Superconductivity approach to the electroweak symmetry breaking

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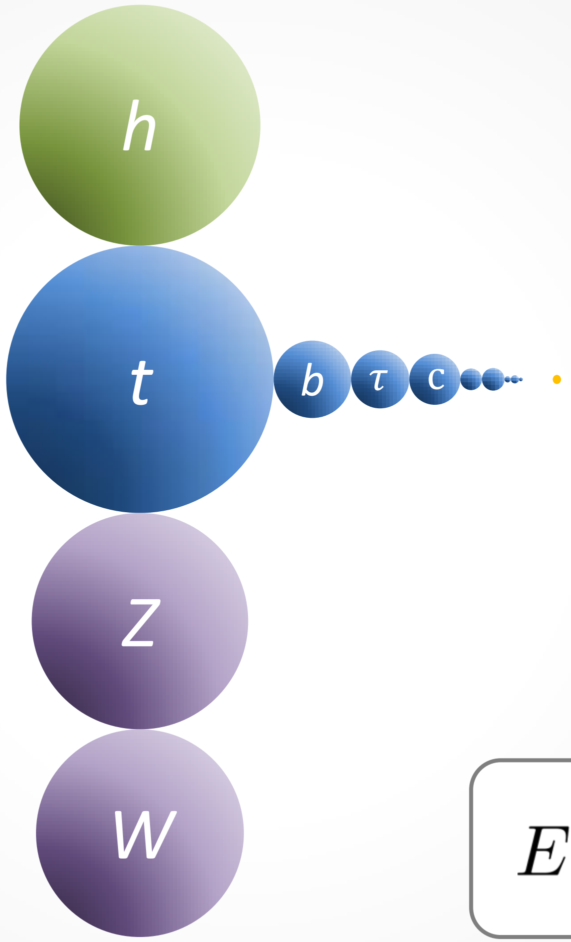
$M_h \doteq 125\,\mathrm{GeV}$

$m_t \doteq 172\,\mathrm{GeV}$

$v \doteq 246\,\mathrm{GeV}$



$$E = \sqrt{p^2 + m^2}$$

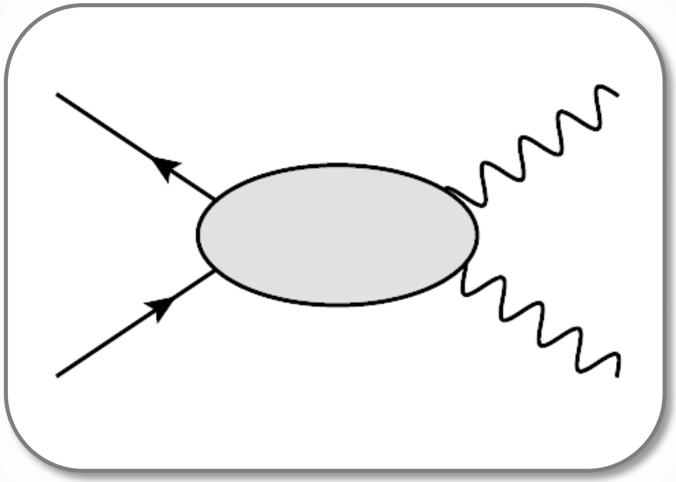
Basics

of quantum field
theory approach

Quantum field theory

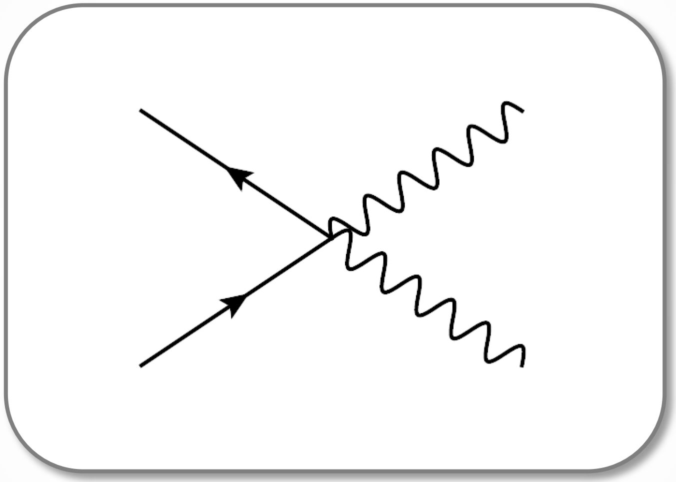
Particle process:

$$ff \longrightarrow VV$$



Quantum field theory

$$\mathcal{L} \supset G_{ffVV}\,\bar{f}fV_\mu^\dagger V^\mu$$



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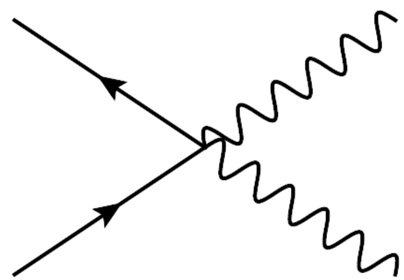
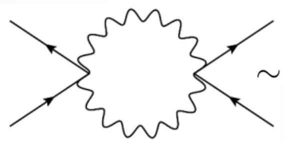
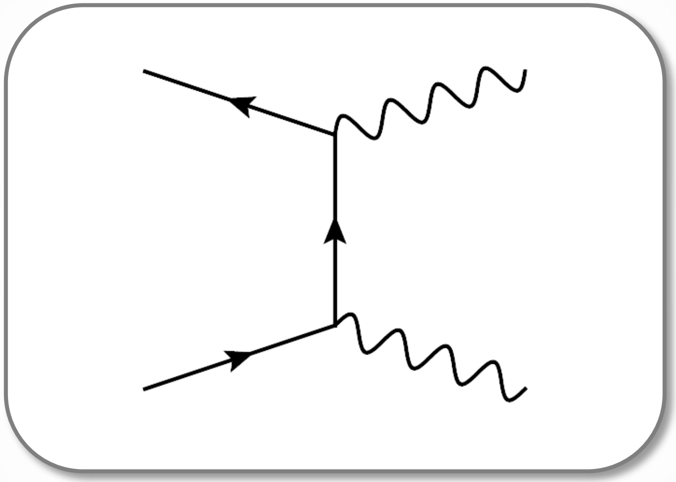


Renormalization:  $\sim \int_\Lambda \frac{\mathrm{d}^4 k}{(k^2)^2} \sim \log \Lambda$ $\qquad \mathcal{L} \overset{!}{\supset} G_{ffff}\, (\bar{f} f)^2$

Non-renormalizable theory contains infinite number of terms.

Quantum field theory

$$\mathcal{L} \supset G_{ffV}\, \bar{f}\gamma_\mu f V^\mu$$



Quantum field theory

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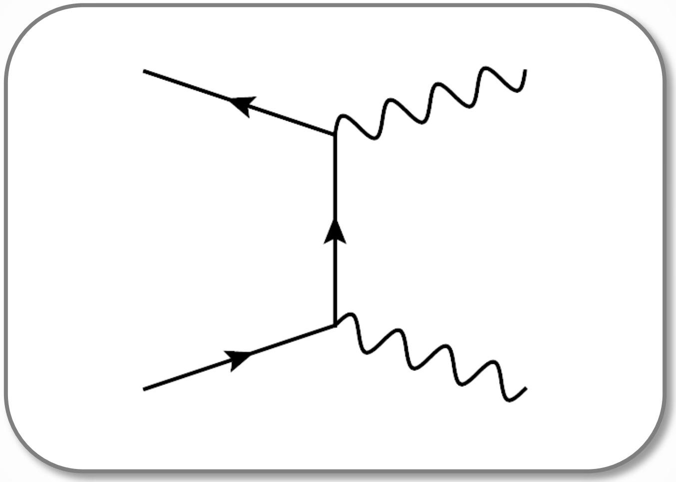
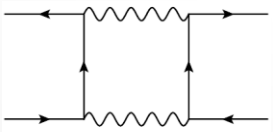


Renormalization:  does not require counter-terms.

(If *A* is massless!)

Renormalizable theory contains finite number of terms.

Gauge principle – massless vector bosons

Introducing vector bosons in a renormalizable way.

QED: massless photons local (gauge) $\mathrm{U}(1)_{\mathrm{em}}$ symmetry
QCD: massless gluons local (gauge) $\mathrm{SU}(3)_{\mathrm{c}}$ symmetry

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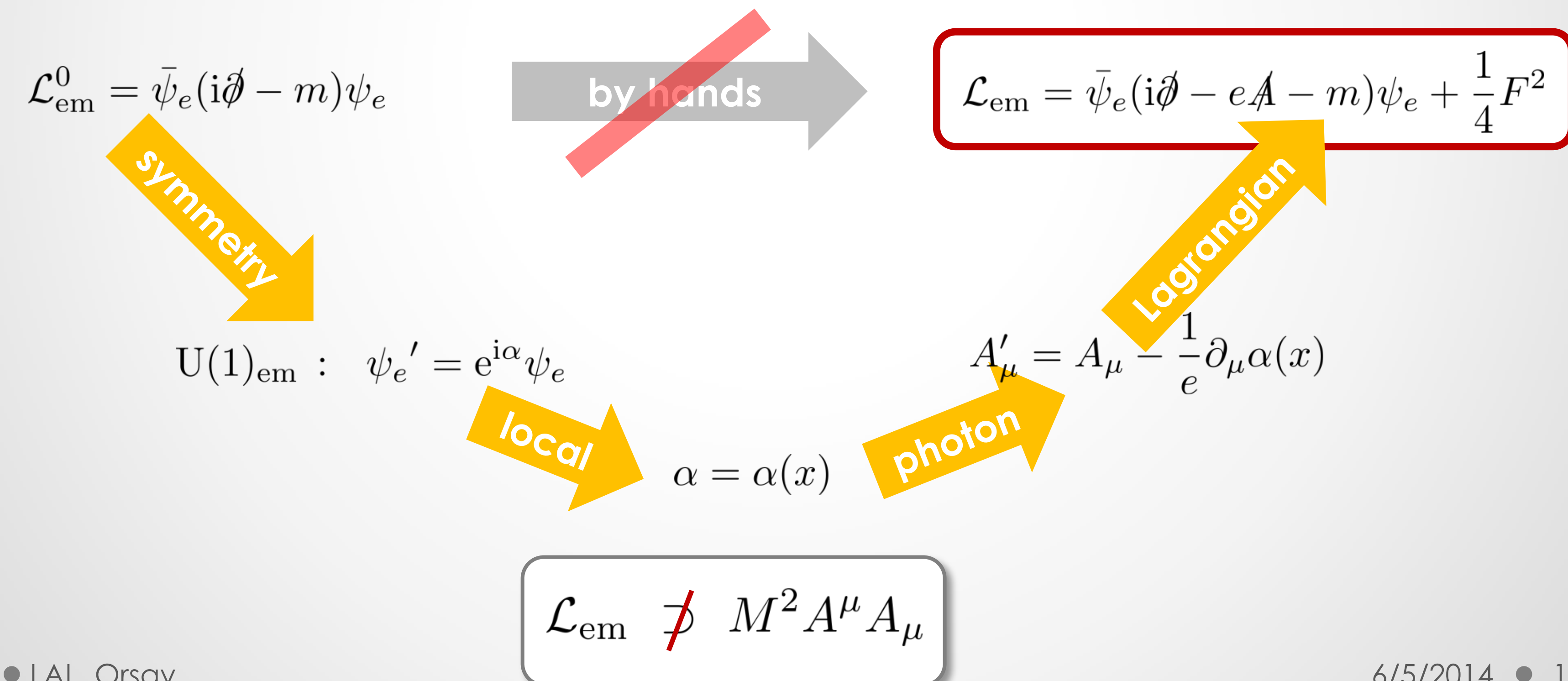
Massless vector bosons cannot be introduced consistently without the gauge principle.



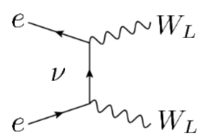
Gauge principle – massive vector bosons

Weak interactions: massive W,Z bosons

Gauge principle – massive vector bosons

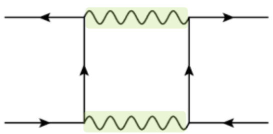
Weak interactions: massive W,Z bosons

By hands as Proca fields $$\Delta^{\mu\nu} = \frac{-g^{\mu\nu} + q^\mu q^\nu / M^2}{q^2 - M^2} \sim \mathcal{O}(q^0)$$



$\sim E^2$

Tree-level unitarity fails.



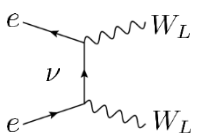
$$\sim \int_\Lambda \frac{d^4k}{\not k \not k} \sim \Lambda^2$$

Nonrenormalizable

Gauge principle – massive vector bosons

Weak interactions: massive W,Z bosons

By hands as Proca fields **Nonrenormalizable**



$\sim E^2$

Tree-level unitarity fails.

By gauge principle: The tree-level unitarity is improved.

But:

$$\mathcal{L} \not\ni M_W^2 W^\mu W_\mu$$

Spontaneous breaking of electroweak gauge symmetry

Lagrangian is invariant, but not the ground state.
Equations of motions are invariant, but not their solutions.

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

GOLDSTONE THEOREM
Nambu—Goldstone bosons

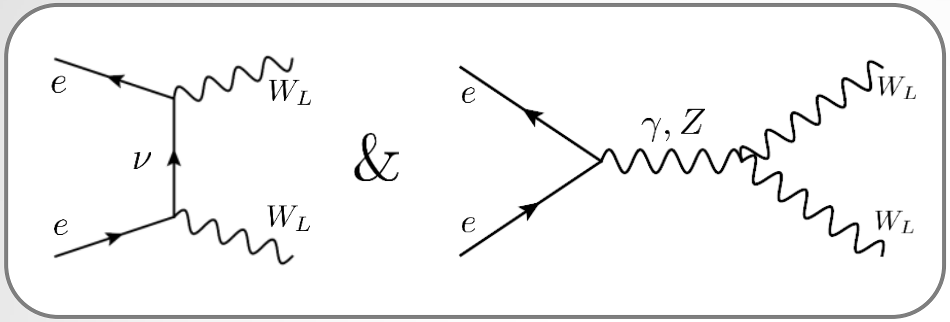
Anderson-Englert-Higgs (ABEGHHK'tH) **mechanism:**

massless gauge boson + massless Nambu-Goldstone boson = massive vector boson
(**2 d.o.f.**) + (**1 d.o.f.**) = (**3 d.o.f.**)

Renormalizable way

Higgs boson

Tree-level unitarity



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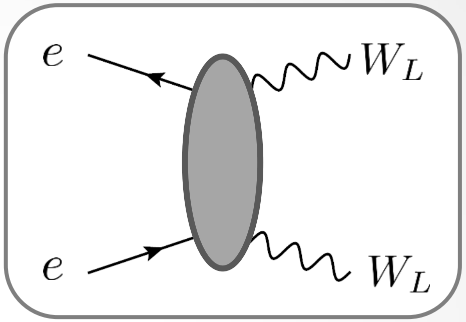
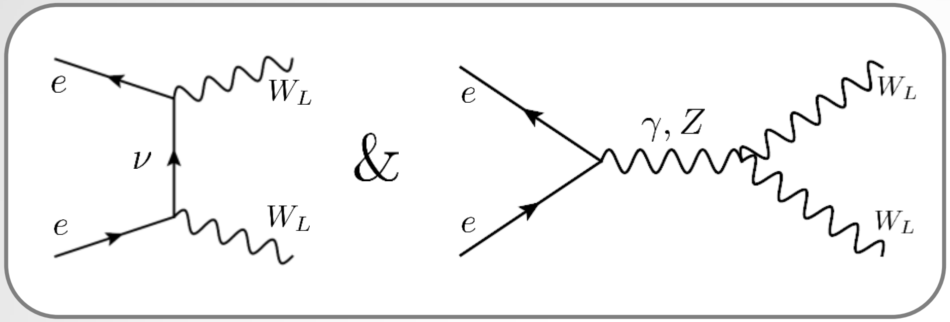


$\sim \mathcal{O}(1)$

$\sim \; E$ $\qquad$ $\sim \; E$

Higgs boson

Tree-level unitarity



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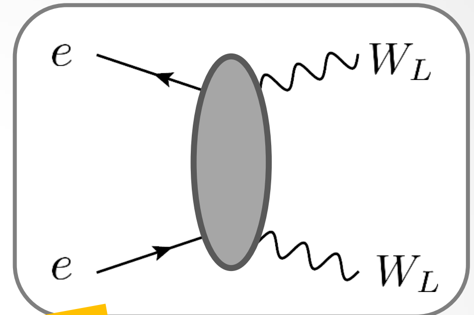
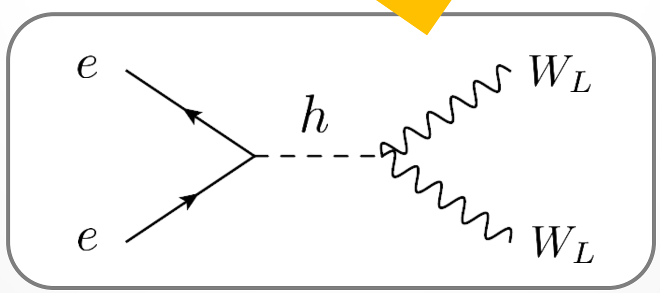


$\sim \mathcal{O}(1)$

$\sim \; E$ $\sim \; E$

Simplest way: introduce neutral scalar



$\sim \; E$

If there is no such new particle(s), the *W* and *Z* stay massless.

Gauge model of elementary particles

$\mathrm{SU}(3)_c$ → gluons

$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_{\mathrm{em}}$ → γ, W, Z

EXTREMELY SUCCESSFUL

Single electroweak scale

$$v \doteq 246\,\mathrm{GeV}$$

$$M_W^2 = g^2 v^2$$

$$M_Z^2 = (g^2 + g'^2) v^2$$

Fermion field content

FITS

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Fermion field content

FITS

But what causes the Higgs mechanism?

Fermion Masses

$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is not only **gauge** but also **chiral**!

Fermion mass terms are forbidden:

$$m_e \bar{\psi}_e \psi_e = m_e(\bar{\psi}_{eL}\psi_{eR} + \bar{\psi}_{eR}\psi_{eL})$$

$$\begin{aligned} \mathrm{U}(1)_Y :\ \psi'_{eR} &= \mathrm{e}^{-2\mathrm{i}\alpha}\psi_{eR} \\ \psi'_{eL} &= \mathrm{e}^{-1\mathrm{i}\alpha}\psi_{eL} \end{aligned}$$

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How to get fermion masses?

Standard Model

Introduce elementary complex scalar doublet.

gauge bosons: $g^2 W^\mu H^\dagger H W_\mu$

fermions: $y \bar{\psi}_L H \psi_R + \mathrm{h.c.}$

Standard Model

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gauge bosons: $g^2\langle H\rangle^2 W^\mu W_\mu$

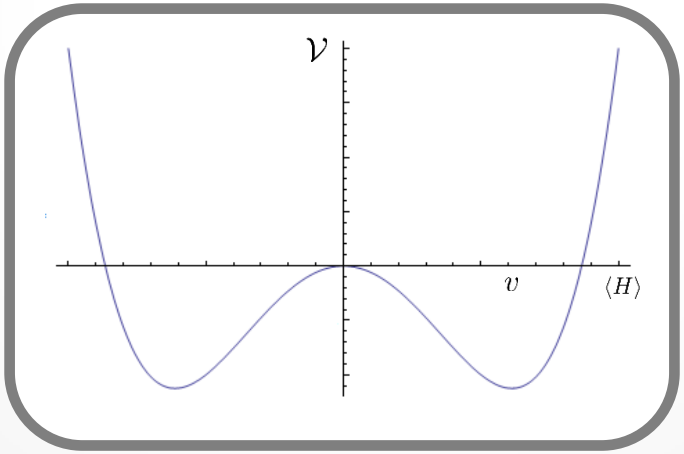
fermions: $y\langle H\rangle\bar{\psi}_L\psi_R + \text{h.c.}$

$\langle H\rangle \propto v$

$M_W^2 \propto g^2 v^2$

$m_\psi \propto yv$

$$\mathcal{V} = -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2$$



Notice that fermion masses are not important for successful EWSB.

Superconductivity

We know similar situation.

Meisner effect: photons are massive in the bulk of superconductor.

$$\mathrm{U}(1)_{\mathrm{em}}$$

is spontaneously broken.

Ginzburg—Landau theory describes it by complex order parameter field,

$$F \supset \alpha|\phi|^2 + \frac{\beta}{2}|\phi|^4 + \frac{1}{2m_e}|(-i\hbar\nabla - 2e\mathbf{A})\phi|^2$$

which can develop nonzero value $|\phi|^2 = -\frac{\alpha}{\beta} \overset{\text{if}}{>} 0$

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PHENOMENOLOGICAL DESCRIPTION

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Bardeen—Cooper—Schrieffer theory: $\phi \sim \psi^e_{\mathbf{k}}\psi^e_{-\mathbf{k}}$

electrons acquire a gap

$$E = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta|^2}$$

Standard Model

Electroweak symmetry breaking: W, Z bosons are massive.

$$\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L$$

is spontaneously broken.

Standard Model describes it by complex Higgs field,

$$\mathcal{L} \supset \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + D^\mu H^\dagger D_\mu H$$

which develops nonzero v.e.v. $H^\dagger H = -\frac{\mu^2}{2\lambda} \overset{\text{if}}{>} 0$

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PHENOMENOLOGICAL DESCRIPTION?

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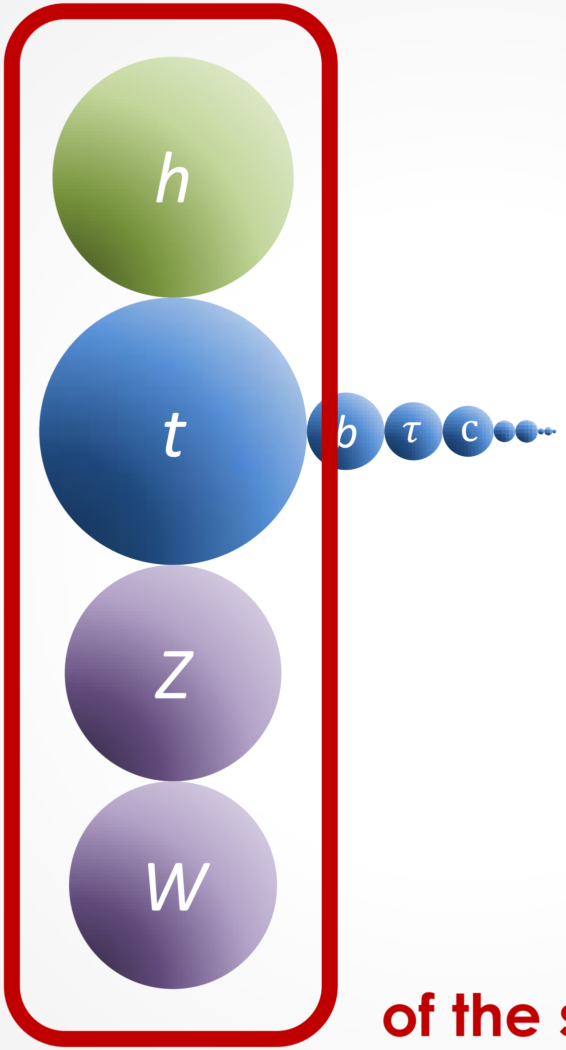
Dynamical EWSB: $H \sim \bar{\Psi}\Psi$

e.g. (E)TC

$$E = \sqrt{\mathbf{p}^2 + m^2}$$

Our approach



of the same order of magnitude

Our approach

new dynamics between quarks and leptons

$$m_t \propto \langle \bar{t}t \rangle\,, \quad m_b \propto \langle \bar{b}b \rangle\,, \quad m_c \propto \langle \bar{c}c \rangle\,, \ \ldots$$
$$m_\tau \propto \langle \bar{\tau}\tau \rangle\,, \quad m_\mu \propto \langle \bar{\mu}\mu \rangle\,, \ \ldots$$

$$v \ \sim \ m_t\,,\ m_b\,,\ m_c\,,\ \ldots$$

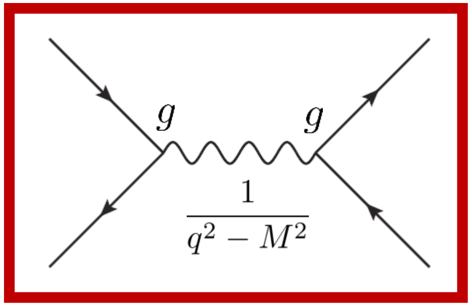
in **complete** analogy
with superconductivity

[Ho82]
[KiMu85]
[Na88]
[MiYa89]
[BaHiLi89]

New dynamics among fermions

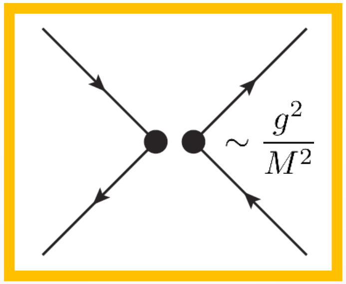
… substituting the Higgs sector of SM.

Renormalizable models:



$$q^2 \ll M^2 \sim \Lambda^2 < \Lambda^2_{\text{Planck}}$$

Simplification:
four-fermion interaction

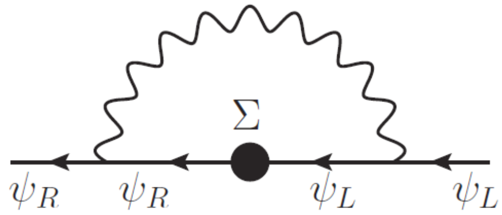


Dynamical fermion mass

Dirac propagator: massless $S_0(p) = \frac{1}{\not p}$ massive $S_m(p) = \frac{1}{\not p - m}$

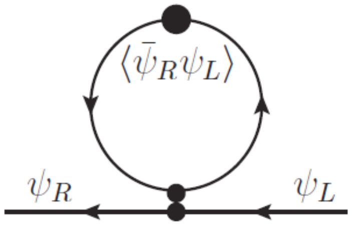
Full propagator: $S(p) = \frac{1}{\not p - \Sigma(p^2)}$

$\Sigma(p^2) \; =$

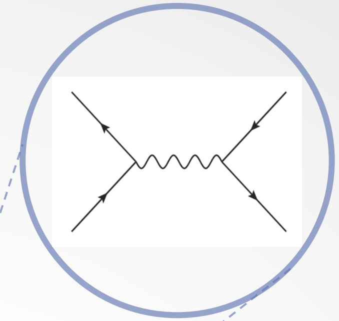


Mass is the pole in the propagator: $\det\left[p^2 - \Sigma^\dagger(p^2)\Sigma(p^2)\right] = 0$

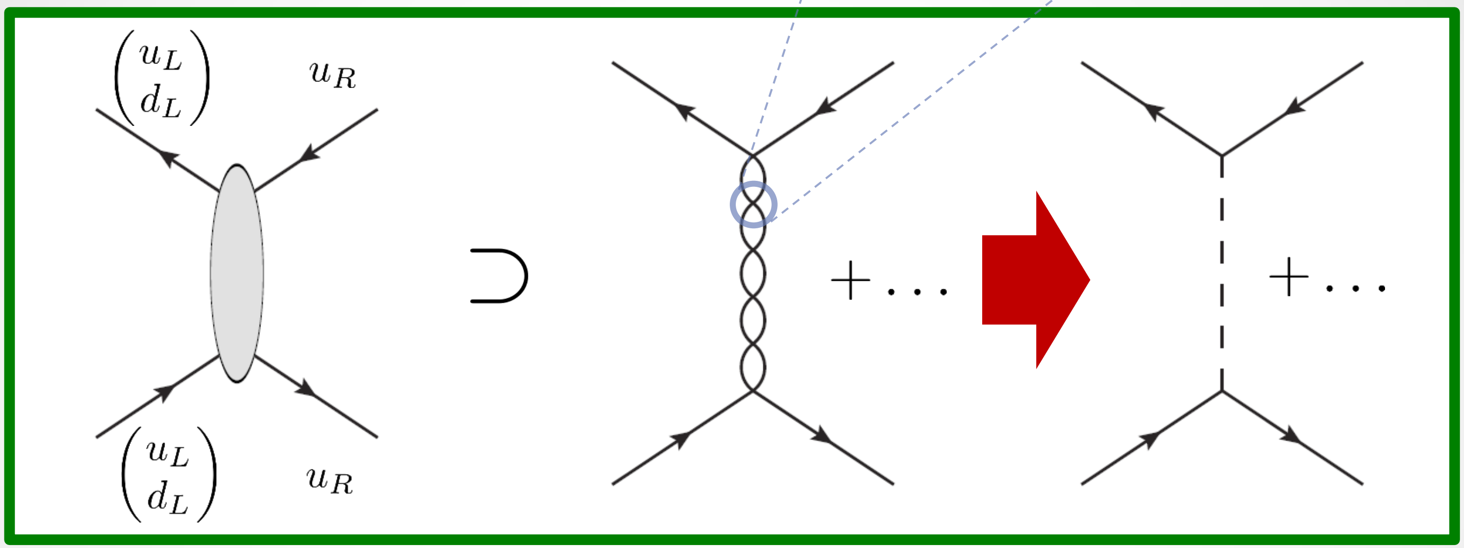
$\langle\bar\psi_R\psi_L\rangle \; =$



Bound states



Full process

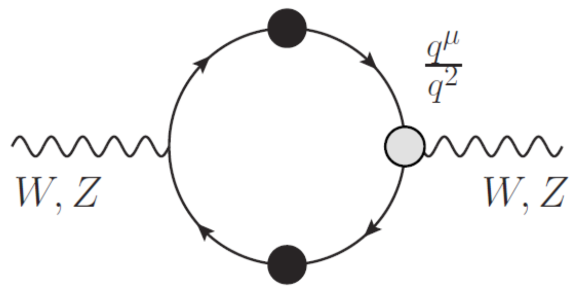


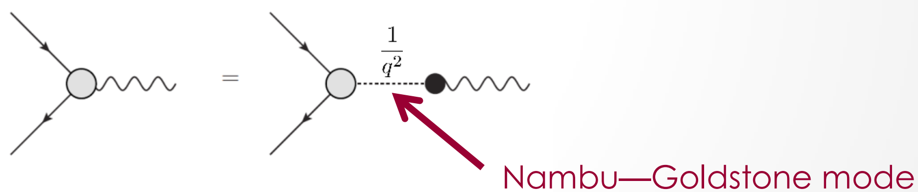
composite scalars:
Nambu—Goldstone bosons, Higgs boson

$$\bar{u}_R q_L = \bar{u}_R \begin{pmatrix} u_L \\ d_L \end{pmatrix} \implies H_u \sim \begin{pmatrix} \bar{u}_R u_L \\ \bar{u}_R d_L \end{pmatrix}$$

Electroweak gauge boson mass generation

Pagels—Stokar formulae

$\Pi(q^2) =$  $\approx -\frac{M^2}{q^2}$



$$\Delta \propto \frac{1}{q^2 + q^2\Pi^2(q^2)} \approx \frac{1}{q^2 - M^2}$$

Low-energy effective description

We expect that the low-energy effective description is the **multi-composite-Higgs-doublet model**.

$$H_t,\ H_b,\ H_\tau,\ H_c,\ H_s,\ H_\mu,\ \ldots$$

Is the scenario viable?

Top-quark condensation model

[MiYa89] [BaHiLi89]

Out of usual Dirac fermions, only the **top-quark** contributes **significantly** to the electroweak scale by its condensate.

single-composite-Higgs-doublet model

Top-quark is too light to saturate the electroweak scale $v_t < 0.68v$

The composite Higgs boson is predicted too heavy $M_h > m_t$

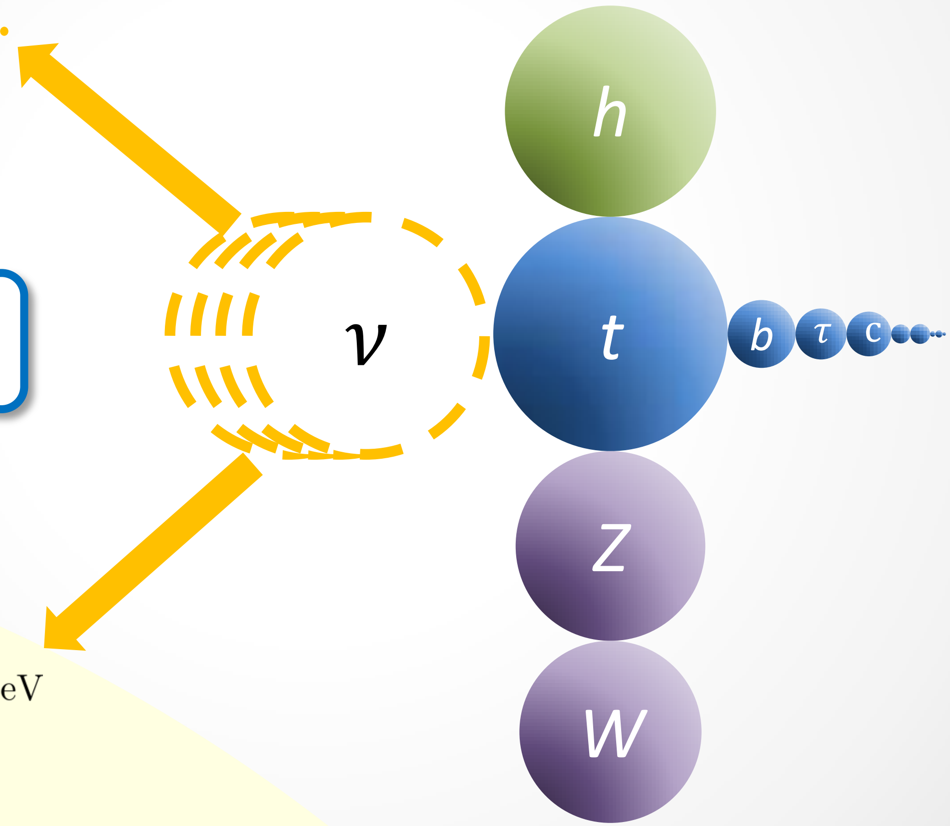
$\Lambda < \Lambda_{\text{Planck}}$

Way out – neutrino condensation

$$m \approx -\frac{m_D^2}{M_R} \sim 0.1\,\mathrm{eV}$$

The seesaw mechanism

$$(\bar{\nu}_L \;\; \bar{\nu}_R^c) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \bar{\nu}_L^c \\ \bar{\nu}_R \end{pmatrix}$$



$$M \approx M_R \sim 10^{14}\,\mathrm{GeV}$$

Top-quark and neutrino conspiracy against the electroweak symmetry

We assume that …

… out of all electroweakly charged fermions, both **top-quark and neutrinos** contribute **significantly** to the electroweak scale by their Dirac masses and the corresponding condensates.

[Ma'91] [AnKeLiRa'03]

Top-quark and neutrino condensation model

two-composite-Higgs-doublet model

$$H_t,\ H_\nu$$

[AS'13]

to reproduce

- the electroweak scale $v \doteq 246\,\mathrm{GeV}$
- the top-quark mass $m_t \doteq 172\,\mathrm{GeV}$
- the neutrino mass $m_\nu \doteq 0.2\,\mathrm{eV}$
- the Higgs boson mass $M_h \doteq 125\,\mathrm{GeV}$

RESULT:

$$y_t \sim 0.6$$

$$M_{H^+} \sim (200-250)\,\mathrm{GeV}$$

$$N_{\nu_R} = \mathcal{O}(10-100)$$

$$\Lambda \sim (10^{17}-10^{18})\,\mathrm{GeV}$$

Conclusions

- Even though the Standard Model is extremely successful, it still worths to look for the origin of the Higgs mechanism.
- The models based on the superconductivity approach are still an option.
- They are ambitious as they link many parts of particle physics.

Lagrangian of the model and composite Higgs doublets

Four-fermion interaction:

$$\mathcal{L} \supset -G_t(\bar{t}_R q_L)(\bar{q}_L t_R) - G_\nu(\sum_{s=1}^{N} \bar{\nu}_R^s \ell_L)(\sum_{s'=1}^{N} \bar{\ell}_L \nu_R^{s'}) - \frac{1}{2} M_R \bar{\nu}_R^s (\nu_R^s)^c + \text{h.c.}$$

Specially designed to provide the simplest seesaw pattern.

The condensation happens at the scale Λ

Two-Higgs-doublet effective description:

$$\mathcal{L}_{\text{eff}} \supset -y_t(\bar{q}_L t_R) H_t - y_\nu(\sum_s \bar{\ell}_L \nu_R^s) H_\nu - \mathcal{V}(H_t, H_\nu)$$

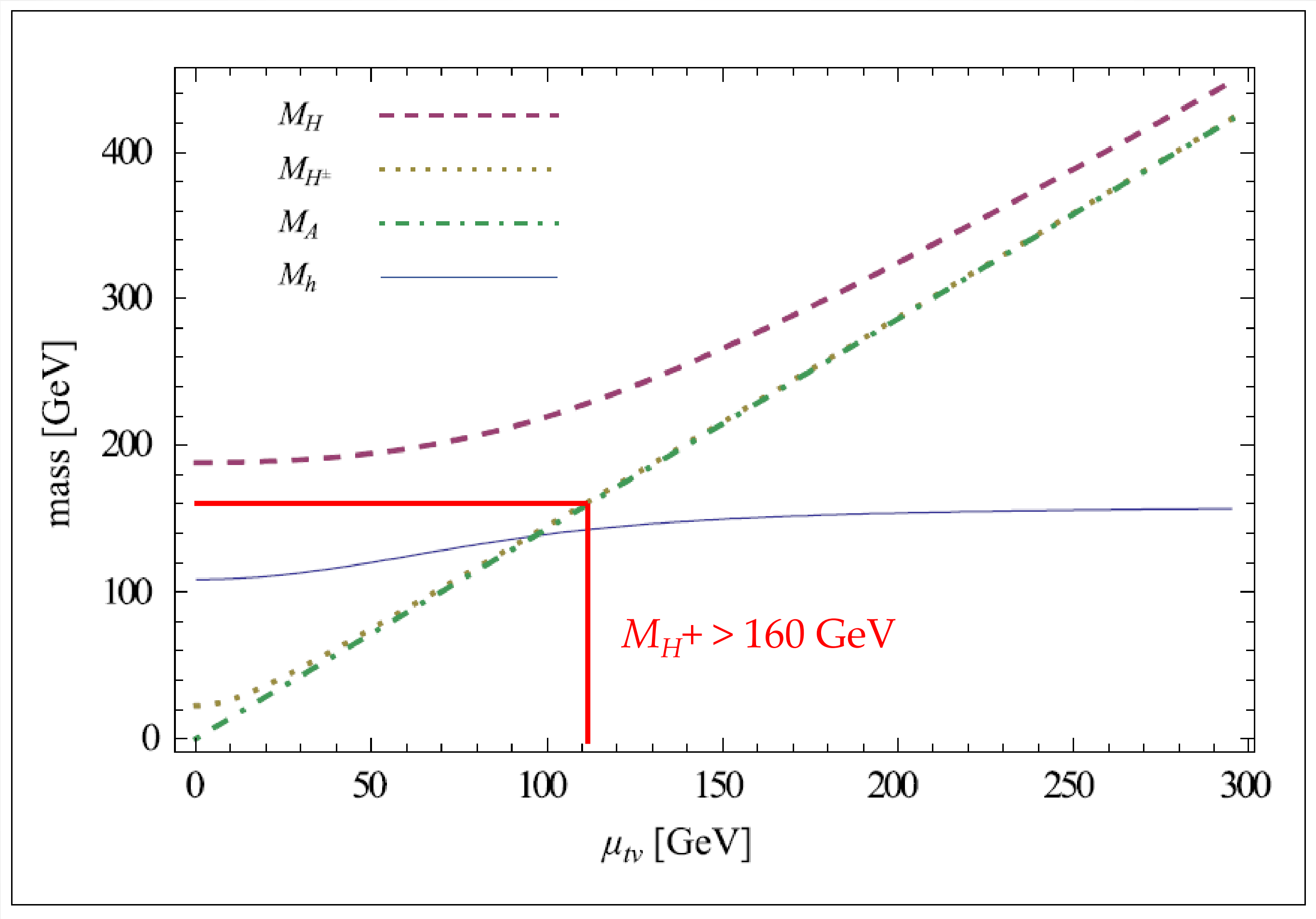
$\mu_{t\nu}$

$$\Lambda_{\text{Planck}} > \Lambda > M_R$$

Mass spectrum of Higgs bosons

$$H_t, H_\nu \longrightarrow h, H, A, H^+$$

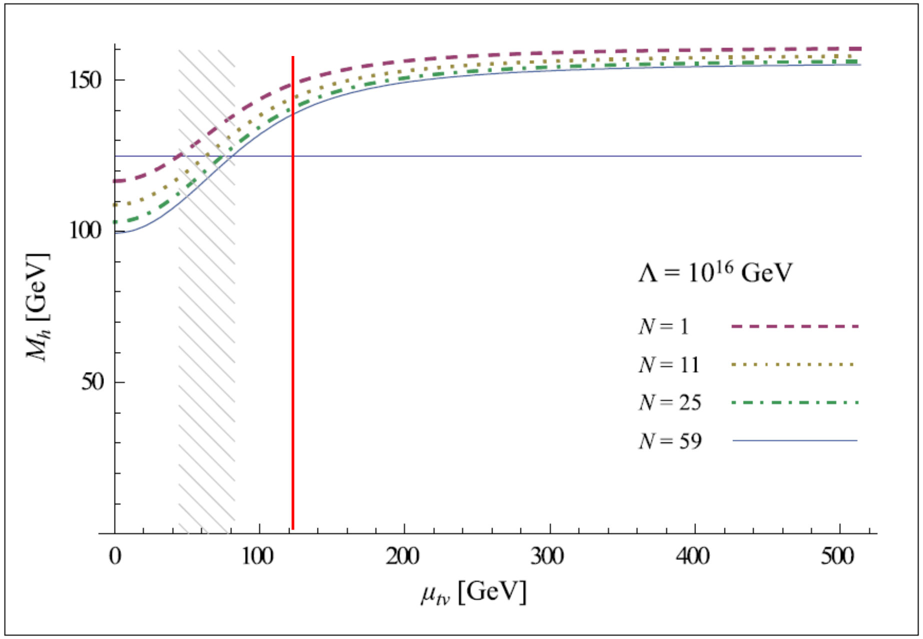
Larger values of $\mu_{t\nu}$ are preferred.

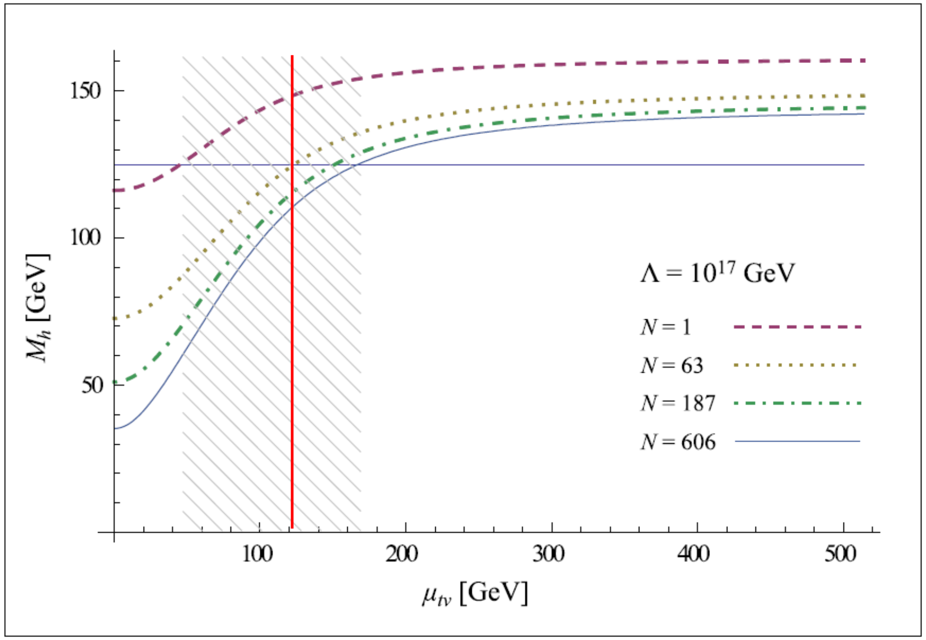


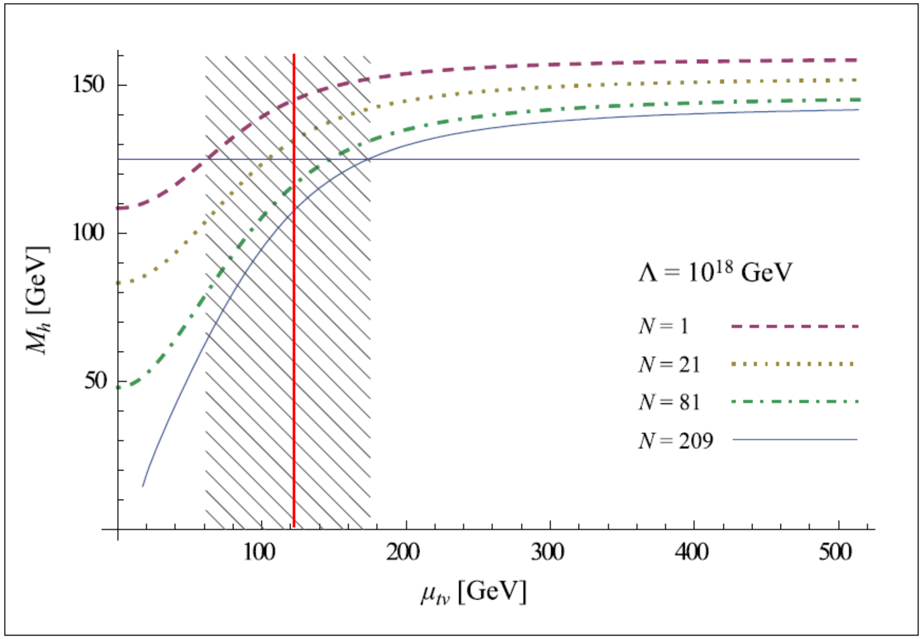
$$N = 1,\ \Lambda = 10^{18}\,\text{GeV},\ M_R \sim 10^{14}\,\text{GeV}$$

The lightest Higgs boson mass depending on the number of right-handed neutrinos

Larger values of $\mu_{t\nu}$ break the condition $\Lambda_{\text{Planck}} > \Lambda > M_R$







Right-handed neutrinos

Their existence is extremely well motivated.

- Adding three right-handed neutrinos is sufficient to reproduce
  - neutrino oscillations
  - dark matter [CaDrScha'13]
  - baryon asymmetry of the Universe
- Right-handed neutrino condensation may drive the inflation of the Universe. [Ba'09]

The number of right-handed neutrinos is not constrained.

Large number of right-handed neutrinos may be welcome.

- $O(100)$ is motivated by some string constructions [ElLe'07]
- $O(10\text{-}100)$ may explain large neutrino mixing [FeKl'12]
- $O(100)$ improves the standard thermal leptogenesis [Ei'08]

We keep a number of the right-handed neutrinos as a free parameter.

In our model their number turns out to be large $O(10\text{-}100)$!