$B \to D^{(*)}$ lattice form factors

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The exclusive semileptonic $B \to D^{(*)}$ decay

Motivations:

- The two decay channels $B \to Dl\nu$ and $B \to D^*l\nu$, where $l$ is an electron or a muon, allow to determine two independent estimates of $|V_{cb}|$. The main theoretical uncertainty on $|V_{cb}|$ comes from the form factors which parametrize the hadronic weak current $\implies$ improve lattice precision.

- inclusive & exclusive
  
  Relying on lattice [FNAL/MILC] determination of $F(1)$:
  
  $$|V_{cb}(excl)| = (39.7 \pm 0.7_{exp} \pm 0.7_{LQCD}) \times 10^{-3}$$

  "$2\sigma$ tension" with the inclusive determination
  
  $$|V_{cb}(incl)| = (41.9 \pm 0.8) \times 10^{-3}$$

  No tension for the heavy flavor sum rule calculations of the form factor $F(1)$, and for results from BaBar09+lattice [Rome ToV] $G(w)$.

- $\tau$ & light leptons


  $$R(D^{(*)}) = \frac{B(B \to D^{(*)}\tau\bar{\nu}_\tau)}{B(B \to D^{(*)}l\bar{\nu}_l)}, \quad \text{where} \quad l = e, \mu$$

  $R(D) = 0.440 \pm 0.058 \pm 0.042 \quad 2.0\sigma$ away from SM: $R(D)^{SM} = 0.297 \pm 0.017$

  $R(D^*) = 0.332 \pm 0.024 \pm 0.018 \quad 2.7\sigma$ away from SM: $R(D^*)^{SM} = 0.252 \pm 0.003$

  Exceed the Standard Model expectations: together, the disagreement is at the $3.4\sigma$ level.
The matrix elements $B \rightarrow D^{(*)}$ of the hadronic weak currents

The matrix elements of the vector and the axial part of the charged weak current can be parametrized through the $h$ form factors:

$$\langle D | V^\mu | B \rangle \frac{1}{\sqrt{M_B M_D}} = (v_B + v_D)^\mu \ h_+(w) + (v_B - v_D)^\mu \ h_-(w)$$

$$\langle D^*_r | V^\mu | B \rangle \frac{1}{\sqrt{M_B M_D^*}} = \varepsilon^{\mu\nu\alpha\beta} v_B^\nu v_D^{*\alpha} \epsilon^{* \beta}_r \ h_V(w)$$

$$\langle D^*_r | A^\mu | B \rangle \frac{1}{\sqrt{M_B M_D^*}} = \epsilon^{* \nu}_r \ [h_{A_1}(w)(1 + w) g^{\mu\nu} - (h_{A_2}(w)v_B^\mu + h_{A_3}(w)v_{D^*}^\mu)v_B^\nu]$$

Where $w \equiv v_{D^(*)} \cdot v_B = \frac{M_B^2 + M_D^{(*)2} - q^2}{2M_B M_D^{(*)}}$ is the product of the four-velocities of the $B$ and the $D^{(*)}$ mesons, and a linear function of the four-momentum transfer $q^2$. 
The exclusive semileptonic decay rate

In the limit of vanishing lepton mass the differential decay rate depends upon a single form factor, which is a combination of the ones describing the current

\[
\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = \text{(fact.)} \times |V_{cb}|^2 (\omega^2 - 1)^{3/2} \left[ G^{B\to D}(\omega) \right]^2
\]

\[
G^{B\to D}(\omega) = h_+(w) - \frac{M_D - M_B}{M_D + M_B} h_-(w)
\]

\[
\frac{d\Gamma(B \to D^*\ell\nu)}{dw} = \text{(fact.)} \times |V_{cb}|^2 \sqrt{w^2 - 1} (1 + w)^2 \lambda(w) \left[ F^{B\to D^*}(w) \right]^2
\]

\[
F^{B\to D^*}(w) = h_{A_1}(w) \sqrt{\frac{H_0^2(w) + H_+^2(w) + H_-^2(w)}{\lambda(w)}}
\]

where

\[
H_0(w) = \frac{w - r - X_3(w) - r X_2(w)}{1 - r}
\]

\[
H_{\pm}(w) = t(w) [1 \pm X_V(w)]
\]

\[
X_V(w) = \sqrt{\frac{w - 1}{w + 1} \frac{h_V(w)}{h_{A_1}(w)}}
\]

\[
X_2(w) = (w - 1) \frac{h_{A_2}(w)}{h_{A_1}(w)}
\]

\[
X_3(w) = (w - 1) \frac{h_{A_3}(w)}{h_{A_1}(w)}
\]

\[
r = \frac{M_{D^*}}{M_B}
\]

\[
t^2(w) = \frac{1 - 2wr + r^2}{(1 - r)^2}
\]

\[
\lambda(w) = 1 + \frac{4w}{w + 1} t^2(w)
\]
Experimental measurements = |(known) \times (CKM elements) \times (hadronic form factor)|^2

One $w$ (or $q^2$) point from the lattice (the normalization of the form factors) is enough to determine $|V_{cb}|$

$B \to D^*$ channel has less experimental uncertainties than the $B \to D$ channel

$\implies$ exclusive $|V_{cb}|$ usually extracted from the lattice form factor $F(1)$
Experimental Results

\[ B \rightarrow Dl\nu : \quad G(w) \]

\[ B \rightarrow D^*l\nu : \quad F(w) \]

Experimental measurements = \( |(\text{known}) \times (\text{CKM elements}) \times (\text{hadronic form factor})|^2 \)

\( w = 1 \) is the easiest point to compute on the lattice, but it requires an extrapolation of experimental data.

In order to minimize errors, lattice results must be computed over the range of non-zero recoil \( w > 1 \) values where the experimental data are more precise.
Despite many lattice collaborations...

Some of the acronyms:

[ETMC] European Twisted Mass collaboration (EU)
[MILC] MIMD (Multiple Instruction Multiple Data) Lattice Computation collaboration (US)
[FNAL] Fermi National Accelerator Laboratory (US)
[QCDSF] QCD Structure Functions (EU,JP)
[UKQCD] United Kingdom QCD collaboration (EU)
[BMWC] Budapest-Marseille-Wuppertal collaboration (EU)
[PACS-CS] Parallel Array Computer System for Computational Sciences collaboration (JP)
[RBC] RIKEN-BNL Research Center (RBRC), Brookhaven National Lab. (BNL) and Columbia Univ. (US)
[JLQCD] Japan Lattice QCD collaboration (JP)
[TWQCD] TaiWan QCD collaboration (TW)
[HSC] Hadron Spectrum Collaboration (US)
[BGR] Bern–Graz–Regensburg collaboration (EU)
[CLS] Coordinated Lattice Simulations (EU)
[HPQCD] High Precision QCD (EU)
[LHP] Lattice Hadron Physics Collaboration.

Apologies for not intentional omissions
Despite many lattice collaborations (all results soon averaged and summarized in FLAG-2 report) . . .

Up to now two efforts to summarize lattice QCD results (only PUBLISHED results):

http://www.latticeaverages.org, by J. Laiho, E. Lunghi, and R. Van de Water


- light and heavy quark data + UT fits with lattice inputs
- \( N_f = 2 + 1 \) results

http://itpwiki.unibe.ch/flag, by Flavianet Lattice Average group (FLAG)


- light quarks only: light quark masses, \( K \) and \( \pi \) physics, LowEnergyContants, . . .
- \( N_f = 2 \) and \( N_f = 2 + 1 \) averaged separately

Both will merge in a wider collaboration to cover ALL lattice data:

latticeaverages + FLAG = FLAG-2

- light and heavy hadron phenomenology
- from collaborations: Alpha, BMW, ETMC, RBC/UKQCD, CLS, Fermilab, HPQCD, JLQCD, MILC, PACS-CS, SWME, . . .
- Review report expected at the end of 2012

FLAG-2 organization

- Advisory Board:
  S. Aoki, C. Bernard, C. Sachrajda
- Editorial Board:
  GC, H. Leutwyler, T. Vladikas, U. Wenger
- Working Groups
  - Quark masses
  - \( V_{us}, V_{ud} \)
  - LEC
  - \( B_K \)
  - \( \alpha_s \)
  - \( f_B, B_B \)
  - \( B \to H\nu \)

[G Colangelo plenary talk @ Lattice 2012]
...unquenched results for $B \to D^{(*)} l \nu$ only from FNAL/MILC. Some history after $\sim2000$:

Quenched@($w = 1$) $\rightarrow$ Unquenched@($w = 1$) $\rightarrow$ Quenched@($w \geq 1$) $\rightarrow$ Unquenched@($w \geq 1$)

- $B \to D^*$
  - 2001 Quenched calculation at zero recoil  
  - 2008 Quenched calculation at non-zero recoil  
  - 2008 Unquenched 2+1 at zero recoil  
  - 2010 FNAL/MILC update at Lattice  

- $B \to D$
  - 1999 Quenched calculation at zero recoil  
  - 2004 Unquenched 2+1 calculation at zero recoil  
  - 2007 Quenched calculation at non-zero recoil  
  - 2011 Unquenched 2+1 non-zero recoil  
    [SW Qiu et al. [FNAL/MILC], arXiv:1111.0677, Lattice 2011]
  - 2012 FNAL/MILC, and update at Lattice  
    [JA Bailey et al. [FNAL/MILC], arXiv:1202.6346, PhysRevD.85.11450]
    [JA Bailey et al. [FNAL/MILC], arXiv:1206.4992, PhysRevLett.109.071802]
    [SW Qiu et al. [FNAL/MILC], arXiv:1211.2247, Lattice 2012]
QCD (and $B$ physics in particular) is a multi–scale problem ($m_u, m_d, m_s, m_c, m_b, \Lambda_{QCD}$) simulations are computational expensive

Lattice is an IR/UV regulator

\[
\begin{align*}
\text{InfraRed cutoff} & : \quad \Lambda_{IR} = 1/L \\
\text{UltraViolet cutoff} & : \quad \Lambda_{UV} = 1/a
\end{align*}
\]

The propagation of a heavy quark needs large volumes and fine lattice spacings to control the Finite Volume Effects and Discretization Errors:

\[
e^{-M\pi L} \ll 1 \quad L \gtrsim 6 \text{ fm} \quad \Lambda_{IR} = 1/L \lesssim 33 \text{ MeV} \\
am_{heavy} \ll 1 \quad \alpha \lesssim 0.05 \text{ fm} \quad \Lambda_{UV} = 1/a \gtrsim 4 \text{ GeV}
\]

\[
N_{points} = L/a \simeq 120
\]

The simulated masses:

\[
\begin{align*}
\text{extrapolated from nearby} & \quad \text{extrapolated} \\
m_{ud}^{\text{sim}} & > m_{ud}^{\text{phys}} \\
m_s^{\text{sim}} & \approx m_s^{\text{phys}} \\
m_c^{\text{sim}} & \approx m_c^{\text{phys}} \\
m_b^{\text{sim}} & < m_b^{\text{phys}}
\end{align*}
\]

\[
\text{CPU cost}[Tflops \times \text{years}] = N \left( \frac{20 \text{ MeV}}{m} \right)^\alpha \left( \frac{L}{3 \text{ fm}} \right)^\beta \left( \frac{0, 1\text{fm}}{a} \right)^\gamma = \mathcal{O}(1) \quad \alpha \sim 1-2 \quad \beta \sim 5 \quad \gamma \sim 4-6
\]
Recent dynamical fermion lattice simulations

All points relative to $N_f = 2 + 1$ except when explicitly indicated: (2), (2+1+1)

Discretization effects in percentage the size of finite-volume effects

Heavy quarks on the lattice

For the accessible lattices the UV cut-off is smaller than the $b$ quark mass

$$\implies \text{large and uncontrolled discretization errors } \propto (am_b)^n$$

Various approaches introduced to manage heavy quarks on the lattice:

- **Effective field theories** for heavy quarks
  - Tuning of parameters of the lattice action: in lattice perturbation theory or by matching QCD non-perturbatively on a small volume
  - non perturbative HQET [ALPHA]
    - static quarks $+ 1/m$ corrections as insertions
  - NRQCD [FNAL/MILC, HPQCD]
    - non-relativistic quark action
  - Fermilab [FNAL/MILC, HPQCD]
    - improved action with breaking of the time space symmetry

  “Interpolation” to $b$ from the charm region and the static limit [ETMC, ALPHA, Becirevic et al. . . .]
  - Interpolation to bottom with fitting functions motivated by HQET

  “Direct” relativistic $b$

  - HISQ action: [FNAL/MILC, HPQCD]
    - High improvement and $am$-dependent coefficients
  - SSM Step Scaling method [Rome ToV]
    - Simulation of $b$ on small volume and evolution in the volume
The Fermilab method

The Fermilab approach consists in simulating the following action with $am_0 \simeq 1$

$$S = \sum_n \bar{\psi}_n \left[ m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - r_t \frac{a D_0^2}{2} - r_s \frac{a \vec{D}^2}{2} + c_B \frac{i \sigma_{ij} F_{ij}}{4} + c_E \frac{i \sigma_{0i} F_{0i}}{2} \right] \psi_n$$

i.e. the Symanzik improved effective action for quarks with $|a\vec{p}| \ll 1$

with mass dependent coefficients usually computed perturbatively

- the number of parameters in the action can be reduced to 3

- and can be determined non-perturbatively by matching QCD on a small volume
The Step Scaling Method

A finite size scaling procedure

\[
\mathcal{O}(m_b, m_l; L = \infty) = \mathcal{O}(m_b, m_l; L_0) \quad \frac{\mathcal{O}(m_b, m_l; 2L_0)}{\mathcal{O}(m_b, m_l; L_0)} \quad \frac{\mathcal{O}(m_b, m_l; 4L_0)}{\mathcal{O}(m_b, m_l; 2L_0)} \quad \cdots
\]

- the step scaling functions $\sigma$'s calculated at lower values of the high energy scale

\[
\mathcal{O}(m_b, m_l; L_0) \leftarrow m_b = m_b^{phys}
\]

\[
\sigma(m_b, m_l; nL_0) \leftarrow m_b \leq \frac{m_b^{phys}}{n}
\]

- The extrapolation of the step scaling functions is much easier than the extrapolation of the observable itself

\[
\mathcal{O}(m_b, m_l; L) = \mathcal{O}^0(m_l; L) \left[ 1 + \frac{\mathcal{O}^1(m_l; L)}{m_b} \right]
\]

\[
\sigma(m_b, m_l; L) = \frac{\mathcal{O}^0(m_l; 2L)}{\mathcal{O}^0(m_l; L)} \left[ 1 + \frac{\mathcal{O}^1(m_l; 2L) - \mathcal{O}^1(m_l; L)}{m_b} \right]
\]
Basic ingredients of SSM

- **Finite Volume Scheme:** Schrödinger functional, i.e. Dirichlet boundary condition in time ($m_{light} = 0$ on the lattice)

- **continuous momenta** (with flavour-twisted boundary conditions) to reach "small" $w$ values
  
  $$\psi(x + iL) = e^{i\theta_i} \psi(x) \quad \theta_0 = 0$$

  $$p_i = \frac{2\pi \theta_i}{L} + \frac{2\pi n_i}{L}, \quad n \in \mathbb{Z}^3, \quad \theta_i \in [0, 1]$$

- Form factors calculated with good precision entirely in terms of double ratios of three point correlation functions

  \[
  \frac{\langle M_F | J^\mu | M_I \rangle}{2\sqrt{E_F E_I}} = \sqrt{\text{different colours} = \text{different flavours}}
  \]

  \[
  \langle PV P \rangle^\mu_{i f} = \hat{Z}_V \sum_{\vec{x}} \langle P_{li} \; V^\mu_{if} (x) \; P'_{fl} \rangle \quad \langle V V V \rangle^I_{i f} = \hat{Z}_V \sum_{\vec{x}} \langle V^I_{li} \; V^\mu_{if} (x) \; V'_{f I} \rangle
  \]

  \[
  \langle PVV \rangle^\mu_{i f} = \hat{Z}_V \sum_{\vec{x}} \langle P_{li} \; V^\mu_{if} (x) \; V'_{f I} \rangle \quad \langle P A V \rangle^I_{i f} = \hat{Z}_A \sum_{\vec{x}} \langle P_{li} \; A^\mu_{if} (x) \; V'_{f I} \rangle
  \]

- **SSM finite size recursive procedure to estimate FVE**
[Rome ToV] $B \to D^* \ell \nu$ at non-zero recoil: the small volume

$$F^{B \to D^*}(w) = h_{A_1}(w) \sqrt{\frac{H_0^2(w) + H_+^2(w) + H_-^2(w)}{\lambda(w)}}$$

$$F^{B \to D^*}(w; L_2) = F^{B \to D^*}(w; L_0) \sigma^{B \to D^*}(w; L_0, L_1) \sigma^{B \to D^*}(w; L_1, L_2)$$

The discretization errors on the small volume are under control
[Rome ToV] $B \to D^* \ell \nu$: first volume step

step scaling functions are very flat: extrapolated values differ from simulated ones by a few per mille

$$\sigma^P \to D^* (w; L_0, L_1) = \frac{F^P \to D^* (w; L_1)}{F^P \to D^* (w; L_0)}$$

$$F^B \to D^* (w; L_2) = F^B \to D^* (w; L_0) \sigma^B \to D^* (w; L_0, L_1) \sigma^B \to D^* (w; L_1, L_2)$$
step scaling functions are very flat: extrapolated values differ from simulated ones by a few per mille

\[
\sigma^{P\to D^*}(w; L_1, L_2) = \frac{F^{P\to D^*}(w; L_2)}{F^{P\to D^*}(w; L_1)}
\]

\[
F^{B\to D^*}(w; L_2) = F^{B\to D^*}(w; L_0) \sigma^{B\to D^*}(w; L_0, L_1) \sigma^{B\to D^*}(w; L_1, L_2)
\]
One $q^2$ point from the lattice, the zero-recoil $q^2_{\text{max}}$ ($w = 1$) is the easiest to compute

$$F(1) = 0.9077(51)_M(88)C(84)g(84)\chi(90)H(30)Q(30)Z(33)$$

- Fermilab action for $b$ and $c$ quarks
- Asqtad staggered action for light valence quarks
- 2+1 rooted staggered sea quarks

Mild light quark mass dependence

reproduced from [S.-W. Qiu et al. [Fermilab/MILC], arXiv:1011.2166]
\[ F(1) = h_{A_1}(1) = 0.9077(51)_{\text{stat}}(88)g(84)\chi(90)a(30)Z(33)\kappa \]

### Error Budget

<table>
<thead>
<tr>
<th>label</th>
<th>percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{stat}</td>
<td>0.5 %</td>
</tr>
<tr>
<td>\textit{g}</td>
<td>1 %</td>
</tr>
<tr>
<td>\textit{\chi}</td>
<td>0.9 %</td>
</tr>
<tr>
<td>\textit{a}</td>
<td>1 %</td>
</tr>
<tr>
<td>\textit{Z}</td>
<td>0.3 %</td>
</tr>
<tr>
<td>\textit{\kappa}</td>
<td>0.3 %</td>
</tr>
</tbody>
</table>

Total 1.7 % errors summed in quadrature

1.7% error on \( F(1) \) translates in 16% on \( (1 - F(1)) \)

\[ |V_{cb}(\text{excl})| \text{ from BaBar+Belle+FNAL/MILC determination of } F(1): \]

\[ |V_{cb}(\text{excl})| = (39.54 \pm 0.50_{\text{exp}} \pm 0.74_{LQCD}) \times 10^{-3} \]

“2\(\sigma\) tension” with the inclusive determination

\[ |V_{cb}(\text{incl})| = (41.9 \pm 0.8) \times 10^{-3} \]

still lacking of \( B \rightarrow D^*l\nu \) at non-zero recoil
From $B \to Dl\nu$ it comes an independent determination of $|V_{cb}|$

$$G(1) = 1.074(18)_{stat}(16)_{syst}$$

turns into the determination

$$|V_{cb}| = (39.70 \pm 1.42_{exp} \pm 0.89_{LQCD}) \times 10^{-3}$$

[The Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/]

2\% error on $G(1)$ translates in 30\% on $(1 - G(1))$
Summary of $w = 1$ results [FNAL/MILC] + [Rome ToV]

reproduced from [C Tarantino, arXiv:0807.2944]

The present accuracy on $|V_{cb}|$ is at the 2\% level

<table>
<thead>
<tr>
<th></th>
<th>$f f @ (w = 1)$</th>
<th>$F(1)$</th>
<th>$G(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rome ToV [$n_f = 0$]</td>
<td></td>
<td>$0.917 \pm 0.008 \pm 0.005$</td>
<td>$1.026 \pm 0.017$</td>
</tr>
<tr>
<td>FNAL/MILC [$n_f = 2 + 1$]</td>
<td></td>
<td>$0.9077 \pm 0.0051 \pm 0.0158$</td>
<td>$1.074 \pm 0.018 \pm 0.016$</td>
</tr>
</tbody>
</table>
**[Fermilab/MILC] \( B \to Dl\nu \) at non-zero recoil**

\( B \to Dl\nu \) form factors recently determined at non-zero recoil to improve the precision with respect to the previous 2004 result \( @ (w = 1) \)

Simulated kinematic range \( 1 \leq w < 1.15 \), extrapolation to the full range \( 1 \leq w < 1.6 \) using the \( z \) expansion (unitarity and analyticity):

\[
z(w) = \frac{\sqrt{1 + w} - \sqrt{2}}{\sqrt{1 + w} + \sqrt{2}}
\]

It represents a conformal map \( w \to z \):

\( w \in [1, 1.6] \to z \in [0, 0.064] \)

\[
f(z) = \frac{1}{P(z)\phi(z)} \sum_{n=0}^{\infty} a_n z^n
\]

\( P(z) \) Blaschke factor, \( \phi \) outer function

---

reproduced from [S.-W. Qiu et al. [Fermilab/MILC], arXiv:1206.4992]

reproduced from [S.-W. Qiu et al. [Fermilab/MILC], arXiv:1211.2247]
$F^{B \rightarrow D^*}(w = 1.075) = 0.877(18)(04)$

$|V_{cb}|(@w = 1.075) = 37.4(8)(5) \times 10^{-3}$

Full parametrization of matrix elements:
also results for each form factor
$h_{A_1}(w), h_{A_2}(w), h_{A_3}(w), h_V(w)$

<table>
<thead>
<tr>
<th>$w$</th>
<th>$F(w)$ [Rome ToV]</th>
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<tr>
<td>1.000</td>
<td>$0.917 \pm 0.008 \pm 0.005$</td>
</tr>
<tr>
<td>1.010</td>
<td>$0.913 \pm 0.009 \pm 0.005$</td>
</tr>
<tr>
<td>1.025</td>
<td>$0.905 \pm 0.010 \pm 0.005$</td>
</tr>
<tr>
<td>1.050</td>
<td>$0.892 \pm 0.013 \pm 0.004$</td>
</tr>
<tr>
<td>1.070</td>
<td>$0.880 \pm 0.017 \pm 0.004$</td>
</tr>
<tr>
<td>1.075</td>
<td>$0.877 \pm 0.018 \pm 0.004$</td>
</tr>
<tr>
<td>1.100</td>
<td>$0.861 \pm 0.023 \pm 0.004$</td>
</tr>
</tbody>
</table>
$B \to D\ell\nu$: theory vs. experiment

$G^{B \to D}(w = 1.2) = 0.853(21)$

$|V_{cb}|(@w = 1.2) = 41.4(1.3)(1.4)(1.0) \times 10^{-3}$

no tension with inclusive determination $(41.9 \pm 0.8) \times 10^{-3}$

comparison with BaBar 2010 data

| $w$  | $G(w)$ [Rome ToV] | $|V_{cb}|G(w) \times 10^3$ [BaBar 10] | $|V_{cb}| \times 10^3$ |
|------|------------------|-------------------------------------|---------------------|
| 1.00 | 1.026 ± 0.017    | 40.9 ± 5.7 ± 1.3                    | 40.9 ± 5.7 ± 1.3 ± 0.8 |
| 1.03 | 1.001 ± 0.019    | 40.2 ± 5.0 ± 1.3                    | 40.7 ± 5.1 ± 1.3 ± 0.6 |
| 1.05 | 0.987 ± 0.015    | 38.3 ± 3.3 ± 1.3                    | 40.6 ± 3.5 ± 1.4 ± 0.5 |
| 1.10 | 0.943 ± 0.011    | 35.3 ± 1.1 ± 1.2                    | 41.4 ± 1.3 ± 1.4 ± 1.0 |
| 1.20 | 0.853 ± 0.021    |                                     |                     |
B → D\ell\nu: the τ channel

Full parametrization of matrix elements:
also results for each form factor \( h_+ (w), h_- (w) \) \( \implies \) byproduct: the τ channel

In the case \( \ell = \tau \) the mass of the lepton cannot be neglected and the differential decay rate is given by

\[
\frac{d\Gamma_{B \to D \tau \bar{\nu}_\tau}}{dw} = \frac{d\Gamma_{B \to D (e,\mu) \nu_e,\mu}}{dw} \left( 1 - \frac{r_\tau^2}{t(w)} \right)^2 \left\{ \left( 1 + \frac{r_\tau^2}{2t(w)} \right) + \frac{3r_\tau^2}{2t(w)} \frac{w + 1}{w - 1} \right\} [\Delta(w)]^2
\]

\[
\Delta(w) = \frac{1}{G(w)} \left[ \frac{1 - r}{1 + r} h_+ (w) - \frac{w - 1}{w + 1} h_- (w) \right]
\]

\[
r_\tau = \frac{m_\tau}{M_B}, \quad r = \frac{M_D}{M_B}, \quad t(w) = 1 + r^2 - 2rw, \quad 1 \leq w \leq \frac{M_B^2 + M_D^2 - m_\tau^2}{2M_B M_D}
\]

\[
R(D) = \frac{B(B \to D\tau\bar{\nu}_\tau)}{B(B \to Dl\bar{\nu}_l)}, \quad \text{where} \quad l = e, \mu
\]

\[
R(D) = 0.440 \pm 0.058 \pm 0.042 \quad 2.0 \sigma \text{ away from SM: } R(D)_{SM} = 0.297 \pm 0.017
\]

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<td>SM</td>
<td>0.297 ± 0.017</td>
</tr>
<tr>
<td>SM: FNAL/MILC</td>
<td>0.316 ± 0.014</td>
</tr>
<tr>
<td>SM: Rome ToV</td>
<td>0.279 ± 0.012</td>
</tr>
<tr>
<td>BaBar</td>
<td>0.440 ± 0.072</td>
</tr>
</tbody>
</table>


Presently: do the quenched numbers have any relevance?

Quenched form factors are in very good agreement with $N_f = 2 + 1$ at zero recoil.

The ratio $F_{B \to D^*} (w) / G_{B \to D} (w)$ is in good agreement with experimental data.
*Conclusions & outlooks*

- FNAL/MILC calculations provide $\sim 2\%$ relative accuracy for $B \to D(\ast)\ell\nu$ at zero recoil, the analysis for $w > 1$ is in progress.
- We are waiting for unquenched results from other collaborations to assess the lattice systematic errors.
- The Step Scaling Method is a viable possibility, it works very well in the quenched approximation.
- Form factors can/must be calculated at $w > 1$ with good accuracy:
  - avoid extrapolations on the experimental side $\implies$ much precision on $|V_{cb}|$ determination (in the $B \to D\ell\nu$ channel experimental extrapolations have a big impact on $|V_{cb}|$)
  - allow to check the consistency of $|V_{cb}|$ determination over the full range of $w$ values
  - the shape and the normalization of the form factors represent the complete parametrization of the hadronic currents $\implies$ can be used to obtain lattice-QCD results of many observables, i.e. branching ratio fraction $R(D(\ast))$, longitudinal polarization ratios $P_L(D(\ast))$, $B_s \to \mu^+ \mu^-$...