Particle Physics: The Standard Model

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March 28 and April 4, 2013

- Remember the particle zoo
- γ and e
- today: add μ and τ

Definition

Charged Leptons: e, μ, τ
Leptons: charged leptons plus neutrinos
Jargon: leptons as charged leptons

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ W^{\pm}, Z^{\circ} \\ H \end{matrix}$$

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$$\begin{matrix} \mathrm{u_R} & \mathrm{c_R} & \mathrm{t_R} \\ \mathrm{d_R} & \mathrm{s_R} & \mathrm{b_R} \\ \mathrm{e_R} & \mu_\mathrm{R} & \tau_\mathrm{R} \end{matrix}$$

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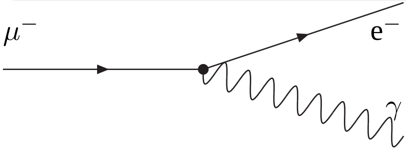
Properties of the μ

$$\begin{array}{lcll} m_0 & = & 0.105\,\mathrm{GeV} & \mu^+e^- \\ \tau & = & (2.197\cdot 10^{-6})\,\mathrm{s} & \mathrm{PSI} \\ c\tau & = & 659\,\mathrm{m} & \end{array}$$



$$\begin{array}{lcl} \mathcal{B}(\mu\to e\gamma) & < & 1.2\cdot 10^{-11} \\ \mathcal{B}(\tau\to e\gamma) & < & 3.3\cdot 10^{-8} \\ \mathcal{B}(\tau\to \mu\gamma) & < & 4.4\cdot 10^{-8} \\ CL & = & 90\% \end{array}$$

Properties of the τ

$$\begin{array}{lcll} m_0 & = & 1.777\,\mathrm{GeV} & e^+e^- \\ \tau & = & (2.906\cdot 10^{-13})\,\mathrm{s} & e^+e^- \\ c\tau & = & 87\,\mu\mathrm{m} & \end{array}$$

Lepton numbers (additive QNs)

	L_e	L_μ	L_τ
e^-	1	0	0
e^+	-1	0	0
μ^-	0	1	0
μ^+	0	-1	0
τ^-	0	0	1
τ^+	0	0	-1
non − leptons	0	0	0

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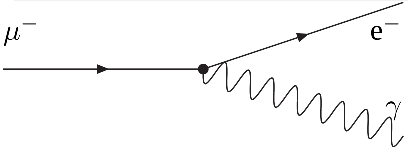

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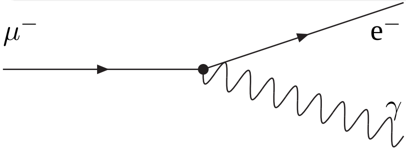

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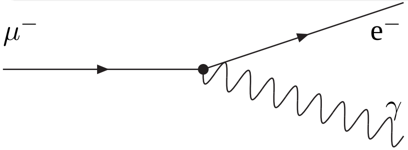

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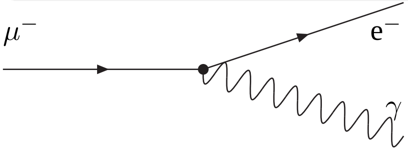

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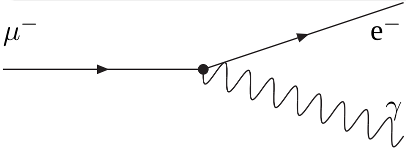

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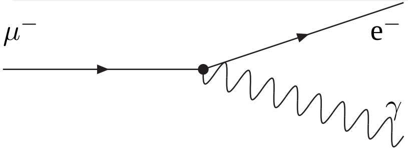

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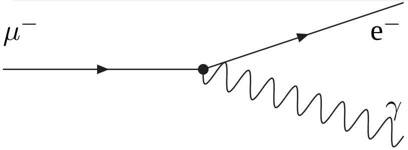

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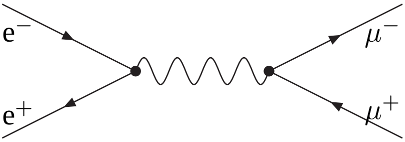

$$e^+(\mathbf{p_2})e^-(\mathbf{p_1}) \rightarrow \mu^+(\mathbf{p_4})\mu^-(\mathbf{p_3})$$

- $L_e^i = 1 - 1 = 0 = L_e^f$
- $L_\mu^i = 0 = 1 - 1 = L_\mu^f$
- Initial state
- Final state
- Photon Propagator

Transition Amplitude

$$\begin{aligned}
\frac{1}{i}T_{fi} &= \frac{1}{i}\left[\bar{v}(\mathbf{p_2})(-ie\gamma^\mu)u(\mathbf{p_1})\frac{-ig_{\mu\nu}}{k^2}\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)v(\mathbf{p_4})\right] \\
&= e^2[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\frac{g_{\mu\nu}}{s}\bar{u}(\mathbf{p_3})\gamma^\nu v(\mathbf{p_4})] \\
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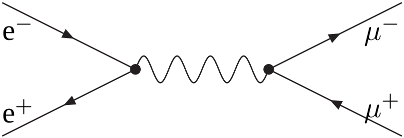

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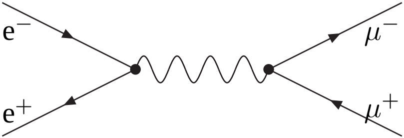

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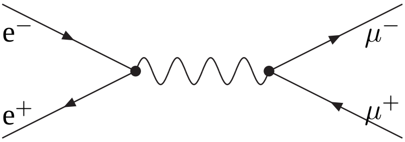

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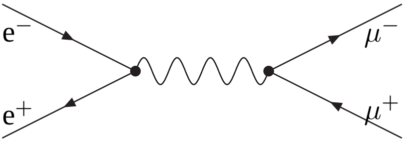

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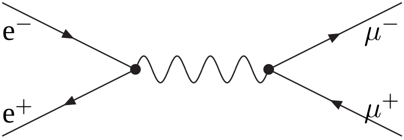

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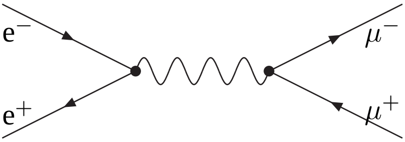

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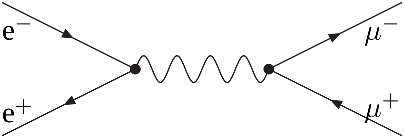
$$= e^2[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\frac{g_{\mu\nu}}{s}\bar{u}(\mathbf{p_3})\gamma^\nu v(\mathbf{p_4})]$$

$$= \frac{e^2}{s}[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\mu v(\mathbf{p_4})]$$

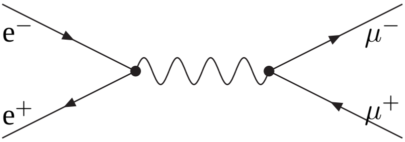


$$e^+(\mathbf{p_2})e^-(\mathbf{p_1}) \to \mu^+(\mathbf{p_4})\mu^-(\mathbf{p_3})$$

- $L_e^i = 1 - 1 = 0 = L_e^f$
- $L_\mu^i = 0 = 1 - 1 = L_\mu^f$
- Initial state
- Final state
- Photon Propagator

Transition Amplitude

$$\begin{aligned}
\frac{1}{i}T_{fi} &= \frac{1}{i}[\bar{v}(\mathbf{p_2})(-ie\gamma^\mu)u(\mathbf{p_1})\frac{-ig_{\mu\nu}}{k^2}\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)v(\mathbf{p_4})] \\
&= e^2[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\frac{g_{\mu\nu}}{s}\bar{u}(\mathbf{p_3})\gamma^\nu v(\mathbf{p_4})] \\
&= \frac{e^2}{s}[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\mu v(\mathbf{p_4})]
\end{aligned}$$



$e^+(\mathbf{p_2})e^-(\mathbf{p_1}) \rightarrow \mu^+(\mathbf{p_4})\mu^-(\mathbf{p_3})$

- $L_e^i = 1 - 1 = 0 = L_e^f$
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Transition Amplitude

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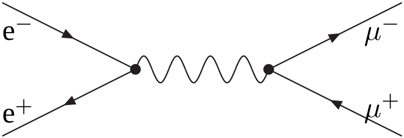

$$e^+(\mathbf{p_2})e^-(\mathbf{p_1}) \rightarrow \mu^+(\mathbf{p_4})\mu^-(\mathbf{p_3})$$

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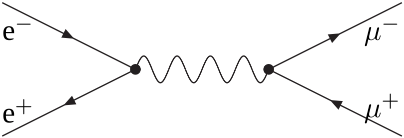

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Transition Amplitude

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&= e^2[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\frac{g_{\mu\nu}}{s}\bar{u}(\mathbf{p_3})\gamma^\nu v(\mathbf{p_4})] \\
&= \frac{e^2}{s}[\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\mu v(\mathbf{p_4})]
\end{aligned}$$

Useful Formula

$$
\begin{aligned}
\gamma_0 &= g_{\mu 0}\gamma^0 &= \gamma^0 \\
\gamma_k &= g_{\mu k}\gamma^k &= -\gamma^k \\
\bar{u} &= u^\dagger\gamma^0 &= u^\dagger\gamma_0
\end{aligned}
$$

Insert

$$
\begin{aligned}
& [\bar{v}(\mathbf{p_2})\gamma^\nu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\nu v(\mathbf{p_4})]^\dagger \\
= & [v^\dagger(\mathbf{p_2})\gamma^0\gamma^\nu u(\mathbf{p_1})u^\dagger(\mathbf{p_3})\gamma^0\gamma_\nu v(\mathbf{p_4})]^\dagger \\
= & [v^\dagger(\mathbf{p_4})(\gamma_\nu)^\dagger(\gamma^0)^\dagger(u^\dagger)^\dagger(\mathbf{p_3})u^\dagger(\mathbf{p_1})(\gamma^\nu)^\dagger(\gamma^0)^\dagger(v^\dagger)^\dagger(\mathbf{p_2})] \\
= & [v^\dagger(\mathbf{p_4})(\gamma_\nu)^\dagger(\gamma^0)^\dagger u(\mathbf{p_3})u^\dagger(\mathbf{p_1})(\gamma^\nu)^\dagger(\gamma^0)^\dagger v(\mathbf{p_2})] \\
= & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu\gamma^0\gamma^0\gamma^0\gamma^0 u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu\gamma^0\gamma^0\gamma^0\gamma^0 v(\mathbf{p_2})] \\
= & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu v(\mathbf{p_2})]
\end{aligned}
$$

Useful Formula

Insert

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& [\bar{v}(\mathbf{p_2})\gamma^\nu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\nu v(\mathbf{p_4})]^\dagger \\
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= & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu v(\mathbf{p_2})] \\
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\end{aligned}
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Useful Formula

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= \; & [v^\dagger(\mathbf{p_4})(\gamma_\nu)^\dagger(\gamma^0)^\dagger(u^\dagger)^\dagger(\mathbf{p_3})u^\dagger(\mathbf{p_1})(\gamma^\nu)^\dagger(\gamma^0)^\dagger(v^\dagger)^\dagger(\mathbf{p_2})] \\
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= \; & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu\gamma^0\gamma^0\gamma^0\gamma^0 u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu\gamma^0\gamma^0\gamma^0\gamma^0 v(\mathbf{p_2})] \\
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\end{aligned}
$$

Useful Formula

$$
\begin{aligned}
(\gamma^\mu)^\dagger &= \gamma^0\gamma^\mu\gamma^0 \\
(\gamma_\mu)^\dagger &= g_{\mu\nu}(\gamma^\nu)^\dagger = g_{\mu\nu}(\gamma^0\gamma^\nu\gamma^0) = \gamma^0\gamma_\mu\gamma^0
\end{aligned}
$$

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= & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu\gamma^0\gamma^0\gamma^0\gamma^0 u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu\gamma^0\gamma^0\gamma^0\gamma^0 v(\mathbf{p_2})] \\
= & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu v(\mathbf{p_2})] \\
= & [\bar{v}(\mathbf{p_4})\gamma_\nu u(\mathbf{p_3})\bar{u}(\mathbf{p_1})\gamma^\nu v(\mathbf{p_2})]
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\end{aligned}$$

Useful Formula

$$\gamma^0\gamma^0 = 1_4$$

Insert

$$
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& [\bar{v}(\mathbf{p_2})\gamma^\nu u(\mathbf{p_1})\bar{u}(\mathbf{p_3})\gamma_\nu v(\mathbf{p_4})]^\dagger \\
= & [v^\dagger(\mathbf{p_2})\gamma^0\gamma^\nu u(\mathbf{p_1})u^\dagger(\mathbf{p_3})\gamma^0\gamma_\nu v(\mathbf{p_4})]^\dagger \\
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= & [v^\dagger(\mathbf{p_4})\gamma^0\gamma_\nu u(\mathbf{p_3})u^\dagger(\mathbf{p_1})\gamma^0\gamma^\nu v(\mathbf{p_2})] \\
= & [\bar{v}(\mathbf{p_4})\gamma_\nu u(\mathbf{p_3})\bar{u}(\mathbf{p_1})\gamma^\nu v(\mathbf{p_2})]
\end{aligned}
$$

Formula

Matrix Element

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{e^4}{4s^2}\sum[\bar{v}_a(\mathbf{p_2})\gamma^\mu_{ab}u_b(\mathbf{p_1})\bar{u}_c(\mathbf{p_3})\gamma_{\mu cd}v_d(\mathbf{p_4})]\\
&\quad [\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef}u_f(\mathbf{p_3})\bar{u}_g(\mathbf{p_1})\gamma^\nu_{gh}v_h(\mathbf{p_2})]\\
&= \frac{e^4}{4s^2}\sum v_h(\mathbf{p_2})\bar{v}_a(\mathbf{p_2})\gamma^\mu_{ab}u_b(\mathbf{p_1})\bar{u}_g(\mathbf{p_1})\gamma^\nu_{gh}\\
&\quad u_f(\mathbf{p_3})\bar{u}_c(\mathbf{p_3})\gamma_{\mu cd}v_d(\mathbf{p_4})\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef}\\
&= \frac{e^4}{4s^2}\sum_s Tr(v(\mathbf{p_2})\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\bar{u}(\mathbf{p_1})\gamma^\nu)\\
&\quad Tr(u(\mathbf{p_3})\bar{u}(\mathbf{p_3})\gamma_\mu v(\mathbf{p_4})\bar{v}(\mathbf{p_4})\gamma_\nu)\\
&= \frac{e^4}{4s^2}Tr(\not{\mathbf{p}}_2\gamma^\mu\not{\mathbf{p}}_1\gamma^\nu)Tr(\not{\mathbf{p}}_3\gamma_\mu\not{\mathbf{p}}_4\gamma_\nu)\\
&= \frac{8e^4}{s^2}[(\mathbf{p_1p_4})(\mathbf{p_2p_3})+(\mathbf{p_1p_3})(\mathbf{p_2p_4})]
\end{aligned}
$$

Formula

Matrix Element

$$
\begin{aligned}
|\mathcal{M}|^2 &= \frac{e^4}{4s^2} \sum [\bar{v}_a(\mathbf{p_2})\gamma^{\mu}_{ab} u_b(\mathbf{p_1}) \bar{u}_c(\mathbf{p_3})\gamma_{\mu cd} v_d(\mathbf{p_4})] \\
&\quad [\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef} u_f(\mathbf{p_3}) \bar{u}_g(\mathbf{p_1})\gamma^{\nu}_{gh} v_h(\mathbf{p_2})] \\
&= \frac{e^4}{4s^2} \sum v_h(\mathbf{p_2})\bar{v}_a(\mathbf{p_2})\gamma^{\mu}_{ab} u_b(\mathbf{p_1})\bar{u}_g(\mathbf{p_1})\gamma^{\nu}_{gh} \\
&\quad u_f(\mathbf{p_3})\bar{u}_c(\mathbf{p_3})\gamma_{\mu cd} v_d(\mathbf{p_4})\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef} \\
&= \frac{e^4}{4s^2} \sum_s Tr(v(\mathbf{p_2})\bar{v}(\mathbf{p_2})\gamma^{\mu} u(\mathbf{p_1})\bar{u}(\mathbf{p_1})\gamma^{\nu}) \\
&\quad Tr(u(\mathbf{p_3})\bar{u}(\mathbf{p_3})\gamma_{\mu} v(\mathbf{p_4})\bar{v}(\mathbf{p_4})\gamma_{\nu}) \\
&= \frac{e^4}{4s^2} Tr(\not{\mathbf{p}}_2\gamma^{\mu} \not{\mathbf{p}}_1\gamma^{\nu}) Tr(\not{\mathbf{p}}_3\gamma_{\mu} \not{\mathbf{p}}_4\gamma_{\nu}) \\
&= \frac{8e^4}{s^2} [(\mathbf{p_1 p_4})(\mathbf{p_2 p_3}) + (\mathbf{p_1 p_3})(\mathbf{p_2 p_4})]
\end{aligned}
$$

Formula

$$\sum_{ff} M_{ff} = Tr(M)$$

Matrix Element

$$
\begin{aligned}
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&\quad [\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef}u_f(\mathbf{p_3})\bar{u}_g(\mathbf{p_1})\gamma^\nu_{gh}v_h(\mathbf{p_2})] \\
&= \frac{e^4}{4s^2}\sum v_h(\mathbf{p_2})\bar{v}_a(\mathbf{p_2})\gamma^\mu_{ab}u_b(\mathbf{p_1})\bar{u}_g(\mathbf{p_1})\gamma^\nu_{gh} \\
&\quad u_f(\mathbf{p_3})\bar{u}_c(\mathbf{p_3})\gamma_{\mu cd}v_d(\mathbf{p_4})\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef} \\
&= \frac{e^4}{4s^2}\sum_s Tr(v(\mathbf{p_2})\bar{v}(\mathbf{p_2})\gamma^\mu u(\mathbf{p_1})\bar{u}(\mathbf{p_1})\gamma^\nu) \\
&\quad Tr(u(\mathbf{p_3})\bar{u}(\mathbf{p_3})\gamma_\mu v(\mathbf{p_4})\bar{v}(\mathbf{p_4})\gamma_\nu) \\
&= \frac{e^4}{4s^2} Tr(\not{\mathbf{p}}_2\gamma^\mu \not{\mathbf{p}}_1\gamma^\nu) Tr(\not{\mathbf{p}}_3\gamma_\mu \not{\mathbf{p}}_4\gamma_\nu) \\
&= \frac{8e^4}{s^2}[(\mathbf{p_1}\mathbf{p_4})(\mathbf{p_2}\mathbf{p_3}) + (\mathbf{p_1}\mathbf{p_3})(\mathbf{p_2}\mathbf{p_4})]
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|\mathcal{M}|^2 &= \frac{e^4}{4s^2} \sum [\bar{v}_a(\mathbf{p_2})\gamma^{\mu}_{ab} u_b(\mathbf{p_1})\bar{u}_c(\mathbf{p_3})\gamma_{\mu cd} v_d(\mathbf{p_4})] \\
&\quad [\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef} u_f(\mathbf{p_3})\bar{u}_g(\mathbf{p_1})\gamma^{\nu}_{gh} v_h(\mathbf{p_2})] \\
&= \frac{e^4}{4s^2} \sum v_h(\mathbf{p_2})\bar{v}_a(\mathbf{p_2})\gamma^{\mu}_{ab} u_b(\mathbf{p_1})\bar{u}_g(\mathbf{p_1})\gamma^{\nu}_{gh} \\
&\quad u_f(\mathbf{p_3})\bar{u}_c(\mathbf{p_3})\gamma_{\mu cd} v_d(\mathbf{p_4})\bar{v}_e(\mathbf{p_4})\gamma_{\nu ef} \\
&= \frac{e^4}{4s^2} \sum_s Tr(v(\mathbf{p_2})\bar{v}(\mathbf{p_2})\gamma^{\mu} u(\mathbf{p_1})\bar{u}(\mathbf{p_1})\gamma^{\nu}) \\
&\quad Tr(u(\mathbf{p_3})\bar{u}(\mathbf{p_3})\gamma_{\mu} v(\mathbf{p_4})\bar{v}(\mathbf{p_4})\gamma_{\nu}) \\
&= \frac{e^4}{4s^2} Tr(\not{\mathbf{p}}_2\gamma^{\mu}\not{\mathbf{p}}_1\gamma^{\nu}) Tr(\not{\mathbf{p}}_3\gamma_{\mu}\not{\mathbf{p}}_4\gamma_{\nu}) \\
&= \frac{8e^4}{s^2}[(\mathbf{p_1 p_4})(\mathbf{p_2 p_3}) + (\mathbf{p_1 p_3})(\mathbf{p_2 p_4})]
\end{aligned}
$$

Formula

$$\sum u\bar{u} \quad = \quad \sum v\bar{v} = \not{\mathbf{p}}$$

Matrix Element

$$
\begin{aligned}
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Formula

$$Tr(\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta) = 4(g^{\alpha\beta}g^{\gamma\delta} + g^{\alpha\delta}g^{\beta\gamma} - g^{\alpha\gamma}g^{\beta\delta})$$

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Formula

$$
\begin{aligned}
(\mathbf{p_1}-\mathbf{p_3})^2 &= \mathbf{p_1}^2+\mathbf{p_3}^2-2\mathbf{p_1}\mathbf{p_3}\\
&= -2\mathbf{p_1}\mathbf{p_3}\\
&= -2\left(\frac{\sqrt{s}}{2}\frac{\sqrt{s}}{2}-\frac{\sqrt{s}}{2}\frac{\sqrt{s}}{2}\cos\theta\right)\\
(\mathbf{p_1}-\mathbf{p_4})^2 &= -2\tfrac{s}{4}(1+\cos\theta)
\end{aligned}
$$

Differential Cross section

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} &= |\mathcal{M}|^2\frac{1}{64\pi^2 s}\\
&= \frac{8e^4}{64\pi^2 s^3}[(\mathbf{p_1}\mathbf{p_4})(\mathbf{p_2}\mathbf{p_3})+(\mathbf{p_1}\mathbf{p_3})(\mathbf{p_2}\mathbf{p_4})]\\
&= \frac{2\alpha^2}{s^3}[\tfrac{s}{4}(1+\cos\theta)\cdot\tfrac{s}{4}(1+\cos\theta)\\
&+ \tfrac{s}{4}(1-\cos\theta)\cdot\tfrac{s}{4}(1-\cos\theta)]\\
&= \frac{2\alpha^2}{s^3}[\tfrac{s^2}{16}(1+\cos\theta)^2+\tfrac{s^2}{16}(1-\cos\theta)^2]\\
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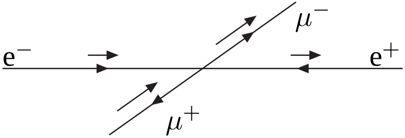
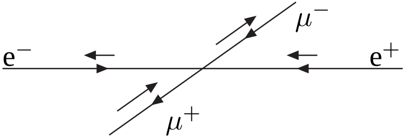
&= \frac{\alpha^2}{4s}[1+\cos^2\theta]
\end{aligned}
$$

$$\frac{d\sigma}{d\Omega} \sim (1-\cos\theta)^2 + (1+\cos\theta)^2$$

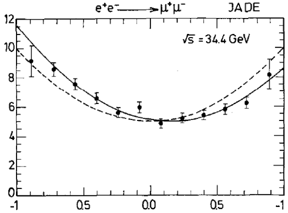
- Do the two terms have a particular meaning?
- Only the spin can lead to an angular distribution that is not flat
- Photon: Spin-1, mass zero $\rightarrow$ 2 dofs: ± 1
- classical ED: 2 polarizations, no restframe...

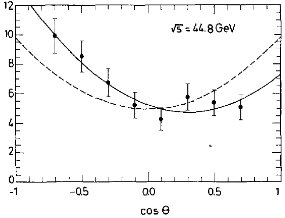


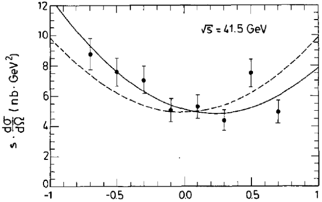
$$\begin{aligned} \theta(\mu^-, e^-) &= 0 \\ 1+\cos\theta &= 2 \quad \text{Probmax} \end{aligned}$$



$$\begin{aligned} \theta(\mu^-, e^-) &= 0 \\ 1-\cos\theta &= 0 \quad \text{Probmin} \end{aligned}$$

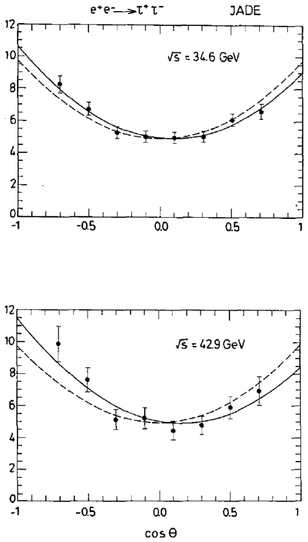


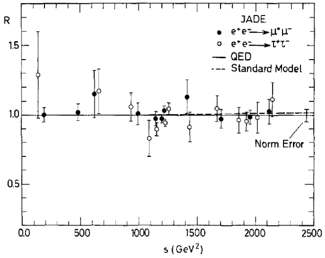




$e^+e^- \to \mu^+\mu^-$

- JADE detector at PETRA
- $s \cdot \frac{d\sigma}{d\Omega}$ scale invariant
- low $s \to (1 + \cos^2\theta)$
- higher $s \to$ asymmetry not QED





$e^+e^- \to \tau^+\tau^-$

- Small mass dependence at high $\sqrt{s}$
- Lepton universality
- Agreement with QED

Bohr

$$
\begin{aligned}
\vec{\mu} &= Current \cdot Surface \cdot \vec{n} \\
&= \frac{e}{t} \cdot \pi r^2 \cdot \vec{n} \\
&= \frac{e}{2\pi r/v} \cdot \pi r^2 \cdot \vec{n} \\
&= \frac{e}{2m}(mvr)\vec{n} \\
&= \frac{e}{2m}(\hbar \ell)\vec{n} \\
&= \mu_B \ell \vec{n} \\
\mu_B &= 5.8 \cdot 10^{-5} eV/T
\end{aligned}
$$

Intrinsic magnetic moment:

$$\vec{\mu} = g \cdot \mu_B \cdot \vec{S}$$

Definition

g is the gyromagnetic ratio

Dirac

$$
\begin{aligned}
\vec{J} &= \vec{L} + \vec{S} \\
&= \vec{L} + \frac{1}{2}\vec{\sigma} \\
\vec{\mu} &= \frac{1}{2}\int \vec{x} \times \vec{j} \\
\vec{j} &= -e\bar{\psi}\vec{\gamma}\psi \\
\langle f|\vec{\mu}|f\rangle &\sim \frac{1}{2}\langle f|\vec{j}|f\rangle \\
&= \frac{-e}{2}\langle f|\bar{\psi}\vec{\gamma}\psi|f\rangle \\
&= \frac{-e}{2}\langle f|\vec{L} + \vec{\sigma}|f\rangle \\
&= \frac{-e}{2}\langle f|\vec{L} + g\vec{S}|f\rangle
\end{aligned}
$$

- The magnetic moment is anti-parallel with the Spin
- Dirac predicts $g = 2$!

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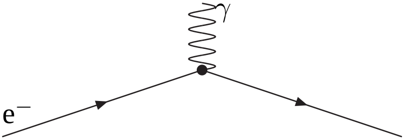
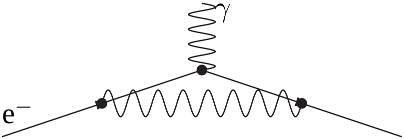
and QFT?



Interaction with an external field: LO



Interaction with an external field: NLO

Electromagnetic current

$$
\begin{aligned}
& -e\bar{u}\gamma^\mu u \\
= \; & -\frac{e}{2m}\bar{u}[(p'+p)^\mu + i(p'-p)_\nu \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)]u \\
= \; & -\frac{e}{2m}\bar{u}[(p'+p)^\mu + i(p'-p)_\nu \sigma^{\mu\nu})u
\end{aligned}
$$

Charge conservation

$$-\frac{e}{2m}\bar{u}(p'+p)^\mu u$$
$$\rightarrow \quad \bar{u}_r u_s = 2m\delta_{rs}$$
$$= \quad -e(p'+p)^\mu$$
$$\rightarrow \quad \mu = 0$$
$$= \quad -e2E$$
$$\textit{conserved}$$

Spin dependent part

$$-\frac{e}{2m}\bar{u}i\sigma^{\mu\nu}uA_\mu(p'-p)_\nu$$
$$\rightarrow \quad (p'-p)_0 = 0$$
$$\rightarrow \quad \sigma^{00} = 0$$
$$\sim \quad -\frac{e}{2m}\bar{u}i\epsilon_{ijk}\sigma_k A_i(p'-p)_j$$
$$\sim \quad \sigma \cdot \vec{\nabla} \times \vec{A}$$
$$\sim \quad \vec{\sigma} \cdot \vec{B}$$

Charge conservation

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& -\frac{e}{2m}\bar{u}(p'+p)^\mu u \\
\rightarrow \quad & \bar{u}_r u_s = 2m\delta_{rs} \\
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\rightarrow \quad & \mu = 0 \\
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& conserved
\end{aligned}$$

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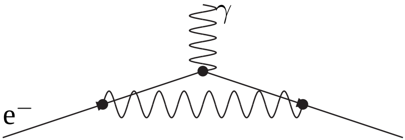
\sim \quad & \sigma \cdot \vec{\nabla} \times \vec{A} \\
\sim \quad & \vec{\sigma} \cdot \vec{B}
\end{aligned}$$



leads to:

$$\begin{aligned}
\Delta\mu &\sim \alpha/\pi \cdot \frac{e}{2m} \\
g &= 2 + \alpha/\pi \\
a &= \frac{g-2}{2} \\
&= \frac{1}{2}\frac{\alpha}{\pi} \\
&\sim 10^{-3}
\end{aligned}$$

Order	*Diagrams*
1	1
2	7
3	72
4	891
5	12672

QED prediction a_e

$$a_e = 1159652182.79 \cdot 10^{-12} \pm 7.79 \cdot 10^{-12}$$

8th order: Phys. Rev. Lett. 99, 110406 (2007)

Electron Precession in B-field

$$
\begin{aligned}
mv_p^2/r &= ev_pB \\
mv_p/r &= eB \\
m\omega r/r &= eB \\
\omega_0 &= eB/m \\
m &\rightarrow m\gamma \\
\omega_C &= \omega_0/\gamma
\end{aligned}
$$

Spin Precession in B-field

$\Delta E = g\mu_B B = \hbar\omega_L$

$\omega_L = g(eB)/(2m) = \frac{1}{2}g\omega_0$

Relativistic corrections (Thomas):

$\omega_P = \omega_L - \omega_T = \frac{g}{2}\omega_0 - \frac{\gamma-1}{\gamma}\omega_0$

Phase difference

$\Delta\omega = \omega_L - \omega_0 = a_e\omega_0$

Relativistic:

$\Delta\omega = \omega_P - \omega_C = a_e\omega_0$

a_e

$a_e = 0$: Spin in phase with electron rotation

$a_e \neq 0$: Spin precession not in phase with precession of particle in B-field

Electron Precession in B-field

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mv_p/r &= eB \\
m\omega r/r &= eB \\
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\end{aligned}$$

Spin Precession in B-field

$\Delta E = g\mu_B B = \hbar\omega_L$

$\omega_L = g(eB)/(2m) = \frac{1}{2}g\omega_0$

Relativistic corrections (Thomas):

$\omega_P = \omega_L - \omega_T = \frac{g}{2}\omega_0 - \frac{\gamma-1}{\gamma}\omega_0$

Phase difference

$\Delta\omega = \omega_L - \omega_0 = a_e\omega_0$

Relativistic:

$\Delta\omega = \omega_P - \omega_C = a_e\omega_0$

a_e

$a_e = 0$: Spin in phase with electron rotation

$a_e \neq 0$: Spin precession not in phase with precession of particle in B-field

Electron Precession in B-field

$$
\begin{aligned}
mv_p^2/r &= ev_pB \\
mv_p/r &= eB \\
m\omega r/r &= eB \\
\omega_0 &= eB/m \\
m &\rightarrow m\gamma \\
\omega_C &= \omega_0/\gamma
\end{aligned}
$$

Spin Precession in B-field

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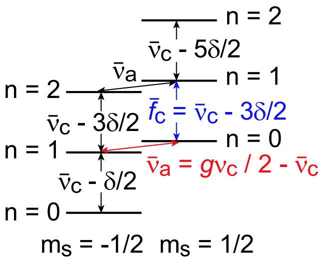
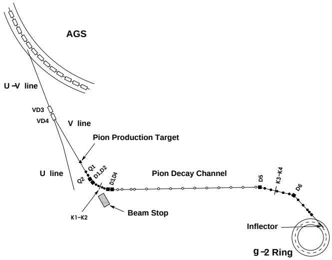


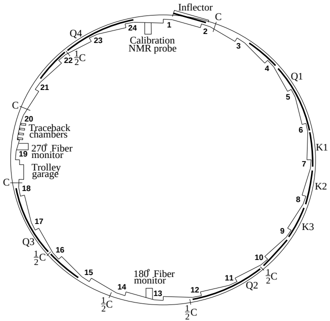
- Penning trap electrons (small scale experiment)
- δ/ν_C: relativistic shift
- f Cyclotron : 149 GHz
- f Anomaly : 173 MHZ

a_e

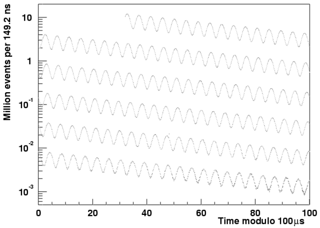
$$a_{\rm e} = 115965218073(28) \cdot 10^{-14}$$
$$\alpha^{-1} = 137.035999084(51)$$

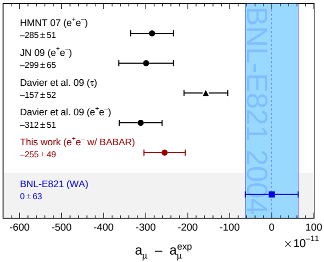
- test QED to 10^{-13}
- determine α to 0.37ppb ($\approx 10^{-9}$)
- natural scale: $m_{\rm e} \approx 0.5 MeV$





- muon liftime penning trap not feasible
- 24GeV protons to produce pions (next week) which decay to muons
- muons decay to electrons
- calorimeters detect the electrons
- excellent knowledge of B-field necessary





- electron counting rate varies as function of the precession of the spin
- natural scale of experiment $m_\mu \approx 0.105 GeV$

- Hadronic contribution (non QED) important (695)
- Prediction is mixture of calculation and measurement
- Supersymmetry?