Higgs production via gluon fusion

Babis Anastasiou
ETH ZURICH
Higgs Hunting, Paris, July 2010

in collaboration with A. Lazopoulos
• Introduction

• QCD

• HQET and EWK

• New physics

• Experimental vs Theoretical Observables

• The ultimate precision

• How many perturbative orders?

• What scale?

• What parton distribution functions?

• What strong coupling value?

• The importance of top-quark loops?

• Bottom quark loops?

• Bottom quarks as initial state?

• Are new physics and QCD factorized?

• Are event generators reliable in estimating the ratio of measured Higgs signals vs the total?
The gluon fusion process

LHC or Tevatron beams

HIGGS

Heavy quark (color)
The end of a Higgs boson

\[ \Gamma (H \rightarrow f \bar{f}) = \frac{M_H}{8\pi} \left( \frac{M_f}{v} \right)^2 N_c \left( 1 - \frac{4M_f^2}{M_H^2} \right)^{\frac{3}{2}} \]

\[ \Gamma (H \rightarrow WW) = \frac{M_H}{16\pi} \left( \frac{M_H}{v} \right)^2 \left( 1 - \frac{4M_W^2}{M_H^2} \right)^{\frac{1}{2}} \times \left[ 1 - 4 \left( \frac{M_W^2}{M_H^2} \right) + 12 \left( \frac{M_W^2}{M_H^2} \right)^2 \right] \]

\[ \Gamma (H \rightarrow ZZ) = \frac{M_H}{32\pi} \left( \frac{M_H}{v} \right)^2 \left( 1 - \frac{4M_Z^2}{M_H^2} \right)^{\frac{1}{2}} \times \left[ 1 - 4 \left( \frac{M_Z^2}{M_H^2} \right) + 12 \left( \frac{M_Z^2}{M_H^2} \right)^2 \right] \]
Two photon decay

(Light Higgs)

$$N_c Q_t^2 (4/3)$$

Small decay width

Probes the electroweak content of the vacuum. Sensitive to new heavy gauge bosons.
A field theory laboratory

- Simple colorless final state, fixed order QCD perturbative calculations at LO, NLO, NNLO, ...
- Diverse energy scales (twice-quark mass, Higgs mass, Higgs recoil energy)
- HQET effective theory, factorization, resummation
Gluon fusion QCD

- **Total cross-section at NLO**
  (Dawson; Spira, Djouadi, Graudenz, Zerwas; ...)

- **Total cross-section at NNLO**
  (Harlander, Kilgore; CA, Melnikov; Ravindran, Smith, van Neerven, ...)

- **Threshold resummation**
  (Catani, de Florian, Grazzini, Nason; Moch, Vogt; Laanen, Magnea; Kulesza, Sterman; Idilbi, Xi, Ma, Juan; Ravindran; Ahrens, Becher, Neubert)

- **Transverse momentum resummation**
  (Bozzi, Catani, de Florian, Grazzini)
Gluon fusion EWK

- Two-loop light fermion amplitude \((\text{Aglietti, Bonziani, Degrassi, Vicini})\)
- Full two-loop EWK amplitude \((\text{Actis, Passarino, Sturm, Uccirati})\)
- Three-loop mixed QCD and EWK \((\text{CA, Boughezal, Petriello})\)
- One-loop EWK, with \(P_t > 0\) \((\text{Keung, Petriello})\)
Heavy top quark expansion

• Beyond the leading term (Chetyrkin, Kniehl, Steinhuser; Kraemer, Laenen, Spira) in the heavy quark-mass expansion at NNLO (Harlander, Mantler, Ozeren; Pak, Rogal, Steinhauser)

• High energy limit (Marzani, Ball, del Duca, Forte, Vicini)
Differential NNLO

- Fully Exclusive Higgs Production
  (CA, Melnikov, Petriello; CA, Dissertori, Stoeckli)

- HNNLO method
  (Catani, Grazzini; Grazzini)

- Fully differential NLO cross-section with exact mass quark effects
  (CA, Bucherer, Kunszt)
In summary

• Full NLO QCD (very sizable)
• Full NNLO HQET (less sizable)
• Glimpses at NNNLO and all-orders in HQET with resummation methods (generally small)
• Electroweak corrections (quite small)

• ALL indispensable in order to trust the error estimates on our predictions for the number of Higgs boson signals that may be detected at the LHC and Tevatron.
A manual to reliable precision for all inclusive and differential gluon fusion cross-sections

- Compute through NNLO QCD
- Choose a renomalization/factorization scale which captures the physics of the process
- Include parton densities through the same order and take into account their uncertainties
- Include exact quark mass effects through NLO
- Include known electroweak corrections (in complex mass scheme)
What scale?

- An almost philosophical question without “theory data”

- Large scales \( \sim (2 \text{ Mhiggs}) \) fail to capture higher order effects for the total cross-section @ Tevatron

- Small scales \( \sim (\text{Mhiggs}/2) \) are much better

**Total cross-section @ Tevatron**

\( \text{Mhiggs} = 165 \text{ GeV} \)
What scale?

- Average transverse momentum is zero at LO.
- Large scales $\sim (2\text{ Mhiggs})$ fail to capture higher order effects for $<\text{Pt}>$ @ Tevatron.
- Small scales $\sim (\text{Mhiggs}/2)$ are much better.

$\langle p_{t,H}\rangle$ (GeV) as a function of $\mu_F/m_H$ at $m_H=165\text{ GeV}$ for Tevatron ($\sqrt{s}=1.96\text{ TeV}$)

Average $pt$ @ Tevatron

Mhiggs = 165 GeV
What scale?

• Large scales $\sim (2 \, \text{Mhiggs})$ fail to capture higher order effects for the total cross-section @ LHC

• Small scales $\sim (\text{Mhiggs}/2)$ are much better

Total cross-section @ LHC 7 TeV

$\text{Mhiggs} = 120 \, \text{GeV}$
What scale?

- Average transverse momentum is zero at LO.
- Large scales $\sim (2 \, M_{\text{higgs}})$ fail to capture higher order effects for $<Pt>$ @ LHC.
- Small scales $\sim (M_{\text{higgs}}/2)$ are much better.

**Average pt @ LHC 7 TeV**

$M_{\text{higgs}} = 120 \text{ GeV}$
What scale?

- Large scales $\sim (2 \text{ Mhiggs})$ fail to capture higher order effects for the total cross-section @ LHC
- Small scales $\sim (\text{Mhiggs}/2)$ are much better

**Total cross-section @ LHC 7 TeV**

$\text{Mhiggs} = 500 \text{ GeV}$
What scale?

- Average transverse momentum is zero at LO.

- Large scales $\sim (2 \text{ Mhiggs})$ fail to capture higher order effects for $<\text{Pt}>$ at Tevatron.

- Small scales $\sim (\text{Mhiggs}/2)$ are much better.

\[ \langle p_{T,H} \rangle (\text{GeV}) \text{ as a function of } \frac{\mu_F}{m_H} \text{ at } m_H = 500\text{GeV for LHC } (\sqrt{s}=7\text{TeV}) \]

Average $p_t$ @ LHC 7 TeV

$\text{Mhiggs} = 500$ GeV
Missing higher orders?

- A small scale leads to a faster convergence from NLO to NNLO.
- Are we missing even higher order effects?
- A lot is known from threshold resummation methods.
- Small mu scales are safe!
Missing higher orders?

- A small scale leads to a faster convergence from NLO to NNLO.
- Are we missing even higher order effects
- A lot is known from threshold resummation methods.

- Small mu scales are safe!
Missing higher orders?

NNLO seems to be a conservative estimate of the cross-section.
Is threshold dominant?

\[ \sigma = \int dx_1 dx_2 f_1(x_1) f_2(x_2) \hat{\sigma} \left( z = \frac{m_h^2}{\hat{s}} \right) \]

<table>
<thead>
<tr>
<th>( \sigma ) [pb]</th>
<th>( \delta(1-z) )</th>
<th>( \left[ \frac{\log^n(1-z)}{1-z} \right]_+ )</th>
<th>Regular</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO@NNLO</td>
<td>14.87</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NLO@NNLO</td>
<td>8.07</td>
<td>5.06</td>
<td>5.91</td>
</tr>
<tr>
<td>NNLO@NNLO</td>
<td>1.55</td>
<td>2.84</td>
<td>6.81</td>
</tr>
</tbody>
</table>

Closer look on logarithmically enhanced terms:

<table>
<thead>
<tr>
<th>( \sigma ) [pb]</th>
<th>n=3</th>
<th>n=2</th>
<th>n=1</th>
<th>n=0</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO@NNLO</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NLO@NNLO</td>
<td>0</td>
<td>0</td>
<td>5.06</td>
<td>0</td>
</tr>
<tr>
<td>NNLO@NNLO</td>
<td>6.19</td>
<td>0.67</td>
<td>-0.47</td>
<td>-3.53</td>
</tr>
</tbody>
</table>

LHC 14 TeV
Mhiggs = 120 GeV
\( \mu = 120 \) GeV

No suppression from NLO to NNLO

No suppression of subleading logs
Is threshold dominant?

\[
\sigma = \int dx_1 dx_2 f_1(x_1) f_2(x_2) z \left( \hat{\sigma} \left( z = \frac{m_h^2}{s} \right) \right)
\]

Many equivalent definitions of threshold logs: can reduce the “regular part”

<table>
<thead>
<tr>
<th>(\sigma ) [pb]</th>
<th>(\delta(1-z))</th>
<th>(\left[ \frac{\log^n (1-z)}{1-z} \right]_+)</th>
<th>Regular</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO@NNLO</td>
<td>14.87</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NLO@NNLO</td>
<td>8.07</td>
<td>10.04</td>
<td>0.92</td>
</tr>
<tr>
<td>NNLO@NNLO</td>
<td>1.55</td>
<td>8.48</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Closer look on logarithmically enhanced terms:

<table>
<thead>
<tr>
<th>(\sigma ) [pb]</th>
<th>(n=3)</th>
<th>(n=2)</th>
<th>(n=1)</th>
<th>(n=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO@NNLO</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NLO@NNLO</td>
<td>0</td>
<td>0</td>
<td>10.04</td>
<td>0</td>
</tr>
<tr>
<td>NNLO@NNLO</td>
<td>13.81</td>
<td>1.43</td>
<td>-0.93</td>
<td>-5.83</td>
</tr>
</tbody>
</table>

Same observations on threshold dominance and convergence
The Higgs+1-jet subprocess

- If threshold logs are not the full reason for the large K-factors, then what?
- Their magnitude depends strongly on a jet-veto.
- Higgs+1-jet subprocess is important.
- Interesting to see the application of the ideas on large K-factors by Rubin, Salam, Sapeta on Higgs production

Figure 2: A) a LO contribution to Z+jet production; B) and C) two contributions that are NLO corrections to Z+jet observables but whose topology is that of a dijet event with additional radiation of a soft or collinear Z-boson either from a final-state quark (B) or an initial-state one (C).
Parton Densities

- PDF uncertainties have surprised us at times!

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3833</td>
<td>0.3988</td>
<td>0.3943</td>
<td>0.3444</td>
</tr>
</tbody>
</table>

- Estimate of alpha_s uncertainty (Martin, Stirling, Thorne, Watt)

- Comparable or bigger uncertainty than scale choice

\[ 389.0 \text{ fb} \pm 8.1\% \text{ (scale)} \pm 13.6\% (\alpha_s + \text{pdf}) \]

@ TEVATRON \quad M_{Higgs} = 165 \text{ GeV}
PDF set differences

- **Three NNLO pdf sets**: Martin, Striling, Thorne, Watt (MSTW) Alekhin, Bluemlein, Klein, Moch (ABKM) Jimenez, Reya (JR)

- Important differences beyond estimated uncertainties which affect Higgs cross-sections, especially @ Tevatron

- No compelling reason to choose one set over the others, but MSTW fits on Tevatron jet data and their alpha_s is very close to world average

- For Tevatron and theorists: Need to check compatibility of all pdf sets with observables sensitive to high-x at NNLO
Precision of HQET

- At LO and NLO in the strong coupling expansion, we can compute total and fully differential (HPro) cross-sections in the full SM theory.

- We only need to employ HQET for the NNLO coefficient, which amounts to about 20% of the total.

- A “mistake” of 10% due to finite top and bottom mass effects at NNLO gives a very forgiving error of less than 2% for the full LO+NLO+NNLO.
Combining theory uncertainties

- PDF uncertainties have or are made to have a statistical interpretation \((\text{Gaussian priors/combination in quadrature})\)

- Other theory uncertainties parameterize the ignorance of theorists, reflecting a safe interval of cross-section estimates which should contain a hypothetical perfect (all orders) theoretical calculation \((\text{flat priors / linear combination})\)
“New” Higgs Physics

• In the standard model, the higgs mass is the only missing parameter to describe all LHC phenomena

• But Higgs physics is new physics!

• Standard Model is likely to fail in describing Higgs cross-sections in the presence of relatively light new states (eg. Low, Rattazzi, Vichi; Falkowski; Barbieri, Rychkov; Kitano, Nomura; Dermisek, Low;...)

• Higgs cross-sections will be precision tests to be passed by any successful theory of LHC phenomena
Beyond the Standard

• Can we derive a mass exclusion limit for a BSM scalar Higgs from a SM analysis?

• Very often yes, if QCD corrections and shapes of signal discriminants are model independent.

• If we can compute the gluon fusion cross-section in BSM as accurately as in the SM.
... Many possibilities!

<table>
<thead>
<tr>
<th>particles in different representations of the Lorentz group</th>
<th>particles of different mass in the loops</th>
<th>particles in different colour representations</th>
<th>different structure of the Higgs coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>quarks</td>
<td><img src="#" alt="Diagram of quarks" /></td>
<td>singlets, triplets, octets</td>
<td>$\sim \bar{\psi}\psi$</td>
</tr>
<tr>
<td>squarks</td>
<td><img src="#" alt="Diagram of squarks" /></td>
<td>fundamental, adjoint</td>
<td>$\sim \bar{\psi}\gamma_5\psi$</td>
</tr>
<tr>
<td>Majorana fermions</td>
<td><img src="#" alt="Diagram of Majorana fermions" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Separating new physics

- Experiments (LEP, Tevatron, ..) indicate that new particles must be heavy, while the Higgs is light.

- This allows for an effective-theory approach:

\[
L_{\text{eff}} = -\frac{\alpha_s}{4\pi} C H G_{\mu\nu}^a G^{a\mu\nu}
\]

\[
\left( C_0 + \left( \frac{\alpha_s}{\pi} \right) C_1 + \left( \frac{\alpha_s}{\pi} \right)^2 C_2 + \ldots \right) \left( \begin{array}{c} \text{QCD only!} \\ \text{factorization of QCD and NP effects} \end{array} \right)
\]

depends on the specific model.
Matching calculations through NNLO

Expansion by subgraphs (Chetyrkin; Gorishny; V.A. Smirnov)
+ small momentum expansion (Fleischer; Tarasov):

\[
\begin{align*}
\text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\end{tikzpicture}} & = & \text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\draw[red, thick] (v3) edge[loop above] (v3);
\end{tikzpicture}} \\
= & C_0 \cdot \text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\end{tikzpicture}} \\
\text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\draw[red, thick] (v3) edge[loop above] (v3);
\end{tikzpicture}} & = & \text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\draw[red, thick] (v3) edge[loop above] (v3);
\end{tikzpicture}} + \text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\draw[red, thick] (v3) edge[loop above] (v3);
\end{tikzpicture}} \\
= & \frac{\alpha_s}{\pi} C_1 \cdot \text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\end{tikzpicture}} + C_0 \cdot \text{\begin{tikzpicture}[baseline=-0.25cm]
\node (v1) at (0,0) [circle,draw] {};
\node (v2) at (1,0) [circle,draw] {};
\node (v3) at (0.5,0.5) [circle,draw] {};
\node (v4) at (0.5,-0.5) [circle,draw] {};
\draw (v1) -- (v2);
\draw (v3) -- (v4);
\end{tikzpicture}}.
\end{align*}
\]
Testing BSM via gluon fusion

- BSM predictions for the gluon fusion cross-section at NNLO can be tested

- 4th generation (CA, Boughezal, Furlan), studied at Tevatron.

- Color octets (Boughezal, Petriello)

Future challenge: models with significant Higgs coupling to light fermions
Theory vs Experimental Signals

- The experimental searches aim to measure Higgs signals to a region of phase-space which is not a necessarily democratic representation of the total cross-section.

- Do we trust experimental efficiencies of parton-shower Monte-Carlo’s?

- How do we quantify the precision on their predicted efficiencies?

- Can we assign the precision of the NNLO, resummed NNLL, NNNLO approx, etc total cross-section to the “targeted signal cross-section” (eg, most significant bins of a neural network on a likelihood analysis)?
Example from CDF

- Break up total NNLO cross-section into 0, 1, and 2 jet bins (Pt,jet = 20 gev). The theory precision degrades from the 0-jet to the 1-jet and the 2-jet bin.

\[
\frac{\Delta N_{\text{inc}}(\text{scale})}{N_{\text{inc}}} = 66.5\% \cdot (+5\%\,{-9\%}) + 28.6\% \cdot (+24\%\,{-22\%}) + 4.9\% \cdot (78\%\,{-41\%}) = (+14.0\%\,{-14.3\%})
\]

- Apply slightly different e.g. lepton selections in the various jet-bins, which are more severe in the 0-jet bin.

\[
\frac{\Delta N_{\text{signal}}(\text{scale})}{N_{\text{signal}}} = 60\% \cdot (+5\%\,{-9\%}) + 29\% \cdot (+24\%\,{-22\%}) + 11\% \cdot (78\%\,{-41\%}) = (+18.5\%\,{-16.3\%})
\]

- Theory uncertainty for the accepted signal events is different than for the total number before cuts.

(CA,Dissertori,Grazzini, Stoeckli,Webber)
Differential Theory

- Lesson: check theory uncertainty on the kinematic bins which drive exclusion

- An NNLO computation of a neural net is as simple as for a rapidity distribution. (CA, Dissertori, Grazzini, Stoeckli, Webber)

- Highly recommended for the low statistics CDF, D0 and first LHC analyses.
Generators differ!

- Pythia has a smaller jet-veto and isolation acceptance than HERWIG and MC@NLO
- HERWIG and MC@NLO closer to NNLO

<table>
<thead>
<tr>
<th>$\sigma_{\text{acc}}/\sigma_{\text{incl}}$</th>
<th>Trigger</th>
<th>+ Jet-Veto</th>
<th>+ Isolation</th>
<th>All Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNLO ($\mu = m_H/2$)</td>
<td>44.7%</td>
<td>39.4% (88.1%)</td>
<td>36.8% (93.4%)</td>
<td>27.8% (75.5%)</td>
</tr>
<tr>
<td>NNLO ($\mu = 2 m_H$)</td>
<td>44.9%</td>
<td>41.8% (93.1%)</td>
<td>40.7% (97.4%)</td>
<td>31.0% (76.2%)</td>
</tr>
<tr>
<td>MC@NLO ($\mu = m_H/2$)</td>
<td>44.4%</td>
<td>38.1% (85.8%)</td>
<td>35.3% (92.5%)</td>
<td>26.5% (75.2%)</td>
</tr>
<tr>
<td>MC@NLO ($\mu = 2 m_H$)</td>
<td>44.8%</td>
<td>38.8% (86.7%)</td>
<td>35.9% (92.5%)</td>
<td>27.0% (75.2%)</td>
</tr>
<tr>
<td>HERWIG</td>
<td>46.7%</td>
<td>40.8% (87.4%)</td>
<td>37.8% (92.7%)</td>
<td>28.6% (75.7%)</td>
</tr>
<tr>
<td>PYTHIA</td>
<td>46.6%</td>
<td>37.9% (81.3%)</td>
<td>32.2% (85.0%)</td>
<td>24.4% (75.8%)</td>
</tr>
</tbody>
</table>
• How many perturbative orders?
• What scale?
• What parton distribution functions?
• What strong coupling value?
• The importance of finite mass in top-quark and bottom quark loops?
• Are new physics and QCD factorized?
• Are event generators reliable in estimating the ratio of measured Higgs signals vs the total?
• What is the ultimate precision?
• Room for improvements?