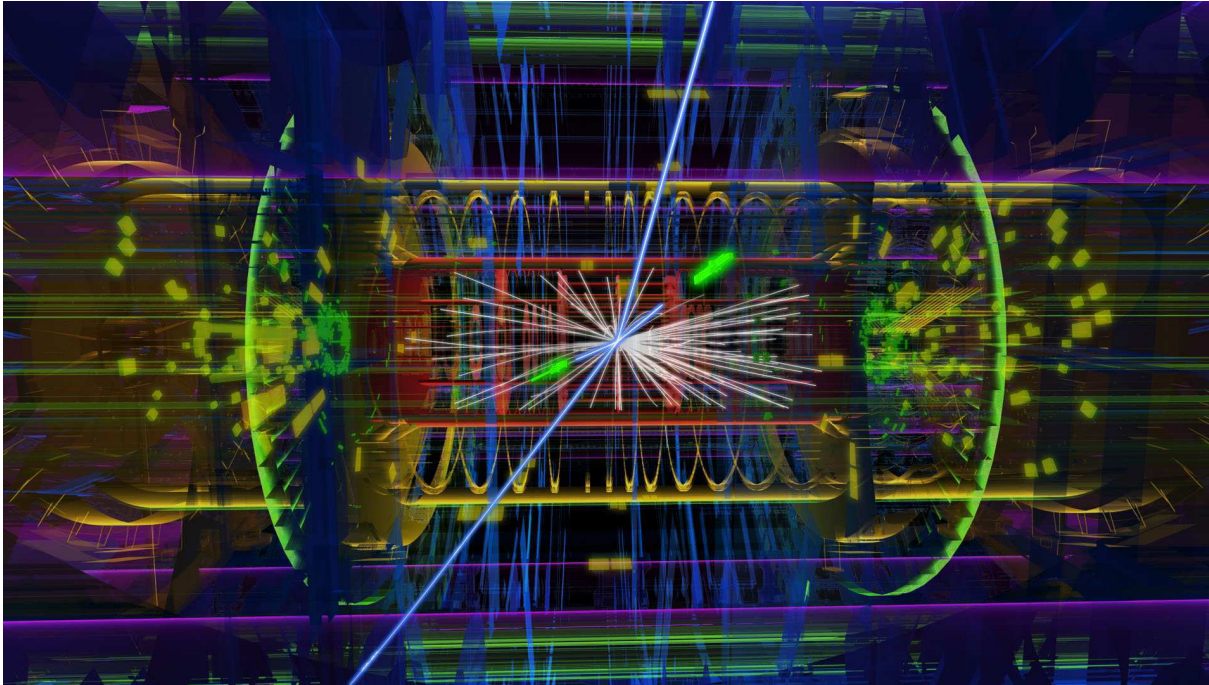


Electroweak Di-boson Production at the LHC



Stefan Dittmaier
Albert-Ludwigs-Universität Freiburg

(based on collaboration with B.Biedermann, M.Billoni, A.Denner, M.Hecht, L.Hofer, B.Jäger, C.Pasold, L.Salfelder, C.Speckner)

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WW / WZ / ZZ production

$W\gamma$ / $Z\gamma$ production

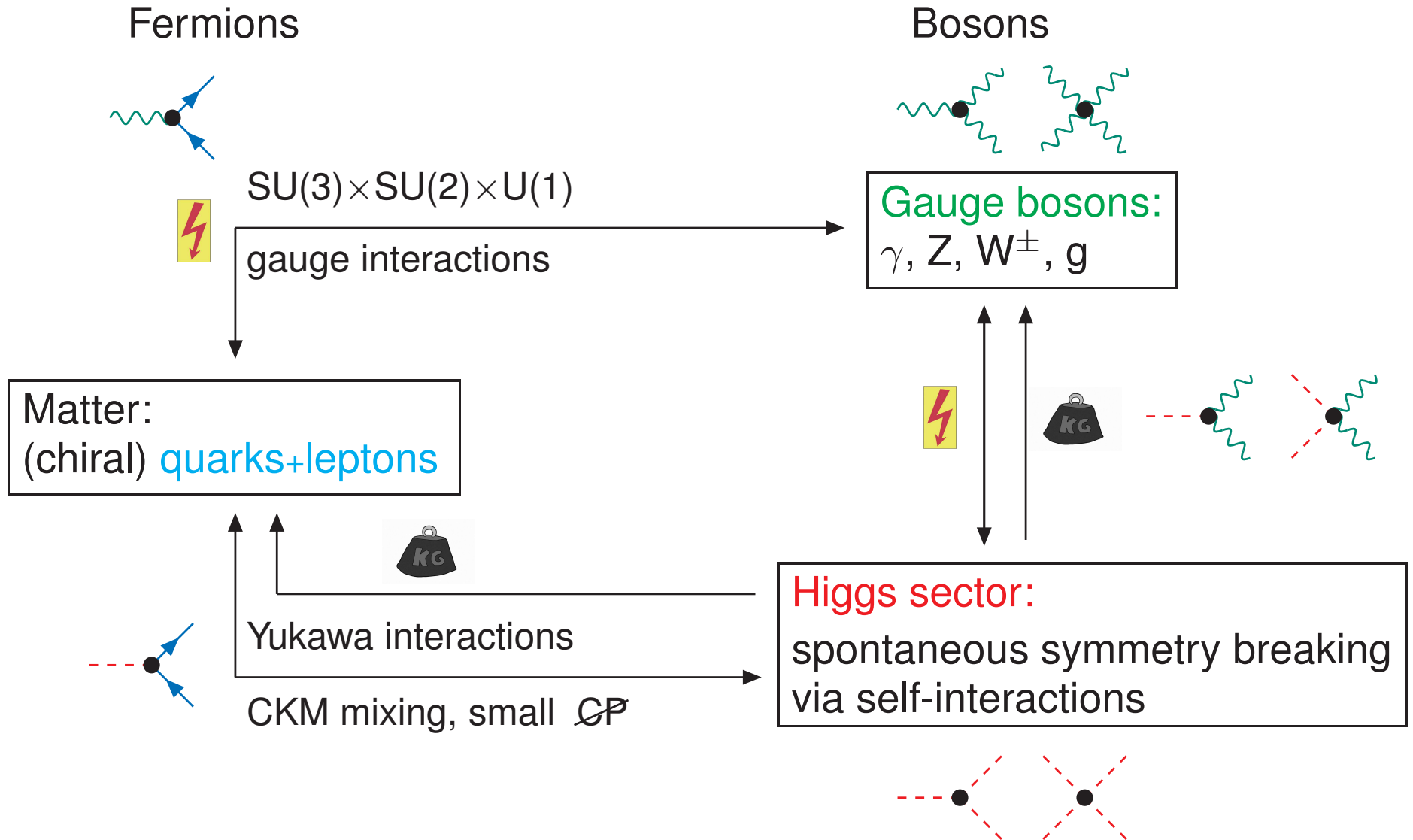
Conclusions



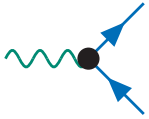
Introduction



Structure and elementary interactions of the Standard Model



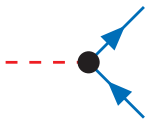
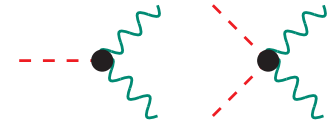
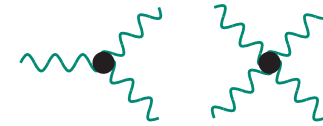
Structure and elementary interactions of the Standard Model



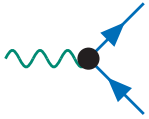
Test of the model

\Leftrightarrow Exp. reconstruction of the elementary couplings

Feynman rules



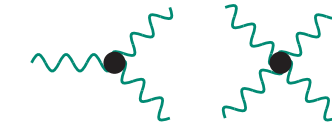
Structure and elementary interactions of the Standard Model



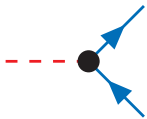
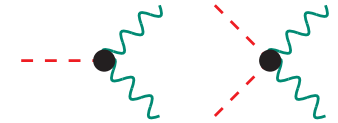
Test of the model

⇔ Exp. reconstruction of the elementary couplings

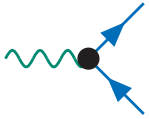
Feynman rules



Building blocks for particle reactions

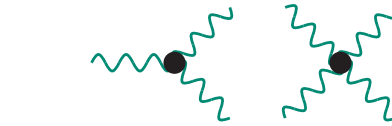


Structure and elementary interactions of the Standard Model



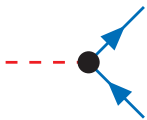
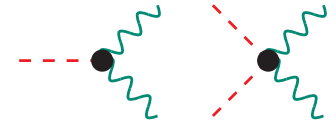
Test of the model

⇔ Exp. reconstruction of the elementary couplings



Feynman rules

Building blocks for particle reactions



Standard Model extensions → more fields, more particles, more interactions, ...

Feynman rules derived from SM Lagrangian:



↪ Recapitulate EW gauge interactions !

Gauge-boson self-interactions

↪ induced by gauge-invariant Yang–Mills Lagrangian

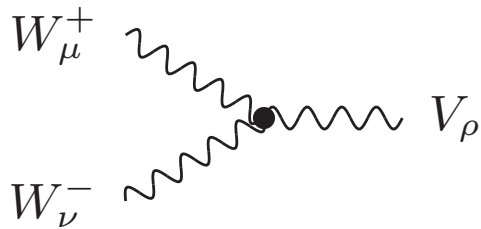
$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu},$$

with the field-strength tensors

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

⇒ Feynman rules for gauge-boson self-interactions:

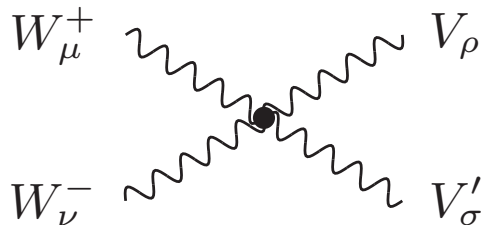
(fields and momenta incoming)



$$ieC_{WWV} \left[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu \right]$$

$$\text{with } C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_W}{s_W}$$

→ testable in di-boson production $ee/pp \rightarrow VV$

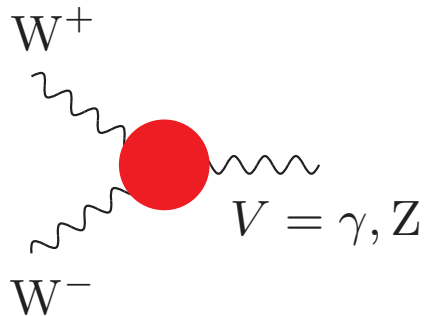


$$ie^2 C_{WWVV'} \left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho} \right]$$

$$\text{with } C_{W^2\gamma^2} = -1, \quad C_{W^2\gamma Z} = \frac{c_W}{s_W}, \quad C_{W^2Z^2} = -\frac{c_W^2}{s_W^2}, \quad C_{W^4} = \frac{1}{s_W^2}$$

→ testable in tri-boson production $ee/pp \rightarrow VVV$
and vector-boson scattering $pp(VV \rightarrow VV) \rightarrow VV + 2\text{jets}$

General parametrization (C- and P-conserving):



$$\mathcal{L}_{VWW} = -ieg_{VWW} \left\{ g_1^V (W_{\mu\nu}^+ W^{-,\mu} V^\nu - W^{-,\mu\nu} W_\mu^+ V_\nu) + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\rho\mu}^+ W_\nu^{-,\mu} V^{\nu\rho} \right\}$$

Meaning for static W^+ bosons:

$$Q_W = eg_1^\gamma = \text{electric charge } (= e \text{ by charge conservation})$$

$$\mu_W = \frac{e}{2M_W} (g_1^\gamma + \kappa_\gamma + \lambda_\gamma) = \text{magnetic dipole moment}$$

$$q_W = -\frac{e}{M_W^2} (\kappa_\gamma - \lambda_\gamma) = \text{electric quadrupole moment}$$

Standard Model values:

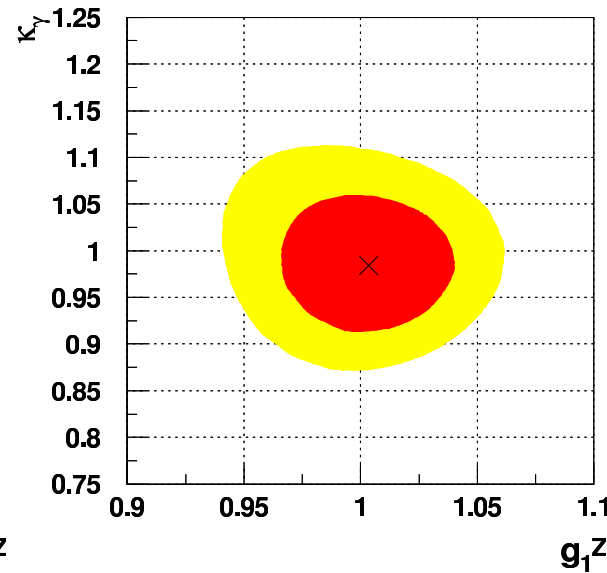
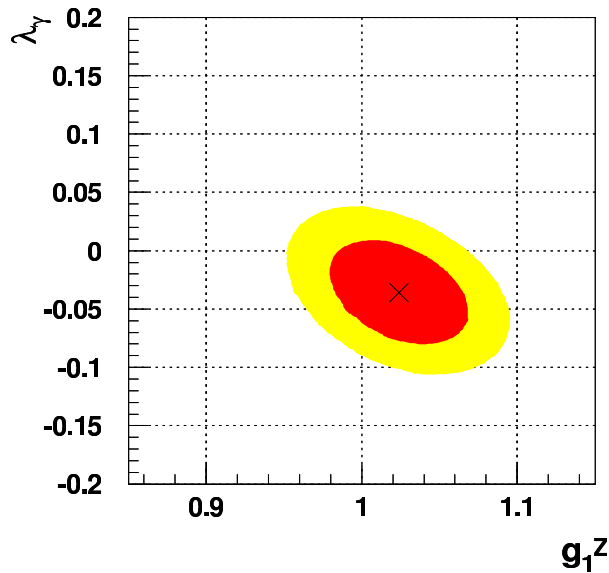
$$g_1^V = \kappa_V = 1, \quad \lambda_V = 0$$

Restriction to $SU(2) \times U(1)$ -symmetric dim-6 operators:

$$\kappa_Z = g_1^Z - (\kappa_\gamma - 1) \tan^2 \theta_W, \quad \lambda_Z = \lambda_\gamma$$

LEP2 constraints on charged TGCs from $e^+e^- \rightarrow WW \rightarrow 4f$

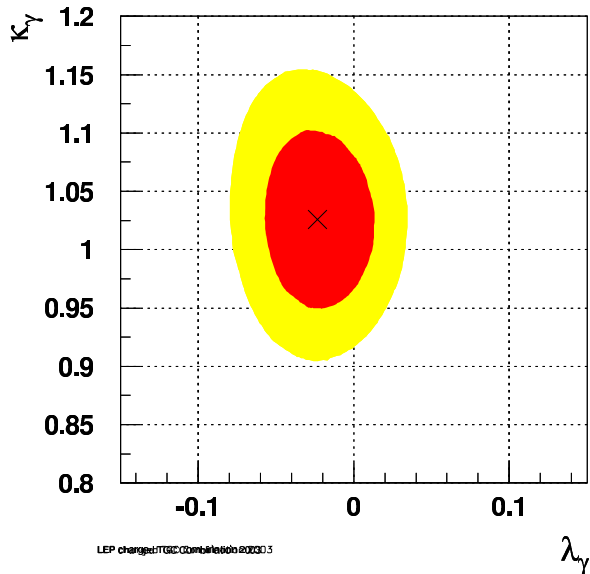
LEPEWWG '04



$$\Delta g_1^Z = -0.009^{+0.022}_{-0.021}$$

$$\Delta \kappa_\gamma = -0.016^{+0.042}_{-0.047}$$

$$\lambda_\gamma = -0.016^{+0.021}_{-0.023}$$



LEP Preliminary

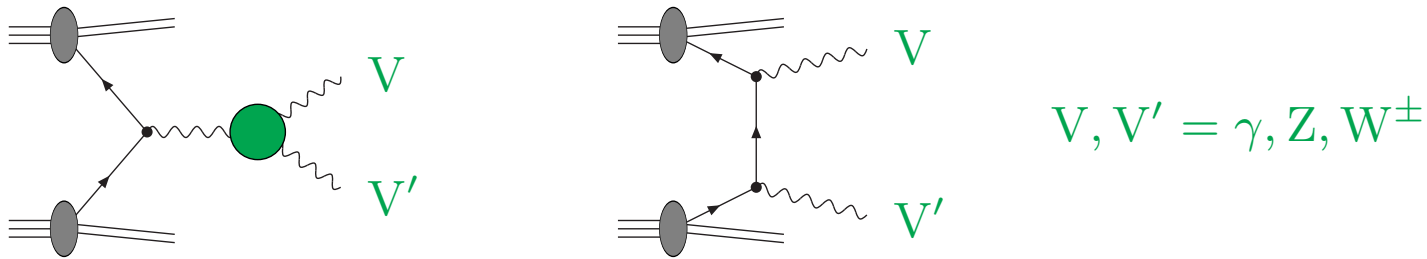
- 95% c.l.
- 68% c.l.
- × 2d fit result

Standard Model values verified
at the level of 2–4%

Similar results from Tevatron and LHC Run 1

LHC will tighten limits further !

EW di-boson production at the LHC



Physics issues:

- triple-gauge-boson couplings, especially at high momentum transfer
 - ◇ EW corrections significant
 - ◇ anomalous TGC: “formfactor approach” to switch off unitarity violation
↪ element of arbitrariness, avoid when possible
- important background processes
 - ◇ to Higgs production, $H \rightarrow WW^*/ZZ^* \rightarrow 4f$
↪ invariant masses below VV thresholds,
proper description of off-shell $V^*V^* \rightarrow 4f$ production required !
 - ◇ to searches at high invariant masses
↪ EW corrections

State-of-the-art predictions

$W\gamma/Z\gamma$ (with leptonic decays)

- NNLO QCD [Grazzini, Kallweit, Rathlev '14,'15](#)
- NLO EW [Denner, S.D., Hecht, Pasold '14,'15](#)

WW, WZ, ZZ

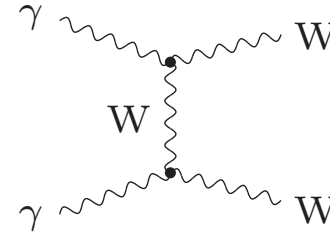
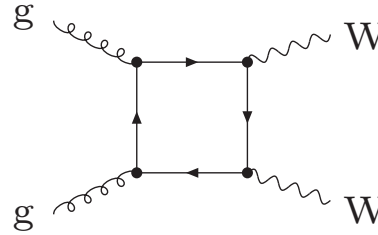
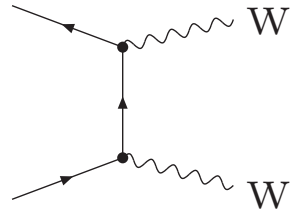
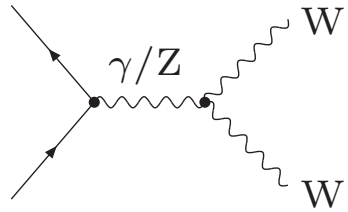
- NNLO QCD
 - ◇ ZZ (on-shell and off-shell) [Cascioli et al. '14; Grazzini, Kallweit, Rathlev '15](#)
 - ◇ WW (on-shell) [Gehrmann et al. '14](#)
 - ◇ $gg \rightarrow VV \rightarrow 4$ leptons [Binoth et al. '05,'06](#)
+ NLO corrections for on-shell V 's [Caola et al. '15](#)
- NLO EW
 - ◇ stable W/Z bosons [Bierweiler, Kasprzik, Kühn '12/'13](#)
[Baglio, Le, Weber '13](#)
 - ◇ $pp \rightarrow WW \rightarrow 4$ leptons in DPA [Billoni, S.D., Jäger, Speckner '13](#)
 - ◇ approximative inclusion in **HERWIG++** [Gieseke, Kasprzik, Kühn '14](#)
 - ◇ full off-shell calculations in progress ($pp \rightarrow \mu^+ \mu^- e^+ e^-$ completed) [Biedermann et al. '16](#)

WW / WZ / ZZ production

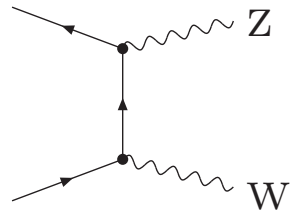
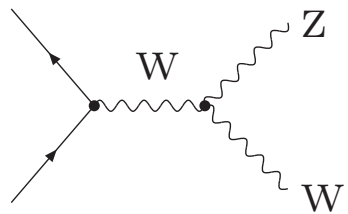


Complementarity in WW / WZ / ZZ production

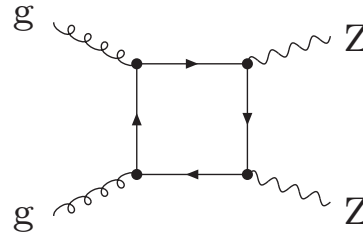
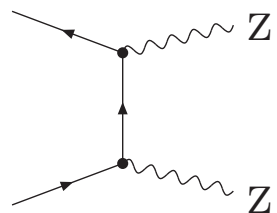
WW production:



WZ production:

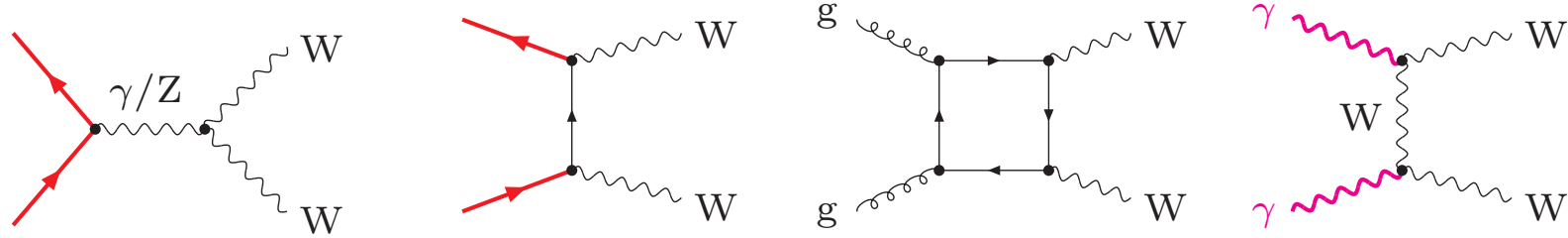


ZZ production:

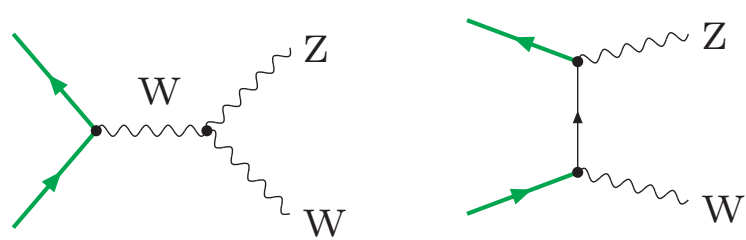


Complementarity in WW / WZ / ZZ production

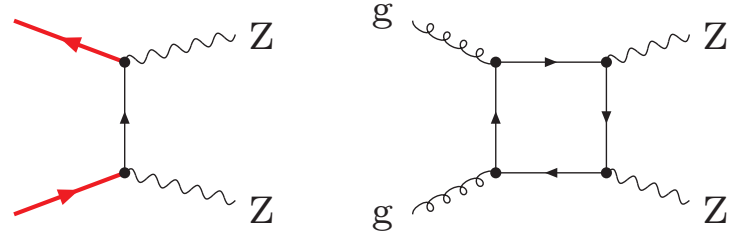
WW production:



WZ production:



ZZ production:

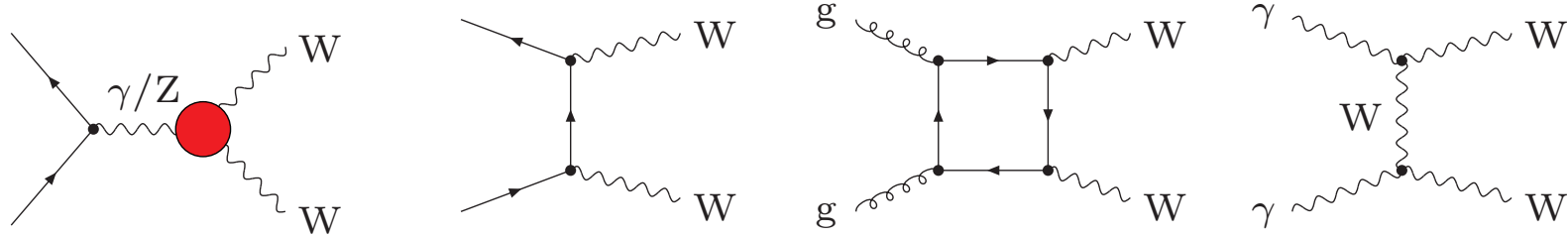


Sensitivity to different PDF combinations:

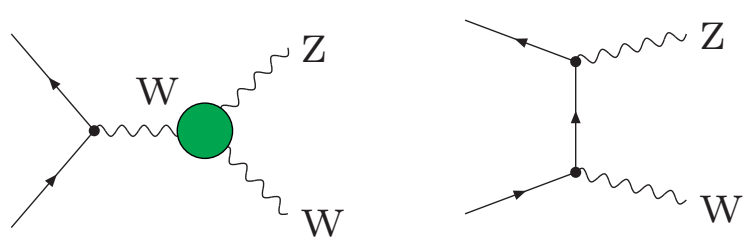
- $q\bar{q}$ in WW/ZZ
- $u\bar{d}/d\bar{u}$ in W^+Z/W^-Z
- $\gamma\gamma$ in WW

Complementarity in WW / WZ / ZZ production

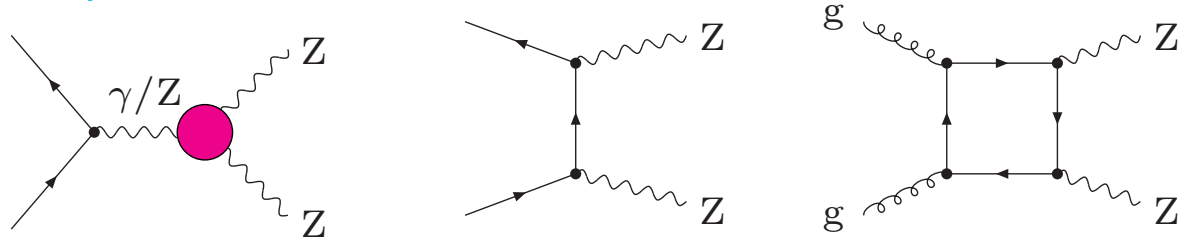
WW production:



WZ production:



ZZ production:

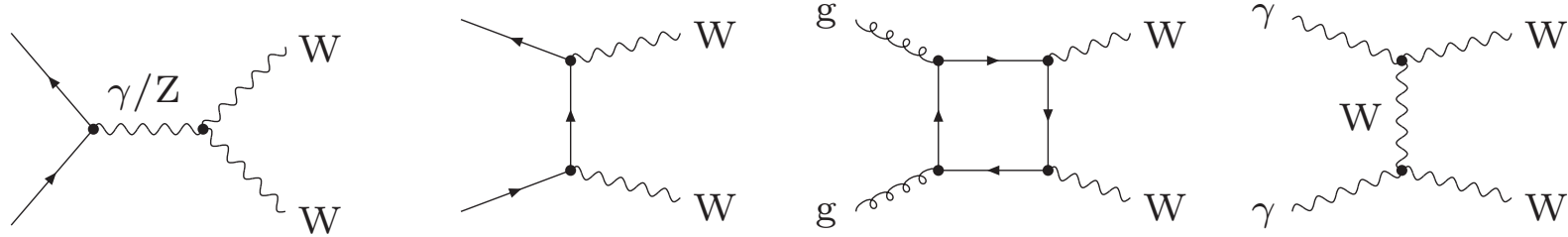


Sensitivity to different anomalous TGCs:

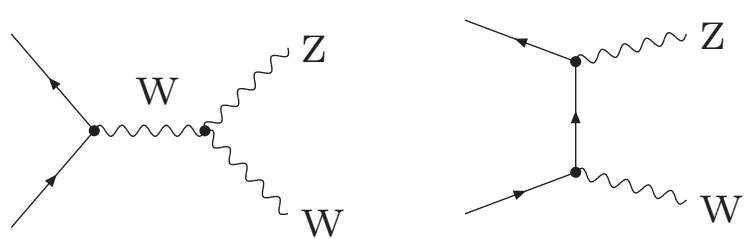
- overlay of $\gamma WW/ZWW$ in WW
- only ZWW in WZ
- $\gamma ZZ/ZZZ$ in ZZ

Complementarity in WW / WZ / ZZ production

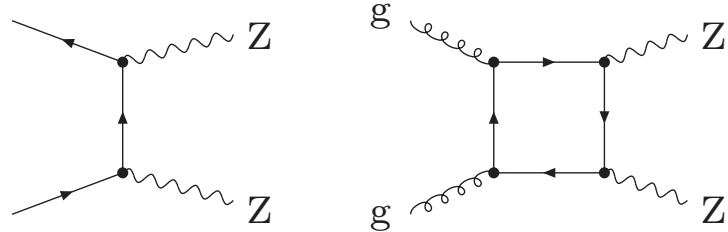
WW production:



WZ production:

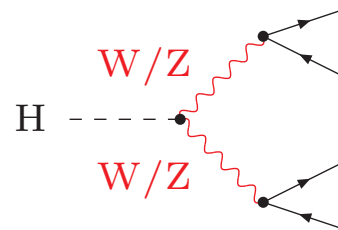


ZZ production:



Background to Higgs production
in channel $H \rightarrow WW^*/ZZ^* \rightarrow 4f$

↪ off-shell calculation
particularly important for **WW/ZZ** !

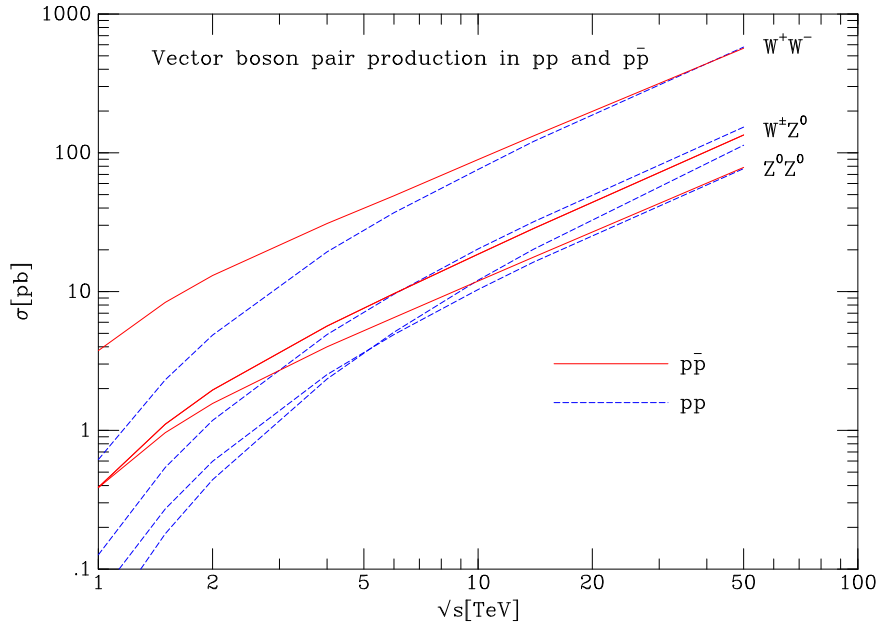


QCD corrections to WW, WZ, ZZ, W γ , Z γ production

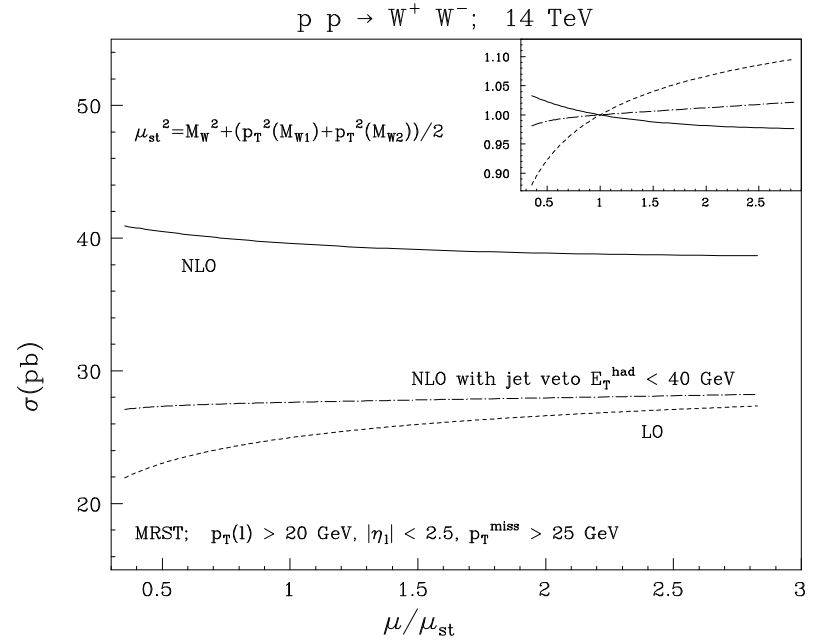
NLO QCD calculated (including leptonic W/Z decays)

Baur, Han, Ohnemus '93-'98
 Dixon, Kunszt, Signer '99
 Campbell, R.K.Ellis '99
 DeFlorian, Signer '00

Campbell, R.K.Ellis et al. '99

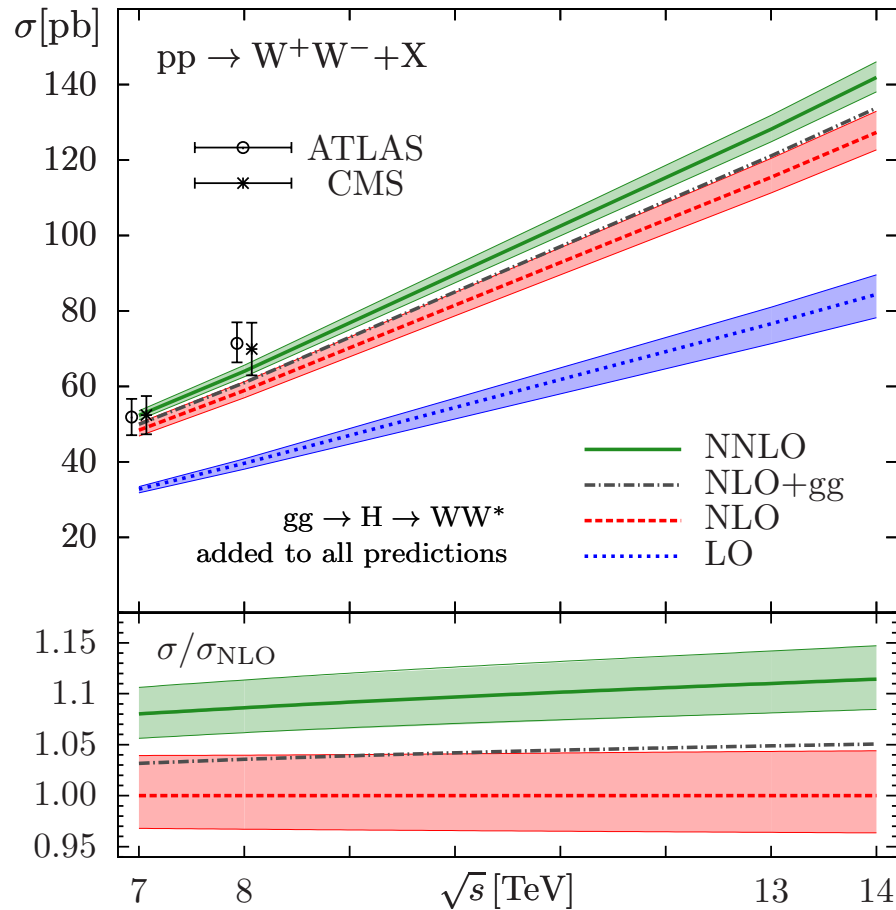


Haywood et al. '00



Large positive corrections due to jet radiation, i.e. $VV + \text{jet}$ production

- reduction of corrections and scale dependence by jet veto: $p_{T,\text{jet}} < \text{cut} ?$
 \hookrightarrow include QCD resummation for veto
- NNLO QCD corrections important

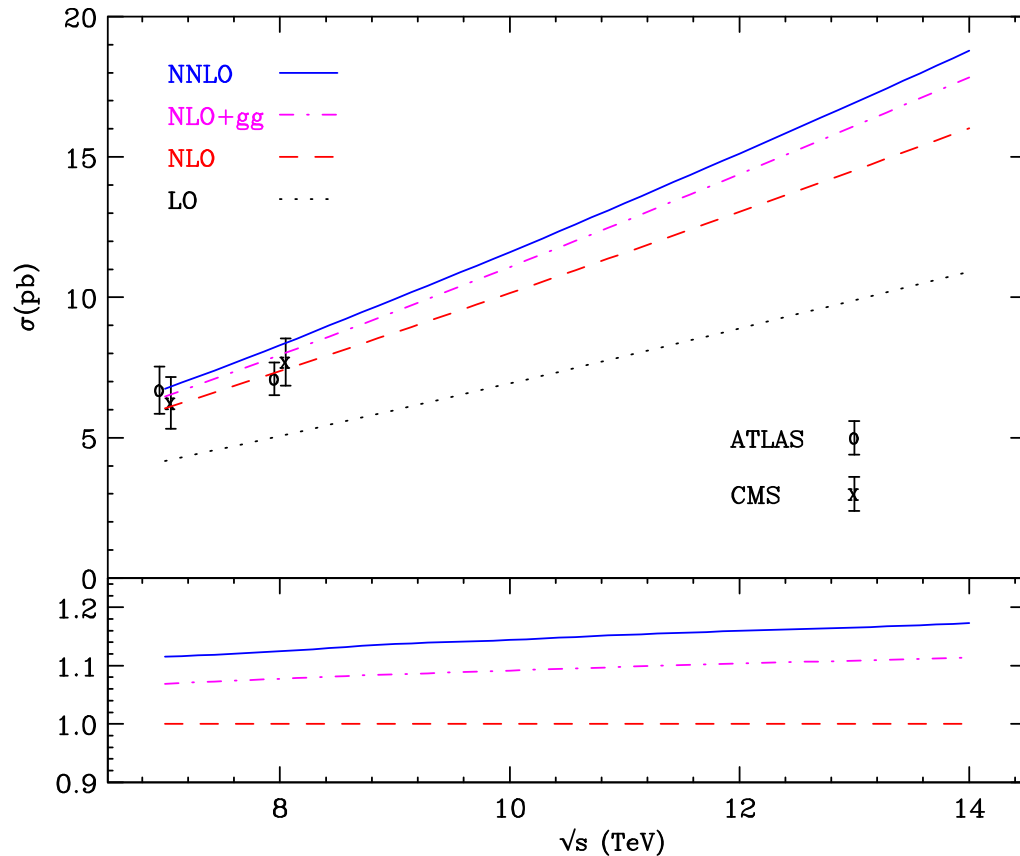


Subtlety:

Separation of single- t and $t\bar{t}$ contributions @ NNLO QCD

\hookrightarrow b-jet veto, etc.

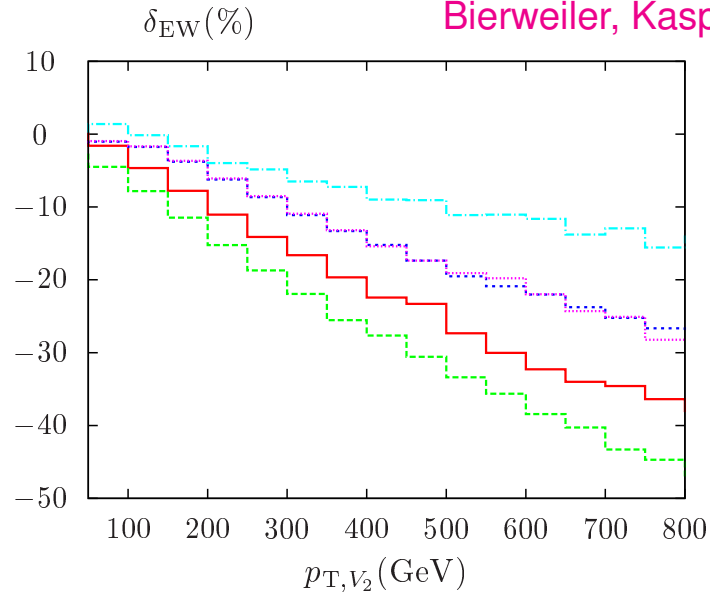
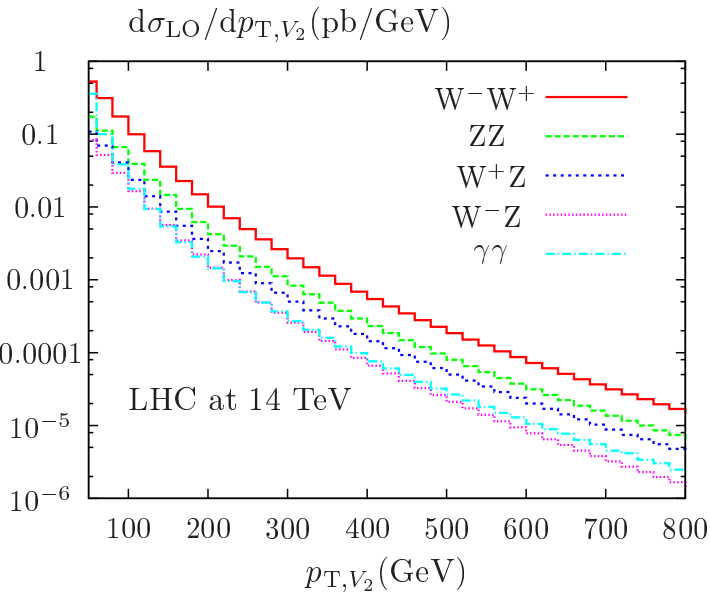
- good agreement of experimental results with NNLO QCD
- NNLO QCD correction $\sim 7(12)\%$ @ 8(13) TeV, scale uncertainty $\lesssim 3\%$
- gg contribution $\sim 7(8)\%$ @ 8(13) TeV
- LHC run 2: higher energy & higher statistics \rightarrow EW corrections important



- good agreement of experimental results with NNLO QCD
- NNLO QCD correction $\sim 12(17)\%$ @ 8(13) TeV, scale uncertainty $\lesssim 3\%$
- gg contribution $\sim 7(10)\%$ @ 8(13) TeV
- LHC run 2: higher energy & higher statistics \rightarrow EW corrections important

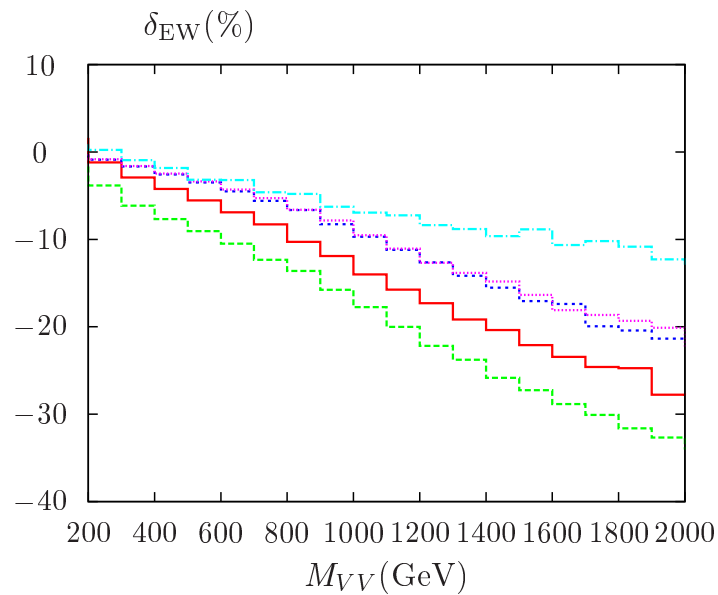
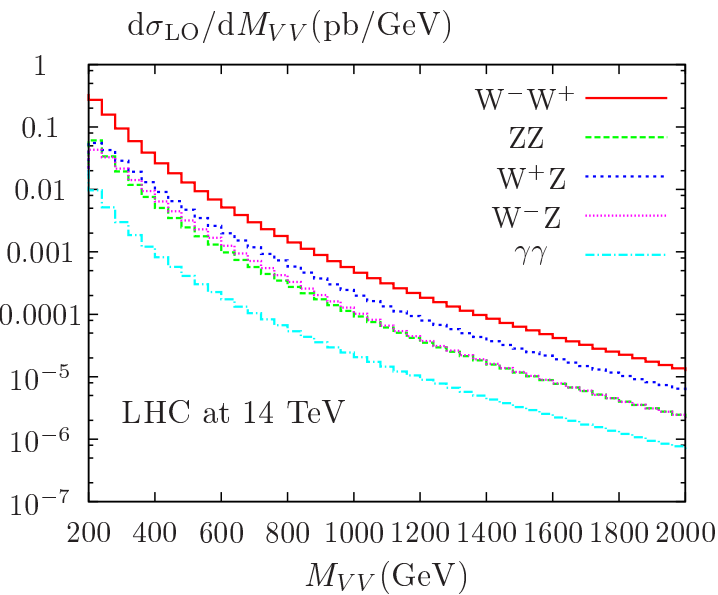
EW corrections to massive di-boson production

Bierweiler, Kasprzik, Kühn '13



EW corrections

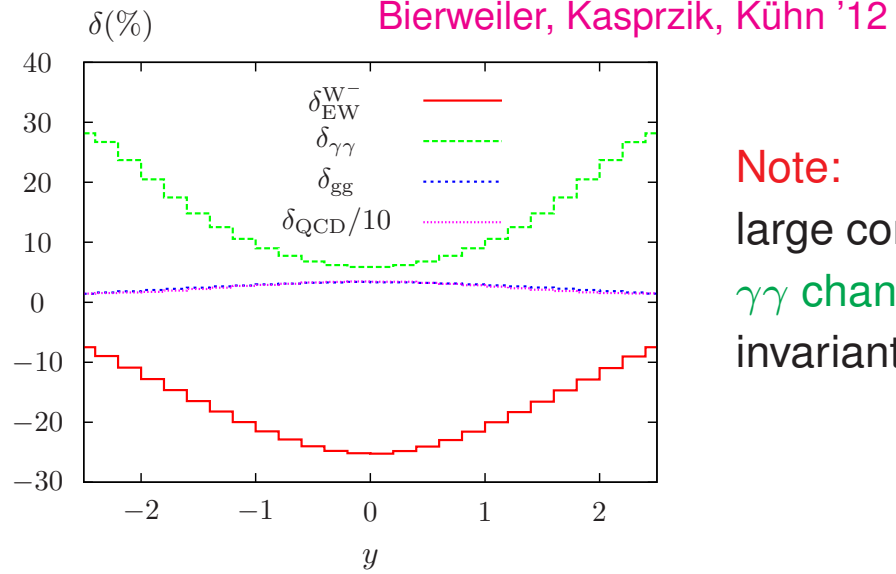
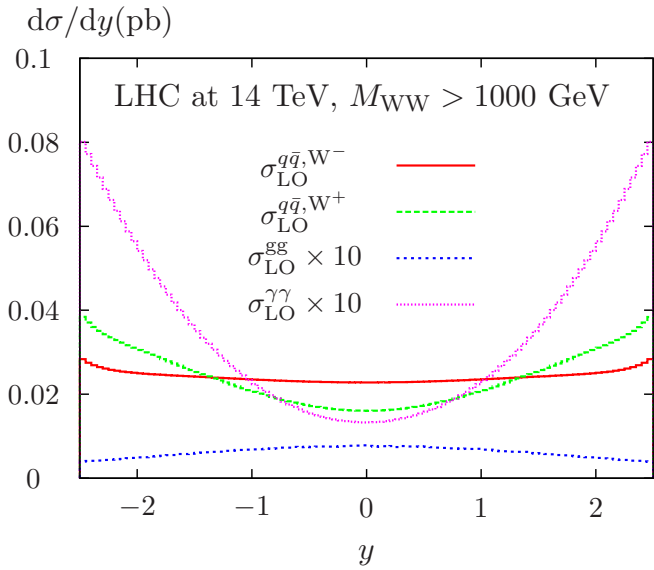
- small for integrated XS
- growing in distributions for larger scales



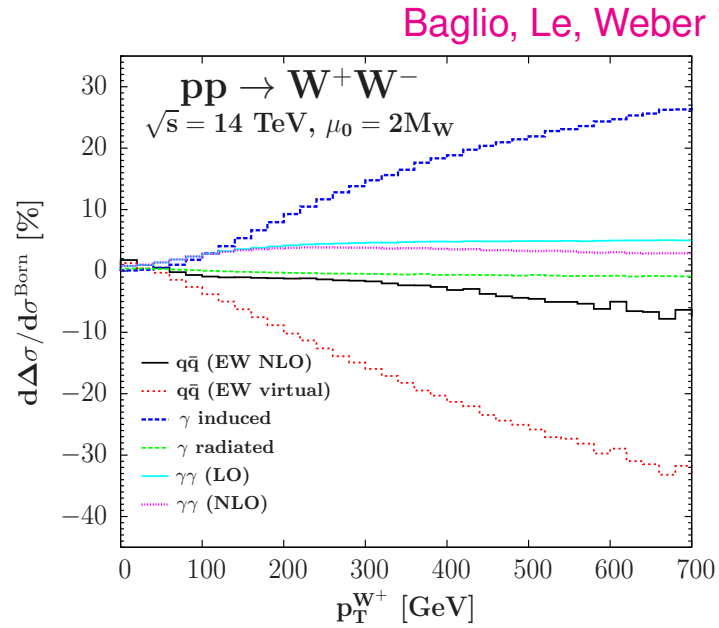
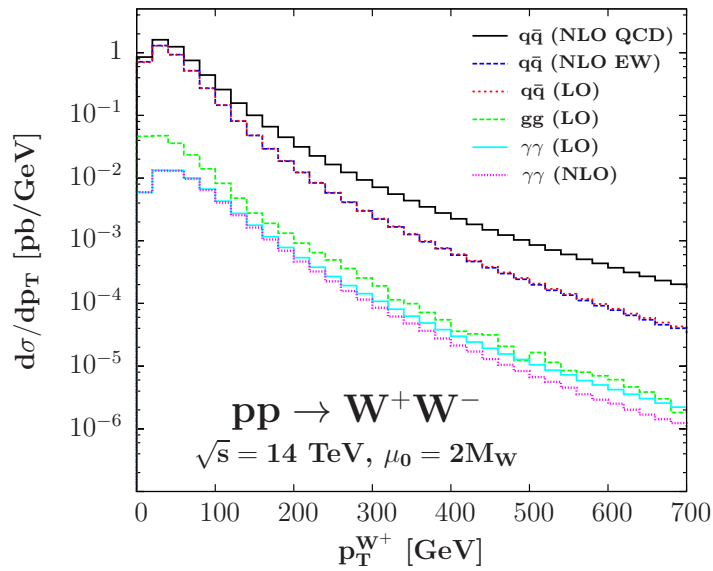
Note:

- M_{VV} not accessible for W final states
- on-shell approximation not applicable for $M_{VV} < M_{V_1} + M_{V_2}$

Survey of corrections to WW production (stable/on-shell W bosons)



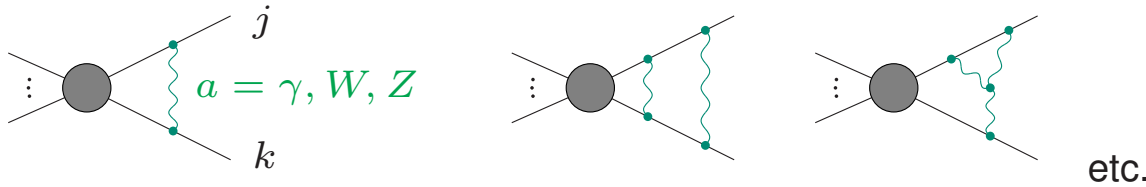
Note:
large contribution by $\gamma\gamma$ channel for high invariant WW masses!



Large impact of $q\gamma$ collisions?

Electroweak corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



+ sub-leading logarithms from collinear singularities

Typical impact on $2 \rightarrow 2$ reactions at $\sqrt{s} \sim 1$ TeV:

$$\delta_{LL}^{1\text{-loop}} \sim -\frac{\alpha}{\pi s_W^2} \ln^2\left(\frac{s}{M_W^2}\right) \simeq -26\%, \quad \delta_{NLL}^{1\text{-loop}} \sim +\frac{3\alpha}{\pi s_W^2} \ln\left(\frac{s}{M_W^2}\right) \simeq 16\%$$

$$\delta_{LL}^{2\text{-loop}} \sim +\frac{\alpha^2}{2\pi^2 s_W^4} \ln^4\left(\frac{s}{M_W^2}\right) \simeq 3.5\%, \quad \delta_{NLL}^{2\text{-loop}} \sim -\frac{3\alpha^2}{\pi^2 s_W^4} \ln^3\left(\frac{s}{M_W^2}\right) \simeq -4.2\%$$

⇒ Corrections still relevant at 2-loop level

Note: differences to QED / QCD where Sudakov log's cancel

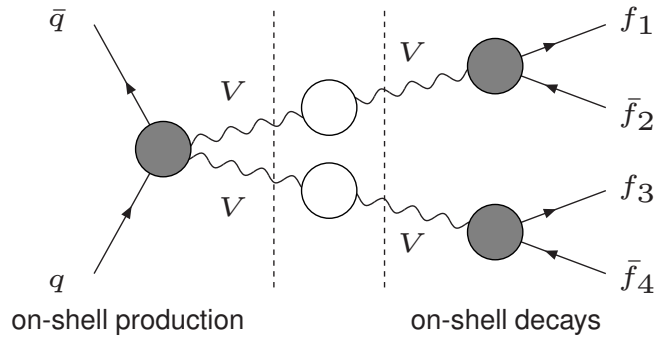
- massive gauge bosons W, Z can be reconstructed
 ⇔ no need to add “real W, Z radiation”
- non-Abelian charges of W, Z are “open” → Bloch–Nordsieck theorem not applicable

Extensive theoretical studies at fixed perturbative (1-/2-loop) order and suggested resummations via evolution equations

Beccaria et al.; Beenakker, Werthenbach; Ciafaloni, Comelli; Denner, Pozzorini; Fadin et al.; Hori et al.; Melles; Kühn et al., Denner et al., Manohar et al. '00–

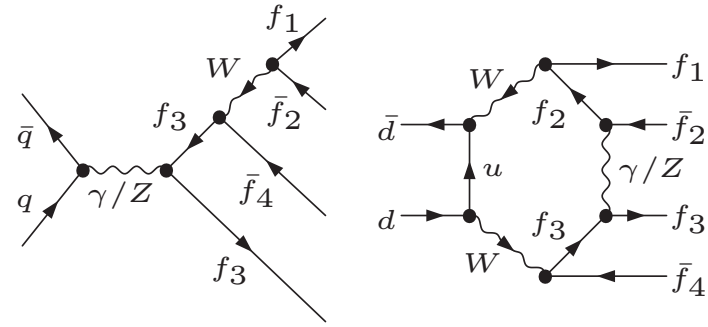
EW corrections with leptonic W/Z decays

Double-pole approximation (DPA)



vs.

Full off-shell $q\bar{q} \rightarrow 4f$ calculation



- expansion about resonance poles
 \hookrightarrow factorizable & non-factorizable corrs.

- not many diagrams ($2 \rightarrow 2$ production)

+ numerically fast

- validity only for $\sqrt{\hat{s}} > 2M_V + \mathcal{O}(\Gamma_V)$

- error estimate for $\sqrt{\hat{s}} \lesssim 0.5-1$ TeV:

$$\Delta \sim \frac{\alpha}{\pi} \frac{\Gamma_V}{M_V} \log(\dots) \sim 0.5-2\%$$

- off-shell calculation with complex-mass scheme

- many off-shell diagrams ($\sim 10^3$ /channel)

- very CPU intensive

+ NLO accuracy everywhere

- global error estimate:

$$\Delta \sim \delta_{\text{NNLO EW}} \sim \delta_{\text{NLO EW}}^2$$

Approaches compared for $e^+e^- \rightarrow WW \rightarrow 4f$

Denner, S.D., Roth, Wieders '05

New: $pp \rightarrow WW \rightarrow 4f$

Biedermann et al. (in preparation)

Collier is hosted by Hepforge, IPPP Durham



**A Complex One-Loop Library
with Extended
Regularizations**

Authors

Ansgar Denner *Universität Würzburg, Germany*
 Stefan Dittmaier *Universität Freiburg, Germany*
 Lars Hofer *Universitat de Barcelona, Spain*

To be released soon!

Features of the library

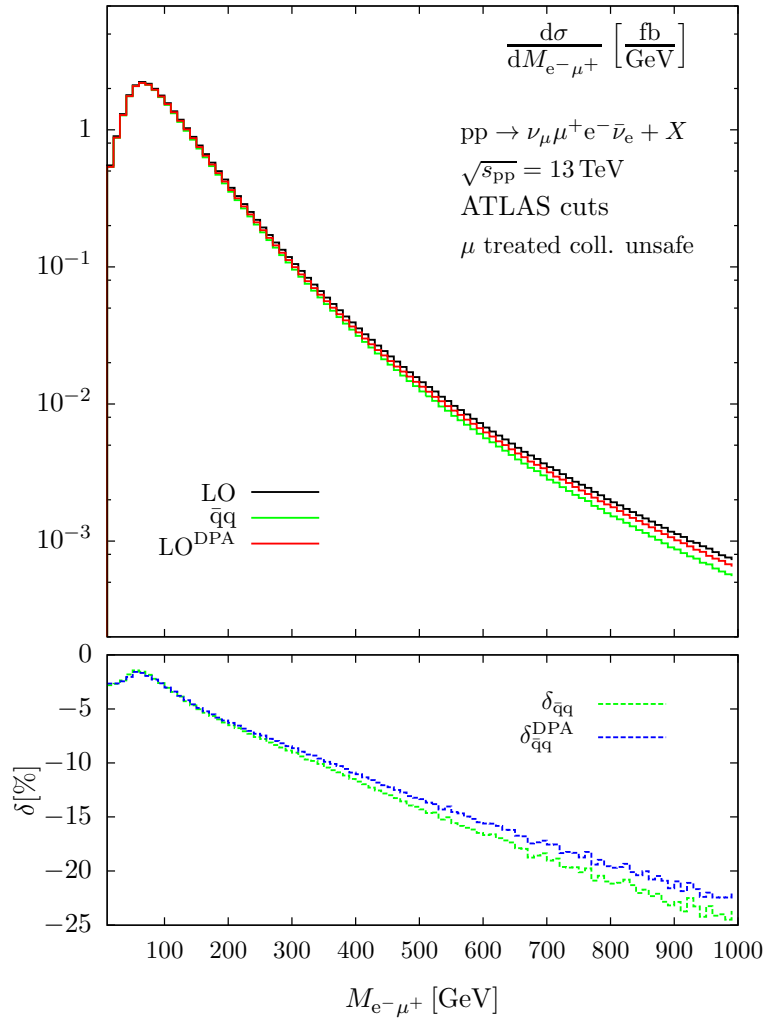
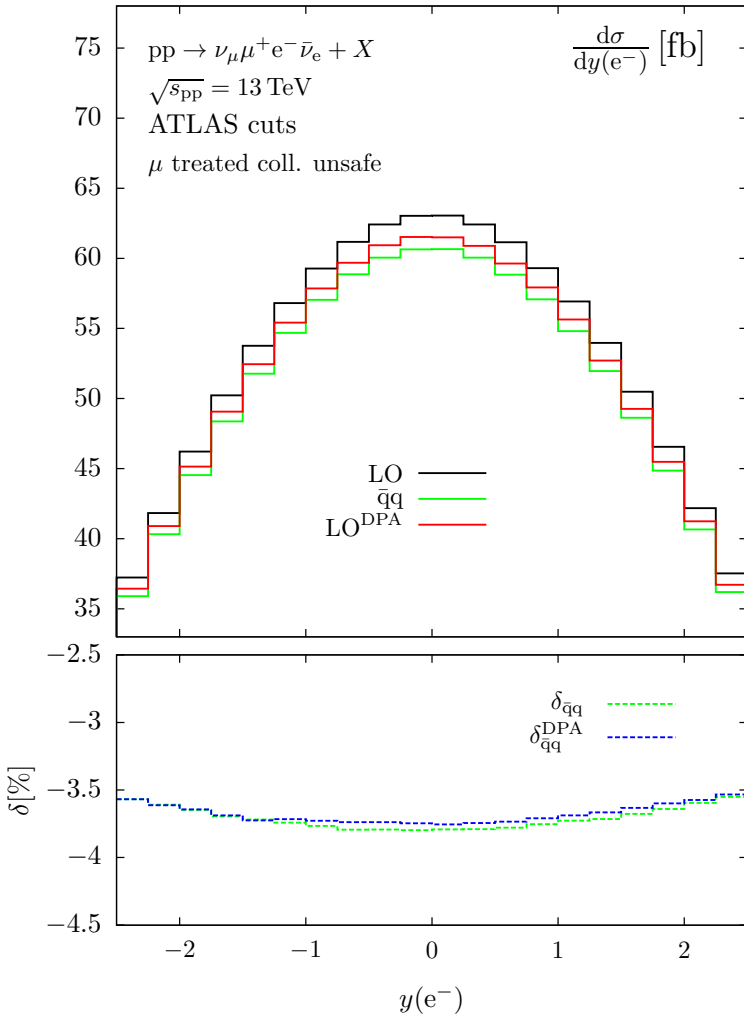
COLLIER is a fortran library for the numerical evaluation of one-loop scalar and tensor integrals appearing in perturbative relativistic quantum field theory with the following features:

- ✧ scalar and tensor integrals for high particle multiplicities
- ✧ dimensional regularization for ultraviolet divergences
- ✧ dimensional regularization for soft infrared divergences
(mass regularization for abelian soft divergences is supported as well)
- ✧ dimensional regularization or mass regularization for collinear mass singularities
- ✧ complex internal masses (for unstable particles) fully supported
(external momenta and virtualities are expected to be real)
- ✧ numerically dangerous regions (small Gram or other kinematical determinants)
cured by dedicated expansions
- ✧ two independent implementations of all basic building blocks allow for internal cross-checks
- ✧ cache system to speed up calculations

If you use Collier for a publication, please cite all the references listed [here!](#)

Rapidity and invariant-mass distributions

Preliminary!

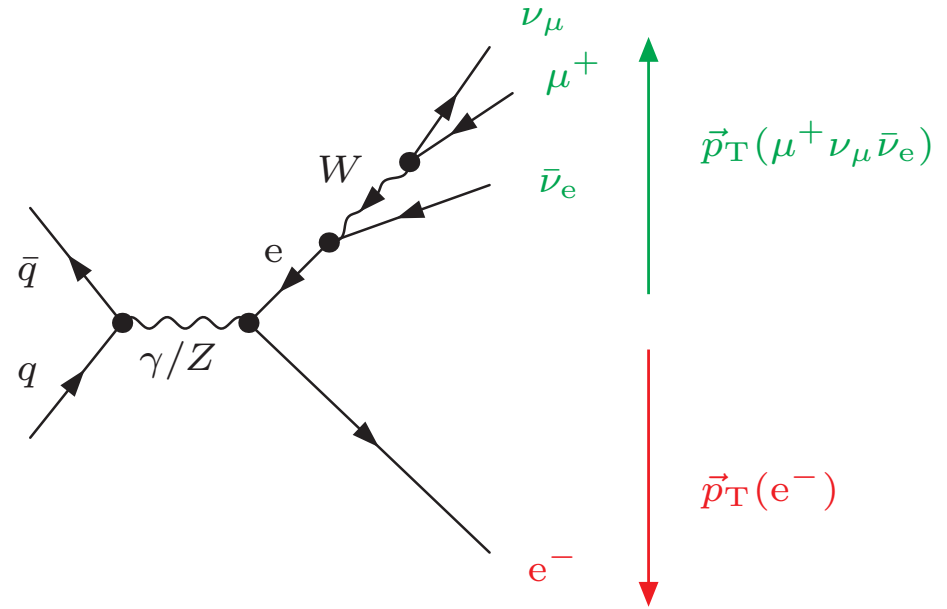
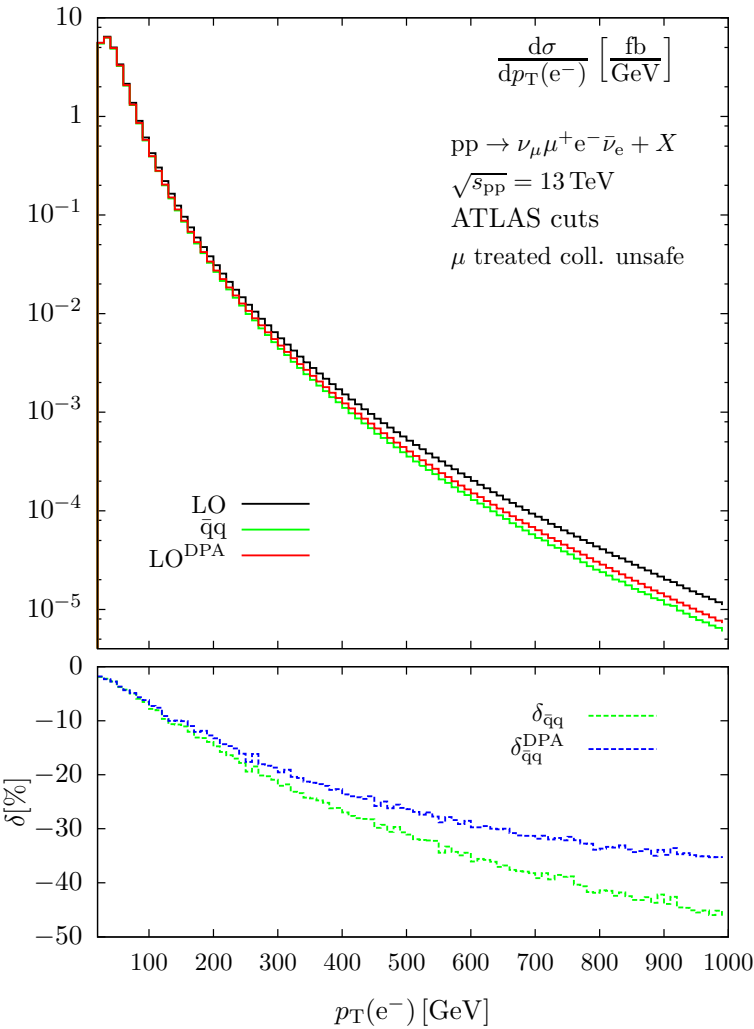


Level of agreement as expected

(dominance of doubly-resonant diagrams)

Transverse-momentum distribution of a single lepton

Preliminary!

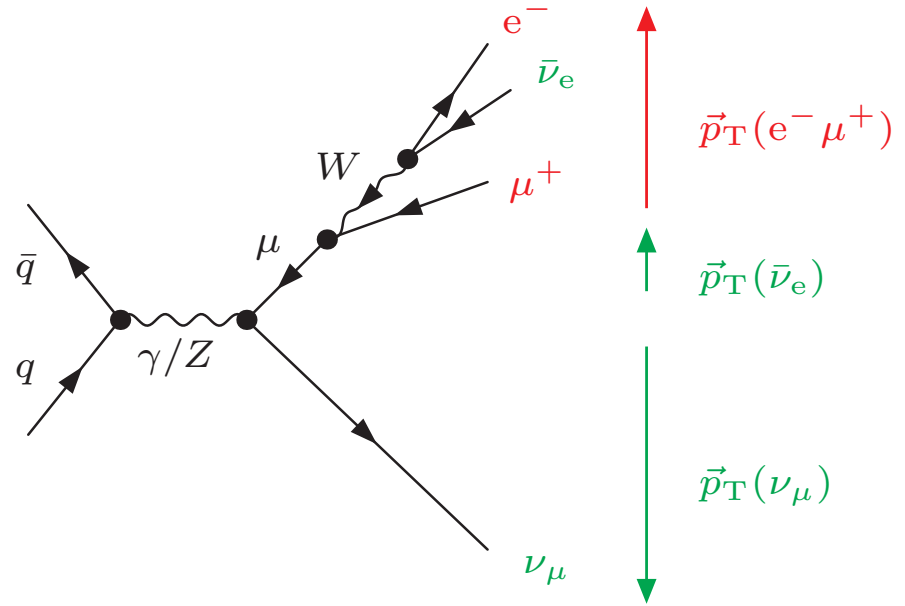
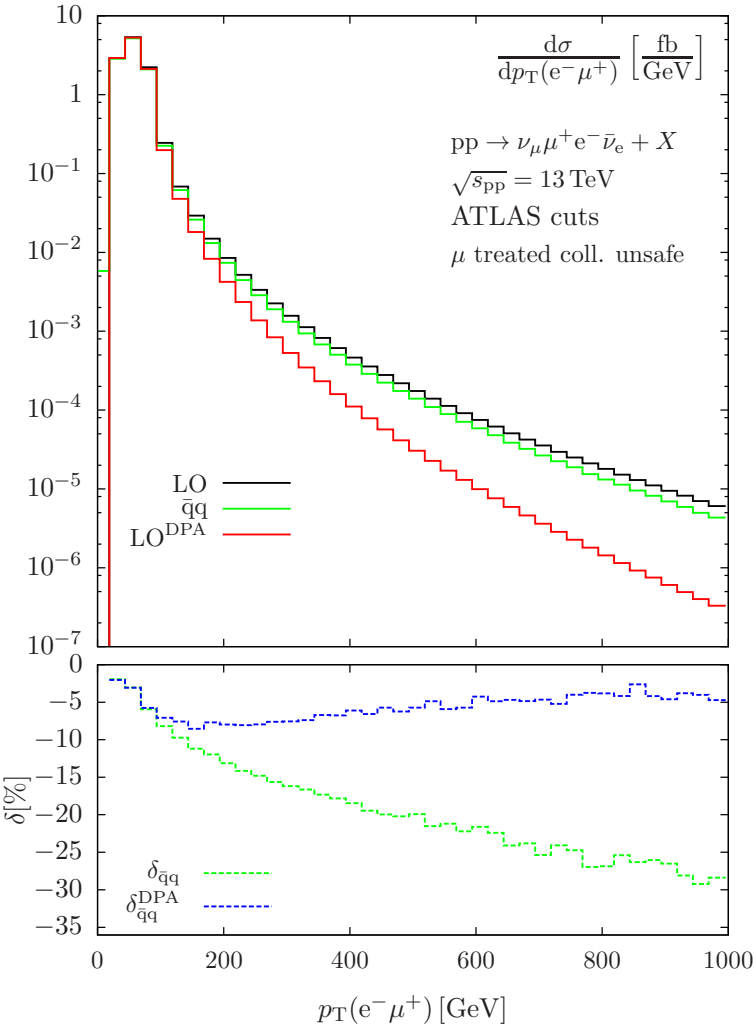


Impact of singly-resonant diagrams where e^- takes recoil from $(\mu^+ \nu_\mu \bar{\nu}_e)$

Agreement degrades for $p_T \gtrsim 300 \text{ GeV}$, since off-shell diagrams get enhanced

Transverse-momentum distribution of the charged lepton pair

Preliminary!



Dominance of singly-resonant diagrams where $(e^- \mu^+)$ recoil against $(\nu_\mu \bar{\nu}_e)$

DPA fails for $p_T \gtrsim 200 \text{ GeV}$, since off-shell production dominates!

A Higgs background study for $pp \rightarrow \mu^+ \mu^- e^+ e^- + X$ Biedermann et al. '16

Setup (inspired by ATLAS)

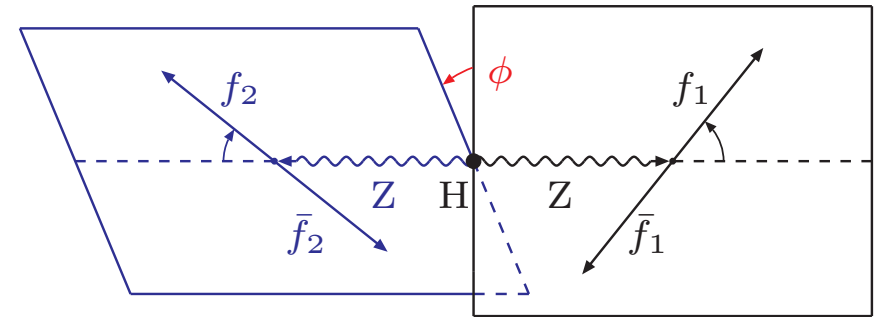
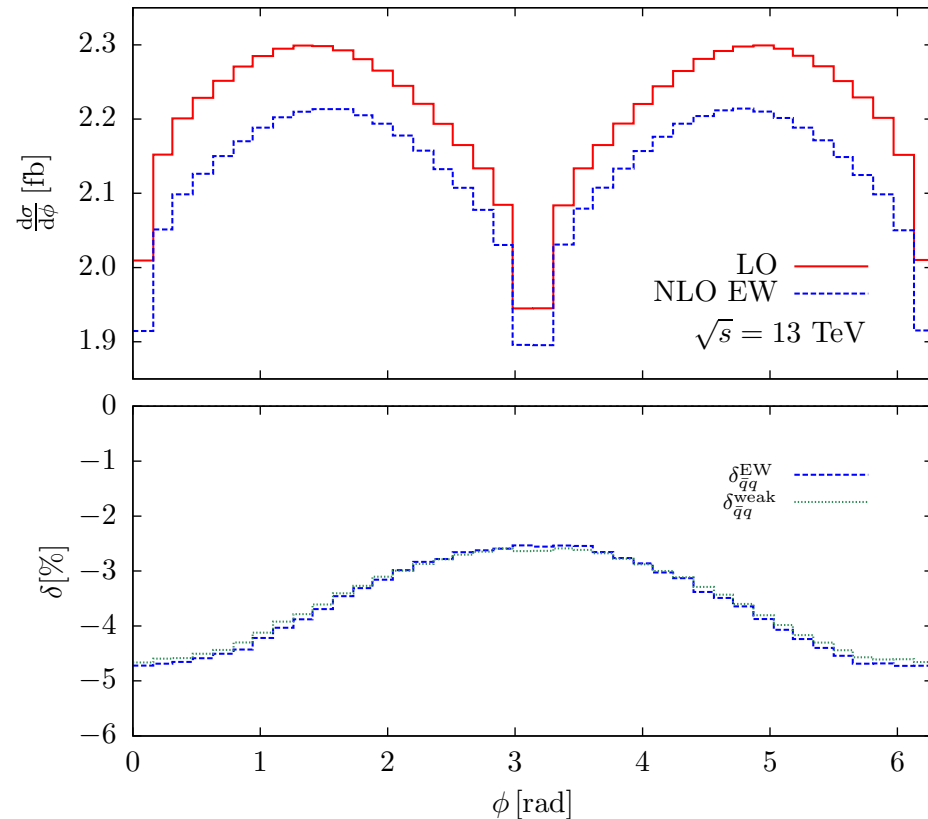
$$p_T(\ell_i) > 6 \text{ GeV}, \quad |y(\ell_i)| < 2.5, \quad \Delta R(\ell_i, \ell_j) = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2} > 0.2$$
$$40 \text{ GeV} < M_{\ell_1^+ \ell_1^-} < 120 \text{ GeV}, \quad 12 \text{ GeV} < M_{\ell_2^+ \ell_2^-} < 120 \text{ GeV}$$

Integrated cross section

\sqrt{s} [TeV]	$\sigma_{\bar{q}q}^{\text{LO}}$ [fb]	$\delta_{\bar{q}q}^{\text{EW}}$ [%]	$\delta_{\bar{q}q}^{\text{weak}}$ [%]
7	7.3293(4)	-3.4	-3.3
8	8.4704(2)	-3.5	-3.4
13	13.8598(3)	-3.6	-3.6
14	14.8943(8)	-3.6	-3.6

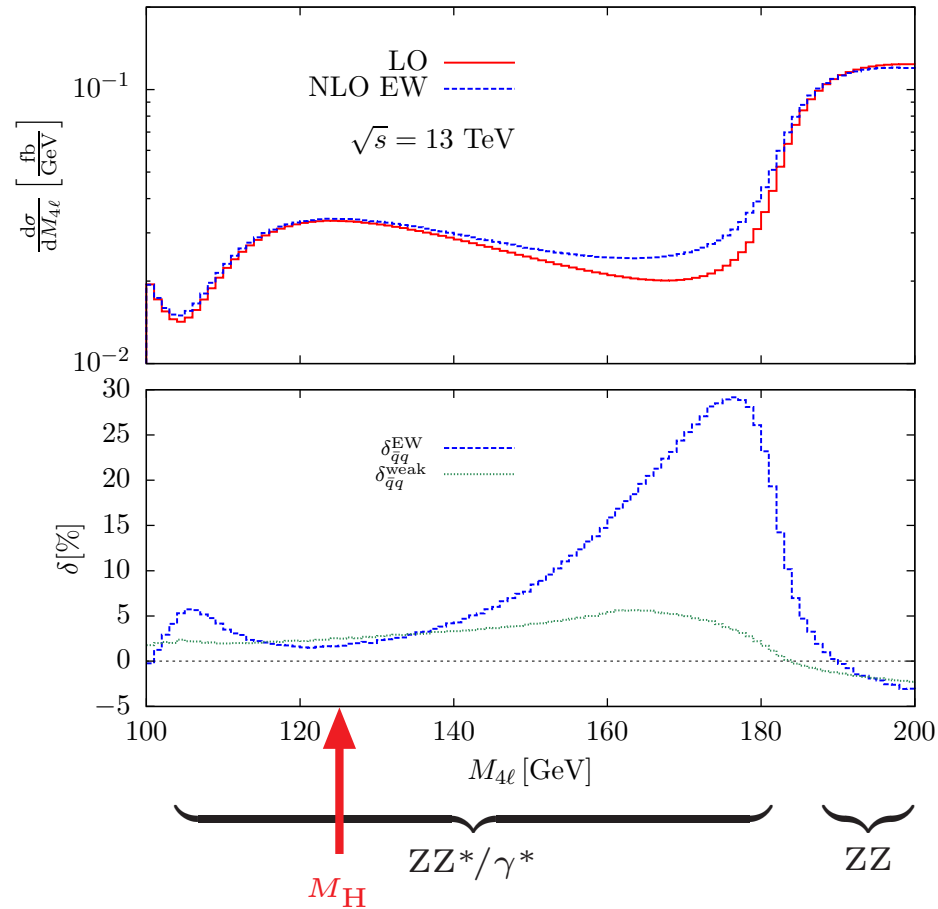
- weak corrections moderate
- photonic corrections negligible
(due to inclusiveness of the event selection)
- deviation of δ from on-shell ZZ calculation $\sim 1\%$

Distribution in the angle between decay planes



- observable dominated by resonant ZZ production
- corrections mostly resemble corrections to the integrated cross section
- photonic corrections extremely suppressed
(insensitivity of ϕ to collinear photon emission off leptons)

Distribution in the 4-lepton invariant mass

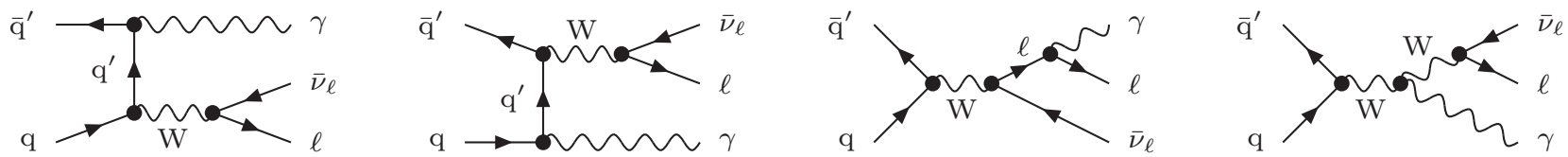


- large radiative tails in photonic corrections below thresholds (known effect ...)
- weak corrections $\sim \mathcal{O}(5\%)$
with sign change in transition from on-shell ZZ to off-shell ZZ^*/γ^* region

$W\gamma / Z\gamma$ production

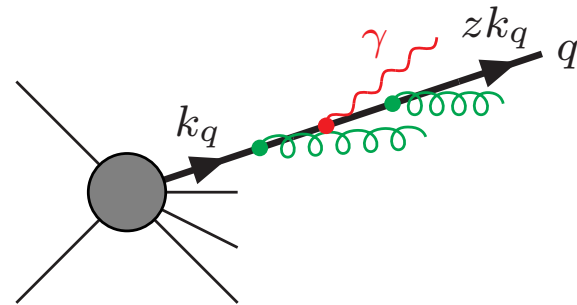


Example of $W\gamma$ production



Issues / physics goals:

- clean **photon–jet separation**
 \hookrightarrow quark-to-photon fragmentation function
Glover, Morgan '94
 or Frixione isolation Frixione '98



- stronger bounds on **anomalous $WW\gamma$ coupling**:

$$\begin{aligned}
 \begin{array}{l} W_{\mu}^{+}(q) \\ W_{\nu}^{-}(\bar{q}) \end{array} \begin{array}{l} \text{---} \\ \text{---} \end{array} \text{---} \gamma_{\rho}(p) &= e \left\{ \bar{q}^{\mu} g^{\nu\rho} \left(\Delta\kappa^{\gamma} + \lambda^{\gamma} \frac{q^2}{M_W^2} \right) - q^{\nu} g^{\mu\rho} \left(\Delta\kappa^{\gamma} + \lambda^{\gamma} \frac{\bar{q}^2}{M_W^2} \right) \right. \\
 &\quad \left. + (\bar{q}^{\rho} - q^{\rho}) \frac{\lambda^{\gamma}}{M_W^2} \left(p^{\mu} p^{\nu} - \frac{1}{2} g^{\mu\nu} p^2 \right) \right\} \times \left(1 + \frac{M_{W\gamma}^2}{\Lambda^2} \right)^2
 \end{aligned}$$

ATLAS limits '12: $\Delta\kappa^{\gamma} = 0.41$, $\lambda^{\gamma} = 0.074$ for $\Lambda = 2 \text{ TeV}$

Photon–jet separation via photon fragmentation function $D_{q \rightarrow \gamma}$ Glover, Morgan '94

Why?

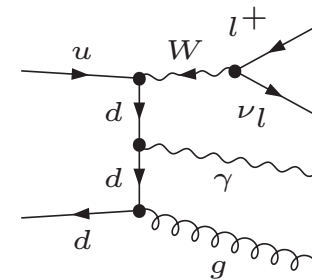
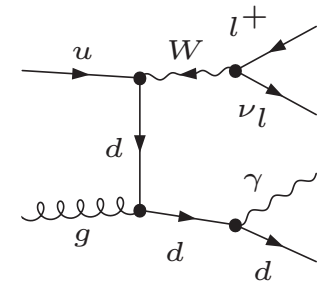
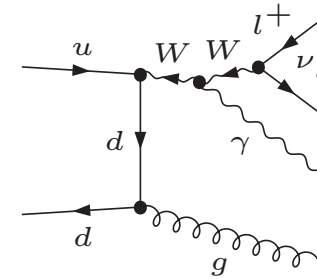
- QCD radiation cannot be suppressed by cuts
 \hookrightarrow treat at least soft/collinear jets inclusively

- separation of collinear quarks and photons leads to IR-unstable corrections $\propto \ln(m_q^2/Q^2)$
 \hookrightarrow recombine collinear quarks and photons

- quark and gluon jets cannot be distinguished event by event
 \hookrightarrow common recombination required for quarks/gluons with photons

\Rightarrow $\underbrace{(\mathbf{g}_{\text{hard}} + \boldsymbol{\gamma}_{\text{soft}})}_{\text{EW corr. to } X+\text{jet}}$ and $\underbrace{(\mathbf{g}_{\text{soft}} + \boldsymbol{\gamma}_{\text{hard}})}_{\text{QCD corr. to } X+\boldsymbol{\gamma}}$ both appear as 1 jet

Problem: signatures of $X+\text{jet}$ and $X+\boldsymbol{\gamma}$ overlap !



Solution:

- **idea:** declare photon/jet systems as photon or jet according to energy share

- determine photon energy fraction $z_\gamma = \frac{E_\gamma}{E_{\text{jet}} + E_\gamma}$ of photon/jet system

↪ event selection:

$$z_\gamma > z_0: \text{ photon}$$

$$z_\gamma < z_0: \text{ jet} \quad (\text{typical value } z_0 = 0.7)$$

- **but:** cut on z_γ destroys inclusiveness needed for KLN theorem

↪ collinear singularity $\propto \alpha \ln m_q$ remains (but are universal!)

- absorb universal collinear singularity in “fragmentation function” $D_{q \rightarrow \gamma}(z_\gamma)$

↪ subtract convolution of LO cross section with

$$D_{q \rightarrow \gamma}^{\overline{\text{MS}}}(z_\gamma, \mu_{\text{fact}}) \Big|_{\text{mass.reg.}} = \frac{\alpha Q_q^2}{2\pi} P_{q \rightarrow \gamma}(z_\gamma) \left[\ln \frac{m_q^2}{\mu_{\text{fact}}^2} + 2 \ln z_\gamma + 1 \right] \leftarrow \text{cancels coll. singularities}$$

$$+ D_{q \rightarrow \gamma}^{\text{ALEPH}}(z_\gamma, \mu_{\text{fact}}) \leftarrow \text{non-perturbative part fitted to ALEPH data}$$

where $P_{q \rightarrow \gamma}(z_\gamma) = \frac{1+(1-z_\gamma)^2}{z_\gamma} =$ quark-to-photon splitting function

Idea: suppress jets inside collinear cone around photons:

$$p_{T,\text{jet}} < \varepsilon p_{T,\gamma} \left(\frac{1 - \cos R_{\gamma\text{jet}}}{1 - \cos R_0} \right) \quad (R_0 = \text{fixed cone size})$$

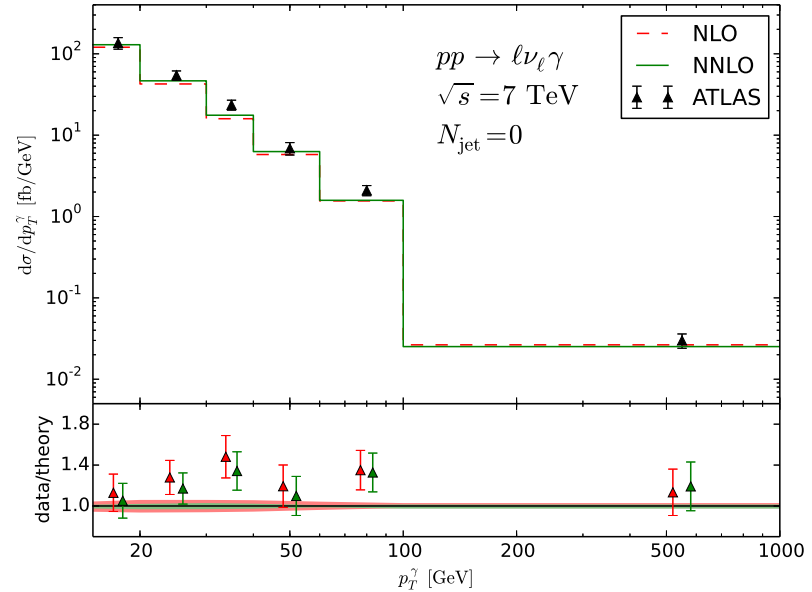
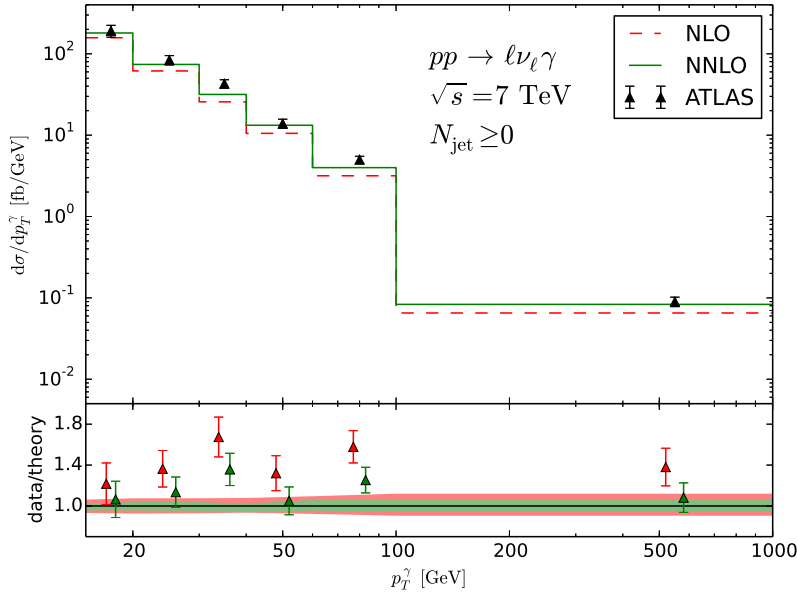
- photon and jet collinear ($R_{\gamma\text{jet}} \rightarrow 0$) \rightarrow event discarded
- photon soft or collinear to beams ($p_{T,\gamma} \rightarrow 0$) \rightarrow event discarded
- jet soft or collinear beams ($p_{T,\text{jet}} \rightarrow 0$) \rightarrow event kept \Rightarrow IR safety

Comments:

- Frixione isolation simple to implement theoretically, but problematic experimentally
- cleaner isolation of non-perturbative effects by fragmentation function
- approximate relation between the two methods:

$$z_\gamma \sim \frac{p_{T,\gamma}}{p_{T,\gamma} + p_{T,\text{jet}}} > \frac{1}{1 + \varepsilon \frac{1 - \cos R_{\gamma\text{jet}}}{1 - \cos R_0}} \sim \frac{1}{1 + \varepsilon} \quad \text{for } R_{\gamma\text{jet}} \sim R_0$$

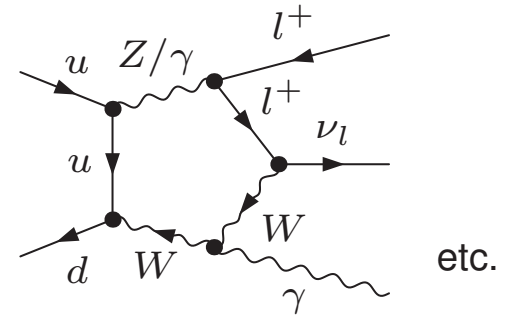
\hookrightarrow methods yield quite similar results for $z_0 \sim \frac{1}{1 + \varepsilon}$



- good agreement of experimental results with NNLO QCD (no EW corrections included)
- QCD uncertainties: (for small/moderate $p_{T,\gamma}$)
 scale: 4–5%, PDF: 1–2% (increasing with $p_{T,\gamma}$)
- LHC run 2: higher energy reach & higher statistics
 ↪ EW corrections important

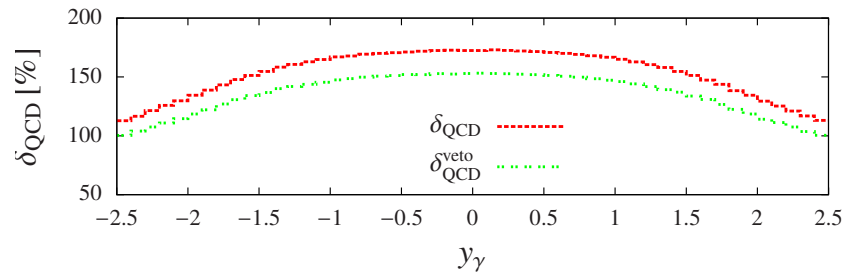
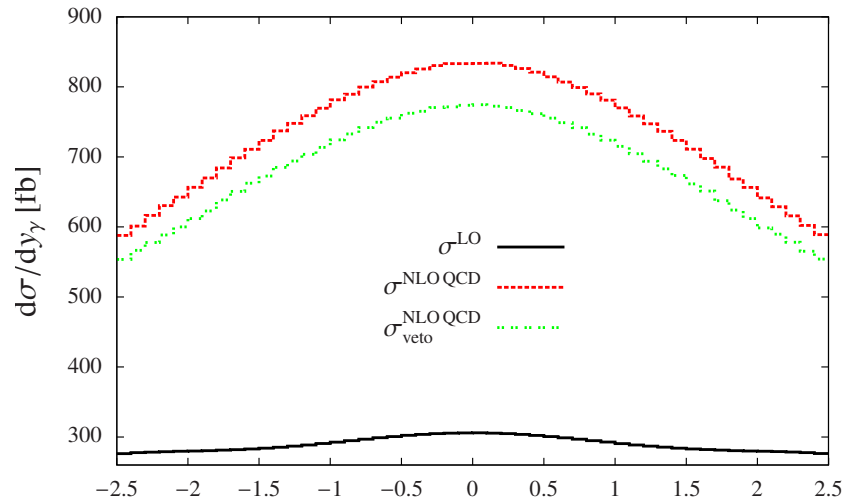
- NLO EW corrections calculated with full W off-shell/decay effects (complex-mass scheme)

↪ more + more complicated diagrams than in QCD

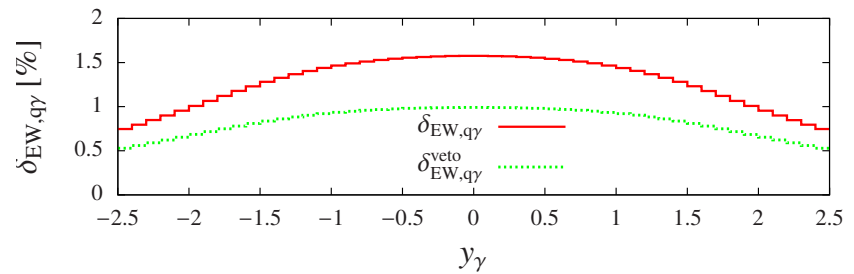
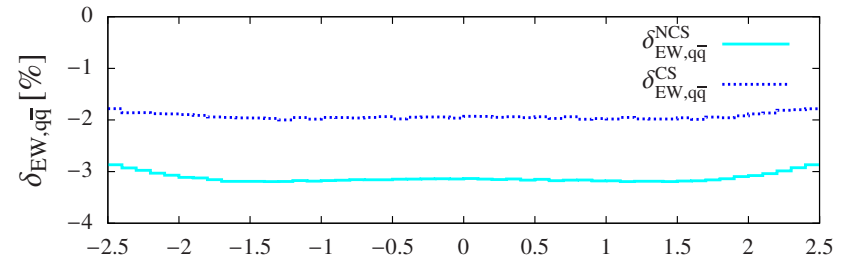


- particular focus on:
 - ◇ high energies (e.g. large p_T):
large EW corrections ↔ sensitivity to anomalous couplings
↪ missing corrections could fake anomalous couplings
 - ◇ photon-induced contributions

$pp \rightarrow l^+ \nu_l \gamma$ (jet)



$\sqrt{s} = 14 \text{ TeV}$



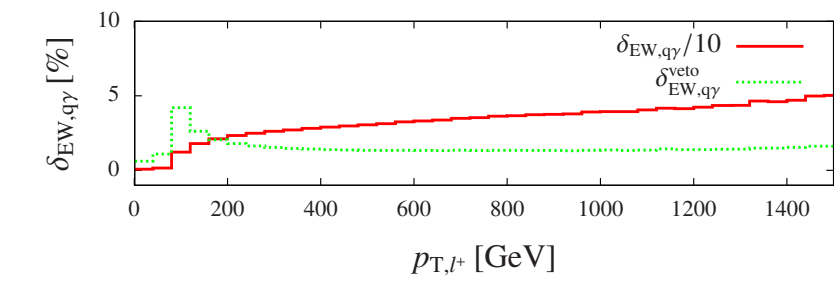
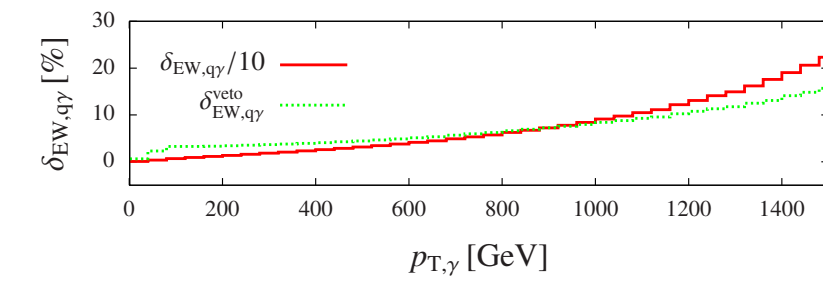
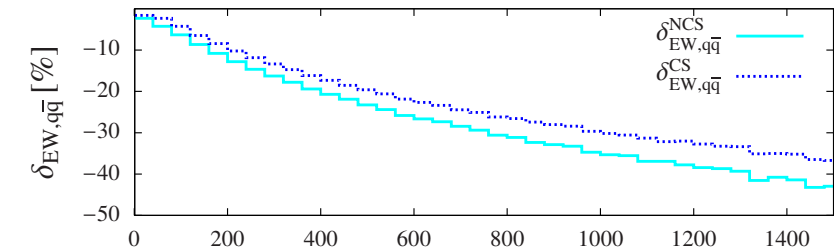
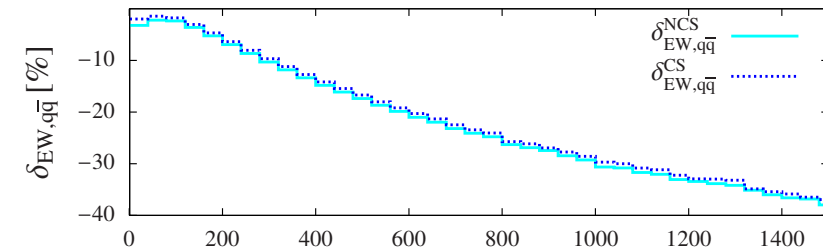
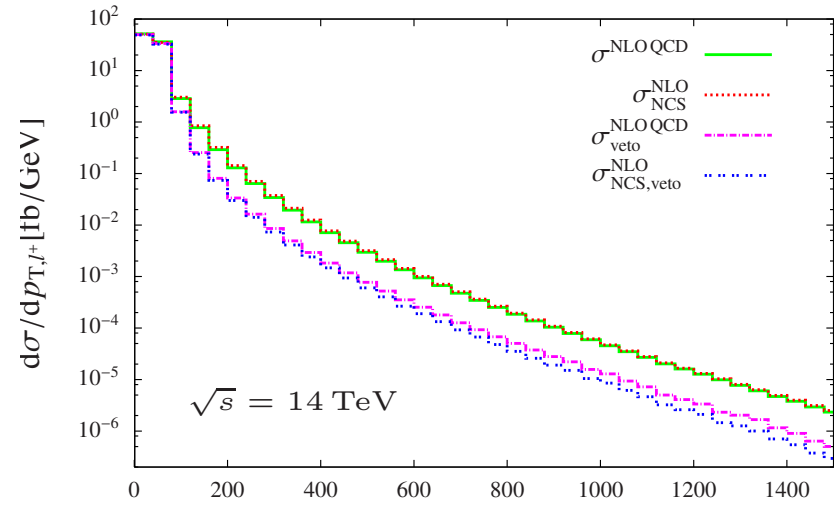
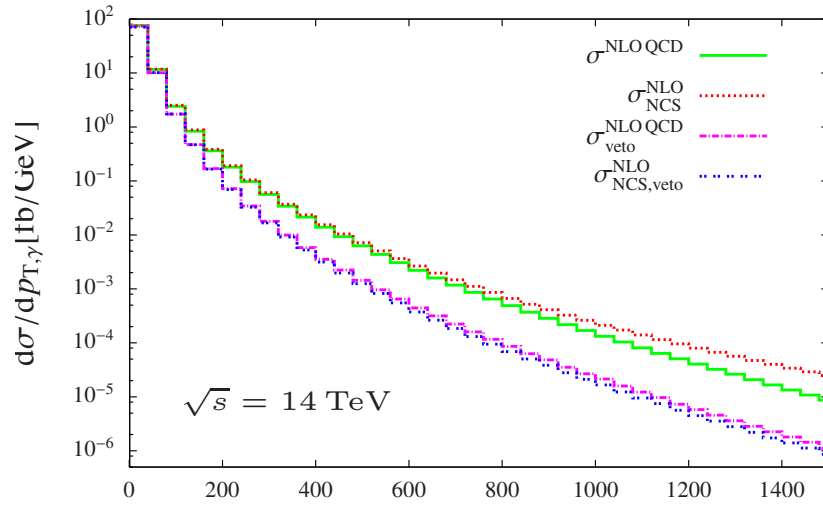
- huge QCD corrections ($\sim 100\%$), only mildly reduced by jet veto $p_{T,\text{jet}} < 100 \text{ GeV}$
- EW corrections and $q\gamma$ channels (few %) small and flat (CS=collinear-safe, NCS=non-collinear-safe)
 \hookrightarrow resemble corrections to integrated cross section

p_T distributions in $W\gamma$ production – EW corrections

Denner, S.D., Hecht, Pasold '14

$$pp \rightarrow l^+ \nu_l \gamma (\gamma/\text{jet})$$

$$pp \rightarrow l^+ \nu_l \gamma (\gamma/\text{jet})$$

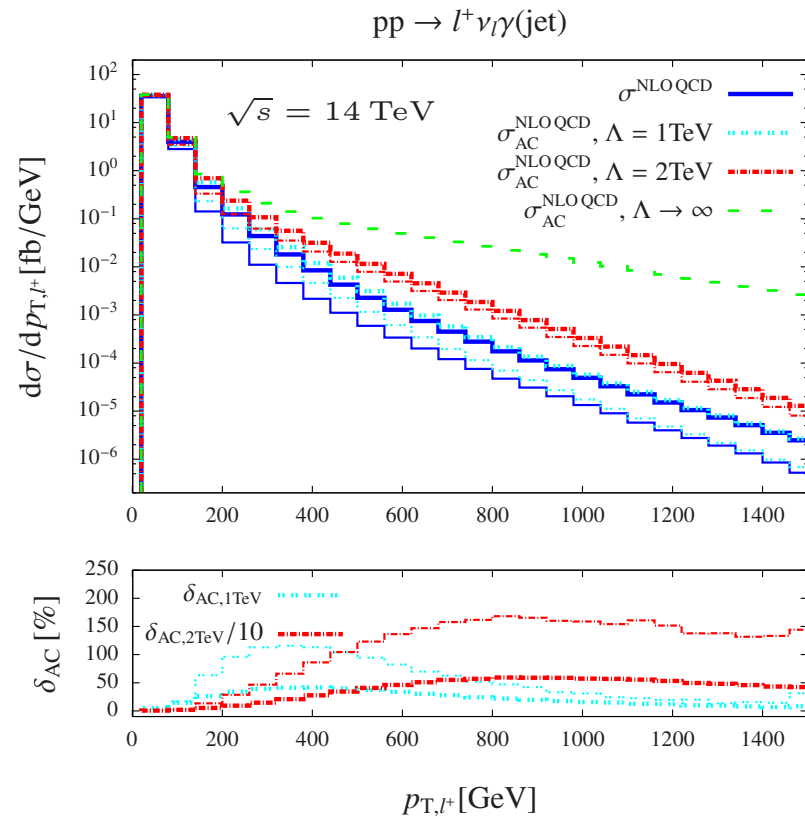
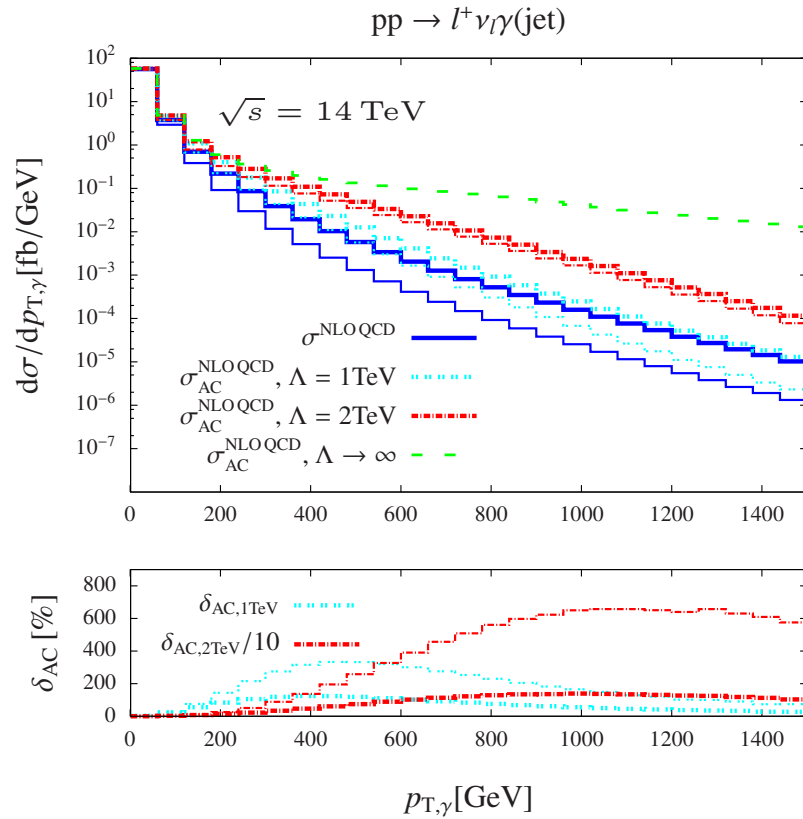


- EW corrections $\sim -30\%$ in TeV range

(CS=collinear-safe, NCS=non-collinear-safe)

- γ -induced corrections non-negligible in TeV range (even with jet veto)

\hookrightarrow reduction of γ PDF uncertainties mandatory !



- results shown without and with jet veto on $p_{T,\text{jet}} > 100 \text{ GeV}$
- ATLAS values of 2012 used: $\Delta\kappa^\gamma = 0.41$, $\lambda^\gamma = 0.074$
 \hookrightarrow much tighter limits expected at LHC run 2

Conclusions



Di-boson production at the LHC

- sensitive to **non-Abelian gauge-boson self-interactions**
 - ↪ LHC will tighten constraints on anomalous triple-gauge-boson couplings
- **important background**
 - ◇ to new-physics searches and
 - ◇ to precision Higgs analyses in $pp \rightarrow H \rightarrow WW/ZZ \rightarrow 4f$

⇒ **Precise predictions with QCD and EW corrections required**

Recent progress at the precision frontier

- **NNLO QCD** corrections to direct VV' production (V s partially off-shell)
- **NLO QCD** corrections to loop-induced channels $gg \rightarrow VV$ (on-shell)
- **NLO EW** corrections to direct VV' production with off-shell V s
 - ↪ particularly important at the TeV scale
- QCD resummations, QCD parton-shower matching (not discussed here)

⇒ **SM predictions in good shape!**

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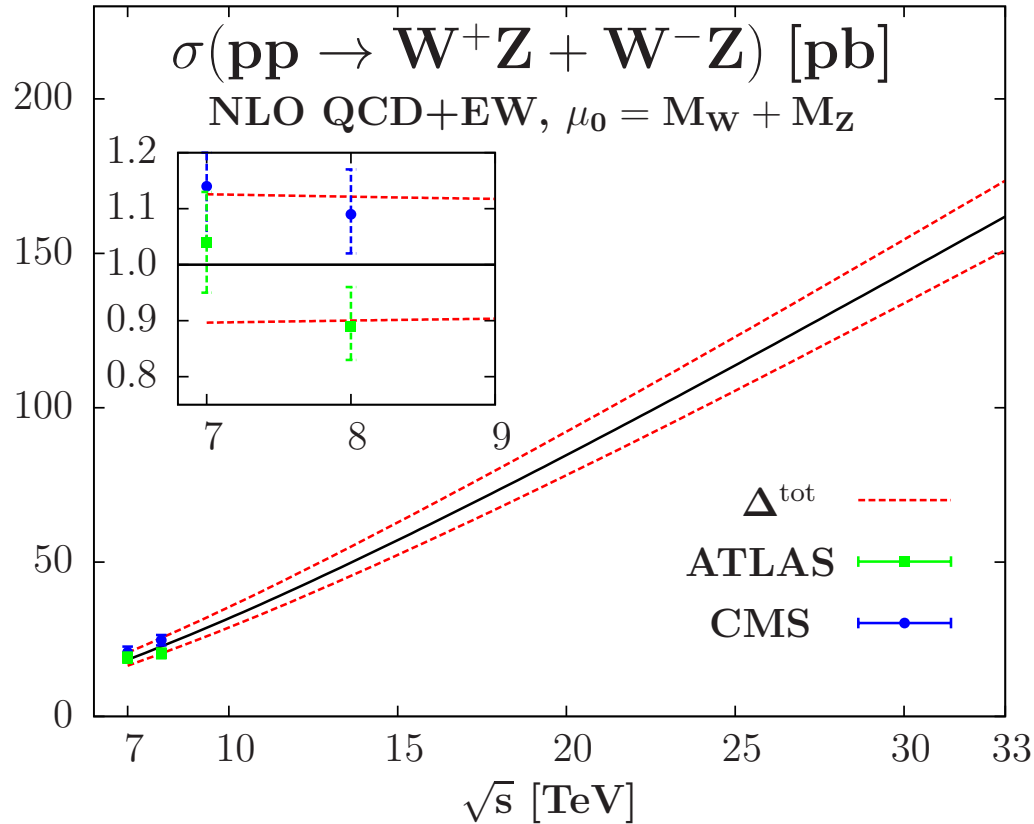
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⇒ **SM predictions in good shape!**

“~~In football as in watchmaking,~~ talent and elegance mean nothing without rigour and precision.”
particle theory [Lionel Messi]

Backup slides



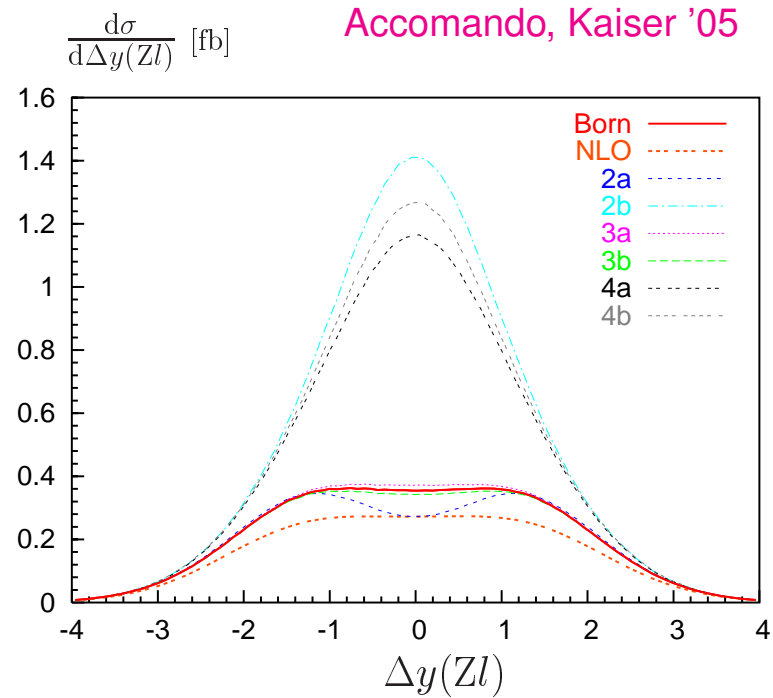
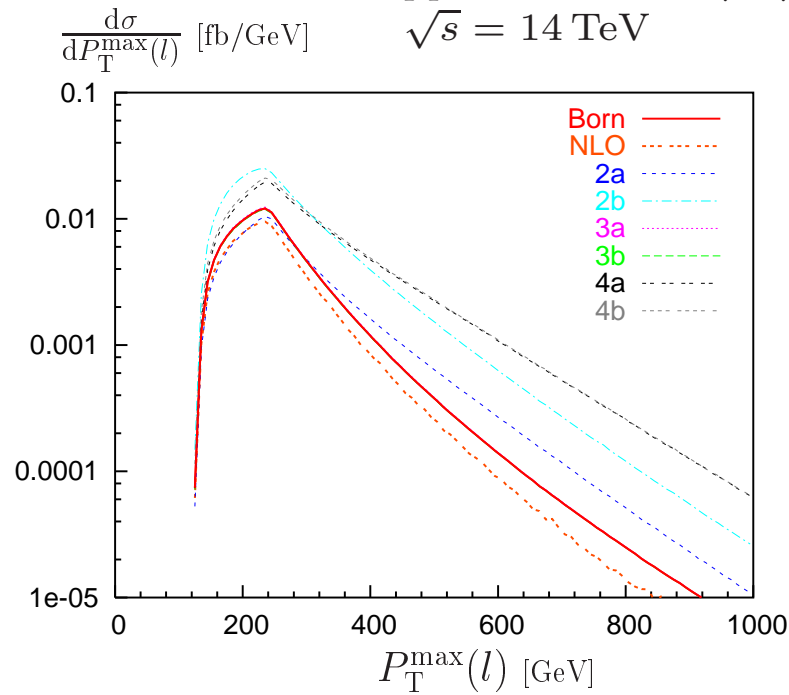


- good agreement of experimental results with NLO QCD
- NLO QCD scale uncertainty $\sim 3\%$, $\Delta_{\text{PDF}+\alpha_s} \sim 4\%$
- LHC run 2: higher energy & higher statistics
 \hookrightarrow NNLO QCD and NLO EW corrections important

EW corrections vs. anomalous TGCs in gauge-boson pair production

$$pp \rightarrow WZ \rightarrow e\nu_e\mu^+\mu^-$$

$$\sqrt{s} = 14 \text{ TeV}$$



Accomando, Kaiser '05

Scenario	Δg_1^Z	$\Delta \kappa_\gamma$	λ_γ
Born	0	0	0
2a/2b	± 0.02	0	0
3a/3b	0	± 0.04	0
4a/4b	0	0	± 0.02

$$\lambda_Z = \lambda_\gamma, \quad \Delta \kappa_Z = \Delta g_1^Z - \tan^2 \theta_W \Delta \kappa_\gamma$$

formfactor rescaling ($\Lambda = 1 \text{ TeV}$):

$$\Delta Y \rightarrow \frac{\Delta Y}{(1 + \hat{s}/\Lambda^2)^2}, \quad \Delta Y = \Delta g_1^Z, \Delta \kappa_\gamma, \lambda_\gamma$$

- Note:**
- EW corrections and anomalous couplings distort distributions
 - neglect of EW corrections can mimick anomalous couplings

Gauge-invariance issues in EW multi-boson production



Gauge invariance implies...

- **Slavnov–Taylor or Ward identities**
= algebraic relations of or between Greens functions
↔ guarantee cancellation of unitarity-violating terms,
crucial for proof of unitarity of S -matrix
- **Nielsen identities** (compensation of gauge-fixing artefacts)
↔ gauge-parameter independence of S -matrix
although Greens function (e.g. self-energies) are gauge dependent

Both statements hold order by order in standard perturbation theory !

Implications:

- **Resonances** require Dyson summation of resonant propagators
↔ perturbative orders mixed → **gauge invariance jeopardized !**
Gauge-invariance-violating terms $\propto \Gamma$ are formally of higher order,
but can be dramatically enhanced if unitarity cancellations disturbed
- **Anomalous couplings** potentially enhanced
if effective operator not gauge invariant

Important Ward identities for processes with EW gauge bosons:

Elmg. U(1) gauge invariance implies

$$k^\mu \gamma_\mu \text{ (wavy line)} \rightarrow \text{circle} \rightarrow \begin{matrix} F_1 \\ \vdots \\ F_n \end{matrix} = 0 \quad \text{for any on-shell fields } F_l$$

↪ Identity becomes crucial for collinear light fermions:

for fermion momenta $p_1 \sim c p_2$:

$$\begin{matrix} p_1 \\ \swarrow \\ \bullet \\ \nwarrow \\ p_2 \end{matrix} \begin{matrix} \xrightarrow{k = p_1 - p_2} \\ \text{wavy line} \end{matrix} = \bar{u}_2(p_2) \gamma^\mu u_1(p_1) \propto k^\mu$$

A typical situation: quasi-real space-like photons

$$\begin{matrix} e \rightarrow \text{---} \bullet \text{---} e \\ \downarrow \text{wavy line } \gamma \\ \downarrow k \\ \text{circle} \end{matrix} \sim \frac{1}{k^2} k^\mu T_\mu^\gamma \quad \text{for } k^2 \rightarrow \mathcal{O}(m_e^2) \ll E^2$$

Identity $k^\mu T_\mu^\gamma = 0$ needed to cancel $1/k^2$,

otherwise gauge-invariance-breaking terms enhanced by E^2/m_e^2 ($\sim 10^{10}$ for LEP2)

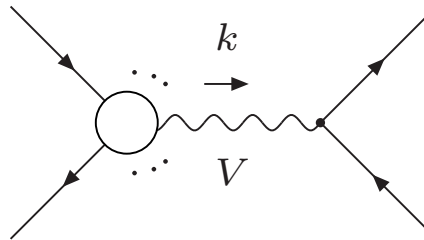
Electroweak SU(2) gauge invariance implies

$$\begin{aligned}
 k^\mu \text{---} Z_\mu \text{---} \text{---} \text{---} \begin{matrix} F_1 \\ \vdots \\ F_n \end{matrix} &= iM_Z \text{---} \chi \text{---} \text{---} \begin{matrix} F_1 \\ \vdots \\ F_n \end{matrix} \\
 k^\mu \text{---} W_\mu^\pm \text{---} \text{---} \text{---} \begin{matrix} F_1 \\ \vdots \\ F_n \end{matrix} &= \pm M_W \text{---} \phi^\pm \text{---} \text{---} \begin{matrix} F_1 \\ \vdots \\ F_n \end{matrix}
 \end{aligned}$$

$F_l = \text{on-shell fields}$
 $\chi, \phi^\pm = \text{would-be Goldstone fields}$

A typical situation: high-energetic quasi-real longitudinal vector bosons

↪ fermion current attached to $V(k)$ again $\propto k^\mu$



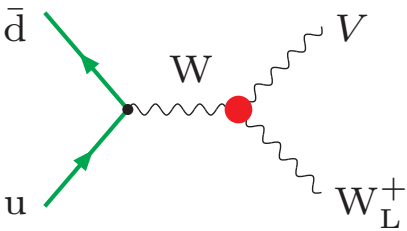
$$\sim \frac{1}{k^2 - M_V^2} k^\mu T_\mu^V \quad \text{for } k^0 \gg M_V$$

Identity $k^\mu T_\mu^V = c_V M_V T^S$ needed to cancel factor k^0 ,
 otherwise gauge-invariance/unitarity-breaking terms enhanced by k^0/M_V

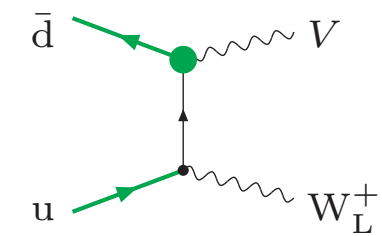
For on-shell V :
$$\varepsilon_{V_L}^\mu(k) = \frac{k^\mu}{M_V} + \mathcal{O}(M_V/k^0)$$

Illustration of unitarity cancellations for WV production ($V = Z/\gamma$)

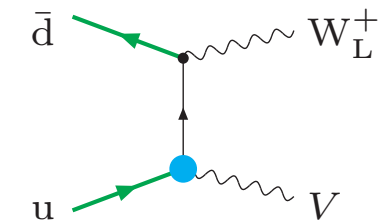
Leading behaviour of amplitudes with $\varepsilon_{W_L^+}^\mu(k) = \frac{k^\mu}{M_W} + \dots$ for $k^0 \gg M_W$:



$$\sim \frac{-ie^2 g_{VWW}}{2\sqrt{2}s_W M_W} [\bar{v}_d \not{\omega} - u_u] \left\{ g_1^V \left[\varepsilon_V^{*\mu} - k^\mu \frac{\varepsilon_V^* \cdot k}{s} \right] + \kappa_V \left[\varepsilon_V^{*\mu} + k^\mu \frac{\varepsilon_V^* \cdot k}{s} \right] \right\}$$



$$\sim \frac{ie^2 g_{V^- dd}}{\sqrt{2}s_W M_W} [\bar{v}_d \not{\omega} - u_u], \quad g_{Zdd}^- = -\frac{s_W}{c_W} Q_d - \frac{1}{2s_W c_W}, \quad g_{\gamma dd}^- = -Q_d$$



$$\sim \frac{-ie^2 g_{V^- uu}}{\sqrt{2}s_W M_W} [\bar{v}_d \not{\omega} - u_u], \quad g_{Zuu}^- = -\frac{s_W}{c_W} Q_u + \frac{1}{2s_W c_W}, \quad g_{\gamma uu}^- = -Q_u$$

Cancellation (unitarity!) of sum demands:

$$g_{V^- dd}^- - g_{V^- uu}^- - \frac{g_{VWW}}{2} (g_1^V + \kappa_V) \stackrel{!}{=} 0, \quad g_1^V \stackrel{!}{=} \kappa_V$$

\hookrightarrow SM provides unique solution: $g_1^Z = \kappa_Z = g_1^\gamma = \kappa_\gamma = 1$

Note: no constraint on coupling λ_V , since effective operator gauge invariant !

Width schemes for LO calculations and gauge invariance

Naive propagator substitutions in full tree-level amplitudes:

$$\frac{1}{k^2 - m^2} \rightarrow \frac{1}{k^2 - m^2 + im\Gamma(k^2)} \quad \text{in all propagators}$$

- constant width $\Gamma(k^2) = \text{const.}$ \rightarrow U(1) respected, SU(2) “mildly” violated
- running width $\Gamma(k^2) \neq \text{const.}$ \rightarrow U(1) and SU(2) violated
 \hookrightarrow results can be totally wrong !

Fudge factor approaches:

Multiply full amplitudes without widths with

$$\text{factors } \frac{p^2 - m^2}{p^2 - m^2 + im\Gamma} \text{ for each potentially resonant propagator}$$

\hookrightarrow gauge invariant, but spurious factors of $\mathcal{O}(\Gamma/m)$

Complex-mass scheme: (see lecture 1)

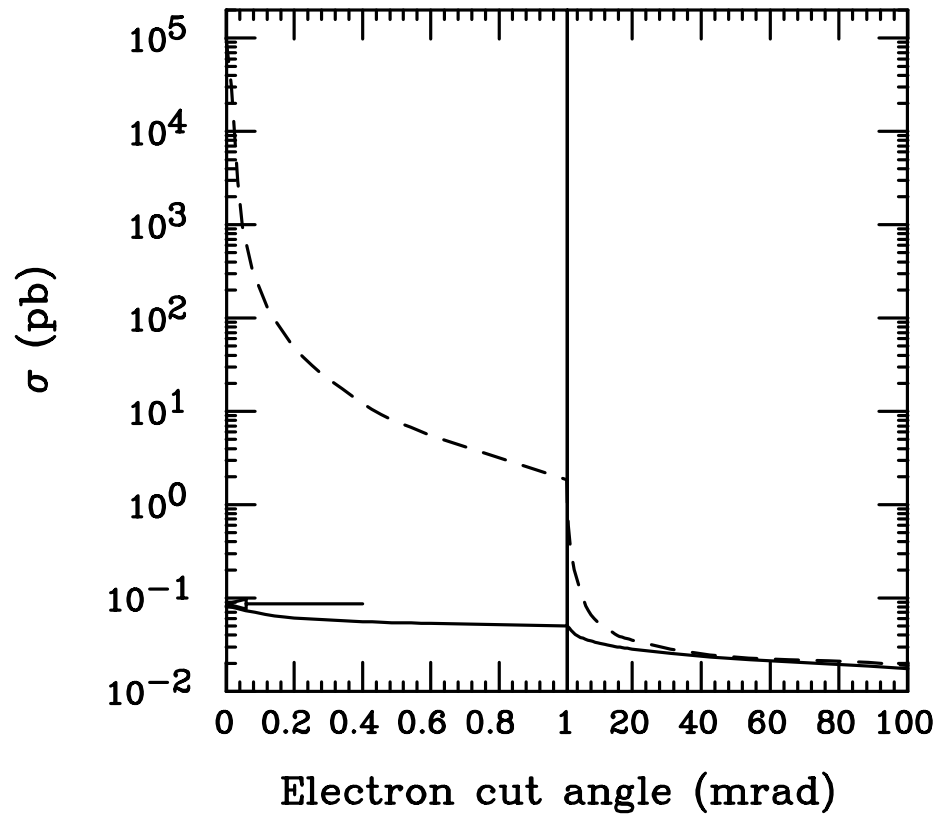
Consistent use of complex masses everywhere (including couplings)

$$\text{For W/Z bosons: } M_V^2 \rightarrow \mu_V^2 = M_V^2 - iM_V\Gamma_V, \quad V = W, Z$$

$$\text{complex weak mixing angle: } c_W^2 = 1 - s_W^2 = \frac{\mu_W^2}{\mu_Z^2}$$

\hookrightarrow gauge invariance fully respected

An example: $e^-e^+ \rightarrow e^- \bar{\nu}_e u \bar{d}$ result of Kurihara, Perret-Gallix, Shimizu '95



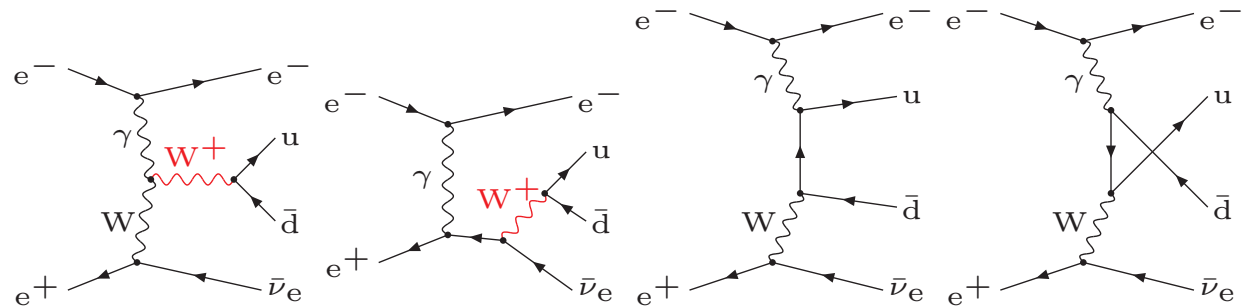
$$\sqrt{s} = 180 \text{ GeV}$$

solid: gauge-invariant
(fudge factor) scheme

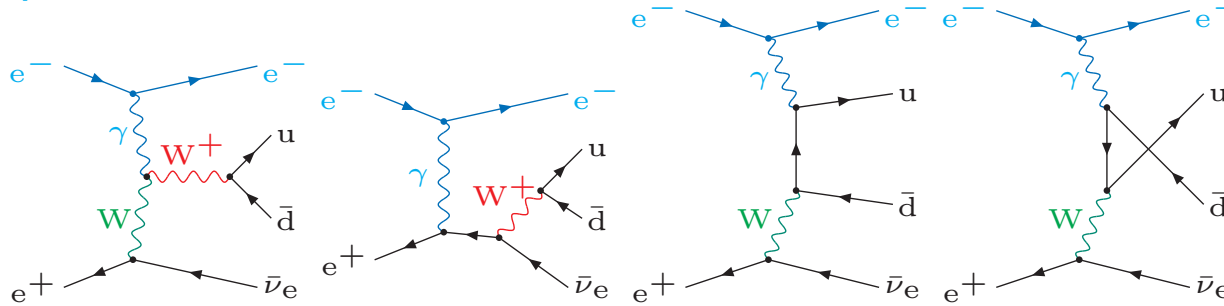
dashed: constant width
only in resonant propagator

↪ crude U(1) gauge-invariance
violation

Dominant diagrams:
nearly real photon !



Example continued:



Partial amplitude from above “photon diagrams”:

$$\mathcal{M}_\gamma = Q_e e \bar{u}_e(k_e) \gamma^\mu u_e(p_e) \frac{1}{k_\gamma^2} T_\mu^\gamma$$

Elmg. Ward identity:

$$0 \stackrel{!}{=} k_\gamma^\mu T_\mu^\gamma \propto (p_+^2 - p_-^2) Q_W P_W(p_+^2) P_W(p_-^2) + Q_e P_W(p_+^2) - (Q_d - Q_u) P_W(p_-^2)$$

With $Q_W = Q_e = Q_d - Q_u$ and $P_W(p^2) = [p^2 - M_W^2 + iM_W \Gamma_W(p^2)]^{-1}$
 one obtains: $\Gamma_W(p_+^2) \stackrel{!}{=} \Gamma_W(p_-^2)$

↪ Elmg. gauge invariance demands

common width on s - and t -channel propagators in “naive fixed width scheme”

Examples from e^+e^- physics: RACOONWW (Denner et al. '99-'01) and LUSIFER (S.D., Roth '02)

- σ [fb] for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$

\sqrt{s}	189 GeV	500 GeV	2 TeV	10 TeV
constant width	703.5(3)	237.4(1)	13.99(2)	0.624(3)
running width	703.4(3)	238.9(1)	34.39(3)	498.8(1)
complex mass	703.1(3)	237.3(1)	13.98(2)	0.624(3)

- σ [fb] for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu + \gamma$ (separation cuts for “visible” γ : $E_\gamma, \theta_{\gamma f} > \text{cut}$)

$\sqrt{s} =$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	224.0(4)	83.4(3)	6.98(5)	0.457(6)
running width	224.6(4)	84.2(3)	19.2(1)	368(6)
complex mass	223.9(4)	83.3(3)	6.98(5)	0.460(6)

- σ [fb] for $e^+e^- \rightarrow \nu_e\bar{\nu}_e\mu^-\bar{\nu}_\mu u\bar{d}$ (phase-space cuts applied)

\sqrt{s}	500 GeV	800 GeV	2 TeV	10 TeV
constant width	1.633(1)	4.105(4)	11.74(2)	26.38(6)
running width	1.640(1)	4.132(4)	12.88(1)	12965(12)
complex mass	1.633(1)	4.104(3)	11.73(1)	26.39(6)

Gauge-invariant width schemes @ NLO

Problem much more complicated than at LO ! (would fill own lectures)

Complex-Mass Scheme (CMS) Denner, S.D., Roth, Wieders '05

- complex, but straightforward renormalization
- NLO everywhere in phase space
- loop integrals with complex masses

Pole Approximation (PA) (= leading term of pole expansion)

- corrections decomposed into two types
 - ◇ factorizable: corrections to on-shell production / decay
 - ◇ non-factorizable: soft photon/gluon exchange between production / decays
- NLO in neighbourhood of resonances
- PA involves less diagrams than CMS → higher multiplicities possible

Effective Field Theories Beneke et al. '03,'04; Hoang,Reisser '04

- involves pole expansions → NLO in neighbourhood of resonances
- formal elegance → e.g. combination with resummations

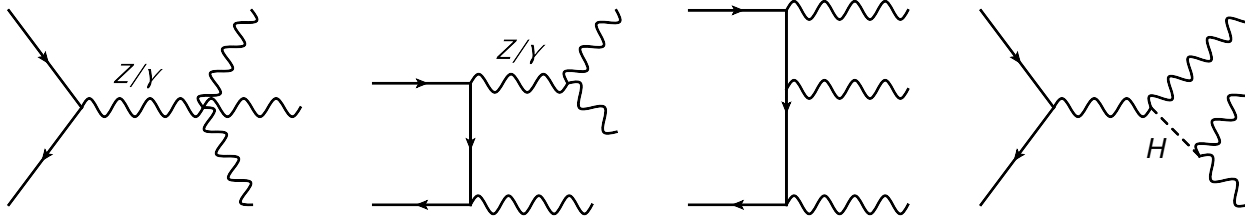
↪ **For details & examples see literature ...**

Outlook to electroweak tri-boson production



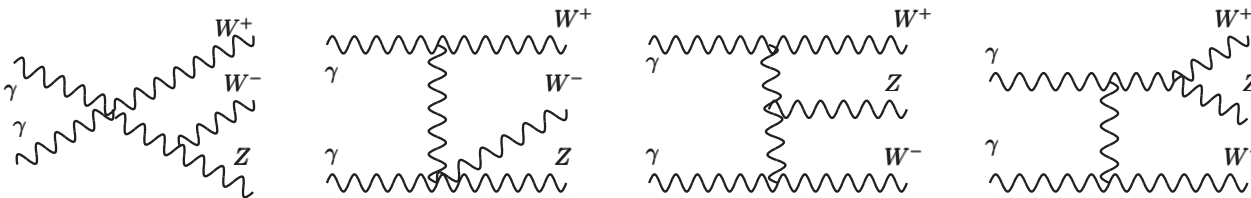
Electroweak tri-boson production – overview

Typical LO diagrams (example of WWZ production):



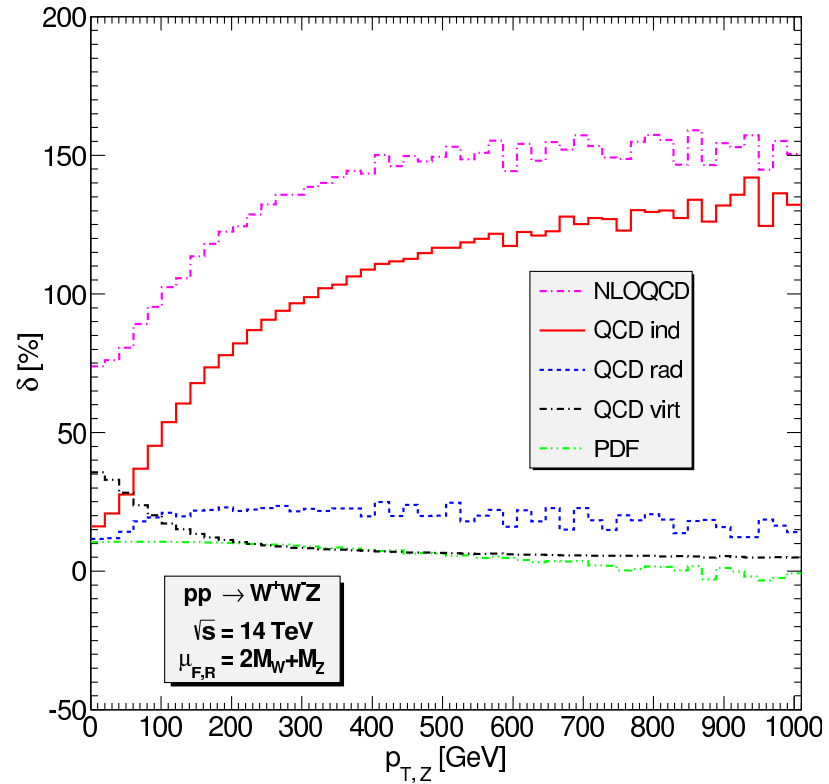
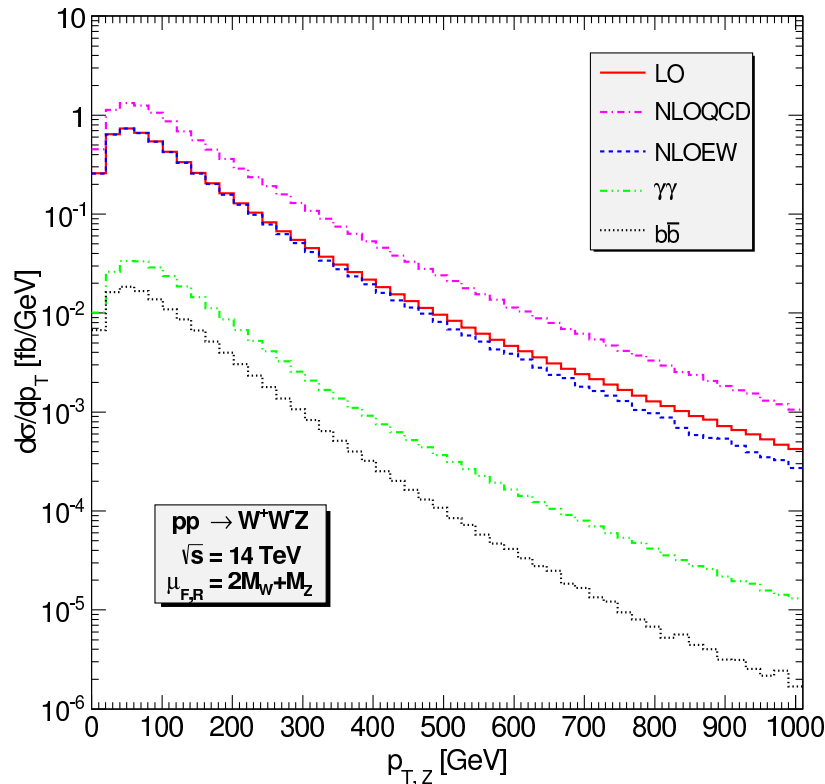
- **similarity/complementarity to vector-boson scattering** (crossed kinematics)
 - ◇ sensitivity to quartic gauge couplings
 - ◇ sensitivity to electroweak symmetry breaking
- background to WH/ZH production with $H \rightarrow VV^*$ (if accessible)

$\gamma\gamma$ channel for WWZ/ γ production:



QCD corrections to WWZ production

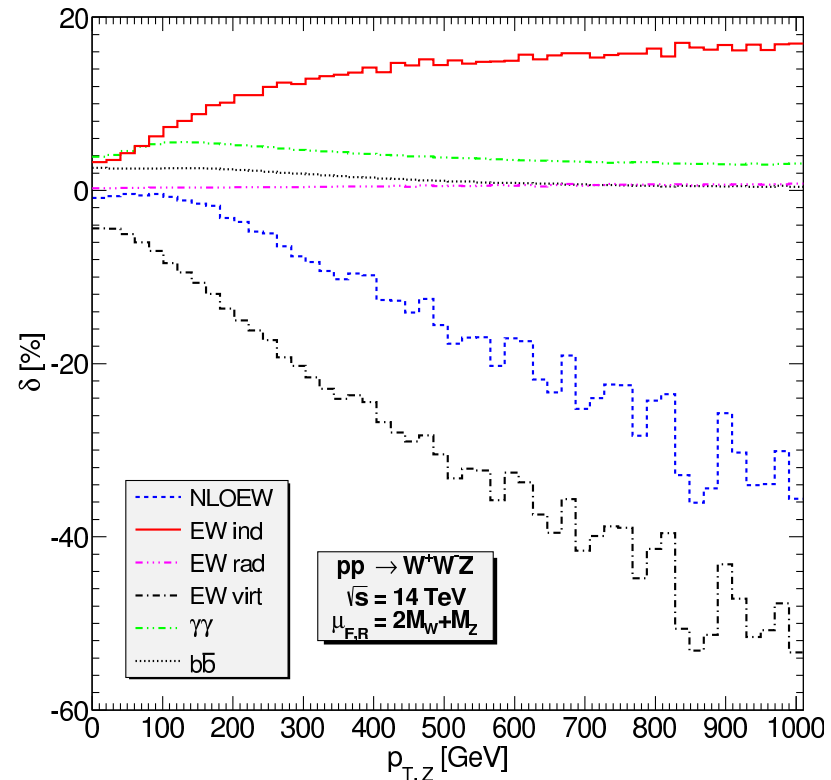
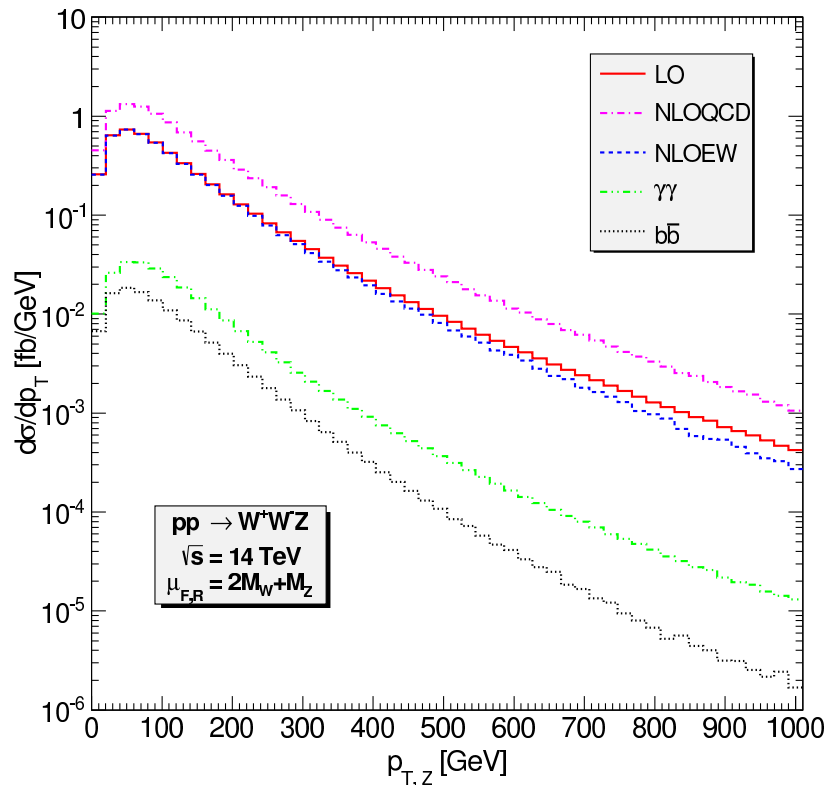
Nhung, Le, Weber '13



- inclusive cross section $\sim 200 \text{ fb}$ @ $\sqrt{s} = 14 \text{ TeV}$
- QCD corrections $\sim 100\%$
 - ◇ other final states WWW, WZZ, etc. known to NLO QCD as well
Lazopoulos, et al. '07; Hankele/Zepfenfeld '07; Binoth et al. '08; Nhung et al. '13
 - ◇ analyze possible jet vetoes
 - ◇ W/Z decays partially included in NWA

EW corrections to WWZ production

Nhung, Le, Weber '13



- sizeable EW corrections in TeV range (as expected from di-boson case)
- EW corrections only known for on-shell (stable) WWZ production
 - ↪ homework for theorists to ...
 - ◇ consider the other cases WWW, WZZ, etc. as well
 - ◇ include W/Z decays
 - ◇ combine NLO QCD (known) and EW corrections