$B \to D^{(*)}$ lattice form factors

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The exclusive semileptonic $B \to D^{(*)}$ decay

Motivations:

- The two decay channels $B \to Dl\nu$ and $B \to D^* l\nu$, where l is an electron or a muon, allow to determine two independent estimates of $|V_{cb}|$
  The main theoretical uncertainty on $|V_{cb}|$ comes from the form factors which parametrize the hadronic weak current $\Longrightarrow$ improve lattice precision

- inclusive & exclusive
  Relying on lattice [FNAL/MILC] determination of $F(1)$:

$$|V_{cb}(excl)| = (39.7 \pm 0.7_{exp} \pm 0.7_{LQCD})\, 10^{-3}$$

  "2σ tension" with the inclusive determination

$$|V_{cb}(incl)| = (41.9 \pm 0.8)\, 10^{-3}$$

  No tension for the heavy flavor sum rule calculations of the form factor $F(1)$, and for results from BaBar09+lattice [Rome ToV] $G(w)$
- τ & light leptons

[BABAR collab. arXiv:1205.5442, Phys. Rev. Lett. 109, 101802 (2012)]

$$R(D^{(*)}) = \frac{B(B \to D^{(*)}\tau\bar{\nu}_\tau)}{B(B \to D^{(*)}l\bar{\nu}_l)}, \qquad \text{where} \quad l = e,\, \mu$$

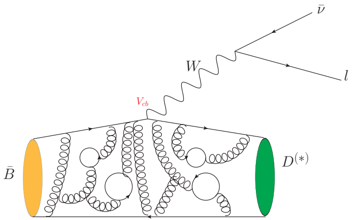
$R(D) = 0.440 \pm 0.058 \pm 0.042$ 2.0σ away from SM: $R(D)_{SM} = 0.297 \pm 0.017$

$R(D^*) = 0.332 \pm 0.024 \pm 0.018$ 2.7σ away from SM: $R(D^*)_{SM} = 0.252 \pm 0.003$

Exceed the Standard Model expectations: together, the disagreement is at the 3.4σ level

The matrix elements $B \to D^{(*)}$ of the hadronic weak currents

The matrix elements of the vector and the axial part of the charged weak current can be parametrized through the h form factors:



$$\frac{\langle D|\ V^{\mu}\ |B\rangle}{\sqrt{M_B M_D}} = (v_B + v_D)^{\mu}\ h_{+}(w) + (v_B - v_D)^{\mu}\ h_{-}(w)$$

$$\frac{\langle D_r^*|V^{\mu}|B\rangle}{\sqrt{M_B M_{D^*}}} = \varepsilon^{\mu\nu\alpha\beta}\ v_B^{\nu} v_{D^*}^{\alpha} \epsilon^{\star\beta}_{r}\ h_V(w)$$

$$\frac{\langle D_r^*|A^{\mu}|B\rangle}{\sqrt{M_B M_{D^*}}} = \epsilon^{\star\nu}_{r}\ \left[h_{A_1}(w)(1+w)g^{\mu\nu} - (h_{A_2}(w)v_B^{\mu} + h_{A_3}(w)v_{D^*}^{\mu})v_B^{\nu}\right]$$

Where $w \equiv v_{D^{(*)}} \cdot v_B = \frac{M_B^2 + M_{D^{(*)}}^2 - q^2}{2 M_B M_{D^{(*)}}}$ is the product of the four-velocities of the B and the $D^{(*)}$ mesons, and a linear function of the four-momentum transfer q^2

The exclusive semileptonic decay rate

In the limit of vanishing lepton mass the differential decay rate depends upon a single form factor, which is a combination of the ones describing the current

$$\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = (\text{fact.}) \times |V_{cb}|^2 (\omega^2 - 1)^{\frac{3}{2}} \left[G^{B\to D}(\omega)\right]^2$$

$$\boxed{G^{B\to D}(\omega) = h_+(w) - \frac{M_D - M_B}{M_D + M_B} h_-(w)}$$

$$\frac{d\Gamma(B \to D^\star\ell\nu)}{dw} = (\text{fact.}) \times |V_{cb}|^2 \sqrt{w^2 - 1}(1+w)^2 \lambda(w) \left[F^{B\to D^\star}(w)\right]^2$$

$$\boxed{F^{B\to D^\star}(w) = h_{A_1}(w)\sqrt{\frac{H_0^2(w) + H_+^2(w) + H_-^2(w)}{\lambda(w)}}}$$

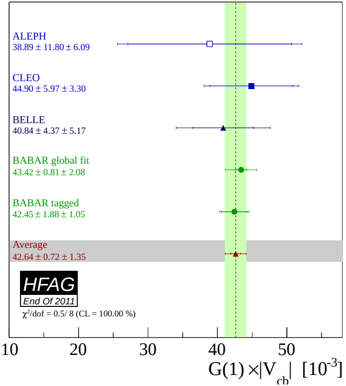
where

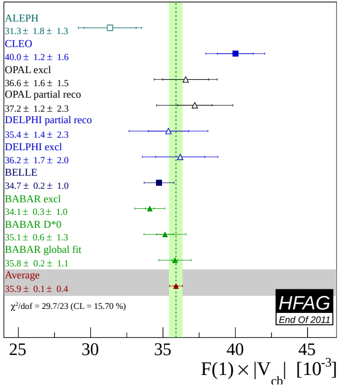
$$H_0(w) = \frac{w - r - X_3(w) - rX_2(w)}{1-r} \qquad H_\pm(w) = t(w)\left[1 \pm X_V(w)\right]$$

$$X_V(w) = \sqrt{\frac{w-1}{w+1}} \frac{h_V(w)}{h_{A_1}(w)} \qquad X_2(w) = (w-1)\frac{h_{A_2}(w)}{h_{A_1}(w)} \qquad X_3(w) = (w-1)\frac{h_{A_3}(w)}{h_{A_1}(w)}$$

$$r = M_{D^*}/M_B \qquad t^2(w) = \frac{1 - 2wr + r^2}{(1-r)^2} \qquad \lambda(w) = 1 + \frac{4w}{w+1} t^2(w)$$

$|V_{cb}|$





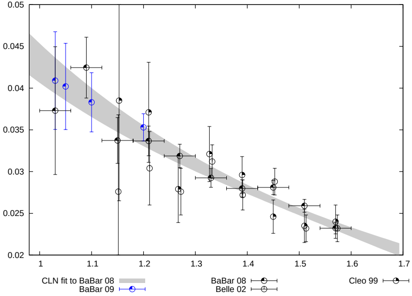
Experimental measurements = | (known) × (CKM elements) × (hadronic form factor) $|^2$

One w (or q^2) point from the lattice (the normalization of the form factors) is enough to determine $|V_{cb}|$

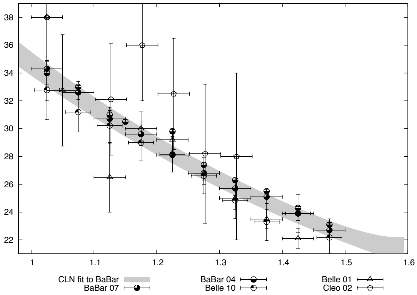
$B \to D^*$ channel has less experimental uncertainties than the $B \to D$ channel
$\Longrightarrow$ exclusive $|V_{cb}|$ usually extracted from the lattice form factor $F(1)$

Experimental Results

$B \to D l \nu$: $G(w)$



$B \to D^* l \nu$: $F(w)$



Experimental measurements = | (known) x (CKM elements) x (hadronic form factor) $|^2$

$w = 1$ is the easiest point to compute on the lattice, but it requires an extrapolation of experimental data

In order to minimize errors, lattice results must be computed over
the range of non-zero recoil $w > 1$ values where the experimental data are more precise

Despite many lattice collaborations . . .

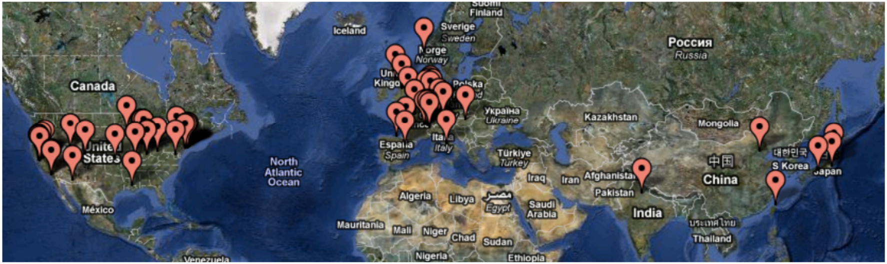


image from [E Lunghi plenary talk @Lattice 2011]

Some of the acronyms:

[ETMC] European Twisted Mass collaboration (EU)
[MILC] MIMD (Multiple Instruction Multiple Data) Lattice Computation collaboration (US)
[FNAL] Fermi National Accelerator Laboratory (US)
[QCDSF] QCD Structure Functions (EU,JP)
[UKQCD] United Kingdom QCD collaboration (EU)
[BMWc] Budapest-Marseille-Wuppertal collaboration (EU)
[PACS-CS] Parallel Array Computer System for Computational Sciences collaboration (JP)
[RBC] RIKEN-BNL Research Center (RBRC), Brookhaven National Lab. (BNL) and Columbia Univ. (US)
[JLQCD] Japan Lattice QCD collaboration (JP)
[TWQCD] TaiWan QCD collaboration (TW)
[HSC] Hadron Spectrum Collaboration (US)
[BGR] Bern–Graz–Regensburg collaboration (EU)
[CLS] Coordinated Lattice Simulations (EU)
[HPQCD] High Precision QCD (EU)
[LHP] Lattice Hadron Physics Collaboration.

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Apologies for not intentional omissions

. . . Despite many lattice collaborations (all results soon averaged and summarized in FLAG-2 report !) . . .

Up to now two efforts to summarize lattice QCD results (only PUBLISHED results):

http://www.latticeaverages.org, by J. Laiho, E. Lunghi, and R. Van de Water

[J Laiho, R S Van de Water, E Lunghi, arXiv:0910.2928, Phys.Rev.D81:034503, 2010]

- light and heavy quark data + UT fits with lattice inputs
- $N_f = 2 + 1$ results

http://itpwiki.unibe.ch/flag, by Flavianet Lattice Average group (FLAG)

[G Colangelo et al. [FLAG working group], arXiv:1011.4408, Eur.Phys.J.C71:1695,2011]

- light quarks only: light quark masses, K and π physics, LowEnergyContants, . . .
- $N_f = 2$ and $N_f = 2 + 1$ averaged separatly

Both will merge in a wider collaboration to cover ALL lattice data:

latticeaverages+ FLAG= FLAG-2

- light and heavy hadron phenomenology
- from collaborations: Alpha, BMW, ETMC, RBC/UKQCD, CLS, Fermilab, HPQCD, JLQCD, MILC, PACS-CS, SWME, . . .
- Review report expected at the end of 2012

FLAG-2 organization

- Advisory Board: S. Aoki, C. Bernard, C. Sachrajda
- Editorial Board: GC, H. Leutwyler, T. Vladikas, U. Wenger
- Working Groups
  - Quark masses — L. Lellouch, T. Blum, V. Lubicz
  - V_{us}, V_{ud} — A. Jüttner, T. Kaneko, S. Simula
  - LEC — S. Dürr, H. Fukaya, S. Necco
  - B_K — H. Wittig, J. Laiho, S. Sharpe
  - α_s — R. Sommer, T. Onogi, J. Shigemitsu
  - f_B, B_B — A. El Khadra, Y. Aoki, M. Della Morte
  - $B \to H\ell\nu$ — R. Van de Water, E. Lunghi, C. Pena

[G Colangelo plenary talk @ Lattice 2012]

...unquenched results for $B \to D^{(*)} l\nu$ only from FNAL/MILC. Some history after ∼2000:

Quenched@$(w = 1)$ → Unquenched@$(w = 1)$ → Quenched@$(w \geq 1)$ → Unquenched@$(w \geq 1)$

$B \to D^*$

- 2001 Quenched calculation at zero recoil
  [S Hashimoto et al. [FNAL], arXiv:hep-ph/0110253, Phys.Rev. D66 (2002) 014503]
- 2008 Quenched calculation at non-zero recoil
  [GMdD et al. [Rome ToV], arXiv:0807.2944, Nucl.Phys.B807:373-395,2009]
- 2008 Unquenched 2+1 at zero recoil
  [C Bernard et al. [FNAL/MILC], arXiv:0808.2519, Phys.Rev.D79:014506,2009]
- 2010 FNAL/MILC update at Lattice
  [S W Qiu et al. [FNAL/MILC], arXiv:1011.2166, PoS Lattice 2010:311,2010]

$B \to D$

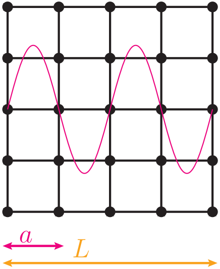
- 1999 Quenched calculation at zero recoil
  [S Hashimoto et al. [FNAL], arXiv:hep-ph/9906376 , Phys.Rev.D61:014502,1999]
- 2004 Unquenched 2+1 calculation at zero recoil
  [M Okamoto et al. [FNAL], arXiv:hep-lat/0409116, Nucl. Phys. Proc. Suppl. 140, 461 (2005)]
- 2007 Quenched calculation at non-zero recoil
  [GMdD et al. [Rome ToV], arXiv:0707.0582, Phys.Lett.B655:45-49,2007]
  [GMdD et al. [Rome ToV], arXiv:0707.0587, JHEP0710:062,2007]
- 2011 Unquenched 2+1 non-zero recoil
  [SW Qiu et al. [FNAL/MILC], arXiv:1111.0677, Lattice 2011]
- 2012 FNAL/MILC, and update at Lattice
  [JA Bailey et al. [FNAL/MILC], arXiv:1202.6346, PhysRevD.85.11450]
  [JA Bailey et al. [FNAL/MILC], arXiv:1206.4992, PhysRevLett.109.071802]
  [SW Qiu et al. [FNAL/MILC], arXiv:1211.2247, Lattice 2012]

Multi scale problem

QCD (and B physics in particular) is a multi–scale problem $(m_u, m_d, m_s, m_c, m_b, \Lambda_{QCD})$
$\Longrightarrow$ simulations are computational expensive

Lattice is an IR/UV regulator

InfraRed cutoff	$\Lambda_{IR} = 1/L$
UltraViolet cutoff	$\Lambda_{UV} = 1/a$



The propagation of a heavy quark needs large volumes and fine lattice spacings to control the Finite Volume Effects and Discretization Errors :

$e^{-M_\pi L} \ll 1$	$L \gtrsim 6\,\mathrm{fm}$	$\Lambda_{IR} = 1/L \lesssim 33\,\mathrm{MeV}$
$am_{heavy} \ll 1$	$a \lesssim 0.05\,\mathrm{fm}$	$\Lambda_{UV} = 1/a \gtrsim 4\,\mathrm{GeV}$

$$N_{points} = L/a \simeq 120$$

The simulated masses:

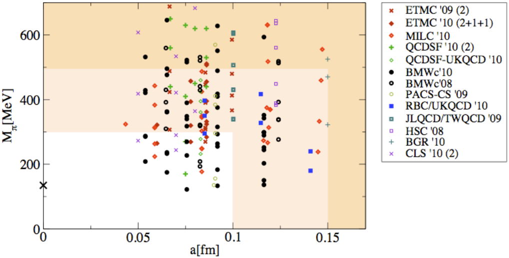
m_{ud}^{sim}	$\gtrsim$	m_{ud}^{phys}	extrapolated from nearby
m_s^{sim}	$\simeq$	m_s^{phys}	interpolated
m_c^{sim}	$\simeq$	m_c^{phys}	interpolated
m_b^{sim}	$<$	m_b^{phys}	extrapolated

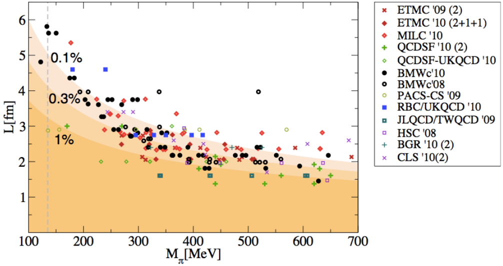
$$CPU\ cost[Tflops \times years] = \underbrace{N}_{\mathcal{O}(1)} \underbrace{\left(\frac{20\,\mathrm{MeV}}{m}\right)^{\alpha}}_{\alpha\sim 1-2} \underbrace{\left(\frac{L}{3\,\mathrm{fm}}\right)^{\beta}}_{\beta\sim 5} \underbrace{\left(\frac{0,1\mathrm{fm}}{a}\right)^{\gamma}}_{\gamma\sim 4-6}$$

Recent dynamical fermion lattice simulations

All points relative to $N_f = 2 + 1$ except when explicitly indicated: (2), (2+1+1)

discretization effects





in percentage the size of finite-volume effects

reproduced from [Z. Fodor and C. Hoelbling arXiv:1203.4789, Rev.Mod.Phys. 84, 449 (2012)]

Heavy quarks on the lattice

For the accessible lattices the UV cut-off is smaller than the b quark mass
$\Longrightarrow$ large and uncontrolled discretization errors $\propto (am_b)^n$

Various approaches introduced to manage heavy quarks on the lattice:

Effective field theories for heavy quarks
Tuning of parameters of the lattice action: in lattice perturbation theory or by matching QCD non–perturbatively on a small volume

- non perturbative HQET [ALPHA]
  static quarks $+ 1/m$ corrections as insertions
- NRQCD [FNAL/MILC, HPQCD]
  non-relativistic quark action
- Fermilab [FNAL/MILC, HPQCD]
  improved action with breaking of the time space symmetry

"Interpolation" to b from the charm region and the static limit [ETMC, ALPHA, Becirevic et al. ...]

- *Interpolation to bottom with fitting functions motivated by HQET*

"Direct" relativistic b

- HISQ action: [FNAL/MILC, HPQCD]
  High improvement and am-dependent coefficients
- SSM Step Scaling method [Rome ToV]
  Simulation of b on small volume and evolution in the volume

The Fermilab method

The Fermilab approach consists in simulating the following action with $am_0 \simeq 1$

[El-Khadra et al Phys.Rev.D55:3933, 1997]
[Aoki et al Prog.Theor.Phys.109:383, 2003]
[Oktay,Kronfeld arXiv:0803.0523]

$$S = \sum_n \bar{\psi}_n \left[m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - r_t \frac{a D_0^2}{2} - r_s \frac{a \vec{D}^2}{2} + c_B \frac{i \sigma_{ij} F_{ij}}{4} + c_E \frac{i \sigma_{0i} F_{0i}}{2} \right] \psi_n$$

i.e. the Symanzik improved effective action for quarks with $|a\vec{p}| \ll 1$
with **mass dependent** coefficients usually computed perturbatively

- the number of parameters in the action can be reduced to **3**

  [Christ,Li,Lin Phys.Rev.D76:074505,2007]

- and can be determined non-perturbatively by matching QCD on a **small volume**

  [Lin,Christ Phys.Rev.D76:074506,2007]

The Step Scaling Method

A finite size scaling procedure

[Guagnelli,Palombi,Petronzio,Tantalo Phys.Lett.B546:237,2002]

$$\mathcal{O}(m_b, m_l; L = \infty) \quad = \quad \mathcal{O}(m_b, m_l; L_0) \quad \underbrace{\frac{\mathcal{O}(m_b, m_l; 2L_0)}{\mathcal{O}(m_b, m_l; L_0)}}_{\sigma(m_b, m_l; L_0)} \quad \frac{\mathcal{O}(m_b, m_l; 4L_0)}{\mathcal{O}(m_b, m_l; 2L_0)} \quad \ldots$$

- the step scaling functions σ's calculated at lower values of the high energy scale

$$\mathcal{O}(m_b, m_l; L_0) \leftarrow m_b = m_b^{phys}$$

$$\sigma(m_b, m_l; nL_0) \leftarrow m_b \leq \frac{m_b^{phys}}{n}$$

- The extrapolation of the step scaling functions is much easier than the extrapolation of the observable itself

$$\mathcal{O}(m_b, m_l; L) = \mathcal{O}^0(m_l; L) \left[1 + \frac{\mathcal{O}^1(m_l; L)}{m_b}\right]$$

$$\sigma(m_b, m_l; L) = \frac{\mathcal{O}^0(m_l; 2L)}{\mathcal{O}^0(m_l; L)} \left[1 + \frac{\mathcal{O}^1(m_l; 2L) - \mathcal{O}^1(m_l; L)}{m_b}\right]$$

Basic ingredients of SSM

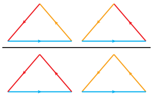
- Finite Volume Scheme:
  Schrödinger functional, i.e. Dirichlet boundary condition in time ($m_{light} = 0$ on the lattice)
- continuous momenta (with flavour-twisted boundary conditions) to reach "small" w values

  $$\psi(x + \hat{\imath}L) = e^{i\theta_i}\psi(x) \qquad \theta_0 = 0$$

  $$p_i = \frac{2\pi\theta_i}{L} + \frac{2\pi n_i}{L}, \quad n \in Z^3, \quad \theta_i \in [0, 1[$$

- form factors calculated with good precision entirely in terms of double ratios of three point correlation functions

$$\frac{\langle \mathtt{M}_F | J^\mu | \mathtt{M}_I \rangle}{2\sqrt{E_F E_I}} = \sqrt{}$$



different colours =
= different flavours

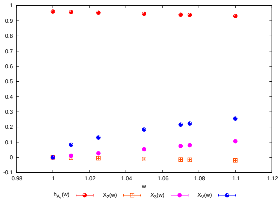
currents $J^\mu = \mathcal{V}^\mu, \mathcal{A}^\mu$
mesons $\mathtt{M} = P, V$

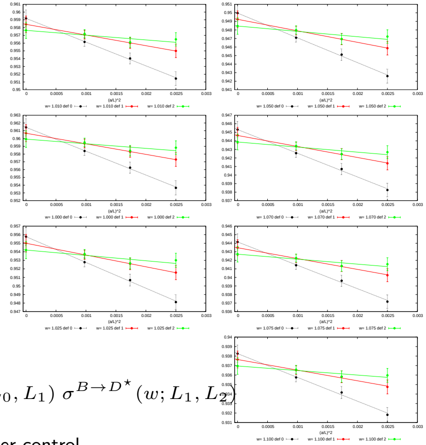
$$\langle PVP \rangle^{\mu}_{if} = \hat{Z}_V \sum_{\vec{x}} \langle \mathbf{P}_{li} \, \mathcal{V}^{\mu}_{if}(x) \, \mathbf{P'}_{fl} \rangle \qquad \langle VVV \rangle^{I\mu I}_{if} = \hat{Z}_V \sum_{\vec{x}} \langle \mathbf{V}^{I}_{li} \, \mathcal{V}^{\mu}_{if}(x) \, \mathbf{V'}^{I}_{fl} \rangle$$

$$\langle PVV \rangle^{\mu I}_{if} = \hat{Z}_V \sum_{\vec{x}} \langle \mathbf{P}_{li} \, \mathcal{V}^{\mu}_{if}(x) \, \mathbf{V'}^{I}_{fl} \rangle \qquad \langle PAV \rangle^{\mu I}_{if} = \hat{Z}_A \sum_{\vec{x}} \langle \mathbf{P}_{li} \, \mathcal{A}^{\mu}_{if}(x) \, \mathbf{V'}^{I}_{fl} \rangle$$

- SSM finite size recursive procedure to estimate FVE

[Rome ToV] $B \to D^\star \ell \nu$ at non-zero recoil: the small volume





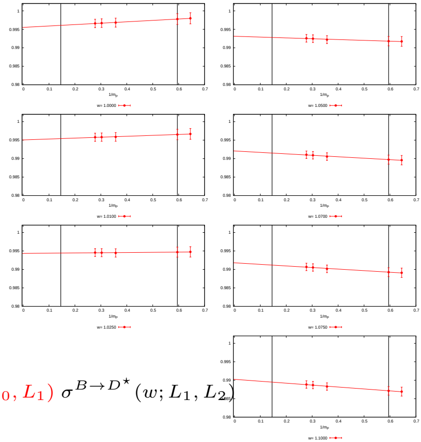
$$F^{B\to D^\star}(w) =$$

$$h_{A_1}(w)\sqrt{\frac{H_0^2(w)+H_+^2(w)+H_-^2(w)}{\lambda(w)}}$$

$$F^{B\to D^\star}(w;L_2) = {\color{red}F^{B\to D^\star}(w;L_0)}\ \sigma^{B\to D^\star}(w;L_0,L_1)\ \sigma^{B\to D^\star}(w;L_1,L_2)$$

The discretization errors on the small volume are under control

[Rome ToV] $B \to D^{\star}\ell\nu$: first volume step



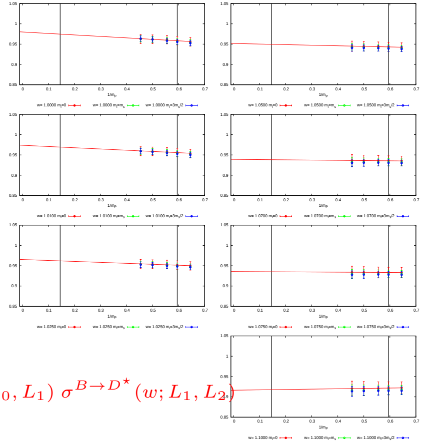
step scaling functions are very flat: extrapolated values differ from simulated ones by a few per mille

$$\sigma^{P\to D^{\star}}(w; L_0, L_1) = \frac{F^{P\to D^{\star}}(w; L_1)}{F^{P\to D^{\star}}(w; L_0)}$$

$$F^{B\to D^{\star}}(w; L_2) = F^{B\to D^{\star}}(w; L_0)\ \sigma^{B\to D^{\star}}(w; L_0, L_1)\ \sigma^{B\to D^{\star}}(w; L_1, L_2)$$

[Rome ToV] $B \to D^\star \ell\nu$*: second volume step*

step scaling functions are very flat: extrapolated values differ from simulated ones by a few per mille



$$\sigma^{P\to D^\star}(w;L_1,L_2) = \frac{F^{P\to D^\star}(w;L_2)}{F^{P\to D^\star}(w;L_1)}$$

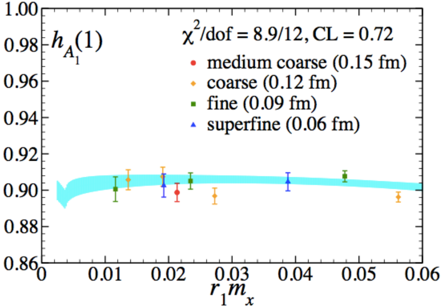
$$F^{B\to D^\star}(w;L_2) = F^{B\to D^\star}(w;L_0)\,\sigma^{B\to D^\star}(w;L_0,L_1)\,\sigma^{B\to D^\star}(w;L_1,L_2)$$

[FNAL/MILC] $B \to D^* l\nu$ at zero recoil(I)

One q^2 point from the lattice, the zero-recoil q^2_{max} $(w = 1)$ is the easiest to compute

$$F(1) = 0.9077(51)_{MC}(88)_g(84)_\chi(90)_{HQ}(30)_Z(33)_\kappa$$

- Fermilab action for b and c quarks
- asqtad staggered action for light valence quarks
- 2+1 rooted staggered sea quarks



reproduced from [S.-W. Qiu et al. [Fermilab/MILC], arXiv:1011.2166]

- Mild light quark mass dependence

[FNAL/MILC] $B \to D^* l\nu$ at zero recoil (II)

$$F(1) = h_{A_1}(1) = 0.9077(51)_{stat}(88)_g(84)_\chi(90)_a(30)_Z(33)_\kappa$$

Error Budget

label		percentage
$stat$	statistics	0,5 %
g	$g_{D^*D\pi}$ coupling	1 %
χ	chiral extrapolation	0,9 %
a	discretization error	1 %
Z	renormalization and matching	0,3 %
κ	κ_c and κ_b tuning	0,3 %
	Total	1,7 %
		errors summed in quadrature

$1,7\%$ error on $F(1)$ translates in 16% on $(1 - F(1))$

$|V_{cb}(excl)|$ from BaBar+Belle+FNAL/MILC determination of $F(1)$:

[The Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/]

$$|V_{cb}(excl)| = (39.54 \pm 0.50_{exp} \pm 0.74_{LQCD})\, 10^{-3}$$

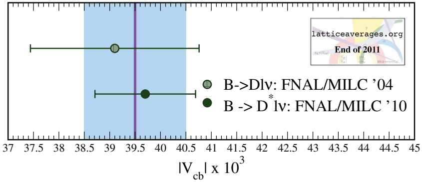
"2σ tension" with the inclusive determination

$$|V_{cb}(incl)| = (41.9 \pm 0.8)\, 10^{-3}$$

still lacking of $B \to D^* l\nu$ at non-zero recoil

[FNAL/MILC] $B \to Dl\nu$ at zero recoil

From $B \to Dl\nu$ it comes an independent determination of $|V_{cb}|$



$$G(1) = 1.074(18)_{stat}(16)_{syst}$$

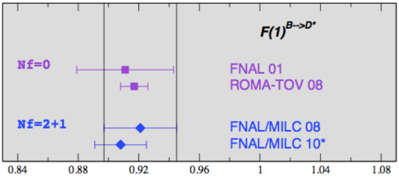
turns into the determination

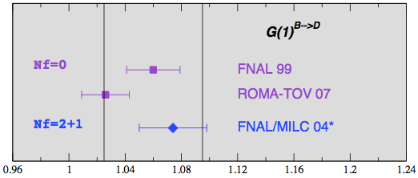
$$|V_{cb}| = (39.70 \pm 1.42_{exp} \pm 0.89_{LQCD})\, 10^{-3}$$

[The Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/]

$2,2\%$ error on $G(1)$ translates in 30% on $(1 - G(1))$

Summary of $w = 1$ results [FNAL/MILC] + [Rome ToV]





reproduced from [C Tarantino, arXiv:0807.2944]

The present accuracy on $|V_{cb}|$ is at the 2% level

$ff@(w=1)$

	$F(1)$	$G(1)$
Rome ToV$[n_f = 0]$	$0.917 \pm 0.008 \pm 0.005$	1.026 ± 0.017
FNAL/MILC$[n_f = 2+1]$	$0.9077 \pm 0.0051 \pm 0.0158$	$1.074 \pm 0.018 \pm 0.016$

[Fermilab/MILC] $B \to D l \nu$ at non-zero recoil

$B \to D l \nu$ form factors recently determined at non-zero recoil to improve the precision with respect to the previous 2004 result $@(w = 1)$

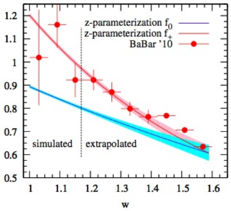
Simulated kinematic range $1 \leq w < 1.15$, extrapolation to the full range $1 \leq w < 1.6$ using the z expansion (unitarity and analyticity): **[Caprini,Lellouch,Neubert Nucl.Phys.B530:153,1998]**

$$z(w) = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}$$

It represents a conformal map $w \to z$:
$w \in [1,\, 1.6] \to z \in [0,\, 0.064]$

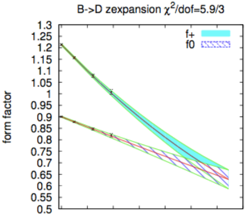
$$f(z) = \frac{1}{P(z)\phi(z)} \sum_{n=0}^{\infty} a_n z^n$$

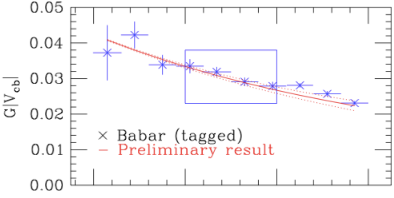
$P(z)$ Blaschke factor, ϕ outer function



reproduced from [S.-W. Qiu et al. [Fermilab/MILC], arXiv:1206.4992]

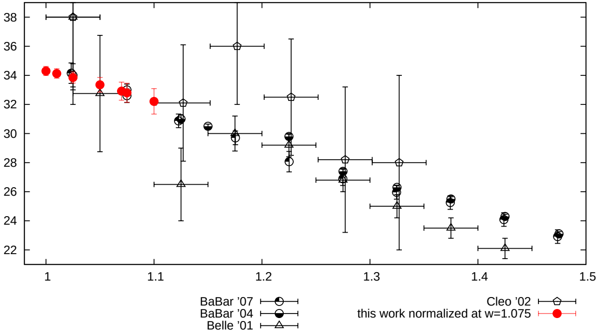
reproduced from [S.-W. Qiu et al. [Fermilab/MILC], arXiv:1211.2247]





[Rome ToV] $B \to D^\star \ell\nu$: theory vs. experiment

[GMdD, Petronzio, Tantalo [Rome ToV], arXiv:0807.2944, Nucl.Phys.B807:373-395,2009]



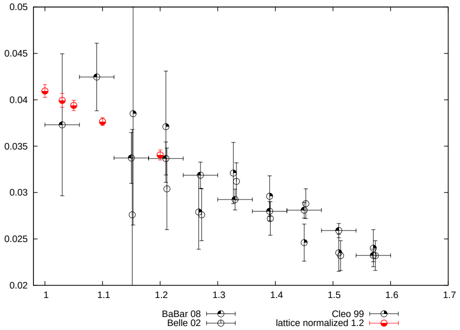
$F^{B\to D^\star}(w = 1.075) = 0.877(18)(04)$

$$|V_{cb}|(@w = 1.075) \; = \; 37.4(8)(5) \times 10^{-3}$$

Full parametrization of matrix elements:
also results for each form factor
$h_{A_1}(w)$,$h_{A_2}(w)$,$h_{A_3}(w)$, $h_V(w)$

w	$F(w)$ [Rome ToV]
1.000	$0.917 \pm 0.008 \pm 0.005$
1.010	$0.913 \pm 0.009 \pm 0.005$
1.025	$0.905 \pm 0.010 \pm 0.005$
1.050	$0.892 \pm 0.013 \pm 0.004$
1.070	$0.880 \pm 0.017 \pm 0.004$
1.075	$0.877 \pm 0.018 \pm 0.004$
1.100	$0.861 \pm 0.023 \pm 0.004$

[GMdD, Molinaro, Petronzio, Tantalo[Rome ToV], arXiv:0707.0582, Phys.Lett.B655:45-49,2007]
[GMdD, Petronzio, Tantalo[Rome ToV], arXiv:0707.0587, JHEP0710:062,2007]



$G^{B\to D}(w=1.2) = 0.853(21)$

$|V_{cb}|(@w=1.2) \; = \; 41.4(1.3)(1.4)(1.0)\times 10^{-3}$

no tension with inclusive determination $(41.9 \pm 0.8)10^{-3}$

comparison with BaBar 2010 data

w	$G(w)$ [Rome ToV]	$\lvert V_{cb}\rvert G(w)\, 10^3$ [BaBar 10]	$\lvert V_{cb}\rvert\, 10^3$
1.00	1.026 ± 0.017		
1.03	1.001 ± 0.019	$40.9 \pm 5.7 \pm 1.3$	$40.9 \pm 5.7 \pm 1.3 \pm 0.8$
1.05	0.987 ± 0.015	$40.2 \pm 5.0 \pm 1.3$	$40.7 \pm 5.1 \pm 1.3 \pm 0.6$
1.10	0.943 ± 0.011	$38.3 \pm 3.3 \pm 1.3$	$40.6 \pm 3.5 \pm 1.4 \pm 0.5$
1.20	0.853 ± 0.021	$35.3 \pm 1.1 \pm 1.2$	$41.4 \pm 1.3 \pm 1.4 \pm 1.0$

$B \to D\ell\nu$: the τ channel

[GMdD, Petronzio, Tantalo[Rome ToV], arXiv:0707.0587, JHEP0710:062,2007]

Full parametrization of matrix elements:
also results for each form factor $h_+(w)$,$h_-(w)$ $\Longrightarrow$ byproduct: the τ channel

In the case $\ell = \tau$ the mass of the lepton cannot be neglected and the differential decay rate is given by

$$\frac{d\Gamma^{B\to D\tau\nu_\tau}}{dw} = \frac{d\Gamma^{B\to D(e,\mu)\nu_{e,\mu}}}{dw}\left(1-\frac{r_\tau^2}{t(w)}\right)^2\left\{\left(1+\frac{r_\tau^2}{2t(w)}\right)+\frac{3r_\tau^2}{2t(w)}\,\frac{w+1}{w-1}\,[\Delta(w)]^2\right\}$$

$$\boxed{\Delta(w) = \frac{1}{G(w)}\left[\frac{1-r}{1+r}\,h_+(w) - \frac{w-1}{w+1}\,h_-(w)\right]}$$

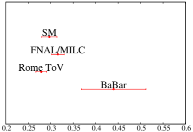
$$r_\tau=\frac{m_\tau}{M_B},\qquad r=\frac{M_D}{M_B},\qquad t(w)=1+r^2-2rw,\qquad 1\le w\le\frac{M_B^2+M_D^2-m_\tau^2}{2M_BM_D}$$

$$R(D) = \frac{B(B\to D\tau\bar\nu_\tau)}{B(B\to Dl\bar\nu_l)},\qquad \text{where}\quad l=e,\ \mu$$

$R(D) = 0.440 \pm 0.058 \pm 0.042$ 2.0σ away from SM: $R(D)_{SM} = 0.297 \pm 0.017$

[S Fajfer et al., arXiv:1203.2654, Phys.Rev. D85 (2012) 094025] ...

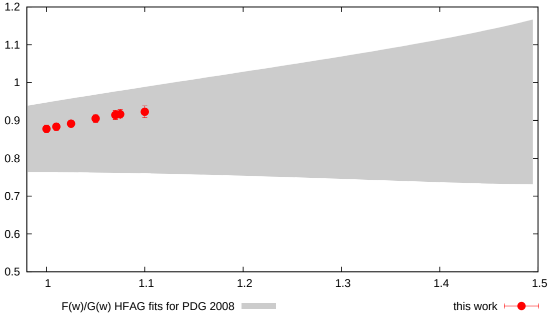
	$R(D)$	σ away from BaBar
SM	0.297 ± 0.017	$2.0\,\sigma$
SM: FNAL/MILC	0.316 ± 0.014	$1.7\,\sigma$
SM: Rome ToV	0.279 ± 0.012	$2.3\,\sigma$
BaBar	0.440 ± 0.072	



[JA Bailey et al. [FNAL/MILC], arXiv:1206.4992, PhysRevLett.109.071802]
[BABAR collab. arXiv:1205.5442, Phys. Rev. Lett. 109, 101802 (2012)]

presently: do the quenched numbers have any relevance?

[GMdD, Petronzio, Tantalo [Rome ToV], arXiv:0807.2944, Nucl.Phys.B807:373-395,2009]



- quenched form factors are in very good agreement with $N_f = 2 + 1$ at zero recoil
- the ratio $F^{B \to D^\star}(w)/G^{B \to D}(w)$ is in good agreement with experimental data

Conclusions & outlooks

- FNAL/MILC calculations provide $\sim 2\%$ relative accuracy for $B \to D^{(\star)}\ell\nu$ at zero recoil, the analysis for $w > 1$ is in progress
- We are waiting for unquenched results from other collaborations to assess the lattice systematic errors
- The Step Scaling Method is a viable possibility, it works very well in the quenched approximation
- Form factors can/must be calculated at $w > 1$ with good accuracy
  - avoid extrapolations on the experimental side $\Longrightarrow$ much precision on $|V_{cb}|$ determination (in the $B \to D\ell\nu$ channel experimental extrapolations have a big impact on $|V_{cb}|$)
  - allow to check the consistency of $|V_{cb}|$ determination over the full range of w values
  - the shape and the normalization of the form factors represent the complete parametrization of the hadronic currents $\Longrightarrow$ can be used to obtain lattice-QCD results of many observables, i.e. branching ratio fraction $R(D^{(*)})$, longitudinal polarization ratios $P_L(D^{(*)})$, $B_s \to \mu^+\mu^- \ldots$