# Off-shell and interference effects in Higgs physics 

Kirill Melnikov<br>KIT TTP/ JHU



Based on collaboration with F. Caola

## The Higgs boson

The new particle discovered at the LHC two years ago appears to be the Higgs boson of the Standard Model. Indeed, its production and decay rates, its spin and parity, as well as its mass, are consistent with the Standard Model expectations. Further studies of these quantities with higher precision are important and will be performed during Run 2.

Most of these studies involve production of the on-shell Higgs boson. This is because the onshell production is definitely the largest signal and the cleanest one to interpret. However, the off-shell production of the Higgs is also interesting. It allows us to constrain the width of the Higgs boson, look for the internal structure of loop-induced Higgs boson couplings and constrain higher-dimensional operators that may affect Higgs production and decay.




## The Higgs boson width

In the Standard Model, the width of 126 GeV Higgs boson is extremely small, it is just 4.2 MeV. It is almost impossible to measure it directly at any collider, with the exception of the muon one. At hadron and electron colliders, one can measure Higgs branching to invisible final states in the Bjorken process and then infer the total width from there. At an e+e- and muon collider, a measurement of the Higgs width with a few percent precision can probably be achieved.



## The Higgs boson width at the LHC

To measure the width directly, we typically study invariant mass distribution of the resonance decay products in the vicinity of a resonance and fit it to the Breit-Wigner formula.
Unfortunately, since the invariant mass of the Higgs decay products can be reconstructed with (for these purposes) poor resolution, the LHC is only sensitive to the Higgs width in a few GeV range. The current direct limit on the Higgs width by the CMS collaboration is $\Gamma_{H}<3.4 \mathrm{GeV}$. The ultimate reach is estimated to be between 1 and 3 GeV .

To get into an MeV range for the Higgs width measurement, we need to improve the sensitivity of our methods by a factor of a thousand! Because of that, measuring the Higgs width at the LHC with any degree of precision was always considered an utopian endeavor.



## Can we tightly constrain the Higgs width at the LHC?

## From rates to couplings: degeneracies

Existing direct constraints on the Higgs boson width, imply that the Higgs boson is a narrow resonance; as such it is mainly produced on-shell. This feature leads to a peculiar relation between production rates, Higgs boson couplings and the Higgs width.


$$
\sigma_{i \rightarrow H \rightarrow f} \sim \frac{g_{i}^{2} g_{f}^{2}}{\Gamma_{H}}
$$

Unfortunately, such a relation makes it impossible to extract the couplings and the width separately, from the measured on-shell cross-sections. Indeed, any on-shell cross-section is invariant under a simultaneous re-scaling of the Higgs couplings and the Higgs width

$$
g \rightarrow \xi g, \quad \Gamma_{H} \rightarrow \xi^{4} \Gamma_{H} \quad \Rightarrow \quad \sigma_{H} \rightarrow \sigma_{H}
$$

Since the width of the Higgs boson is practically unconstrained, extraction of the Higgs couplings from production/decay rates suffers from significant ambiguity.

To resolve the ambiguity, we need to either measure the width of the Higgs boson or the Higgs couplings independently of each other.

## Couplings from off-shell production

One can try to measure the couplings of the Higgs boson when it is produced off-shell. The off-shell cross-section is proportional to couplings and is independent of the width, resolving the width/couplings ambiguity.

$$
\left.\sigma_{i \rightarrow H \rightarrow f} \sim \int \frac{\mathrm{~d} s g_{i}^{2} g_{f}^{2}}{\left(s-m_{h}\right)^{2}+m_{h}^{2} \Gamma_{h}^{2}}\right|_{s \gg m_{h}^{2}} \rightarrow \frac{g_{i}^{2} g_{f}^{2}}{s}
$$

The immediate problem with this idea is that off-shell contribution to Higgs boson production is expected to be extremely small.

However, Kauer and Passarino pointed out that a significant enhancement in the off-shell Higgs production rate exists, making the invariant mass distribution very different from the expected Breit-Wigner shape.


Kauer, Passarino

## Higgs decays to ZZ

One can use this enhancement in the off-shell Higgs production to resolve couplings/width degeneracy. The cleanest final state is ZZ (four leptons), so it is natural to look there.

Caola, K.M.
In this case, the off-shell rate appears to be significant because decay to two on-shell gauge bosons opens up and because the cross-section for producing two longitudinally polarized Z bosons in decays of (strongly) off-shell Higgs is large.

Kauer, Passarino

$$
\mathcal{A}_{H^{*} \rightarrow Z_{L} Z_{L}} \sim \frac{s}{v} \quad \frac{\left|\mathcal{A}_{H^{*} \rightarrow Z_{L} Z_{L}}\right|^{2}}{\left(s-m_{h}^{2}\right)^{2}+m_{h}^{2} \Gamma_{h}^{2}} \rightarrow \text { const, } \quad s \gg m_{h}^{2}
$$

For large invariant masses of the Z boson pair, the amplitude squared becomes independent of ZZ invariant mass, enhancing the off-shell production significantly. Off-shell cross-section is large; it is close to ten percent of the resonance cross-section.


$$
\sigma_{H}\left(m_{Z Z}>160 \mathrm{GeV}\right) \approx 0.1 \sigma_{H}
$$



## Higgs decay to ZZ

The off-shell production cross-section does not depend on the Higgs width but does depend on the Higgs couplings to initial state particles (gluons) and final state particles ( Z bosons). This implies that if we change both the width of the Higgs and its couplings to other particles in such a way that the resonance cross-section does not change, the off-shell production cross-section changes proportionally to the Higgs width.
$\sigma_{H} \sim \frac{g_{H \rightarrow g g}^{2} g_{H \rightarrow Z Z}^{2}}{\Gamma_{H}} ; \quad \sigma_{\text {off }} \sim g_{H \rightarrow g g}^{2} g_{H \rightarrow Z Z}^{2}$.


The current direct upper bound on the Higgs width is 3.4 GeV (CMS) which is 820 times larger than the Standard Model value. If the width were actually that large, Higgs couplings to gluons and ZZ should be different from their SM values to ensure agreement of the on-shell cross-section. However, once couplings are modified, one should expect a very large number of additional off-shell events that exceed by almost a factor four the total number of ZZ events observed by the CMS!

$$
N_{\text {off }} \approx 0.1 \times N_{\text {peak }} \times 820 \sim 1600 \gg N_{4 l}^{\text {total }}
$$



Therefore, one can already put meaningful bounds on the Higgs width using current data on ZZ final states !

## Complication: signal-background interference

Production of two Z bosons in collisions of two gluons can occur either directly or through the Higgs boson. The two amplitudes interfere destructively (essentially, unitarity cancellations in the Standard Model). The interferenceis negligible at the peak (narrow resonance) but it is significant $(-50 \%)$ off the peak.

Kauer, Passarino; Ellis, Campbell, Williams


For our purposes, it is important that the scaling of the interference with the width differs from the scaling of the off-shell cross-section, since dependence of the interference on the Higgs couplings is weaker.

$$
\sigma_{\mathrm{int}} \sim \mathcal{A}_{g g \rightarrow H^{*} \rightarrow Z Z} \sim g^{2} \sim \sqrt{\Gamma_{H}}
$$

## Magnitude of various contributions

How large is the total sample of events we have now and how large are different contributions to pp -> ZZ for realistic selection cuts?

Consider CMS 4-lepton events as an example. CMS observes 451 ZZ (41) events in the invariant mass range between 100 and 800 GeV ; while $432(30) \mathrm{ZZ}$ events were expected.

Most of these events come from quark-antiquark and gluon annihilation to Z-pairs; the third largest contribution is the resonance Higgs production. Off-shell production of the Higgs and its interference with gg -> ZZ production were not even included in early CMS analysis because they are small in the Standard Model and because they do not affect properties of the Higgs resonance (off-shell effects, no impact on the Higgs properties extracted from peak cross-sections).


$$
\begin{aligned}
& N_{q q \rightarrow Z Z} \approx N_{\mathrm{tot}} \\
& N_{g g} \sim 10^{-1} \times N_{\mathrm{tot}} \\
& N_{H} \sim 5 \times 10^{-2} \times N_{\mathrm{tot}} \\
& N_{\mathrm{off}} \sim 10^{-2} N_{\mathrm{tot}} \\
& N_{\mathrm{int}} \sim-2 \times 10^{-2} N_{\mathrm{tot}}
\end{aligned}
$$

## Constraining the width

However, if we float the width of the Higgs boson, the number of expected events changes; as we have seen, the off-shell Higgs production cross-section scales as the width and the interference scales as the square root of the width. Considering ZZ invariant mass range from 100 GeV to 800 GeV , we find a new estimate for the number of events

$$
N_{\exp }=432+2.78 \frac{\Gamma_{H}}{\Gamma_{H}^{S M}}-5.95 \sqrt{\frac{\Gamma_{H}}{\Gamma_{\mathrm{H}}^{\mathrm{SM}}}} \pm 31
$$

Requiring that observed (451) and expected number of events do not differ by more than two standard deviations, we derive an upper bound on the Higgs boson width

$$
\left|N_{\text {nobs }}-\bar{N}_{\mathrm{exp}}\right|<62 \quad \Gamma_{H}<43 \Gamma_{H}^{\mathrm{SM}}=181 \mathrm{MeV}(95 \% \mathrm{C} . \mathrm{L})
$$

Caola, K.M.; Campbell, Ellis,Williams


Campbell, Ellis, Williams

The analysis can be improved by focusing on the region of higher invariant masses. This is because the off-shell Higgs production is significant beyond 200 GeV while there is large negative interference below 200 GeV ; removing contribution of that region, improves the constraint. For example, selecting events with 4lepton invariant mass higher than 300 GeV , we find $\Gamma_{H}<25.2 \Gamma_{H}^{S M}<105 \mathrm{MeV}(95 \%$ C.L. $)$

Caola, K.M.; Campbell, Ellis,Williams

## Further improvements in the analysis

In principle, one can imagine further steps to improve the analyses by accounting for the shape of 41 invariant mass distribution and the angular distributions of leptons.

Indeed, as one can see from the plots below, shapes of the invariant mass distribution ( $\mathrm{gg}->\mathrm{ZZ}$ only) change for different values of the width. This is particular clear at small(ish) invariant masses where interference is large; at higher invariant masses, the invariant mass distribution develops a bump whose position is not sensitive to the width.

Angular distributions of leptons may be helpful to select events with longitudinally-polarized Z's -- these are the Higgs-related events in the off-shell tail of 41-invariant mass distribution. The qq -> ZZ background should have higher fraction of transversely polarized Z-bosons, so focusing on longitudinal polarization may help to effectively reduce $q q->Z Z$ background (MELA cut in CMS).


## Using kinematic discriminants to constrain the width

A standard way to pick up differences in the kinematics of interesting (signal) and uninteresting (background) events is to use kinematic discriminants. Kinematic discriminants are functions of matrix elements of signals and backgrounds that become large for events where signal is large and background is small; this allows us to find relevant places in multiparticle phase-space to look at, to optimize signal/background separation. A discriminant for this problem was suggested by Campbell, Ellis and Williams. Events with large width develop a feature at large values of the CEW discriminant.

$$
D_{S}=\log \left[\frac{P_{H}}{P_{g g}+P_{q q}}\right] \quad P_{i} \sim\left|M_{i}\right|^{2}
$$



By selecting events with Ds $>1$, one can improve bounds on the width by a factor 1.6 , compared to an analysis with 300 GeV cut on ZZ invariant mass. Campbell et al find that

$$
\Gamma_{H}<15.7 \Gamma_{H}^{\mathrm{SM}}(95 \% \mathrm{C} . \mathrm{L} .)
$$

can be reached. Further improvements should be possible by fitting Ds distribution, rather than simply cutting on it.

## The Higgs width constraint: CMS

CMS collaboration has presented results of the actual width measurement using off-shell ZZ production. The measurement includes both ZZ and WW channels.


|  |  | $4 \ell$ | $2 \ell 2 v$ |
| :---: | :--- | :---: | :---: |
| (a) | total $\operatorname{gg}\left(\Gamma_{\mathrm{H}}=\Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $1.8 \pm 0.3$ | $9.6 \pm 1.5$ |
|  | gg signal component $\left(\Gamma_{\mathrm{H}}=\Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $1.3 \pm 0.2$ | $4.7 \pm 0.6$ |
|  | gg background component | $2.3 \pm 0.4$ | $10.8 \pm 1.7$ |
| (b) | total gg $\left(\Gamma_{\mathrm{H}}=10 \times \Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $9.9 \pm 1.2$ | $39.8 \pm 5.2$ |
| (c) | total VBF $\left(\Gamma_{\mathrm{H}}=\Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $0.23 \pm 0.01$ | $0.90 \pm 0.05$ |
|  | VBF signal component $\left(\Gamma_{\mathrm{H}}=\Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $0.11 \pm 0.01$ | $0.32 \pm 0.02$ |
|  | VBF background component | $0.35 \pm 0.02$ | $1.22 \pm 0.07$ |
| (d) | total VBF $\left(\Gamma_{\mathrm{H}}=10 \times \Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $0.77 \pm 0.04$ | $2.40 \pm 0.14$ |
| (e) | $\mathrm{q} \overline{\mathrm{q}}$ background | $9.3 \pm 0.7$ | $47.6 \pm 4.0$ |
| (f) | other backgrounds | $0.05 \pm 0.02$ | $35.1 \pm 4.2$ |
| $(\mathrm{a}+\mathrm{c}+\mathrm{e}+\mathrm{f})$ | total expected $\left(\Gamma_{\mathrm{H}}=\Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $11.4 \pm 0.8$ | $93.2 \pm 6.0$ |
| $(\mathrm{~b}+\mathrm{d}+\mathrm{e}+\mathrm{f})$ | total expected $\left(\Gamma_{\mathrm{H}}=10 \times \Gamma_{\mathrm{H}}^{\mathrm{SM}}\right)$ | $20.1 \pm 1.4$ | $124.9 \pm 7.8$ |
|  | observed | 11 | 91 |

$$
\Gamma_{\mathrm{H}}<5.4 \Gamma_{\mathrm{H}, \mathrm{SM}}=22 \mathrm{MeV} @ 95 \mathrm{CL}
$$

A very impressive result; more than two orders of magnitude improvement compared to the direct ( on peak) bound of the width.

## The Higgs width constraint: ATLAS

A similar measurement was performed by the ATLAS collaboration. Analyses in ZZ and WW channels combined. ATLAS analysis emphasized the dependence of the final bound on the width on the (unknown) size of QCD corrections to gg->ZZ background process.

| Process | $220 \mathrm{GeV}<m_{4 \ell}<1000 \mathrm{GeV}$ | $400 \mathrm{GeV}<m_{4 \ell}<1000 \mathrm{GeV}$ |
| :---: | :---: | :---: |
| $g g \rightarrow H^{*} \rightarrow Z Z(S)$ | $2.2 \pm 0.5$ | $1.1 \pm 0.3$ |
| $g g \rightarrow Z Z(\mathrm{~B})$ | $30.7 \pm 7.0$ | $2.7 \pm 0.7$ |
| $g g \rightarrow\left(H^{*} \rightarrow\right) Z Z$ | $29.2 \pm 6.7$ | $2.3 \pm 0.6$ |
| $g g \rightarrow\left(H^{*} \rightarrow\right) Z Z\left(\mu_{\text {off-shell }}=10\right)$ | $40.2 \pm 9.2$ | $9.0 \pm 2.5$ |
| VBF $H^{*} \rightarrow Z Z(S)$ | $0.2 \pm 0.0$ | $0.1 \pm 0.0$ |
| $\operatorname{VBF} Z Z(\mathrm{~B})$ | $2.2 \pm 0.1$ | $0.7 \pm 0.0$ |
| $\operatorname{VBF}\left(H^{*} \rightarrow\right) Z Z$ | $2.0 \pm 0.1$ | $0.6 \pm 0.0$ |
| $\operatorname{VBF}\left(H^{*} \rightarrow\right) Z Z\left(\mu_{\text {off-shell }}=10\right)$ | $3.0 \pm 0.2$ | $1.4 \pm 0.1$ |
| $q \bar{q} \rightarrow Z Z$ | $168 \pm 13$ | $21.3 \pm 2.1$ |
| Reducible backgrounds | $1.4 \pm 0.1$ | $0.1 \pm 0.0$ |
| Total Expected $(\mathrm{SM})$ | $200 \pm 15$ | $24.3 \pm 2.2$ |
| Observed | 182 | 18 |

$\Gamma_{\mathrm{H}}<4.8-7.7 \Gamma_{\mathrm{H}, \mathrm{SM}}=20-32 \mathrm{MeV} @ 95 \mathrm{CL}$
(depending on the assumed K-factor for gg->ZZ background)

## Invisible branching and the Higgs width: ATLAS

Important information about the Higgs boson width can be obtained from experimental constraints on the branching ratio of Higgs decays to invisible final states, $\mathrm{Br}(\mathrm{H}->$ inv $)<$ 0.75. Current constraints are obtained assuming the Standard Model production crosssection ratio for $\mathrm{pp}->\mathrm{ZH}$ but the re-scaling of couplings violates this assumption.


Within out framework where all couplings and the width change coherently, the bound on the width from invisible branching needs to be reconsidered.

$$
\mathrm{Br}_{\mathrm{inv}}=1-\xi^{-2} \quad \sqrt{\frac{\Gamma_{H}}{\Gamma_{H, \mathrm{SM}}}}-1<0.75 \quad \Gamma_{H} \leq 3.1 \Gamma_{H, \mathrm{SM}}
$$

The constraint on the width from invisible branching ratio appear to be somewhat stronger than the direct constraints from the off-shell production that we just discussed.

## General comments

1) A suggestion that the Higgs boson width can be constrained at the LHC came unexpectedly; it showed that it is possible to use subtle quantum mechanical effects ( off-shell, interference) to get to interesting physics. Hopefully, we will see more examples of that type in the future.
2) CMS/ATLAS measurements prove that it is possible -- in practice -- to constrain the Higgs boson width using off-shell production of Z and W pairs. The results are very promising and appear to constrain the width to be in $\mathrm{O}(20) \mathrm{MeV}$ range, to be compared with $\mathrm{O}(1) \mathrm{GeV}$ direct constraint.
3) It is important to get the logic of the measurement correctly: by going off-shell, we measure couplings. No width enters the off-shell physics. We infer the information about the width from the off-shell cross-section once couplings are known.
4) Even with all statistical tricks ( likelihood etc.), at its core, this is a counting experiment that requires understanding of yields rather than shapes. Proper theoretical predictions for signal, background and interferences are therefore very important.

If we see an enhancement, is it the width?

## Enhancement at large ZZ invariant masses

The main idea of the Higgs width measurement method is that excessive events at high-invariant mass of Z-boson pairs are interesting and may be related to Higgs physics. Interpretation of such excesses in terms of limits on the Higgs boson width is possible, as we have seen, but may require some care since it forces us to relate couplings measured at different invariant masses. The problem is that in any Quantum field Theory couplings "run". A weak, logarithmic running is probably not so important at this point.

In general, a relation between on- and off-shell couplings may become less straightforward if the HZZ vertex contains anomalous couplings or the Hgg vertex receives significant contributions from light degrees of freedom. In these cases the couplings change so strong that it is incorrect to assume that they are equal on- and off-shell.

It was pointed out by various authors that these effects can tame or mimic the coupling/ width enhancement. Luckily, it seems that many such effects can be constrained from various on-shell measurements, as I will discuss shortly.


## Example: anomalous HZZ coupling

Anomalous HZZ couplings may lead to an increase in the number of events in the off-shell tail that, however, is independent of the "width".


## Example: additional Higgs bosons

A possible physics illustration of the operator analysis is provided by a model with two Higgs bosons; the role of the second (heavier) Higgs boson is to restore violations of unitarity that occur if couplings of the discovered Higgs boson to top quarks and Z-bosons are changed.

Logan et al (2014)


The amplitude becomes equal to the SM one for invariant masses higher than masses of both Higgs bosons, so the enhancement is gone. While this example shows that it is possible to hide the Higgs width enhancement from the off-shell measurements, it is done at the expense of introducing even more exciting physics that is detectable in the high-energy tail!

## Example: light colored singlets in the loop



Englert, Spannowsky (2014)
Light particles in HGG vertex induce powerlike running of the coupling constants and, therefore, can change the relation between onand off-shell couplings. However, one can check for them in a different way.

| $m_{\phi}$ | $\mu(h$ peak $)$ | $\Gamma_{h} / \Gamma_{h}^{\mathrm{SM}}$ | $\bar{\sigma} / \bar{\sigma}^{\mathrm{SM}}[m(4 \ell) \geq 330 \mathrm{GeV}]^{a}$ |
| :---: | :---: | :---: | :---: |
| 70 GeV | $\simeq 1.0$ | $\simeq 5$ | $-2 \%$ |
| 170 GeV | $\simeq 1.0$ | $\simeq 4.7$ | $+80 \%$ |
| 170 GeV | $\simeq 1.0$ | $\simeq 1.7$ | $+6 \%$ |

${ }^{a}$ We impose the cut set used by CMS [18] without the MELA cut [35].

For example, light particles coupled to the Higgs boson and charged under $\mathrm{SU}(3)$-color can be detected by studying the Higgs $\mathrm{p}_{\mathrm{T}}$ distribution (probably, need theoretical improvement, full $m_{t}$ dependence).

## Future prospects

## Weak boson fusion

It is possible to apply a similar analysis to other production channels of the Higgs boson, most notably to the production of the Higgs boson in weak boson fusion.

Recent analysis by Ellis and Campbell suggests that the most promising channel to look for off-shell events is the equal sign W-boson production because of the tiny background. The Higgs boson in this case appears in the $t$-channel.


Standard VBF cuts are applied; 4-lepton invariant (or transverse) mass is required to be smaller than 300 GeV .

Results from Run I constrain the width to be smaller than 60 times its Standard Model values ( $\mathrm{O}(1)$ event observed by ATLAS)

Number of events with 100 inverse fb for realistic selection cuts

$$
\begin{aligned}
l^{+} l^{+} \nu \nu: & N_{\text {off }}=38.8-18.3 \kappa_{V}^{2}+8.3 \kappa_{V}^{4} \\
l^{-} l^{-} \bar{\nu} \bar{\nu}: & N_{\text {off }}=11.5-4.1 \kappa_{V}^{2}+1.8 \kappa_{V}^{4}
\end{aligned}
$$

Limits expected from future LHC runs

$$
\begin{array}{cc}
\Gamma_{H}<4.4 \times \Gamma_{H}^{S M} & \left(100 \mathrm{fb}^{-1} \text { data }\right), \\
\Gamma_{H}<3.2 \times \Gamma_{H}^{S M} & \left(300 \mathrm{fb}^{-1} \text { data }\right) .
\end{array}
$$

## Backgrounds determine the ultimate reach

The ultimate reach of this method to constrain the width is determined by how well the number of ZZ events at high invariant mass can be predicted in the Standard Model. This requires NNLO computations for qq->ZZ, two-loop NLO QCD computations for gg -> ZZ and the signal-background interference. Electroweak corrections may be also sizable, at high invariant mass.

Two-loop computations are not easy. However, recently there appeared to be a breakthrough with two groups completing the necessary scalar integrals. These results where already used to construct the NNLO QCD predictions for ZZ production cross-section (Grazzini et al.). It was found that the corrections are at the level of 12 to 14 percent depending on the center-of-mass energy with the residual scale dependence at the level of three percent.

Further down the road are computations of NLO QCD corrections to $\mathrm{gg}->\mathrm{ZZ}$ and to the interference. When everything is completed, the quality of the Standard Model prediction for the off-shell ZZ production will be extremely high. A residual theoretical uncertainty for $\mathrm{pp}->\mathrm{ZZ}$ at the level of just a few percent can probably be reached within a year or two.


## Signal-background interference and the width measurement in $\gamma \gamma$ channel

## Signal-background interference in $\mathrm{H}->\gamma \gamma$ channel

Martin; Dixon, Li (2013)


Consider full pp-> $\gamma \gamma$ process; the amplitude is the sum of the "signal' and the background. The interference of the signal and background amplitudes contain two distinct terms: the term that is asymmetric across the resonant peak and the term that is proportional to the width.

The asymmetric term usually averages to zero when cross-sections are considered. The term proportional to the width is usually very small. However, the width suppressed term is multiplied by the background amplitude and if the background amplitude is large, the width suppression can be overcome.

In case of the Higgs production, the background amplitude is one-loop and the signal amplitude is two-loop which, potentially, makes the interference term significant.

## The interference contribution to the cross-section is small

How large are the interference effects for the Higgs boson production in the Standard Model?


$$
\left|A_{i \rightarrow f}\right|^{2}=\frac{|S|^{2} m_{h}^{2}}{\left(s-m_{h}^{2}\right)^{2}+\Gamma_{h}^{2} m_{h}^{2}}\left[1+\frac{2\left(s-m_{h}^{2}\right)}{m_{h}^{2}} \operatorname{Re}\left(\frac{B^{*}}{S}\right)+\frac{2 \Gamma_{h}}{m_{h}} \operatorname{Im}\left(\frac{B^{*}}{S}\right)\right]+|B|^{2}
$$

Naively, the interference is loop enhanced (S -> 2 loop, B-> 1 loop)

$$
S \sim \frac{\alpha_{s} \alpha m_{h}^{2}}{(4 \pi v)^{2}}, \quad B \sim \frac{g_{s}^{2} e^{2}}{(4 \pi)^{2}} \quad\left[\sigma_{\text {int }} / \sigma_{H}\right]_{\text {naive }} \approx \frac{2 \Gamma_{h}}{m_{h}} \frac{(4 \pi v)^{2}}{m_{h}^{2}} \approx 0.1
$$

In reality: interference starts at two-loop (no imaginary part for $\pm \pm$ background amplitude at one-loop). As a consequence, small effect ( $\sim$ few percent) in the SM.

## Features of the interference

The remaining contribution to the cross-section is provided by the interference term proportional to the real part of the background amplitude. This contribution seems to be irrelevant for the production cross-section of a narrow resonance (asymmetric across the peak). However, it may lead to other interesting effects.

Martin (2012)

$$
\begin{aligned}
& \left|A_{i \rightarrow f}\right|^{2}=\frac{|S|^{2} m_{h}^{2}}{\left(s-m_{h}^{2}\right)^{2}+\Gamma_{h}^{2} m_{h}^{2}}\left[1+\frac{2\left(s-m_{h}^{2}\right)}{m_{h}^{2}} \operatorname{Re}\left(\frac{B^{*}}{S}\right)+\frac{2 \Gamma_{h}}{m_{h}} \operatorname{Im}\left(\frac{B^{*}}{S}\right)\right]+|B|^{2} \\
& \sigma_{o f f} \sim \frac{2 \operatorname{Re}\left(S B^{*}\right)}{s-m_{h}^{2}}, \quad\left(s-m_{h}^{2}\right) \gg m_{h} \Gamma_{h}
\end{aligned}
$$

## The real part of the interference leads to the mass-shift

Suppose we define the Higgs boson mass as the first normalized moment of the invariant mass distribution. In this case, the extraction of the mass is affected by real part of interference. The mass shift is independent of the width but it dependents on the environmental parameters (energy resolution). One can expect $\mathrm{O}(100 \mathrm{MeV})$ shift that is similar to the ultimate precision of the Higgs mass measurement at the LHC.


$$
\begin{gathered}
\frac{d \sigma}{d s}=\frac{A}{\left(s-m_{h}^{2}\right)^{2}+m_{h}^{2} \Gamma_{h}^{2}}+\frac{\left(s-m_{h}^{2}\right) I}{\left(s-m_{h}^{2}\right)^{2}+m_{h}^{2} \Gamma_{h}^{2}} \\
\left\langle M^{2}\right\rangle=\frac{1}{\sigma_{0}} \int d s s \frac{d \sigma}{d s}=m_{h}^{2}+\frac{I}{\sigma_{0}} \int_{s_{-}}^{s^{+}} d s \\
\delta m_{h}=\frac{2 I \delta}{\sigma_{0}}, \quad s_{ \pm}=\left(m_{h} \pm \delta\right)^{2}
\end{gathered}
$$

Martin (2012)

## Establishing the existence mass shift

To establish the existence of the mass shift, one compares mass measurements in $\mathrm{H}->\gamma \gamma$ (affected by the interference) with the "control mass", e.g. as obtained from H->ZZ (no loop enhancement, tiny mass shift). Alternatively, one may study the Higgs bosons "mass" in H-> $\gamma \gamma$ in dependence of the Higgs boson transverse momentum.

With $300 / \mathrm{fb}$ collected luminosity one becomes sensitive to $\mathrm{O}(100) \mathrm{MeV}$ mass shifts caused by the interference.
[Martin (20I2); Dixon and Li; de Florian et al (2013)]


## $\Delta \mathrm{M}_{\mathrm{ZZ}, \gamma \gamma}$ : model-independent determination of $\Gamma_{\mathrm{H}}$

[Dixon and Li (2013)]
It is possible to use the measured value of the mass shift to constrain the Higgs boson width. Indeed $\sigma_{h} \sim g_{i}{ }^{2} g_{f}{ }^{2} / \Gamma_{h}, \sigma_{i} \sim g_{i} g_{f}$. Therefore, assuming $\sigma_{h}=\sigma_{h}{ }^{\mathrm{SM}}(\mathrm{LHC})$, we conclude that the interference should scale as the square root of the width. The mass difference is proportional to the interference and, therefore, also scales as the square root of the width.

$$
\delta m_{h}=\frac{2 \sigma_{i} \delta}{\sigma_{h}} \sim 100 \mathrm{MeV} \times \sqrt{\frac{\Gamma_{h}}{\Gamma_{h, s m}}}
$$



Existing measurements of the Higgs boson mass differences by ATLAS and CMS are not very consistent. Constraints on the Higgs width from these measurements are not conclusive at the moment.

$$
\begin{aligned}
\Delta m_{H H} & =1.47 \pm 0.67 \pm 0.18 \mathrm{GeV},[\text { ATLAS }] \\
\Delta m_{H H} & =-0.9 \pm 0.4 \pm 0.2 \mathrm{GeV},[\mathrm{CMS}]
\end{aligned}
$$

Currently, the off-shell ZZ production offers significantly higher sensitivity to the Higgs width than the interferometry in $\gamma \gamma$ channel. However, the advantage of the latter is that no large off-shell extrapolation needs to be performed. For this reason, measurements in ZZ and $\gamma \gamma$ are complimentary and both need to be pursued.

## Conclusions

Interesting effects in Higgs physics come from subtle phenomena, such as off-shell production and interference of scattering amplitudes.

In the four-lepton channel, large effects are caused by the decay of an ''off-shell Higgs" to longitudinal Z bosons at large invariant masses. This leads to a plateau of Higgs-induced events. Measuring the number of events at the high-invariant mass region probes Higgs couplings to gluons and Z bosons and is independent of the Higgs width. The measured value of the Higgs on-shell production cross-section is then used to infer the value of the Higgs width.

Already with the current data, one can argue that the Higgs width can not exceed $\mathrm{O}(15-20)$ times the SM value and significant improvements in this result are very likely. In fact, the very recent CMS and ATLAS measurements suggest $\quad \Gamma_{H}<\mathcal{O}(7) \Gamma_{H}^{\mathrm{SM}}$

Further advances in constraining the Higgs width using this method will require very precise theoretical predictions for ZZ production in proton collisions ( the recent progress with multi-loop computations makes this well within reach), detailed studies of on-shell couplings to Z's and gluons as well as careful exploration of four-lepton invariant mass shape, to constrain possible effects of higher dimensional operators and light degrees of freedom.

It is important to look for the off-shell production in $\mathrm{H}+\mathrm{jet}$ and $\mathrm{H}+2$ jet channels, study weak boson fusion and explore width constraints that follow from di-photon final states.

Campbell, Ellis, Furlan, Rontsch; Buschman, Goncalves, Kuttimalai, Schonherr, Krauss, Plehn Altogether, this is a rich research program that requires strong collaboration between theory and experiment and will, hopefully, lead to interesting insights into Higgs physics during the LHC Run II.

