D-meson resonances in $D\pi$ scattering from lattice QCD

*Workshop on B decay into D** and related issues*

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[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Outline

- which D-mesons are "easy" to simulate and which not
- strategies for lattice simulations of observed D-mesons
- lattice results for the masses and widths of D-mesons
- comparison to experiment

ab-initio D-meson spectroscopy

- Which hadron masses can lattice QCD compute easily and reliably ?

  for hadrons that can not decay strongly : D

  D* (does not have enough phase space to decay on lattice)

  in this case : m=E when P=0

- But: all other D-mesons can decay strongly ; they are resonances !

  in some cases m=E applicable for very narrow resonances

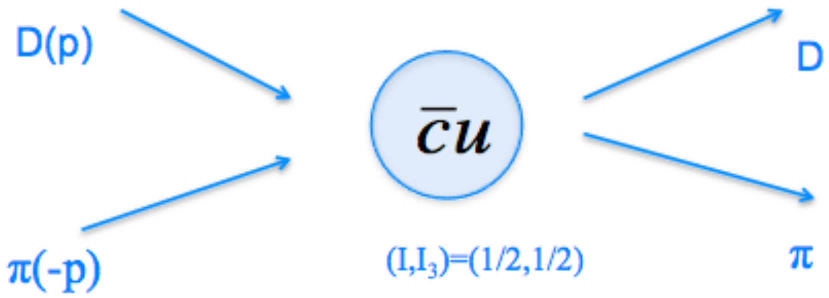
  but not for broad resonances!

- caution: all simulations of D-meson resonances assumed m=E up to now!

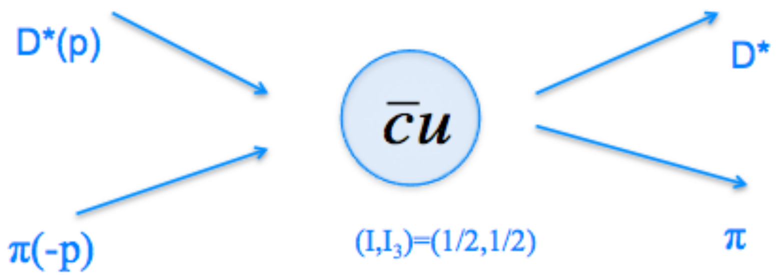
Resonances require:

simulation of scattering on the lattice

Dπ scattering

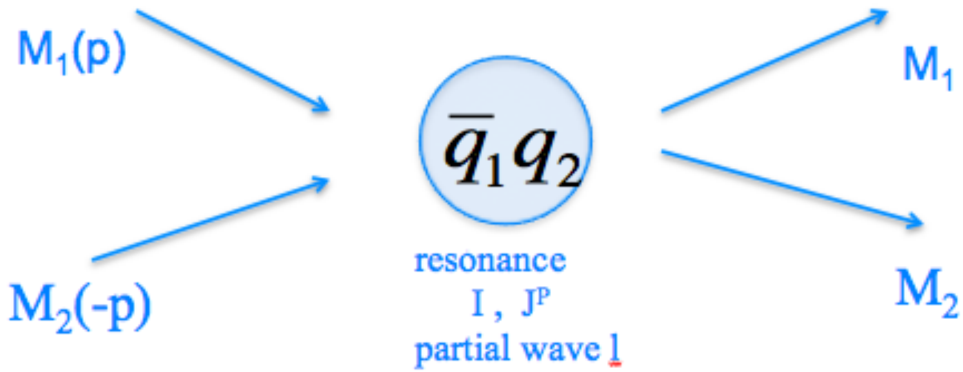


D*π scattering



Strategy for extracting resonance m and Γ

… in experiment, continuum and lattice.

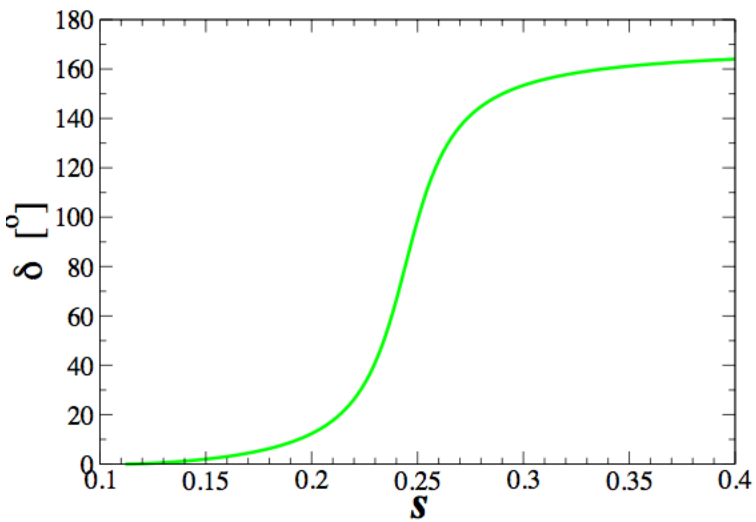


$$a = \frac{-\sqrt{s}\,\Gamma(s)}{s - m^2 + i\sqrt{s}\,\Gamma(s)} = \frac{1}{2i}\left(e^{2i\delta(s)} - 1\right)$$

$$m = \sqrt{s} \quad \text{where} \quad \delta = 90^o$$

$$\Gamma = \Gamma(s = m^2)$$

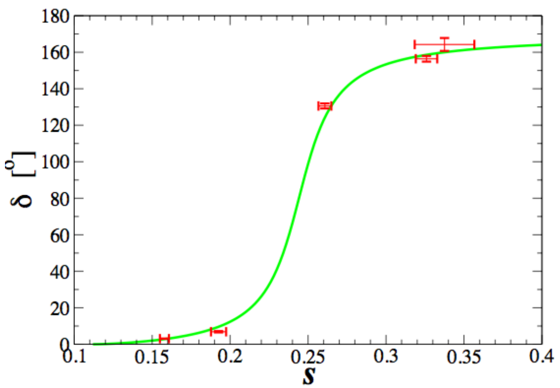
$$s \equiv E^2 - P^2$$



Problem: lattice studies of resonances in infancy !

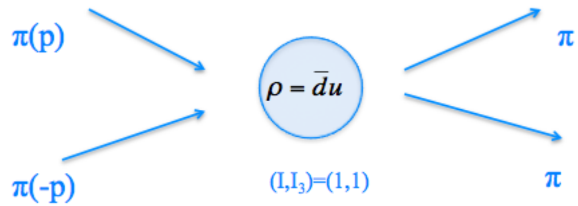
Only hadronic resonance that was properly simulated before 2012: ρ

LAT: [Lang, Mohler, S. P., Vidmar, 2011]



$J^{PC} = 1^{--}$

$l = 1$ (p - wave)



Several groups simluated ρ : Aoki et al. (2007), Gockeler et al (2008), Feng et al, Frison et al. (2010), Feng et al (2011), Aoki et al (2011), Lang et al (2011), Pelissier et al (2011, 2012)

Idea: Simulate other scattering in resonant channels

In particular: D-meson resonances in Dπ and D*π scattering

by default: our simulation is exploratory

Lattice simulation

- 280 gauge config with dynamical u,d quarks (generated by A. Hasenfratz)

  thanks !!

  $N_f = 2 \quad a = 0.1239 \pm 0.0013\ fm \quad a^{-1} = 1.58 \pm 0.02\ GeV$

  $N_L^3 \times N_T = 16^3 \times 32 \qquad L \approx 2\ fm \qquad T = 4\ fm \qquad m_\pi \approx 266\ MeV$

- small volume allows us to use powerful but costly distillation method (Peardon et. at, 2009)

- dynamical u, d , valence u,d,s : Improved Wilson Clover

  valence c: Fermilab method [El-Khadra et al. 1997]

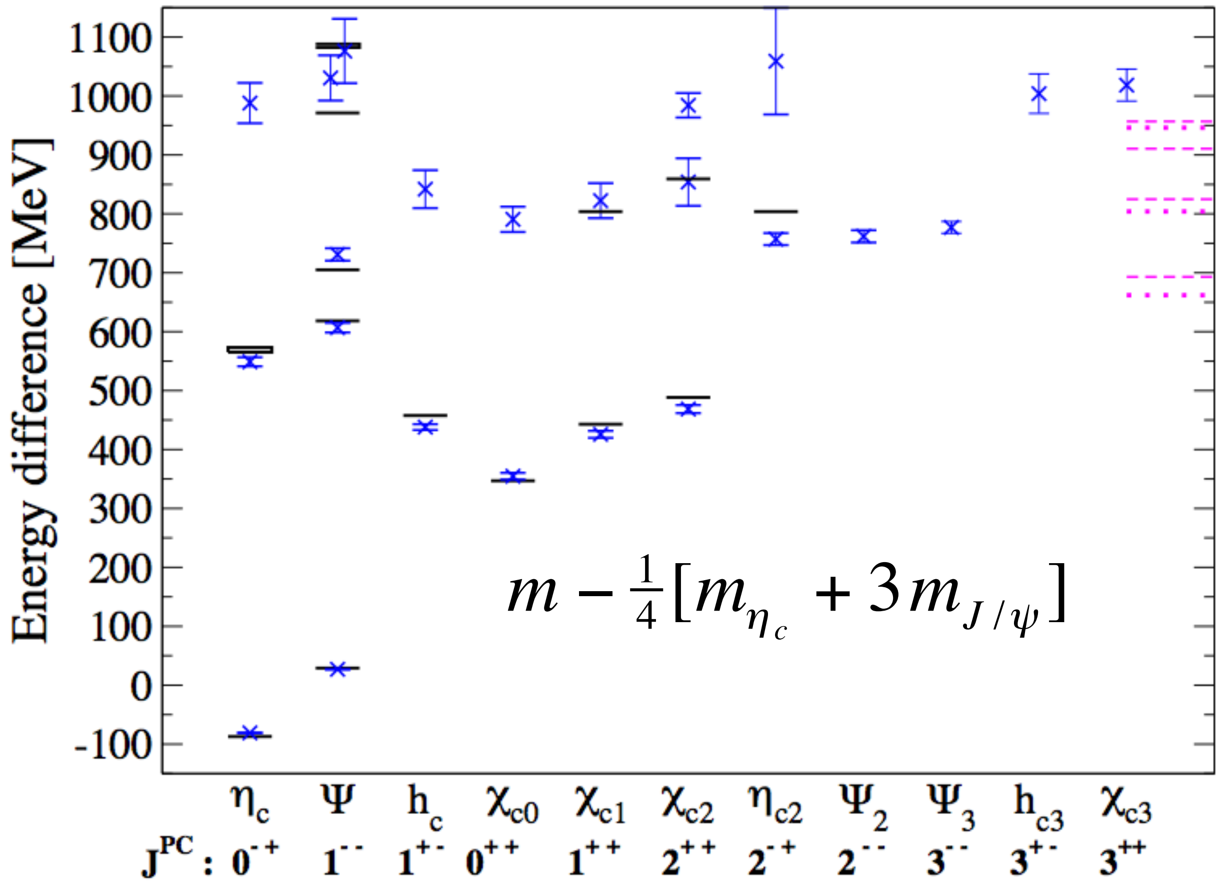
  a set using r0

  m_c set using $\frac{1}{4}[M_2(\eta_c) + 3M_2(J/\psi)]_{lat} = \frac{1}{4}[M(\eta_c) + 3M(J/\psi)]_{\exp}$

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Charmonium spectrum:

verification of charm quark treatment (Fermilab method)



D-meson resonances: introduction

- interpretation assuming conventional quark-antiquark structure

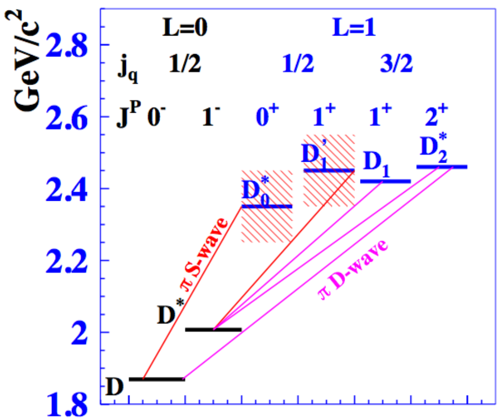
only **1S and 1P** <u>c</u>u states well established and confirmed n exp

for $m_c=\infty$ [Isgur & Wise, 1991] :

- two 1P states decay only in S-wave ➔ broad

  we treat those two as resonances

- two 1P states decay only in D-wave ➔ narrow in exp



➔ even narrower on our lat

with mpi=266 MeV

we treat those as stable: M=E(L)

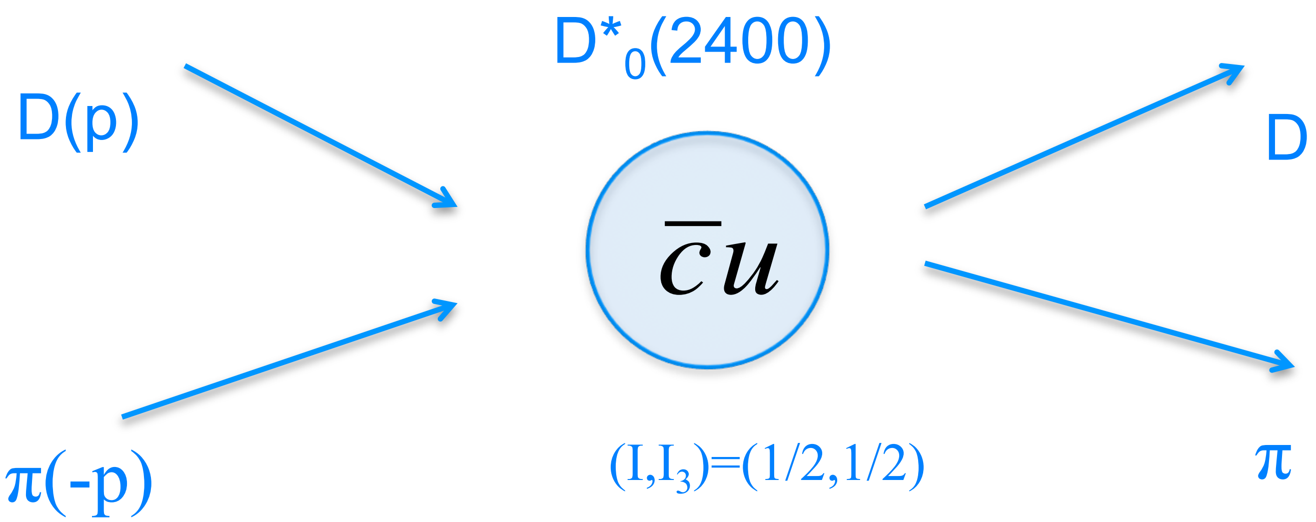
taken from
Belle PRD(2004)

- <u>radial and orbital excitations</u> [Babar 2010]:

(need confirmation), O=quark-antiquark

Dπ scattering

$I=1/2$, s-wave, $J^P=0^+$



[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Dπ scattering : I=1/2, s-wave, $J^P=0^+$

$D^*_0(2400)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 267\ MeV \quad \bar{c}u$?

$\bar{c}\bar{s}su \pm \bar{c}\bar{d}du$?

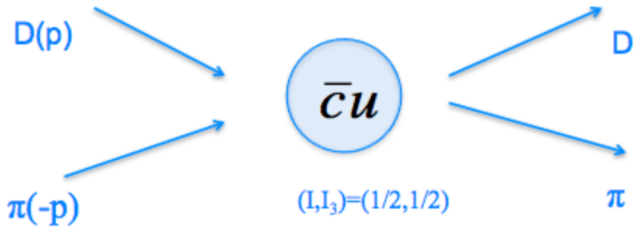
$D_{s0}(2317)$ $\exp: M \approx 2318\ MeV \quad \Gamma \approx 0\ MeV \quad \bar{c}s$?

$\bar{c}\bar{u}us \pm \bar{c}\bar{d}ds$?

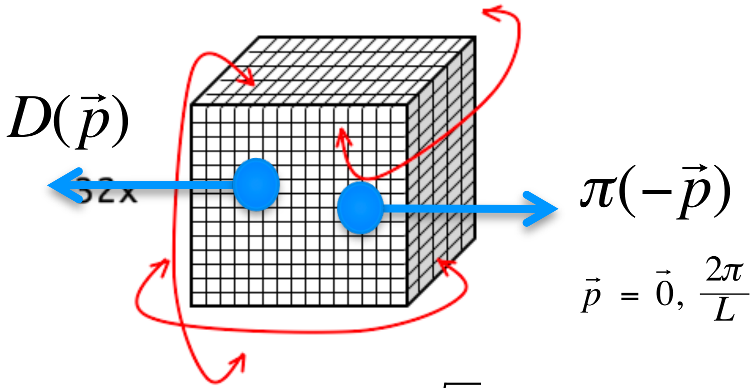
degeneracy between non-strange and strange partners
not naively expected for conventional quark-antiquark

interesting to see if lattice QCD reproduces correct masses and widths of these two states

Dπ scattering on lattice



$J^P = 0^+, \quad P = 0$



$\vec{p} = \vec{0},\ \frac{2\pi}{L}\vec{e}_z$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle$$

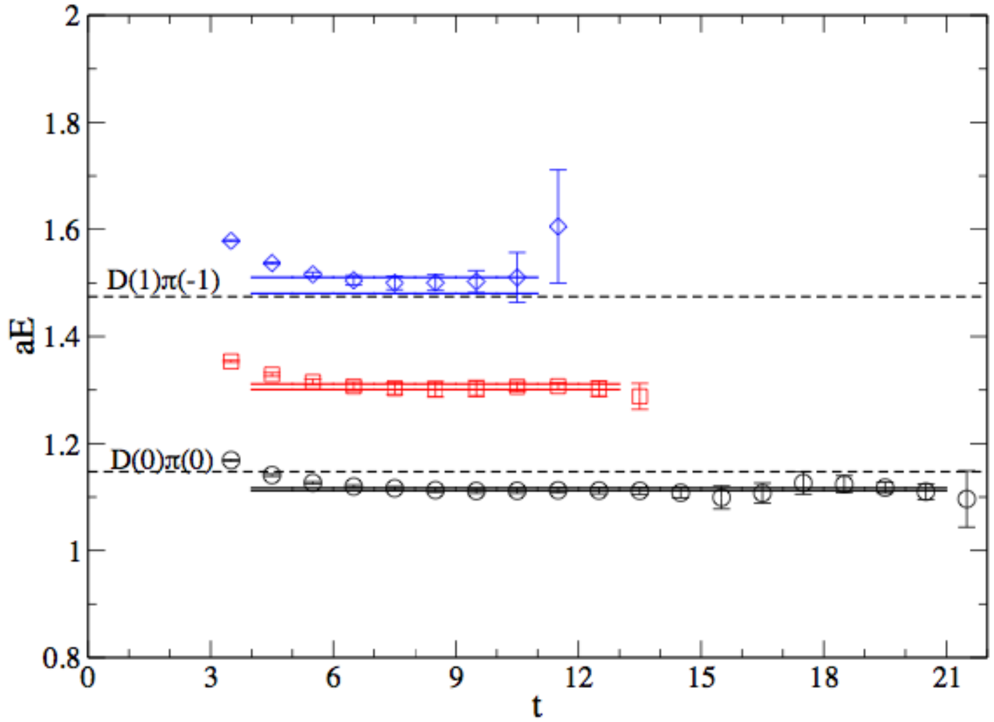
$$\mathcal{O} = \begin{cases} D(\vec{p})\pi(-\vec{p}) = \sqrt{\tfrac{2}{3}}\,[\bar{c}\gamma_5 d]\ [\bar{d}\gamma_5 u] + \sqrt{\tfrac{1}{6}}\,[\bar{c}\gamma_5 u]\ [\bar{u}\gamma_5 u - \bar{d}\gamma_5 d] \\ \bar{c}u \\ \bar{c}\gamma_i D_i u \\ \bar{c}\gamma_t \gamma_i D_i u \\ \bar{c} D_i D_i u \end{cases}$$

D=covariant derivative

6x6 correlators computed using powerful distillation method [Peardon et a. 2009]

$$C_{ij}(t) = \sum_n A_n^{ij}\, e^{-E_n t}$$

Dπ scattering in s-wave, J^P=0+ resulting energy levels



additional level due to $D_0^*(2400)$

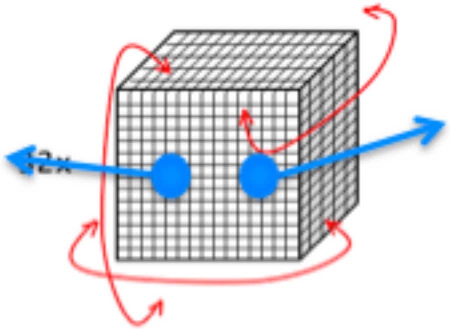
$$E(L) = \sqrt{m_D^2 + \vec{p}^2} + \sqrt{m_\pi^2 + \vec{p}^2} + \Delta E$$

Fermilab analog

$\vec{p} = \vec{0}, \quad \frac{2\pi}{L}\vec{e}_z$

due to the strong interaction

Energy shift ΔE renders δ(E)

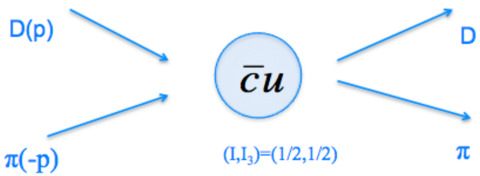


$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

E(L) ➔ δ (E) [Luscher 1986] , for p1+p2=0

E(L) ➔ δ (E) [Leskovec, S.P., PRD 2012] , for p1+p2≠0, m1≠m2

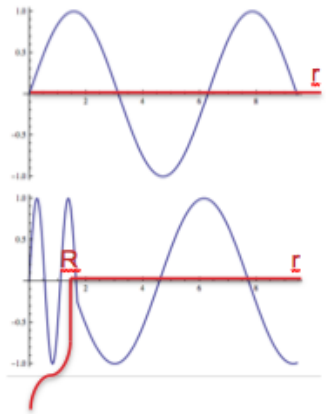
Reaminder: meaning of δ(E)



$$\sigma \propto \frac{\sin^2\delta}{p_{cms}^2}$$

phase shift in QM:

$u(r) = r\,\psi(r)$



example: l = 0

red: V(r > R) = 0

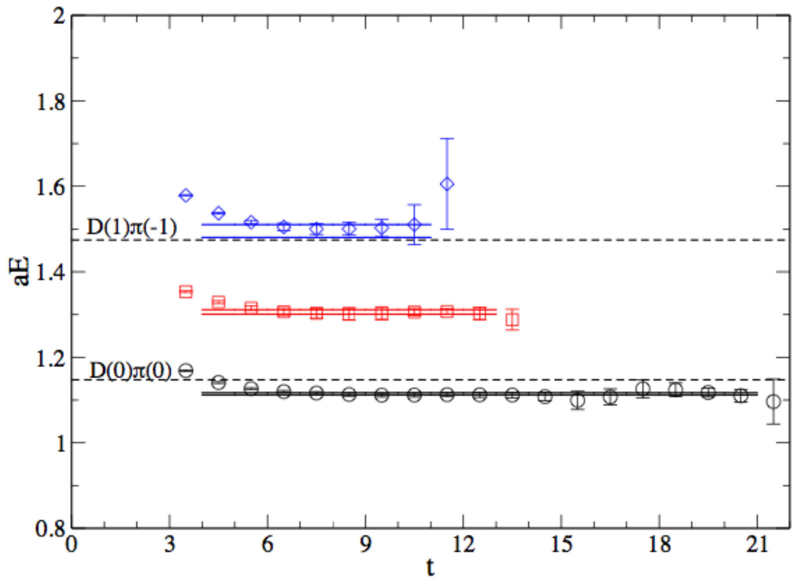
$$\psi(r) \propto \frac{\sin(kr)}{r}$$

$$\psi(r) \propto \frac{\sin(kr+\delta)}{r} \quad r > R$$

unknown $\quad r < R$

potential V(r),
V(r>R)=0

Dπ scattering: resulting levels and phase shifts



δ~173±12^0

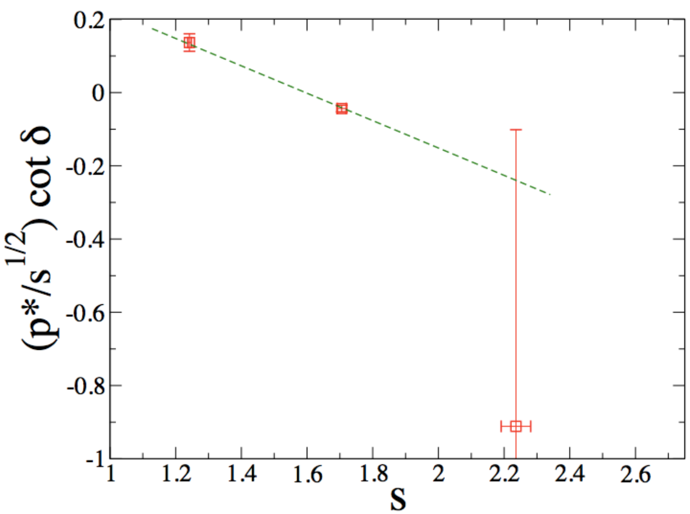
δ~103^0

δ~ 41^0 i → $a_{D\pi}^{I=1/2} = \lim_{p\to 0} \frac{\tan\delta(p)}{p} = 0.81 \pm 0.14\ fm$

additional level due to $D_0^*(2400)$

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Dπ scattering:
extracting resonance parameters for $D_0(2400)$



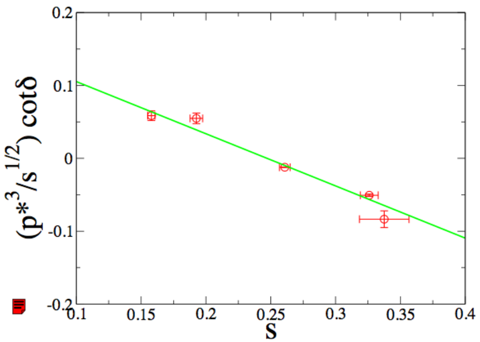
$$s = E^2$$

$$a = \frac{-\sqrt{s}\,\Gamma(s)}{s - m^2 + i\sqrt{s}\,\Gamma(s)} = \frac{1}{2i}\left(e^{2i\delta} - 1\right)$$

$$\sqrt{s}\,\Gamma(s)\cot\delta(s) = m^2 - s, \quad \Gamma(s) = \frac{p}{s} g^2$$

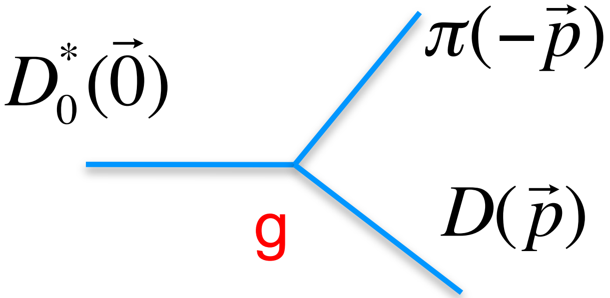
$$\frac{p}{\sqrt{s}}\cot\delta = \frac{1}{g^2}(m^2 - s)$$

For comparison, our result for rho:
there one can check linear behavior.



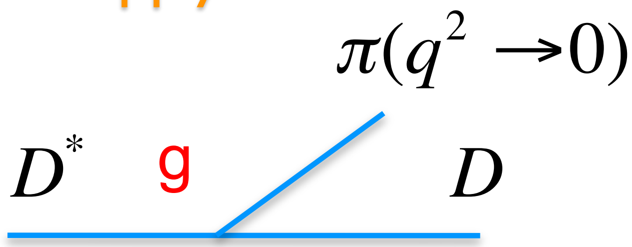
	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

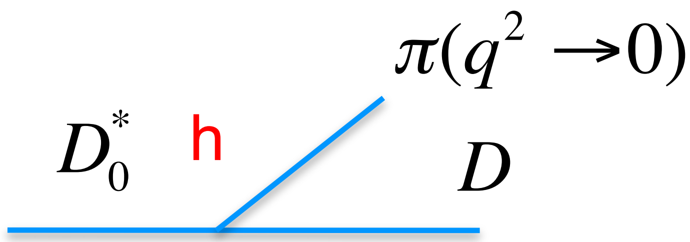
our g applies to kinematic situation in decay



$$\Gamma \equiv g^2 \frac{p}{s} \quad \text{(our definition)}$$

note: this is **different** defintion of coupling than g ,h that apply to emission of soft pion

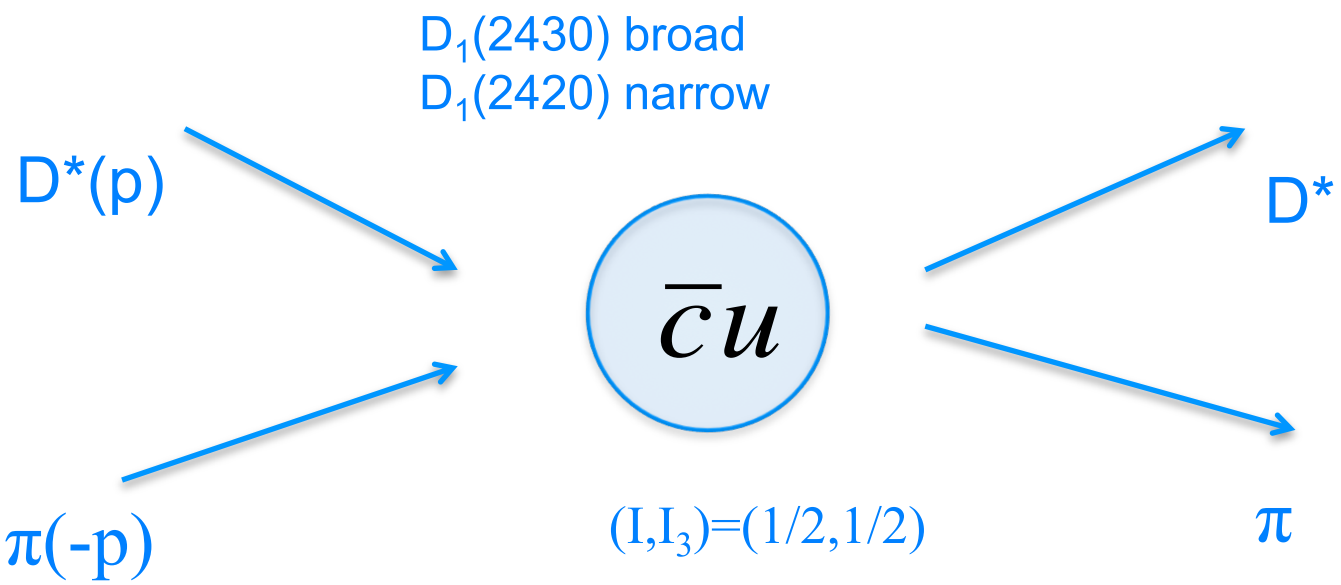




determined for example:
Becirvevic, Chang, Yaouanc
arXiv:1203.0167

D*π scattering

I=1/2, s-wave, $J^P=1^{++}$



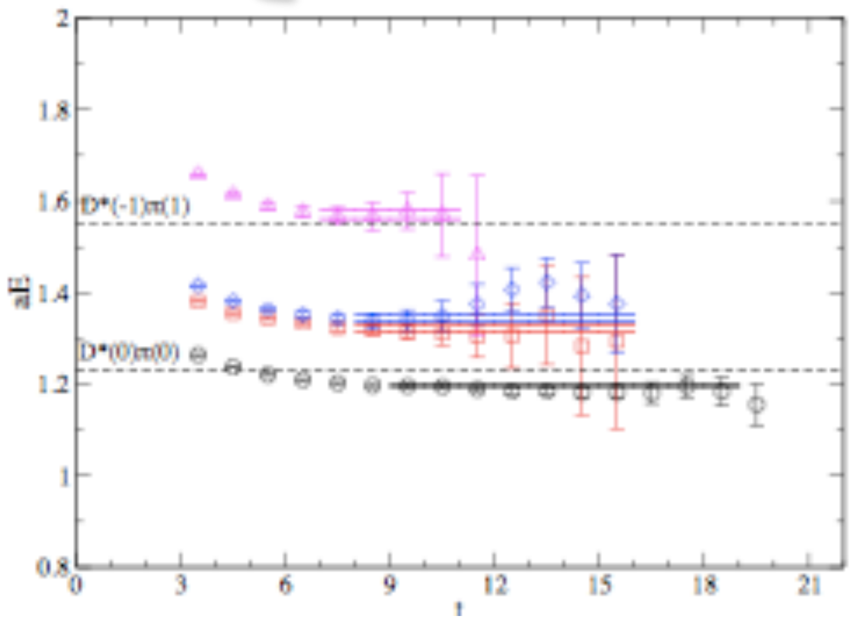
[Mohler, S. P., Woloshyn, arXiv:1208.4059]

$D^*\pi$ scattering: I=1/2, s-wave, $J^P=1^{++}$

exp:
$D_1(2430)$ broad
$D_1(2420)$ narrow

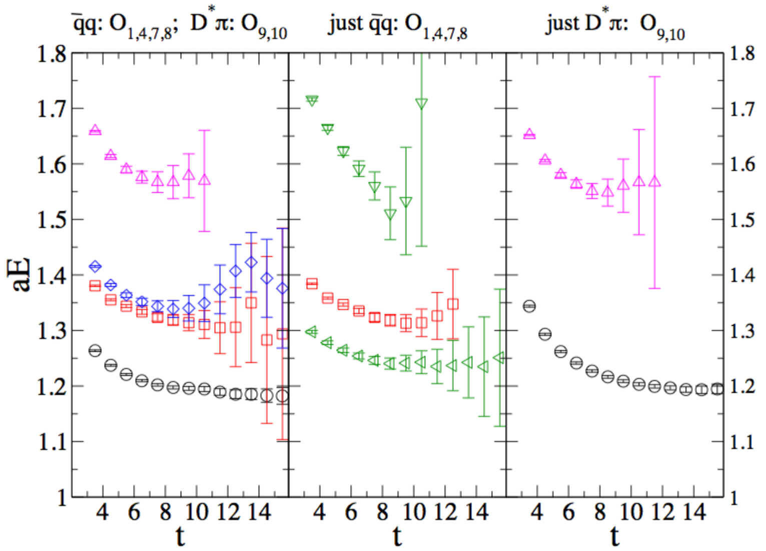
$$\mathcal{O} = \begin{cases} D_i^*(\vec{\mathrm{p}})\pi(-\vec{\mathrm{p}}) = \sqrt{\tfrac{2}{3}}\,[\bar{c}\gamma_i d]\,[\bar{d}\gamma_5 u] + \sqrt{\tfrac{1}{6}}\,[\bar{c}\gamma_i u]\,[\bar{u}\gamma_5 u - \bar{d}\gamma_5 d] \\ \bar{q}\gamma_i\gamma_5 q \\ \bar{q}\epsilon_{ijk}\gamma_j\overrightarrow{\nabla}_k q \\ \bar{q}\epsilon_{ijk}\gamma_t\gamma_j\overrightarrow{\nabla}_k q \\ \bar{q}\overleftarrow{\nabla}_i\gamma_i\gamma_5\overrightarrow{\nabla}_i q \\ \bar{q}\overleftarrow{\Delta}\gamma_i\gamma_5\overrightarrow{\Delta} q \\ \bar{q}\overleftarrow{\Delta}\epsilon_{ijk}\gamma_j\overrightarrow{\nabla}_k q \\ \bar{q}\overleftarrow{\Delta}\epsilon_{ijk}\gamma_t\gamma_j\overrightarrow{\nabla}_k q \\ \bar{q}|\epsilon_{ijk}|\gamma_5\gamma_j\overrightarrow{D}_k q \end{cases}$$

$$\vec{\mathrm{p}} = \vec{0},\ \tfrac{2\pi}{L}\vec{e}_z$$



$$a_{D^*\pi}^{I=1/2} = \lim_{p\to 0}\frac{\tan\delta(p)}{p} = 0.81 \pm 0.17\ fm$$

analysis/approximation inspired by $m_c=\infty$ limit



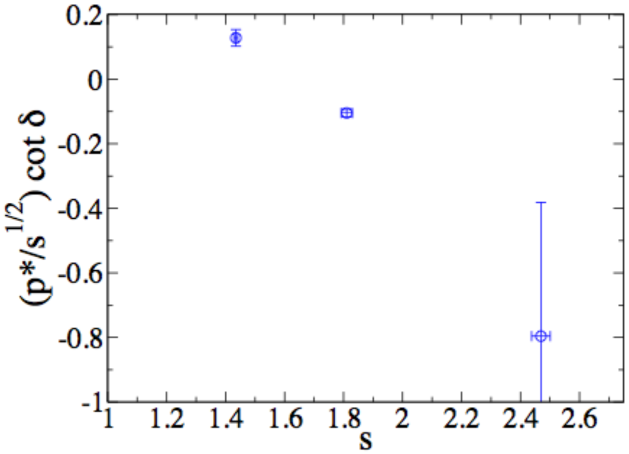
[Isgur & Wise, 1991]

- Blue expected to decay only in S-wave since present only when D(0)pi(0) in the basis.
- Then red expected to decay only in D-wave

blue level: broad $D_1(2430)$
treat as resonance

red level: narrow $D_1(2420)$
assume m=E

s-wave



$$\Gamma(s) = \frac{p}{s} g^2 \qquad \frac{p}{\sqrt{s}} \cot\delta = \frac{1}{g^2}(m^2 - s)$$

results for $D_1(2430)$

	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

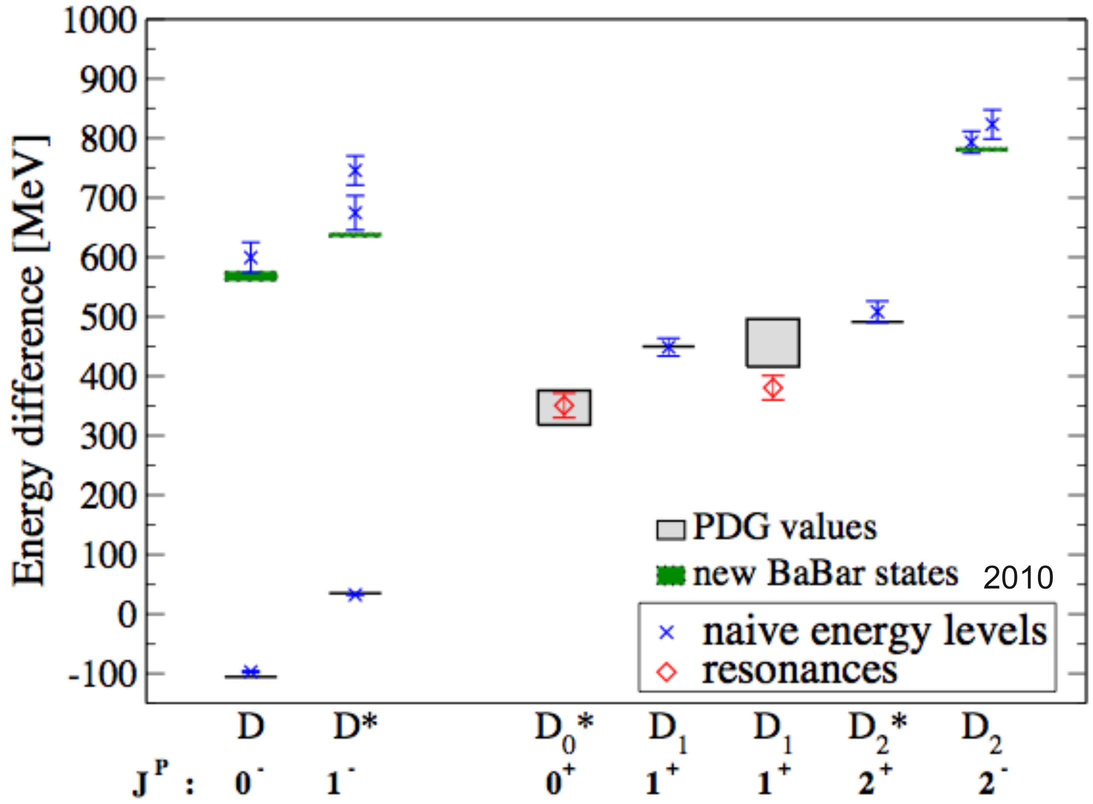
resulting D-meson spectrum

energy difference ≡

$m - \frac{1}{4}[m(D) + 3m(D^*)]$

$\exp: \frac{1}{4}[m(D) + 3m(D^*)]$

$= 1.97\ GeV$



red diamonds: our lat results for resonance masses from scattering study
blue crosses: our lattice results for other resonances: m=E(L), O= qbar q

D-mesons with other J^P

J^P:

0-

$$\bar{q}\gamma_5 q'$$
$$\bar{q}\gamma_t\gamma_5 q'$$
$$\bar{q}\gamma_t\gamma_i\gamma_5 \overrightarrow{\nabla}_i q'$$
$$\bar{q}\gamma_i\gamma_5 \overrightarrow{\nabla}_i q'$$
$$\bar{q}\overleftarrow{\nabla}_i\gamma_5 \overrightarrow{\nabla}_i q'$$
$$\bar{q}\overleftarrow{\nabla}_i\gamma_t\gamma_5 \overrightarrow{\nabla}_i q'$$

1-

$$\bar{q}\gamma_i q'$$
$$\bar{q}\gamma_t\gamma_i q'$$
$$\bar{q}\overrightarrow{\nabla}_i q'$$
$$\bar{q}\epsilon_{ijk}\gamma_j\gamma_5 \overrightarrow{\nabla}_k q'$$
$$\bar{q}\gamma_t \overrightarrow{\nabla}_i q'$$
$$\bar{q}\epsilon_{ijk}\gamma_t\gamma_j\gamma_5 \overrightarrow{\nabla}_k q'$$
$$\bar{q}\overleftarrow{\nabla}_i\gamma_i \overrightarrow{\nabla}_i q'$$
$$\bar{q}\overleftarrow{\nabla}_i\gamma_t\gamma_i \overrightarrow{\nabla}_i q'$$

2-

$$\bar{q}_s|\epsilon_{ijk}|\gamma_j\gamma_5 \overrightarrow{\nabla}_k q'$$
$$\bar{q}|\epsilon_{ijk}|\gamma_t\gamma_j\gamma_5 \overrightarrow{\nabla}_k q'$$

2+

$$\bar{q}|\epsilon_{ijk}|\gamma_j \overrightarrow{\nabla}_k q'$$
$$\bar{q}|\epsilon_{ijk}|\gamma_t\gamma_j \overrightarrow{\nabla}_k q'$$

naive treatment (like all simulations up to now)

just quark-antiquark interpolators

assuming **m=E** (applicable for narrow states)

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

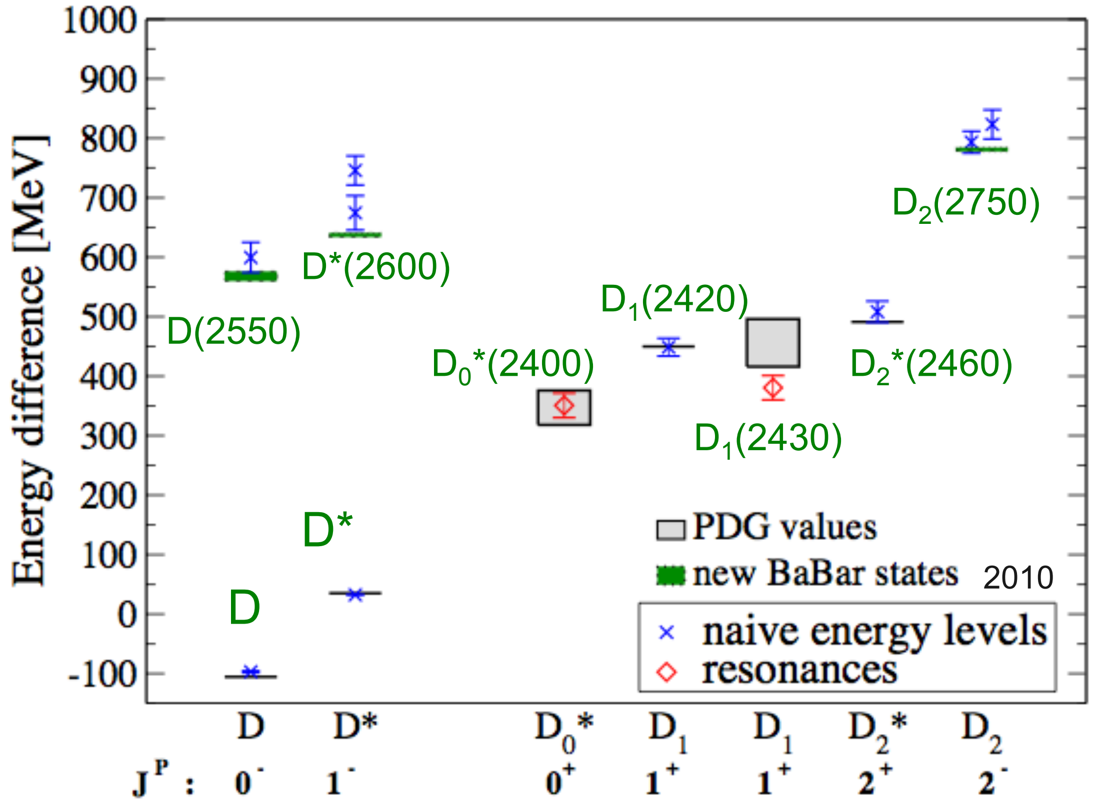
resulting D-meson spectrum

note: m_c fixed by charmonium, so all masses are lattice pre/post-dictions

energy difference ≡

$$m - \tfrac{1}{4}[m(D)+3m(D^*)]$$

$$\exp: \tfrac{1}{4}[m(D)+3m(D^*)]$$
$$= 1.97\ GeV$$



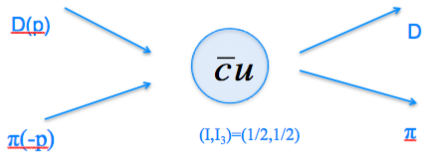
Babar 2010: not clear whether $D_2(2750)$ and $D_2^*(2760)$ the same states
D*pi D pi

We use a possible assignment of $D_2(2750)$ in plot above.

Conclusions

- lattice can "easily" calculate masses of hadrons, that do not decay strongly
- however: large majority of hadrons decay strongly; they are resonances
- simulations of resonances are in infancy (only rho explored up to now)
- I presented *exploratory* simulation of two broad D-meson resonances

  caution: simulation on single Nf=2 ensemble (mpi=266 MeV, rather small volume)



$D_0^*(2400)$ resonance

$J^P=0^+$

agreement with exp without additional valence ss

	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

$D_1(2430)$ resonance

$J^P=1^+$

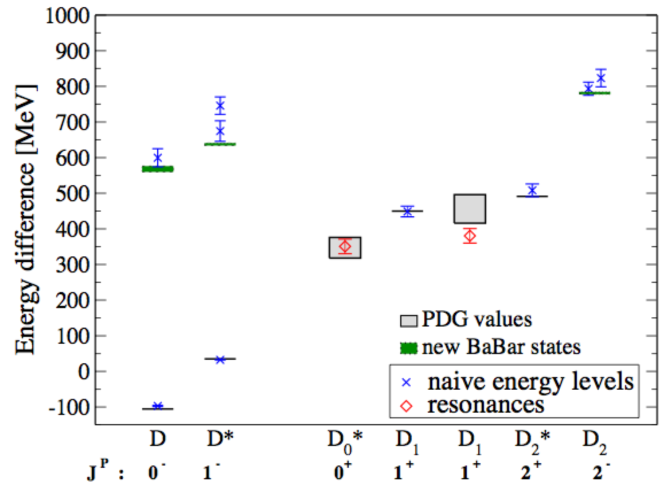
$$\Gamma(s) = \frac{p}{s} g^2$$

	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

Conclusions (continued)

- other excited states treated naively: m=E

  in reality applicable for narrow states

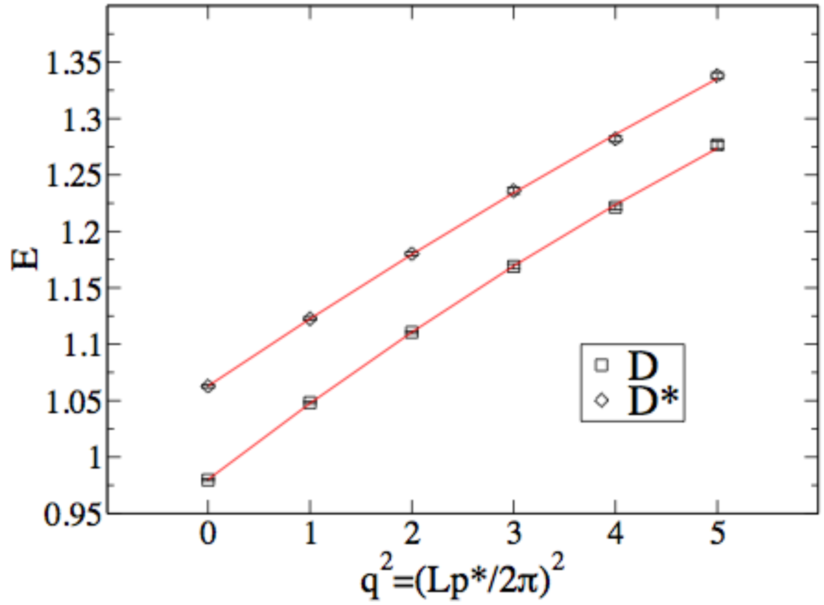


- qualitative agreement with experimental spectrum
- encouraging results for further lattice studies of resonances in general

  for more detailed simulations of D-mesons

Backup slides

Dispersion relation for D and D*

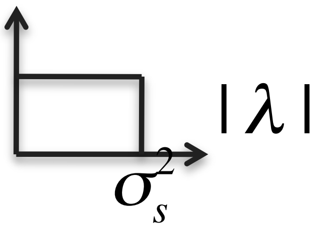


$$E(p) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(\mathbf{p}^2)^2}{8M_4^3} + \dots$$

Distillation method with LapH quark smearing

- Proposed in: *A novel qaurk-field creation operator construction for hadrons in LQCD*
  [Peardon et al (HSC), PRD80 (2009) 054506]

- Laplacian Heaviside (LapH) smearing:



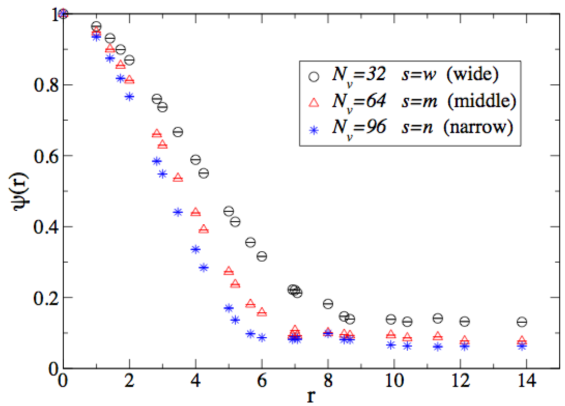
$$q_s = \Theta(\sigma_s^2 + \nabla^2)\, q = \sum_{k=1}^{N_v} v^{(k)}(t)\, v^{(k)+}(t)\;\, q$$

spectral representation

$$\nabla^2(t)\, v^{(k)}(t) = \lambda^{(k)}(t)\, v^{(k)}(t)$$

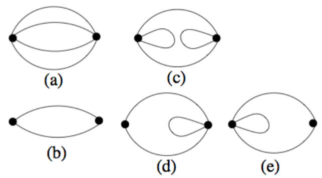
- We use three smearing widths for quarks

$$N_v = 96 \text{ for } s = n \text{ (narrow)} ,$$
$$N_v = 64 \text{ for } s = m \text{ (middle)}$$
$$N_v = 32 \text{ for } s = w \text{ (wide)} ,$$



$$\Psi(r) = \sum_{\mathbf{x},t} \sqrt{\mathrm{Tr}_c[\ \Box_{\mathbf{x},\mathbf{x}+\mathbf{r}}(t)\ \Box_{\mathbf{x},\mathbf{x}+\mathbf{r}}(t)\ ]}$$

All correlators can be expressed in terms of τ



- Perambulator: $\tau^{k'k}(t',t)$

  it is a propagator from eigenvector $v^k(t)$ on time-slice t

  to eigenvector $v^{k'}(t')$ on time-slice t'

- We computed and saved perambulators for all

  $t=1,..,N_T$, $t'=1,..,N_T$; $k=1,\ldots,N_v$, $k'=1,..,N_v$, $N_v=96$

  allows "all-to-all" treatment

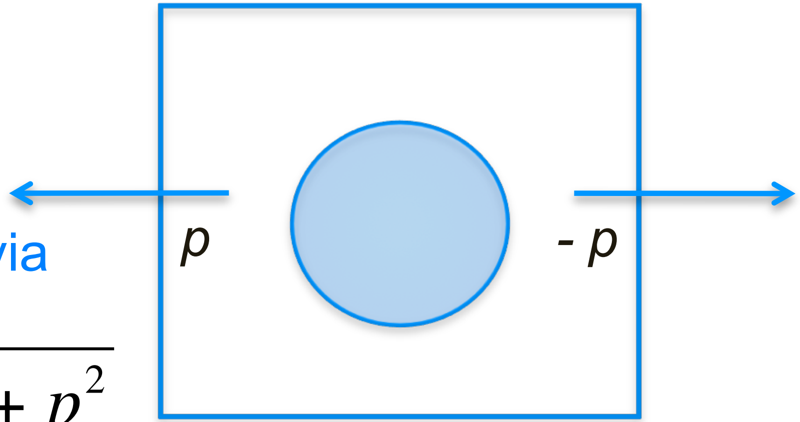
- Analytic expressions for correlators in terms of τ
- averaging correlators over all t_i

  and all directions of momenta and rho polarization

$$C_{jk}(t=t_f-t_i)=\sum_{t_i=1,..,N_T}\sum_{\mathbf{A}\ \text{or}\ \mathbf{d}} C_{jk}(t_f,t_i)$$

Extracting δ(p) from E_n at $p_1+p_2=0$ [Luscher]

- extract $E_n(L)$
- E_n renders p in "outside" region via



$$E = \sqrt{s} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

- p contains info on δ(p)

$$\tan\delta(s) = \frac{\pi^{3/2} q}{Z_{00}(1;q^2)} \qquad q \equiv \frac{L}{2\pi} p$$

$$Z_{00}(1;q^2) \equiv \sum_{\vec{n}\in N^3} \frac{1}{\vec{n}^2 - q^2}$$

[Lang, Leskovec, Mohler, S. P., arXiv:1207.3204, PRD 86]

s-wave scattering lengths

$$a_0 \equiv \lim_{p\to 0} \frac{\tan\delta(p)}{p}$$

at our mpi=266 MeV, mK, mD

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

our lat. sim.	a_0 [fm]	a_0/μ [GeV^{-2}]
K π , I=3/2	-0.140 ± 0.018	-3.94 ± 0.52
K π , I=1/2	0.636 ± 0.090	17.9 ± 2.5
D π , I=1/2	0.81 ± 0.14	17.7 ± 3.1
D*π , I=1/2	0.81 ± 0.17	17.6 ± 3.6

→ $r_{eff} \sim 0$

D-pi: only indirect lattice determination

- from D→π semileptonic form factors [Flynn, Nieves 2007]
- from LOC [Liu, Orginos, Meissner et al 2012]

I=1/2	our result	Flynn & Nieves
a_0/μ [GeV^{-2}]	17.7 ± 3.1	15.9 ± 2.2

[Weinberg's current algebra 1966]
scattering of pion on any particle

$$\frac{a_0^{I=1/2}}{\mu} = \frac{1}{2\pi F_\pi^2} \approx 10\ GeV^{-2}$$

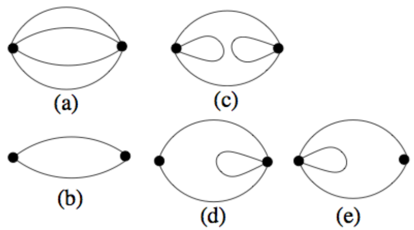
$$\frac{a_0^{I=3/2}}{\mu} = -\frac{1}{2} \times \frac{a_0^{I=1/2}}{\mu}$$

[Mohler, S. P., Woloshyn, arXiv:1208.4059]

Dπ scattering: I=1/2, s-wave, $J^P=0^+$

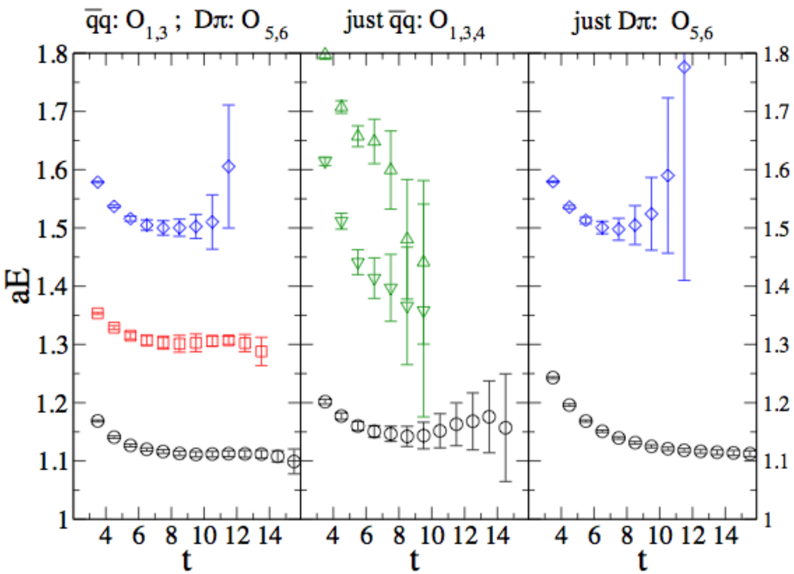
exp: $D_0^*(2400)$

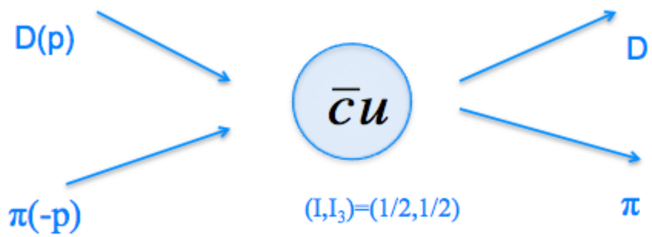
interpolators : 4 quark-antiquark, 2 meson-meson



$$\bar{q}q$$
$$\bar{q}\gamma_i\overrightarrow{\nabla}_i q$$
$$\bar{q}\gamma_t\gamma_i\overrightarrow{\nabla}_i q$$
$$\bar{q}\overleftarrow{\nabla}_i\overrightarrow{\nabla}_i q$$

$$\sqrt{\tfrac{2}{3}}D^-(0)\pi^+(0)+\sqrt{\tfrac{1}{3}}\bar{D}^0(0)\pi^0(0)\,,$$
$$\sum_i\sqrt{\tfrac{2}{3}}D^-(\mathbf{e}_i)\pi^+(-\mathbf{e}_i)+\sqrt{\tfrac{1}{3}}\bar{D}^0(\mathbf{e}_i)\pi^0(-\mathbf{e}_i)$$





Lattice irrep	Quantum numbers J^{PC} in irrep	Interpolator label	Operator
$A1^-$	$0^-, 4^-, \ldots$	1	$\bar{q}\gamma_5 q'$
		2	$\bar{q}\gamma_t\gamma_5 q'$
		3	$\bar{q}\gamma_t\gamma_i\gamma_5\overrightarrow{\nabla}_i q'$
		4	$\bar{q}\gamma_i\gamma_5\overrightarrow{\nabla}_i q'$
		5	$\bar{q}\overleftarrow{\nabla}_i\gamma_5\overrightarrow{\nabla}_i q'$
		6	$\bar{q}\overleftarrow{\nabla}_i\gamma_t\gamma_5\overrightarrow{\nabla}_i q'$
$A1^+$	$0^+, 4^+, \ldots$	1	$\bar{q}q'$
		2	$\bar{q}\gamma_i\overrightarrow{\nabla}_i q'$
		3	$\bar{q}\gamma_t\gamma_i\overrightarrow{\nabla}_i q'$
		4	$\bar{q}\overleftarrow{\nabla}_i\overrightarrow{\nabla}_i q'$
T_1^-	$1^-, 3^-, 4^-, \ldots$	1	$\bar{q}\gamma_i q'$
		2	$\bar{q}\gamma_t\gamma_i q'$
		3	$\bar{q}\overrightarrow{\nabla}_i q'$
		4	$\bar{q}\epsilon_{ijk}\gamma_j\gamma_5\overrightarrow{\nabla}_k q'$
		5	$\bar{q}\gamma_t\overrightarrow{\nabla}_i q'$
		6	$\bar{q}\epsilon_{ijk}\gamma_t\gamma_j\gamma_5\overrightarrow{\nabla}_k q'$
		7	$\bar{q}\overleftarrow{\nabla}_i\gamma_i\overrightarrow{\nabla}_i q'$
		8	$\bar{q}\overleftarrow{\nabla}_i\gamma_t\gamma_i\overrightarrow{\nabla}_i q'$
T_1^+	$1^+, 3^+, 4^+, \ldots$	1	$\bar{q}\gamma_i\gamma_5 q'$
		2	$\bar{q}\epsilon_{ijk}\gamma_j\overrightarrow{\nabla}_k q'$
		3	$\bar{q}\epsilon_{ijk}\gamma_t\gamma_j\overrightarrow{\nabla}_k q'$
		4	$\bar{q}\gamma_t\gamma_i\gamma_5 q'$
		5	$\bar{q}\gamma_5\overrightarrow{\nabla}_i q'$
		6	$\bar{q}\gamma_t\gamma_5\overrightarrow{\nabla}_i q'$
		7	$\bar{q}\overleftarrow{\nabla}_i\gamma_i\gamma_5\overrightarrow{\nabla}_i q'$
		8	$\bar{q}\overleftarrow{\nabla}_i\gamma_t\gamma_i\gamma_5\overrightarrow{\nabla}_i q'$
T_2^-	$2^-, 3^-, 4^-, \ldots$	1	$\bar{q}_s\lvert\epsilon_{ijk}\rvert\gamma_j\gamma_5\overrightarrow{\nabla}_k q'$
		2	$\bar{q}\lvert\epsilon_{ijk}\rvert\gamma_t\gamma_j\gamma_5\overrightarrow{\nabla}_k q'$
T_2^+	$2^+, 3^+, 4^+, \ldots$	1	$\bar{q}\lvert\epsilon_{ijk}\rvert\gamma_j\overrightarrow{\nabla}_k q'$
		2	$\bar{q}\lvert\epsilon_{ijk}\rvert\gamma_t\gamma_j\overrightarrow{\nabla}_k q'$

TABLE XII. Table of $u\bar{c}$ interpolators used for D mesons; in addition we use $D\pi$ [21] and $D^*\pi$ [23] interpolators for irreps A_1^+ and T_1^+. Interpolators are sorted by irreducible representation of the octahedral group O_h and by the parity quantum number P. The reduced lattice symmetry implies an infinite number of continuum spins in each irreducible representation of O_h. For operators, repeated roman indices indicate summation. The quantity γ_t denotes the Dirac matrix for the time direction.