

# Software development in AppStat

AppStat: Applied Statistics and Machine Learning

*AppStat: Apprentissage Automatique et Statistique Appliquée*

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Service Informatique  
Nov 30, 2010

# Overview

- Introduction
  - me
  - the team
  - collaborations
- Scientific projects → software
  - discriminative learning → boosting → [multiboost.org](http://multiboost.org)
  - inference, Monte-Carlo integration → adaptive MCMC → integration into root (save it for next time)

## Scientific path

Hungary	1989 – 94	M.Eng. Computer Science	BUTE
	1994 – 95	research assistant	BUTE
Canada	1995 – 99	Ph.D. Computer Science	Concordia U
	2000	postdoc	Queen's U
	2001 – 06	assistant professor	U of Montreal
France	2006 –	research scientist (CR1)	CNRS / U Paris Sud

- **Research interests:** machine learning, pattern recognition, signal processing, applied statistics
- **Applications:** image and music processing, bioinformatics, software engineering, grid control, experimental physics

# The team

R. Busa-Fekete (postdoc)  
2008 -  
- boosting  
- optimization  
- SysBio



B. Kégl (team leader)  
2006 -  
- boosting  
- MCMC  
- Auger



D. Benbouzid (Ph.D. student)  
2010 -  
- boosting  
- JEM EUSO



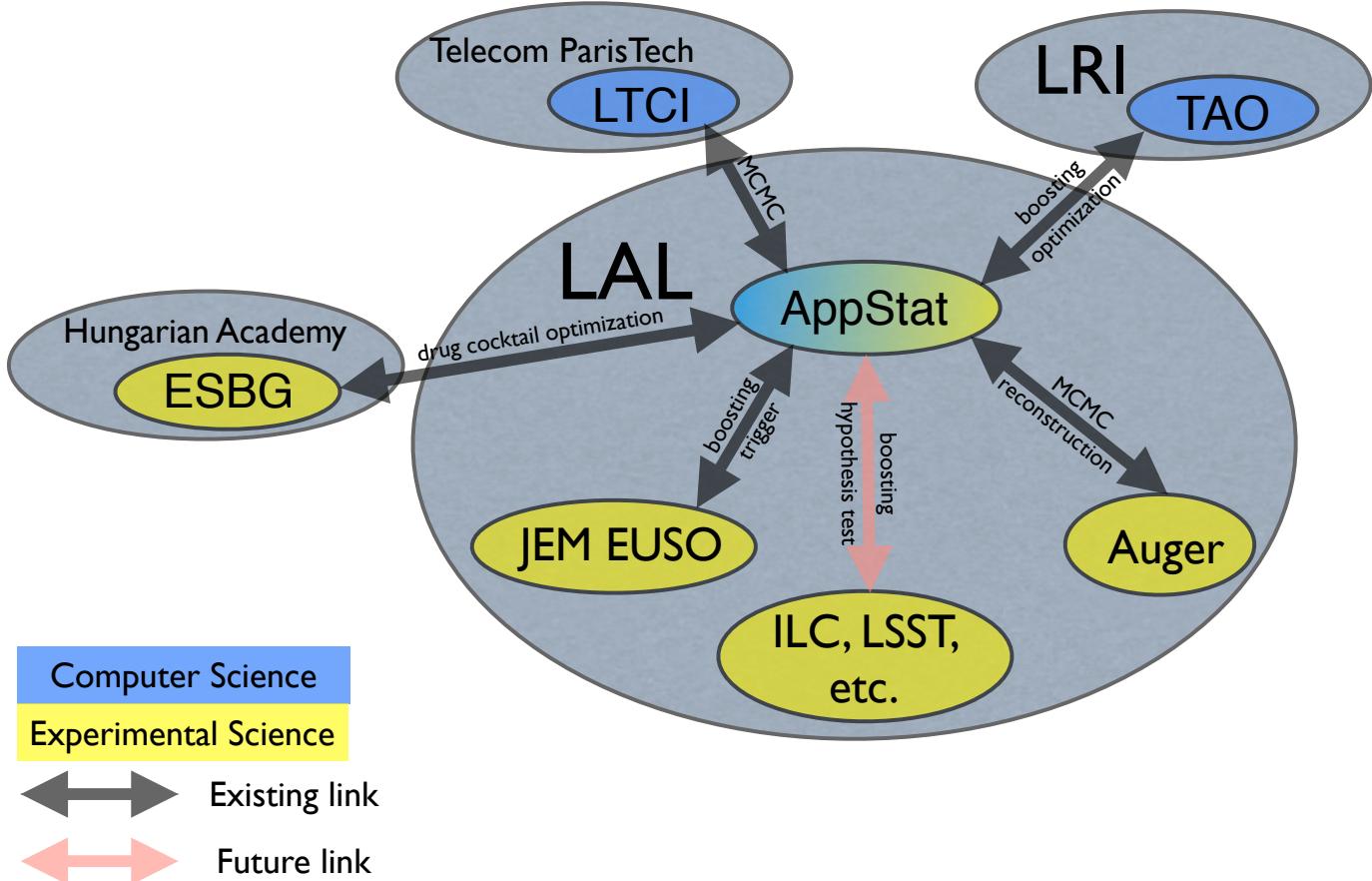
R. Bardenet (Ph.D student)  
2009 -  
- MCMC  
- optimization  
- Auger



F-D. Collin (software engineer; 01/12/2010)  
- multiboost.org  
- MCMC in root  
- system integration

D. Garcia (postdoc; 01/01/2011)  
- generative models  
- Auger / JEM EUSO  
- tutoring

# Collaborations



# Funding

- ANR “jeune chercheur” **MetaModel**
  - 2007–2010, 150K€
- ANR “COSINUS” **Siminole**
  - 2010–2014, 1043K€ (658K€ at LAL)
- MRM **Grille Paris Sud**
  - 2010–2012, 60K€ (31K€ at LAL)

# Siminole within ANR COSINUS

- COSINUS = Conception and Simulation
  - Theme 1: simulation and supercomputing
  - Theme 2: conception and optimization
  - Theme 3: large-scale data storage and processing
- Siminole
  - principal theme: Theme 2
  - secondary theme: Theme 1

# Siminole within ANR COSINUS

- Simulation: third pillar of scientific discovery
- Improving simulation
  - algorithmic development inside the simulator
  - implementation on high-end computing devices
  - our approach: control the number of calls to the simulator

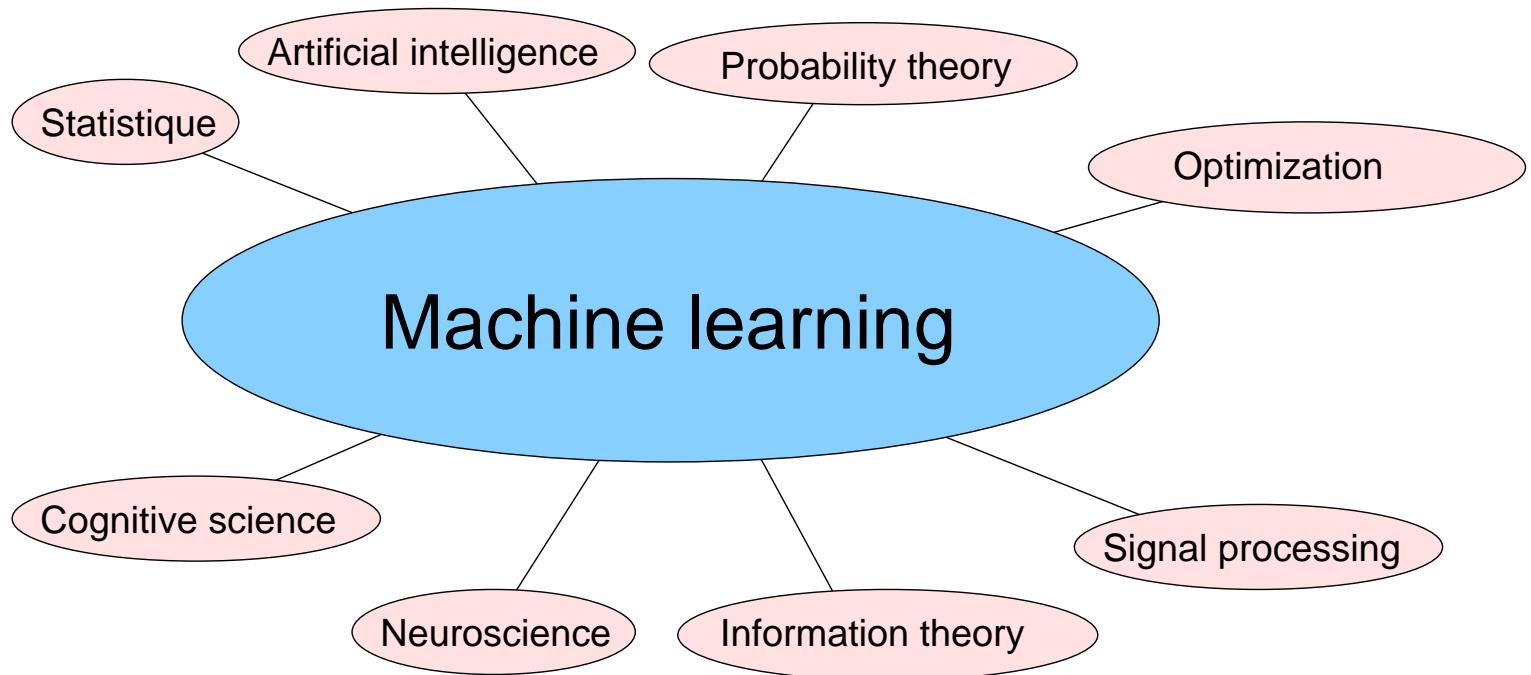
# Siminole within ANR COSINUS

- Optimization
  - simulate from  $f(x)$ , find  $\max_x f(x)$
- Inference
  - simulate from  $p(x|\theta)$ , find  $p(\theta|x)$
- Discriminative learning
  - simulate from  $p(x, \theta)$ , find  $\theta = f(x)$

# Discriminative learning → boosting → multiboost.org

- Discriminative learning (classification)
  - Infer  $f(\mathbf{x}) : \mathbb{R}^d \rightarrow 1, \dots, K$  from a database  $\mathcal{D} = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$
- boosting, AdaBoost
  - one of the state-of-the-art classification algorithms
- multiboost.org
  - our implementation

# Machine learning at the crossroads



# Machine Learning

- From a **statistical** point of view
  - **non-parametric** fitting, **capacity/complexity** control
  - large **dimensionality**
  - **large data** sets, **computational** issues
  - mostly **classification** (categorization, discrimination)

# Discriminative learning

- observation vector:  $\mathbf{x} \in \mathbb{R}^d$
- class label:  $y \in \{-1, 1\}$  – binary classification
- class label:  $y \in \{1, \dots, K\}$  – multi-class classification
- classifier:  $g : \mathbb{R}^d \mapsto \{-1, 1\}$
- discriminant function:  $f : \mathbb{R}^d \mapsto [-1, 1]$

$$g(\mathbf{x}) = \begin{cases} 1, & \text{if } f(\mathbf{x}) \geq 0, \\ -1, & \text{if } f(\mathbf{x}) < 0 \end{cases}$$

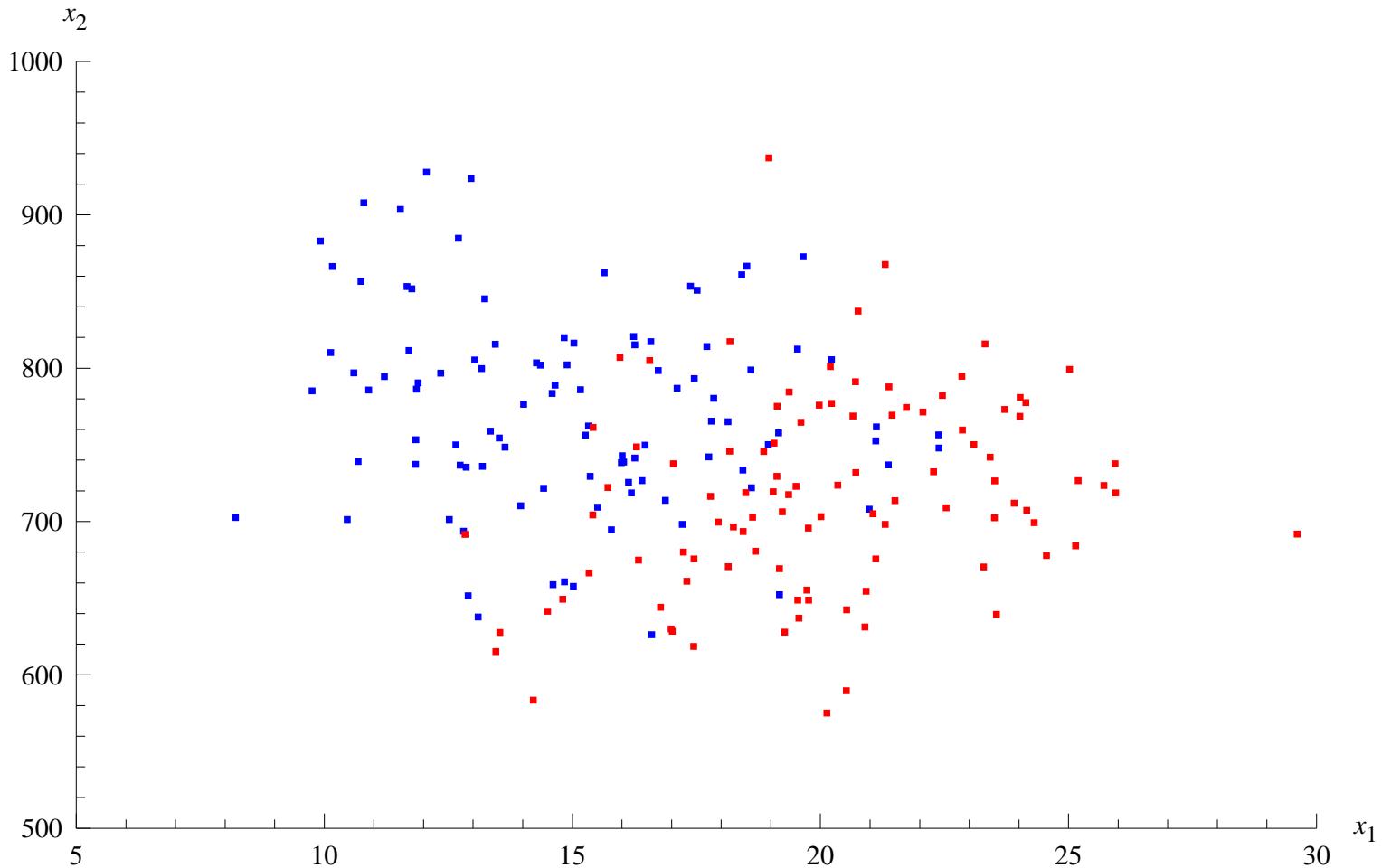
# Discriminative learning

- Inductive learning

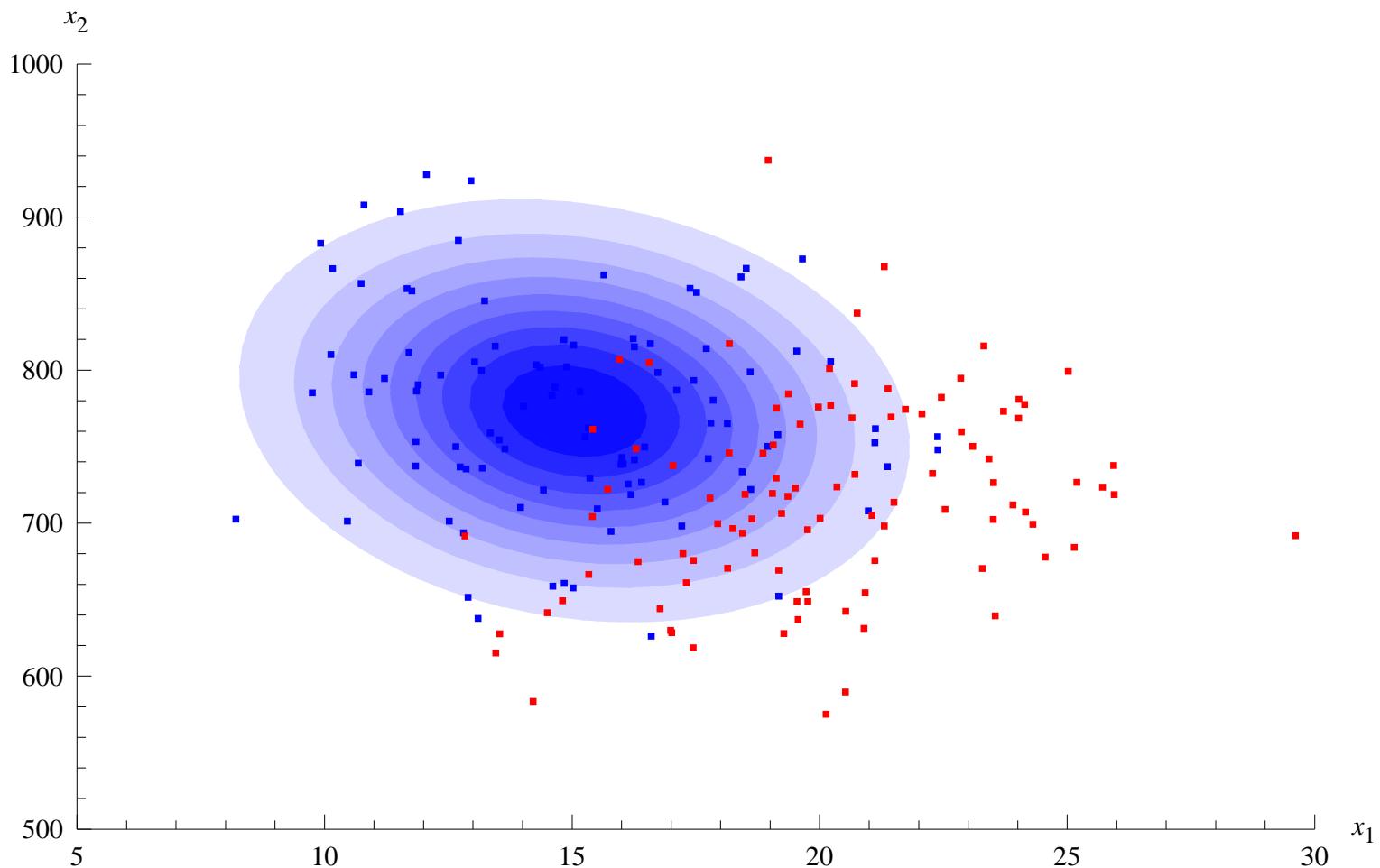
- training sample:  $D_n = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$
- function set:  $\mathcal{F}$
- learning algorithm:  $\text{ALGO} : (\mathbb{R}^d \times \{-1, 1\})^n \mapsto \mathcal{F}$ 
$$\text{ALGO}(D_n) \rightarrow f$$

- goal: small generalization error  $P[f(\mathbf{X}) \neq Y]$

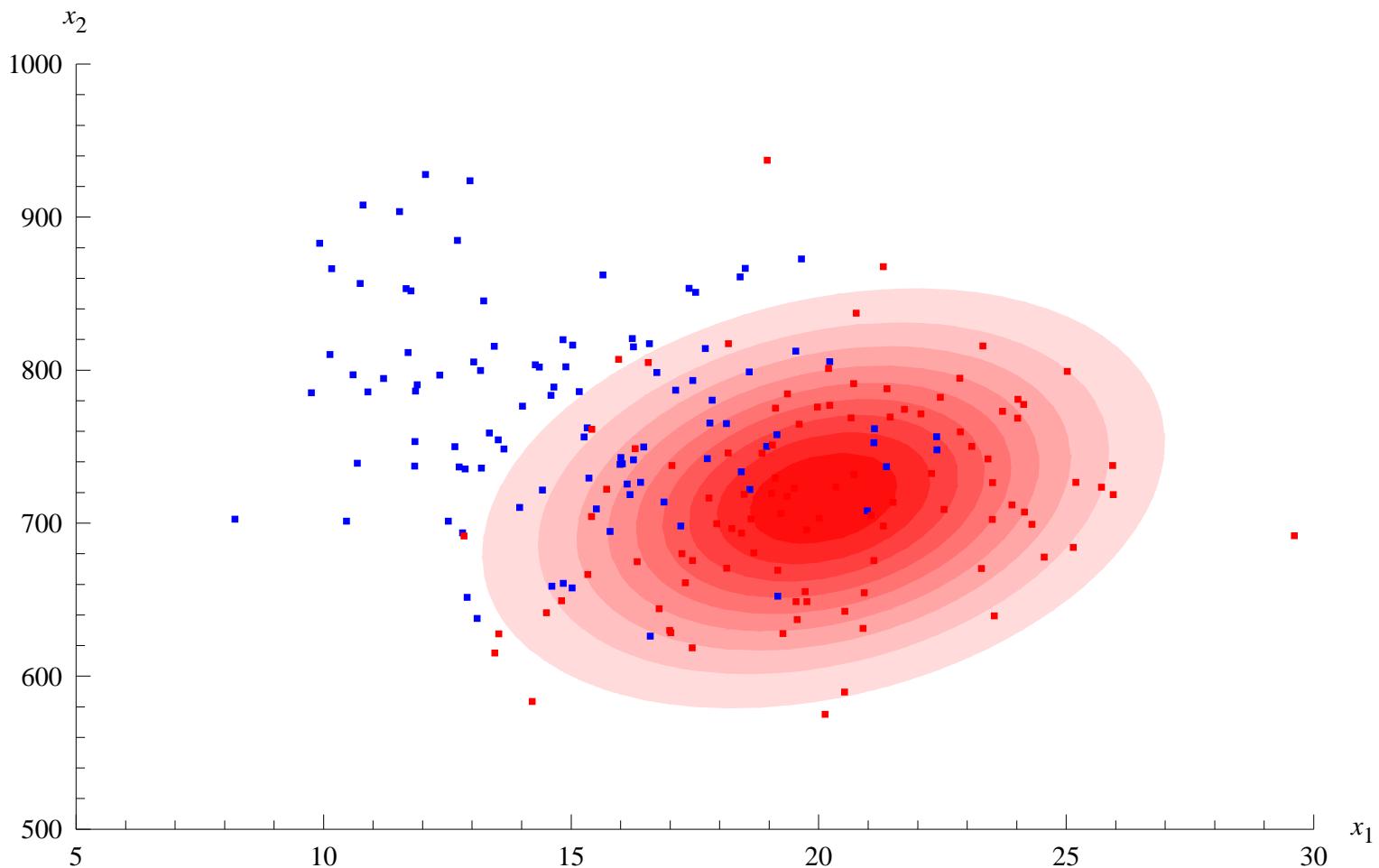
## Data for two-class classification problem



## 2-D Gaussian fit for class 1



## 2-D Gaussian fit for class 2



# Classification

- Terminology

- Conditional densities:  $p(\mathbf{x}|Y = 1)$ ,  $p(\mathbf{x}|Y = -1)$
- Prior probabilities:  $p(Y = 1)$ ,  $p(Y = -1)$
- Posterior probabilities:  $p(Y = 1|\mathbf{x})$ ,  $p(Y = -1|\mathbf{x})$

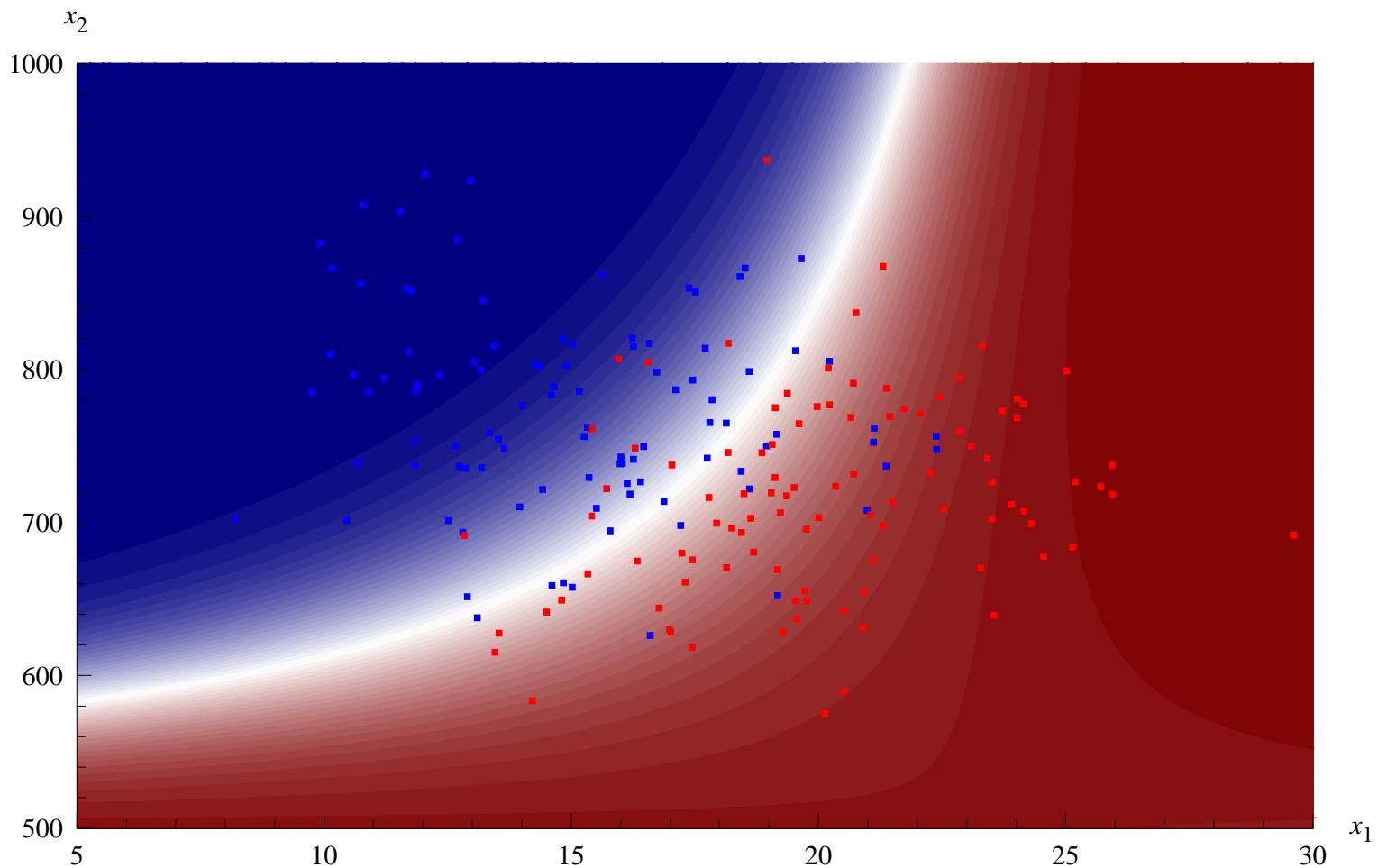
- Bayes theorem:

$$p(Y = 1|\mathbf{x}) = \frac{p(\mathbf{x}|Y = 1)p(Y = 1)}{p(\mathbf{x})} \sim p(\mathbf{x}|Y = 1)p(Y = 1)$$

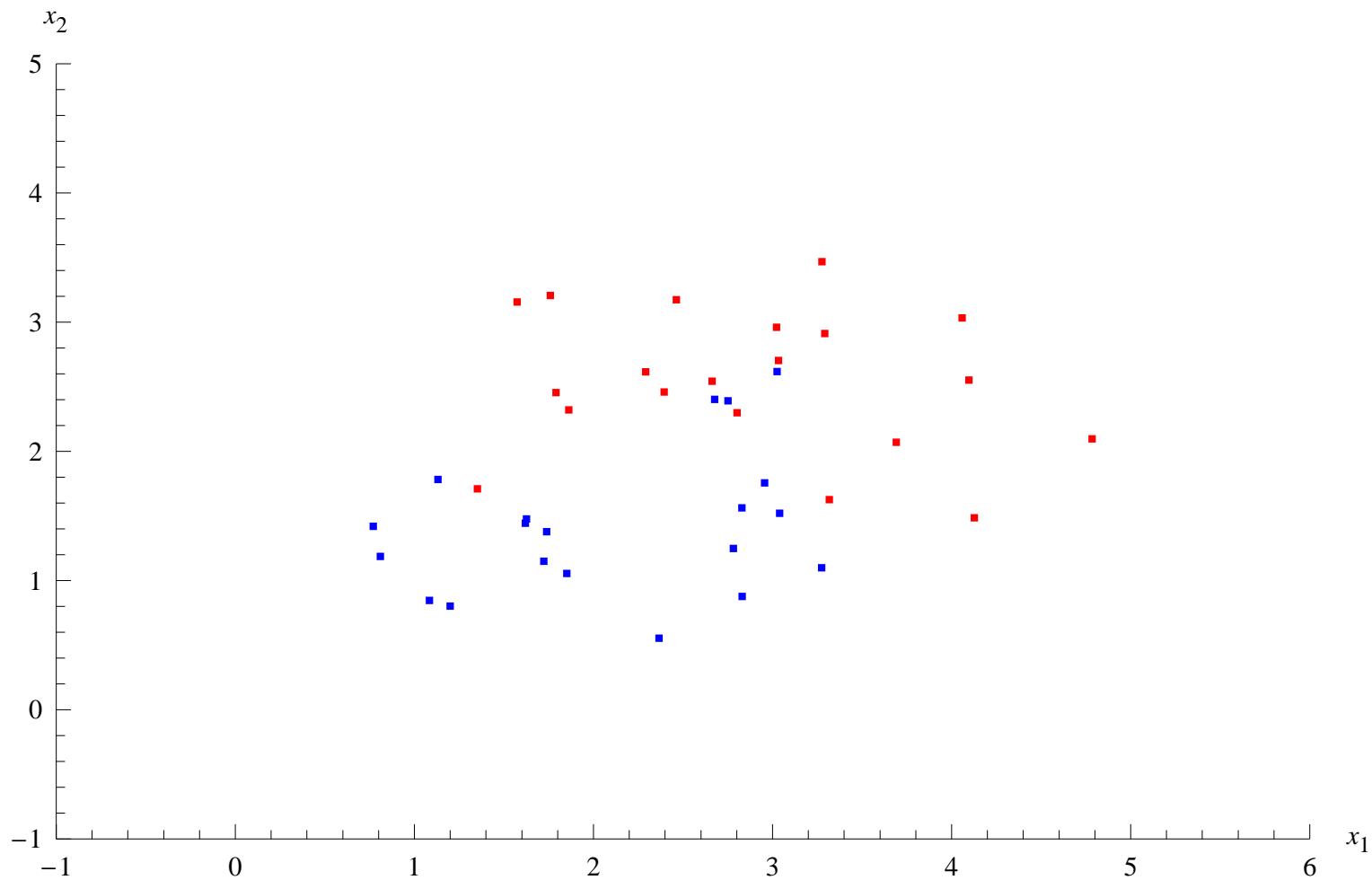
- Decision:

$$g(\mathbf{x}) = \begin{cases} 1 & \text{if } \frac{p(\mathbf{x}|Y=1)p(Y=1)}{p(\mathbf{x}|Y=-1)p(Y=-1)} > 1, \\ -1 & \text{otherwise.} \end{cases}$$

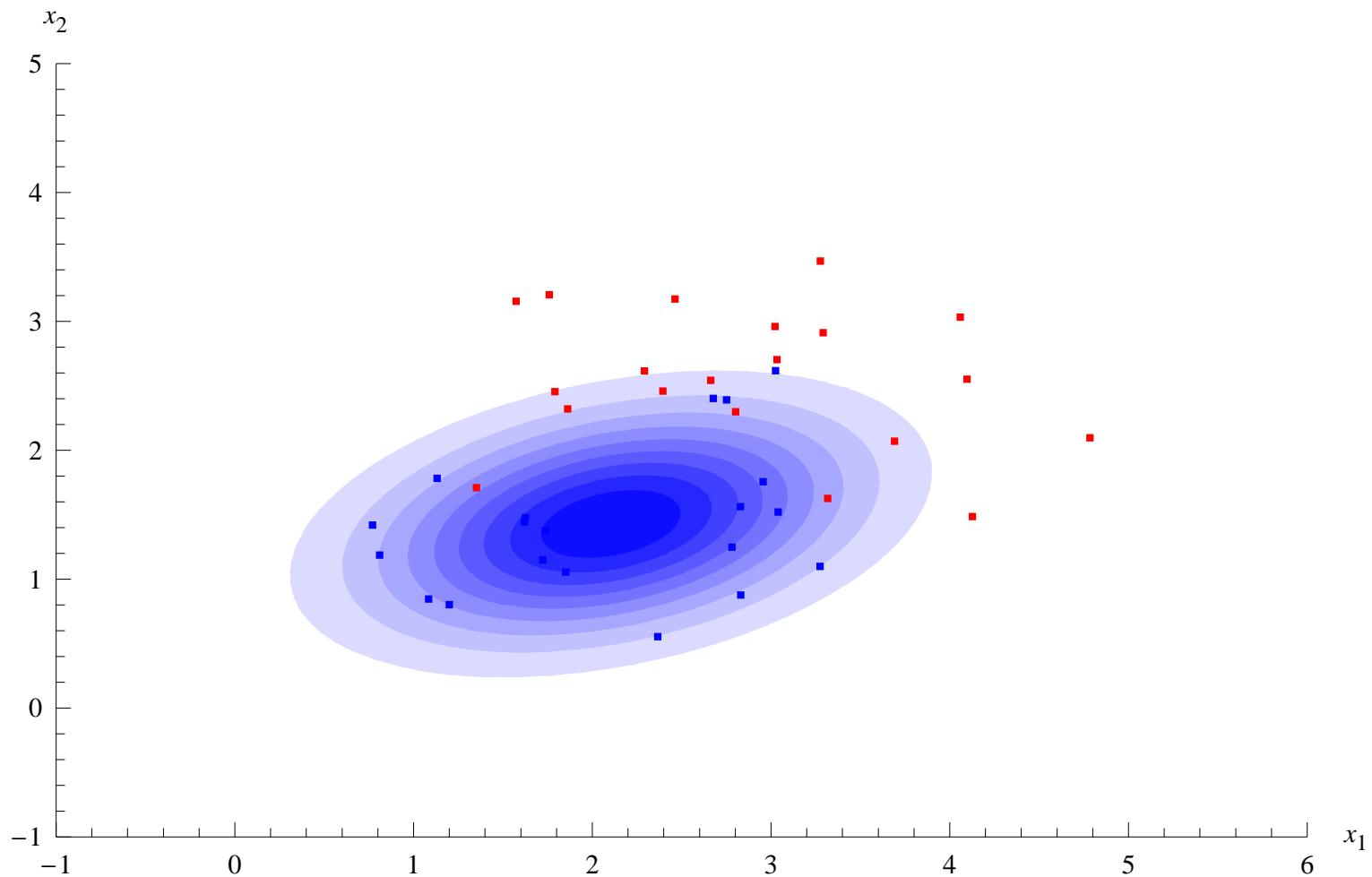
## Discriminant function with Gaussian fits



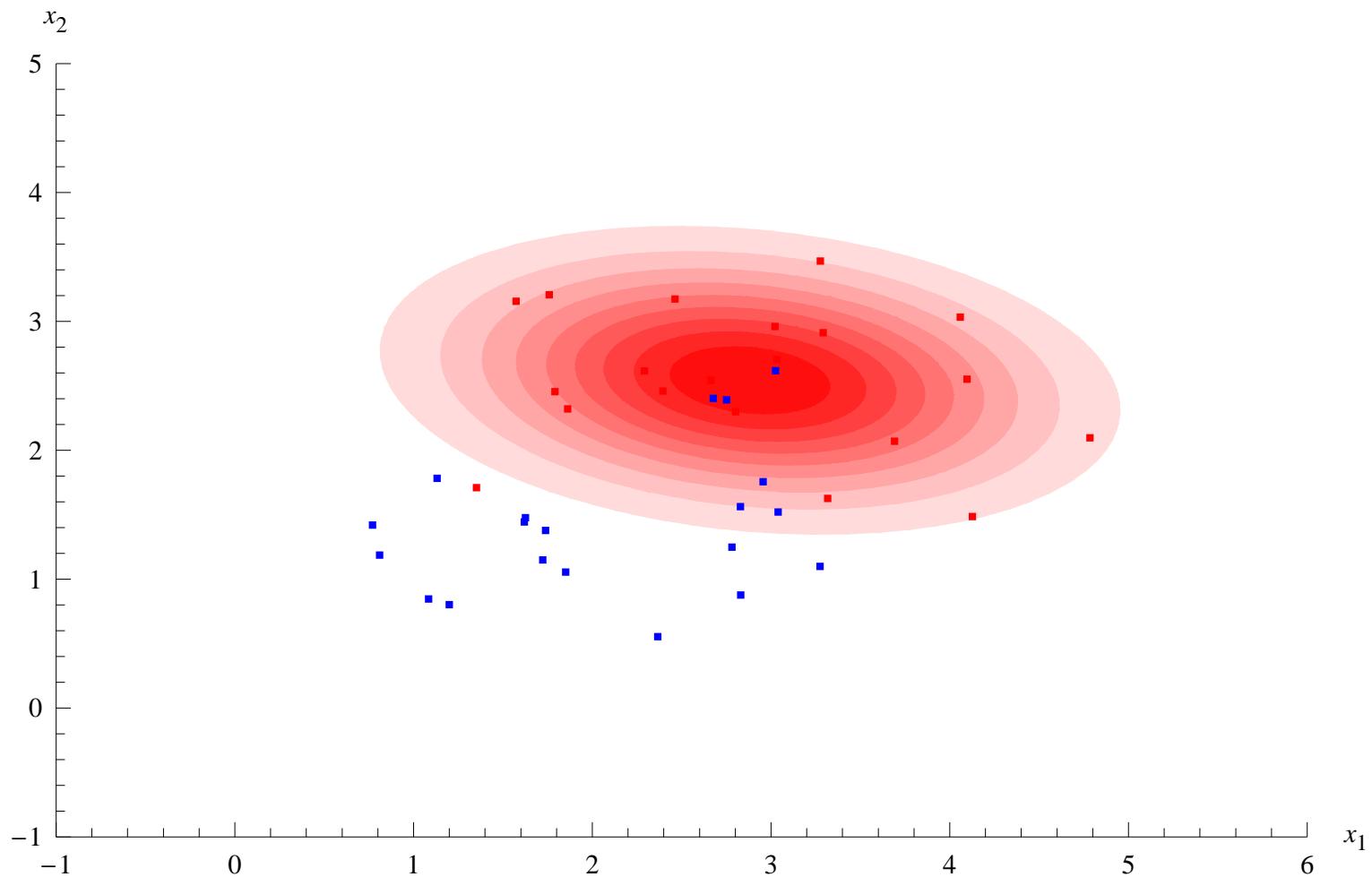
## 'Two Moons' data for two-class classification problem



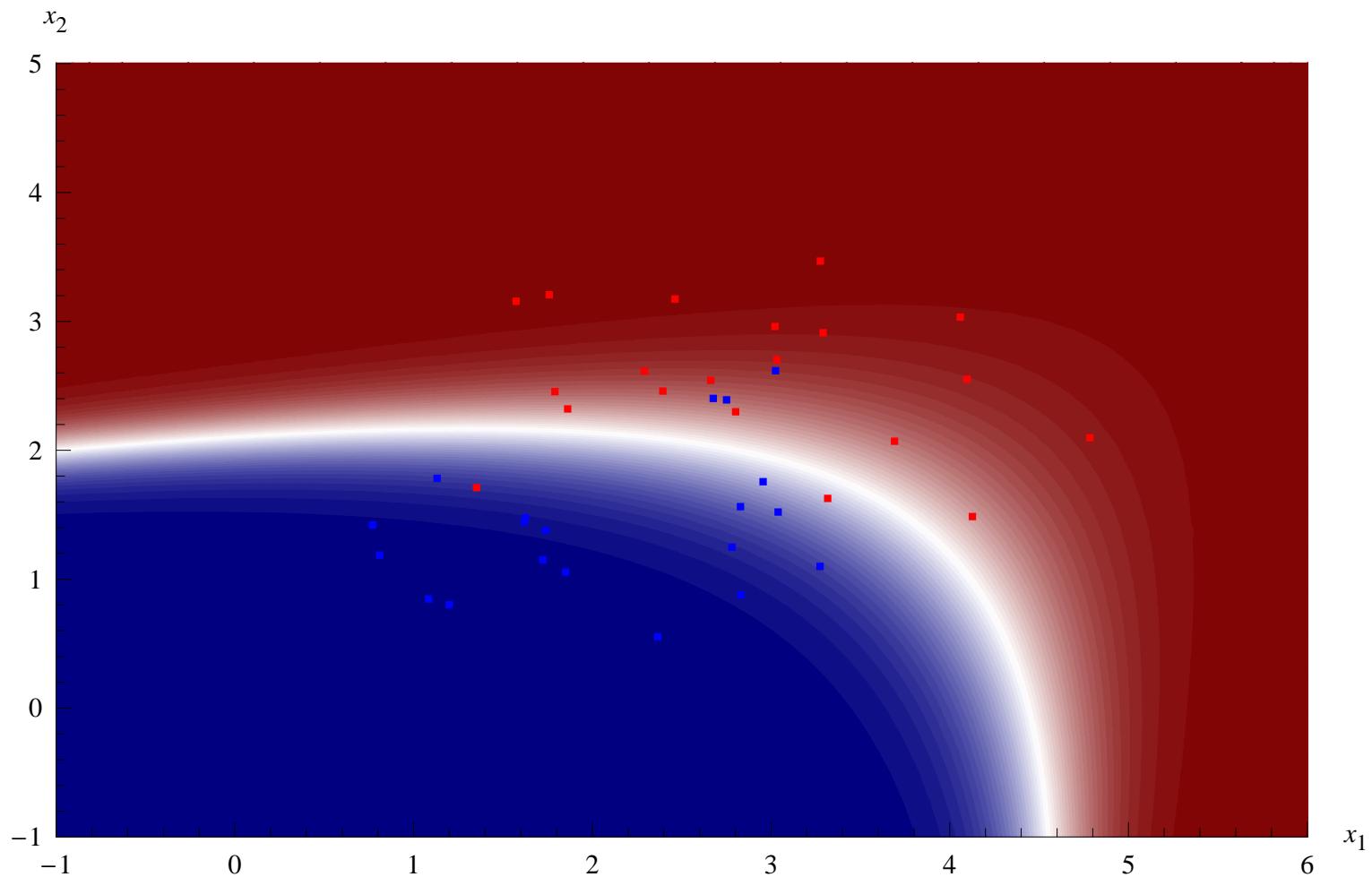
## 2-D Gaussian fit for class 1

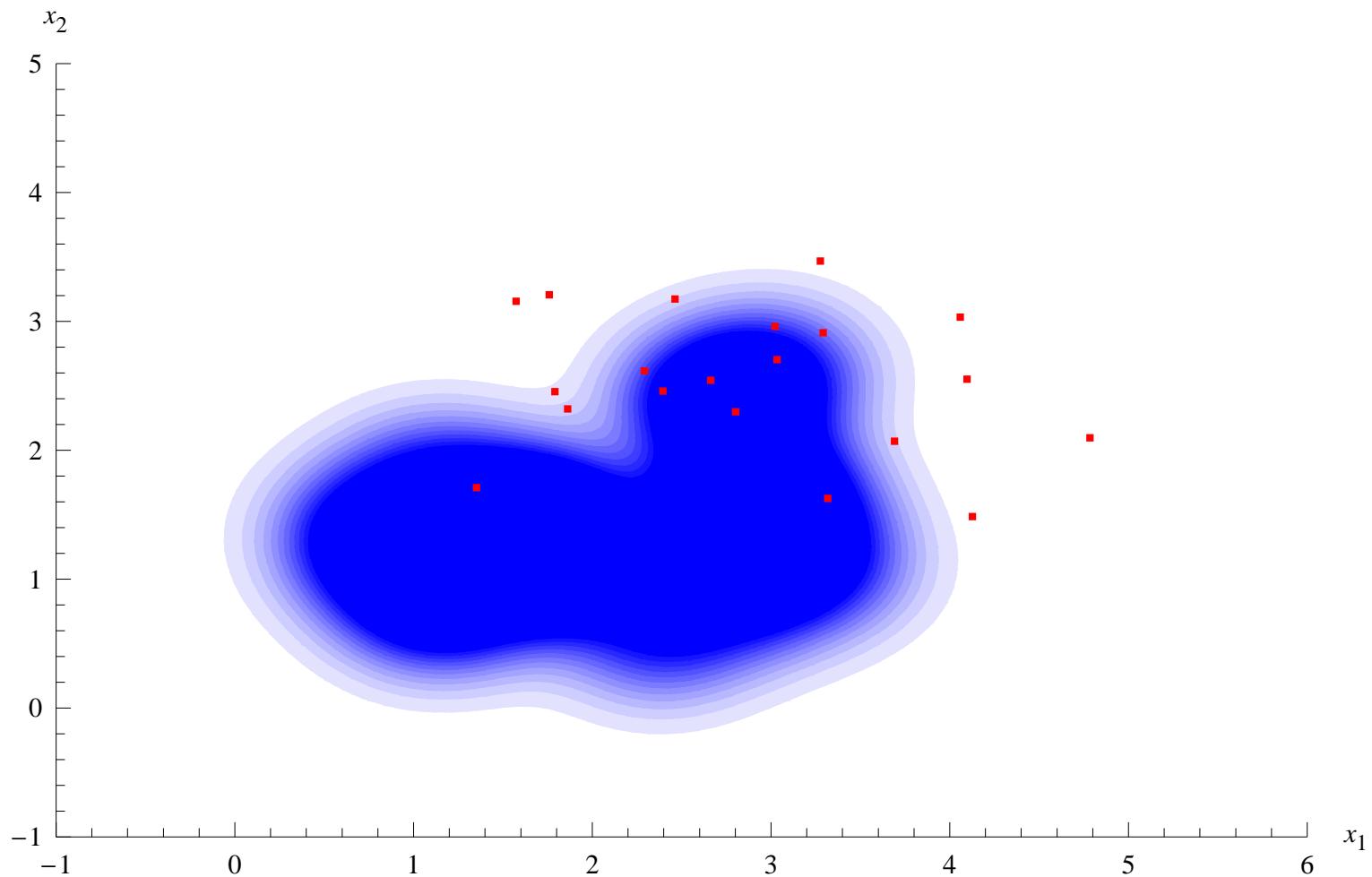


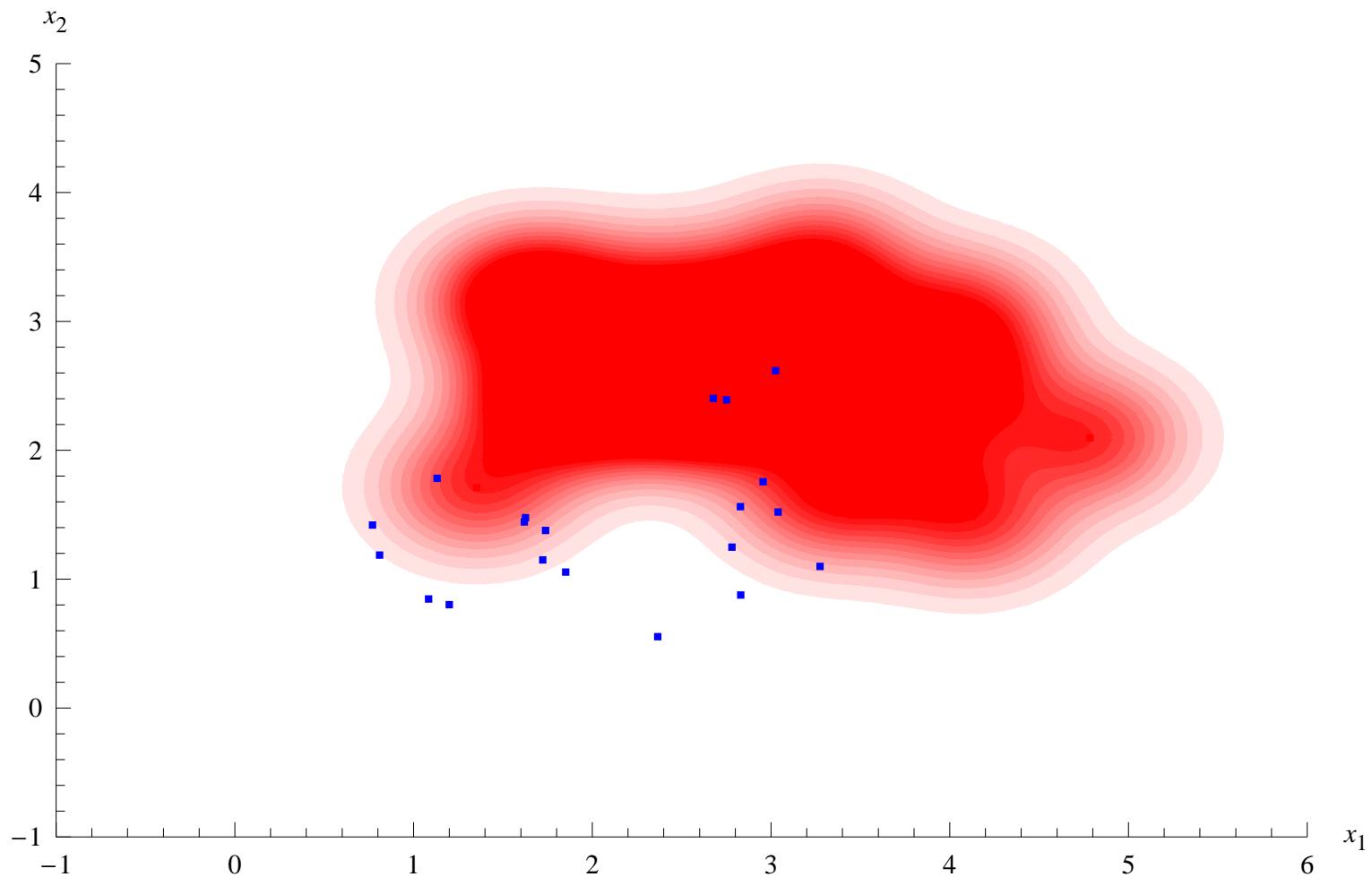
## 2-D Gaussian fit for class 2

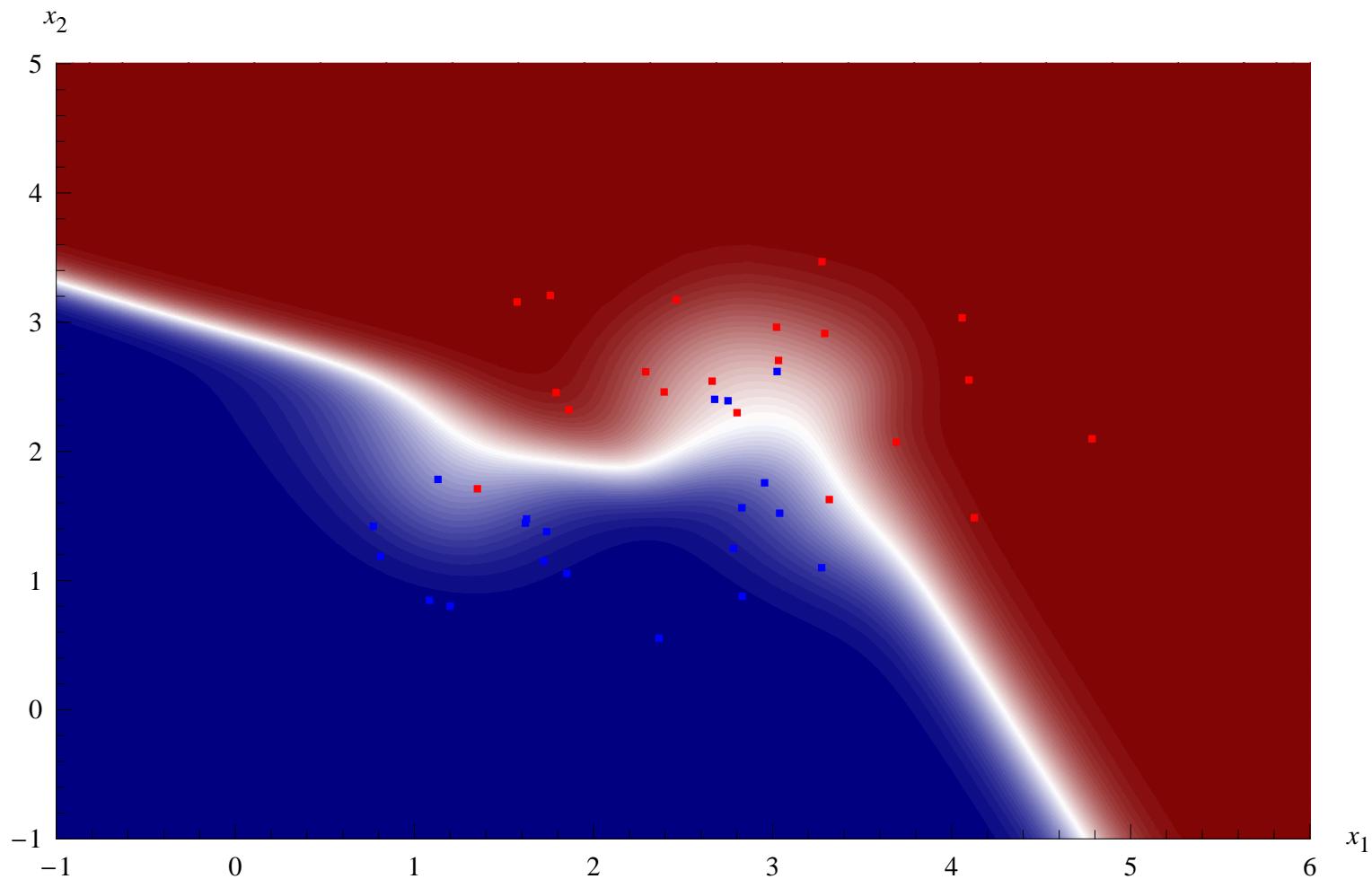


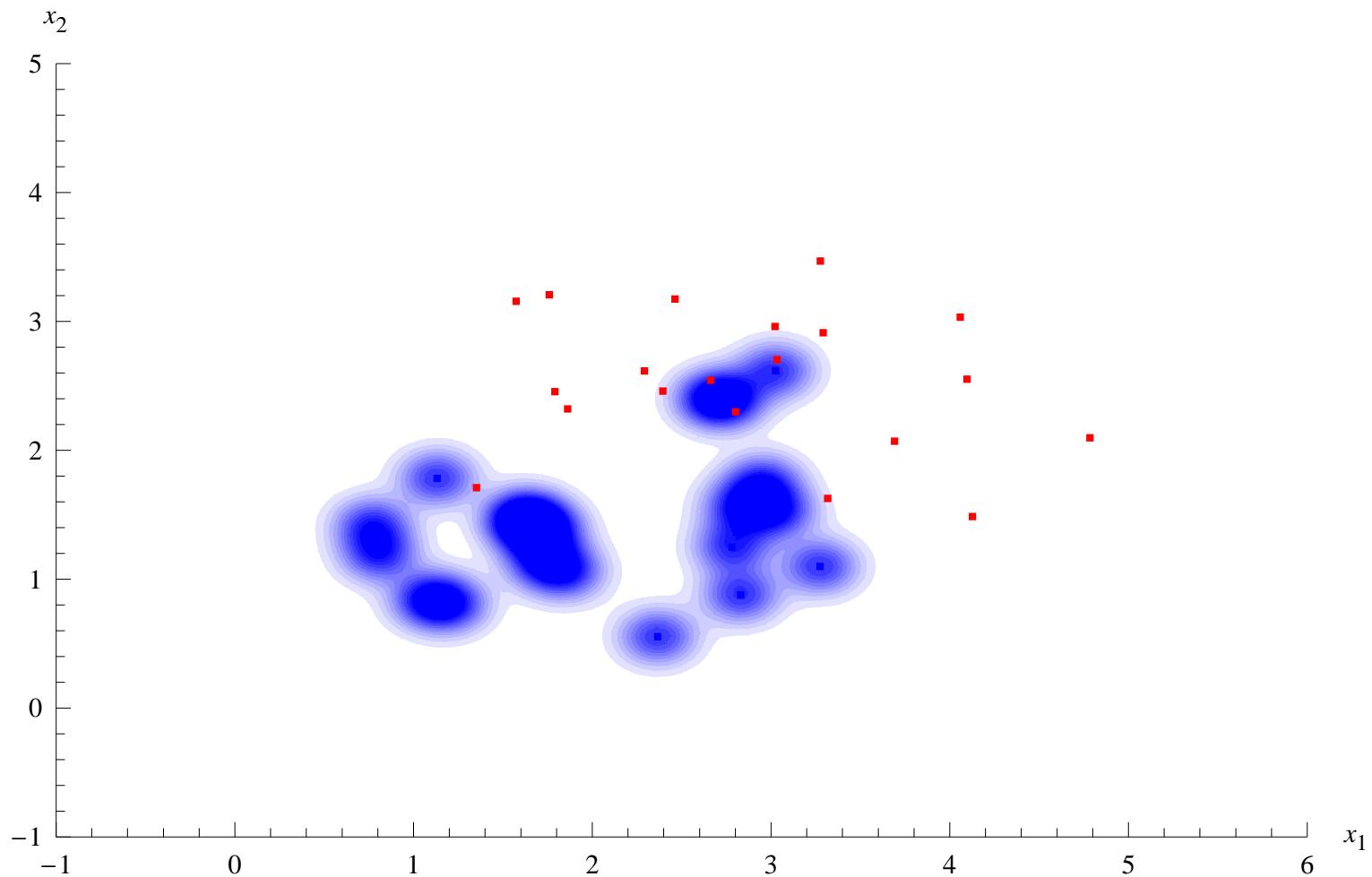
## Discriminant function with Gaussian fits

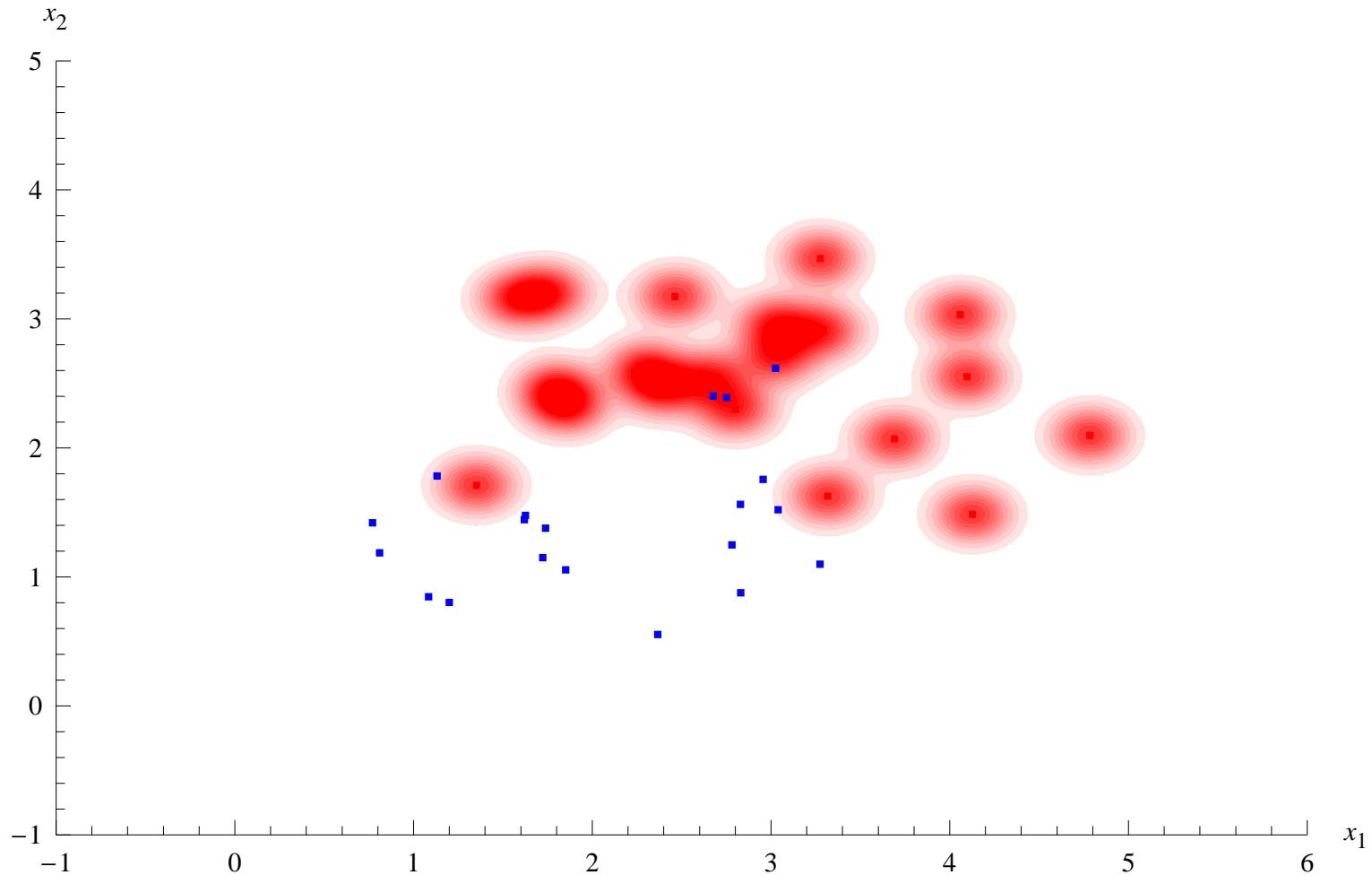


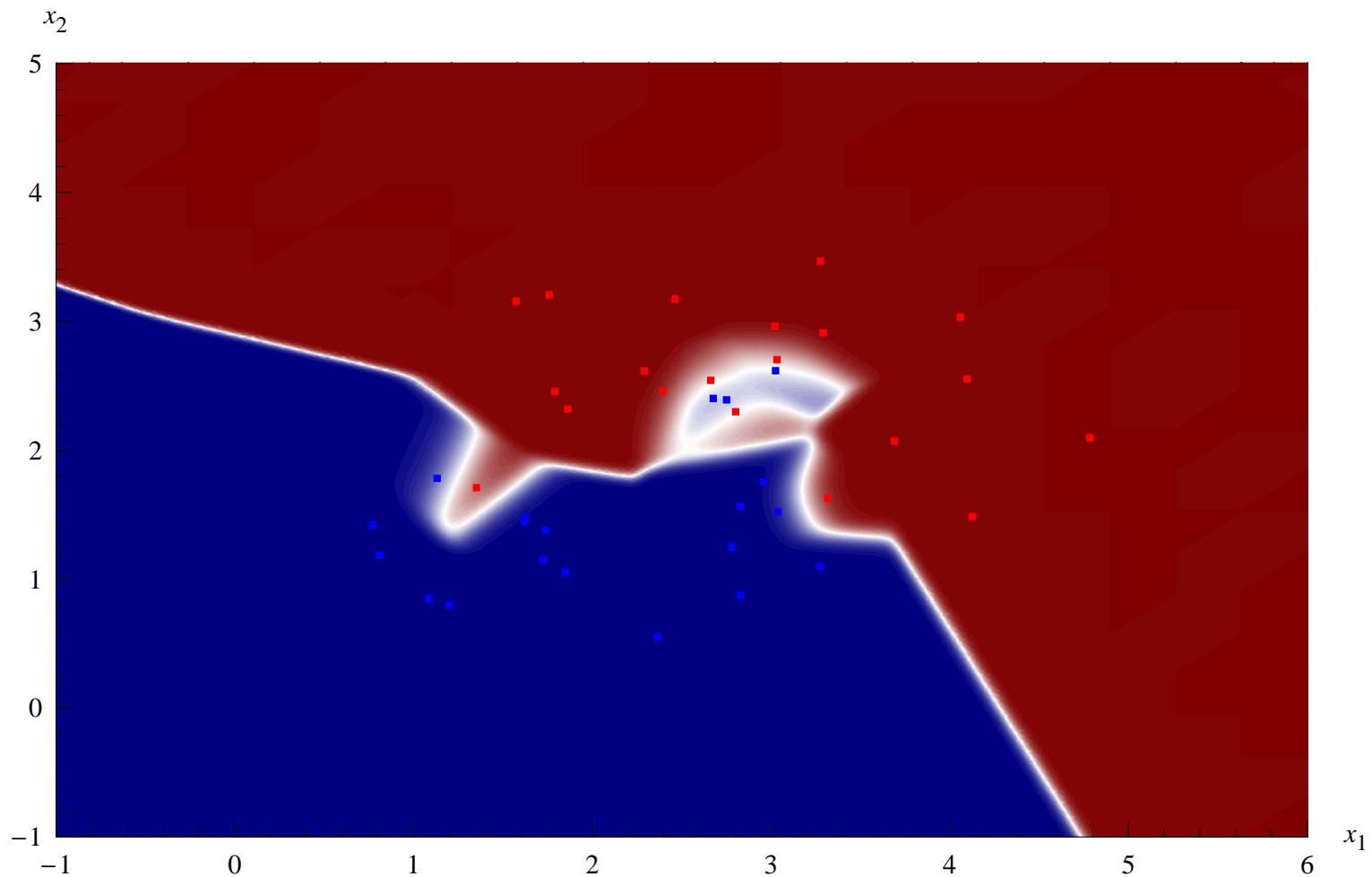
2-D Parzen fit for class 1,  $h = 0.12$ 

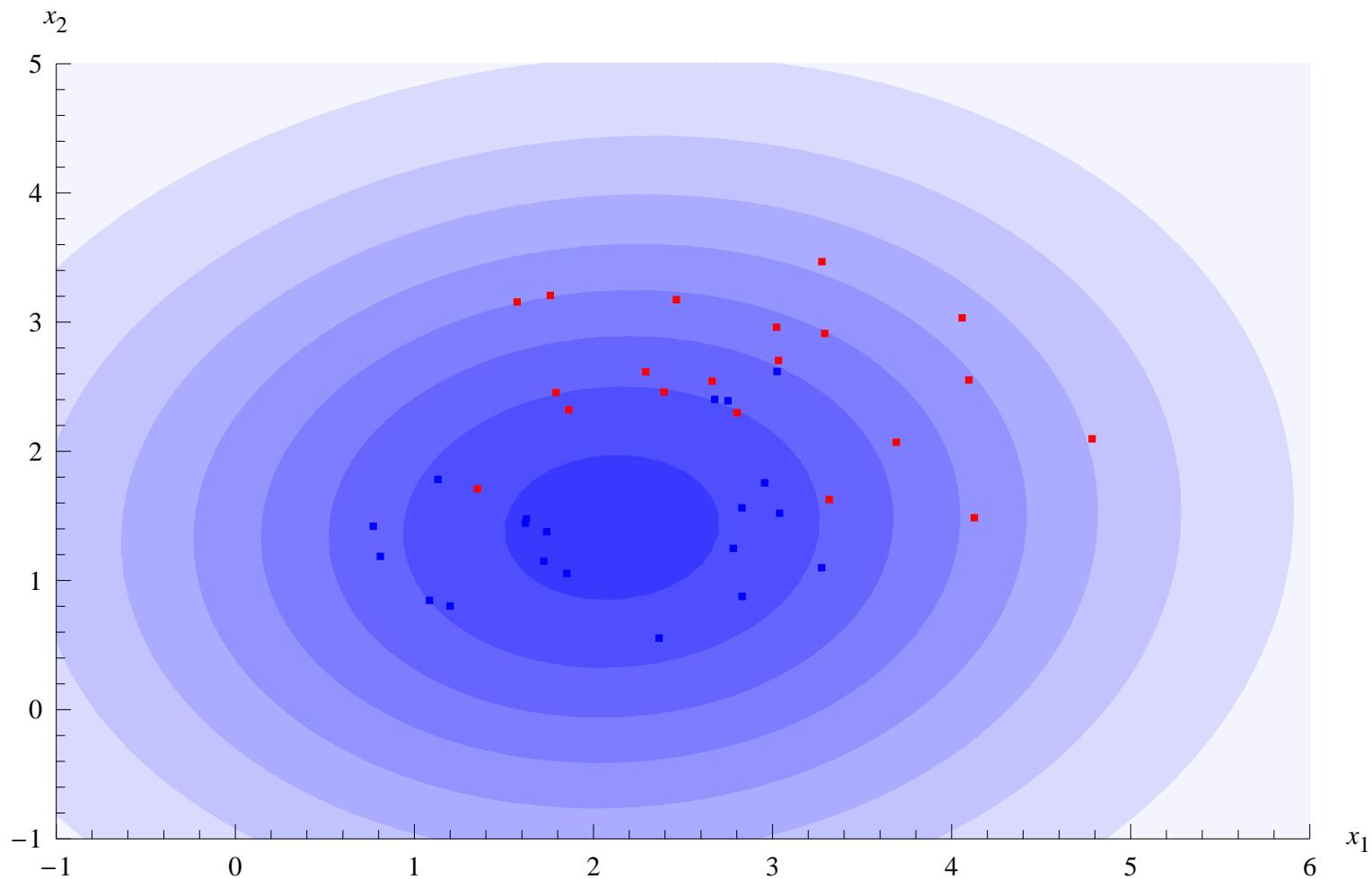
2-D Parzen fit for class 2,  $h = 0.12$ 

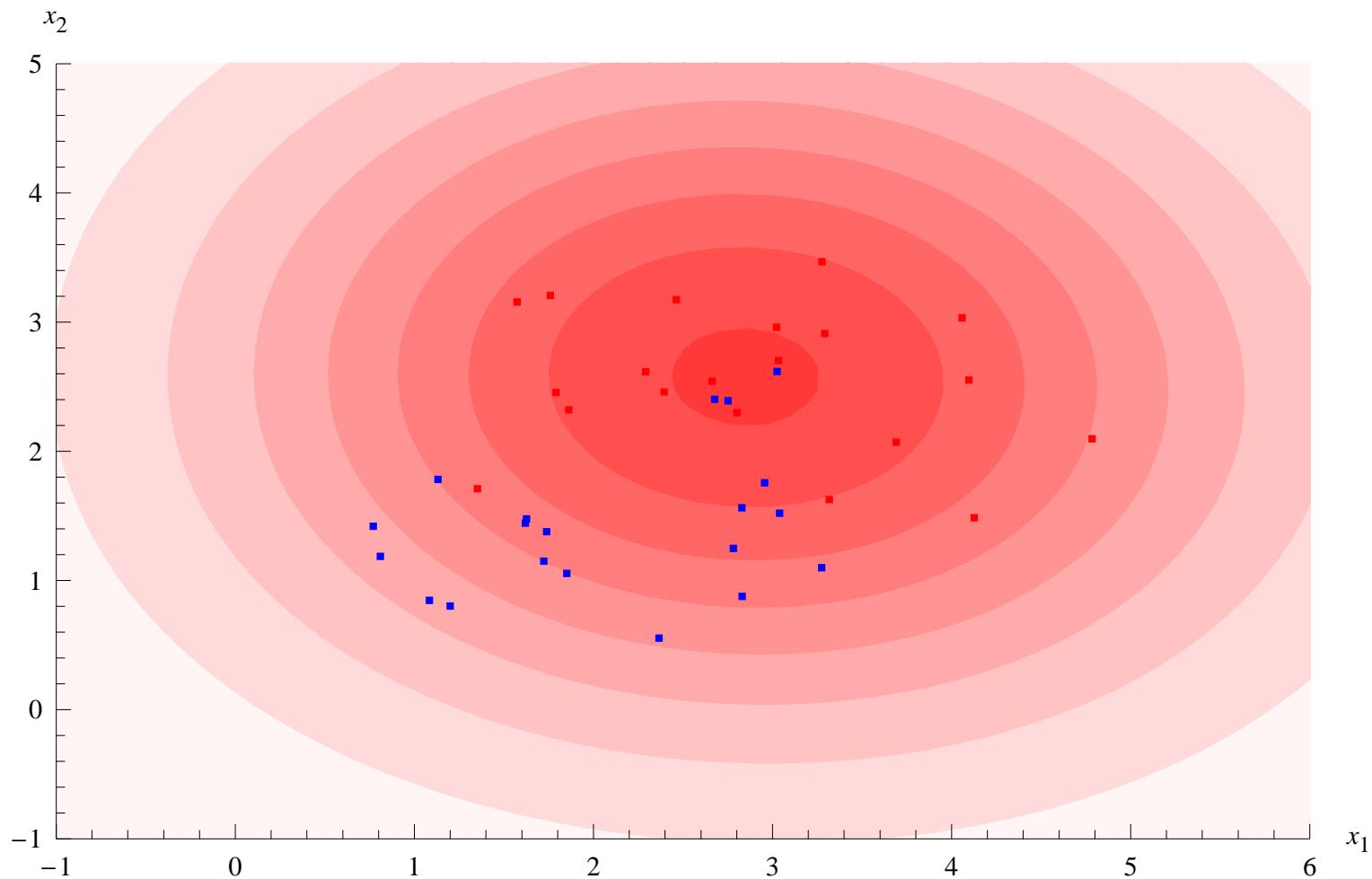
Discriminant function with Parzen fits,  $h = 0.12$ 

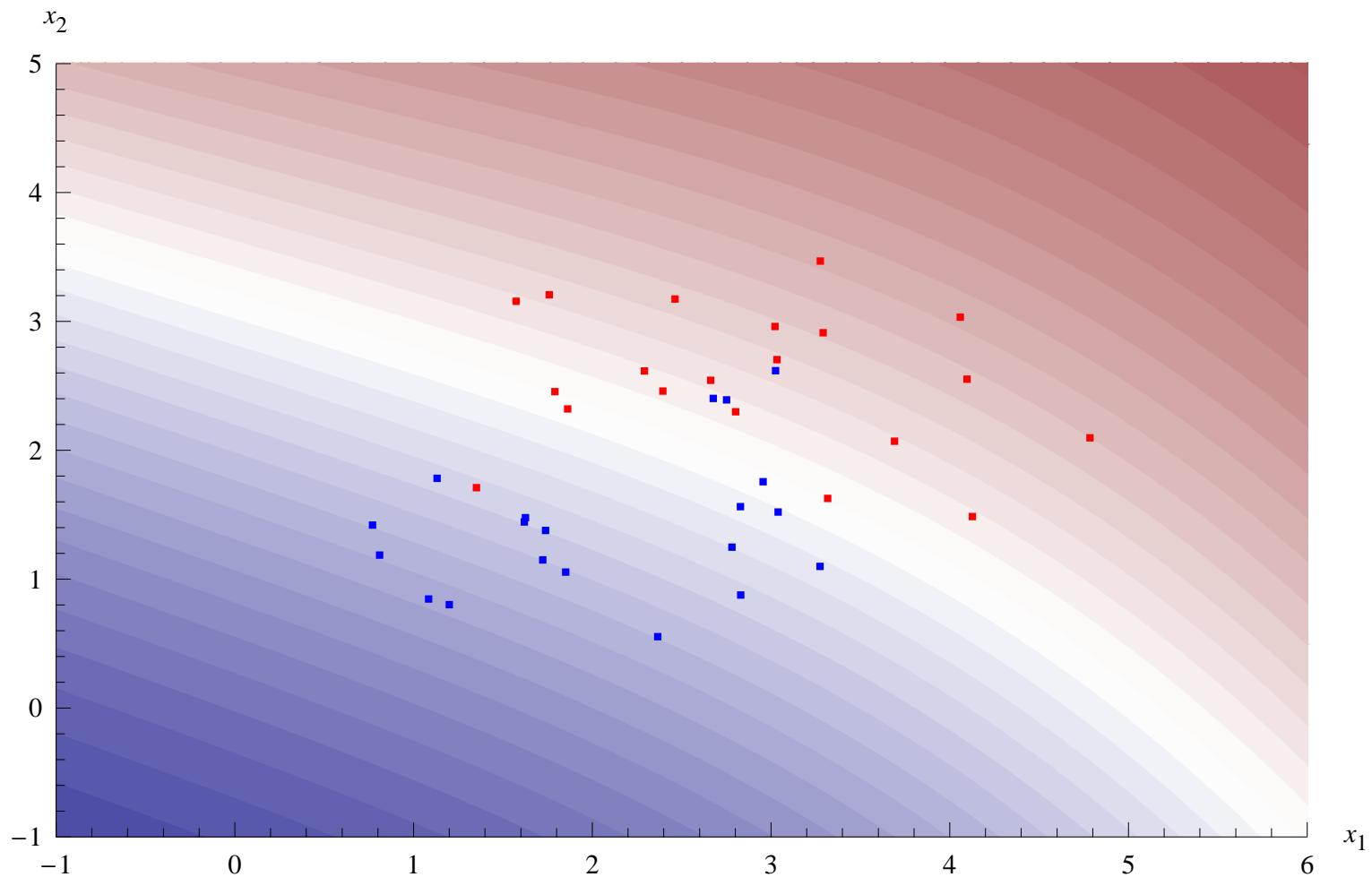
2-D Parzen fit for class 1,  $h = 0.02$ 

2-D Parzen fit for class 2,  $h = 0.02$ 

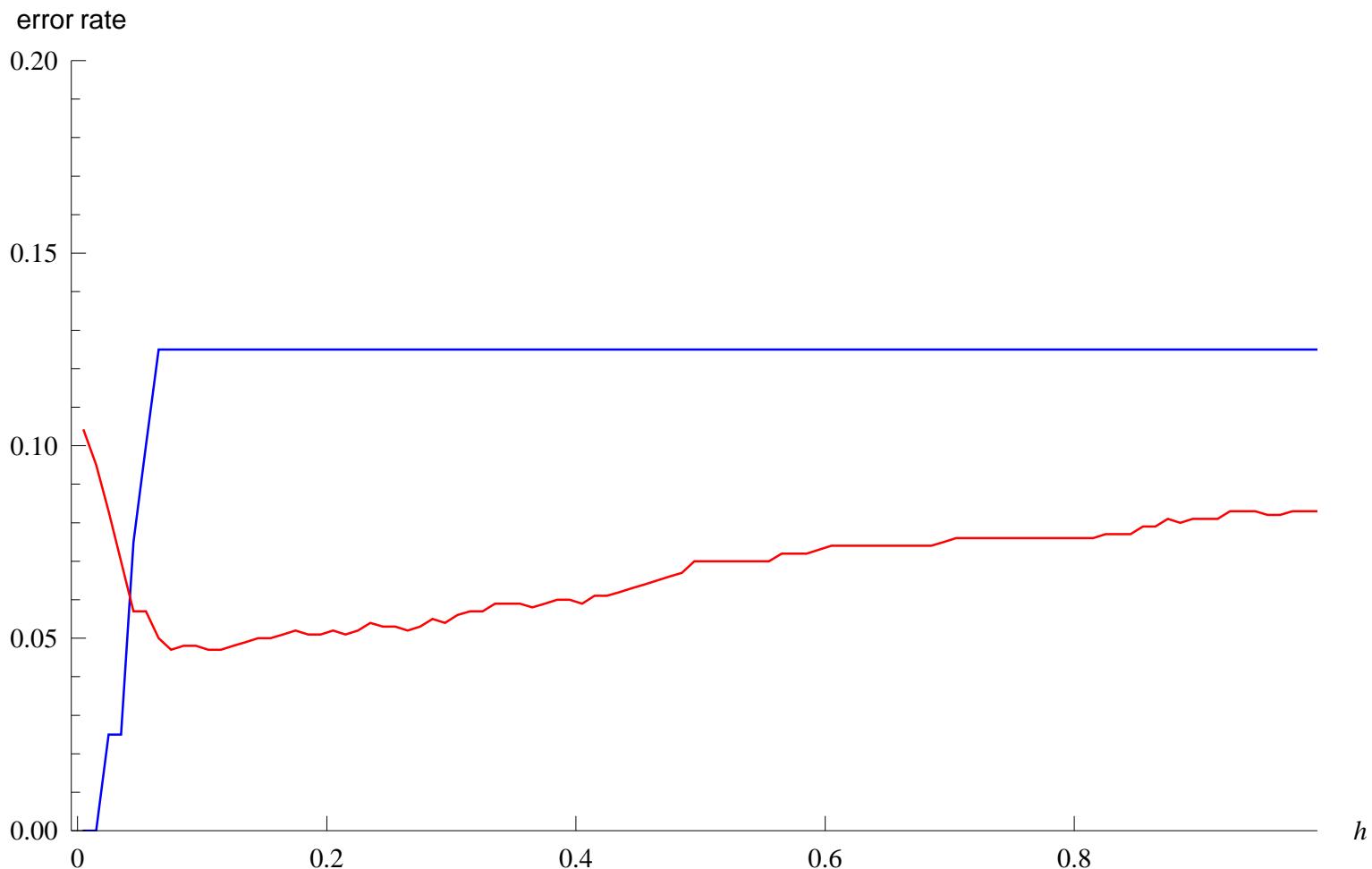
Discriminant function with Parzen fits,  $h = 0.02$ 

2-D Parzen fit for class 1,  $h = 3$ 

2-D Parzen fit for class 2,  $h = 3$ 

Discriminant function with Parzen fits,  $h = 3$ 

## Training and test error rates for Parzen fits with different bandwidths



# Non-parametric fitting

- Capacity control, regularization
  - trade-off between approximation error and estimation error
  - complexity grows with data size
  - no need to correctly guess the function class

# Curse of dimensionality

- Capacity/complexity control becomes a real issue in **high-dimensional** spaces
  - in a 10000-dimensional space a **linear** function has **10000 parameters!**
- Examples
  - images
  - music
  - language, text
  - bioinfo (genetics, proteomics)

# Machine learning problems

- Common goal: predict the future
  - make inferences on unknown future observations
- Non-supervised learning
  - density estimation  $p(\text{obs})$
  - clustering, dimensionality reduction, one-class learning
- Supervised learning
  - Classification :  $f(\text{obs}) \mapsto \text{category}$
  - Regression :  $f(\text{obs}) \mapsto \text{response}$

# The supervised learning model

- observation vector:  $\mathbf{x} \in \mathbb{R}^d$
- class label:  $y \in \{-1, 1\}$  (or  $y \in \{1, \dots, K\}$ )
- classifier:  $g : \mathbb{R}^d \rightarrow \{-1, 1\}$
- Discriminant function:  $f : \mathbb{R}^d \rightarrow [-1, 1]$

- $\longrightarrow$  classifier

$$g(\mathbf{x}) = \begin{cases} 1, & \text{if } f(\mathbf{x}) \geq 0, \\ -1, & \text{if } f(\mathbf{x}) < 0 \end{cases}$$

- decision boundary:  $\{\mathbf{x} : f(\mathbf{x}) = 0\}$

# The supervised learning model

- Learning by **experience**, with a **supervisor**

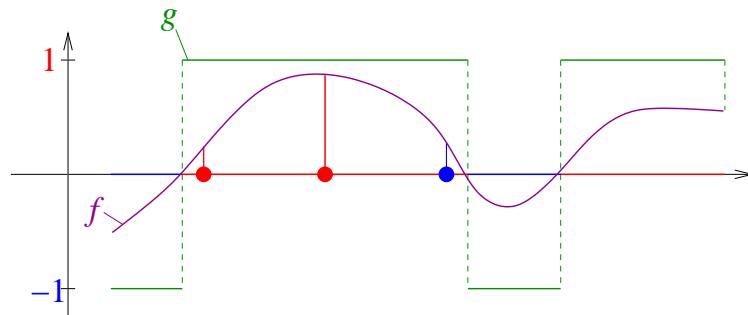
- **training set** :  $D_n = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)\}$
- **function class** :  $\mathcal{F}$
- **learning algorithm** :  $\text{ALGO} : (\mathbb{R}^d \times \{-1, 1\})^n \rightarrow \mathcal{F}$

$$\text{ALGO}(D_n) \mapsto f$$

- goal: small **generalization error**  $R(g) = P[g(\mathbf{X}) \neq Y] = P[f(\mathbf{X})Y \leq 0]$
- learning principle: minimize the **training error**

$$\widehat{R}(g) = \frac{1}{n} \sum_{i=1}^n \mathbb{I}\{g(\mathbf{x}_i) \neq y_i\}$$

# The supervised learning model

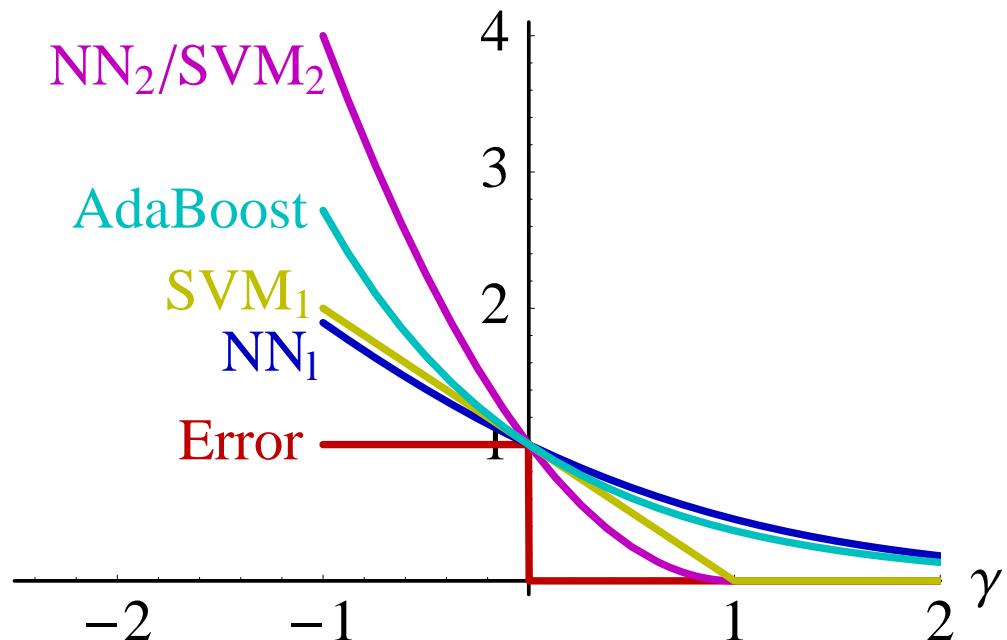


- Margin:  $\gamma = y \cdot f(\mathbf{x})$ 
  - classification error  $\equiv$  negative margin
  - the magnitude of a positive margin quantifies the confidence
  - learning principle: minimize a smooth loss function over the margin

$$\widehat{R}_\gamma(f) = \frac{1}{n} \sum_{i=1}^n L(f(\mathbf{x}_i) y_i)$$

# The supervised learning model

- Margin loss functions



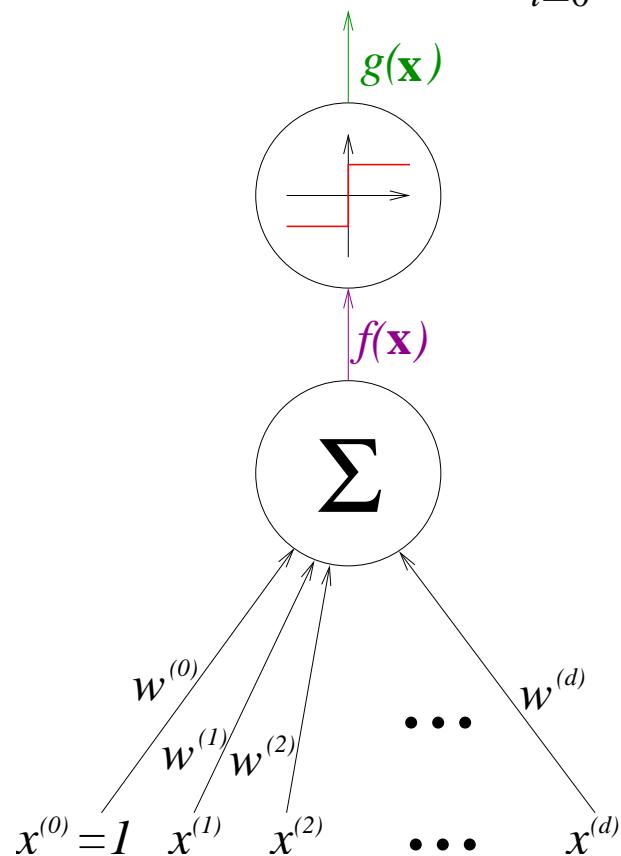
# History

- Algorithms

- 1958: Perceptron [Rosenblatt, '58] – [Minsky–Papert '69]
- 1986: Multilayer perceptrons (neural networks) and the back-propagation algorithm [Rumelhart–Hinton–Williams, '86]
- 1995: Support vector machines [Boser–Guyon–Vapnik, '92], [Cortes–Vapnik, '95]
- 1997: boosting, AdaBoost [Freund, '95], [Freund–Schapire, '97]

# The perceptron

- Linear discriminant functions:  $f(\mathbf{x}) = \sum_{i=0}^d w^{(i)} \cdot x^{(i)} = \langle \mathbf{w}, \mathbf{x} \rangle$



# The perceptron

- Linear discriminant functions:  $f(\mathbf{x}) = \sum_{i=0}^d w^{(i)} \cdot x^{(i)} = \langle \mathbf{w}, \mathbf{x} \rangle$
- Algorithm
  - simple iterative error correction
  - convergence if the data is linearly separable
  - oscillation for linearly non-separable data

# Generalized linear discriminant functions

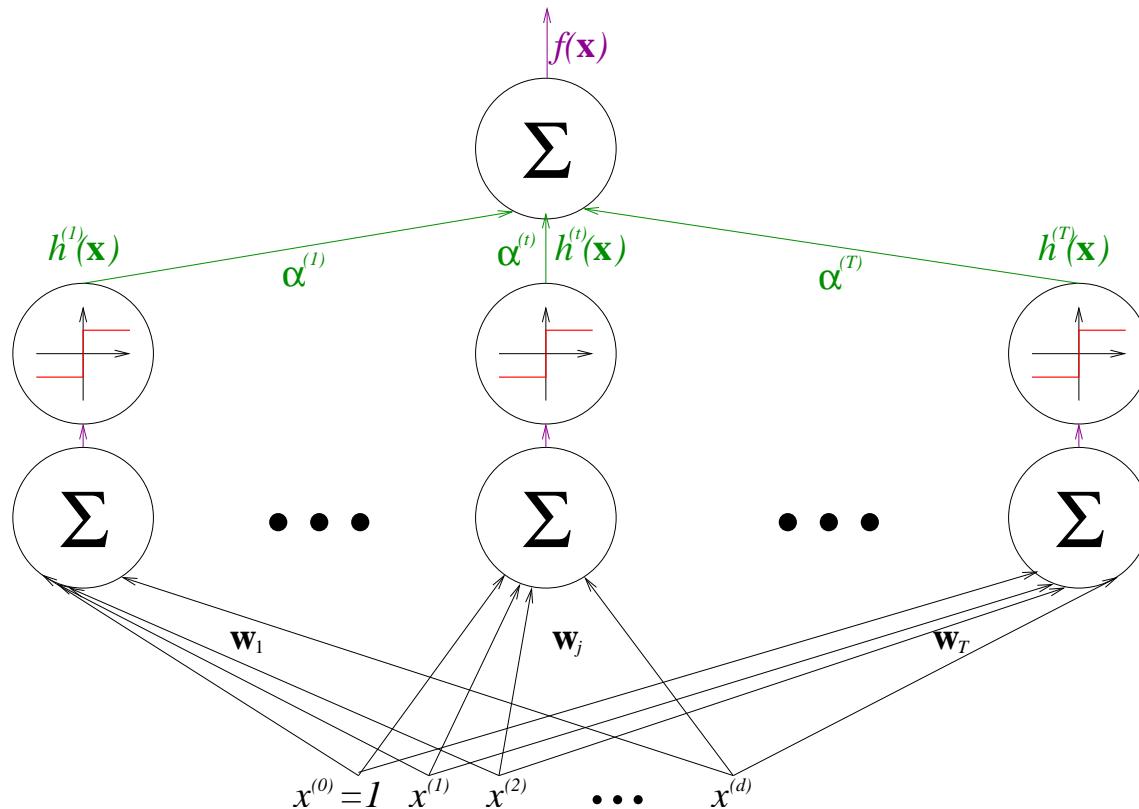
- Model:

$$f(\mathbf{x}) = \sum_{j=1}^N \alpha^{(j)} h^{(j)}(\mathbf{x})$$

- $h^{(j)} : \mathbb{R}^d \rightarrow [-1, 1]$ 
  - simple classifiers/discriminant functions, features, experts
- $\alpha^{(j)} \in \mathbb{R}^+$ 
  - weight of the expert  $h^{(j)}$  in the final vote

# Multilayer perceptron (neural net)

- Model:  $f(\mathbf{x}) = \sum_{j=1}^N \alpha^{(j)} \sigma(\langle \mathbf{w}_j, \mathbf{x} \rangle)$



# Multilayer perceptron (neural net)

- Model:  $f(\mathbf{x}) = \sum_{j=1}^N \alpha^{(j)} \sigma(\langle \mathbf{w}_j, \mathbf{x} \rangle)$
- Algorithm:
  - gradient descent optimization
  - differentiable error functions → margin loss
  - differentiable activation function  $\sigma$ : the sigmoid
  - local minima, “engineering”, parameters to tune

# Support vector machine

- Model:

$$f(\mathbf{x}) = \sum_{j \in I_{\text{sv}}} \alpha^{(j)} y_j K(\mathbf{x}_j, \mathbf{x})$$

- $I_{\text{sv}} \subset \{1, \dots, n\}$  is the set of support vectors
- $K(\cdot, \cdot)$  is a similarity function (kernel)
- goal: classification boundary equidistant from classes
- “sophisticated nearest neighbor”
- slow and complex quadratic programming optimization
- turn-key algorithm, very limited parameter tuning

# Support vector machine

- Model:

$$f(\mathbf{x}) = \sum_{j \in I_{\text{sv}}} \alpha^{(j)} y_j K(\mathbf{x}_j, \mathbf{x})$$

- Kernel:

- $K(\mathbf{x}, \mathbf{x}') = \langle \mathbf{x}, \mathbf{x}' \rangle \longrightarrow f(\mathbf{x}) \text{ is linear}$

- $K(\mathbf{x}, \mathbf{x}') = (1 + \langle \mathbf{x}, \mathbf{x}' \rangle)^d \longrightarrow f(\mathbf{x}) \text{ is a polynomial of degree } d$

- $K(\mathbf{x}, \mathbf{x}') = \exp(-1/h \|\mathbf{x} - \mathbf{x}'\|^2) \longrightarrow f(\mathbf{x}) \text{ is a Gaussian mixture } (\rightarrow \text{Parzen})$

# AdaBoost

- Model:

$$f(\mathbf{x}) = \sum_{j=1}^N \alpha^{(j)} h^{(j)}(\mathbf{x})$$

- no restriction on the form of  $h^{(j)}(\mathbf{x})$
- often “decision stumps” :

$$h_{\ell,\theta}(\mathbf{x}) = \begin{cases} +1 & \text{if } \mathbf{x}^{(\ell)} \geq \theta, \\ -1 & \text{otherwise} \end{cases}$$

where  $\mathbf{x} = (x^{(1)}, \dots, x^{(d)})$

# AdaBoost

- Intuitive elementary algorithm
  - add one expert at a time
  - add the best expert on training points mis-classified by previous experts
  - weight of the expert chosen proportionally to its correctness

# AdaBoost

- Weighting over the training points  $w_1, \dots, w_n$

- normalized:  $\sum_{i=1}^n w_i = 1$
- initialized uniformly :  $\mathbf{w} = (1/n, \dots, 1/n)$
- if  $\mathbf{x}_i$  is mis-classified by  $h^{(j)}$ , increase  $w_i$
- otherwise, decrease  $w_i$
- “difficult” training points get larger weights gradually

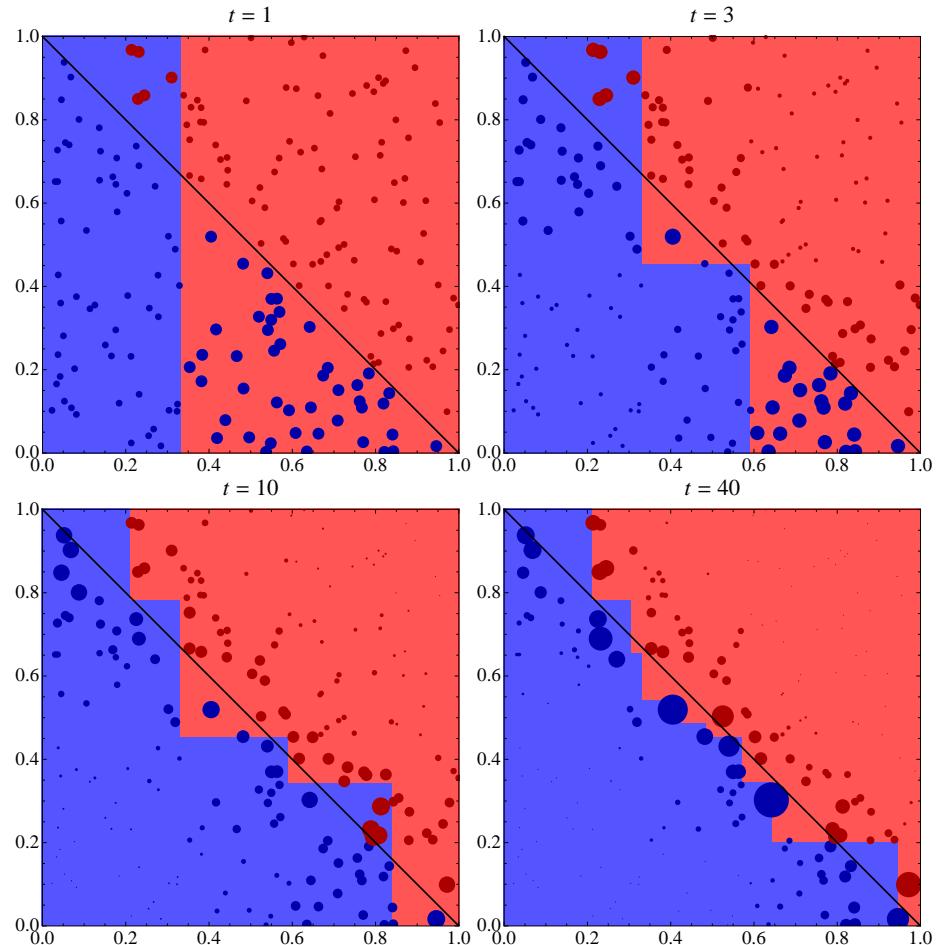
# AdaBoost [Freund – Schapire '97]

**ADABoost**( $D_n = \{(\mathbf{x}_i, y_i)\}_{i=1}^n, \text{BASE}(\cdot, \cdot), T$ )

```

1    $\mathbf{w}^{(1)} \leftarrow (1/n, \dots, 1/n)$             $\triangleright$  initial weights
2   for  $t \leftarrow 1$  to  $T$ 
3      $h^{(t)} \leftarrow \text{BASE}(D_n, \mathbf{w}^{(t)})$        $\triangleright$  calling the base learner
4      $\gamma^{(t)} \leftarrow \sum_{i=1}^n w_i^{(t)} h^{(t)}(\mathbf{x}_i) y_i$      $\triangleright$  edge =  $1 - 2 \times \text{error}$ 
5      $\alpha^{(t)} \leftarrow \frac{1}{2} \ln \left( \frac{1 + \gamma^{(t)}}{1 - \gamma^{(t)}} \right)$      $\triangleright$  coefficient of  $h^{(t)}$ 
6     for  $i \leftarrow 1$  to  $n$             $\triangleright$  re-weighting the points
7       if  $h^{(t)}(\mathbf{x}_i) \neq y_i$  then
8          $w_i^{(t+1)} \leftarrow w_i^{(t)} \frac{1}{1 - \gamma^{(t)}}$ 
9       else
10         $w_i^{(t+1)} \leftarrow w_i^{(t)} \frac{1}{1 + \gamma^{(t)}}$ 
11   return  $f^{(T)}(\cdot) = \sum_{t=1}^T \alpha^{(t)} h^{(t)}(\cdot)$ 
```

# AdaBoost



# AdaBoost

- Algorithm

- extremely simple learning, limited parameter tuning
- fast
- intuitive interpretation: weighted vote of experts
- the choice of the pool of experts captures the a-priori knowledge
- no restriction on the form of the experts
- label noise can be a problem

## multiboost.org

- Multi-class multi-label boosting software
  - based on ADABOOST.MH [Schapire-Singer '99]
  - started by [Norman Casagrande](#) (M.Sc. student in Montreal, now with last.fm)
  - multi-platform C++
  - command-line UI, easy-to-use for a non-expert
  - adapting to a new data type is easy for an advanced user
  - tons of features
  - scales nicely

- Plan

- going **beyond classification**: regression, ranking, collaborative filtering, reinforcement learning
- **technical** improvements: multicore, GPU, grid, memory handling, etc.
- **redesign**: orthogonal features → templates
- implement a **software development cycle** (tests, etc.), can be tricky to balance between research and production

- Plan for F-D.

- get acquainted with Machine Learning through understanding the code (of course, we'll be there)
- first concrete task: port it to multi-core (we have good understanding how) than to GPU (we have a vague understanding, more challenging to F-D.)
- gradually implement a software development cycle
- redesign