Coupling to the radially excited charm

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November 28, 2012

Summary

Subject of our study

- Radially excited $D_{(s)}$ meson
- First attempt to determine mass and coupling:
  - on the lattice
  - in the continuum limit
- Caution: results still preliminary!

Contents

Phenomenological motivation

BaBar studies of $D^*\pi$ production invariant mass distribution

Two peaks reported (arXiv:1009.2076):

$$m_1 = 2539 \pm 8\,\mathrm{MeV}, \qquad \Gamma_1 = 130 \pm 18\,\mathrm{MeV}$$
$$m_2 = 2752 \pm 3\,\mathrm{MeV}, \qquad \Gamma_2 = 71 \pm 13\,\mathrm{MeV}$$

m_1 believed to be the $2S$ radially excited D'

Analysis of $B^- \to D_s^{(*)+} K^- \ell^- \bar{\nu}_\ell$ and of $B^- \to D_s^{(*)+} K^- \pi^-$

- BaBar (2010), Belle (2012): $\mathcal{B}(B^- \to D_s^{(*)+} K^- \ell^- \bar{\nu}_\ell) \sim 0.06\%$
- Belle: peak in invariant mass distribution of $D_s^{(*)+} K^-$:
  - around 2.6 GeV when associated to leptons
  - around $2.8 - 3.0$ GeV when associated to π

Identification of radial excitation

- How to interpret the peaks: which one is the radial excitation?
- What is its coupling?

Phenomenological motivation & Theoretical predictions

Relevance of the D'^0 coupling determination

$$A_{\text{fact.}}(B^- \to \bar{D}'^0 \pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[a_1 f_\pi (m_B^2 - m_{D'}^2) F_0^{B\to D'}(m_\pi^2) + a_2 f_{D'} (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_{D'}^2) \right]$$

If the ratio of class-1 and class-3 decays was measured:
$\to$ $f_{D'}$ would be needed for theoretical prediction

Theoretical studies of D' system

- Mass of D':
  - Godfrey, Isgur, 1985: $m_{D'} = 2.58$ GeV with quark model
  - Mohler and Woloshyn (arXiv:1103.5506) on the lattice: single lattice spacing, various sea quark masses: $m_{D'} = 2.6(3)$ GeV
- General statement on $f_{D'}$: $f_{D'} > f_D$ (from covariant quark model, lattice QCD in the static limit)

Further studies are needed!

Motivation: why to use lattice QCD?

Basically because... Lattice QCD = QCD!

- Non-perturbative computation starting from first principles
- Reach of the method limited only by available computing power

Last years developments (since 2005)

- Unquenching of *Up* and *Down* quarks (assuming isospin symmetry)
- Extrapolation toward physical pion mass (not directly accessible normally)
- Extrapolation toward the continuum limit

Nowadays working area and next-to-come developments

- Full unquenching of all the quarks (also beyond isospin symmetry)
- Calculation performed directly at the physical pion mass
- Improved continuum limit thanks to more sophisticated regularizations
- Electromagnetic corrections taken into account

Our Setup: Simulation Parameters

Fermionic part: Twisted mass QCD at maximal twist

$$S^{ferm} = \bar{\psi}^a \left[\left(1/2\kappa + K[U]\right) \delta^{ab} + \mathbf{i}\mu\gamma_5\tau_3^{ab} \right] \psi^b, \qquad a, b \in \{\psi_{up}, \psi_{down}\}$$

Original Wilson formulation, with **additional mass term**

No additive mass renormalization, automatic $\mathcal{O}(a)$ improvement

Gauge field configurations

- Provided by European Twisted Mass Collaboration
  9 countries, $\sim$20 universities and research centers
- Unquenched *Up Down* quarks
- Publicly available on International Lattice Data Grid

Subset used for this work

- Single lattice spacing (4 available)
- Single light quark mass (pion mass $\sim$400 MeV)

How to compute decay constants

Decay constant computation when $D^{(\prime)}$ at rest

$$\langle 0| \left(A + \not{V}\right)_0^{ren} |D^{(\prime)}(p=0)\rangle = M_D f_D, \qquad \langle 0| V_0^{ren} |D^{(\prime)}\rangle = 0$$

With Twisted mass regularization

$$\langle 0| A_0^{ren} |D^{(\prime)}\rangle = \frac{m_l + m_c}{M_D} \langle 0| P |D^{(\prime)}\rangle, \quad A_0 = \bar{c}\gamma_0\gamma_5 d, \; P = i\bar{c}\gamma_5 d$$

Decay constant is expressed in terms of RGI product

$$f_{D^{(\prime)}} = \frac{(m_l + m_c) \langle 0| P |D^{(\prime)}\rangle}{M^2_{D^{(\prime)}}}$$

We must compute Pseudoscalar matrix element between $|0\rangle$ and $|D\rangle$:

$$Z_{D^{(\prime)}} \equiv \langle 0| P |D^{(\prime)}\rangle$$

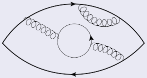
this can be obtained from 2 points correlation function fit

Spectral decomposition of two points functions

Basic ingredient: two points correlation functions

$$O(x) \equiv \bar{c}(x)\gamma_5 q(x)\,. \quad q = \{u/d,\, s,\, c\}$$

$$C(\tau) = \sum_{\vec{x}} \left\langle O^\dagger(\vec{x},\tau)\, O(\vec{0},0) \right\rangle =$$



Determination of ground state properties

Inserting the sum of projectors over all the states $\sum_i |D^{(i)}\rangle\langle D^{(i)}|$:

$$C(\tau) = \sum_i \left| \langle 0|\, O |D^{(i)}\rangle \right|^2 \frac{e^{-M_{D^{(i)}}\tau}}{2M_{D^{(i)}}} \quad \xrightarrow{\tau\to\infty} \quad |\underbrace{\langle 0|\, O\,|D\rangle}_{Z_D}|^2 \frac{e^{-M_D\tau}}{2M_D}$$

Information on excited states extracted by looking at intermediate times:

$$0 \ll t \ll t_{ground}$$

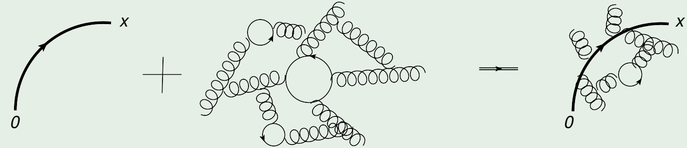
Two points correlation function computation

Wick contraction

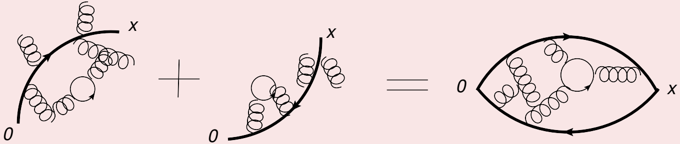
$$C(\tau) = \sum_{\vec{x}} \left\langle O_\Gamma^\dagger(\vec{x},\tau)\, O_\Gamma\left(\vec{0},0\right)\right\rangle \underset{Wick}{=} \mathrm{Tr}\left[\Gamma S_l\left(\vec{x},\tau;\vec{0},0\right)\Gamma S_c\left(\vec{0},0;\vec{x},\tau\right)\right]$$

Solving Dirac equation on gauge background → full quark propagator

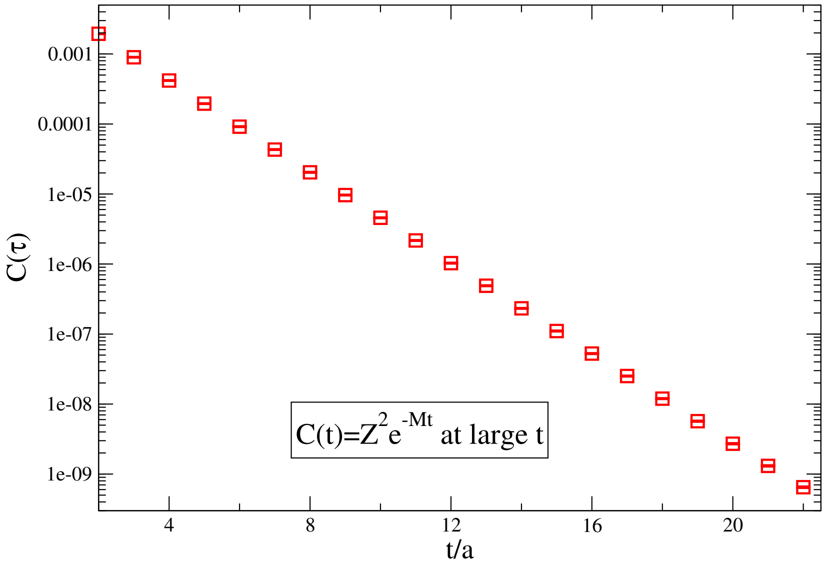
$$D_q(y,x)\cdot S_q(x,0) = \delta_{y,0} \qquad D_q = \left(\tfrac{1}{2\kappa} + K[U]\right)\mathbf{1} + i m_q \gamma_5 \tau_3$$



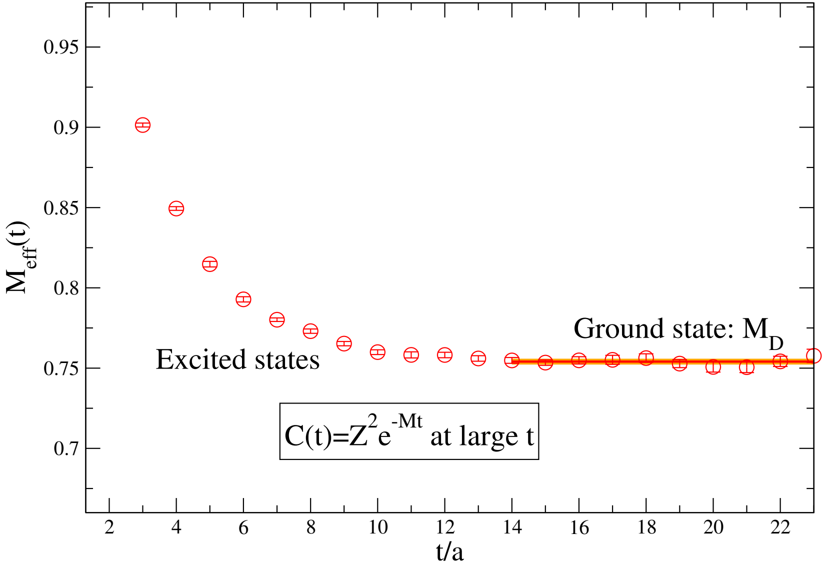
Combine 2 propagators with Dirac structures → 2 points functions



Two points correlation function (log scale)



Effective mass: $M_{eff}(\tau) \equiv \log C(\tau+1) - \log C(\tau)$



How to access to $f_{D'}$?

Access to excited states

Coming back to spectral decomposition

$$C\left(t\right)\equiv\sum_{\vec{x}}\left\langle O^{\dagger}\left(\vec{x},\tau\right)O\left(\vec{0},0\right)\right\rangle=\sum_{P}|\underbrace{\left\langle 0\right|O\left|P\right\rangle}_{Z(P)}|^{2}\frac{e^{-M_{P}t}}{2M_{P}}$$

Limiting to two lightest states

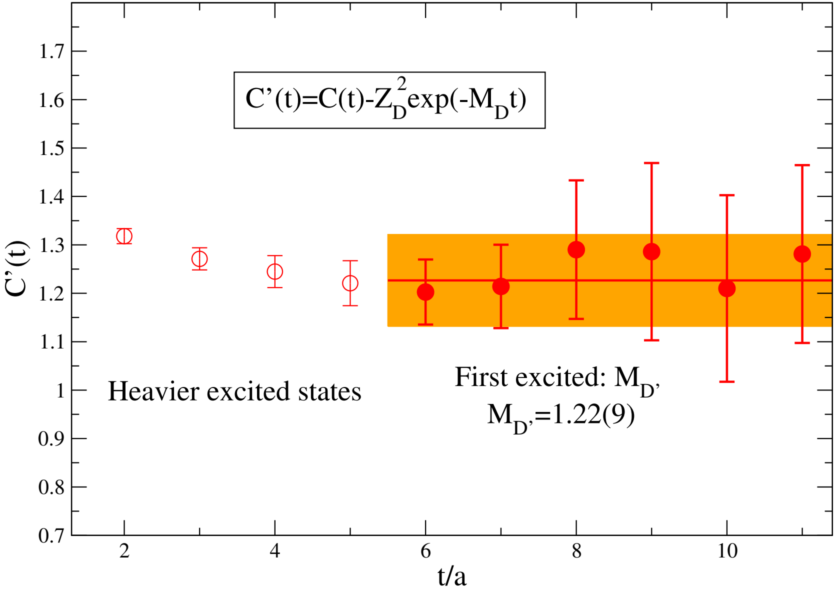
$$C\left(t\right)=Z^{2}\left(D\right)\frac{e^{-M_{D}t}}{2M_{D}}+Z^{2}\left(D'\right)\frac{e^{-M_{D'}t}}{2M_{D'}}\xrightarrow[t\to\infty]{}Z^{2}\left(D\right)\frac{e^{-M_{D}t}}{2M_{D}}$$

We determine $Z\left(D\right)$ and M_{D} by fitting $C\left(t\right)$ at large time

Subtract D state contribution from $C\left(t\right)$

$$C'\left(t\right)\equiv C\left(t\right)-Z^{2\,fit}\left(D\right)\frac{e^{-M_{D}^{fit}t}}{2M_{D}^{fit}}\xrightarrow[t\to\infty]{}Z^{2}\left(D'\right)\frac{e^{-M_{D'}t}}{2M_{D'}}$$

$C'(t)$ effective mass



Eigenvalue problem

How to build different interpolating operators

- $O_i(x) = \bar{\Psi}_i(x)\gamma_5\Psi_i(x)$ built in terms of $\Psi_i(x) = G_i(x,y)\,\psi(y)$
- G is a convolution operator (smearing)

Consider a $n \times n$ matrix of correlation functions

$$C_{ij}(\tau) \equiv \sum_{\vec{x}} \left\langle O_i^\dagger(\vec{x},\tau)\, O_j\left(\vec{0},0\right)\right\rangle = \sum_P \underbrace{\langle 0|\, O_i^\dagger\,|P\rangle}_{Z_i^*(P)} \underbrace{\langle P|\, O_j\,|0\rangle}_{Z_j(P)} \frac{e^{-M_P\tau}}{2M_P}$$

Diagonalization of correlation matrix

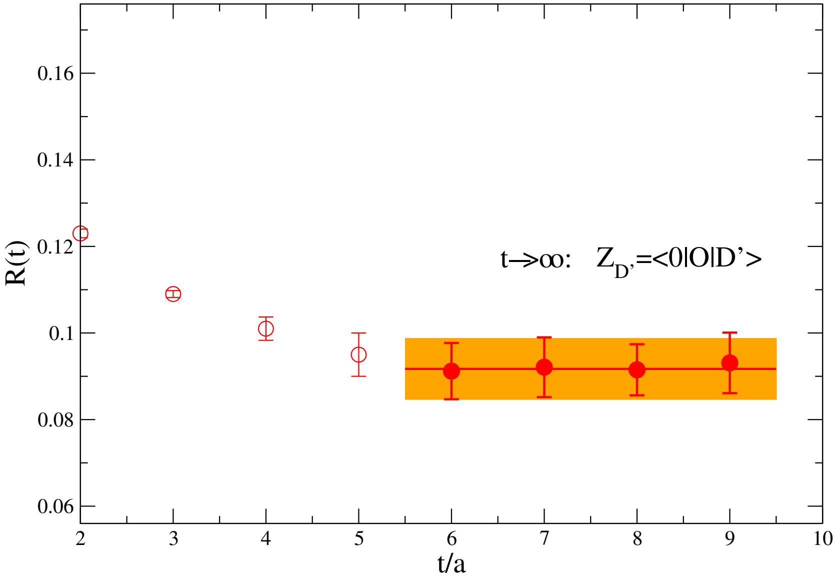
It can be shown that eigenvalues $\lambda_k(t)$ of matrix $C(t)$behaves like:

$$\lambda_0(\tau) = Z_D^* \exp(-M_D\tau),\ \lambda_1(\tau) = Z_{D'}^* \exp(-M_{D'}\tau)\ \ldots$$

$\vec{v}_k(t)$ define "optimal" interpolating operator coupling uniquely to state k:

$$O_D = \vec{v}_0\vec{O}, \quad O_{D'} = \vec{v}_1\vec{O}, \ \ldots$$

Determining $f_{D'}$: $R_{D'}(\tau) = \left(\vec{v}_1^T \cdot C(\tau) \cdot \vec{v}_1 \times e^{M_{D'}\tau}\right)^{-1/2}$



Conclusions

Preliminary results

- From the value of $Z_{D'}$ and Z_D: $\frac{f_{D'}}{f_D} = 0.78(6)$
- Relative mass difference: $\frac{M_{D'} - M_D}{M_D} = 59(6)\%$

Caveat: everything very preliminary!

- Single lattice spacing
- Not extrapolated to the physical pion mass

Future development

- Analyze the other 3 lattice spacings and take the chiral extrapolation
- All correlators already computed: expect the results in the next weeks

Thank you!