Particle Physics: The Standard Model

Dirk Zerwas

LAL
zerwas@lal.in2p3.fr

May 10, 2012

- The Particles
- $W^\pm$ couples to $SU(2)_L$ doublets
- Z°: no FCNC (Z° cannot change flavor just like γ)
- Assumption MassEigenstates=EWEigenstates
- Why?

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\gamma$$
$$g$$
$$W^\pm, Z^\circ$$
$$H$$

- The Particles
- $\mathrm{W}^{\pm}$ couples to $SU(2)_L$ doublets
- Z°: no FCNC (Z° cannot change flavor just like γ)
- Assumption MassEigenstates=EWEigenstates
- Why?

$$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$$

$$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$$

$$\begin{matrix} \mathrm{u_R} & \mathrm{c_R} & \mathrm{t_R} \\ \mathrm{d_R} & \mathrm{s_R} & \mathrm{b_R} \\ \mathrm{e_R} & \mu_\mathrm{R} & \tau_\mathrm{R} \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ \mathrm{W}^{\pm}, \mathrm{Z}^{\circ} \\ \mathrm{H} \end{matrix}$$

- The Particles
- $\mathrm{W}^{\pm}$ couples to $SU(2)_L$ doublets
- Z°: no FCNC (Z° cannot change flavor just like γ)
- Assumption MassEigenstates=EWEigenstates
- Why?

$$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$$

$$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$$

$$\begin{matrix} \mathrm{u_R} & \mathrm{c_R} & \mathrm{t_R} \\ \mathrm{d_R} & \mathrm{s_R} & \mathrm{b_R} \\ \mathrm{e_R} & \mu_\mathrm{R} & \tau_\mathrm{R} \end{matrix}$$

$$\gamma$$
$$g$$
$$\mathrm{W}^{\pm}, \mathrm{Z}^{\circ}$$
$$\mathrm{H}$$

- Another perspective: meson lifetimes
- Weak decays of mesons differ by orders of magnitude?
- How can the s decay weakly? $m_c > m_s$?
- Keep leptons untouched
- Introduce the CKM matrix

Properties of the c

$$
\begin{aligned}
\mathrm{m}_0 &= 1.27\mathrm{GeV} \\
\tau &= (1.040 \cdot 10^{-12})\mathrm{s} \quad c\bar{d} \\
c\tau &= 311.8\mu\mathrm{m}
\end{aligned}
$$

Properties of the s

$$
\begin{aligned}
\mathrm{m}_0 &= 100 \pm 25\mathrm{MeV} \\
\tau &= (1.24 \cdot 10^{-8})\mathrm{s} \quad u\bar{s} \\
c\tau &= 3.7\mathrm{m}
\end{aligned}
$$

- Another perspective: meson lifetimes
- Weak decays of mesons differ by orders of magnitude?
- How can the s decay weakly? $m_{\mathrm{c}} > m_{\mathrm{s}}$?
- Keep leptons untouched
- Introduce the CKM matrix

Properties of the c

$$\begin{aligned} \mathrm{m}_0 &= 1.27\mathrm{GeV} \\ \tau &= (1.040 \cdot 10^{-12})\mathrm{s} \quad c\bar{d} \\ c\tau &= 311.8\mu\mathrm{m} \end{aligned}$$

Properties of the s

$$\begin{aligned} \mathrm{m}_0 &= 100 \pm 25\mathrm{MeV} \\ \tau &= (1.24 \cdot 10^{-8})\mathrm{s} \quad u\bar{\mathrm{s}} \\ c\tau &= 3.7\mathrm{m} \end{aligned}$$

- Another perspective: meson lifetimes
- Weak decays of mesons differ by orders of magnitude?
- How can the s decay weakly? $m_\mathrm{c} > m_\mathrm{s}$?
- Keep leptons untouched
- Introduce the CKM matrix

Properties of the c

$$
\begin{aligned}
\mathrm{m}_0 &= 1.27\mathrm{GeV} \\
\tau &= (1.040 \cdot 10^{-12})\mathrm{s} \quad c\bar{d} \\
c\tau &= 311.8\mu\mathrm{m}
\end{aligned}
$$

Properties of the s

$$
\begin{aligned}
\mathrm{m}_0 &= 100 \pm 25\mathrm{MeV} \\
\tau &= (1.24 \cdot 10^{-8})\mathrm{s} \quad u\bar{\mathrm{s}} \\
c\tau &= 3.7\mathrm{m}
\end{aligned}
$$

- Another perspective: meson lifetimes
- Weak decays of mesons differ by orders of magnitude?
- How can the s decay weakly? $m_\mathrm{c} > m_\mathrm{s}$?
- Keep leptons untouched
- Introduce the CKM matrix

Properties of the c

$$\begin{aligned} \mathrm{m}_0 &= 1.27\mathrm{GeV} \\ \tau &= (1.040 \cdot 10^{-12})\mathrm{s} \quad c\bar{d} \\ c\tau &= 311.8\mu\mathrm{m} \end{aligned}$$

Properties of the s

$$\begin{aligned} \mathrm{m}_0 &= 100 \pm 25\mathrm{MeV} \\ \tau &= (1.24 \cdot 10^{-8})\mathrm{s} \quad u\bar{\mathrm{s}} \\ c\tau &= 3.7\mathrm{m} \end{aligned}$$

- Another perspective: meson lifetimes
- Weak decays of mesons differ by orders of magnitude?
- How can the s decay weakly? $m_\mathrm{c} > m_\mathrm{s}$?
- Keep leptons untouched
- Introduce the CKM matrix

Properties of the c

$$\begin{aligned} \mathrm{m}_0 &= 1.27\mathrm{GeV} \\ \tau &= (1.040 \cdot 10^{-12})\mathrm{s} \quad c\bar{d} \\ c\tau &= 311.8\mu\mathrm{m} \end{aligned}$$

Properties of the s

$$\begin{aligned} \mathrm{m}_0 &= 100 \pm 25\mathrm{MeV} \\ \tau &= (1.24 \cdot 10^{-8})\mathrm{s} \quad u\bar{\mathrm{s}} \\ c\tau &= 3.7\mathrm{m} \end{aligned}$$

- Another perspective: meson lifetimes
- Weak decays of mesons differ by orders of magnitude?
- How can the s decay weakly? $m_\mathrm{c} > m_\mathrm{s}$?
- Keep leptons untouched
- Introduce the CKM matrix

Properties of the c

$$\begin{aligned} \mathrm{m}_0 &= 1.27\,\mathrm{GeV} \\ \tau &= (1.040 \cdot 10^{-12})\,\mathrm{s} \quad c\bar{d} \\ c\tau &= 311.8\,\mu\mathrm{m} \end{aligned}$$

Properties of the s

$$\begin{aligned} \mathrm{m}_0 &= 100 \pm 25\,\mathrm{MeV} \\ \tau &= (1.24 \cdot 10^{-8})\,\mathrm{s} \quad u\bar{\mathrm{s}} \\ c\tau &= 3.7\,\mathrm{m} \end{aligned}$$

Definition

d is the mass Eigenstate

d′ is the isospin partner of u

V: unitary 3 × 3 matrix $V^\dagger V = 1_3$

Cannot simplify: masses not equal

$$\mathcal{L}_{Yuk} = -\bar{u}m_u u - \bar{c}m_c c - \bar{t}m_t t - \bar{d}m_d d - \bar{s}m_s s - \bar{b}m_b b$$

$$= -\begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

Definition

d is the mass Eigenstate

d' is the isospin partner of u

V: unitary 3×3 matrix $\mathrm{V}^\dagger \mathrm{V} = 1_3$

Cannot simplify: masses not equal

$$
\begin{aligned}
\mathcal{L}_{Yuk} &= -\bar{\mathrm{u}} m_{\mathrm{u}} \mathrm{u} - \bar{\mathrm{c}} m_{\mathrm{c}} \mathrm{c} - \bar{\mathrm{t}} m_{\mathrm{t}} \mathrm{t} - \bar{\mathrm{d}} m_{\mathrm{d}} \mathrm{d} - \bar{\mathrm{s}} m_{\mathrm{s}} \mathrm{s} - \bar{\mathrm{b}} m_{\mathrm{b}} \mathrm{b} \\
&= -\begin{pmatrix} \bar{\mathrm{u}} & \bar{\mathrm{c}} & \bar{\mathrm{t}} \end{pmatrix} \begin{pmatrix} m_{\mathrm{u}} & 0 & 0 \\ 0 & m_{\mathrm{c}} & 0 \\ 0 & 0 & m_{\mathrm{t}} \end{pmatrix} \begin{pmatrix} \mathrm{u} \\ \mathrm{c} \\ \mathrm{t} \end{pmatrix} \\
&\quad -\begin{pmatrix} \bar{\mathrm{d}} & \bar{\mathrm{s}} & \bar{\mathrm{b}} \end{pmatrix} \begin{pmatrix} m_{\mathrm{d}} & 0 & 0 \\ 0 & m_{\mathrm{s}} & 0 \\ 0 & 0 & m_{\mathrm{b}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}
\end{aligned}
$$

Definition

d is the mass Eigenstate
d' is the isospin partner of u
V: unitary 3×3 matrix $\mathrm{V}^\dagger \mathrm{V} = 1_3$
Cannot simplify: masses not equal

$$\begin{aligned}
\mathcal{L}_{Yuk} &= -\bar{\mathrm{u}} m_\mathrm{u} \mathrm{u} - \bar{\mathrm{c}} m_\mathrm{c} \mathrm{c} - \bar{\mathrm{t}} m_\mathrm{t} \mathrm{t} - \bar{\mathrm{d}} m_\mathrm{d} \mathrm{d} - \bar{\mathrm{s}} m_\mathrm{s} \mathrm{s} - \bar{\mathrm{b}} m_\mathrm{b} \mathrm{b} \\
&= -\begin{pmatrix} \bar{\mathrm{u}} & \bar{\mathrm{c}} & \bar{\mathrm{t}} \end{pmatrix} \begin{pmatrix} m_\mathrm{u} & 0 & 0 \\ 0 & m_\mathrm{c} & 0 \\ 0 & 0 & m_\mathrm{t} \end{pmatrix} \begin{pmatrix} \mathrm{u} \\ \mathrm{c} \\ \mathrm{t} \end{pmatrix} \\
&\quad -\begin{pmatrix} \bar{\mathrm{d}}' & \bar{\mathrm{s}}' & \bar{\mathrm{b}}' \end{pmatrix} \mathrm{V} \begin{pmatrix} m_\mathrm{d} & 0 & 0 \\ 0 & m_\mathrm{s} & 0 \\ 0 & 0 & m_\mathrm{b} \end{pmatrix} \mathrm{V}^\dagger \begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix}
\end{aligned}$$

Properties of V

- Cabibbo, Kobayashi, Maskawa
- V complex: $3 \times 3 \times 2$
- $VV^\dagger = 1_3$: 9 constraints
- 5 phases absorbed
- 3 real mixing angles, 1 complex phase

Cabibbo

(2 generations):

$$\begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

Wolfenstein

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Diagonal entries dominate

Properties of V

- Cabibbo, Kobayashi, Maskawa
- V complex: $3 \times 3 \times 2$
- $VV^\dagger = 1_3$: 9 constraints
- 5 phases absorbed
- 3 real mixing angles, 1 complex phase

Cabibbo

(2 generations):

$$\begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

Wolfenstein

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Diagonal entries dominate

Properties of V

- Cabibbo, Kobayashi, Maskawa
- V complex: $3 \times 3 \times 2$
- $VV^\dagger = 1_3$: 9 constraints
- 5 phases absorbed
- 3 real mixing angles, 1 complex phase

Cabibbo

(2 generations):

$$\begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

Wolfenstein

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Diagonal entries dominate

Properties of V

- Cabibbo, Kobayashi, Maskawa
- V complex: $3 \times 3 \times 2$
- $VV^\dagger = 1_3$: 9 constraints
- 5 phases absorbed
- 3 real mixing angles, 1 complex phase

Cabibbo

(2 generations):

$$\begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

Wolfenstein

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Diagonal entries dominate

Charged Current

$$\begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L' \\ s_L' \\ b_L' \end{pmatrix}$$

$$= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu V \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Decays

$$\begin{array}{llll} d & \rightarrow & u + W^- & V_{11} \\ s & \rightarrow & u + W^- & V_{12} \\ b & \rightarrow & u + W^- & V_{13} \end{array}$$

- $V_{11} = 0.98$, $V_{12} = 0.2$
- charmed mesons: $V_{11}^2 G^2 \rightarrow 0.96 G^2$
- strange mesons: $V_{12}^2 G^2 \rightarrow 0.04 G^2$ longer lifetime

Charged Current

$$\begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L{}' \\ s_L{}' \\ b_L{}' \end{pmatrix}$$

$$= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu V \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Decays

$$d \rightarrow u + W^- \quad V_{11}$$

$$s \rightarrow u + W^- \quad V_{12}$$

$$b \rightarrow u + W^- \quad V_{13}$$

- $V_{11} = 0.98$, $V_{12} = 0.2$
- charmed mesons: $V_{11}^2 G^2 \rightarrow 0.96 G^2$
- strange mesons: $V_{12}^2 G^2 \rightarrow 0.04 G^2$ longer lifetime

Charged Current

$$\begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L' \\ s_L' \\ b_L' \end{pmatrix}$$

$$= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu V \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Decays

$$\begin{array}{cccc} d & \rightarrow & u + W^- & V_{11} \\ s & \rightarrow & u + W^- & V_{12} \\ b & \rightarrow & u + W^- & V_{13} \end{array}$$

- $V_{11} = 0.98$, $V_{12} = 0.2$
- charmed mesons: $V_{11}^2 G^2 \rightarrow 0.96 G^2$
- strange mesons: $V_{12}^2 G^2 \rightarrow 0.04 G^2$ longer lifetime

Charged Current

$$(\bar{u}_L \quad \bar{c}_L \quad \bar{t}_L)\gamma^\mu \begin{pmatrix} d_L' \\ s_L' \\ b_L' \end{pmatrix}$$

$$= (\bar{u}_L \quad \bar{c}_L \quad \bar{t}_L)\gamma^\mu V \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

Decays

$$\begin{array}{cccc} d & \rightarrow & u + W^- & V_{11} \\ s & \rightarrow & u + W^- & V_{12} \\ b & \rightarrow & u + W^- & V_{13} \end{array}$$

- $V_{11} = 0.98$, $V_{12} = 0.2$
- charmed mesons: $V_{11}^2 G^2 \rightarrow 0.96 G^2$
- strange mesons: $V_{12}^2 G^2 \rightarrow 0.04 G^2$ longer lifetime

Neutral Current

Neutral currents couple to right- and left-handed quarks:

$$\begin{pmatrix} \bar{\mathrm{d}}' & \bar{\mathrm{s}}' & \bar{\mathrm{b}}' \end{pmatrix} \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix}$$

$$= \begin{pmatrix} \bar{\mathrm{d}} & \bar{\mathrm{s}} & \bar{\mathrm{b}} \end{pmatrix} \mathrm{V}^\dagger \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \mathrm{V} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$

$$= \begin{pmatrix} \bar{\mathrm{d}} & \bar{\mathrm{s}} & \bar{\mathrm{b}} \end{pmatrix} \mathrm{V}^\dagger \mathrm{V} \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$

$\mathrm{V}^\dagger \mathrm{V} = 1_3$: does not have a Lorentz index, only family index

Neutral Current

Neutral currents couple to right- and left-handed quarks:

$$\begin{pmatrix} \bar{d}' & \bar{s}' & \bar{b}' \end{pmatrix} \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

$$= \begin{pmatrix} \bar{d} & \bar{s} & \bar{b} \end{pmatrix} V^\dagger \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= \begin{pmatrix} \bar{d} & \bar{s} & \bar{b} \end{pmatrix} V^\dagger V \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V^\dagger V = 1_3$: does not have a Lorentz index, only family index

Neutral Current

Neutral currents couple to right- and left-handed quarks:

$$\begin{pmatrix} \bar{d}' & \bar{s}' & \bar{b}' \end{pmatrix} \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

$$= \begin{pmatrix} \bar{d} & \bar{s} & \bar{b} \end{pmatrix} V^\dagger \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= \begin{pmatrix} \bar{d} & \bar{s} & \bar{b} \end{pmatrix} V^\dagger V \gamma^\mu \left(-\frac{1}{2}\frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_W\right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V^\dagger V = 1_3$: does not have a Lorentz index, only family index

And the up-type sector?

$$
\begin{aligned}
&\begin{pmatrix} \bar{u}_L' & \bar{c}_L' & \bar{t}_L' \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L' \\ s_L' \\ b_L' \end{pmatrix} \\
&= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} V_2^\dagger \gamma^\mu \mathrm{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \\
&= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu V_2^\dagger \mathrm{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}
\end{aligned}
$$

define: $V_3 = V_2^\dagger \mathrm{V}$

$V_3 V_3^\dagger = (V_2^\dagger \mathrm{V})(V_2^\dagger \mathrm{V})^\dagger = V_2^\dagger \mathrm{V}\mathrm{V}^\dagger V_2 = 1$

$\rightarrow$ 1 matrix sufficient

And the up-type sector?

$$\begin{pmatrix} \bar{u}'_L & \bar{c}'_L & \bar{t}'_L \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L{}' \\ s_L{}' \\ b_L{}' \end{pmatrix}$$

$$= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} V_2^\dagger \gamma^\mu \mathrm{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

$$= \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \gamma^\mu V_2^\dagger \mathrm{V} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

define: $V_3 = V_2^\dagger \mathrm{V}$

$V_3 V_3^\dagger = (V_2^\dagger \mathrm{V})(V_2^\dagger \mathrm{V})^\dagger = V_2^\dagger \mathrm{V}\mathrm{V}^\dagger V_2 = 1$

$\rightarrow$ 1 matrix sufficient

- C: transforms particles into anti-particles
- P: inverts momentum

EM Interactions

	e^-	$\rightarrow$	γe^-
	L	$\rightarrow$	$-1R$
P	e^-	$\rightarrow$	γe^-
	R	$\rightarrow$	$+1L$
C	e^+	$\rightarrow$	γe^+
	L	$\rightarrow$	$-1R$
CP	e^+	$\rightarrow$	γe^+
	R	$\rightarrow$	$+1L$

EW Interactions

	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	L	$\rightarrow$	LLR
P	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	R	$\rightarrow$	RRL
C	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	L	$\rightarrow$	LLR
CP	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	R	$\rightarrow$	RRL

CPT always conserved

- C: transforms particles into anti-particles
- P: inverts momentum

EM Interactions

	e^-	$\longrightarrow$	γe^-
	L	$\longrightarrow$	$-1R$
P	e^-	$\longrightarrow$	γe^-
	R	$\longrightarrow$	$+1L$
C	e^+	$\longrightarrow$	γe^+
	L	$\longrightarrow$	$-1R$
CP	e^+	$\longrightarrow$	γe^+
	R	$\longrightarrow$	$+1L$

EW Interactions

	μ^-	$\longrightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	L	$\longrightarrow$	LLR
P	μ^-	$\longrightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	R	$\longrightarrow$	RRL
C	μ^+	$\longrightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	L	$\longrightarrow$	LLR
CP	μ^+	$\longrightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	R	$\longrightarrow$	RRL

CPT always conserved

- *C*: transforms particles into anti-particles
- *P*: inverts momentum

EM Interactions

	e^-	$\rightarrow$	γe^-
	L	$\rightarrow$	$-1R$
P	e^-	$\rightarrow$	γe^-
	R	$\rightarrow$	$+1L$
C	e^+	$\rightarrow$	γe^+
	L	$\rightarrow$	$-1R$
CP	e^+	$\rightarrow$	γe^+
	R	$\rightarrow$	$+1L$

EW Interactions

	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	L	$\rightarrow$	LLR
P	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	R	$\rightarrow$	RRL
C	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	L	$\rightarrow$	LLR
CP	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	R	$\rightarrow$	RRL

CPT always conserved

- C: transforms particles into anti-particles
- P: inverts momentum

EM Interactions

	e^-	$\rightarrow$	γe^-
	L	$\rightarrow$	$-1R$
P	e^-	$\rightarrow$	γe^-
	R	$\rightarrow$	$+1L$
C	e^+	$\rightarrow$	γe^+
	L	$\rightarrow$	$-1R$
CP	e^+	$\rightarrow$	γe^+
	R	$\rightarrow$	$+1L$

EW Interactions

	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	L	$\rightarrow$	LLR
P	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	R	$\rightarrow$	RRL
C	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	L	$\rightarrow$	LLR
CP	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	R	$\rightarrow$	RRL

CPT always conserved

- C: transforms particles into anti-particles
- P: inverts momentum

EM Interactions

	e^-	$\rightarrow$	γe^-
	L	$\rightarrow$	$-1R$
P	e^-	$\rightarrow$	γe^-
	R	$\rightarrow$	$+1L$
C	e^+	$\rightarrow$	γe^+
	L	$\rightarrow$	$-1R$
CP	e^+	$\rightarrow$	γe^+
	R	$\rightarrow$	$+1L$

EW Interactions

	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	L	$\rightarrow$	LLR
P	μ^-	$\rightarrow$	$\nu_{\mu_L} e^- \bar{\nu}_{e_L}$
	R	$\rightarrow$	RRL
C	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	L	$\rightarrow$	LLR
CP	μ^+	$\rightarrow$	$\bar{\nu}_{\mu_L} e^+ \nu_{e_L}$
	R	$\rightarrow$	RRL

CPT always conserved

- *C*: transforms particles into anti-particles
- *P*: inverts momentum

EM Interactions

$$\begin{array}{llll} & e^- & \rightarrow & \gamma e^- \\ & L & \rightarrow & -1R \\ P & e^- & \rightarrow & \gamma e^- \\ & R & \rightarrow & +1L \\ C & e^+ & \rightarrow & \gamma e^+ \\ & L & \rightarrow & -1R \\ CP & e^+ & \rightarrow & \gamma e^+ \\ & R & \rightarrow & +1L \end{array}$$

EW Interactions

$$\begin{array}{llll} & \mu^- & \rightarrow & \nu_{\mu_L} e^- \bar{\nu}_{e_L} \\ & L & \rightarrow & LLR \\ P & \mu^- & \rightarrow & \nu_{\mu_L} e^- \bar{\nu}_{e_L} \\ & R & \rightarrow & RRL \\ C & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} e^+ \nu_{e_L} \\ & L & \rightarrow & LLR \\ CP & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} e^+ \nu_{e_L} \\ & R & \rightarrow & RRL \end{array}$$

CPT always conserved

- *C*: transforms particles into anti-particles
- *P*: inverts momentum

EM Interactions

$$\begin{array}{llll} & e^- & \rightarrow & \gamma e^- \\ & L & \rightarrow & -1R \\ P & e^- & \rightarrow & \gamma e^- \\ & R & \rightarrow & +1L \\ C & e^+ & \rightarrow & \gamma e^+ \\ & L & \rightarrow & -1R \\ CP & e^+ & \rightarrow & \gamma e^+ \\ & R & \rightarrow & +1L \end{array}$$

EW Interactions

$$\begin{array}{llll} & \mu^- & \rightarrow & \nu_{\mu_L} e^- \bar{\nu}_{e_L} \\ & L & \rightarrow & LLR \\ P & \mu^- & \rightarrow & \nu_{\mu_L} e^- \bar{\nu}_{e_L} \\ & R & \rightarrow & RRL \\ C & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} e^+ \nu_{e_L} \\ & L & \rightarrow & LLR \\ CP & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} e^+ \nu_{e_L} \\ & R & \rightarrow & RRL \end{array}$$

CPT always conserved

- C: transforms particles into anti-particles
- P: inverts momentum

EM Interactions

$$\begin{array}{llll} & \mathrm{e}^- & \rightarrow & \gamma \mathrm{e}^- \\ & L & \rightarrow & -1R \\ P & \mathrm{e}^- & \rightarrow & \gamma \mathrm{e}^- \\ & R & \rightarrow & +1L \\ C & \mathrm{e}^+ & \rightarrow & \gamma \mathrm{e}^+ \\ & L & \rightarrow & -1R \\ CP & \mathrm{e}^+ & \rightarrow & \gamma \mathrm{e}^+ \\ & R & \rightarrow & +1L \end{array}$$

EW Interactions

$$\begin{array}{llll} & \mu^- & \rightarrow & \nu_{\mu_L} \mathrm{e}^- \bar{\nu}_{\mathrm{e}_L} \\ & L & \rightarrow & LLR \\ P & \mu^- & \rightarrow & \nu_{\mu_L} \mathrm{e}^- \bar{\nu}_{\mathrm{e}_L} \\ & R & \rightarrow & RRL \\ C & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} \mathrm{e}^+ \nu_{\mathrm{e}_L} \\ & L & \rightarrow & LLR \\ CP & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} \mathrm{e}^+ \nu_{\mathrm{e}_L} \\ & R & \rightarrow & RRL \end{array}$$

CPT always conserved

- C: transforms particles into anti-particles
- P: inverts momentum

EM Interactions

$$\begin{array}{llll} & e^- & \rightarrow & \gamma e^- \\ & L & \rightarrow & -1R \\ P & e^- & \rightarrow & \gamma e^- \\ & R & \rightarrow & +1L \\ C & e^+ & \rightarrow & \gamma e^+ \\ & L & \rightarrow & -1R \\ CP & e^+ & \rightarrow & \gamma e^+ \\ & R & \rightarrow & +1L \end{array}$$

EW Interactions

$$\begin{array}{llll} & \mu^- & \rightarrow & \nu_{\mu_L} e^- \bar{\nu}_{e_L} \\ & L & \rightarrow & LLR \\ P & \mu^- & \rightarrow & \nu_{\mu_L} e^- \bar{\nu}_{e_L} \\ & R & \rightarrow & RRL \\ C & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} e^+ \nu_{e_L} \\ & L & \rightarrow & LLR \\ CP & \mu^+ & \rightarrow & \bar{\nu}_{\mu_L} e^+ \nu_{e_L} \\ & R & \rightarrow & RRL \end{array}$$

CPT always conserved

Neutral Kaons

$$K^\circ = |\bar{s}d\rangle$$
$$\bar{K}^\circ = -|s\bar{d}\rangle$$

-: strong Isospin anti-particle

Charge Conjugation

$$C K^\circ = C(|\bar{s}d\rangle)$$
$$= |s\bar{d}\rangle$$
$$= -\bar{K}^\circ$$
$$C\bar{K}^\circ = -|\bar{s}d\rangle$$
$$= -K^\circ$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi-\theta, \pi+\phi) = (-1)^\ell Y(\theta, \phi)$
- multiplicative: $P(p_1 p_2) = P(p_1) \cdot P(p_2)$
- Spinor: $\gamma^0 \psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

Neutral Kaons

$$K^\circ = |\bar{s}d\rangle$$
$$\bar{K}^\circ = -|s\bar{d}\rangle$$

-: strong Isospin anti-particle

Charge Conjugation

$$CK^\circ = C(|\bar{s}d\rangle)$$
$$= |s\bar{d}\rangle$$
$$= -\bar{K}^\circ$$
$$C\bar{K}^\circ = -|\bar{s}d\rangle$$
$$= -K^\circ$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi-\theta,\pi+\phi) = (-1)^\ell Y(\theta,\phi)$
- multiplicative: $P(p_1 p_2) = P(p_1)\cdot P(p_2)$
- Spinor: $\gamma^0\psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

Neutral Kaons

$$K^\circ = |\bar{s}d\rangle$$
$$\bar{K}^\circ = -|s\bar{d}\rangle$$

-: strong Isospin anti-particle

Charge Conjugation

$$CK^\circ = C(|\bar{s}d\rangle)$$
$$= |s\bar{d}\rangle$$
$$= -\bar{K}^\circ$$
$$C\bar{K}^\circ = -|\bar{s}d\rangle$$
$$= -K^\circ$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi - \theta, \pi + \phi) = (-1)^\ell Y(\theta, \phi)$
- multiplicative: $P(p_1 p_2) = P(p_1) \cdot P(p_2)$
- Spinor: $\gamma^0 \psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

Neutral Kaons

$$K^\circ = |\bar{s}d\rangle$$
$$\bar{K}^\circ = -|s\bar{d}\rangle$$

-: strong Isospin anti-particle

Charge Conjugation

$$CK^\circ = C(|\bar{s}d\rangle) = |s\bar{d}\rangle = -\bar{K}^\circ$$
$$C\bar{K}^\circ = -|\bar{s}d\rangle = -K^\circ$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi-\theta,\pi+\phi) = (-1)^\ell Y(\theta,\phi)$
- multiplicative: $P(p_1p_2) = P(p_1)\cdot P(p_2)$
- Spinor: $\gamma^0\psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

Neutral Kaons

$$\begin{aligned} K^\circ &= |\bar{s}d\rangle \\ \bar{K}^\circ &= -|s\bar{d}\rangle \end{aligned}$$

-: strong Isospin anti-particle

Charge Conjugation

$$\begin{aligned} CK^\circ &= C(|\bar{s}d\rangle) \\ &= |s\bar{d}\rangle \\ &= -\bar{K}^\circ \\ C\bar{K}^\circ &= -|\bar{s}d\rangle \\ &= -K^\circ \end{aligned}$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi-\theta,\pi+\phi) = (-1)^\ell Y(\theta,\phi)$
- multiplicative: $P(p_1 p_2) = P(p_1)\cdot P(p_2)$
- Spinor: $\gamma^0\psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

Neutral Kaons

$$\mathrm{K}^\circ = |\bar{\mathrm{s}}\mathrm{d}\rangle$$
$$\bar{\mathrm{K}}^\circ = -|\mathrm{s}\bar{\mathrm{d}}\rangle$$

-: strong Isospin anti-particle

Charge Conjugation

$$\begin{aligned} C\mathrm{K}^\circ &= C(|\bar{\mathrm{s}}\mathrm{d}\rangle) \\ &= |\mathrm{s}\bar{\mathrm{d}}\rangle \\ &= -\bar{\mathrm{K}}^\circ \\ C\bar{\mathrm{K}}^\circ &= -|\bar{\mathrm{s}}\mathrm{d}\rangle \\ &= -\mathrm{K}^\circ \end{aligned}$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi - \theta, \pi + \phi) = (-1)^\ell Y(\theta, \phi)$
- multiplicative: $P(p_1 p_2) = P(p_1) \cdot P(p_2)$
- Spinor: $\gamma^0 \psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

Neutral Kaons

$$\mathrm{K}^\circ = |\bar{s}d\rangle$$
$$\bar{\mathrm{K}}^\circ = -|s\bar{d}\rangle$$

-: strong Isospin anti-particle

Charge Conjugation

$$C\mathrm{K}^\circ = C(|\bar{s}d\rangle) = |s\bar{d}\rangle = -\bar{\mathrm{K}}^\circ$$
$$C\bar{\mathrm{K}}^\circ = -|\bar{s}d\rangle = -\mathrm{K}^\circ$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi - \theta, \pi + \phi) = (-1)^\ell Y(\theta, \phi)$
- multiplicative: $P(p_1 p_2) = P(p_1) \cdot P(p_2)$
- Spinor: $\gamma^0 \psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

Neutral Kaons

$$K^\circ = |\bar{s}d\rangle$$
$$\bar{K}^\circ = -|s\bar{d}\rangle$$

-: strong Isospin anti-particle

Charge Conjugation

$$CK^\circ = C(|\bar{s}d\rangle) = |s\bar{d}\rangle = -\bar{K}^\circ$$
$$C\bar{K}^\circ = -|\bar{s}d\rangle = -K^\circ$$

not Eigenstates of C

Parity

- $(-1)^\ell$ from $Y_{\ell m}(\pi - \theta, \pi + \phi) = (-1)^\ell Y(\theta, \phi)$
- multiplicative: $P(p_1 p_2) = P(p_1) \cdot P(p_2)$
- Spinor: $\gamma^0 \psi$ (DIRAC equation)
  - $\gamma^0 u(\mathbf{p}') = u(\mathbf{p})$
  - $\gamma^0 v(\mathbf{p}') = -v(\mathbf{p})$
- relative (-1) between particle and anti-particle

CP

$$
\begin{aligned}
CP\mathrm{K}^\circ &= (-1)\cdot(-1)^\ell(-\bar{\mathrm{K}}^\circ)\\
&= \bar{\mathrm{K}}^\circ\\
CP\bar{\mathrm{K}}^\circ &= (-1)\cdot(-1)^\ell(-\mathrm{K}^\circ)\\
&= \mathrm{K}^\circ
\end{aligned}
$$

CP Eigenstates

$$
\begin{aligned}
(+1):\quad K_1 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ+\bar{\mathrm{K}}^\circ)\\
(-1):\quad K_2 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ-\bar{\mathrm{K}}^\circ)
\end{aligned}
$$

strong prod, weak decay

$\pi^+\pi^-$

$$
\begin{aligned}
C(\pi^+\pi^-) &= \pi^-\pi^+\\
P(\pi^+\pi^-) &= P(\pi^-)P(\pi^+)\\
&= 1\\
CP(\pi^+\pi^-) &= 1
\end{aligned}
$$

$\pi^+\pi^-\pi^0$

$$
\begin{aligned}
C(\pi^0) &= C(\gamma)^2 = 1\\
P(\pi^0) &= -1\\
CP(\pi^+\pi^-\pi^0) &= CP(\pi^+\pi^-)\cdot\\
&\quad CP(\pi^0)\\
&= -1
\end{aligned}
$$

CP

$$\begin{aligned} CP\mathrm{K}^\circ &= (-1)\cdot(-1)^\ell(-\bar{\mathrm{K}}^\circ) \\ &= \bar{\mathrm{K}}^\circ \\ CP\bar{\mathrm{K}}^\circ &= (-1)\cdot(-1)^\ell(-\mathrm{K}^\circ) \\ &= \mathrm{K}^\circ \end{aligned}$$

CP Eigenstates

$$\begin{aligned} (+1):\quad K_1 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ + \bar{\mathrm{K}}^\circ) \\ (-1):\quad K_2 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ - \bar{\mathrm{K}}^\circ) \end{aligned}$$

strong prod, weak decay

$\pi^+\pi^-$

$$\begin{aligned} C(\pi^+\pi^-) &= \pi^-\pi^+ \\ P(\pi^+\pi^-) &= P(\pi^-)P(\pi^+) \\ &= 1 \\ CP(\pi^+\pi^-) &= 1 \end{aligned}$$

$\pi^+\pi^-\pi^0$

$$\begin{aligned} C(\pi^0) &= C(\gamma)^2 = 1 \\ P(\pi^0) &= -1 \\ CP(\pi^+\pi^-\pi^0) &= CP(\pi^+\pi^-)\cdot \\ &\quad CP(\pi^0) \\ &= -1 \end{aligned}$$

CP

$$
\begin{aligned}
CP\mathrm{K}^\circ &= (-1)\cdot(-1)^\ell(-\bar{\mathrm{K}}^\circ) \\
&= \bar{\mathrm{K}}^\circ \\
CP\bar{\mathrm{K}}^\circ &= (-1)\cdot(-1)^\ell(-\mathrm{K}^\circ) \\
&= \mathrm{K}^\circ
\end{aligned}
$$

CP Eigenstates

$$
\begin{aligned}
(+1): \quad K_1 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ + \bar{\mathrm{K}}^\circ) \\
(-1): \quad K_2 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ - \bar{\mathrm{K}}^\circ)
\end{aligned}
$$

strong prod, weak decay

$\pi^+\pi^-$

$$
\begin{aligned}
C(\pi^+\pi^-) &= \pi^-\pi^+ \\
P(\pi^+\pi^-) &= P(\pi^-)P(\pi^+) \\
&= 1 \\
CP(\pi^+\pi^-) &= 1
\end{aligned}
$$

$\pi^+\pi^-\pi^0$

$$
\begin{aligned}
C(\pi^0) &= C(\gamma)^2 = 1 \\
P(\pi^0) &= -1 \\
CP(\pi^+\pi^-\pi^0) &= CP(\pi^+\pi^-)\cdot CP(\pi^0) \\
&= -1
\end{aligned}
$$

CP

$$
\begin{aligned}
CP\mathrm{K}^\circ &= (-1)\cdot(-1)^\ell(-\bar{\mathrm{K}}^\circ)\\
&= \bar{\mathrm{K}}^\circ\\
CP\bar{\mathrm{K}}^\circ &= (-1)\cdot(-1)^\ell(-\mathrm{K}^\circ)\\
&= \mathrm{K}^\circ
\end{aligned}
$$

CP Eigenstates

$$
\begin{aligned}
(+1):\quad K_1 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ + \bar{\mathrm{K}}^\circ)\\
(-1):\quad K_2 &= \frac{1}{\sqrt{2}}(\mathrm{K}^\circ - \bar{\mathrm{K}}^\circ)
\end{aligned}
$$

strong prod, weak decay

$\pi^+\pi^-$

$$
\begin{aligned}
C(\pi^+\pi^-) &= \pi^-\pi^+\\
P(\pi^+\pi^-) &= P(\pi^-)P(\pi^+)\\
&= 1\\
CP(\pi^+\pi^-) &= 1
\end{aligned}
$$

$\pi^+\pi^-\pi^0$

$$
\begin{aligned}
C(\pi^0) &= C(\gamma)^2 = 1\\
P(\pi^0) &= -1\\
CP(\pi^+\pi^-\pi^0) &= CP(\pi^+\pi^-)\cdot\\
&\quad CP(\pi^0)\\
&= -1
\end{aligned}
$$

Lifetimes

Kaon mass: $494 MeV$

$$K_1 \rightarrow \pi^+\pi^-$$
$$\tau_S = 0.9 \cdot 10^{-10} s$$
$$K_2 \rightarrow \pi^+\pi^-\pi^0$$
$$\tau_L = 5.2 \cdot 10^{-8} s$$

phase space:

$$m(\pi^+\pi^-) \approx 280 MeV$$
$$m(\pi^+\pi^-\pi^0) \approx 420 MeV$$

K_2 was initially “overlooked”

Time dependence

Decay is described by weak Eigenstates with a well-defined lifetime:

$$|K_1(t)\rangle = |K_1(0)\rangle \exp^{-iM_S t} \exp^{-\Gamma_S t/2}$$
$$|K_2(t)\rangle = |K_2(0)\rangle \exp^{-iM_L t} \exp^{-\Gamma_L t/2}$$

strong as f(weak):

$$K^\circ = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle)$$
$$\bar{K}^\circ = \frac{1}{\sqrt{2}}(|K_1\rangle - |K_2\rangle)$$

Lifetimes

Kaon mass: $494 MeV$

$$\begin{aligned} K_1 &\rightarrow \pi^+\pi^- \\ \tau_S &= 0.9 \cdot 10^{-10} s \\ K_2 &\rightarrow \pi^+\pi^-\pi^0 \\ \tau_L &= 5.2 \cdot 10^{-8} s \end{aligned}$$

phase space:

$$\begin{aligned} m(\pi^+\pi^-) &\approx 280 MeV \\ m(\pi^+\pi^-\pi^0) &\approx 420 MeV \end{aligned}$$

K_2 was initially “overlooked”

Time dependence

Decay is described by weak Eigenstates with a well-defined lifetime:

$$\begin{aligned} |K_1(t)\rangle &= |K_1(0)\rangle \exp^{-iM_S t} \exp^{-\Gamma_S t/2} \\ |K_2(t)\rangle &= |K_2(0)\rangle \exp^{-iM_L t} \exp^{-\Gamma_L t/2} \end{aligned}$$

strong as f(weak):

$$\begin{aligned} \mathrm{K}^\circ &= \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle) \\ \bar{\mathrm{K}}^\circ &= \frac{1}{\sqrt{2}}(|K_1\rangle - |K_2\rangle) \end{aligned}$$

Oscillation

$A(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\bar{\mathrm{K}}^\circ$:

$$
\begin{aligned}
A(t) &= \langle \bar{\mathrm{K}}^\circ(t) | \mathrm{K}^\circ(t=0) \rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t}\exp^{-\Gamma_S t/2} - \exp^{-iM_L t}\exp^{-\Gamma_L t/2}) \\
A(t)A^\star(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}
$$

- $\Gamma_S \gg \Gamma_L$: decay with Γ_L
- oscillation with frequency $\Delta M = M_L - M_S$

Oscillation

$A(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\overline{\mathrm{K}}^\circ$:

$$
\begin{aligned}
A(t) &= \langle \overline{\mathrm{K}}^\circ(t) | \mathrm{K}^\circ(t=0)\rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t}\exp^{-\Gamma_S t/2} - \exp^{-iM_L t}\exp^{-\Gamma_L t/2}) \\
A(t)A^\star(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}
$$

- $\Gamma_S \gg \Gamma_L$: decay with Γ_L
- oscillation with frequency $\Delta M = M_L - M_S$

Oscillation

$A(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\overline{\mathrm{K}}^\circ$:

$$
\begin{aligned}
A(t) &= \langle \overline{\mathrm{K}}^\circ(t) | \mathrm{K}^\circ(t=0)\rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t}\exp^{-\Gamma_S t/2} - \exp^{-iM_L t}\exp^{-\Gamma_L t/2}) \\
A(t)A^\star(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}
$$

- $\Gamma_S \gg \Gamma_L$: decay with Γ_L
- oscillation with frequency $\Delta M = M_L - M_S$

Oscillation

$A(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\overline{\mathrm{K}}^\circ$:

$$
\begin{aligned}
A(t) &= \langle \overline{\mathrm{K}}^\circ(t) | \mathrm{K}^\circ(t=0)\rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t}\exp^{-\Gamma_S t/2} - \exp^{-iM_L t}\exp^{-\Gamma_L t/2}) \\
A(t)A^\star(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}
$$

- $\Gamma_S \gg \Gamma_L$: decay with Γ_L
- oscillation with frequency $\Delta M = M_L - M_S$

Oscillation

$A(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\overline{\mathrm{K}}^\circ$:

$$\begin{aligned}
A(t) &= \langle \overline{\mathrm{K}}^\circ(t) | \mathrm{K}^\circ(t=0)\rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t}\exp^{-\Gamma_S t/2} - \exp^{-iM_L t}\exp^{-\Gamma_L t/2}) \\
A(t)A^\star(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}$$

- $\Gamma_S \gg \Gamma_L$: decay with Γ_L
- oscillation with frequency $\Delta M = M_L - M_S$

Oscillation

$A(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\overline{\mathrm{K}}^\circ$:

$$\begin{aligned}
A(t) &= \langle \overline{\mathrm{K}}^\circ(t) | \mathrm{K}^\circ(t=0)\rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t}\exp^{-\Gamma_S t/2} - \exp^{-iM_L t}\exp^{-\Gamma_L t/2}) \\
A(t)A^\star(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}$$

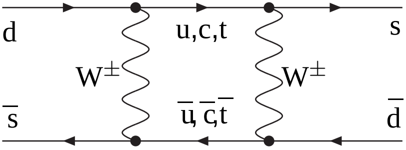
- $\Gamma_S \gg \Gamma_L$: decay with Γ_L
- oscillation with frequency $\Delta M = M_L - M_S$

Oscillation

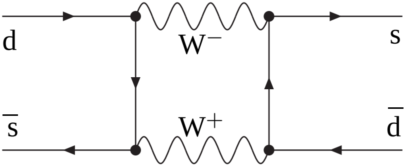
$A(t)$: amplitude to produce at $t = 0$ a K° and find at t a $\overline{\mathrm{K}}^\circ$:

$$
\begin{aligned}
A(t) &= \langle \overline{\mathrm{K}}^\circ(t) | \mathrm{K}^\circ(t=0)\rangle \\
&= \tfrac{1}{2}(\langle K_1(t)| - \langle K_2(t)|)(|K_1(t=0)\rangle + |K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\langle K_1(t)|K_1(t=0)\rangle - \langle K_2(t)|K_2(t=0)\rangle) \\
&= \tfrac{1}{2}(\exp^{-iM_S t}\exp^{-\Gamma_S t/2} - \exp^{-iM_L t}\exp^{-\Gamma_L t/2}) \\
A(t)A^\star(t) &= \tfrac{1}{4}(\exp^{-\Gamma_S t} + \exp^{-\Gamma_L t} - 2\cos(\Delta M t)\exp^{-(\Gamma_L+\Gamma_S)t/2})
\end{aligned}
$$

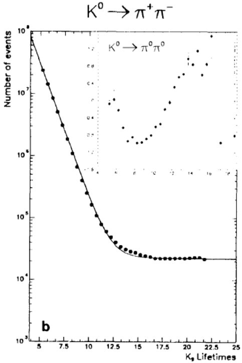
- $\Gamma_S \gg \Gamma_L$: decay with Γ_L
- oscillation with frequency $\Delta M = M_L - M_S$



Follow fermion line: transition between generations inevitable!

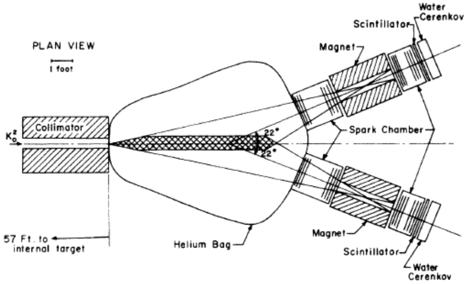


Need *CKM* non-diagonal: $\sim \sin^2 \theta_C$



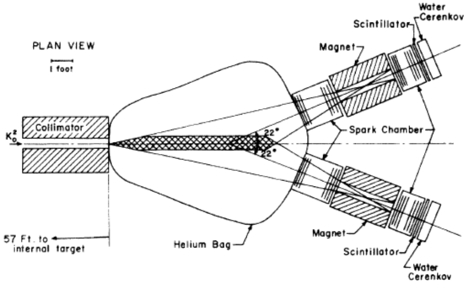
- need interference term!
- $\Delta M \sim 3.5 \cdot 10^{-6} eV$

All settled?



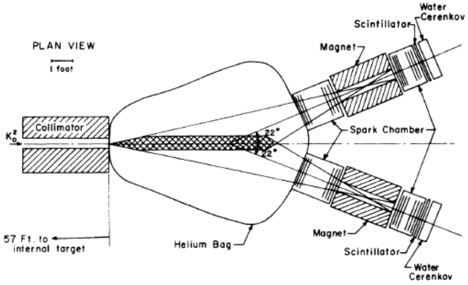
- BNL AGS 30GeV protons
- K_1 die out (can be regenerated)
- expect no $\pi^+\pi^-$ decays at the K° mass (theoretically)
- experimentally: combinatorics
- use angle between beam and reconstructed $\pi^+\pi^-$ system
- no peak expected

All settled?



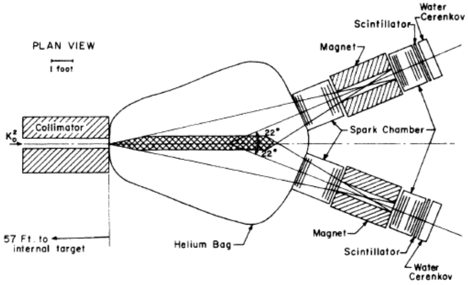
- BNL AGS 30GeV protons
- K_1 die out (can be regenerated)
- expect no $\pi^+\pi^-$ decays at the K° mass (theoretically)
- experimentally: combinatorics
- use angle between beam and reconstructed $\pi^+\pi^-$ system
- no peak expected

All settled?



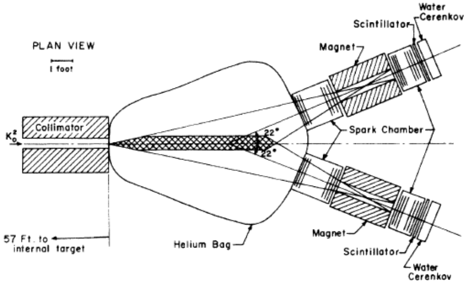
- BNL AGS 30GeV protons
- K_1 die out (can be regenerated)
- expect no $\pi^+\pi^-$ decays at the K° mass (theoretically)
- experimentally: combinatorics
- use angle between beam and reconstructed $\pi^+\pi^-$ system
- no peak expected

All settled?



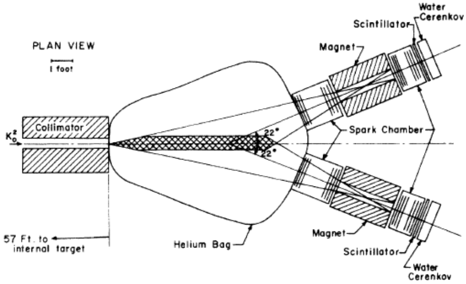
- BNL AGS 30GeV protons
- K_1 die out (can be regenerated)
- expect no $\pi^+\pi^-$ decays at the K° mass (theoretically)
- experimentally: combinatorics
- use angle between beam and reconstructed $\pi^+\pi^-$ system
- no peak expected

All settled?

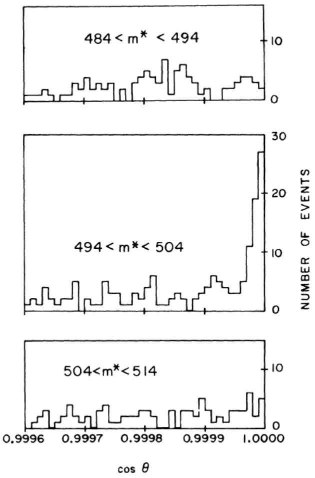


- BNL AGS 30GeV protons
- K_1 die out (can be regenerated)
- expect no $\pi^+\pi^-$ decays at the K° mass (theoretically)
- experimentally: combinatorics
- use angle between beam and reconstructed $\pi^+\pi^-$ system
- no peak expected

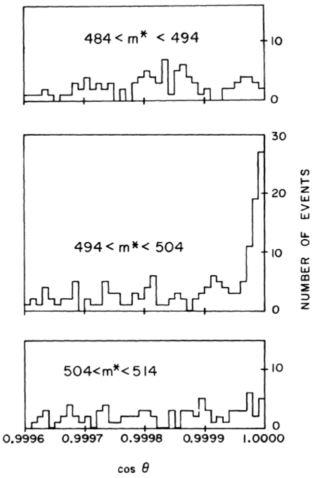
All settled?



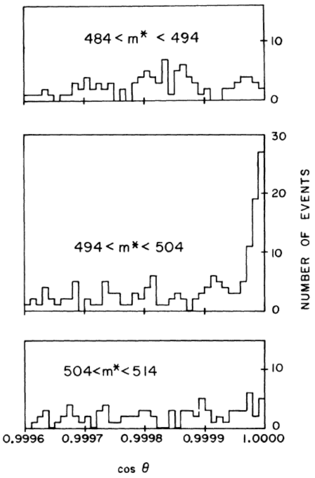
- BNL AGS 30GeV protons
- K_1 die out (can be regenerated)
- expect no $\pi^+\pi^-$ decays at the K° mass (theoretically)
- experimentally: combinatorics
- use angle between beam and reconstructed $\pi^+\pi^-$ system
- no peak expected



- PEAK!!!
- level: 10^{-3}
- *CP* must be violated!
- *CKM* has a complex phase



- PEAK!!!
- level: 10^{-3}
- *CP* must be violated!
- *CKM* has a complex phase



- PEAK!!!
- level: 10^{-3}
- *CP* must be violated!
- *CKM* has a complex phase

Kaon description

$$K_S = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle)$$
$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}(\epsilon|K_1\rangle + |K_2\rangle)$$

CP violation in mixing

$$|\epsilon| = \sqrt{\frac{\Gamma_L(\pi^+\pi^-)}{\Gamma_S(\pi^+\pi^-)}} = 2.268 \pm 0.023 \cdot 10^{-3}$$

CP violation in decay

$$Re(\tfrac{\epsilon'}{\epsilon}) = \frac{1}{6}\left(1 - \frac{\Gamma_L(\pi^0\pi^0)\Gamma_S(\pi^+\pi^-)}{\Gamma_S(\pi^0\pi^0)\Gamma_L(\pi^+\pi^-)}\right)$$
$$(NA31) = 23 \pm 6.5 \cdot 10^{-4}$$
$$(FNAL) = 7.4 \pm 5.9 \cdot 10^{-4}$$
$$(FNAL) = 28 \pm 4.1 \cdot 10^{-4}$$
$$(NA48) = 18.5 \pm 7.3 \cdot 10^{-4}$$

- *CP* violation discovered
- good for our existence
- $\alpha_S(\mathrm{K}^\circ)$!

Kaon description

$$K_S = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle)$$
$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}(\epsilon|K_1\rangle + |K_2\rangle)$$

CP violation in mixing

$$|\epsilon| = \sqrt{\frac{\Gamma_L(\pi^+\pi^-)}{\Gamma_S(\pi^+\pi^-)}}$$
$$= 2.268 \pm 0.023 \cdot 10^{-3}$$

CP violation in decay

$$Re(\frac{\epsilon'}{\epsilon}) = \frac{1}{6}(1 - \frac{\Gamma_L(\pi^0\pi^0)\Gamma_S(\pi^+\pi^-)}{\Gamma_S(\pi^0\pi^0)\Gamma_L(\pi^+\pi^-)})$$
$$(NA31) = 23 \pm 6.5 \cdot 10^{-4}$$
$$(FNAL) = 7.4 \pm 5.9 \cdot 10^{-4}$$
$$(FNAL) = 28 \pm 4.1 \cdot 10^{-4}$$
$$(NA48) = 18.5 \pm 7.3 \cdot 10^{-4}$$

- *CP* violation discovered
- good for our existence
- $\alpha_S(\mathrm{K}^\circ)$!

Kaon description

$$K_S = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle)$$
$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}(\epsilon|K_1\rangle + |K_2\rangle)$$

CP violation in mixing

$$|\epsilon| = \sqrt{\frac{\Gamma_L(\pi^+\pi^-)}{\Gamma_S(\pi^+\pi^-)}}$$
$$= 2.268 \pm 0.023 \cdot 10^{-3}$$

CP violation in decay

$$Re(\frac{\epsilon'}{\epsilon}) = \frac{1}{6}(1 - \frac{\Gamma_L(\pi^0\pi^0)\Gamma_S(\pi^+\pi^-)}{\Gamma_S(\pi^0\pi^0)\Gamma_L(\pi^+\pi^-)})$$
$$(NA31) = 23 \pm 6.5 \cdot 10^{-4}$$
$$(FNAL) = 7.4 \pm 5.9 \cdot 10^{-4}$$
$$(FNAL) = 28 \pm 4.1 \cdot 10^{-4}$$
$$(NA48) = 18.5 \pm 7.3 \cdot 10^{-4}$$

- *CP* violation discovered
- good for our existence
- $\alpha_S(K^\circ)$!

Kaon description

$$K_S = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_1\rangle + \epsilon|K_2\rangle)$$
$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}(\epsilon|K_1\rangle + |K_2\rangle)$$

CP violation in mixing

$$|\epsilon| = \sqrt{\frac{\Gamma_L(\pi^+\pi^-)}{\Gamma_S(\pi^+\pi^-)}}$$
$$= 2.268 \pm 0.023 \cdot 10^{-3}$$

CP violation in decay

$$Re(\frac{\epsilon'}{\epsilon}) = \frac{1}{6}(1 - \frac{\Gamma_L(\pi^0\pi^0)\Gamma_S(\pi^+\pi^-)}{\Gamma_S(\pi^0\pi^0)\Gamma_L(\pi^+\pi^-)})$$
$$(NA31) = 23 \pm 6.5 \cdot 10^{-4}$$
$$(FNAL) = 7.4 \pm 5.9 \cdot 10^{-4}$$
$$(FNAL) = 28 \pm 4.1 \cdot 10^{-4}$$
$$(NA48) = 18.5 \pm 7.3 \cdot 10^{-4}$$

- *CP* violation discovered
- good for our existence
- $\alpha_S(\mathrm{K}^\circ)$!

B-sector

- Flavour oscillation in all neutral systems
- $m_{\mathrm{B}^\circ} \sim 5GeV \gg m_{\mathrm{K}^\circ} \sim 0.5GeV$
- lifetime (tag)

Experiments

large production

- dedicated machine: $\mathrm{e}^+\mathrm{e}^-$
- or pp
- good PID

- BABAR@SLAC PEP-II
- BELLE@KEK-B
- asymmetric colliders
  - e^-: 9.1GeV
  - e^+: 3.4GeV
- Υ^{4s}:
  - resonance $\mathrm{b\bar{b}}$
  - decay to $\mathrm{B_d^\circ \bar{B}_d^\circ}$ and $\mathrm{B_s^\circ \bar{B}_s^\circ}$
  - 250μm need great vertex detector

B-sector

- Flavour oscillation in all neutral systems
- $m_{\mathrm{B}^\circ} \sim 5GeV \gg m_{\mathrm{K}^\circ} \sim 0.5GeV$
- lifetime (tag)

Experiments

large production

- dedicated machine: $\mathrm{e}^+\mathrm{e}^-$
- or pp
- good PID

- BABAR@SLAC PEP-II
- BELLE@KEK-B
- asymmetric colliders
  - e^-: 9.1GeV
  - e^+: 3.4GeV
- Υ^{4s}:
  - resonance $\mathrm{b}\bar{\mathrm{b}}$
  - decay to $\mathrm{B}_\mathrm{d}^\circ\bar{\mathrm{B}}_\mathrm{d}^\circ$ and $\mathrm{B}_\mathrm{s}^\circ\bar{\mathrm{B}}_\mathrm{s}^\circ$
  - 250μm need great vertex detector

B-sector

- Flavour oscillation in all neutral systems
- $m_{B^\circ} \sim 5GeV \gg m_{K^\circ} \sim 0.5GeV$
- lifetime (tag)

Experiments

large production

- dedicated machine: e^+e^-
- or pp
- good PID

- BABAR@SLAC PEP-II
- BELLE@KEK-B
- asymmetric colliders
  - e^-: 9.1GeV
  - e^+: 3.4GeV
- Υ^{4s}:
  - resonance $b\bar{b}$
  - decay to $B_d^\circ \bar{B}_d^\circ$ and $B_s^\circ \bar{B}_s^\circ$
  - 250μm need great vertex detector

Back to *CKM*

- V complex
- $\mathrm{V}^\dagger \mathrm{V} = 1_3$: 9 equations
- 6 equations with complex $= 0$
- 2-coordinate plane: triangle
- α, β, γ

Unitary triangle

$$(\mathrm{V}^\dagger \mathrm{V})_{31} = 0 = V^\star_{ub} V_{ud} + V^\star_{cb} V_{cd} + V^\star_{tb} V_{td}$$

Measurements

- Δm_s
- Δm_d
- $\bar{\mathrm{B}}^\circ \to \pi^+\pi^-$
- $\bar{\mathrm{B}}^\circ \to J/\psi K_S$
- $B^+ \to DK^+$
- $B \to \tau\nu$
- overconstrained

Back to *CKM*

- V complex
- $\mathrm{V}^\dagger \mathrm{V} = 1_3$: 9 equations
- 6 equations with complex $= 0$
- 2-coordinate plane: triangle
- α, β, γ

Unitary triangle

$$
\begin{aligned}
(\mathrm{V}^\dagger \mathrm{V})_{31} &= 0 \\
&= V_{ub}^\star V_{ud} + V_{cb}^\star V_{cd} + V_{tb}^\star V_{td}
\end{aligned}
$$

Measurements

- Δm_s
- Δm_d
- $\bar{\mathrm{B}}^\circ \rightarrow \pi^+\pi^-$
- $\bar{\mathrm{B}}^\circ \rightarrow J/\psi K_S$
- $B^+ \rightarrow DK^+$
- $B \rightarrow \tau\nu$
- overconstrained

Back to *CKM*

- V complex
- $V^\dagger V = 1_3$: 9 equations
- 6 equations with complex $= 0$
- 2-coordinate plane: triangle
- α, β, γ

Unitary triangle

$$(V^\dagger V)_{31} = 0$$
$$= V^\star_{ub} V_{ud} + V^\star_{cb} V_{cd} + V^\star_{tb} V_{td}$$

Measurements

- Δm_s
- Δm_d
- $\bar{B}^\circ \to \pi^+\pi^-$
- $\bar{B}^\circ \to J/\psi K_S$
- $B^+ \to DK^+$
- $B \to \tau\nu$
- overconstrained

Back to *CKM*

- V complex
- $\mathrm{V}^\dagger \mathrm{V} = 1_3$: 9 equations
- 6 equations with complex $= 0$
- 2-coordinate plane: triangle
- α, β, γ

Unitary triangle

$$
\begin{aligned}
(\mathrm{V}^\dagger \mathrm{V})_{31} &= 0 \\
&= V^\star_{ub} V_{ud} + V^\star_{cb} V_{cd} + V^\star_{tb} V_{td}
\end{aligned}
$$

Measurements

- Δm_s
- Δm_d
- $\overline{\mathrm{B}}^\circ \to \pi^+\pi^-$
- $\overline{\mathrm{B}}^\circ \to J/\psi K_S$
- $B^+ \to DK^+$
- $B \to \tau\nu$
- overconstrained

- relationship angles-V: Problem Solving
- all measurements in agreement
- no sign of BSM
- impressive progress in 10 years
- D0 like-sign di-muons?