

# Particle Physics: The Standard Model

Dirk Zerwas

LAL  
zerwas@lal.in2p3.fr

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- charged leptons and photon
- quarks and gluon
- neutrinos
- $W^\pm, Z^0$
- H

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

$$\begin{array}{ccc} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{array}$$

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## History

- 1896 Henri Becquerel:  $\beta$  decay
- 1899 Ernest Rutherford: distinguishes  $\alpha$  and  $\beta$  rays
- 1914 James Chadwick: the  $\beta$  decay has a continuous spectrum
- 1930 Wolfgang Pauli: postulates the neutrino (ballroom)
- 1933 Enrico Fermi: contact interaction
- 1953 Frederick Reines:  $\bar{\nu}_{eL} + p \rightarrow n + e^+$
- 1956 Lee, Yang, Wu, Garwin et al: Parity violation
- 1961 Glashow, Salam, Weinberg, Higgs, EBKGH
- 1973 Lagarrigue, Faissner: neutral currents ( $Z^0$  t-channel)
- 1984 Rubbia, van der Meer :  $W^\pm, Z^0$
- 2012 discovery of the Higgs boson

$$\begin{aligned}
 n &\rightarrow p + e^- + \bar{\nu}_{eL} \\
 T_{fi} &\sim G(\bar{p}\gamma^\mu n)(\bar{e}\gamma_\mu\nu) \\
 &\sim G(\bar{p}\gamma^\mu n)\frac{1}{q^2 - m^2}(\bar{e}\gamma_\mu\nu)
 \end{aligned}$$

- QED:  $m^2 = 0 \rightarrow \frac{1}{q^2}$
- if  $m^2 \gg q^2$  neglect  $q^2$
- constant in momentum space  $\rightarrow$  Dirac function in space-time: contact interaction
- $G \sim 10^{-5} \text{GeV}^{-2} \ll \alpha_{EM}$

## Currents

$\bar{\psi}\psi$	scalar	S
$\bar{\psi}\gamma^\mu\psi$	vector	V
$\bar{\psi}\sigma^{\mu\nu}\psi$	tensor	T
$\bar{\psi}\gamma^\mu\gamma_5\psi$	axial vector	A
$\bar{\psi}\gamma_5\psi$	pseudo scalar	PS

QED: V

EW: V - A

V - A violates parity  
(experiment)

V - A quark/lepton level, not  
hadron level

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## Chirality

Chirality is the handed-ness of the particle:

$$\begin{aligned}\psi &= P_L\psi + P_R\psi \\ &= \psi_L + \psi_R\end{aligned}$$

## Definitions

- Helicity:  $\vec{\sigma} \cdot \vec{p}$
- $m = 0$ : Helicity = chirality
- $m = 0$ :  $\psi$  and  $\gamma_5\psi$  solve DIRAC

## Weyl basis

$$\begin{aligned}\gamma_5 &= i\gamma^0\gamma^1\gamma^2\gamma^3 \\ \gamma_5^2 &= 1 \\ 0 &= \gamma_5\gamma^\mu + \gamma^\mu\gamma_5 \\ \gamma_5 &= \begin{pmatrix} -1_2 & 0 \\ 0 & 1_2 \end{pmatrix} \\ \gamma^0 &= \begin{pmatrix} 0 & 1_2 \\ 1_2 & 0 \end{pmatrix} \\ \gamma^{0\dagger} &= \gamma^0 \\ \gamma_5^\dagger &= \gamma_5\end{aligned}$$

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## Left and Right chirality

$$\begin{aligned}
 P_L &= \frac{1}{2}(1 - \gamma_5) \\
 P_R &= \frac{1}{2}(1 + \gamma_5) \\
 P_L + P_R &= 1 \\
 P_L^2 &= \frac{1}{2}(1 - \gamma_5)\frac{1}{2}(1 - \gamma_5) \\
 &= \frac{1}{4}(1 - \gamma_5)(1 - \gamma_5) \\
 &= \frac{1}{4}(1 - \gamma_5 - \gamma_5 + \gamma_5^2) \\
 &= \frac{1}{4}(1 - \gamma_5 - \gamma_5 + 1) \\
 &= \frac{1}{2}(1 - \gamma_5) \\
 P_L P_R &= \frac{1}{2}(1 - \gamma_5)\frac{1}{2}(1 + \gamma_5) \\
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$$\begin{aligned}
 P_L \psi &= P_L \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \\
 &= \psi_L \\
 \bar{\psi} P_L &= (\psi_L^\dagger \gamma^0 P_L \psi)^\dagger \\
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- $m = 0$  helicity conserved  $\rightarrow \sigma \cdot \vec{p}$  good QN
- particle ( $\mathbf{p}$ )  $\rightarrow$  anti-particle  $-\mathbf{p}$ :  $\sigma \rightarrow -\sigma$
- $\psi_R$  right-(left)handed (anti-)particle
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EM current

$$\begin{aligned}
 j^\mu &= -e\bar{\psi}\gamma^\mu\psi \\
 &= -e\bar{\psi}(P_L + P_R)\gamma^\mu(P_L + P_R)\psi \\
 &= -e\bar{\psi}P_L\gamma^\mu P_L\psi - e\bar{\psi}P_R\gamma^\mu P_R\psi - e\bar{\psi}P_R\gamma^\mu P_L\psi - e\bar{\psi}P_L\gamma^\mu P_R\psi \\
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Perfect symmetry under parity:  $\vec{p} \rightarrow -\vec{p}$

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- weak interaction: Left is not equal to Right
- use vector bosons
- ask for local gauge invariance
- remember that  $U(1)_{EM}$  is QED and was extremely successful
- unify electromagnetic and weak interactions
- $SU(2) \times U(1)$
- $SU(2)$ : three generators (gauge bosons)
- $U(1)$ : one generators (gauge boson)
- $SU(2)$  vector bosons must be massive (Fermi-contact interaction)
- massive vector bosons lead to a non-renormalizable theory



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The free Lagrangian ( $\mathcal{L}_0$ ) Remember QCD:GaugeGroup  $SU(3)$ 

Gaugebosons 8

Lorentz – Vectors  $G_\mu^a(\mathbf{x})$ Field – Tensor  $G_{\mu\nu}^a = \partial_\mu G_\nu^a(\mathbf{x}) - \partial_\nu G_\mu^a(\mathbf{x}) - g_S f_{abc} G_\mu^b(\mathbf{x}) G_\nu^c(\mathbf{x})$ Structure  $[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}] = if_{abc} \frac{\lambda_c}{2}$  $SU(2)_L$ :GaugeGroup  $SU(2)$ 

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Free Lagrangian Gauge Fields  $SU(2)_L \times U(1)_Y$ 

$$\begin{aligned}\mathcal{L}_0 &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} \\ &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr}(W_{\mu\nu} W^{\mu\nu}) \\ W_{\mu\nu} &= W_{\mu\nu}^a \frac{T_a}{2}\end{aligned}$$

## Organize the Dirac Fields

$$\begin{aligned}e_L(\mathbf{x}) &= P_L e(\mathbf{x}) \\ \bar{e}_L(\mathbf{x}) &= (\gamma^0 P_L e(\mathbf{x}))^\dagger \\ \ell(\mathbf{x}) &= \begin{pmatrix} \nu_{eL}(\mathbf{x}) \\ e_L(\mathbf{x}) \\ e_R(\mathbf{x}) \end{pmatrix}\end{aligned}$$

No right-handed neutrinos

## Define the weak Hypercharge

hypercharge left  $\neq$  right:

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## Interaction Lagrangian

$$\mathcal{L}' = -\bar{\ell}\gamma^\mu (g_2 W_\mu^a \frac{T_a}{2} + g_1 B_\mu Y) \ell$$



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## Investigate the Interaction

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 &= -g_2(\bar{\nu}_{eL}, \bar{e}_L)\gamma^{\mu}(W_{\mu}^1 \frac{T_1}{2} + W_{\mu}^2 \frac{T_2}{2} + W_{\mu}^3 \frac{T_3}{2}) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \\
 &\quad + \frac{1}{2}g_1 \bar{\nu}_{eL}\gamma^{\mu}B_{\mu}\nu_{eL} + \frac{1}{2}g_1 \bar{e}_L\gamma^{\mu}B_{\mu}e_L - y_R g_1 \bar{e}_R\gamma^{\mu}B_{\mu}e_R \\
 &= -\frac{g_2}{\sqrt{2}}(W_{\mu}^+ \bar{\nu}_{eL}\gamma^{\mu}e_L + W_{\mu}^- \bar{e}_L\gamma^{\mu}\nu_{eL}) \\
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Introduce a complex scalar doublet:

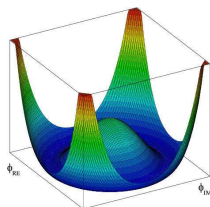
$$\phi(\mathbf{x}) = \begin{pmatrix} \phi_1(\mathbf{x}) \\ \phi_2(\mathbf{x}) \end{pmatrix}$$

### Free Lagrangian

$$\begin{aligned} \mathcal{L}_0 &= (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi) \\ V(\phi) &= \kappa \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \end{aligned}$$

Theory must be stable:  $\lambda > 0$

Minimum not at 0:  $\kappa = -\mu^2 < 0$



The ground state is not unique:

$$\phi = \exp\left(i\frac{\tau_a}{2}\varphi_a\right) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix}$$

Choose  $\varphi = 0 \rightarrow SU(2)$   
symmetry is broken

## Yukawa terms

$$\mathcal{L}_Y = -y_e \bar{e}_R \phi^\dagger \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

*h.c.*

$$\begin{aligned} & -y_e (\bar{\nu}_{eL}, e_L) \phi e_R \\ & = -y_e (\bar{e}_R \phi_1^\dagger \nu_{eL} + \bar{e}_R \phi_2^\dagger e_L) \\ & \quad -y_e (\bar{\nu}_{eL} \phi_1 e_R + \bar{e}_L \phi_2 e_R) \end{aligned}$$

Deduce the hypercharge:

$$\begin{aligned} e_L & \rightarrow e_R + \phi_2 \\ -\frac{1}{2} & = y_H + -1 \\ y_H & = \frac{1}{2} \end{aligned}$$

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$$\begin{aligned} \partial_\mu \phi & \rightarrow \partial_\mu \phi + ig_2 W_\mu^a \frac{\tau_a}{2} \phi \\ & \quad + ig_1 B_\mu y_H \phi \\ \partial_\mu \phi^\dagger & \rightarrow \partial_\mu \phi^\dagger - \phi^\dagger ig_2 W_\mu^a \frac{\tau_a}{2} \\ & \quad - \phi^\dagger ig_1 B_\mu y_H \end{aligned}$$

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$$\phi^T = (0, \sqrt{\frac{\mu^2}{2\lambda}}) = (0, \frac{v}{\sqrt{2}})$$

## Yukawa terms

$$\mathcal{L}_Y = -y_e \bar{e}_R \phi^\dagger \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

*h.c.*

$$\begin{aligned} & -y_e (\bar{\nu}_{eL}, e_L) \phi e_R \\ & = -y_e (\bar{e}_R \phi_1^\dagger \nu_{eL} + \bar{e}_R \phi_2^\dagger e_L) \\ & \quad -y_e (\bar{\nu}_{eL} \phi_1 e_R + \bar{e}_L \phi_2 e_R) \end{aligned}$$

Deduce the hypercharge:

$$\begin{aligned} e_L & \rightarrow e_R + \phi_2 \\ -\frac{1}{2} & = y_H + -1 \\ y_H & = \frac{1}{2} \end{aligned}$$

## Minimal Substitution

$$\begin{aligned} \partial_\mu \phi & \rightarrow \partial_\mu \phi + ig_2 W_\mu^a \frac{\tau_a}{2} \phi \\ & \quad + ig_1 B_\mu y_H \phi \\ \partial_\mu \phi^\dagger & \rightarrow \partial_\mu \phi^\dagger - \phi^\dagger ig_2 W_\mu^a \frac{\tau_a}{2} \\ & \quad - \phi^\dagger ig_1 B_\mu y_H \end{aligned}$$

Calculate the interaction terms

$$\begin{aligned} & \phi^\dagger (-ig_2 W_\mu^a \frac{\tau_a}{2} - ig_1 B_\mu y_H) \\ & (+ig_2 W_\mu^a \frac{\tau_a}{2} + ig_1 B_\mu y_H) \phi \end{aligned}$$

$$\phi^T = (0, \sqrt{\frac{\mu^2}{2\lambda}}) = (0, \frac{v}{\sqrt{2}})$$

$$\begin{aligned}
& (0, \sqrt{\frac{\mu^2}{2\lambda}}) (-ig_2 W_\mu^a \frac{\tau_a}{2} - ig_1 B_\mu y_H) (+ig_2 W_\mu^a \frac{\tau_a}{2} + ig_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix} \\
&= (0, \sqrt{\frac{\mu^2}{2\lambda}}) (g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H) (g_2 W_\mu^a \frac{\tau_a}{2} + g_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix} \\
&= (0, \sqrt{\frac{\mu^2}{2\lambda}}) \begin{pmatrix} \frac{2g_2 g_1 A_\mu + (g_2^2 - g_1^2) Z_\mu}{2\sqrt{g_1^2 + g_2^2}} & \frac{g_2}{\sqrt{2}} W_\mu^+ \\ \frac{g_2}{\sqrt{2}} W_\mu^- & -\frac{\sqrt{g_1^2 + g_2^2}}{2} Z_\mu \end{pmatrix} \\
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&= \frac{g_2^2 v^2}{4} W_\mu^- W^{\mu+} + \frac{(g_1^2 + g_2^2) v^2}{8} Z_\mu Z^\mu
\end{aligned}$$

The weak bosons have acquired a mass!

$$\begin{aligned}
& (0, \sqrt{\frac{\mu^2}{2\lambda}}) (-ig_2 W_\mu^a \frac{\tau_a}{2} - ig_1 B_\mu y_H) (+ig_2 W_\mu^a \frac{\tau_a}{2} + ig_1 B_\mu y_H) \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} \end{pmatrix} \\
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## Charged lepton masses

$$\begin{aligned}
 \mathcal{L}_Y &= -y_e(\bar{e}_R\phi_1^\dagger\nu_{eL} + \bar{e}_R\phi_2^\dagger e_L) - y_e(\bar{\nu}_{eL}\phi_1 e_R + \bar{e}_L\phi_2 e_R) \\
 &= -y_e(\bar{e}_R\frac{v}{\sqrt{2}}e_L) - y_e(\bar{e}_L\frac{v}{\sqrt{2}}e_R) \\
 &= -y_e\frac{v}{\sqrt{2}}(\bar{e}_R e_L + \bar{e}_L e_R) \\
 &= -y_e\frac{v}{\sqrt{2}}(\bar{e}e)
 \end{aligned}$$

## Masses

$$\begin{aligned}
 m_e &= y_e\frac{v}{\sqrt{2}} \\
 m_{W^\pm}^2 &= \frac{g_2^2 v^2}{4} = \frac{e^2 v^2}{4 \sin^2 \theta_W} \\
 m_{Z^0}^2 &= \frac{(g_1^2 + g_2^2)v^2}{4} = \frac{e^2 v^2}{4 \sin^2 \theta_W \cos^2 \theta_W} \quad \mathcal{L} \rightarrow EQM \\
 \frac{m_{W^\pm}^2}{m_{Z^0}^2} &= \cos^2 \theta_W
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 \end{aligned}$$

• Quantum Numbers weak Isospin $SU(2)_L$ of fermions	$I^W$	$I_3^W$	$Y$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$
	$\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{6}$			
• weak hypercharge	$\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$
• $Q = I_3^W + Y$	0	0	$\frac{2}{3}$	$u_R$	$c_R$	$t_R$
	0	0	$-\frac{1}{3}$	$d_R$	$s_R$	$b_R$
	0	0	$-1$	$e_R$	$\mu_R$	$\tau_R$

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	$0$	$0$	$-\frac{1}{3}$	$d_R$	$s_R$	$b_R$
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• $Q = I_3^W + Y$	0	0	$\frac{2}{3}$	$u_R$	$c_R$	$t_R$
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	0	0	$-1$	$e_R$	$\mu_R$	$\tau_R$

$$\bar{e}_L = (\gamma^0 P_L e)^\dagger = e^\dagger P_L^\dagger \gamma^0 = e^\dagger P_L \gamma^0 = e^\dagger \gamma^0 P_R = \bar{e} P_R$$

## The interactions

$$\begin{aligned} \mathcal{L}' = & -\frac{e}{\sqrt{2} \sin \theta_W} (W_\mu^+ \bar{\nu}_{eL} \gamma^\mu e_L + W_\mu^- \bar{e}_L \gamma^\mu \nu_{eL}) \\ & -\frac{e}{\sin \theta_W \cos \theta_W} Z_\mu \left[ \frac{1}{2} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \frac{1}{2} \bar{e}_L \gamma^\mu e_L \right. \\ & \left. - \sin^2 \theta_W (-\bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R) \right] \\ & - e A_\mu (-\bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R) \end{aligned}$$

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## Electromagnetic Current

$$\begin{aligned} \mathcal{L}' = & -e A_\mu (-\bar{e} P_R \gamma^\mu P_L e - \bar{e} P_L \gamma^\mu P_R e) \\ = & -e A_\mu \bar{e} \gamma^\mu Q e = -e A_\mu \bar{e} \gamma^\mu (I_3^W + Y) e \\ = & -e A_\mu j_{EM}^\mu \end{aligned}$$

$$\bar{e}_L = (\gamma^0 P_L e)^\dagger = e^\dagger P_L^\dagger \gamma^0 = e^\dagger P_L \gamma^0 = e^\dagger \gamma^0 P_R = \bar{e} P_R$$

## The interactions

$$\begin{aligned} \mathcal{L}' = & -\frac{e}{\sqrt{2} \sin \theta_W} (W_\mu^+ \bar{\nu}_{eL} \gamma^\mu e_L + W_\mu^- \bar{e}_L \gamma^\mu \nu_{eL}) \\ & -\frac{e}{\sin \theta_W \cos \theta_W} Z_\mu \left[ \frac{1}{2} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \frac{1}{2} \bar{e}_L \gamma^\mu e_L \right. \\ & \left. - \sin^2 \theta_W (-\bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R) \right] \\ & - e A_\mu (-\bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R) \end{aligned}$$

## Charged Current

$$\begin{aligned} \mathcal{L}' = & -\frac{e}{\sqrt{2} \sin \theta_W} (W_\mu^+ \bar{\nu}_{eL} \gamma^\mu P_L e + W_\mu^- \bar{e} P_R \gamma^\mu \nu_{eL}) \\ = & -\frac{e}{\sqrt{2} \sin \theta_W} (W_\mu^+ \bar{\nu}_{eL} \gamma^\mu P_L e + W_\mu^- \bar{e} \gamma^\mu P_L \nu_{eL}) \\ = & -\frac{e}{\sqrt{2} \sin \theta_W} (W_\mu^+ j_{CC}^\mu + W_\mu^- j_{CC}^{\mu \dagger}) \end{aligned}$$



$$\bar{e}_L = (\gamma^0 P_L e)^\dagger = e^\dagger P_L^\dagger \gamma^0 = e^\dagger P_L \gamma^0 = e^\dagger \gamma^0 P_R = \bar{e} P_R$$

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## Neutral Current

$$\begin{aligned} \mathcal{L}' = & -\frac{e}{\sin \theta_W \cos \theta_W} Z_\mu [\bar{\nu}_{eL} \gamma^\mu I_3^W \nu_{eL} + \bar{e}_L \gamma^\mu I_3^W e_L \\ & - \sin^2 \theta_W j_{EM}^\mu] \\ = & -\frac{e}{\sin \theta_W \cos \theta_W} Z_\mu j_{NC}^\mu \end{aligned}$$