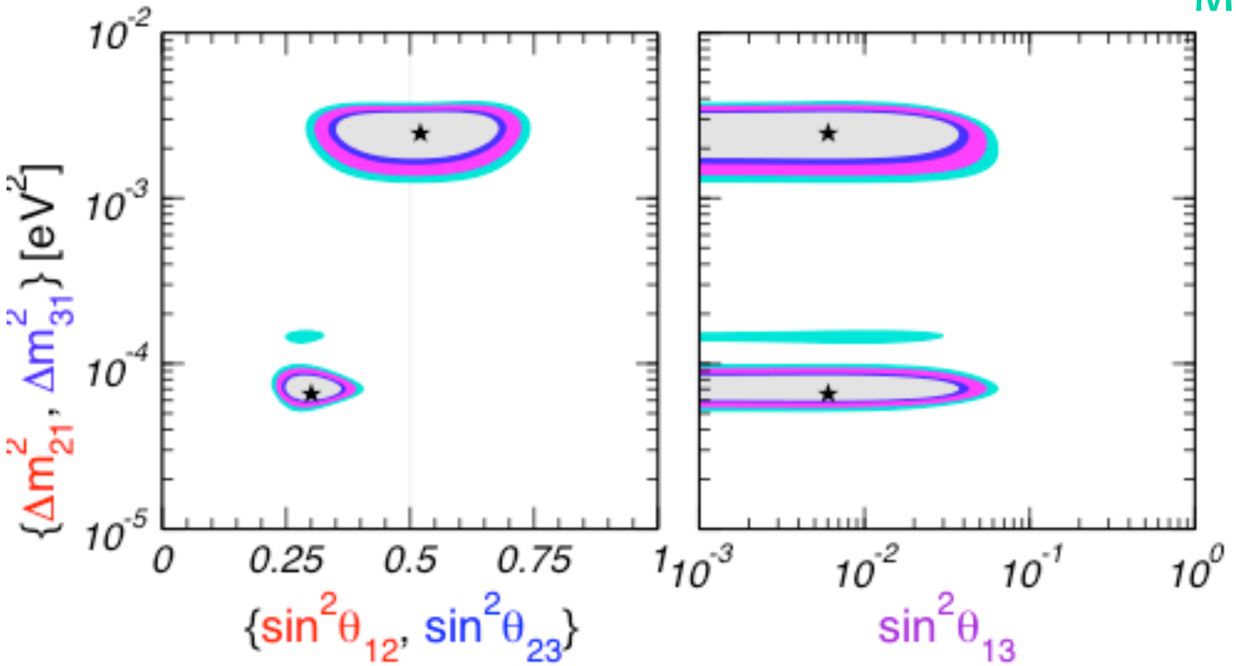
$\Delta m^2_{atm} > 0$ normal hierarchy

Neutrino oscillation parameters

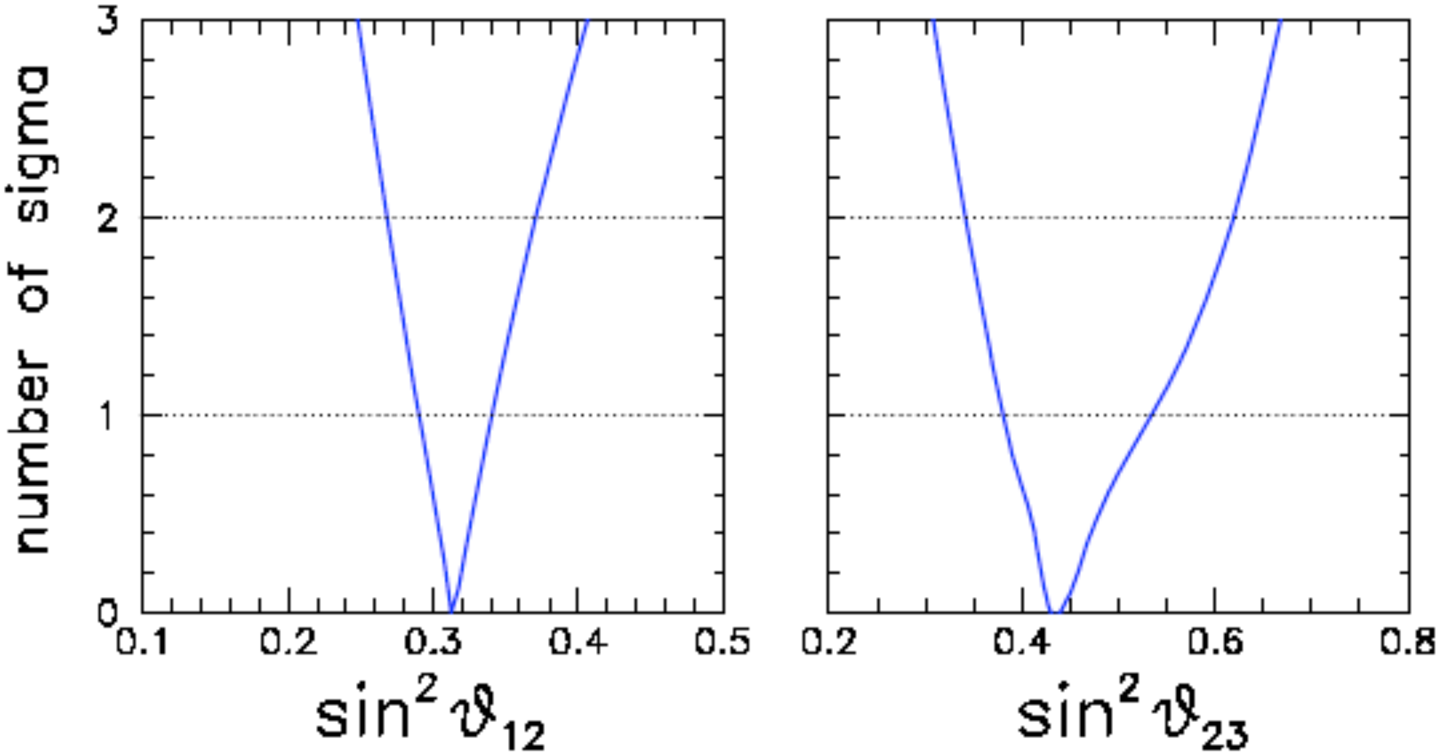
- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ	5σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	6.9	6.0–8.4	5.4–9.5	2.1–28
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	≤ 0.035	≤ 0.054	≤ 0.11

Maltoni et al '04



Fogli et al '05



$$\delta m^2 = 7.92\,(1^{+0.09}_{-0.09}) \times 10^{-5}\ \mathrm{eV}^2$$

$$\Delta m^2 = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3}\ \mathrm{eV}^2$$

$$\sin^2\theta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

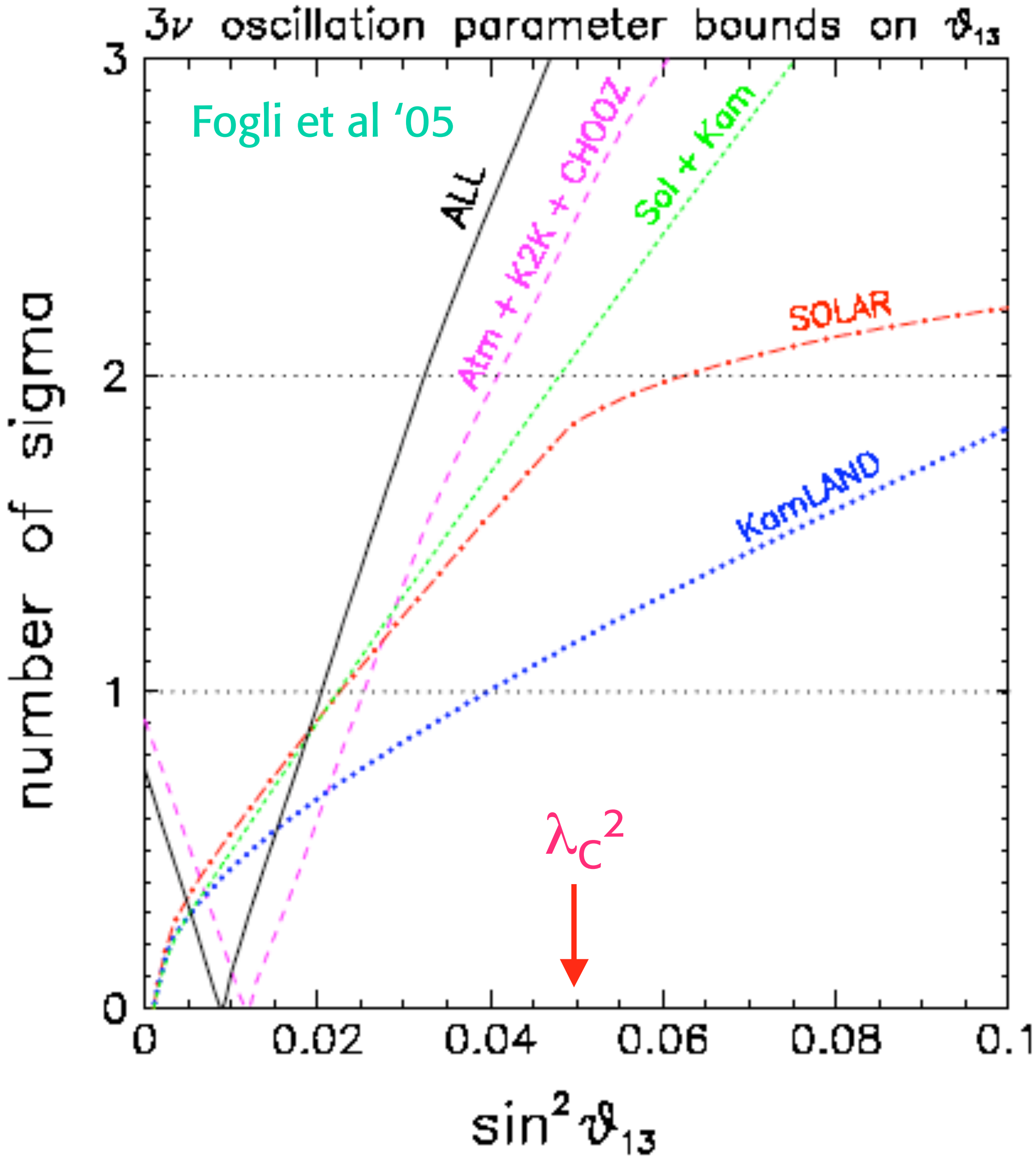
$$\sin^2\theta_{23} = 0.44\,(1^{+0.41}_{-0.22})$$

$$\sin^2\theta_{13} < 3.2 \times 10^{-2}$$

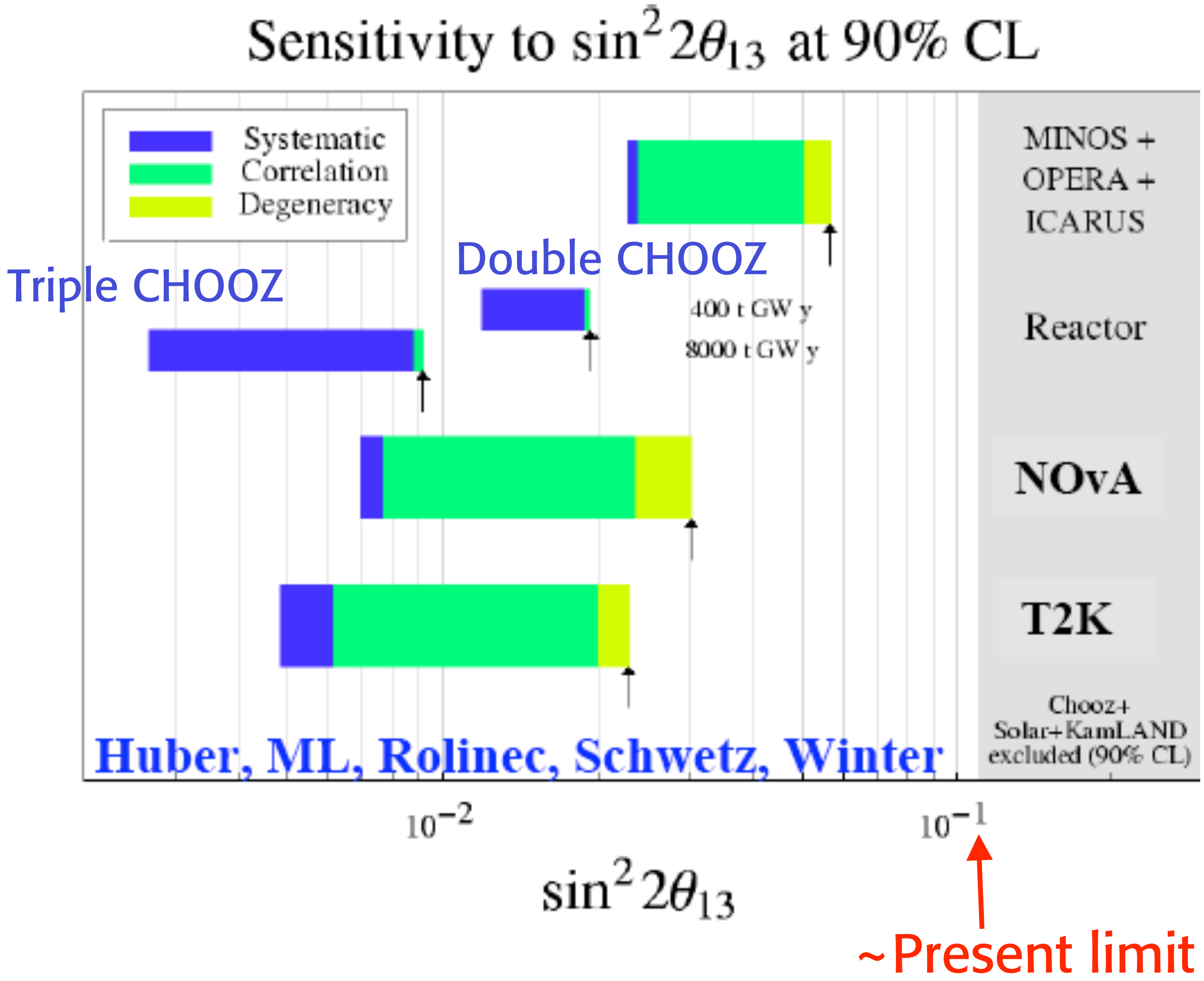
2σ ranges 95%

very precise (KamLAND)
very close to 1/3

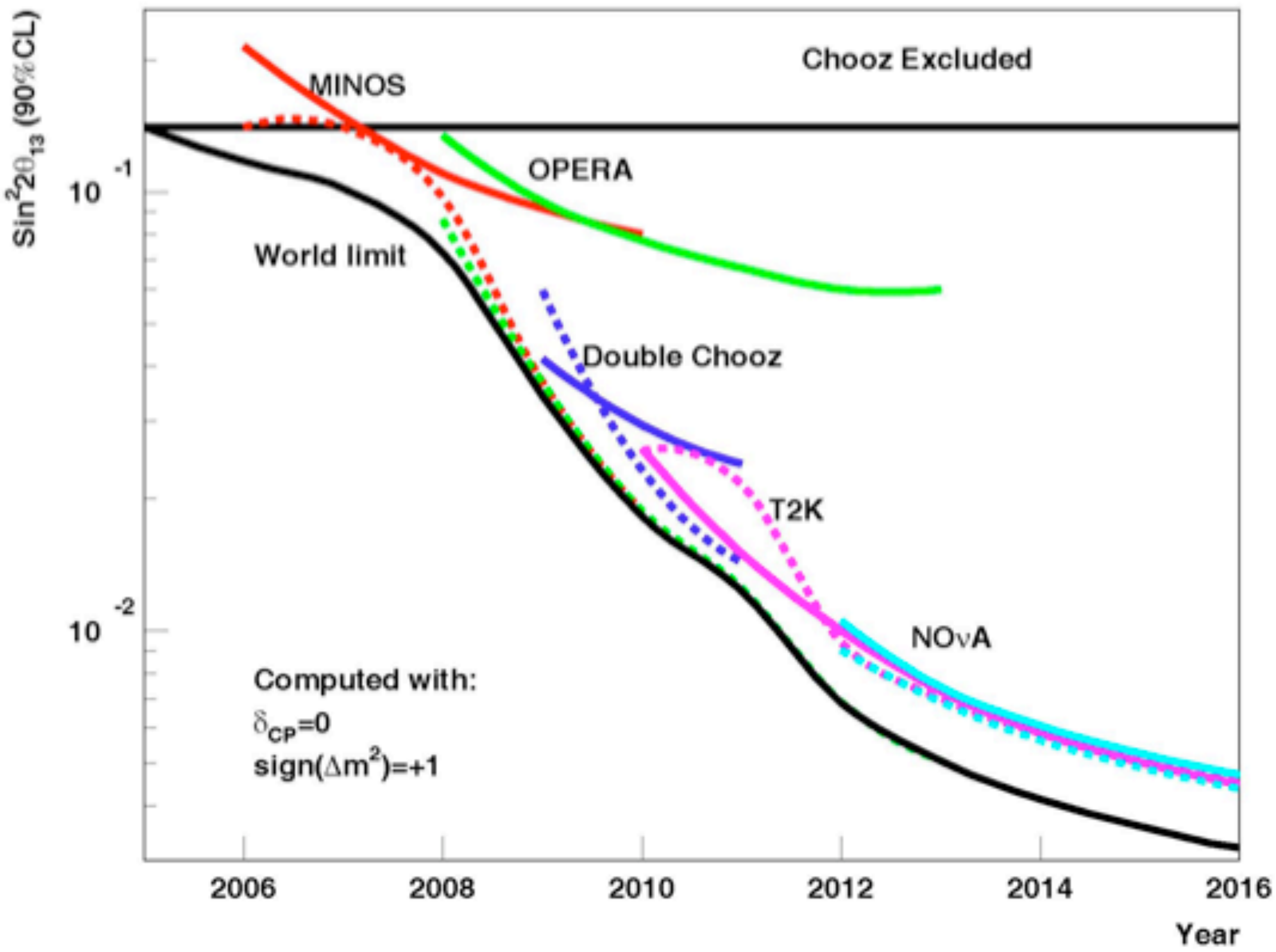
θ_{13} bounds

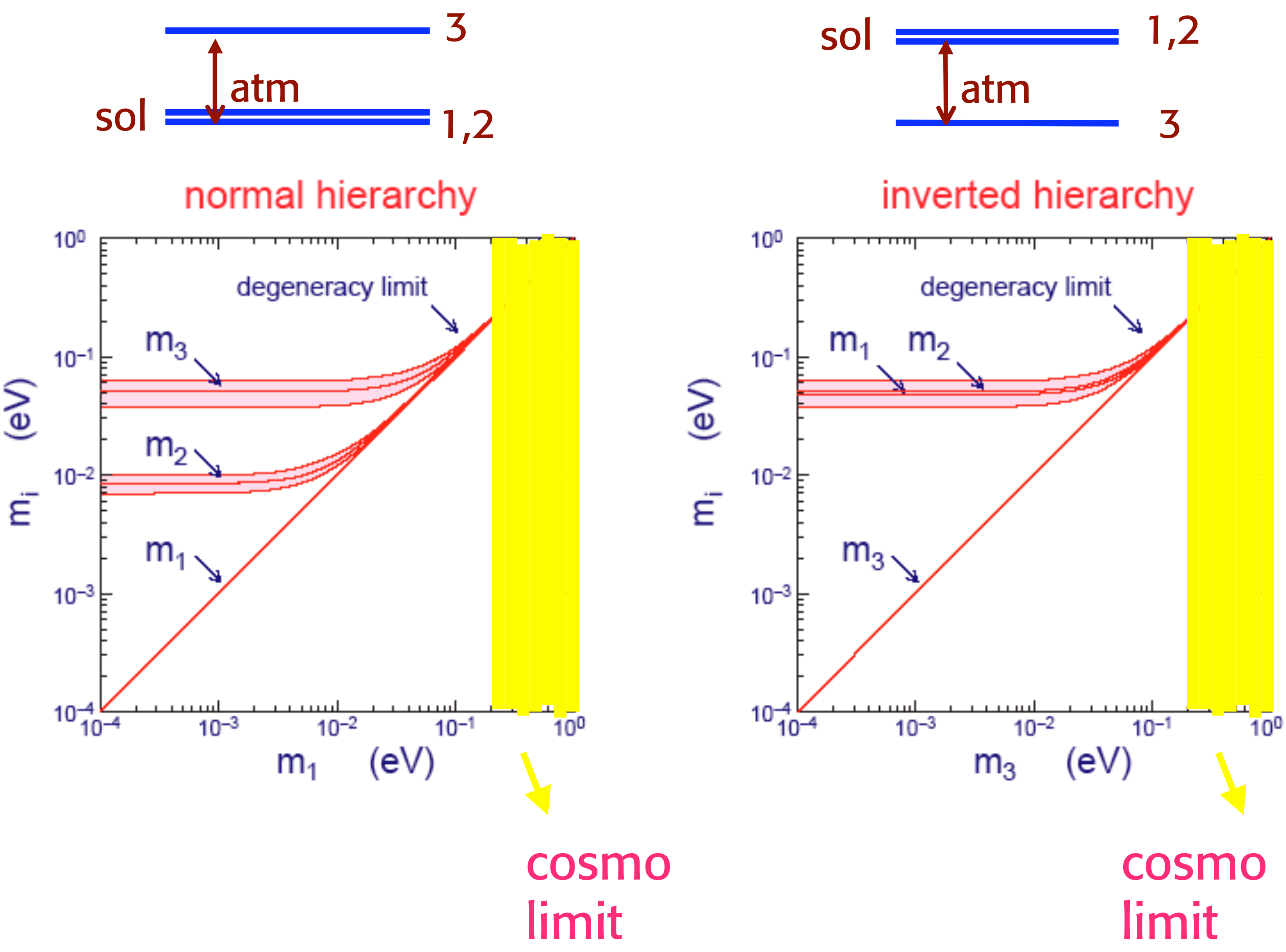


Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)



A possible time map for $\sin^2 2\theta_{13}$





Only moderate degeneracy allowed

0νββ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}|=c_{13}^2\,[m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

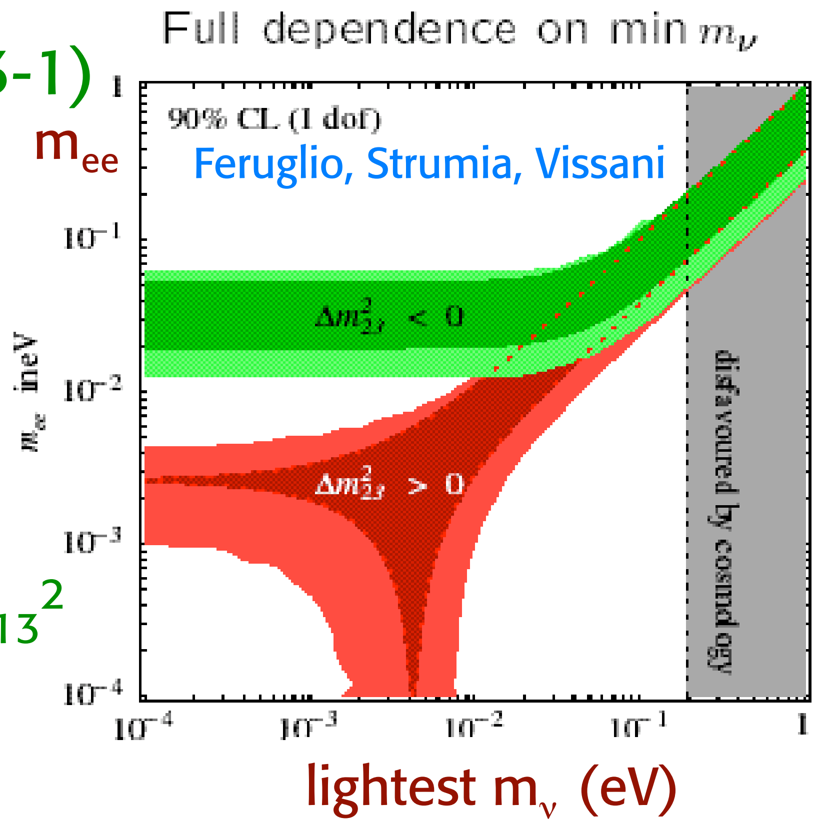
$|m_{ee}|\sim|m|\,(0.3\text{ -}1)\leq< 0.23\text{-}1$ eV

IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim(1.6\text{-}5)\;10^{-2}$ eV

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) 10^{-3} eV



Present exp. limit: $m_{ee}< 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

Baryogenesis

$n_B/n_\gamma \sim 10^{-10}$, $n_B >> n_{Bbar}$

Conditions for baryogenesis: (Sacharov '67)

- B non conservation (obvious)
- C, CP non conserv'n (B-B^{bar} odd under C, CP)
- No thermal equilib'm ($n = \exp[\mu - E/kT]$; $\mu_B = \mu_{Bbar}$, $m_B = m_{Bbar}$ by CPT

If several phases of BG exist at different scales the asymm. created by one out-of-equilib'm phase could be erased in later equilib'm phases: BG at lowest scale best

Possible epochs and mechanisms for BG:

- At the weak scale in the SM Excluded
- At the weak scale in the MSSM Disfavoured
- Near the GUT scale via Leptogenesis Very attractive

Possible epochs for baryogenesis

- BG at the weak scale: $T_{EW} \sim 0.1$- 10 TeV

Rubakov, Shaposhnikov; Cohen, Kaplan, Nelson; Quiros….

In SM:

- B non cons. by instantons ('t Hooft)
  (non pert.; negligible at T=0 but large at $T=T_{EW}$
  B-L conserved!

- CP violation by CKM phase. Not enough
  By general consensus far too small.

- Out of equilibrium during the EW phase trans.
  Needs strong 1st order phase trans. (bubbles)
  Only possible for $m_H <\sim 40$ GeV
  Now excluded by LEP

Is BG at the weak scale possible in MSSM?

- In principle additional sources of CP violation

  But so far no signal at beauty factories

- Constraint on m_H modified by presence of extra scalars with strong couplings to Higgs sector (e.g. s-top)

- Requires:
  m_h<80-100 GeV; $m_{s\text{-topl}}$<m_t; $tg\beta \sim 1.2$-5 preferred

  Espinosa, Quiros, Zwirner; Giudice; Myint; Carena, Quiros, Wagner; Laine; Cline, Kainulainen; Farrar, Losada…..

Much disfavoured by LEP

Baryogenesis by decay of heavy Majorana ν's

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al …..

Only survives if Δ(B-L)≠is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

Large neutrino mixings can induce observable $\tau \to \mu\gamma$ and $\mu \to e\gamma$ transitions

In fact, in SUSY models large lepton mixings induce large s-lepton mixings via RG effects (boosted by the large Yukawas of the 3rd family)

Detailed predictions depend on the model structure and the SUSY parameters.
Lopsided models tend to lead to the largest rates.

Typical values: $B(\mu \to e\gamma) \sim 10^{-11} - 10^{-14}$ (now: $\sim 10^{-11}$)
$B(\tau \to \mu\gamma) < \sim 10^{-7}$ (now: $\sim 10^{-7}$)

See, e.g., ••••• Lavignac, Masina, Savoy'02
Masiero, Vempati, Vives'03; Babu, Dutta, Mohapatra'03;
Babu, Pati, Rastogi'04; Blazek, King '03; Petcov et al '04;
Barr '04 ••••••

Model building

Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,

G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103,

hep-ph/0610165.

G.A, R. Franceschini, hep-ph/0512202.

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048];

G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

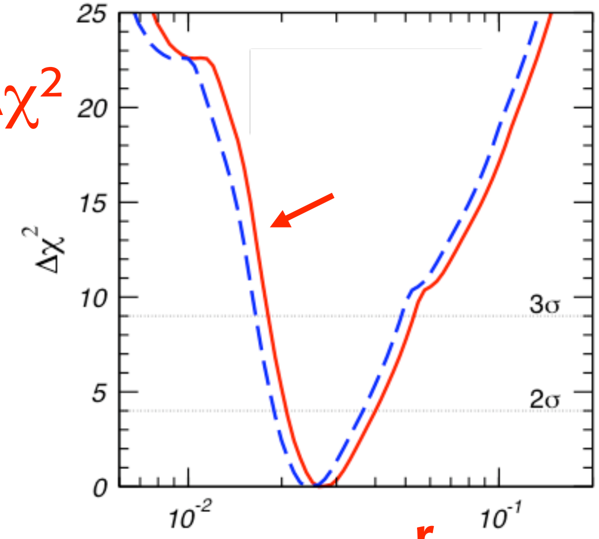
G.A, hep-ph/0611117.

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is needed for ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

$\Delta\chi^2$



r

Precisely at 2σ: $0.025 < r < 0.049$

or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν

$\longrightarrow$ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

3-σ interval $0.31 < \sin^2\theta_{23} < 0.72$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

Very small θ_{13} theoretically hard

Many viable solutions

Normal models: θ_{23} large but not maximal, θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

Exceptional models: θ_{23} maximal and/or θ_{13} very small and/or a special value for θ_{12}....

Natural models of the “normal” type are not too difficult to build up

It is reasonable to attribute hierarchies in masses and mixings to differences in some flavour quantum number(s).

A simplest flavour (or horizontal) symmetry is $U(1)_F$

For example, some simple models based on see-saw and $U(1)_F$ work for all quark and lepton masses and mixings, are natural and compatible with (SUSY) GUT’s, e.g $SU(5)xU(1)_F$.

Larger flavour symmetry groups have been studied.
They are more predictive but less flexible.
The problem of the "best" flavour group is still open.

The most ambitious models try to combine (SUSY) SO(10) GUT's with a suitable flavour group

Hierarchy for masses and mixings via horizontal $U(1)_F$ charges.

Froggatt, Nielsen '79

Principle: A generic mass term

$\bar{R}_1 m_{12} L_2 H$

is forbidden by U(1)

if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.
If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

$$\bar{R}_1 m_{12} L_2 H \, (\theta/M)^{\overbrace{q1+q2+qH}^{\Delta_{charge}}} \qquad m_{12} \to m_{12}\, \lambda^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0) etc -> many versions

For example: Normal hierarchy

- A crucial point: in the 2-3 sector we need both large m_3-m_2 splitting and large mixing.

  $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
  $m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 8\ 10^{-3}$ eV

- The "theorem" that large Δm_{32} implies small mixing (pert. th.: $\theta_{ij} \sim 1/|E_i - E_j|$) is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

  Det = 0; Eigenvl's: 0, $1+x^2$
  Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For $x \sim 1$
large splitting
and large mixing!

Examples of mechanisms for Det[23]~0

based on see-saw: $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\quad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5): LH for d quarks	$\Longleftrightarrow$	RH for l^- leptons

$$m_d \sim \overset{\bar{5}}{\bar{d}_R}\,\overset{10}{d_L}$$

$$m_e \sim \overset{10}{\bar{e}_R}\,\overset{\bar{5}}{e_L}$$

$$\bar{5} : (\underbrace{\bar{d},\bar{d},\bar{d},}_{R}\ \underbrace{\nu,e^-}_{L})$$

$$\boxed{m_d = m_e^T}$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models large atmospheric mixing arises (at least in part) from the charged lepton sector.

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

SU(5)xU(1)$_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

Offers a simple description of hierarchies, but it is not very predictive (large number of undetermined o(1) parameters)

- SO(10) models could be more predictive, as are non abelian flavour symmetries, eg O(3)$_F$, SU(3)$_F$

Albright, Barr; Babu et al; Bajic et al; Barbieri et al; Buccella et al; King et al; Mohapatra et al; Raby et al; G. Ross et al

Example: Normal Hierarchy

G.A., Feruglio, Masina

1st fam. 2nd 3rd

$q(10)$: $(5, 3, 0)$
$q(\bar{5})$: $(2, 0, 0)$
$q(1)$: $(1,-1, 0)$

Note: not all charges positive
--> det23 suppression

$q(H) = 0$, $q(\bar{H}) = 0$
$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $\langle\theta\rangle/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. 0(1) omitted, only orders of magnitude predicted

$$\begin{array}{c} \bar{5}_i 1_j \\ \swarrow \end{array} \qquad \qquad \qquad \qquad \qquad \qquad \begin{array}{c} 1_i 1_j \\ \swarrow \end{array}$$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix}, \qquad M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}{}^T M_{RR}{}^{-1} m_{\nu D}$

$$m_\nu \sim v_u{}^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \boxed{\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}} \\ \lambda^2 & & \end{bmatrix},$$

$\det_{23} \sim \lambda^2$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

This model works, in the sense that all small parameters are naturally due to various degrees of suppression.
But too many free parameters!!

A very exceptional model

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \rightarrow \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

Fogli et al '05

$\sin^2\theta_{12} = 1/3$: 0.290-0.342

$\sin^2\theta_{23} = 1/2$: 0.39-0.53

$\sin^2\theta_{13} = 0$: < 0.02

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

It is interesting to construct models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) are very interesting
Ma...;
GA, Feruglio hep-ph/0504165, hep-ph/0512103

Alternative models based on $SU(3)_F$ or $SO(3)_F$
Verzielas, G. Ross King

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

Thus A4 transf.s can be written as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

C_1, C_2, C_3, C_4 are equivalence classes

x, x′ in same class if $[x' \sim g x g^{-1}]$ g: group element

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1', 1"

In the (S-diag basis) consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$

(a_1,a_2,a_3) $\xrightarrow{T}$ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$1" = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2\,[a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

while, under S, 1" is inv.

Three singlet inequivalent represent'ns:

Recall:

$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1\text{: } S=1,\ T=1 \\ 1'\text{: } S=1,\ T=\omega \\ 1''\text{: } S=1,\ T=\omega^2 \end{cases}$$

$$\boxed{\begin{aligned} \omega &= \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2} \\ \omega^3 &= 1 \\ 1+\omega+\omega^2 &= 0 \\ \omega^2 &= \omega^* \end{aligned}}$$

The only indep. 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

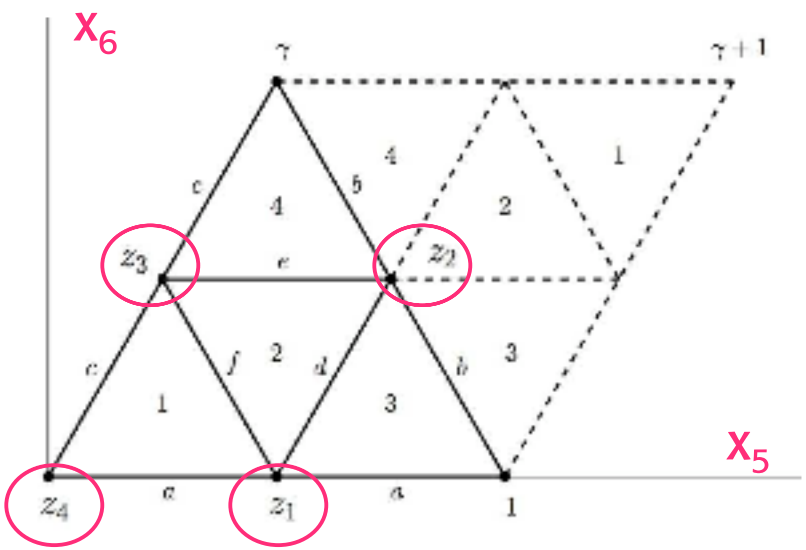
$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

What can be the origin of A4?

G.A., F. Feruglio, hep-ph/0610165

A4 (or some other discrete group) could arise from extra dimensions (by orbifolding with fixed points) as a remnant of 6-dim spacetime symmetry.



$z = x_5 + i x_6$

A torus with identified points:

$z \to z + 1$

$z \to z + \gamma \qquad \gamma = \exp(i\pi/3)$

and a parity $z \to -z$

leads to 4 fixed points

(equivalent to a tethraedron).

There are 4D branes at the fixed points where the SM fields live (additional gauge singlets are in the bulk)

⊕ A4 interchanges the fixed points

Under A4

lepton doublets l ~ 3
e^c, μ^c, τ^c ~ 1, 1′, 1″ respectively
gauge singlet flavons ϕ, ϕ', ξ, (ξ') ~ 3, 3, 1,(1) respectively
driving fields (for SUSY version) ϕ_0, ϕ'_0, ξ_0 ~ 3, 3, 1

Additional symmetries: broken $U(1)_F$ symmetry (ch. lepton masses) with e^c, μ^c, τ^c charges (3 or 4,2,0)
and a discrete symmetry (dep. on versions) : for example
Z: (e^c, μ^c, τ^c)-> -i (e^c, μ^c, τ^c), l -> il, ϕ -> ϕ, (ξ,ϕ') -> - (ξ,ϕ')

The Yukawa interactions in the lepton sector are:

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

Here is without see-saw (with see-saw is also OK: wait!)

Structure of the model

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a\xi(ll) + x_d(\varphi' ll) + h.c. + ...$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l)h_d/\Lambda, \qquad x_a\xi(ll) \sim x_a\xi(lh_u lh_u)/\Lambda^2$$

!!!

$$\boxed{\begin{array}{rcl} \langle\varphi'\rangle & = & (v',0,0) \\ \langle\varphi\rangle & = & (v,v,v) \\ \langle\xi\rangle & = & u \end{array}}$$

$$m_l = v_d\frac{v}{\Lambda}\begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu\omega^2 & y_\mu\omega \\ y_\tau & y_\tau\omega & y_\tau\omega^2 \end{pmatrix}$$

the big plus of A4

Spectrum free.
Diagonalized by U_e:

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix} \qquad l \to \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} l = Vl$$

- From here it follows that U_{HPS} is the mixing matrix

m_ν in the basis of diagonal charged leptons is:

$$m_\nu\big|_{ldiag} \sim V^* \begin{bmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{bmatrix} V^* = \begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

which in turn can be written as:

$$m_\nu\big|_{ldiag} \sim U^T \begin{bmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{bmatrix} U$$

with:

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

The crucial issue is to guarantee the strict alignment

$$
\begin{aligned}
\langle \varphi' \rangle &= (v', 0, 0) \\
\langle \varphi \rangle &= (v, v, v) \\
\langle \xi \rangle &= u
\end{aligned}
$$

We have constructed a number of completely natural versions of the model, e.g.:

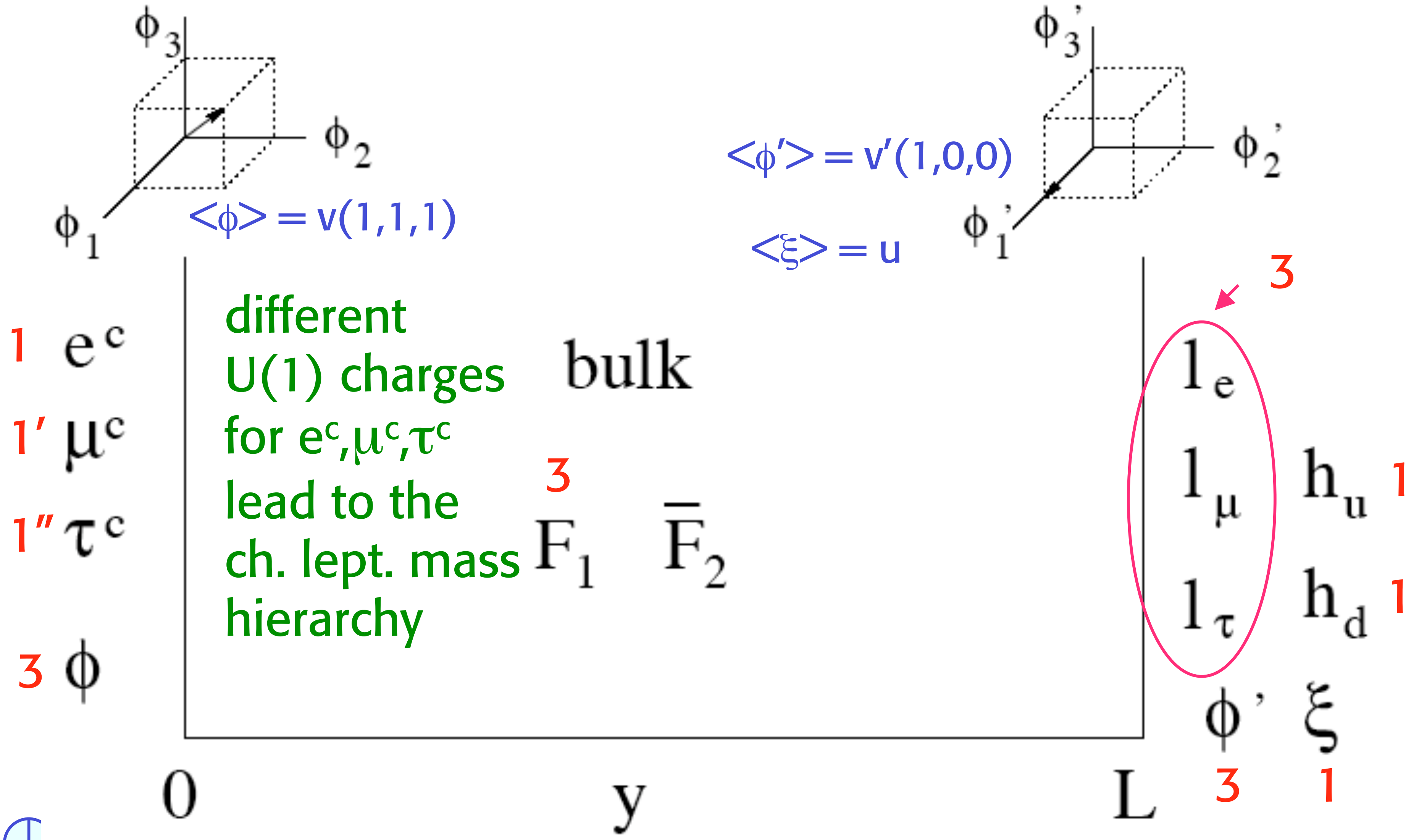
- a version in 5 dimensions (economic in flavon fields)
- a SUSY version in 4-dim (with more fields)

We first briefly discuss the 5-dim version

GA, Feruglio hep-ph/0504165

The model has 1 compactified extra dim. and 2 branes

(crucial issue: guarantee and protect the vev alignment)



In lowest approximation the action is:

$$
\begin{aligned}
S &= \int d^4x dy \Big\{ \Big[iF_1\sigma^\mu\partial_\mu\overline{F}_1 + iF_2\sigma^\mu\partial_\mu\overline{F}_2 + \frac{1}{2}(F_2\partial_y F_1 - \partial_y F_2 F_1 + h.c.) \Big] \\
&- M(F_1F_2 + \overline{F}_1\overline{F}_2) \\
&+ V_0(\varphi)\delta(y) + V_L(\varphi', \xi)\delta(y-L) \\
&+ [Y_e e^c(\varphi F_1) + Y_\mu \mu^c(\varphi F_1)'' + Y_\tau \tau^c(\varphi F_1)' + h.c.]\,\delta(y) \\
&+ \Big[\frac{x_a}{\Lambda^2}\xi(ll)h_u h_u + \frac{x_d}{\Lambda^2}(\varphi' ll)h_u h_u + Y_L(F_2 l)h_d + h.c. \Big]\,\delta(y-L) \Big\} + \ldots
\end{aligned}
$$

a Z-parity has also been imposed

$$
(f^c, l, F, \varphi, \varphi', \xi) \xrightarrow{Z} (-if^c, il, iF, \varphi, -\varphi', -\xi)
$$

After integrating out of the F fields one obtains the required effective 4-dim action

$$
\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots
$$

In the flavour basis:

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

m_ν = U diag(a+d,a,-a+d)U^T (in units of v_u^2/Λ) and U=U_{HPS}

In terms of physical param.s (moderate normal hierarchy):

$$|m_1|^2 = \left[-r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_2|^2 = \frac{1}{8\cos^2\Delta(1-2r)}\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_3|^2 = \left[1 - r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.053\ \mathrm{eV})^2$$

- A moderate fine tuning is needed for r

A version with see-saw is also possible

ν_R is a triplet of A4: $\nu^c \sim 3$ No change for ch leptons

Dirac Majorana

$$w_l = \ldots + y(\nu^c l) + x_A \xi(\nu^c \nu^c) + x_B(\varphi_T \nu^c \nu^c)$$

[Discrete parity Z: $\omega,\omega^2,\omega^2,\omega^2$ for l, ν^c, ϕ_T, ξ respectively]

$m_\nu^D \sim 1$ $M_{RR} \sim \begin{bmatrix} A & 0 & 0 \\ 0 & A & D \\ 0 & D & A \end{bmatrix}$ $m_\nu = m_\nu^{DT} M_{RR}^{-1} m_\nu^D \sim M_{RR}^{-1}$

The mass matrix appears just as the inverse of what was before, so that the mixing matrix is the same.

Eigenvalues are the inverse: one can produce inverse hierarchy with realistic θ_{12}, θ_{23} and very small θ_{13}

The 4-dim SUSY version (written in the T-diag basis)

In this basis the ch. leptons are diagonal!

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

$$\begin{aligned} w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\ &+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2 \end{aligned}$$

and the potential $$V = \sum_i \left| \frac{\partial w}{\partial \phi_i} \right|^2 + m_i^2 |\phi_i|^2 + \ldots$$

The assumed simmetries are summarised here

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2
$U(1)_F$		2q	q	1								

The driving field have zero vev. So the minimization is:

$$\frac{\partial w}{\partial \varphi_{01}^T} = M\varphi_{T1} + \frac{2g}{3}(\varphi_{T1}^2 - \varphi_{T2}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{01}^S} = g_2\tilde{\xi}\varphi_{S1} + \frac{2g_1}{3}(\varphi_{S1}^2 - \varphi_{S2}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{02}^T} = M\varphi_{T3} + \frac{2g}{3}(\varphi_{T2}^2 - \varphi_{T1}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{02}^S} = g_2\tilde{\xi}\varphi_{S3} + \frac{2g_1}{3}(\varphi_{S2}^2 - \varphi_{S1}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{03}^T} = M\varphi_{T2} + \frac{2g}{3}(\varphi_{T3}^2 - \varphi_{T1}\varphi_{T2}) = 0 \qquad \frac{\partial w}{\partial \varphi_{03}^S} = g_2\tilde{\xi}\varphi_{S2} + \frac{2g_1}{3}(\varphi_{S3}^2 - \varphi_{S1}\varphi_{S2}) = 0$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S1}^2 + 2\varphi_{S2}\varphi_{S3}) = 0$$

Solution:

$$\varphi_T = (v_T, 0, 0) \quad , \qquad v_T = -\frac{3M}{2g}$$

$$\begin{aligned} \tilde{\xi} &= 0 \\ \xi &= u \\ \varphi_S &= (v_S, v_S, v_S) \quad , \qquad v_S^2 = -\frac{g_4}{3g_3}u^2 \end{aligned}$$

In the paper
w at NLO is also
studied

NLO corrections studied in detail

to m_l 1st non trivial correction at $o(1/\Lambda^3)$

LO is $1/\Lambda$

to m_ν

LO is $1/\Lambda^2$

$$\frac{x_c}{\Lambda^3}(\varphi_T\varphi_S)'(ll)''h_u h_u \qquad \frac{x_d}{\Lambda^3}(\varphi_T\varphi_S)''(ll)'h_u h_u \qquad \frac{x_e}{\Lambda^3}\xi(\varphi_T ll)h_u h_u$$

to vevs

LO is 1

$$\begin{aligned}
\langle\varphi_T\rangle &\rightarrow (v_T' + \delta v_T, \delta v_T, \delta v_T)\\
\langle\varphi_S\rangle &\rightarrow (v_S + \delta v_1, v_S + \delta v_2, v_S + \delta v_3)\\
\langle\xi\rangle &\rightarrow u\\
\langle\tilde{\xi}\rangle &\rightarrow \delta u'
\end{aligned}$$

δv_T, δv_S, δv_i, $\delta u'$ ~ $o(1/\Lambda)$

All observables get a correction of order $1/\Lambda$

From exp (eg θ_{12}) must be less than 5%

$$0.0022 < \frac{v_S}{\Lambda} \approx \frac{v_T}{\Lambda} \approx \frac{u}{\Lambda} < 0.05$$

In particular $\theta_{13} <\sim 0.05$,
$|tg^2\theta_{23}-1| <\sim 0.05$

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices are BOTH diagonalised by

$$U_u, U_d \sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

As a result VCKM is unity: $V_{CKM} = U_u^{+} U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are HPS and quark mixings ~identity

Corrections are far too small to reproduce quark mixings eg λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$. But even those are essentially the same for u and d quarks)

Note: it not straightforward to embed these models in a GUT: with these assignments A4 does not commute with SU(5)

If $l \sim 3$ then all 5bar ~ 3, so that $d^c_i \sim 3$
if $e^c, \mu^c, \tau^c \sim 1, 1', 1''$ then all $10_i \sim 1, 1', 1''$

Realistic quark mass matrices are not easy to obtain from these assigments

For example, for u quarks at leading order:

$$m_u \sim 1.1 + 1'.1'' + 1''.1' \sim a\, u_1 u_1 + b\, (u_2 u_3 + u_3 u_2)$$

or

$$m_u \sim \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$$

Which implies $|m_c| = |m_t|$ and maximal U_{23}

Main lessons from ν masses and mixings

- ν's are not all massless but their masses are very small
- probably masses are small because ν's are Majorana particles
- then masses are inv. prop. to the large scale M of L n. viol.
- $M \sim m_{\nu R}$ is empirically close to 10^{14}-10^{15} GeV ~ M_{GUT}
  -> ν masses fit well in the SUSY GUT picture
- decays of ν_R with CP & L violation can produce a B-L asymm.
  -> baryogenesis via leptogenesis
- detecting $0\nu\beta\beta$ would prove ν's are Majorana and L is viol.
- ν mixing angles are large except for θ_{13} that is small
- ν's are not a significant component of dark matter in Universe
- there is no contradiction between large ν mixings and small
  q mixings, even in GUT's

Conclusion

From experiment: a good first approximation for quarks:

$$V_{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and for neutrinos

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Models based on A4 indeed lead to this pattern

All this is highly non trivial but no real illumination has followed!!