FROM QED TO THE HIGGS MECHANISM: A SHORT REVIEW

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For a selection of original papers see,

Selected papers on Gauge theory of weak and electromagnetic interactions, CH Lai ed. World Scientific Singapore 1981.

Perturbative Quantum Field Theory: General strategy

The construction of a quantum field theory generally involved three steps:

(1) Construction of the classical theory.
(2) Quantization.
(3) Renormalization.

Moreover, in gauge theories, due to the necessity of gauge fixing, an additional step is required, proving **unitarity**. This is specially complicate in the non-Abelian case, owing to the presence of spinless fermions, the Faddeev–Popov ghosts. Moreover, in presence of spontaneous symmetry breaking and in covariant gauges, the decoupling of would-be massless Goldstone bosons has to be demonstrated.

Only when this program is completed, can perturbative calculations be performed.

Toward a model for weak and electromagnetic interactions

The present model of weak and electromagnetic interactions has involves several ingredients:

(1) Quantum Electrodynamics or Abelian gauge theories.

(2) Spontaneously broken scalar boson and fermion theories.

(3) Non-Abelian gauge theories.

(4) The Abelian Higgs mechanism, which combines Abelian gauge theories and spontaneous symmetry breaking.

(5) The non-Abelian Higgs mechanism, which combines non-Abelian gauge theories and spontaneous symmetry breaking.

Note that the Higgs mechanism contains itself two ingredients: mass given to vector fields and absence (or decoupling) of massless scalar particles.

Below, I will not describe the road from classical electrodynamics (Maxwell equations) to QED, a lengthy process extending over more than 20 years.

Spontaneous symmetry breaking: the classical theory

The idea of spontaneous symmetry breaking originates from statistical physics and the theory of phase transitions. A general framework was provided by Landau (1937).

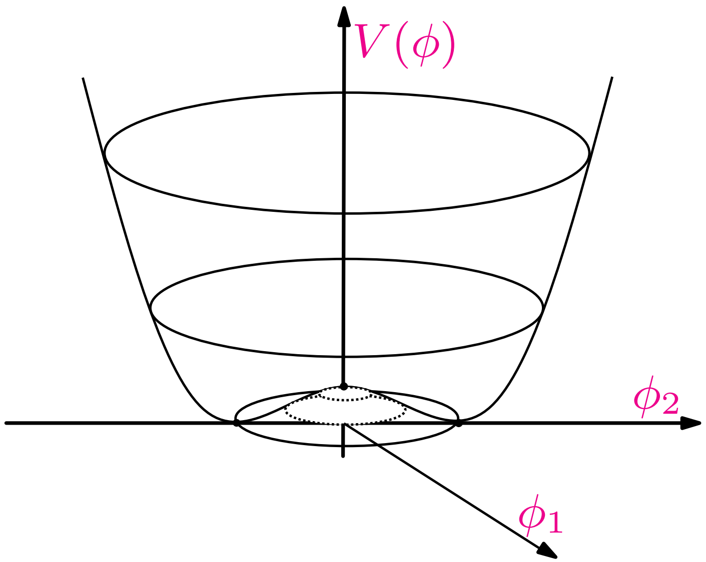

Fig. 1 A $O(2)$ symmetric potential $V(\phi)$ with degenerate minima.

Relativistic quantum field theory

Motivation and quantization

The notion of spontaneous symmetry breaking was introduced in the relativistic theory in the context of approximate $SU(2) \times SU(2)$ chiral symmetry. In particular, spontaneous symmetry breaking implies the existence of massless particles (Nambu–Goldstone bosons) and this was consistent with the small pion mass. Early articles are

Y. Nambu, *Axial Vector Current Conservation in Weak Interactions*, Phys. Rev. Lett. 4 (1960) 380;

J. Goldstone, Nuovo Cimento 19 (1961) 155;

Y. Nambu and G. Jona-Lasinio, *Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I, II*, Phys. Rev. 122 (1961) 345; 124 (1961) 246;

J. Goldstone, A. Salam and S. Weinberg, *Broken symmetries*, Phys. Rev. 127 (1962) 965.

Spontaneous symmetry breaking and renormalization

A simple example of a relativistic quantum field theory in dimension $d = 4$ (time + space) exhibiting spontaneous symmetry breaking is the $O(4)$ symmetric $(\boldsymbol{\phi}^2)^2$ theory for a four-component, self-interacting, scalar boson field $\boldsymbol{\phi}$ corresponding to the action

$$\mathcal{S}(\phi) = \int \mathrm{d}^4 x \left[\tfrac{1}{2} \sum_{\mu} \left(\partial_\mu \boldsymbol{\phi}(x) \right)^2 + \tfrac{1}{2} r \boldsymbol{\phi}^2(x) + \frac{g}{4!} \left(\boldsymbol{\phi}^2(x) \right)^2 \right].$$

When $r < 0$, the minimum of the action is degenerate, corresponding to the sphere $\boldsymbol{\phi}^2 \equiv v^2 = -6r/g$ and one says that the symmetry is spontaneously broken. Shifting by its expectation value $\boldsymbol{\phi} = \mathbf{v} + \boldsymbol{\chi}$, one finds

$$\mathcal{S}(\chi) - \mathcal{S}(v) = \int \mathrm{d}^4 x \left[\tfrac{1}{2} \sum_{\mu} \left(\partial_\mu \boldsymbol{\chi}(x) \right)^2 + \frac{g}{6} \left(\mathbf{v} \cdot \boldsymbol{\chi}(\boldsymbol{x}) \right)^2 \right.$$
$$\left. + \frac{g}{6} \mathbf{v} \cdot \boldsymbol{\chi}(x) \boldsymbol{\chi}^2(x) + \frac{g}{4!} \left(\boldsymbol{\chi}^2(x) \right)^2 \right].$$

The perturbative spectrum thus contains one massive particle and 3 massless particles called Nambu–Goldstone bosons.

While the renormalization of the theory in the symmetric phase is simple, it is more complicate in the broken phase because the Feynman rules are no longer explicitly symmetric. The proof is based on the use of generalized Ward–Takahashi identities.

B.W. Lee, Renormalization of the σ-model, Nuclear Physics B 9 (1969) 649-672.

K. Symanzik, Renormalizable Models with Simple Symmetry Breaking, Commun. math. Phys. 16(1970) 48-80.

Non-Abelian gauge theories

Classical theory

The classical non-Abelian extension of Maxwell's electrodynamics is proposed in

C.N. Yang and R.L. Mills, *Conservation of Isotopic Spin and Isotopic Gauge Invariance*, Phys. Rev. 96 (1954) 191-195.

However, the trial and error method used in the Abelian case to quantify this gauge invariant field theory does not work. This was demonstrated in

R.P. Feynman, *Quantum theory of gravitation*, Acta Phys. Polon. 24 (1963) 697-722,

where it is pointed out that, after the usual QED-like gauge fixing, at one loop, unitarity implies, the addition of spinless fermion quanta. After this work, the problem of quantization of non-Abelian gauge theories remained an open problem.

Abelian and non-Abelian Higgs mechanism

Classical field theory and general considerations

The problem has a long history. The idea that gauge fields could acquire a mass through interactions seems to originate from

J. Schwinger, *Gauge Invariance and Mass*, Phys. Rev. 125 (1962) 397-398, *ibidem Gauge Invariance and Mass II,* Phys. Rev. 128 (1962) 2425-2429.

but the proposal is not very explicit and no reference to spontaneous symmetry breaking is made. Actually, it is illustrated in a subsequent paper by the solution of a $1+1$ dimensional QED model with massless fermions, which, however, in modern thinking rather illustrates confinement.

An example based on spontaneous symmetry breaking is then provided by

P.W. Anderson, *Plasmons, Gauge Invariance and Mass*, Phys. Rev. 130 (1963) 439-442.

Abstract. Schwinger has pointed out that the Yang-Mills vector boson implied by associating a generalized gauge transformation with a conservation law (of baryonic charge, for instance) does not necessarily have zero mass, if a certain criterion on the vacuum fluctuations of the generalized current is satisfied. We show that the theory of plasma oscillations is a simple nonrelativistic example exhibiting all of the features of Schwinger's idea. It is also shown that Schwinger's criterion that the vector field $m \neq 0$ implies that the matter spectrum before including the Yang-Mills interaction contains $m = 0$, but that the example of superconductivity illustrates that the physical spectrum need not.

However, the absence of massless Goldstone bosons is justified by the observed properties of superconductivity and not by a theoretical argument.

Some discussion and confusion about the relevance of the superconducting example, owing to its non-relativistic character, to particle physics follows (*e.g.*, Klein and Lee, Gilbert). The possibility of a relativistic "Higgs mechanism" is again stressed in

P.W. Higgs, *Broken symmetries, massless particles and gauge fields*, Phys. Lett. 12 (1964) 132-133.

It is argued that the presence of gauge fields indeed allows avoiding massless scalar bosons. The argument is given for an Abelian QED-like example in a linearized approximation and in a specific, non-covariant gauge. The extension to non-Abelian theories is vaguely mentioned.

P.W. Higgs, *Broken symmetries and the masses of gauge bosons*, Phys. Rev. Lett. 13 (1964) 508-509.

Compared to the preceding article, the extension to the non-Abelian situation is discussed more explicitly in the form of a $SU(3)$ theory and the spectrum of the gauge fields after some specific symmetry breaking.

In

F. Englert and R. Brout, *Broken symmetry and the mass of vector bosons*, Phys. Rev. Lett. 13 (1964) 321-323,

it is shown that, in the classical limit, in non-Abelian gauge theories, spontaneous symmetry breaking could give masses to gauge bosons. However, nothing is said about the disappearance of massless scalar bosons.

In the article

G.S. Guralnik, C.R. Hagen and T.W.B. Kibble, *Conservation laws and massless particles*, Phys. Rev. Lett. 13 (1964) 585-587,

the authors consider a field theory model, QED coupled to charged scalar particles. They show, in the linear approximation and in a specific gauge, that after spontaneous symmetry breaking, the vector field becomes massive and the Goldstone boson decouples.

Full (i.e., beyond linear approximation) classical argument and quantization

Finally, in

P.W. Higgs, *Spontaneous Symmetry Breakdown without Massless Bosons*, Phys. Rev. 145 (1966) 1156-1163,

The Abelian model, QED coupled to a scalar charged field with spontaneous symmetry breaking, is studied as a true quantum theory, in a specific gauge, in the one-loop approximation. Moreover, it is shown at the classical level, that a gauge transformation can be found that transforms the initial Lagrangian into a Lagrangian with only physical degrees of freedom, a massive "Higgs" field and a massive vector field. This representation is often called "unitary gauge", but leads to non-renormalizable theories.

The classical argument is generalized to the non-Abelian case in

T.W.B. Kibble, *Symmetry Breaking in Non-Abelian Gauge Theories*, Phys. Rev. 155 (1967) 1554-1561.

Non-Abelian gauge theories: Quantization

In contrast with QED (the Abelian case) and as Feynman had already shown, the problem of gauge fixing could not solved by simple heuristic arguments. The quantization, a very difficult problem, was finally achieved in 1967 by

L.D. Faddeev and V.N. Popov, *Feynman diagrams for the Yang–Mills field*, Phys. Lett. 25B (1967) 29-30; *Perturbation theory for gauge-invariant fields*, Kiev report No. ITP 67-36.

See also

B.S. DeWitt, *Quantum theory of gravity II, III*, Phys. Rev. 162 (1967) 1195-1239; *ibidem* 1239-1256.

Towards renormalization

Two articles then triggered an enormous theoretical activity

G. 't Hooft, *Renormalization of massless Yang–Mills fields*, Nucl. Phys. B35 (1971) 173-199.

G. 't Hooft, *Renormalizable Lagrangians for massive Yang–Mills*, Nucl. Phys. B35 (1971) 167-188,

where it is argued that some models based on non-Abelian gauge theories, both in the symmetric phase and in the case of spontaneous symmetry breaking, should be renormalizable.

Abelian Higgs model: Full quantum theory and renormalization

First, motivated by 't Hooft's work, Lee proves rigorously that the model is renormalizable, proving unitarity to all orders, in particular, that it contains no massless scalar boson but instead a massive vector field:

B.W. Lee, *Renormalizable Massive Vector-Meson Theory-Perturbation Theory of the Higgs Phenomenon*, Phys. Rev. D5 (1972) 823-835.

This article also seems to be at the origin of the denominations "Higgs mechanism" and "Higgs boson".

Non-Abelian gauge theories: Renormalization

The first complete proofs of renormalizability then relied on a set of generalized Ward–Takahashi identities derived in

A.A. Slavnov, *Ward identities in gauge theories*, Theor. Math. Phys. 10 (1972) 99-107; J.C. Taylor, *Ward identities*, Nucl. Phys. B33 (1971) 436-444.

These were used in the first proofs of renormalizability of non-Abelian gauge theories in the broken phase in

B.W. Lee and J. Zinn-Justin, *Spontaneously broken gauge symmetries, I, II, III*, Phys. Rev. D5 (1972) 3121-3137, 3137-3155, 3155-3160;

B.W. Lee and J. Zinn-Justin, *Spontaneously broken gauge symmetries, IV General gauge formulation*, Phys. Rev. D7 (1973) 1049-1056.

The complexity of these proofs is a consequence of gauge fixing and spontaneous symmetry breaking, which completely destroy the beautiful geometric structure of the Lagrangian, in particular, the appearance of a number of non-physical particles, would-be Goldstone bosons, Faddeev–Popov ghosts.

Also relevant to this topics are articles by Fradkin and Tyutin, 't Hooft and Veltman, Fujikawa, Lee and Sanda, Ross and Taylor...

BRST symmetry

An additional important technical progress was provided by the observation that the quantized gauge action has an unexpected fermion-like symmetry, now called the BRST symmetry,

C. Becchi, A. Rouet and R. Stora, *The Abelian Higgs–Kibble Model. Unitarity of the S Operator*, Phys. Lett 52B (1974) 344; *ibidem Renormalization of the Abelian Higgs-Kibble Model,* Comm. Math. Phys. 42 (1975) 127; *ibidem Renormalization of gauge theories*, Ann. Phys. (NY) 98 (1976) 287-321.

I. Tyutin, Preprint of Lebedev Physical Institute, 39 (1975).

This observation was immediately used to derive a completely general, much more transparent proof, of renormalizability covering all groups, renormalizable gauges..., based on the ZJ equation:

J. Zinn-Justin (1975), *Renormalization of gauge theories*, Bonn lectures 1974, published in Trends in Elementary Particle Physics, Lecture Notes in Physics 37 pages 1-39, H. Rollnik and K. Dietz eds., Springer Verlag, Berlin.

Gauge independence and unitarity in this general context is dealt with in

J. Zinn-Justin (1975) in Proc. of the 12th School of Theoretical Physics, Karpacz, *Functional and probabilistic methods in quantum field theory* Acta Universitatis Wratislaviensis 368 (1976) 435-453, Saclay preprint t76/048.

Finally, all technical details concerning the theoretical developments described in this talk can be found in

J. Zinn-Justin (1989), *Quantum Field Theory and Critical Phenomena*, 4th edition (2002), Oxford University Press, Oxford.