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Double pole in the neutral Higgs sector

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Motivation

- Degeneracy of the heavy neutral Higgses will be manifested for $H_2^0$ and $H_3^0$ as coherent states.

  $\rightarrow$ mass degeneracy will be revealed as an exceptional point in the complex parameter space.

- Proper treatment of Non-Hermitian systems will ensure correct theoretical predictions in the study of possible sources of CP violation

  $\rightarrow$ CP phases in the Higgs line-shape, may have an unexpected behavior

  at the exact degeneracy of a coherent system
Necessary conditions for a double pole in the Higgs sector

1. multi-Higgses → as 2HDM and MSSM

2. very close in mass at tree level → as in the MSSM $m_H^{(0)} = m_A^{(0)}$
   for $m_A \gg m_Z$ and $\tan \beta \gg 1$.

3. masses larger than the EW scale → including radiative corrections
   $m_h < m_H, m_A$

4. system manifestly non-Hermitian → CP non-invariant Higgs sector: $h, H, A$ CP defined → $H_1, H_2, H_3$ CP-mixed
**Example:** Degeneracy of non-coherent two single poles.

**Figure 1:** two non-coherent states, [Barger, Berger, Gunion, Han 96]

[Bernabeu, Binosi, Papavassiliou, 06]
Example: Degeneracy and coherent states.

Figure 2: two coherent states may have destructive interference.

For general model see [Cacciapaglia, Deandrea, De Curtis 09]
Higgs masses as the poles of the propagator

Neutral heavy Higgs bosons as s-channel resonances

Masses and mixings of the H-A system may be detected as two closely spaced or even overlapping resonances in the s-channel reaction, \( \text{i.e.} \)

\[
\mu^+ \mu^- \rightarrow A^*/H^* \rightarrow f \bar{f}\]

[\text{Pilaftsis 97}, \text{Bernabeu, Binosi, Papavasiliu 06}]

The form of the line shape of this process would indicate the presence (or absence) of CPV in the heavy Higgs system.

In the resonant region, the t-channel amplitude is relatively small and may be ignored.

Then, in the electroweak basis, the transition amplitude matrix between states with CP-violation via resonant Higgs exchange is

\[
\mathcal{T}^{res}(s) = V^P \hat{\Delta}_{H_2-H_3}^{-1}(s) V^D
\]

where we identify the propagator as

\[
\hat{\Delta}_{H_2-H_3}^{-1}(s) = s1_{2 \times 2} - M_{H_2-H_3}^2(s) = \begin{pmatrix} s - (M^2_H - \hat{\Pi}_{HH}(s)) & \hat{\Pi}_{HA}(s) \\ \hat{\Pi}_{HA}(s) & s - (M^2_A - \hat{\Pi}_{AA}(s)) \end{pmatrix}
\]

(3)
Physical masses as the poles of the propagator

The physical masses of the neutral heavy Higgs bosons are identified with the poles of the propagator matrix $\hat{\Delta}_{H_2-H_3}(s)$.

Hence, the masses of the neutral, heavy Higgs bosons are defined as the solutions of the implicit equation

$$\det \left[ \Delta_{H_2-H_3}(s^*) \right] = \det \left[ (s^*) \mathbf{1}_{2\times2} - \mathcal{M}^2_{H_2-H_3}(s^*) \right] = 0. \quad (4)$$

In the physical basis, $\mathcal{M}^2_{H_2-H_3}(s)$ is diagonal,

$$M^2_{H_i}(s^*) - i M_i \Gamma_i(s^*) := \mu^2_{H_i}(s^*). \quad (5)$$

then eq. (4) becomes

$$(s_2 - \mu^2_{H_2}(s_2))(s_3 - \mu^2_{H_3}(s_3)) = 0, \quad (6)$$
Degeneracy of neutral heavy CP non-invariant Higgs bosons

Considering the full s-dependance, the true physical masses are identified with the poles of the propagator. Therefore, the heavy Higgs bosons masses should be defined by the solutions of the implicit equations [Stuart 95],[Bohm,Kaldass and Wickramasekara 02]

\[ \mu_{H_i}^2(s_{H_i}^*) - s_{H_i}^* = 0; \quad i = 2, 3 \] (7)

We say that the two heavy neutral Higgs bosons are mass degenerate if there exist an \( s^* \) such that

\[ s^* - \mu_{H_2}^2(s^*) = 0 \]

\[ \Rightarrow \mu_{H_2}^2(s^*) = \mu_{H_3}^2(s^*), \]

\[ s^* - \mu_{H_3}^2(s^*) = 0 \]

In order to avoid a clumsy notation, we will write the effective squared mass matrix \( \mathcal{M}_{H_2H_3}^2(s) \) in the basis of the Pauli spin matrices as

\[ \mathcal{M}_{H_2H_3}^2(s) = \frac{1}{2} T_{12 \times 2} + (\vec{R} - i\vec{\Gamma}) \cdot \vec{\sigma}. \] (8)

where \( T \) is the trace of \( \mathcal{M}_{H_2H_3}^2 \).
Degeneracy conditions

And we may write the degeneracy condition on the eigenvalues as

\[
\frac{1}{2} \left[ \mu_{H3}(s^*) - \mu_{H2}(s^*) \right] = \sqrt{(\vec{R}_d - i\vec{\Gamma}_d)^2} = 0, \tag{9}
\]

where we have defined the vectors

\[
\vec{R} = \left( \frac{1}{2}(M^2_H - M^2_A), 0, \Re \Delta^2_{HA} \right) \quad \text{and} \quad \vec{\Gamma} = \left( \frac{1}{2}(M_H \Gamma_H - M_A \Gamma_A), 0, \Im \Delta^2_{HA} \right),
\]

The eq.(9) implies \( R^2_d(s^*) = \Gamma^2_d(s^*) \) and \( \vec{R}_d(s^*) \cdot \vec{\Gamma}_d(s^*) = 0 \) for \( \vec{R}_d, \vec{\Gamma}_d \neq 0 \).

From the above conditions we found that the traceless term of the propagator, at degeneracy, will be given by

\[
(\vec{R}_d - i\vec{\Gamma}_d) \cdot \vec{\sigma} = M_d \Gamma_d \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}, \tag{10}
\]
**s-channel transition matrix at degeneracy**

The propagator can be written as

\[
\Delta^{(d)}_{H_2-H_3}(s) = \begin{pmatrix}
\frac{1}{(s-M_d^2+iM_d\Gamma_d)} & \frac{iM_d\Gamma_d}{(s-M_d^2+iM_d\Gamma_d)^2} \\
0 & \frac{1}{(s-M_d^2+iM_d\Gamma_d)}
\end{pmatrix}
\]  

We then can write the resonant transition matrix in the mass representation as

\[
\mathcal{T}^{\text{res}(d)}(s) = (\tilde{V}_P^H, \tilde{V}_A^P) \Delta^{(d)}_{H_2,H_3}(s) \begin{pmatrix}
\tilde{V}_H^D \\
\tilde{V}_A^D
\end{pmatrix}
\]

\[
= \tilde{V}_H^P \frac{1}{s-\mu_d^2(s)} \tilde{V}_H^D + \tilde{V}_A^P \frac{1}{s-\mu_d^2(s)} \tilde{V}_A^D + \tilde{V}_H^P \frac{iM_d\Gamma_d}{(s-\mu_d^2(s))^2} \tilde{V}_A^D
\]

with

\[
\begin{pmatrix}
\tilde{V}_{H}^{P,D} \\
\tilde{V}_{A}^{P,D}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & +i \\ 1 & -i \end{pmatrix} \begin{pmatrix} V_{H}^{P,D} \\
V_{A}^{P,D}\end{pmatrix}
\]
Application for the 2HDM

Can be shown that only two of the neutral Higgs states are degenerated obtaining a \( 2 \times 2 \) mass matrix.

In the decoupling limit, defined by the inequality \([\text{Gunion, Haber 03}]
\[
m^2_A \gg |\lambda_i|v^2,
\]
the, mixing between the light state, \( H_1 (\rightarrow h^0) \), and the heavy states, \( H_2 \) and \( H_3 \), is small, compared with the mixing of the nearly degenerate heavy Higgs states \( H_2 \) and \( H_3 \).

\([\text{Félix-Beltrán,Gómez-Bock,Hernández, Mondragón,Mondragón 2009}]\)

\[
\mathcal{M}^2_{H_2-H_3}(s) = 
\begin{pmatrix}
M^2_H(s) - i M_H \Gamma_H(s) & \Delta^2_{HA}(s) \\
\Delta^2_{HA}(s) & M^2_A(s) - i \Gamma_A M_A(s)
\end{pmatrix}
\]

\([\text{Pilaftsis 98}], [\text{Demir 99}]\)
**An approach for neutral Higgs boson eigenvalues in the 2HDM**

The matrix elements are expressed as functions of the model parameters. In the decoupling limit \( M_A^2 \gg |\lambda_i|v^2 \), we may find a simplifying approach for the relations of the mass matrix elements as [Choi,Kalinowski,Liao and Zerwas 05]

\[
M_H^2 - M_A^2 \approx \lambda v^2 \cos \phi
\]  
\[32\pi [M_H \Gamma_H - M_A \Gamma_A] \approx [\Delta_t + 9\lambda^2 v^2 \cos 2\phi]
\]  
\[Re \Delta_{HA}^2 \approx -\frac{1}{2} \lambda v^2 \sin \phi
\]  
\[32\pi Im \Delta_{HA}^2 \approx -\frac{9}{2} \lambda^2 v^2 \sin 2\phi
\]  

We have taken the magnitudes of all \( \lambda_i \) as same order, and \( \phi \) is the CP violating common phase of the complex couplings. And

\[
\Delta_t = -12M_{H/A}^2 (m_t/v)^2 (1 - \beta_t^2) \beta_t,
\]

is the one loop contribution of the top quark.
**Exceptional point in the mass complex surfaces**

We are able to write explicity the masses of the heavy neutral Higgs bosons as functions of the parameters $\lambda$ and $\phi$ and if further more we neglect the weak $s$ dependence of the elements of $\mathcal{M}_{HA}^2$, we found an approximation for pole position mass. The term under the square root admits a Puiseux expansion series around the exceptional point, [Hernández, Jáuregui and Mondragón 06].

$$\mu_{2,3}^2(\lambda, \phi) = \frac{1}{2} \sqrt{c_1^{(1)}(\lambda - \lambda^*) + c_2^{(1)}(\phi - \phi^*) + ...}$$

(21)

where the degeneracy conditions we get the exceptional point as: $\lambda^* = 0.1075$, $\phi^* = \pi/2$ and $c_k^{(1)}$ are the derivatives of $\mu_{2,3}^2$ with respect to the parameters $\lambda$ and $\phi$.

$$\Re \mu_{2,3}^2 = \pm \frac{1}{2\sqrt{2}} |\zeta|^{1/2} \left[ \sqrt{(\vec{R} \cdot \hat{\zeta})^2 + (\vec{I} \cdot \hat{\zeta})^2 + (\vec{R} \cdot \hat{\zeta})} \right]^{1/2}$$

(22)

$$\Im \mu_{2,3}^2 = \pm \frac{1}{2\sqrt{2}} |\zeta|^{1/2} \left[ \sqrt{(\vec{R} \cdot \hat{\zeta})^2 + (\vec{I} \cdot \hat{\zeta})^2 - (\vec{R} \cdot \hat{\zeta})} \right]^{1/2}$$

(23)

with

$$\vec{R} = (\Re c_1^{(1)}, \Re c_2^{(1)}) \ , \ \vec{I} = (\Im c_1^{(1)}, \Im c_2^{(1)}) \ , \ \hat{\zeta} = \left( \begin{array}{c} \lambda - \lambda^* \\ \phi - \phi^* \end{array} \right).$$

(24)
Unfolding of the exceptional point

The figures show the mass hypersurface representing the imaginary parts of $\mu_{2,3}$ as function of the Lagrangian parameters in the neighbourhood of the exceptional point.

Figure 3: Real and imaginary mass surfaces around the exceptional point
Conclusions

- CP-violating complex couplings allow for the possibility of mixing of $H$, $A$ and degeneracy of the $H_2$, $H_3$ physical states.

- At exact mass degeneracy, the propagator of the system has a combination of one double and two single poles in the complex energy $s$-plane.

- In parameter space the mass surfaces have one branch point of rank one where exact degeneracy occurs.

- At degeneracy, the identification of the two particles will depend strongly on the values of the parameters where the mass surfaces are displayed.

- It is imperative to consider the main $s$-dependent 1-loop diagrams for the Higgs self-energy in order to find the exceptional point in terms of the MSSM parameters.

- These features would make a CP-violating Higgs sector of the MSSM easily discernible from a CP-preserving one.
thank you