Particle Physics: The Standard Model

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- charged leptons and photon
- quarks and gluon
- neutrinos
- $W^\pm$, Z°
- H

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

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History

- 1896 Henri Becquerel: β decay
- 1899 Ernest Rutherford: distinguishes α and β rays
- 1914 James Chadwick: the β decay has a continuous spectrum
- 1930 Wolfgang Pauli: postulates the neutrino (ballroom)
- 1933 Enrico Fermi: contact interaction
- 1953 Frederick Reines: $\bar{\nu}_{e_L} + p \rightarrow n + e^+$
- 1956 Lee, Yang, Wu, Garwin et al: Parity violation
- 1961 Glashow, Salam, Weinberg, Higgs, EBKGH
- 1973 Lagarrigue, Faissner: neutral currents (Z° t-channel)
- 1984 Rubbia, van der Meer : $W^\pm$, Z°
- 2012 discovery of the Higgs?

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$$n \rightarrow p + e^- + \bar{\nu}_{e_L}$$
$$T_{fi} \sim G(\bar{p}\gamma^\mu n)(\bar{e}\gamma_\mu \nu)$$
$$\sim G(\bar{p}\gamma^\mu n)\frac{1}{q^2 - m^2}(\bar{e}\gamma_\mu \nu)$$

- QED: $m^2 = 0 \rightarrow \frac{1}{q^2}$
- if $m^2 \gg q^2$ neglect q^2
- constant in momentum space $\rightarrow$ Dirac function in space-time: contact interaction
- $G \sim 10^{-5} GeV^{-2} \ll \alpha_{EM}$

Currents

$\bar{\psi}\psi$	*scalar*	*S*
$\bar{\psi}\gamma^\mu\psi$	*vector*	*V*
$\bar{\psi}\sigma^{\mu\nu}\psi$	*tensor*	*T*
$\bar{\psi}\gamma^\mu\gamma_5\psi$	*axial vector*	*A*
$\bar{\psi}\gamma_5\psi$	*pseudo scalar*	*PS*

QED: V
EW: $V - A$
$V - A$ violates parity (experiment)
$V - A$ quark/lepton level, not hadron level

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Chirality

Chirality is the handed-ness of the particle:

$$\begin{aligned}\psi &= P_L\psi + P_R\psi \\ &= \psi_L + \psi_R\end{aligned}$$

Definitions

- Helicity: $\vec{\sigma} \cdot \vec{p}$
- $m = 0$: Helicity = chirality
- $m = 0$: ψ and $\gamma_5\psi$ solve DIRAC

Weyl basis

$$\begin{aligned}\gamma_5 &= i\gamma^0\gamma^1\gamma^2\gamma^3 \\ \gamma_5^2 &= 1 \\ 0 &= \gamma_5\gamma^\mu + \gamma^\mu\gamma_5 \\ \gamma_5 &= \begin{pmatrix} -1_2 & 0 \\ 0 & 1_2 \end{pmatrix} \\ \gamma^0 &= \begin{pmatrix} 0 & 1_2 \\ 1_2 & 0 \end{pmatrix} \\ \gamma^{0\dagger} &= \gamma^0 \\ \gamma_5{}^\dagger &= \gamma_5\end{aligned}$$

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Left and Right chirality

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\begin{aligned}
P_L &= \tfrac{1}{2}(1-\gamma_5) \\
P_R &= \tfrac{1}{2}(1+\gamma_5) \\
P_L + P_R &= 1 \\
P_L^2 &= \tfrac{1}{2}(1-\gamma_5)\tfrac{1}{2}(1-\gamma_5) \\
&= \tfrac{1}{4}(1-\gamma_5)(1-\gamma_5) \\
&= \tfrac{1}{4}(1-\gamma_5-\gamma_5+\gamma_5^2) \\
&= \tfrac{1}{4}(1-\gamma_5-\gamma_5+1) \\
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P_L P_R &= \tfrac{1}{2}(1-\gamma_5)\tfrac{1}{2}(1+\gamma_5) \\
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- ψ_R right-(left)handed (anti-)particle
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EM current

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\begin{aligned}
j^\mu &= -e\bar{\psi}\gamma^\mu\psi \\
&= -e\bar{\psi}(P_L + P_R)\gamma^\mu(P_L + P_R)\psi \\
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Perfect symmetry under parity: $\vec{p} \rightarrow -\vec{p}$

- weak interaction: Left is not equal to Right
- use vector bosons
- ask for local gauge invariance
- remember that $U(1)_{EM}$ is QED and was extremely successful
- unify electromagnetic and weak interactions
- $SU(2) \times U(1)$
- $SU(2)$: three generators (gauge bosons)
- $U(1)$: one generators (gauge boson)
- $SU(2)$ vector bosons must be massive (Fermi-contact interaction)
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$GaugeGroup$	$SU(3)$
$Gaugebosons$	8
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$Field-Tensor$	$G^a_{\mu\nu}=\partial_\mu G^a_\nu(\mathbf{x})-\partial_\nu G^a_\mu(\mathbf{x})-g_S f_{abc}G^b_\mu(\mathbf{x})G^c_\nu(\mathbf{x})$
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Free Lagrangian Gauge Fields $SU(2)_L \times U(1)_Y$

$$\begin{aligned} \mathcal{L}_0 &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} \\ &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} Tr(W_{\mu\nu} W^{\mu\nu}) \\ W_{\mu\nu} &= W^a_{\mu\nu} \frac{T_a}{2} \end{aligned}$$

Organize the Dirac Fields

$$\begin{aligned} \mathrm{e_L}(\mathbf{x}) &= P_L \mathrm{e}(\mathbf{x}) \\ \overline{\mathrm{e_L}}(\mathbf{x}) &= (\gamma^0 P_L \mathrm{e}(\mathbf{x}))^\dagger \\ \ell(\mathbf{x}) &= \begin{pmatrix} \nu_{\mathrm{e_L}}(\mathbf{x}) \\ \mathrm{e_L}(\mathbf{x}) \\ \mathrm{e_R}(\mathbf{x}) \end{pmatrix} \end{aligned}$$

No right-handed neutrinos

Define the weak Hypercharge

hypercharge left $\neq$ right:

$$Y = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & y_R \end{pmatrix}$$

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Minimal Substitution

$$\partial_\mu \rightarrow \partial_\mu + ig_2 W^a_\mu \frac{T_a}{2} + ig_1 B_\mu Y$$

Interaction Lagrangian

$$\mathcal{L}' = -\bar{\ell}\gamma^\mu (g_2 W^a_\mu \frac{T_a}{2} + g_1 B_\mu Y)\ell$$

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- use form of Pauli matrices and $W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} \mp i W^2_{\mu})$

Investigate the Interaction

$$\begin{aligned}
\mathcal{L}' &= -\bar{\ell}\gamma^{\mu}(g_2 W^a_{\mu}\frac{T_a}{2} + g_1 B_{\mu} Y)\ell \\
&= -\bar{\ell}\gamma^{\mu}[g_2(W^1_{\mu}\frac{T_1}{2} + W^2_{\mu}\frac{T_2}{2} + W^3_{\mu}\frac{T_3}{2}) + g_1 B_{\mu} Y]\ell \\
&= -g_2(\overline{\nu_{e_L}}, \overline{e_L})\gamma^{\mu}(W^1_{\mu}\frac{\tau_1}{2} + W^2_{\mu}\frac{\tau_2}{2} + W^3_{\mu}\frac{\tau_3}{2})\begin{pmatrix}\nu_{e_L} \\ e_L\end{pmatrix} \\
&\quad + \frac{1}{2}g_1\overline{\nu_{e_L}}\gamma^{\mu}B_{\mu}\nu_{e_L} + \frac{1}{2}g_1\overline{e_L}\gamma^{\mu}B_{\mu}e_L - y_R g_1\overline{e_R}\gamma^{\mu}B_{\mu}e_R \\
&= -\frac{g_2}{\sqrt{2}}(W^+_{\mu}\overline{\nu_{e_L}}\gamma^{\mu}e_L + W^-_{\mu}\overline{e_L}\gamma^{\mu}\nu_{e_L}) \\
&\quad -\frac{1}{2}(g_2 W^3_{\mu} - g_1 B_{\mu})\overline{\nu_{e_L}}\gamma^{\mu}\nu_{e_L} \\
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&= -g_2(\overline{\nu_{e_L}}, \overline{e_L})\gamma^\mu(W_\mu^1 \frac{\tau_1}{2} + W_\mu^2 \frac{\tau_2}{2} + W_\mu^3 \frac{\tau_3}{2})\begin{pmatrix}\nu_{e_L} \\ e_L\end{pmatrix} \\
&\quad + \frac{1}{2} g_1 \overline{\nu_{e_L}}\gamma^\mu B_\mu \nu_{e_L} + \frac{1}{2} g_1 \overline{e_L}\gamma^\mu B_\mu e_L - y_R g_1 \overline{e_R}\gamma^\mu B_\mu e_R \\
&= -\frac{g_2}{\sqrt{2}}(W_\mu^+ \overline{\nu_{e_L}}\gamma^\mu e_L + W_\mu^- \overline{e_L}\gamma^\mu \nu_{e_L}) \\
&\quad -\frac{1}{2}(g_2 W_\mu^3 - g_1 B_\mu)\overline{\nu_{e_L}}\gamma^\mu \nu_{e_L} \\
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&= -g_2(\overline{\nu_{\mathrm{e_L}}}, \overline{\mathrm{e_L}})\gamma^\mu(W^1_\mu \tfrac{\tau_1}{2} + W^2_\mu \tfrac{\tau_2}{2} + W^3_\mu \tfrac{\tau_3}{2}) \begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \\
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&= -\bar{\ell}\gamma^\mu [g_2 (W^1_\mu \frac{T_1}{2} + W^2_\mu \frac{T_2}{2} + W^3_\mu \frac{T_3}{2}) + g_1 B_\mu Y]\ell \\
&= -g_2 (\overline{\nu_{\mathrm{e_L}}}, \overline{\mathrm{e_L}})\gamma^\mu (W^1_\mu \frac{\tau_1}{2} + W^2_\mu \frac{\tau_2}{2} + W^3_\mu \frac{\tau_3}{2}) \begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \\
&\quad + \frac{1}{2} g_1 \overline{\nu_{\mathrm{e_L}}}\gamma^\mu B_\mu \nu_{\mathrm{e_L}} + \frac{1}{2} g_1 \overline{\mathrm{e_L}}\gamma^\mu B_\mu \mathrm{e_L} - y_R g_1 \overline{\mathrm{e_R}}\gamma^\mu B_\mu \mathrm{e_R} \\
&= -\frac{g_2}{\sqrt{2}} (W^+_\mu \overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W^-_\mu \overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}) \\
&\quad - \frac{1}{2}(g_2 W^3_\mu - g_1 B_\mu)\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} \\
&\quad + \frac{1}{2}(g_2 W^3_\mu + g_1 B_\mu)\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - y_R g_1 B_\mu \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R}
\end{aligned}$$

Identify the gauge bosons

- charged bosons: $W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} \mp iW^2_{\mu})$
- neutral boson: $Z_{\mu} = \frac{1}{\sqrt{g_1^2+g_2^2}}(g_2 W^3_{\mu} - g_1 B_{\mu})$
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- A_{μ} and Z_{μ} are orthogonal
- weak angle: $\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}$, $\cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}}$

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B_{\mu} &= \frac{1}{\sqrt{g_1^2+g_2^2}}(g_2 A_{\mu} - g_1 Z_{\mu}) &= \cos\theta_W A_{\mu} - \sin\theta_W Z_{\mu} \\
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$$= -\frac{1}{2}[g_2 \frac{1}{\sqrt{g_1^2+g_2^2}}(g_1 A_\mu + g_2 Z_\mu) - g_1 \frac{1}{\sqrt{g_1^2+g_2^2}}(g_2 A_\mu - g_1 Z_\mu)]\overline{\nu_{e_L}}\gamma^\mu \nu_{e_L}$$
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$$= -\sqrt{g_1^2+g_2^2} Z_\mu [\frac{1}{2}\overline{\nu_{e_L}}\gamma^\mu \nu_{e_L} - \frac{1}{2}\overline{e_L}\gamma^\mu e_L$$
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deduce $\frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}} = g_1\cos\theta_W = g_2 \sin\theta_W = e$

deduce $y_R = -1$

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&\quad -\sin^2\theta_W(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})] \\
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- photon couples to charged particles only
- charged gauge bosons ensure transition between charged leptons and neutrinos
- a neutral gauge boson is predicted
- all gauge bosons are massless

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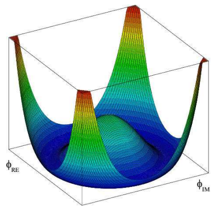
Introduce a complex scalar doublet:

$$\phi(\mathbf{x}) = \begin{pmatrix} \phi_1(\mathbf{x}) \\ \phi_2(\mathbf{x}) \end{pmatrix}$$



Free Lagrangian

$$\begin{aligned} \mathcal{L}_0 &= (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi) \\ V(\phi) &= \kappa \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \end{aligned}$$

Theory must be stable: $\lambda > 0$
Minimum not at 0: $\kappa = -\mu^2 < 0$

The ground state is not unique:

$$\phi = \exp(i \frac{\tau_a}{2} \varphi_a) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

Choose $\varphi = 0 \rightarrow SU(2)$ symmetry is broken

Yukawa terms

$$
\begin{aligned}
\mathcal{L}_Y &= -y_e \overline{\mathrm{e_R}} \phi^\dagger \begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \\
h.c. & \quad -y_e (\overline{\nu_{\mathrm{e_L}}}, \mathrm{e_L}) \phi \mathrm{e_R} \\
&= -y_e (\overline{\mathrm{e_R}} \phi_1^\dagger \nu_{\mathrm{e_L}} + \overline{\mathrm{e_R}} \phi_2^\dagger \mathrm{e_L}) \\
& \quad -y_e (\overline{\nu_{\mathrm{e_L}}} \phi_1 \mathrm{e_R} + \overline{\mathrm{e_L}} \phi_2 \mathrm{e_R})
\end{aligned}
$$

Deduce the hypercharge:

$$
\begin{aligned}
\mathrm{e_L} &\rightarrow \mathrm{e_R} + \phi_2 \\
-\frac{1}{2} &= y_H + -1 \\
y_H &= \frac{1}{2}
\end{aligned}
$$

Minimal Subsitution

$$
\begin{aligned}
\partial_\mu \phi &\rightarrow \partial_\mu \phi + i g_2 W_\mu^a \frac{\tau_a}{2} \phi \\
& \quad + i g_1 B_\mu y_H \phi \\
\partial_\mu \phi^\dagger &\rightarrow \partial_\mu \phi^\dagger - \phi^\dagger i g_2 W_\mu^a \frac{\tau_a}{2} \\
& \quad - \phi^\dagger i g_1 B_\mu y_H
\end{aligned}
$$

Calculate the interaction terms

$$
\begin{aligned}
& \phi^\dagger (-i g_2 W_\mu^a \tfrac{\tau_a}{2} - i g_1 B_\mu y_H) \\
& (+i g_2 W_\mu^a \tfrac{\tau_a}{2} + i g_1 B_\mu y_H) \phi
\end{aligned}
$$

$$
\phi^T = (0, \sqrt{\frac{\mu^2}{2\lambda}}) == (0, \frac{v}{\sqrt{2}})
$$

Yukawa terms

$$\mathcal{L}_Y = -y_e \overline{e_R} \phi^\dagger \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$$

$$h.c. \quad -y_e (\overline{\nu_{e_L}}, e_L) \phi e_R$$

$$= -y_e (\overline{e_R} \phi_1^\dagger \nu_{e_L} + \overline{e_R} \phi_2^\dagger e_L) - y_e (\overline{\nu_{e_L}} \phi_1 e_R + \overline{e_L} \phi_2 e_R)$$

Deduce the hypercharge:

$$e_L \rightarrow e_R + \phi_2$$
$$-\frac{1}{2} = y_H + -1$$
$$y_H = \frac{1}{2}$$

Minimal Subsitution

$$\partial_\mu \phi \rightarrow \partial_\mu \phi + i g_2 W_\mu^a \frac{\tau_a}{2} \phi + i g_1 B_\mu y_H \phi$$

$$\partial_\mu \phi^\dagger \rightarrow \partial_\mu \phi^\dagger - \phi^\dagger i g_2 W_\mu^a \frac{\tau_a}{2} - \phi^\dagger i g_1 B_\mu y_H$$

Calculate the interaction terms

$$\phi^\dagger (-i g_2 W_\mu^a \frac{\tau_a}{2} - i g_1 B_\mu y_H)$$
$$(+i g_2 W_\mu^a \frac{\tau_a}{2} + i g_1 B_\mu y_H) \phi$$

$$\phi^T = (0, \sqrt{\frac{\mu^2}{2\lambda}}) == (0, \frac{v}{\sqrt{2}})$$

Yukawa terms

$$\mathcal{L}_Y = -y_e \overline{\mathrm{e_R}} \phi^\dagger \begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix}$$
$$h.c. \quad -y_e (\overline{\nu_{\mathrm{e_L}}}, \mathrm{e_L}) \phi \mathrm{e_R}$$
$$= -y_e (\overline{\mathrm{e_R}} \phi_1^\dagger \nu_{\mathrm{e_L}} + \overline{\mathrm{e_R}} \phi_2^\dagger \mathrm{e_L}) -y_e (\overline{\nu_{\mathrm{e_L}}} \phi_1 \mathrm{e_R} + \overline{\mathrm{e_L}} \phi_2 \mathrm{e_R})$$

Deduce the hypercharge:

$$\mathrm{e_L} \rightarrow \mathrm{e_R} + \phi_2$$
$$-\frac{1}{2} = y_H + -1$$
$$y_H = \frac{1}{2}$$

Minimal Subsitution

$$\partial_\mu \phi \rightarrow \partial_\mu \phi + i g_2 W_\mu^a \frac{\tau_a}{2} \phi + i g_1 B_\mu y_H \phi$$
$$\partial_\mu \phi^\dagger \rightarrow \partial_\mu \phi^\dagger - \phi^\dagger i g_2 W_\mu^a \frac{\tau_a}{2} - \phi^\dagger i g_1 B_\mu y_H$$

Calculate the interaction terms

$$\phi^\dagger (-i g_2 W_\mu^a \frac{\tau_a}{2} - i g_1 B_\mu y_H)$$
$$(+i g_2 W_\mu^a \frac{\tau_a}{2} + i g_1 B_\mu y_H) \phi$$

$$\phi^T = (0, \sqrt{\frac{\mu^2}{2\lambda}}) == (0, \frac{v}{\sqrt{2}})$$

Yukawa terms

$$\mathcal{L}_Y = -y_e \overline{\mathrm{e}_\mathrm{R}} \phi^\dagger \begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix}$$

$$h.c. \quad -y_e(\overline{\nu_{\mathrm{e_L}}}, \mathrm{e_L})\phi \mathrm{e_R}$$

$$= -y_e(\overline{\mathrm{e}_\mathrm{R}}\phi_1^\dagger \nu_{\mathrm{e_L}} + \overline{\mathrm{e}_\mathrm{R}}\phi_2^\dagger \mathrm{e_L}) -y_e(\overline{\nu_{\mathrm{e_L}}}\phi_1 \mathrm{e_R} + \overline{\mathrm{e_L}}\phi_2 \mathrm{e_R})$$

Deduce the hypercharge:

$$\begin{aligned} \mathrm{e_L} &\rightarrow \mathrm{e_R} + \phi_2 \\ -\frac{1}{2} &= y_H + -1 \\ y_H &= \frac{1}{2} \end{aligned}$$

Minimal Subsitution

$$\begin{aligned} \partial_\mu \phi &\rightarrow \partial_\mu \phi + ig_2 W_\mu^a \tfrac{\tau_a}{2}\phi + ig_1 B_\mu y_H \phi \\ \partial_\mu \phi^\dagger &\rightarrow \partial_\mu \phi^\dagger - \phi^\dagger ig_2 W_\mu^a \tfrac{\tau_a}{2} - \phi^\dagger ig_1 B_\mu y_H \end{aligned}$$

Calculate the interaction terms

$$\phi^\dagger(-ig_2 W_\mu^a \tfrac{\tau_a}{2} - ig_1 B_\mu y_H)(+ig_2 W_\mu^a \tfrac{\tau_a}{2} + ig_1 B_\mu y_H)\phi$$

$$\phi^T = (0, \sqrt{\frac{\mu^2}{2\lambda}}) == (0, \frac{v}{\sqrt{2}})$$

$$(0, \sqrt{\tfrac{\mu^2}{2\lambda}})(-ig_2 W_\mu^a \tfrac{\tau_a}{2} - ig_1 B_\mu y_H)(+ig_2 W_\mu^a \tfrac{\tau_a}{2} + ig_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\tfrac{\mu^2}{2\lambda}})(g_2 W_\mu^a \tfrac{\tau_a}{2} + g_1 B_\mu y_H)(g_2 W_\mu^a \tfrac{\tau_a}{2} + g_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\tfrac{\mu^2}{2\lambda}}) \begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2+g_2^2}}{2} Z_\mu \end{pmatrix}$$

$$\begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2+g_2^2}}{2} Z_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= \frac{g_2^2 v^2}{4} W_\mu^- W^{\mu +} + \frac{(g_1^2+g_2^2) v^2}{8} Z_\mu Z^\mu$$

The weak bosons have acquired a mass!

$$(0, \sqrt{\frac{\mu^2}{2\lambda}})(-ig_2 W_\mu^a \frac{\tau_a}{2} - ig_1 B_\mu y_H)(+ig_2 W_\mu^a \frac{\tau_a}{2} + ig_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\frac{\mu^2}{2\lambda}})(g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H)(g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\frac{\mu^2}{2\lambda}}) \begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2+g_2^2}}{2} Z_\mu \end{pmatrix}$$

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$$= (0, \sqrt{\frac{\mu^2}{2\lambda}})(g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H)(g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\frac{\mu^2}{2\lambda}}) \begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2 + g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2 + g_2^2}}{2} Z_\mu \end{pmatrix}$$

$$\begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2 + g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2 + g_2^2}}{2} Z_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

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$$(0, \sqrt{\frac{\mu^2}{2\lambda}})(-ig_2 W_\mu^a \frac{\tau_a}{2} - ig_1 B_\mu y_H)(+ig_2 W_\mu^a \frac{\tau_a}{2} + ig_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\frac{\mu^2}{2\lambda}})(g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H)(g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\frac{\mu^2}{2\lambda}}) \begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2+g_2^2}}{2} Z_\mu \end{pmatrix}$$

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$$(0, \sqrt{\tfrac{\mu^2}{2\lambda}})(-ig_2 W_\mu^a \tfrac{\tau_a}{2} - ig_1 B_\mu y_H)(+ig_2 W_\mu^a \tfrac{\tau_a}{2} + ig_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\tfrac{\mu^2}{2\lambda}})(g_2 W_\mu^a \tfrac{\tau_a}{2} + g_1 B_\mu y_H)(g_2 W_\mu^a \tfrac{\tau_a}{2} + g_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

$$= (0, \sqrt{\tfrac{\mu^2}{2\lambda}}) \begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2+g_2^2}}{2} Z_\mu \end{pmatrix}$$

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The weak bosons have acquired a mass!

Charged lepton masses

$$\begin{aligned}
\mathcal{L}_Y &= -y_e(\overline{\mathrm{e_R}}\phi_1^\dagger \nu_{\mathrm{e_L}} + \overline{\mathrm{e_R}}\phi_2^\dagger \mathrm{e_L}) - y_e(\overline{\nu_{\mathrm{e_L}}}\phi_1 \mathrm{e_R} + \overline{\mathrm{e_L}}\phi_2 \mathrm{e_R}) \\
&= -y_e(\overline{\mathrm{e_R}}\frac{v}{\sqrt{2}}\mathrm{e_L}) - y_e(\overline{\mathrm{e_L}}\frac{v}{\sqrt{2}}\mathrm{e_R}) \\
&= -y_e\frac{v}{\sqrt{2}}(\overline{\mathrm{e_R}}\mathrm{e_L} + \overline{\mathrm{e_L}}\mathrm{e_R}) \\
&= -y_e\frac{v}{\sqrt{2}}(\overline{\mathrm{e}}\mathrm{e})
\end{aligned}$$

Masses

$$\begin{aligned}
m_{\mathrm{e}} &= y_e\frac{v}{\sqrt{2}} \\
m^2_{\mathrm{W^\pm}} &= \frac{g_2^2 v^2}{4} = \frac{e^2 v^2}{4\sin^2\theta_W} \\
m^2_{\mathrm{Z^\circ}} &= \frac{(g_1^2+g_2^2)v^2}{4} = \frac{e^2 v^2}{4\sin^2\theta_W\cos^2\theta_W} \quad \mathcal{L} \rightarrow EQM \\
\frac{m^2_{\mathrm{W^\pm}}}{m^2_{\mathrm{Z^\circ}}} &= \cos^2\theta_W
\end{aligned}$$

Charged lepton masses

$$
\begin{aligned}
\mathcal{L}_Y &= -y_e(\overline{\mathrm{e_R}}\phi_1^\dagger \nu_{\mathrm{e_L}} + \overline{\mathrm{e_R}}\phi_2^\dagger \mathrm{e_L}) - y_e(\overline{\nu_{\mathrm{e_L}}}\phi_1 \mathrm{e_R} + \overline{\mathrm{e_L}}\phi_2 \mathrm{e_R}) \\
&= -y_e(\overline{\mathrm{e_R}}\frac{v}{\sqrt{2}}\mathrm{e_L}) - y_e(\overline{\mathrm{e_L}}\frac{v}{\sqrt{2}}\mathrm{e_R}) \\
&= -y_e\frac{v}{\sqrt{2}}(\overline{\mathrm{e_R}}\mathrm{e_L} + \overline{\mathrm{e_L}}\mathrm{e_R}) \\
&= -y_e\frac{v}{\sqrt{2}}(\overline{\mathrm{e}}\mathrm{e})
\end{aligned}
$$

Masses

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\begin{aligned}
m_{\mathrm{e}} &= y_e\frac{v}{\sqrt{2}} \\
m^2_{\mathrm{W}^\pm} &= \frac{g_2^2 v^2}{4} = \frac{e^2 v^2}{4\sin^2\theta_W} \\
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\frac{m^2_{\mathrm{W}^\pm}}{m^2_{\mathrm{Z}^\circ}} &= \cos^2\theta_W
\end{aligned}
$$

- Quantum Numbers weak Isospin $SU(2)_L$ of fermions
- weak hypercharge
- $Q = I_3^W + Y$

I^W	I_3^W	Y			
$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	$\frac{1}{6}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$
$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	$-\frac{1}{2}$	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$
0	0	$\frac{2}{3}$	u_R	c_R	t_R
0	0	$-\frac{1}{3}$	d_R	s_R	b_R
0	0	-1	e_R	μ_R	τ_R

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I^W	I_3^W	Y			
$\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{6}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$
$\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$
0	0	$\frac{2}{3}$	u_R	c_R	t_R
0	0	$-\frac{1}{3}$	d_R	s_R	b_R
0	0	-1	e_R	μ_R	τ_R

- Quantum Numbers weak Isospin $SU(2)_L$ of fermions
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- $Q = I_3^W + Y$

I^W	I_3^W	Y			
$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	$\frac{1}{6}$	$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix}$	$\begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix}$	$\begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$
$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	$-\frac{1}{2}$	$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$
0	0	$\frac{2}{3}$	$\mathrm{u_R}$	$\mathrm{c_R}$	$\mathrm{t_R}$
0	0	$-\frac{1}{3}$	$\mathrm{d_R}$	$\mathrm{s_R}$	$\mathrm{b_R}$
0	0	-1	$\mathrm{e_R}$	μ_R	τ_R

$$\overline{\mathrm{e_L}} = (\gamma^0 P_L \mathrm{e})^\dagger = \mathrm{e}^\dagger P_L^\dagger \gamma^0 = \mathrm{e}^\dagger P_L \gamma^0 = \mathrm{e}^\dagger \gamma^0 P_R = \bar{\mathrm{e}} P_R$$

The interactions

$$\begin{aligned}
\mathcal{L}' \quad = \quad & -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W_\mu^- \overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}) \\
& -\frac{e}{\sin\theta_W \cos\theta_W} Z_\mu [\tfrac{1}{2}\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} - \tfrac{1}{2}\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\
& -\sin^2\theta_W(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})] \\
& -eA_\mu(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})
\end{aligned}$$

$$\overline{\mathrm{e_L}} = (\gamma^0 P_L \mathrm{e})^\dagger = \mathrm{e}^\dagger P_L^\dagger \gamma^0 = \mathrm{e}^\dagger P_L \gamma^0 = \mathrm{e}^\dagger \gamma^0 P_R = \bar{\mathrm{e}} P_R$$

The interactions

$$\begin{aligned}
\mathcal{L}' &= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W_\mu^- \overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}) \\
& -\frac{e}{\sin\theta_W \cos\theta_W} Z_\mu [\tfrac{1}{2}\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} - \tfrac{1}{2}\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\
& - \sin^2\theta_W (-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})] \\
& - eA_\mu(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})
\end{aligned}$$

Electromagnetic Current

$$\begin{aligned}
\mathcal{L}' &= -eA_\mu(-\bar{\mathrm{e}} P_R \gamma^\mu P_L \mathrm{e} - \bar{\mathrm{e}} P_L \gamma^\mu P_R \mathrm{e}) \\
&= -eA_\mu \bar{\mathrm{e}}\gamma^\mu Q \mathrm{e} = -eA_\mu \bar{\mathrm{e}}\gamma^\mu (I_3^W + Y)\mathrm{e} \\
&= -eA_\mu j^\mu_{EM}
\end{aligned}$$

$$\overline{\mathrm{e_L}} = (\gamma^0 P_L \mathrm{e})^\dagger = \mathrm{e}^\dagger P_L^\dagger \gamma^0 = \mathrm{e}^\dagger P_L \gamma^0 = \mathrm{e}^\dagger \gamma^0 P_R = \bar{\mathrm{e}} P_R$$

The interactions

$$\begin{aligned}
\mathcal{L}' &= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W_\mu^- \overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}) \\
&\quad -\frac{e}{\sin\theta_W \cos\theta_W} Z_\mu [\tfrac{1}{2}\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} - \tfrac{1}{2}\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\
&\quad - \sin^2\theta_W(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})] \\
&\quad - eA_\mu(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})
\end{aligned}$$

Charged Current

$$\begin{aligned}
\mathcal{L}' &= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu P_L \mathrm{e} + W_\mu^- \bar{\mathrm{e}} P_R \gamma^\mu \nu_{\mathrm{e_L}}) \\
&= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu P_L \mathrm{e} + W_\mu^- \bar{\mathrm{e}} \gamma^\mu P_L \nu_{\mathrm{e_L}}) \\
&= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ j_{CC}^\mu + W_\mu^- j_{CC}^{\mu\,\dagger})
\end{aligned}$$

$$\overline{\mathrm{e_L}} = (\gamma^0 P_L \mathrm{e})^\dagger = \mathrm{e}^\dagger P_L^\dagger \gamma^0 = \mathrm{e}^\dagger P_L \gamma^0 = \mathrm{e}^\dagger \gamma^0 P_R = \bar{\mathrm{e}} P_R$$

The interactions

$$\begin{aligned}\mathcal{L}' &= -\frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \overline{\nu_{\mathrm{e_L}}}\gamma^\mu \mathrm{e_L} + W_\mu^- \overline{\mathrm{e_L}}\gamma^\mu \nu_{\mathrm{e_L}}) \\ &\quad -\frac{e}{\sin\theta_W\cos\theta_W} Z_\mu [\tfrac{1}{2}\overline{\nu_{\mathrm{e_L}}}\gamma^\mu \nu_{\mathrm{e_L}} - \tfrac{1}{2}\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} \\ &\quad - \sin^2\theta_W(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})] \\ &\quad - eA_\mu(-\overline{\mathrm{e_L}}\gamma^\mu \mathrm{e_L} - \overline{\mathrm{e_R}}\gamma^\mu \mathrm{e_R})\end{aligned}$$

Neutral Current

$$\begin{aligned}\mathcal{L}' &= -\frac{e}{\sin\theta_W\cos\theta_W} Z_\mu [\overline{\nu_{\mathrm{e_L}}}\gamma^\mu I_3^W \nu_{\mathrm{e_L}} + \overline{\mathrm{e_L}}\gamma^\mu I_3^W \mathrm{e_L} \\ &\quad - \sin^2\theta_W j_{EM}^\mu] \\ &= -\frac{e}{\sin\theta_W\cos\theta_W} Z_\mu j_{NC}^\mu\end{aligned}$$