Particle Physics: The Standard Model

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- Remember the particle zoo
- charged leptons and photon
- add u, d $SU(2)$-Isospin
- add s $SU(3)$-Flavour
- add gluon (g)
- add the other quarks

$$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$$

$$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$$

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$$\gamma$$
$$g$$
$$\mathrm{W^\pm, Z^\circ}$$
$$\mathrm{H}$$

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Quarks $\mathrm{u, d, c, s, t, b}$
sometimes also called partons

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Properties of the u

$$m_0 = 2.5 \pm 0.7\mathrm{MeV}(2GeV)$$

Properties of the d

$$m_0 = 5.0 \pm 0.8\mathrm{MeV}(2GeV)$$

Properties of the c

$$m_0 = 1.27\mathrm{GeV}$$
$$\tau = (1.040 \cdot 10^{-12})\mathrm{s} \quad \mathrm{c\bar{d}}$$
$$c\tau = 311.8\mu\mathrm{m}$$

Properties of the s

$$m_0 = 100 \pm 25\mathrm{MeV}$$
$$\tau = (1.24 \cdot 10^{-8})\mathrm{s} \quad \mathrm{u\bar{s}}$$
$$c\tau = 3.7\mathrm{m} \quad 1st$$

Properties of the t

$$m_0 = 172.9 \pm 1.0\mathrm{GeV}$$
$$\tau \sim 10^{-23}\ \mathrm{s}$$
$$c\tau \sim 10^{-15}\ \mathrm{m}$$

Properties of the b

$$m_0 = 4.19 \pm 0.12\mathrm{GeV} \quad (\overline{MS})$$
$$\tau = (1.6 \cdot 10^{-12})\mathrm{s} \quad \mathrm{u\bar{b}}$$
$$c\tau = 492\mu\mathrm{m}$$

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History

- 1947: Discovery of the charged pion in cosmic rays
- 1947: V particles (kink plus nothing then Vertex with 2 tracks)
- 1950: neutral pion
- 1960s: lots of new hadronic particles
- attempt to order the zoo
- introduce additional quantum numbers, substructure
- makes only sense if predictions arise from these attempts to order (if number of parameters is equal to the number of particles to be described it is a waste of time)

$SU(2)$

- $SU(2)$: 2×2 matrix
- $UU^\dagger = 1_2$, $det(U) = 1$
- $U = 1 + i \sum_{a=1}^{3} \delta\phi_a \frac{\tau_a}{2}$ with $\tau_a = \sigma_a$

Pions: 140MeV, Spin-0

- $I = 1 \rightarrow \pm 1, 0$
- $I_3|\pi^+\rangle = |\pi^+\rangle$
- $I_3|\pi^-\rangle = -|\pi^-\rangle$
- Kemmer predicted a neutral particle:
- $I_3|\pi^0\rangle = 0|\pi^0\rangle$

Nucleons: 1GeV, Spin-$\frac{1}{2}$

- electron spin: $\pm\frac{1}{2}$
- new QN: Isospin I (behaves spin-like)
- $m_p \approx m_n$
- $I = \frac{1}{2}$
- $I_3|p\rangle = \frac{1}{2}|p\rangle$
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Order with Spin and Isospin: 5 particles described with quantum number

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Order with Spin and Isospin: 5 particles described with quantum number

Baryons

- System of three quarks
- $|p\rangle = |uud\rangle$
- $|n\rangle = |udd\rangle$

Mesons

- System of quark anti-quark
- $|\pi^+\rangle = -|\mathrm{u\bar{d}}\rangle$
- $|\pi^0\rangle = \frac{1}{\sqrt{2}}|\mathrm{u\bar{u}} - \mathrm{d\bar{d}}\rangle$
- $|\pi^-\rangle = |\mathrm{d\bar{u}}\rangle$

Hypercharge

$$Q = I_3 + \frac{1}{2}Y$$

therefore:

$$\begin{aligned} Y &= 2(Q - I_3) \\ Y(\mathrm{u}) &= 2(\tfrac{2}{3} - \tfrac{1}{2}) \\ &= \tfrac{1}{3} \\ Y(\mathrm{d}) &= \tfrac{1}{3} \\ Y(\mathrm{\bar{u}}) &= -Y(u) \\ Y(\mathrm{\bar{d}}) &= -Y(d) \end{aligned}$$

for the anti-quarks both charge AND isospin change signs

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Proof.

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

Charge conjugation:

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Respect Charge ordering (index $1 \leftrightarrow 2$):

$$\begin{pmatrix} \bar{\mathrm{d}}' \\ \bar{\mathrm{u}}' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \bar{\mathrm{d}} \\ \bar{\mathrm{u}} \end{pmatrix}$$

Rewrite to obtain the same rotation matrix as for particles:

$$\begin{pmatrix} -\bar{\mathrm{d}}' \\ \bar{\mathrm{u}}' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -\bar{\mathrm{d}} \\ \bar{\mathrm{u}} \end{pmatrix}$$

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Something strange was observed

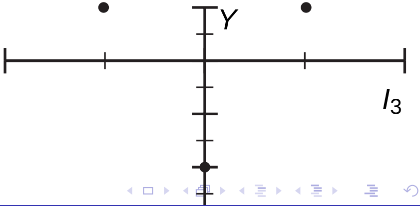
- 1953: production of V^0s in accelerators
- $\pi^- p \to \mathrm{K}^\circ \Lambda \to \pi^+ \pi^- p \pi^-$
- $\sigma \sim 1mb \approx 10^{-31} m^2 \approx (10^{-15} m)^2 = (fm)^2$
- cross section of the order of the geometrical hadron radius
- $\tau \sim 10^{-10} s$
- or: strong interaction $\tau = \frac{1fm}{3{\cdot}10^8 m/s} \approx 10^{-23} s$
- new QN: strangeness (conserved by strong interaction)
  $S(\mathrm{K}^\circ) = +1$, $S(\Lambda) = -1$
- modern formulation: introduce a new quark: s
- introduce a QN: S (strangeness)

- The hypercharge is redefined: $Y = S + B$
- Gell-Mann-Nishijima: $Q = I_3 + \frac{1}{2}Y$

B: Baryonnumber

quarks: $\frac{1}{3}$
anti-quarks: $-\frac{1}{3}$
Mesons (quark-anti-quark systems): 0
Baryons (3quark system): 1

	I	I_3	Y	S	B	Q
u	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$
d	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$-\frac{1}{3}$
s	0	0	$-\frac{2}{3}$	-1	$\frac{1}{3}$	$-\frac{1}{3}$



- Isospin $SU(2)$, hypercharge (a number) $U(1)$ gives $SU(2) \times U(1)$
- Gell-Mann-Ne'eman: $SU(3)$ can be decomposed into $SU(2) \times U(1)$

$SU(3)$

- $|\mathrm{u}\rangle$, $|\mathrm{d}\rangle$, $|\mathrm{s}\rangle$
- $UU^\dagger = 1_3$, $det(U) = 1$
- $U = 1 + i \sum_{a=1}^{8} \delta\phi_a \frac{\lambda_a}{2}$
- $3 \times 3 \times 2 - 9 - 1 = 8$

Gell-Mann Matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

λ_1, λ_2, λ_3 are essentially the Pauli matrices of $SU(2)$, $\frac{1}{2}\lambda_3$ is I_3, $\frac{1}{\sqrt{3}}\lambda_8$ is the hypercharge.

Multiplets: Mesons

ud-Mesons: $2 \times 2 = 4 = 1 + 3$

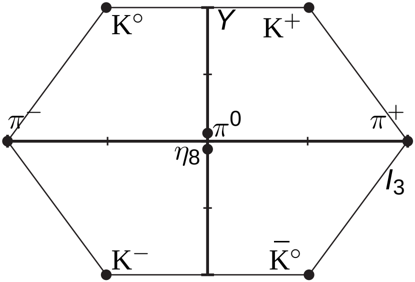
with $I = 1, 0$

uds: Eight-fold way

$3 \times 3 = 1 + 8$

about same mass (!)

$\eta_1 \sim \frac{1}{\sqrt{3}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} + \mathrm{s\bar{s}})$



Navigation with Gell-Mann Matrices

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\begin{array}{lllllll}
I_\pm & = & \frac{1}{2}(\lambda_1 \pm i\lambda_2) & \Delta I_3 & = & \pm 1 & & \mathrm{d} \leftrightarrow \mathrm{u} \\
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U_\pm & = & \frac{1}{2}(\lambda_6 \pm i\lambda_7) & \Delta I_3 & = & \mp\frac{1}{2} & \Delta Y = \pm 1 & \mathrm{s} \leftrightarrow \mathrm{d}
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$$

Multiplets: Mesons

ud-Mesons: $2 \times 2 = 4 = 1 + 3$

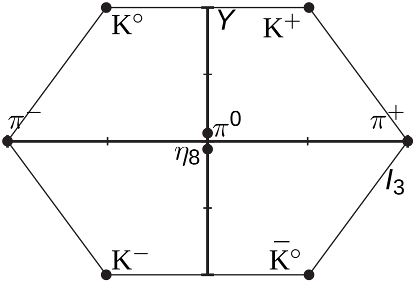
with $I = 1, 0$

uds: Eight-fold way

$3 \times 3 = 1 + 8$

about same mass (!)

$\eta_1 \sim \frac{1}{\sqrt{3}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} + \mathrm{s\bar{s}})$



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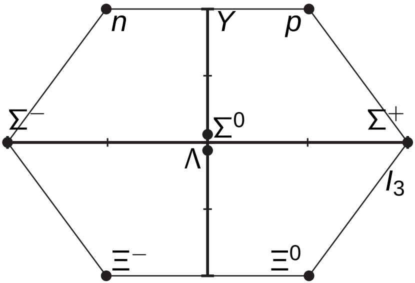
Multiplets: Baryons

uds:

$3 \times 3 \times 3 = 1 + 8 + 8 + 10$



$$\begin{aligned} I_-|u\rangle &= |d\rangle \\ I_-|d\rangle &= 0 \\ I_-|p\rangle &= I_-|u\rangle|u\rangle|d\rangle \\ &= I_-(|u\rangle)|u\rangle|d\rangle + |u\rangle I_-(|u\rangle)|d\rangle + |u\rangle|u\rangle I_-|d\rangle \\ &\sim |d\rangle|u\rangle|d\rangle + |u\rangle|d\rangle|d\rangle \end{aligned}$$

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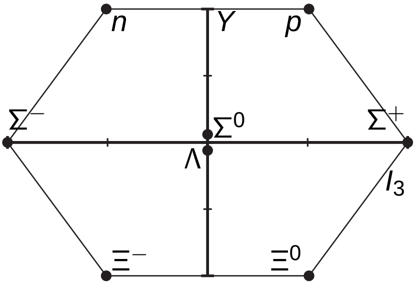

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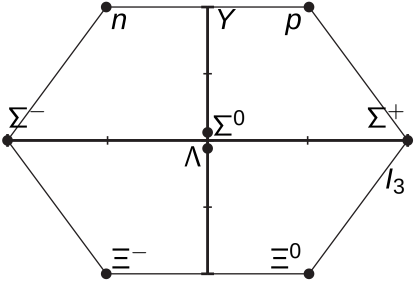

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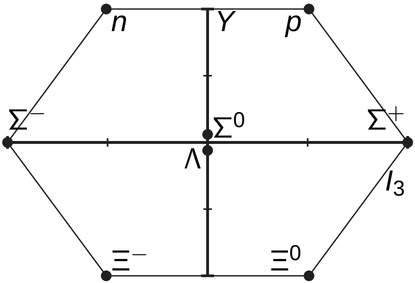

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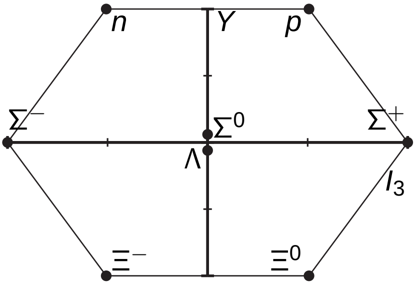

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- $\Delta^{++} = |\text{uuu}\rangle$
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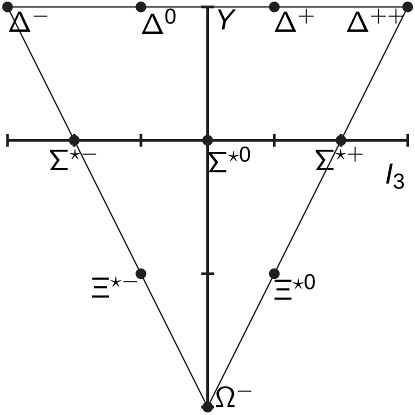

Gell-Mann Okubo

- masses similar within multiplet
- $m_{\Omega^-} = 1672 \neq 1230 = m_{\Delta^{++}}$
- take mass as perturbation

$SU(3)$ – *Flavor* beyond order: relate cross sections via clebsch gordon coefficients

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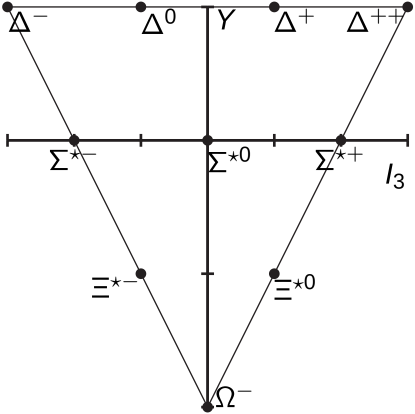

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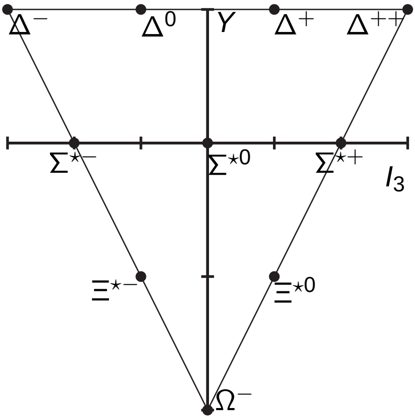

$SU(3) - Flavor$ beyond order: relate cross sections via clebsch gordon coefficients

Adding the spin

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \\ s\uparrow \\ s\downarrow \end{pmatrix}$$

Weight diagram and $SU(6)$ in Problem Solving session

- SI: preserves inner QNs (e.g. S)
- EM: violates I, but preserves I_3: $\pi^0 \rightarrow \gamma\gamma$, $1 \rightarrow 0$, $0 \rightarrow 0$

charm, bottom, top

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- What glue holds the hadrons together?
- Are quarks math or particles?
- photon carrier of EM-interactions
- gluon

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$$\gamma$$
$$g$$

QCD 1972,1973 Gell-Mann, Fritzsch, Leutwyler :
SU(3) – *Color*

$$\mathcal{L}_0 = \bar{\psi}(\mathbf{x})(i\gamma^\mu\partial_\mu - m)\psi(\mathbf{x})$$

extend to 6 quarks (u, d, c, s, t, b):

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The free Lagrangian ($\mathcal{L}_0$)
Remember QED:

GaugeGroup	$U(1)$
Gaugebosons	1
Lorentz – Vector	$A_\mu(\mathbf{x})$
Field – Tensor	$F_{\mu\nu} = \partial_\mu A_\nu(\mathbf{x}) - \partial_\nu A_\mu(\mathbf{x})$

QCD:

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Structure constants of $SU(3)$ (totally anti-symmetric):

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The additional term is characteristic of a non-Abelian theory

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Define a hermitian matrix with the gluon Lorentz-Vectors (λ_a are hermitian, $G^a_\mu(\mathbf{x})$ is real):

$$\mathbf{G}_\mu(\mathbf{x}) = G^a_\mu(\mathbf{x})\frac{\lambda_a}{2} = (G^a_\mu(\mathbf{x}))^\dagger \frac{\lambda_a^\dagger}{2} = \mathbf{G}^\dagger_\mu(\mathbf{x})$$

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Minimal substitution

$$\partial_\lambda \rightarrow \mathbf{D}_\lambda = \partial_\lambda + ig_S \mathbf{G}_\lambda(\mathbf{x}) + iqeA_\lambda(\mathbf{x})$$

where q is the charge of the quark and e is the elementary charge (>0). $q = -1$ for the electron.

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2} Tr(\mathbf{G}_{\mu\nu}(\mathbf{x})\mathbf{G}^{\mu\nu}(\mathbf{x})) + \sum_{j=1}^{6} \bar{\mathbf{q}}^j (i\gamma^\lambda D_\lambda - m_j))\mathbf{q}^j \\ &= -\frac{1}{4} G^a_{\mu\nu}(\mathbf{x}) G^{\mu\nu a}(\mathbf{x}) \\ &+ \sum_{j=1}^{6} \bar{\mathbf{q}}^j (i\gamma^\lambda(\partial_\lambda + ig_S G^a_\lambda \frac{\lambda_a}{2} + iqeA_\lambda(\mathbf{x}) - m_j))\mathbf{q}^j\end{aligned}$$

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The Ω^- puzzle

- $\Omega^- = |sss\rangle$
- $J(\Omega^-) = \frac{3}{2}$ (very difficult)
- $\Omega^- = |s\uparrow s\uparrow s\uparrow\rangle$
- violates Pauli: fermions are anti-symmetric
- deduce hidden quantum number: QCD

Color

- $|u\rangle \to |u\rangle, |u\rangle, |u\rangle$
- $\langle u|u\rangle = \langle u|u\rangle = 0$
- anti-quarks carry anti-color
- particles are white: Mesons $white = C + \bar{C}$, baryons $white = C_1 + C_2 + C_3$

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- $\Omega^- = \epsilon_{ijk} s_i s_j s_k$

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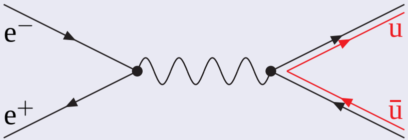
  white $= C_1 + C_2 + C_3$

Final state

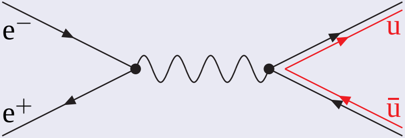


- $e^+e^- \rightarrow \gamma \rightarrow u\bar{u}$
- if color is not measured: sum of color
- $\sigma \sim N_C = 3$
- $\sigma \sim N_C \cdot q^2$

Initial state

- $u\bar{u} \rightarrow \gamma \rightarrow e^+e^-$
- if color is not measured: average
- $\langle u|u\rangle = 1$ $\langle u|u\rangle = 1$
  $\langle u|u\rangle = 1$ $\langle u|u\rangle = 0$
  $\langle u|u\rangle = 0$ $\langle u|u\rangle = 0$
  $\langle u|u\rangle = 0$ $\langle u|u\rangle = 0$
  $\langle u|u\rangle = 0$
- $\sigma \sim \frac{3}{9} = \frac{1}{3}$

Final state

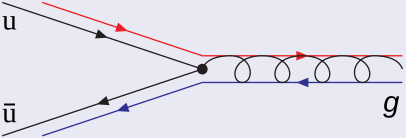


- $e^+e^- \to \gamma \to u\bar{u}$
- if color is not measured: sum of color
- $\sigma \sim N_C = 3$
- $\sigma \sim N_C \cdot q^2$

Initial state

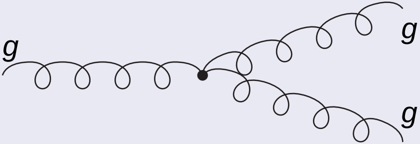
- $u\bar{u} \to \gamma \to e^+e^-$
- if color is not measured: average
- $\langle {\color{green}u}|{\color{green}u}\rangle = 1$ $\langle {\color{red}u}|{\color{red}u}\rangle = 1$
  $\langle {\color{blue}u}|{\color{blue}u}\rangle = 1$ $\langle {\color{red}u}|{\color{green}u}\rangle = 0$
  $\langle {\color{blue}u}|{\color{green}u}\rangle = 0$ $\langle {\color{green}u}|{\color{red}u}\rangle = 0$
  $\langle {\color{blue}u}|{\color{red}u}\rangle = 0$ $\langle {\color{red}u}|{\color{blue}u}\rangle = 0$
  $\langle {\color{green}u}|{\color{blue}u}\rangle = 0$
- $\sigma \sim \frac{3}{9} = \frac{1}{3}$

qqg

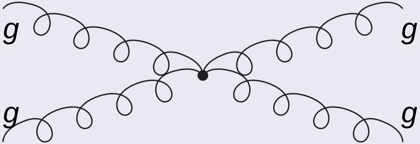


- gluon carries color and anti-color charge
- $g q\bar{q}$ vertex: $\sim g_S$ ($\alpha_S = \frac{g_S^2}{4\pi}$) electric charge irrelevant!

TGV and QGV



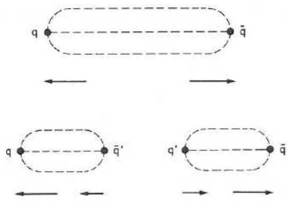
Non-abelian theory: triple gluon vertex $\sim g_S$



four gluon vertex $\sim g_S^2$

Fragmentation

- connection between hadrons and quarks?
- no colored particles observed
- Lund string fragmentation ($V \sim kr$)



- $\sqrt{s} = 1\ \mathrm{GeV}$
- $|\mathrm{K}^+\rangle = |\mathrm{u}\bar{\mathrm{s}}\rangle$
- $m_{\mathrm{K}^+} = 0.494\mathrm{GeV}$
- $\mathrm{e}^+\mathrm{e}^- \to \mathrm{s}\bar{\mathrm{s}} \to \mathrm{K}^+\mathrm{K}^-$
- more difficult at $\sqrt{s} \gg 2m$