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Beyond the Standard Model

GUT's 2007

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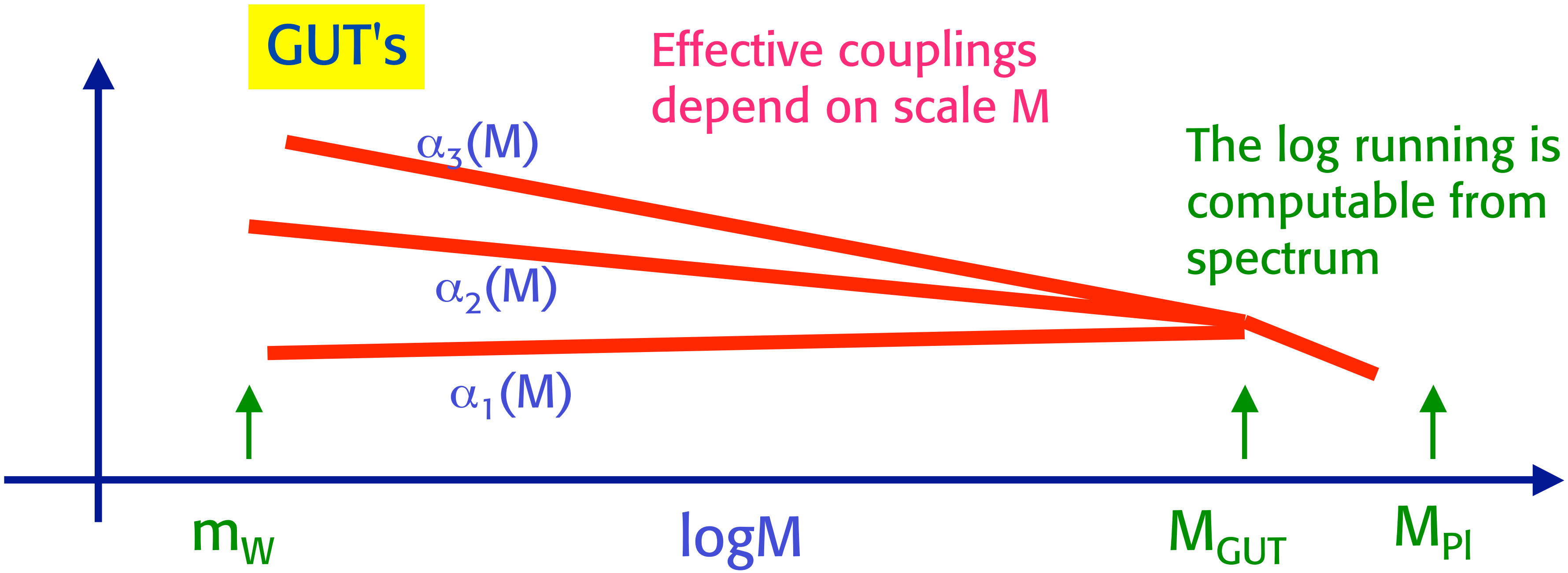
Plan of the lectures

- Experimental Status of the SM
- Problems of the SM (conceptual and empirical)
- Overview of Physics Beyond the SM
    - Supersymmetry
    - Little Higgs Models
    - Extra Dimensions
    - Composite Higgs
- The most accepted BSM: GUT's
- The most established BSM: Neutrino masses

My purpose: give basic facts, describe the most interesting ideas, expand on the most realistic avenues (proceed from real to imaginary)



The large scale structure of particle physics:

- SU(3) ⊗ SU(2) ⊗ U(1) unify at M_{GUT}
- at M_{Pl}: quantum gravity

$$G_{Newton} = \frac{\hbar c}{M_{Pl}^2}$$

$r \sim 10^{-33}$ cm

Superstring theory:
a 10-dimensional non-local, unified theory of all interact's

The really fundamental level

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By now GUT's are part of our culture in particle physics

- Unity of forces: $G \supset SU(3) \otimes SU(2) \otimes U(1)$

  unification of couplings
- Unity of quarks and leptons

  different "directions" in G
- B and L non conservation

  ->p-decay, baryogenesis, ν masses
- Family Q-numbers

  e.g. in SO(10) a whole family in 16
- Charge quantisation: $Q_d = -1/3 \to -1/N_{colour}$

  anomaly cancelation

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Most of us believe that Grand Unification must be a feature of the final theory!

$$G \supset SU(3) \otimes SU(2) \otimes U(1)$$

G commutes with the Poincare' group

→ repres.ns must contain states with same momentum, spin..

We cannot use e^-_L, e^-_R, but need all L or all R.

$$e^-_R \xrightarrow{\text{TCP}} e^+_L$$

We can use e^-_L, e^+_L etc. One family becomes

$3 \times \begin{bmatrix} u \\ d \end{bmatrix}_L$ $\begin{bmatrix} \nu \\ e^- \end{bmatrix}_L$ $3 \times u^{bar}_L$, $3 \times d^{bar}_L$ e^+_L (ν^{bar}_L)

Note that in each family there are 15 (16) two-component spinors

SU(5): $5^{bar} + 10 + (1)$

SO(10): 16

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Content of SU(5) representations

$$\bar{5} = \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ \nu \\ e^- \end{bmatrix} \qquad 10 = \begin{bmatrix} 0 & \bar{u}_3 & \bar{u}_2 & u_1 & d_1 \\ - & 0 & \bar{u}_1 & u_2 & d_2 \\ - & - & 0 & u_3 & d_3 \\ - & - & - & 0 & e^+ \\ - & - & - & - & 0 \end{bmatrix}$$

$$24 = \begin{bmatrix} g & g & g & X_1^{4/3} & Y_1^{1/3} \\ g & g & g & X_2^{4/3} & Y_2^{1/3} \\ g & g & g & X_3^{4/3} & Y_3^{1/3} \\ X_1^{-4/3} & X_2^{-4/3} & X_3^{-4/3} & W^3 & W^+ \\ Y_1^{-1/3} & Y_2^{-1/3} & Y_3^{-1/3} & W^- & B \end{bmatrix}$$

SO(10) is very impressive

A whole family in a single representation 16

$$16 \supset \bar{5} + 10 + 1 \longleftarrow \nu_R$$

SO(10) SU(5)

Too striking not to be a sign! SO(10) must be relevant at least as a classification group.

Different avenues for SO(10) breaking:

We could have:

$$SO(10) \xrightarrow[M_{Pl}]{16} SU(5) \xrightarrow[M_{GUT}]{45} SU(3)x\ SU(2)xU(1)$$

and SU(5) physics is completely preserved

or $\longrightarrow$

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Interesting subgroups of SO(10) are

$$SO(10) \supset SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \qquad 54$$

$$SO(10) \supset SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \qquad 45$$

10x10=1+45+54

These breakings can occur anywhere from M_{GUT} down. Possibility of two steps: M_{GUT} -> $M_{intermediate}$ -> M_{weak}. In this case with $M_{intermediate} \sim 10^{12}$ GeV good coupling unification without SUSY.

PS= Pati-Salam: L as the 4th colour

16: $$\begin{bmatrix} u & u & u & \nu \\ d & d & d & e \end{bmatrix}_L = (4, 2, 1) \qquad \begin{bmatrix} u & u & u & \nu \\ d & d & d & e \end{bmatrix}_R = (4, 1, 2)$$

Also note: $Q = T^3_L + T^3_R + (B-L)/2$

Left-Right symmetry (parity) is broken spontaneously

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The 16 of SO(10) can be generated by 5 spin 1/2 with even number of $s_3 = -1/2$

State	Y	Color	Weak
ν^c	0	$+ + +$	$+ +$
e^c	2	$+ + +$	$- -$
u_r	$1/3$	$- + +$	$+ -$
d_r	$1/3$	$- + +$	$- +$
u_b	$1/3$	$+ - +$	$+ -$
d_b	$1/3$	$+ - +$	$- +$
u_y	$1/3$	$+ + -$	$+ -$
d_y	$1/3$	$+ + -$	$- +$
u^c_r	$-4/3$	$+ - -$	$+ +$
u^c_b	$-4/3$	$- + -$	$+ +$
u^c_y	$-4/3$	$- - +$	$+ +$
d^c_r	$2/3$	$+ - -$	$- -$
d^c_b	$2/3$	$- + -$	$- -$
d^c_y	$2/3$	$- - +$	$- -$
ν	-1	$- - -$	$+ -$
e	-1	$- - -$	$- +$

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In SM the covariant derivative is:

$$D_\mu = \partial_\mu - ie_s \sum_{c=1}^{8} t^c g_\mu^{\ c} - ig \sum_{i=1}^{3} t^i W_\mu^{\ i} - ig' \frac{Y}{2} B_\mu$$

$t^c = \frac{\lambda^c}{2}$ Gell-Mann $\qquad$ $t^i = \frac{\tau}{2}$ Pauli

$Tr(t^c t^{c'}) = 1/2\ \delta^{cc'}$ $\qquad$ $Tr(t^i t^{i'}) = 1/2\ \delta^{ii'}$

$$\alpha_s \equiv \alpha_3 = \frac{e_s^2}{4\pi} \qquad \alpha_W \equiv \alpha_2 = \frac{g^2}{4\pi} \qquad \alpha_1 = \frac{g'^2}{4\pi}$$

In G gauge th. the covariant derivative is:

$$D_\mu = \partial_\mu - ig_G \sum_{A=1}^{d} T^A X^A$$

g_G: symm. coupl.
X^A: G gauge bos'ns $Tr(T^A T^B) \sim \delta^{AB}$

I can always choose the T^A norm'n as:

$Q = t^3 + Y/2$ → $Q = T^3 + bT^0$ $\qquad$ Then $\quad aT^c = \lambda^c/2$

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a,b: const's dep. on G and the 3x2x1 embedding

From $Q = T^3 + bT^0$ we find:

$TrQ^2 = (1+b^2)trT^2$

$Tr(T^A T^B) \sim \delta^{AB}$

$tr(T^3)^2 = tr(T^0)^2 = tr(T^A)^2 = trT^2$

From $aT^c = \lambda^c/2$ we have:

$a^2 TrT^2 = Tr(\lambda^c/2)^2$

tr is over any red. or irred. repr. of G

IF all particles in one family fill one such repres. of G:

$3 \times \begin{bmatrix} u \\ d \end{bmatrix}_L$ $\begin{bmatrix} \nu \\ e^- \end{bmatrix}_L$ $3 \times u^{bar}_L$ $3 \times d^{bar}_L$ e^+_L (ν^{bar}_L)

$$Tr(T^3)^2 = 3 \cdot \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{4}\right) = 2$$

$$TrQ^2 = (3+3) \cdot \left(\frac{4}{9} + \frac{1}{9}\right) + 1 + 1 = \frac{16}{3}$$

$$Tr\left(\frac{\lambda_3}{2}\right)^2 = (2+2) \cdot \left(\frac{1}{4} + \frac{1}{4}\right) = 2$$

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→ $b^2 = 5/3, \; a^2 = 1$

The G-symmetric cov. derivative contains:

$$g_G \sum T^c g^c_\mu + g_G \sum T^i W^i_\mu + g_G T^0 B_\mu$$

or

$$\frac{g_G}{a} \sum t^c g^c_\mu + g_G \sum t^i W^i_\mu + \frac{g_G}{b} \frac{Y}{2} B_\mu$$

comparing with:

$$D_\mu = \partial_\mu - i e_s \sum_{c=1}^{8} t^c g_\mu{}^c - i g \sum_{i=1}^{3} t^i W_\mu{}^i - i g' \frac{Y}{2} B_\mu$$

we find: $\alpha_G = \frac{g_G^2}{4\pi}$ ← the one which is unified

$$\alpha_s \equiv \alpha_3 = \frac{\alpha_G}{a^2} \qquad \alpha_W \equiv \alpha_2 = \alpha_G \qquad \alpha_1 = \frac{\alpha_G}{b^2}$$

(SUSY) GUT's: Coupling Unification at 1-loop

SU(5), SO(10)
$b^2=5/3$
$a=1$

$$\frac{1}{b^2\alpha_1(\mu)} = \frac{1}{\alpha_G(M)} - \beta_1 \ln\frac{M^2}{\mu^2}$$

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_G(M)} - \beta_2 \ln\frac{M^2}{\mu^2}$$

$$\frac{1}{a^2\alpha_3(\mu)} = \frac{1}{\alpha_G(M)} - \beta_3 \ln\frac{M^2}{\mu^2}$$

SM

$$\beta_1 = -\frac{3}{5}\cdot\frac{n_H}{24\pi} + X$$

$$\beta_2 = \frac{11\cdot 2}{12\pi} - \frac{n_H}{24\pi} + X$$

$$\beta_3 = \frac{11\cdot 3}{12\pi} + X$$

SUSY

$$\beta_1 = -\frac{3}{5}\cdot\frac{3n_H}{24\pi} + X$$

$$\beta_2 = \frac{18}{12\pi} - \frac{3n_H}{24\pi} + X$$

$$\beta_3 = \frac{27}{12\pi} + X$$

We take as independent variables

$$(\sin\theta_W)^2 \equiv s_W^2 \quad , \quad \alpha \quad , \quad \alpha_3$$

In terms of them: $\alpha_2 = \frac{\alpha}{s_W^2} \qquad \alpha_1 = \frac{\alpha}{c_W^2}$

From (here $\alpha=\alpha(\mu)$)

$$\frac{1}{b^2\alpha_1} - \frac{1}{\alpha_2} = (\beta_2 - \beta_1)\cdot \ln\frac{M^2}{\mu^2}$$

$$\frac{1}{\alpha_2} - \frac{1}{a^2\alpha_3} = (\beta_3 - \beta_2)\cdot \ln\frac{M^2}{\mu^2}$$

For m=μ the differences vanish e.g. →

$$s_W^2\Big|_{at\ M} = \frac{1}{1+b^2}$$

Setting $b^2=5/3$ and a=1and $n_H=2$ in SUSY:

$$\frac{7}{5}\cdot\left(\frac{3}{5}\cdot\frac{c_W^2}{\alpha} - \frac{s_W^2}{\alpha}\right) = \frac{1}{\pi}\ln\frac{M^2}{\mu^2} = \frac{s_W^2}{\alpha} - \frac{1}{\alpha_3}$$

Equivalently:

$$s_W^2 = \frac{7}{15}\cdot\frac{\alpha}{\alpha_3} + \frac{1}{5} \qquad\qquad \ln\frac{M}{\mu} = \frac{\pi}{10}\cdot\left(\frac{1}{\alpha} - \frac{8}{3}\cdot\frac{1}{\alpha_3}\right)$$

1-loop SUSY:

$$s_W^2 = \frac{7}{15} \cdot \frac{\alpha}{\alpha_3} + \frac{1}{5} \qquad \ln\frac{M}{\mu} = \frac{\pi}{10} \cdot \left(\frac{1}{\alpha} - \frac{8}{3} \cdot \frac{1}{\alpha_3}\right)$$

Suppose we take $\mu \sim 100$ GeV, $s_W^2 \sim 0.23$, $\alpha \sim 1/129$
we obtain $\alpha_3 \sim 0.12$. The measured value at μ is just about 0.12.
(in the SM we would have obtained $\alpha_3 \sim 0.07$)

From the second eq. with $\alpha_3 \sim 0.12$ we find
$M \sim 4\ 10^{16}$ GeV (in SM $M \sim 2\ 10^{15}$ GeV).

From this simple 1-loop approx. we see that SUSY is much better than SM for both unification and p-decay
(p-decay rate scales as M^{-4}).

We now refine the evaluation by taking 2-loop beta functions and threshold corrections into account.

In the SUSY case there is a lot of sensitivity on the number of H doublets ($n_H=2+\delta$)

$$\alpha_3 = \alpha \cdot \frac{56 - 2\delta}{s_W^2 \cdot (120 + 6\delta) - (24 + 3\delta)}$$

δ	n_H	α_3
-2	0	0.068
-1	1	0.086
0	2	0.121
1	3	0.211
2	4	1.120

α_3-> infinity for δ=2.22...

So just 2 doublets are needed in SUSY and this is what is required in the MSSM!

In SM we would need n_H~7 to approach α_3~0.12

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The value of $\alpha_3(\mu)$ for unification, given s^2_W and α, is modified as:

1-loop

$$\alpha_3 = \frac{\alpha_3^{LO}}{1+\alpha_3^{LO}\delta}$$

$$\delta = k + \frac{1}{2\pi}\log\frac{m_{SUSY}}{m_Z} - \frac{3}{5\pi}\log\frac{m_{H_T}}{m_{GUT}^{LO}}$$

$$k = k_2 + k_{SUSY} + k_{GUT}$$

2-loop (→ k_2); thresholds (→ $k_{SUSY} + k_{GUT}$)

$k_2 \sim -0.733$

k_{SUSY} describes the onset of the SUSY threshold at around m_{SUSY}

k_{GUT} describes effects of the splittings inside (in SU(5)) the 24, 5 and 5^{bar}

Beyond leading approx. we define m_{GUT} as the mass of the heavy 24 gauge bosons, while $m_T = m_{HT}$ is the mass of the triplet Higgs

$$5^{bar} = \underset{H_T}{(3,1)} + \underset{H_D}{(1,2)}$$

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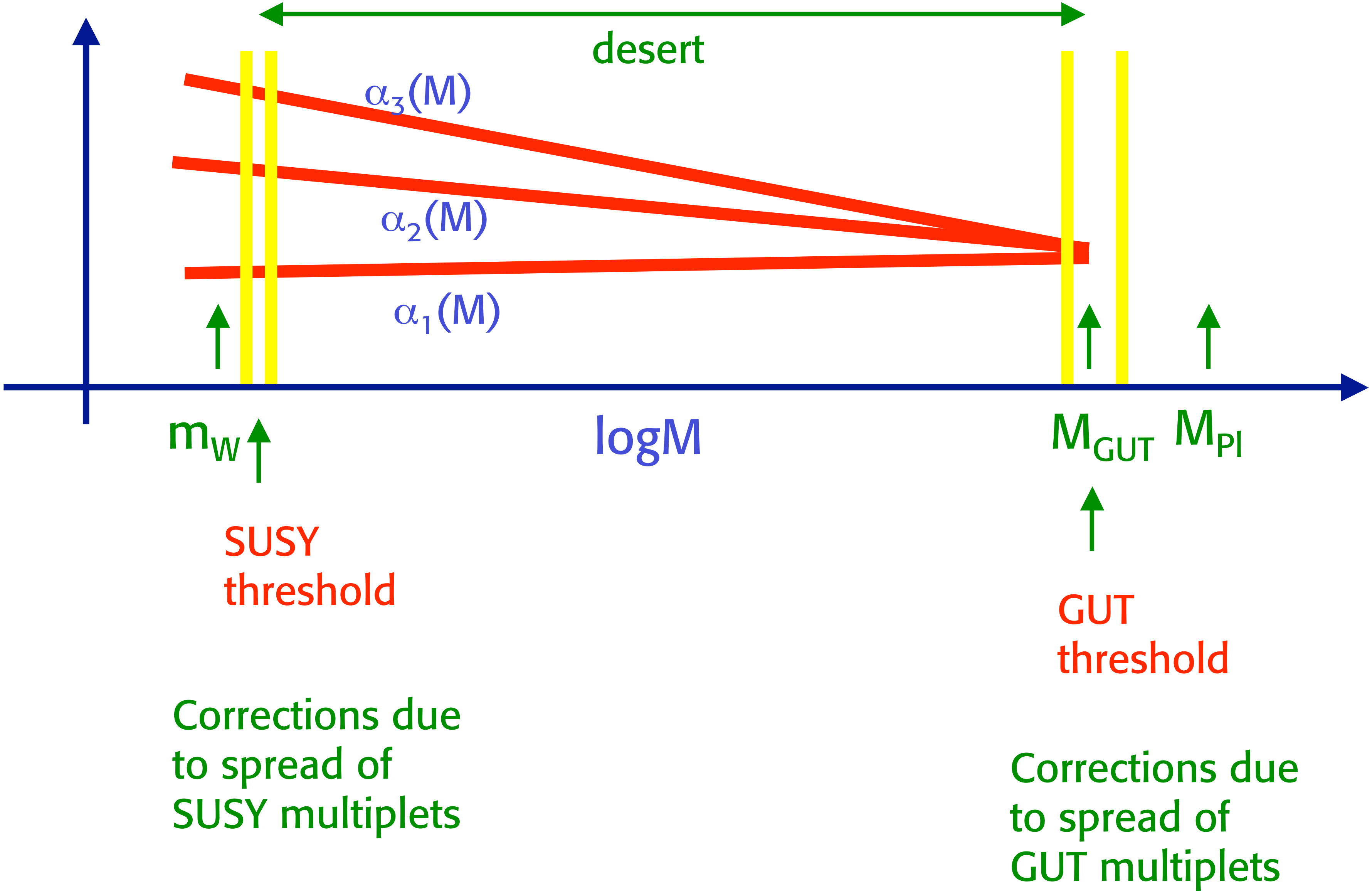
desert
α3(M)
α2(M)
α1(M)
mW
logM
MGUT
MPl
SUSY threshold
GUT threshold
Corrections due to spread of SUSY multiplets
Corrections due to spread of GUT multiplets

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From a representative SUSY spectrum:

sparticle	mass^2
gluinos	$(2.7 m_{1/2})^2$
winos	$(0.8 m_{1/2})^2$
higgsinos	μ^2
extra Higgses	m_H^2
squarks	$m_0^2 + 6 m_{1/2}^2$
(sleptons)$_L$	$m_0^2 + 0.5 m_{1/2}^2$
(sleptons)$_R$	$m_0^2 + 0.15 m_{1/2}^2$

with
$0.8m_0 = 0.8m_{1/2} = 2\mu = m_H = m_{SUSY}$
one finds: $k_{SUSY} \sim -0.510$

The value of k_{GUT} turns out to be negligible for the minimal model ($24+5+5^{bar}$): $k_{GUT} \sim 0$

$k = -0.733 - 0.510 = -1.243$ Minimal Model

This negative k tends to make α_3 too large: we must take m_{SUSY} large and m_T small.

But beware of hierarchy problem and p-decay!

$m_{SUSY} \sim 1$ TeV, $m_T \sim (m_{GUT})^{LO}$ → $\alpha_3 \sim 0.13$ → a bit large!

Similarly: $M_{GUT} \sim 2\ 10^{16}$ GeV

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The Doublet -Triplet Splitting Problem

In SU(5) the superpotential in the Higgs sector is

$$W = aH_{5Bar}\Sigma_{24}H_5 + mH_{5Bar}H_5$$

$$H_5 = \begin{bmatrix} H_{T1} \\ H_{T2} \\ H_{T3} \\ H^+ \\ H^0 \end{bmatrix}$$

H_{T1}, H_{T2}, H_{T3}: colour triplet

H^+, H^0: usual doublet

$$\langle \Sigma \rangle = M \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{-3}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{-3}{2} \end{bmatrix}$$

Higgs masses:

$m_{HT} = + aM + m$

$m_H = -3/2\ aM + m \sim 0$

Since $M \sim m \sim M_{GUT}$ it takes an enormous fine-tuning to set m_H to zero.

SUSY slightly better because once put by hand at tree level is not renormalised.

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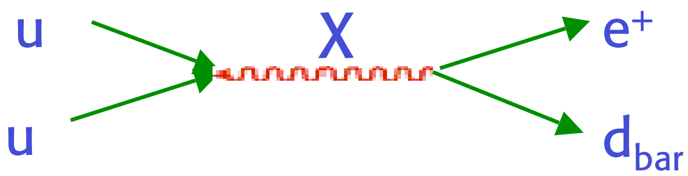
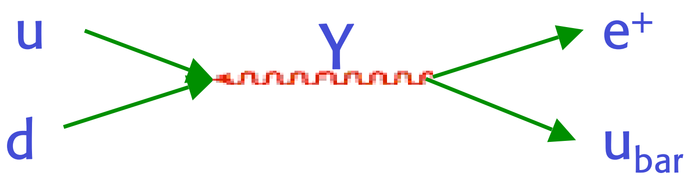
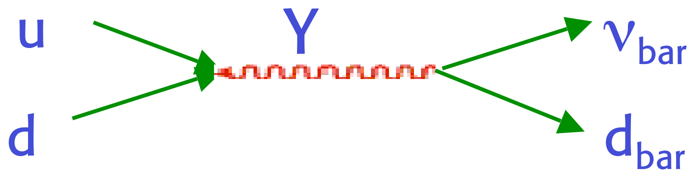
Is a big problem for minimal models (see later)

Proton Decay in SU(5) (no SUSY)

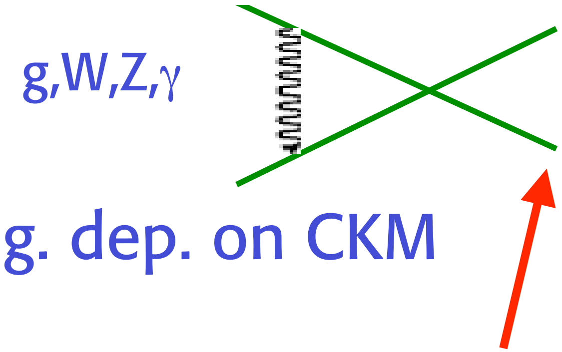






$$\tau_p^{-1} = \Gamma_p \sim \alpha_G^2 \cdot \frac{m_p^5}{M_{X,Y}^4}$$

$p \to e^+\pi^0$, $e^+\omega$, $e^+\rho$..., $\nu^e\pi^+$,...



- Compute the effective 4-f interaction (e.g. dep. on CKM mixing angles)
- Run the vertices from M_{GUT} down to m_p
- Determine $M_{X,Y}$ precisely
- Compute the hadronic matrix element of the 4-f operator (model dep.)

prediction: $\tau_p \sim 10^{30\pm1.7}$ y

exp (SK) $p \to e^+\pi^0$:
$\tau_p/B > 5.0\ 10^{33}$y

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Non-SUSY SU(5) dead!

Proton Decay in Minimal SUSY-SU(5)

M_{GUT} increases: non SUSY: $M_{GUT} \sim 10^{15}$ GeV, SUSY $\sim 10^{16}$ GeV
and gauge mediation becomes negligible:

$\tau_{p\ \mathrm{NON\ SUSY}} \sim 10^{30\pm1.7}\ y < 10^{32}\ y$

$\tau_{p\ \mathrm{SUSY,\ Gauge}} \sim 10^{36}\ y$ $(\tau_p \sim m_{GUT}^4)$

In SUSY coloured Higgs(ino) exchange dominant

← Yukawa Superpot.

$H_{u,d}$: 5 or 5^{bar} H
$G_{u,d}$: matrices in family space

$W_Y = 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H^{bar}{}_d$

in terms of $H_{D,T}$ (doublet or triplet H):

$W_Y = Q G_u u^c H_{Du} + Q G_d d^c H_{Dd} + e^c G_d^T L H_{Dd} +$
$-1/2\ Q G_u Q H_{Tu} + u^c G_u e^c H_{Tu} - Q G_d L H_{Td} + u^c G_d d^c H_{Td}$

The H_D terms -> masses; H_T terms->p-decay

Very rigid:
given the mass constraints p-decay is essentially fixed

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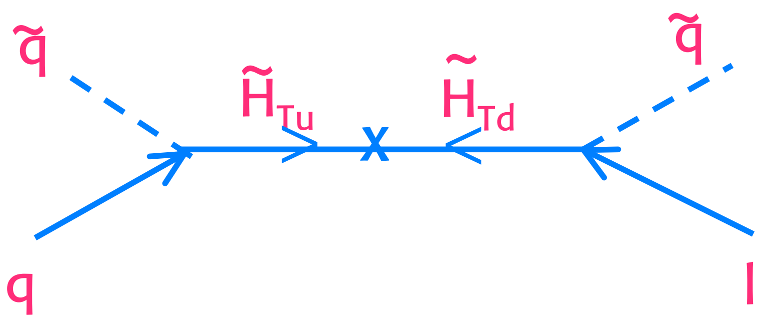
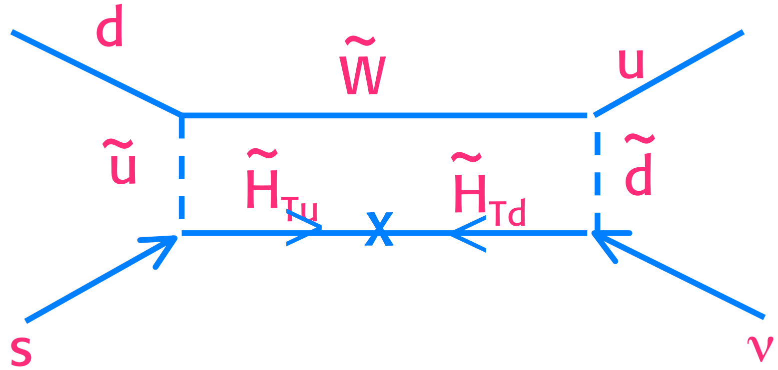




After integration of H_T:

Dominant mode $p \to K^+ \nu^{bar}$

$$W_{eff} = [Q(G_u/2)Q\cdot QG_dL + u^cG_ue^c\cdot u^cG_dd^c]/m_{HT}$$

G_u: symm. 3x3 matrix: 12 real parameters
G_d : 3x3 matrix: 18 real parameters
12+18=30 but we can eliminate 9+9 by separately rotating 10 and 5^{bar} fields
3up +3down or lepton masses ($m_l = m_d^T$ in min. SU(5)) + 3 angles+ 1 phase (V_{CKM}) = 10 real parameters
2 phases are the only left-over freedom (arbitrary phases in the 2 W_{eff} terms)
NOT ENOUGH!

Reminder: Fermion Masses in SU(5)

$m_{Dirac} = R^{bar} m L + h.c$

u: $10 Y^u 10 \cdot H_{5,45bar,50bar}$

d and e: $5^{bar} Y^d 10 \cdot H_{5bar,45}$

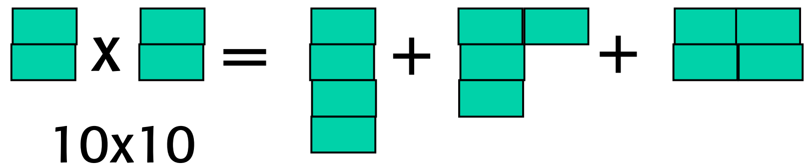
ν_{Dirac}: $5^{bar} Y^\nu 1 \cdot H_5$

In minimal SU(5) one only has H_5
($H_{5bar} = H^+$)

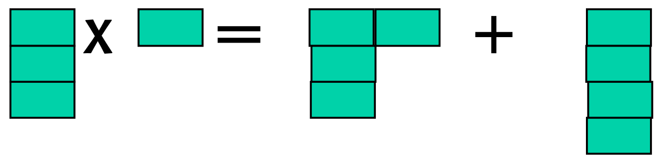
$m_u = Y_u \langle H_5 \rangle$: symmetric
$m_d = m_e^T = Y_d \langle H_5 \rangle$

$10 \times 10 = 5^{bar} + 45 + 50$

$10^{bar} \times 5 = 5^{bar} + 45$



10x10 5^{bar} 45 50



10^{bar}x5 45 5^{bar}

$5^{bar} Y^d 10$ → (d_R, L) (Q, u_R, e_R) → $d_R Q + L e_R +$ ...

$Q = \begin{bmatrix} u \\ d \end{bmatrix}_L$ $L = \begin{bmatrix} \nu \\ e^- \end{bmatrix}_L$

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In Minimal SUSY-SU(5), using W_{eff} one finds

$p \to K^+ \nu^{bar}$ $\tau/B \sim 9\ 10^{32}$ y (exp. > 1.6 10^{33} y at 90%)

Superkamiokande

This is a central value with a spread of about a factor of about 1/3 - 3.

The minimal model perhaps is not yet completely excluded but the limit is certainly quite constraining.

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A "realistic" SUSY-GUT model should possess the properties:

- Coupling Unification
  - * No extra light Higgs doublets
  - * M_{GUT} threshold corrections in the right direction
- Natural doublet-triplet splitting
  - * e.g. missing partner mechanism
- Well compatible with p-decay bounds
  - * No large fine-tuning
- Correct masses and mixings for q,l and ν's
  - * e.g. $m_b = m_\tau$ at m_{GUT} but m_s different than $\neq m_\mu$, m_d different than m_e

Examples

SU(5): Berezhiani, Tavartkiladze; GA, Feruglio, Masina
SO(10): Babu, Pati, Wilczek; Albright, Barr; Raby et al; King, Ross; …..

An example of "realistic" SUSY-SU(5)xU$(1)_F$ model

(GA, Feruglio, Masina JHEP11(2000)040)

The D-T splitting problem is solved by the missing partner mechanism protected from rad. corr's by a flavour symm. U$(1)_F$

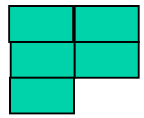
Masiero,Tamvakis; Nanopoulos, Yanagida...

1) We do not want neither the 5.5^{bar} nor the $5.5^{bar}.24$ terms

So, first, we break SU(5) by a 75:

1=X, 75=Y, 5,50=$H_{5,50}$

$$SU(5) \xrightarrow[M_{GUT}]{75} SU(3)x\, SU(2)xU(1)$$

75 = 

2) The 5 5^{bar} Higgs mass term is forbidden by symmetry and masses arise from

$W=M75.75+75.75.75+5.75.50 +5^{bar}.75.50^{bar}+50.50^{bar}.1$

As $50=(8,2)+(6,3)+(6^{bar},1)+(3,2)+(3^{bar},1)+(1,1)$

there is a colour triplet (with right charge) but not a colourless doublet (1,2)

→ the doublet finds no partner and only the triplet gets a large mass

Note: we need a large mass for 50 not to spoil coupling unification. But if the terms $5.75.50 + 5^{bar}.75.50^{bar} + 50.50^{bar}$ are allowed then also the non rin. operator

$$O = c\,\frac{5\cdot\bar{5}\cdot 75\cdot 75}{M_{Pl}}$$

Randall, Csaki

is allowed in the superpotential and gives too large a mass $M_{GUT}^2/M_{Pl} \sim 10^{12}\text{-}10^{13}$GeV

All this is avoided by taking the following $U(1)_F$ charges :

Berezhiani, Tavartkiladze

field:	Y_{75}	H_5	H_{5bar}	H_{50}	H_{50bar}	X_1
F-ch:	0	-2	1	2	-1	-1

All good terms are then allowed:

$W = M75.75 + 75.75.75 + 5.75.50 + 5^{bar}.75.50^{bar} + 50.50^{bar}.1$

while all bad terms like $5.5^{bar}.(X)^n.(Y)^m$, $n,m>0$ are forbidden

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In the SUSY limit <5>, <5^{bar}>, <50>, <50^{bar}>=0 while <Y>~M_{GUT} and <X> is undetermined. Higgs doublets stay massless. Triplet Higgs mix between 5 and 50:

$$m_T = \begin{bmatrix} 0 & 5.50 \\ 5.50 & 50.50 \end{bmatrix} = \begin{bmatrix} 0 & <Y> \\ <Y> & <X> \end{bmatrix}$$

In terms of $m_{T1,2}$ (eigenvalues of $m_T m_T^+$) the relevant mass for p-decay is

$$m_T = \frac{m_{T1} \cdot m_{T2}}{<X>} \sim \frac{<Y>^2}{<X>}$$

When SUSY is broken the doublets get a small mass and <X> is driven at the cut-off between m_{GUT} and m_{Pl}.

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Coupling unification

Recall:

$$\alpha_3 = \frac{\alpha_3^{LO}}{1+\alpha_3^{LO}\delta}$$

1-loop

$$\delta = k + \frac{1}{2\pi}\log\frac{m_{SUSY}}{m_Z} - \frac{3}{5\pi}\log\frac{m_{H_T}^{LO}}{m_{GUT}}$$

$$k = k_2 + \underbrace{k_{SUSY} + k_{GUT}}_{\text{thresholds}}$$

2-loop

k_2 ~ -0.733, k_{SUSY} ~ -0.510 remain the same.
But k_{GUT} ~ 0 for the 24 is now k_{GUT} ~ 1.86 for the 75 (the 50 is unsplit).
So k ~ -1.243 in the minimal model becomes k~+0.614 in this model.

Now α_s would become too small and we need m_{SUSY} small and m_T large

$m_T|_{Realistic}$ ~ 20-30 $m_T|_{minimal}$

good for p-decay!
factor 400-900

Due to 50, 75, SU(5) no more asympt. free: α_s blows up below m_{Pl} (Λ~20-30 M_{GUT})

Not necessarily bad!

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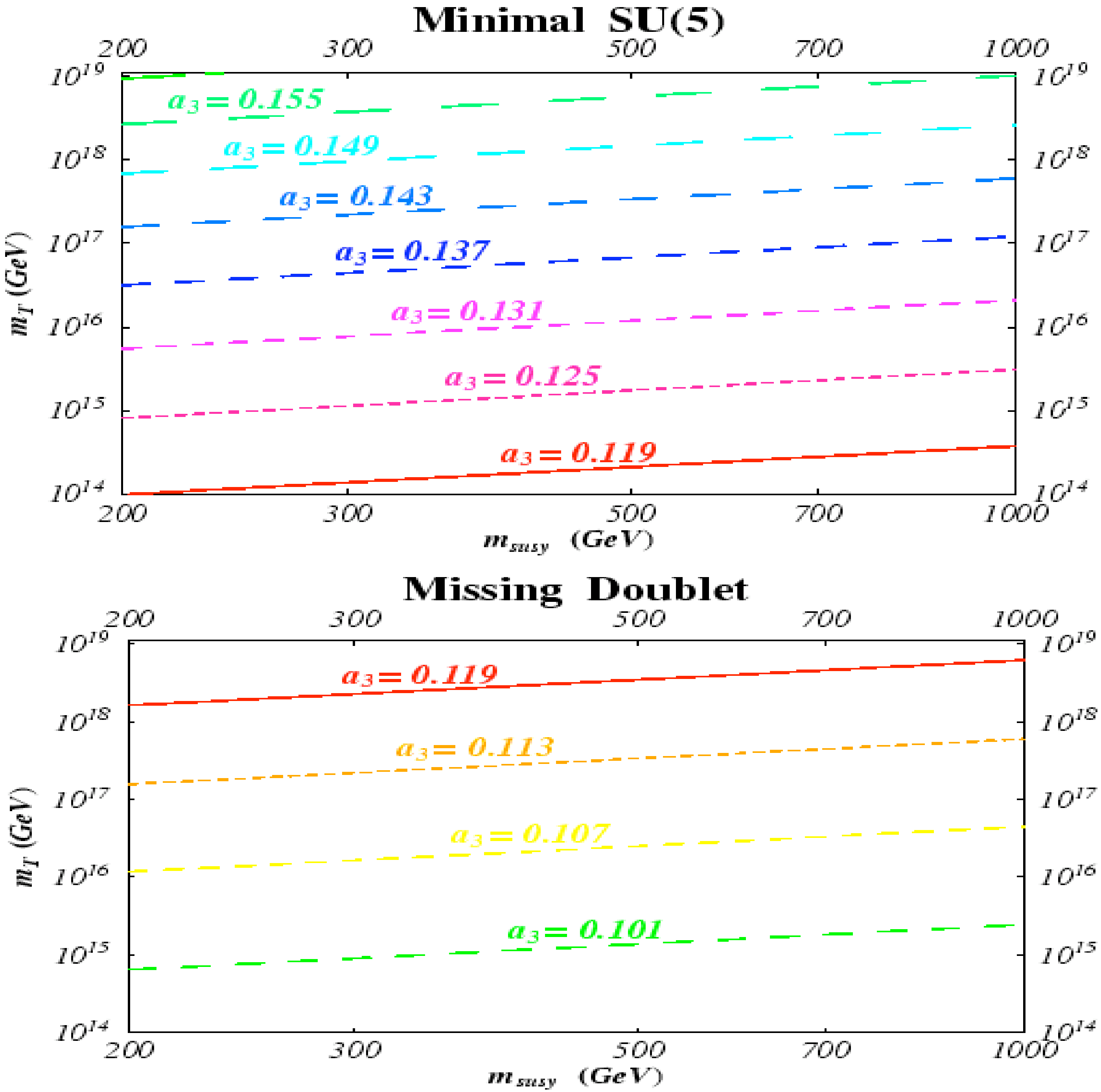

Minimal SU(5)
$a_3 = 0.155$
$a_3 = 0.149$
$a_3 = 0.143$
$a_3 = 0.137$
$a_3 = 0.131$
$a_3 = 0.125$
$a_3 = 0.119$
m_T (GeV)
m_{susy} (GeV)
200 300 500 700 1000
10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19}
Missing Doublet
$a_3 = 0.119$
$a_3 = 0.113$
$a_3 = 0.107$
$a_3 = 0.101$
m_T (GeV)
m_{susy} (GeV)

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Fermion masses

Consider a typical mass term: $10 G_d 5^{bar} H_d$ ← $F(X,Y)$

Recall: X SU(5) singlet, F(X) = -1
Y SU(5) 75, F(Y) = 0

First approximation:
no Y insertions -> F(X,0)

Pattern determined by $U(1)_F$ charges

Froggatt-Nielsen

i,j=family1,2,3

$F(10) = (4,3,1)$ $\quad$ $F(H_u) = -2$

$F(5^{bar}) = (4,2,2)$ $\quad$ $F(H_d) = 1$

$F(1) \quad = (4,-1,0)$

$\lambda_C \sim 0.22$

→ $10_i 5^{bar}_j (\langle X\rangle/\Lambda)^{fi+fj+fH} v_d$

$$m_u = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u \qquad m_d = m_l^T = \begin{bmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{bmatrix} v_d \lambda^4$$

quarks: m_u, m_d, V_{CKM} ~OK, tgβ~o(1)
ch. leptons: $m_d = m_l^T$ broken by Y insertions

1st order:

$m_d \sim G_d + \langle Y\rangle/\Lambda\ F_d$
$m_e^T \sim G_d - 3\langle Y\rangle/\Lambda\ F_d$ ← $10_i 5^{bar}_j \lambda_C^{nij} (\langle Y\rangle/\Lambda)\ v_d$

Proton decay

Higgs triplet exchange

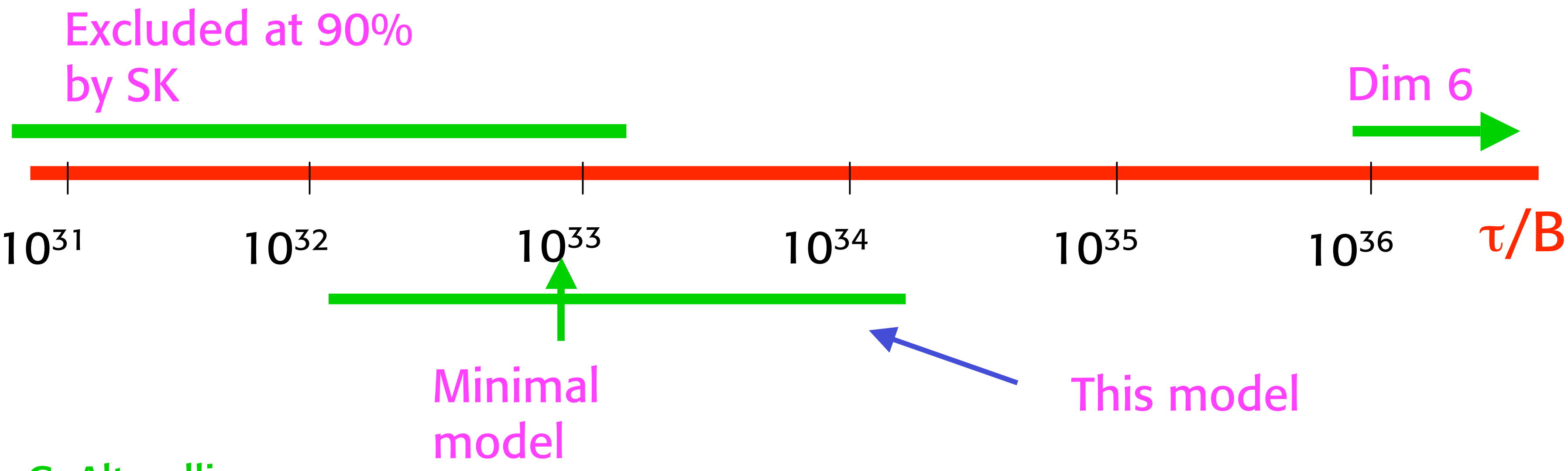
$W_{eff} = [Q(1/2A)Q \cdot QBL + u^c C e^c \cdot u^c D d^c]/m_{HT}$

Advantages w.r.t. minimal SUSY-SU(5)

- Larger m_T by factor 20 -30
- Extra terms: e.g. not only $10G_u 10H_u$ but also $10G_{50} 10H_{50bar}$

(free of mass constraints because $\langle H_{50bar} \rangle = 0$)

Results: $p \to K^+ \nu_{bar}$ (similarly for $p \to \pi^0 e^+$)



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Realistic models also possible in SO(10)

Babu, Pati, Wilczek; Albright, Barr; Raby et al; King, Ross; …..

Most economic models only involve Higgs in 16, 10, 45

Masses also determined by non rinormalisable, higher dimension, operators

A suitable flavour symmetry is needed to allow the required terms and only those

Thus:

- Minimal SUSY-SU(5), -SO(10) models are in trouble

- More realistic models are possible but they tend to be baroque (e.g. large Higgs representations)

Recently a new idea has been developed and looks promising: unification in extra dimensions

[Fayet '84],
Kawamura
GA, Feruglio hep-ph/0102301;
Hall, Nomura;
Hebecker, March-Russell;
Hall, March-Russell, Okui, Smith
Asaka, Buchmuller, Covi
....

Factorised metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

But while for the hierarchy problem R~1/TeV
here we consider R~1/M_{GUT}
(not so large!)
Assumes that SUSY solves the hierarchy problem

Note:

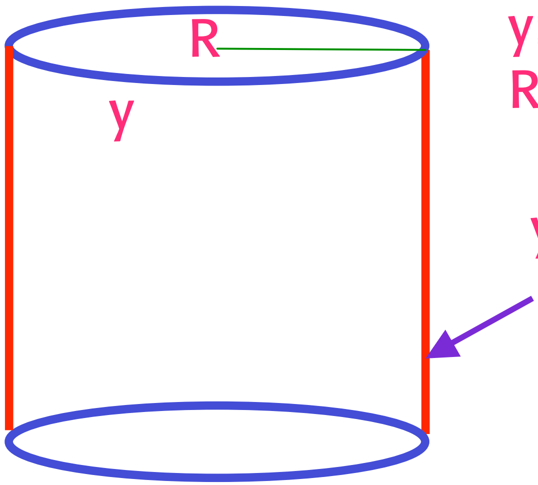
The R-S model with SM particles on the TeV brane and Rm ~ 12 is a possible solution of the hierarchy problem.

But it is not suitable for GUT's because on the TeV brane the cutoff is at TeV

GUT's in RS could be realised if one puts the Higgs on the TeV brane for the hierarchy and all SM gauge and fermion particles in the bulk

Pomarol'00; Agashe, Delgado, Sundrum'02

But the phenomenology of this kind of models Is less transparent and it is not clear that it works



y: extra dimension
R: compact'n radius

y=0 "our" brane

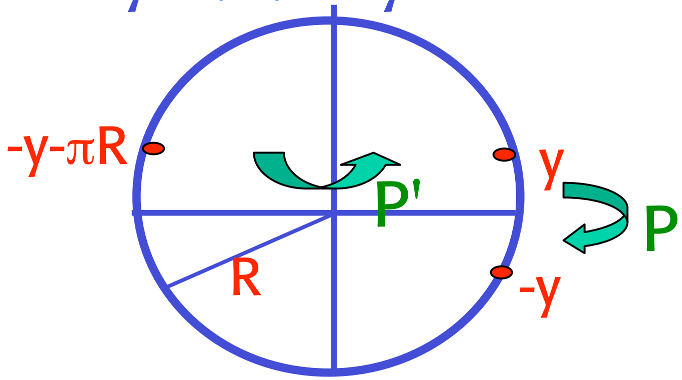
$S/(Z_2 x Z_2')$

Z_2-> P: y $\Longleftrightarrow$ -y

Z_2'-> P': y' $\Longleftrightarrow$ -y'

$y' = y + \pi R/2$

or y $\Longleftrightarrow$ $-y - \pi R$



Diagonal fields in P,P' can be Fourier expanded:

$$\phi_{++}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{++}^{(2n)}(x_\mu)\cos\frac{2ny}{R}$$

$$\phi_{+-}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{+-}^{(2n+1)}(x_\mu)\cos\frac{2n+1}{R}y$$

$$\phi_{-+}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum \phi_{-+}^{(2n+1)}(x_\mu)\sin\frac{2n+1}{R}y$$

$$\phi_{--}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{--}^{(2n+2)}(x_\mu)\sin\frac{2n+2}{R}y$$

Only ϕ_{++},ϕ_{+-} not 0 at y=0

Only ϕ_{++} is massless

A different view of GUT's SUSY-SU(5) in extra dimensions

- In 5 dim. the theory is symmetric under N=2 SUSY and SU(5)

Gauge 24 + Higgs $5+5^{bar}$: N=2 supermultiplets in the bulk

A_M M=0,1,2,3,5 H_u H_d

λ_2 λ_1 h_u h'_u h_d h'_d

Σ H'_u H'_d

24 N=1 chiral multiplets 5 5^{bar}

- Compactification by $S/(Z_2 x Z_2')$ $1/R \sim M_{GUT}$

N=2 SUSY-SU(5) -> N=1 SUSY-SU(3)xSU(2)xU(1)

- Matter 10, 5^{bar}, 1 on the brane (e.g. x_5=y=0) or in the bulk (many possible variations)

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P breaks N=2 SUSY down to N=1 SUSY
but conserves SU(5): on 5 of SU(5) P=(+,+,+,+,+)

P' breaks SU(5) P'=(-,-,-,+,+) $P'T^aP'=T^a$, $P'T^\alpha P'= -T^\alpha$
(T^a: span 3x2x1, T^α : all other SU(5) gen.'s)

P P'	bulk field		mass
++	$A^a_\mu, \lambda^a_2, H^D_u, H^D_d$	← Doublet	2n/R
+ -	$A^\alpha_\mu, \lambda^\alpha_2, H^T_u, H^T_d$	← Triplet	(2n+1)/R
- +	$A^\alpha_5, \Sigma^\alpha, \lambda^\alpha_1, H'^T_u, H'^T_d$		(2n+1)/R
- -	$A^a_5, \Sigma^a, \lambda^a_1, H'^D_u, H'^D_d$		(2n+2)/R

Note:

$$\partial_5 = (-,-)$$

Gauge parameters are also y dep.

$$U = \exp[i\xi^a(x_\mu, y)T^a + i\xi^\alpha(x_\mu, y)T^\alpha]$$

$$\xi^a(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum \xi^a(x_\mu)\cos\frac{2ny}{R}$$

$$\xi^\alpha(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \xi^\alpha(x_\mu)\cos\frac{2n+1}{R}y$$

} both not zero at y=0

$$U = \exp[i\xi^a(x_\mu, y)T^a + i\xi^\alpha(x_\mu, y)T^\alpha]$$

$$\xi^a(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \xi^a(x_\mu)\cos\frac{2ny}{R}$$

$$\xi^\alpha(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \xi^\alpha(x_\mu)\cos\frac{2n+1}{R}y$$

At y=0 both ξ^a and ξ^α not 0: so full SU(5) gauge transf.s, while at y=πR/2 only SU(3)xSU(2)xU(1).

Virtues:

- No baroque 24 Higgs to break SU(5)
- $A^{a(0)}_\mu$, $\lambda^{a(0)}_2$ massless N=1 multiplet
- $A^{a(2n)}_\mu$ eat $\partial_5 A^{a(2n)}_5$ and become massive (n>0)
- Doublet-Triplet splitting automatic and natural: $H^{D(0)}_{u,d}$ massless, $H^{T(0)}_{u,d}$ $m \sim 1/R \sim m_{GUT}$

The brane at $y=0$ (or πR) is a fixed point under P.
There the full SU(5) gauge group operates.
The brane at $y= \pi R/2$ (or $- \pi R/2$) is a fixed point under P'.
There only the SM gauge group operates.

Matter fields (10, 5^{bar}, 1, and the Higgs also) could be either on the bulk, or at $y=0$ or $y= \pi R/2$. Many possibilities

In the bulk must satisfy all symmetries, at $y=0$ must come in N=1 SUSY-SU(5) representations, at $y= \pi R/2$ must only fill N=1 SUSY-SU(3)xSU(2)xU(1) representations

For example, if H^D_u, H^D_d are at $y= \pi R/2$ one can even not introduce H^T_u, H^T_d

A simple option is to take the Higgs in the bulk and the matter 10, 5^{bar}, 1 at y=0, πR.

In our paper we take fully symmetric Yukawa couplings at y=0:

$W_Y = 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H^{bar}_d$

This contains H^D (mass) and H^T (p-decay) interactions:

$W_D = Q G_u u^c \cdot H^D_u + Q G_d d^c \cdot H^D_d + L G_d e^c \cdot H^D_d$

$W_T = Q G_u Q \cdot H^T_u + u^c G_d d^c \cdot H^T_d + Q G_d L \cdot H^T_d + u^c G_u e^c \cdot H^T_u$

P' transforms y=0 into y=πR. We choose P' parities of 10, 5^{bar}, 1 that fix W(y=πR) such that only wanted terms survive in

$$w^{(4)} = \int [\delta(y) + \delta(y - \pi R)] w(y) dy$$

We take Q, u^c, d^c +,+ and L, e^c, ν^c +,-: all mass terms allowed, p-decay forbidden

recall H^D ++, H^T +-

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QQQL, $u^c u^c d^c e^c$, $Q d^c L$, $L e^c L$ all forbidden

With our choice of P' parities the couplings at $y=\pi R$ explicitly break SU(5), in the Yukawa and in the gauge-fermion terms. (SU(5) is only recovered in the limit R-> infinity).
But we get acceptable mass terms and can forbid p-decay completely, if desired.

An alternative adopted by Hall&Nomura is to take:
$y=0$: $W_Y = 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H_d$
$y=\pi R$: $W_Y = -\ 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H_d$
as if the Yukawa coupling was y-dep. not a constant.

Then, by taking $P'(Q,u^c,d^c,L,e^c)=(+\ -\ +\ -\ -)$, SU(5) is fully preserved

One obtains the SU(5) mass relations and p-decay is suppressed but not forbidden.

A different possibility is to put $H^D_{u,d}$ at $y=\pi R/2$ (no triplets) and the matter in the bulk (N=2 SUSY-SU(5) multiplets).

In order to be massless all of them should be ++.
Looks impossible:

PP'	bulk field	mass
++	u^c, e^c, L	2n/R
+ -	Q, d^c	(2n+1)/R
- +	Q', d'^c	(2n+1)/R
- -	u'^c, e'^c, L'	(2n+2)/R

(follows from P=(+++++), P'=(---++))

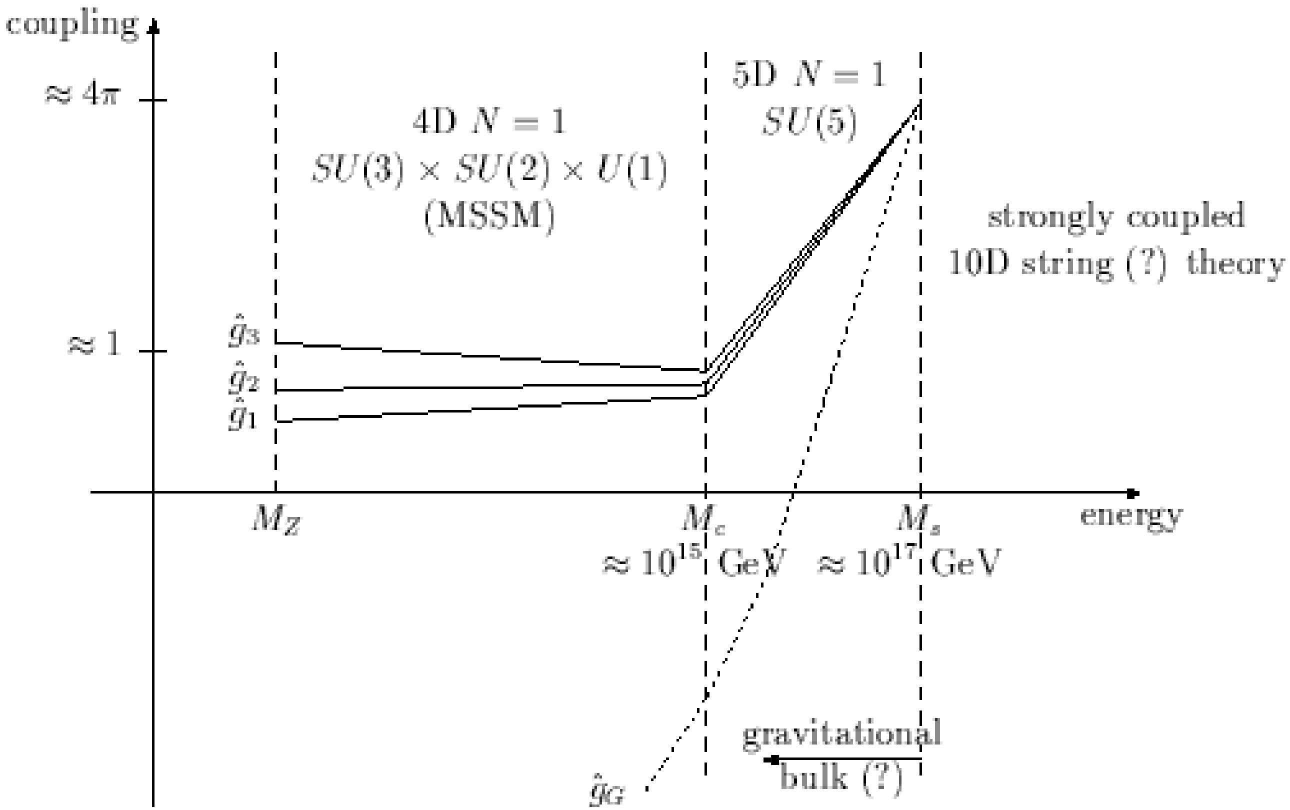
But one can add a duplicate with opposite P':
then we get the full set u^c, e^c, L and Q, d^c at ++

Hebecker,
March-Russell'01

Finally one is free to take some generation in one way, some other in a different way to get flavour hierarchies etc

Coupling unification can be maintained and threshold corrections evaluated

Hall, Nomura
Contino, Pilo,Rattazzi, Trincherini



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SO(10) models can also be constructed

Breaking by orbifolding requires 6-dim and leave an extra U(1)
(the rank is maintained)

Asaka, Buchmuller, Covi
Hall, Nomura

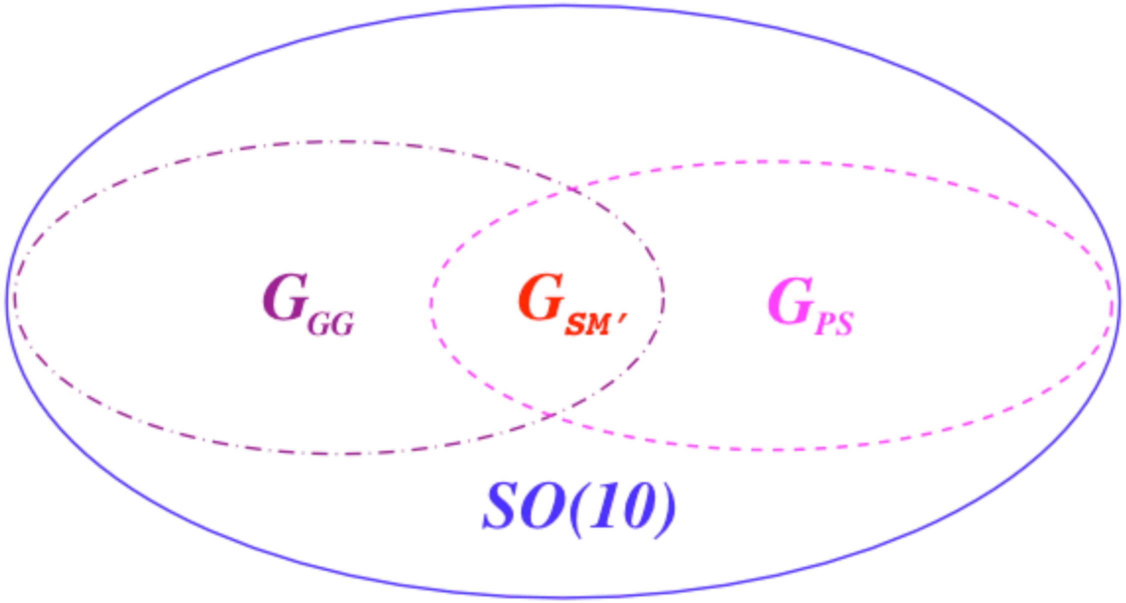
Breaking by BC or mixed orbifolding+BC can be realised in 5 dimensions

Demisek, Mafi;
Kim, Rabi
Albright, Barr
Barr, Dorsner (flipped SU(5))

Breaking SUSY-SO(10) in 6 dim by orbifolding

The ED y, z span a torus T^2 -> $T^2/ZxZ_{PS}xZ_{GG}$

$$G_{PS} = SU(4) \times SU(2) \times SU(2)\,, \quad G_{GG} = SU(5) \times U(1)_X$$



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$G_{SM'}$ = SU(3)xSU(2)xU(1)xU(1)

By using breaking by BC one can stay in 5 dim

S/ZxZ'

Z -> P breaks SUSY
Z' -> P' breaks SO(10) down to $G_{PS} = SU(4)xSU(2)_L xSU(2)_R$

(G_{PS} is the residual symmetry on the hidden brane at $y=\pi R/2$)

On the visible brane at y=0 SO(10) is broken down to SU(5) (lower rank!) by BC acting as Higgs $16+16^{bar}$
(we could use real Higgses localised at y=0 but sending their mass to infinity is more economical)

Thus:

- By realising GUT's in extra-dim we obtain great advantages:
    - No baroque Higgs system
    - Natural doublet-triplet splitting
    - Coupling unification can be maintained (threshold corr.'s can be controlled)
    - P-decay can be suppressed or even forbidden
    - SU(5) mass relations can be maintained, or removed (also family by family)

Conclusion

- SUSY GUT's remain the reference framework
- Minimal models in big troubles
- Realistic models rather baroque
- More sophisticated approaches emerging: eg extra dimension SUSY GUT's