Is naturalness pertinent?

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Higgs Hunting, Orsay 31 July 2015

m^2?

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The Higgs mass term m has a unique status in the Standard Model:

dimensionless parameters: g, g' gauge couplings
λ quartic Higgs coupling
λ_i Yukawa couplings
δ CP-violating phase

dimensionful parameter: m^2

Hence the Higgs mass term sets the scale of the Standard Model.

With respect to which scale?

scale

- Λ_{QCD} : cf. grand unification

- M_P = Planck mass

With respect to which scale?

scale

- Λ_{QCD} : cf. grand unification
- Λ new physics
- M_P = Planck mass

What is naturalness?

Scalar masses are directly sensitive to new physics thresholds because of quantum corrections.

$$\delta m^2 = \lambda \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \sim \frac{\lambda}{16\pi^2} \int^\Lambda dk^2,$$

Hence

$$m^2 = m_0^2 + \alpha\,\lambda\,\frac{\Lambda^2}{16\pi^2}$$

Higgs mass in the fundamental theory

corrections

Hence $\quad m_0^2/\Lambda^2 = -\alpha\lambda/(16\pi^2) + m^2/\Lambda^2$

Take $m = 100$ GeV, $\Lambda = M_P = 10^{19}$ GeV, $\quad 10^{-3} \quad 10^{-34}$

Hence the value m_0 must be in the underlying theory fixed at a precision of 10^{-31}!

Note that Λ is not a regularisation cut-off !
It is a physical scale corresponding to the next physical mass threshold

In practise, the role of the cut-off is played by the mass of a new physics state coupled to the Higgs field h

e.g. new scalar field ϕ of mass M coupled to the Higgs field

$$\delta V = M^2 |\phi|^2 + \xi |\phi|^2 |h|^2$$

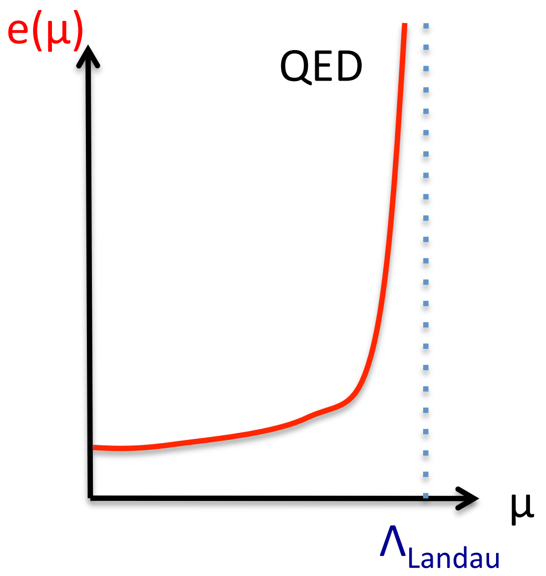
$$\delta m_h^2 \simeq \frac{\xi}{16\pi^2} M^2 \ln M^2/\Lambda^2$$

Is it different with a spin ½ field ? Yes

$$\Delta m^2 \simeq \alpha\, m^2 \log \Lambda^2/m^2$$

Δm^2 vanishes for vanishing m^2 because of a symmetry:

chiral symmetry $\qquad \psi \to \exp(i\alpha\gamma_5)\,\psi$

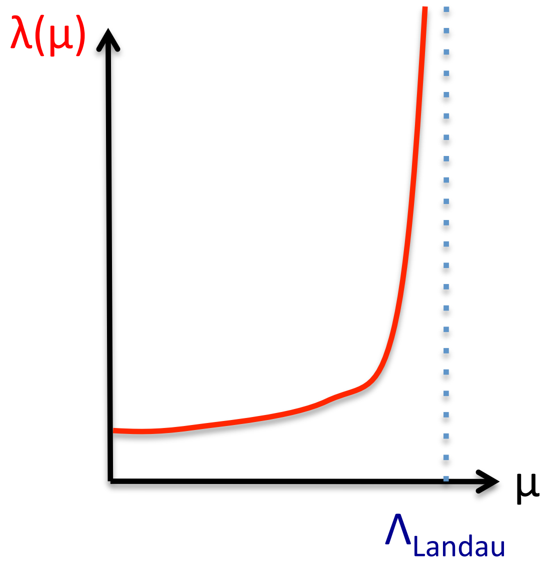


The electron mass is said to be *technically natural*: if it is small with respect to some scale Λ, its radiative corrections are small.

‘t Hooft (1979)

A theory is natural if, for all its parameters *p* which are small with respect to the fundamental scale Λ, the limit $p \to 0$ corresponds to an enhancement of the symmetry of the system.

Example of a scalar field theory :



$$V(\phi) = m^2\phi^2 + \lambda\phi^4$$

• $\lambda = 0$ corresponds to an enhancement of the symmetry (conservation of the number of ϕ particles) $\Rightarrow$ λ is a naturally small parameter.

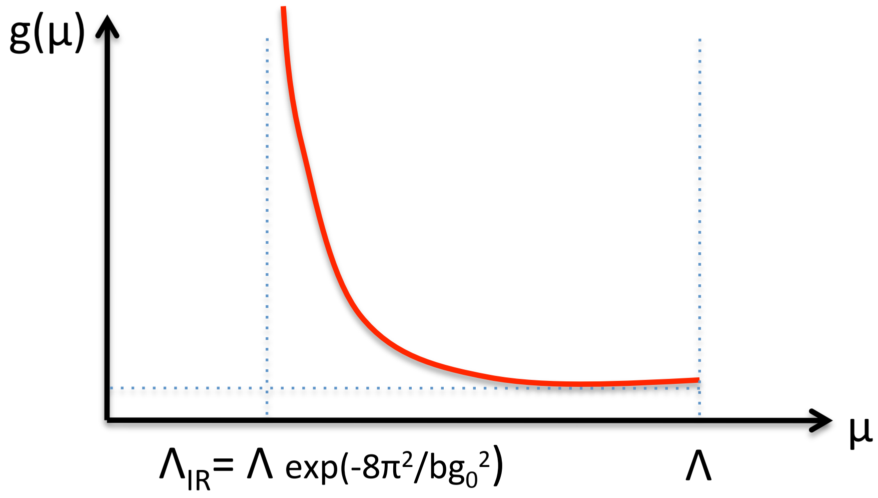
• $m^2 = 0$ corresponds to an enhancement of the classical symmetry (conformal invariance) but it is broken by quantum corrections
$\Rightarrow$ m^2 is a not a naturally small parameter in the quantum theory

• $m^2 = \lambda = 0$ corresponds to an enhancement of the symmetry: $\phi(x) \to \phi(x) + C$
Assume that this is a symmetry of an underlying theory at scale Λ, and that this symmetry is broken by effects described by a small parameter ε:

$$\lambda \sim \varepsilon \qquad m^2 \sim \varepsilon\, \Lambda^2 \quad \Rightarrow \quad \Lambda \sim m/\surd\lambda$$

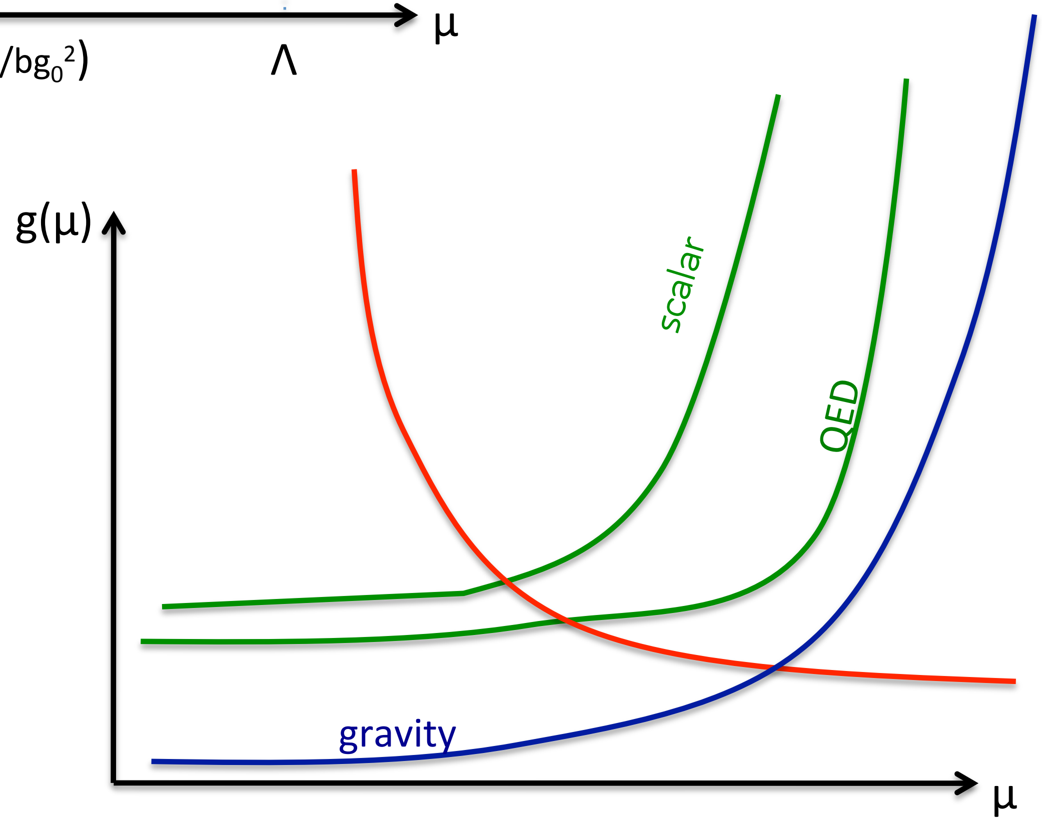
Since $m/\surd\lambda \sim v$, the Standard Model is natural only up to a scale of order v

An example of a natural theory: asymptotically free gauge theory (e.g. QCD)



$m^2 \sim \Lambda_{IR}^2 \ll \Lambda^2$ natural

<u>Note: connection with triviality</u>



asymptotically free

trivial (abelian gauge, scalar)

nonrenormalizable ($G_N\mu^2$)

The Standard Model

$$V(h) = V_0 + m^2 h^2 + \lambda h^4$$

Two serious fine tuning problems:

$V_0/M_P{}^4 < 10^{120}$

$m^2/M_P{}^2 \sim 10^{-34}$

The Standard Model

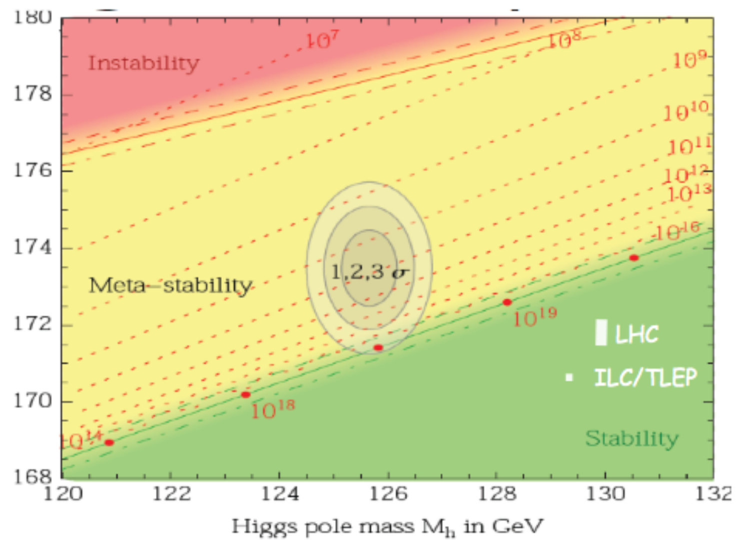
$$V(h) = V_0 + m^2 h^2 + \lambda h^4$$

Two serious fine tuning problems:

And one potential stability problem:

$V_0/M_P^4 < 10^{120}$

$m^2/M_P^2 \sim 10^{-34}$



Solution 1: symmetry

Supersymmetry: symmetry between between and fermions which cancels the quadratic divergences.

Quadratically divergent term prop. to $\sum m^2_{boson} - \sum m^2_{fermion}$

Note: global supersymmetry cancels also the constant term (vacuum energy)

But supersymmetry is not observed in the spectrum: it is (spontaneously) broken

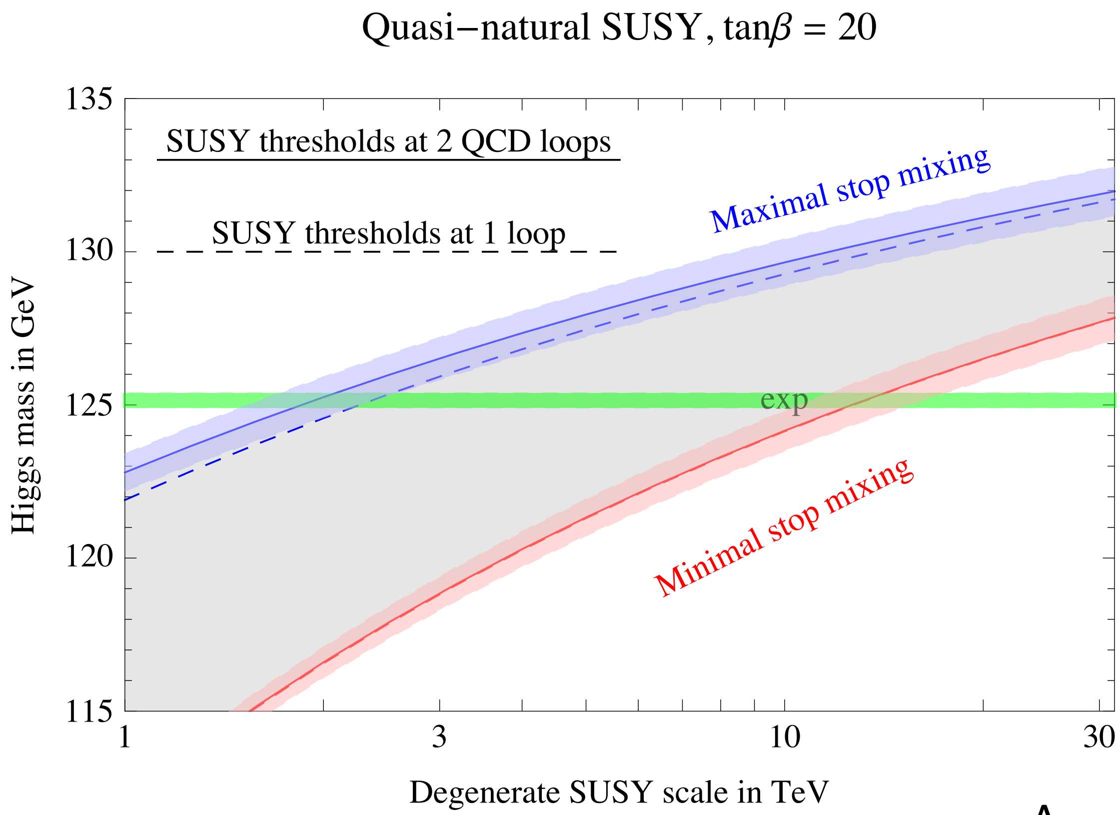
plays the role of the cut-off of new physics

$\Lambda_{\cancel{SUSY}}$

$$\Delta m_h^2 = \frac{3\lambda_t^2}{8\pi^2} \tilde{m}_t^2 \ln \tilde{m}_t^2/\Lambda$$

Fine tuning f is typically given by $\boxed{\delta m_h^2/m_h^2 = 1/f}$

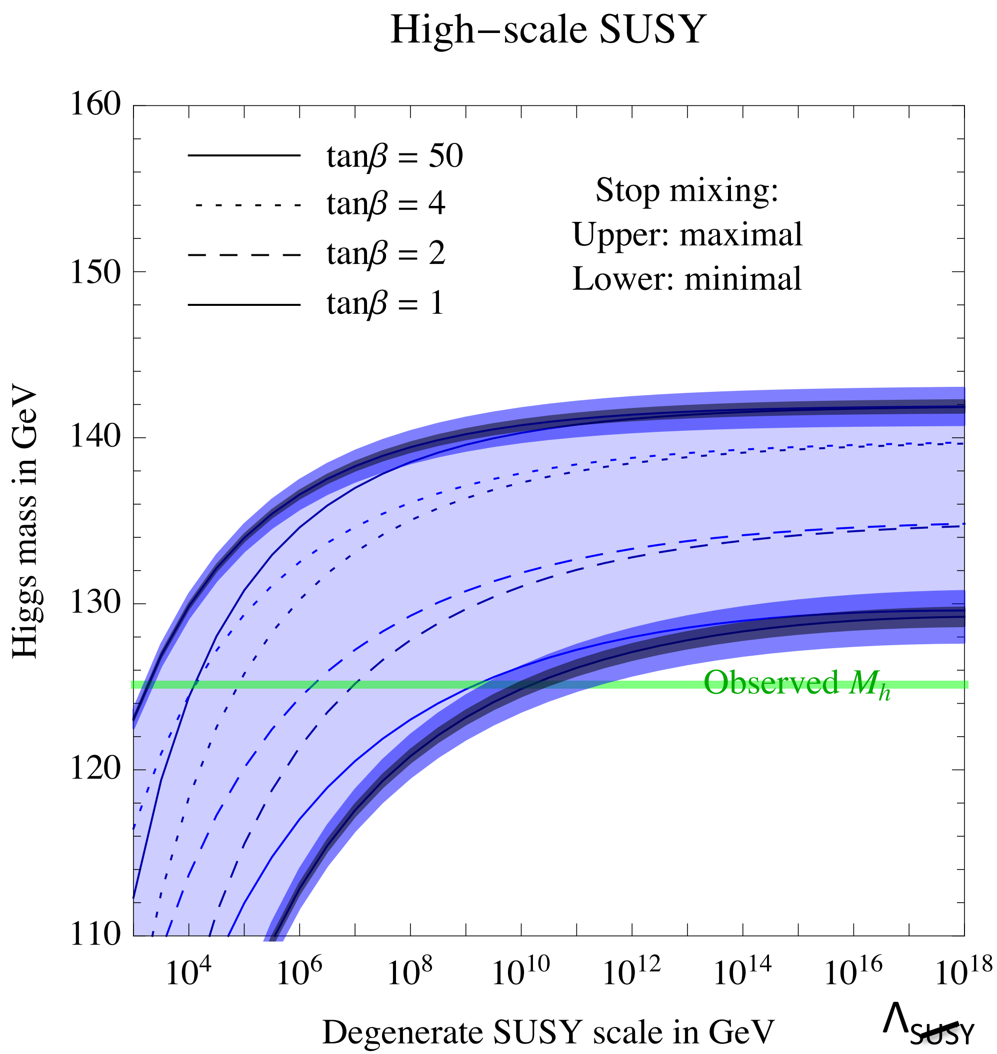
Here $f \sim m_h^2/\tilde{m}_t^2$



$\Lambda_{\cancel{SUSY}}$

requires resumming large $\log(\Lambda_{\cancel{SUSY}} / M_Z)$

Bagnaschi, Giudice, Slavich, Strumia 1407.4081



Bagnaschi, Giudice, Slavich, Strumia 1407.4081

The approach is general:

- Introduce a symmetry to explain naturalness
- This symmetry is not observed at low energy: it is broken at some scale Λ.
- Fine tuning between the parameters of the underlying theory increases with Λ:

$$f \sim m_h^2/\Lambda^2$$

If no new physics shows up, fine tuning increases!

Other possibility: composite Higgs

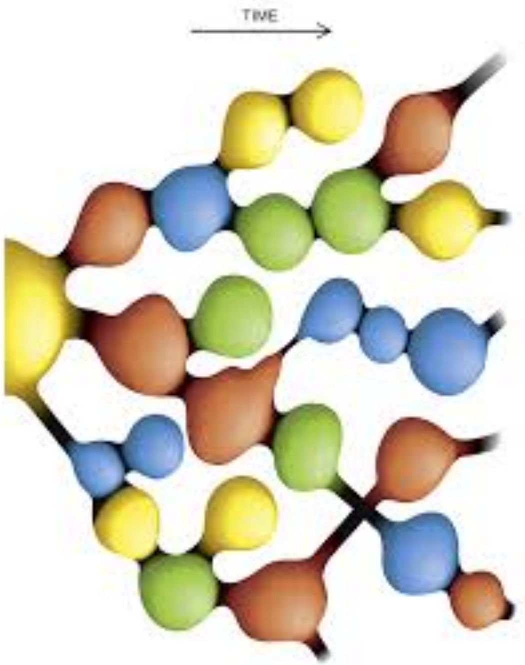
New physics: strong interaction, Higgs = (pseudo-)Goldstone boson

There exists in Nature some fine tuning situations:

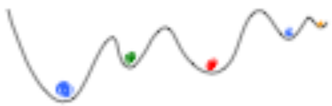
- Proton-neutron mass difference: u, d quark masses

- Apparition of life on Earth: dynamics of planets in the early solar system

Solution 2: anthropic principle

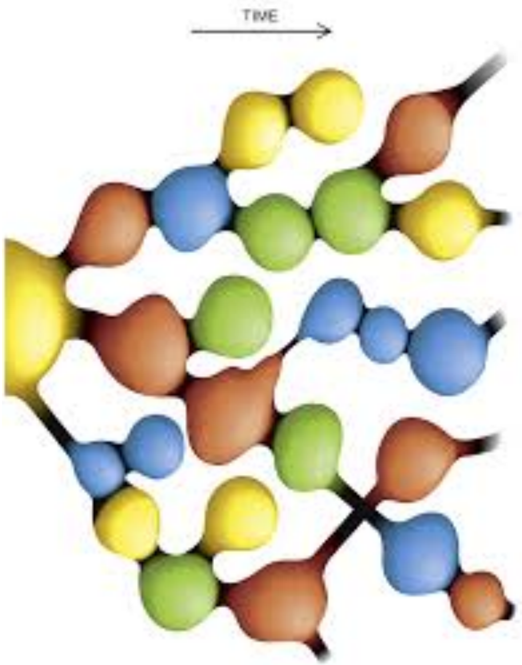


eternal inflation



string theory landscape

Solution 2: anthropic principle



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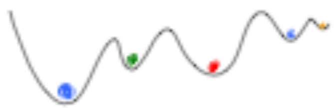
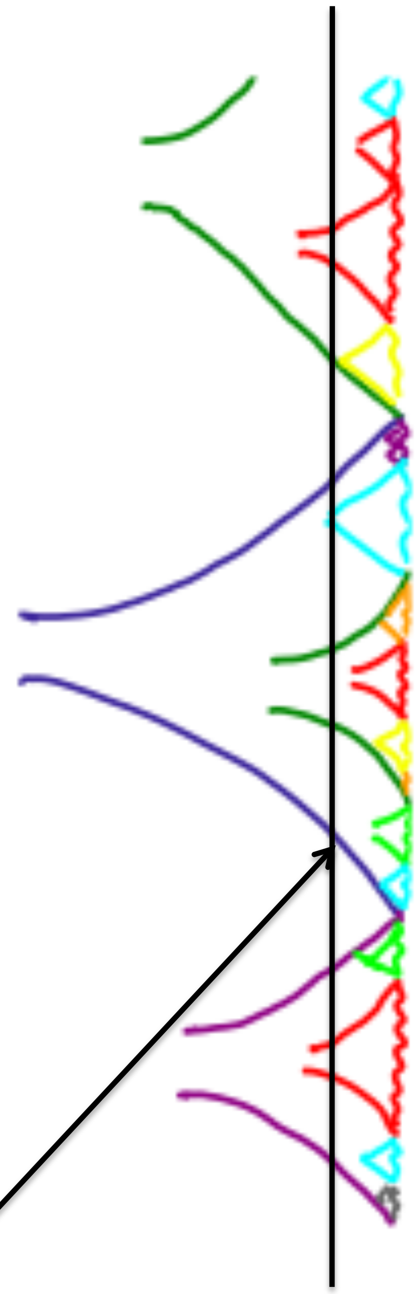


string theory landscape



difficult to define a probability measure

Solution 3: cosmological evolution

We have to understand two severe fine tuning problems:

$M_P^4/\Lambda \sim 10^{120}$

$M_P/m \sim 10^{17}$ m Higgs mass

<u>Dirac large number hypothesis (1937)</u>

Large mass scale ratios are a consequence of the age of the Universe, i.e. are the result of the cosmological evolution.

Dirac concluded that $G \propto 1/t$ which is not observed.

But could we apply similar arguments to fine tuning ratios?

Abbott 1985, Dvali and Vilenkin hep-th/0404043, Dvali hep-th/0410286

The rel(axion)

Graham, Kaplan, Rajendran 1504.0755

Simplest model: SM + QCD axion ϕ

$\Lambda^4 \sim f_\pi \, m_\pi^2 \propto (\lambda_u + \lambda_d) \langle h \rangle$

large mass scale M

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + \underbrace{\boxed{gM^2\phi + g^2\phi^2 + \cdots}}_{V(g\phi)} + \Lambda^4 \cos\frac{\phi}{f}$$

The rel(axion)

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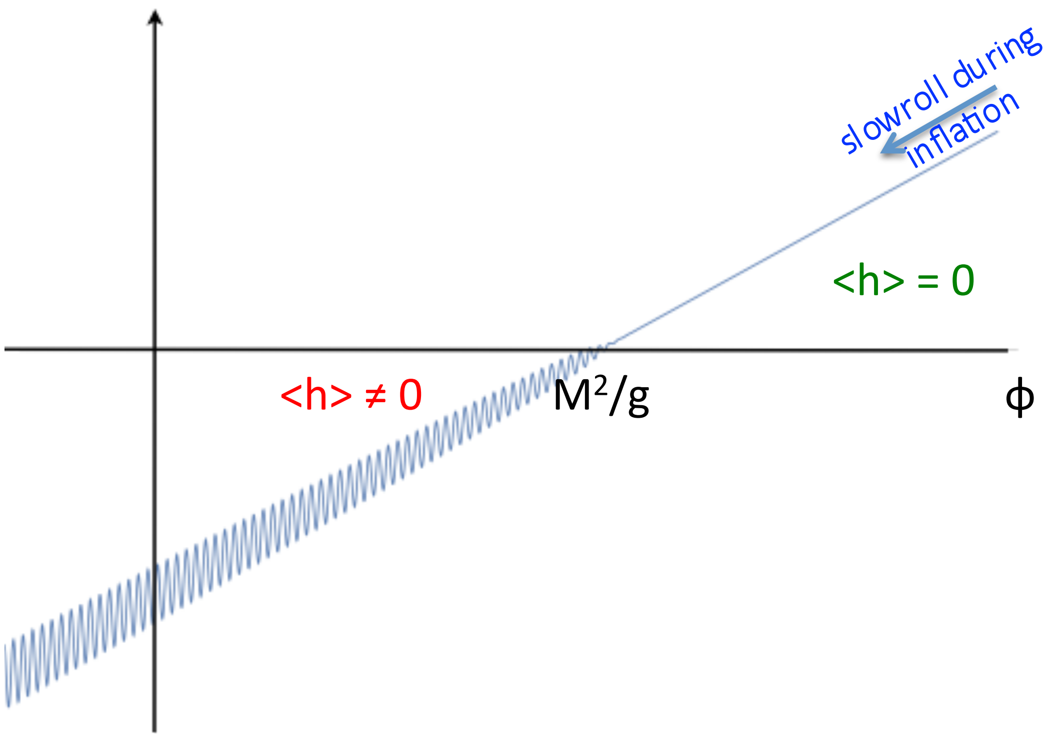
Simplest model: SM + QCD axion ϕ

large mass scale M

$\Lambda^4 \sim f_\pi m_\pi^2 \propto (\lambda_u + \lambda_d)$ <h>

V(gϕ)

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + \boxed{gM^2\phi + g^2\phi^2 + \cdots} + \Lambda^4 \cos\frac{\phi}{f}$$



In the limit g → 0,
shift symmetry: $\phi \to \phi + 2\pi f$
(from continuous symmetry
(in the absence of strong interactions)
$\phi \to \phi$ + cst
g naturally small

Bound on cutoff...

Incompatible with limit on neutron dipole moment

- QCD axion $M < 3 \times 10^8\ \mathrm{GeV}$

- Variations: i) drop the slope

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + \kappa\sigma^2\phi + gM^2\phi + \cdots + \Lambda^4 \cos\frac{\phi}{f}$$

inflaton - drops at end of inflation

$$\boxed{M < 1000\ \mathrm{TeV}\left(\frac{\theta}{10^{-10}}\right)^{\frac{1}{4}}}$$

ii) Use a different strong group and couple ϕ to $G'^{\mu\nu}\tilde{G}'_{\mu\nu}$.

Thus interesting class of modelswhich require further investigation:

- the g parameter is very small (e.g. 10^{-31} GeV): this is natural in a technical senses but why?

- the model requires a long inflation period; connection ϕ field $\leftrightarrow$ inflaton
  very long inflation period needed!

- reheating

- can the ϕ field provide dark matter

see G. Servant talk and Espinosa, Grojean, Panico, Pomarol, Pujolas, Servant 1506.09217

Conclusions

Naturalness is a very useful tool for theorists to search for theories beyond the Standard Model, given the non-naturalness of the Standard Model.

Non-naturalness is the price to pay for having a fundamental scalar.

Naturalness is only a search tool. 

It does not lead to any no go theorem (and never did).

The fine tuning of today may be the dynamics of tomorrow.

Because gravity is involved in some of the fine tunings that we encounter, cosmology i.e. the fact that we live in a dynamical universe may play a role in solving the mystery.

Is naturalness pertinent?

Maybe