Economics, Complexity and Agent Based Models

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Economics and Complexity

A relevant issue: Price Formation mechanisms

The Brock and Hommes Model
Economics and Complexity

What is Economics about?

Explaining emergence of order from disorder…
…in social phenomena

Disorder: self-interested and interacting agents
Order: some stable and persistent behaviour
Economics and Complexity

What is Economics about?

Explaining emergence of order from disorder...
...in social phenomena

- Disorder: self-interested and interacting agents
- Order: some *stable* and *persistent* behaviour
Examples

- How do market prices and interest rates emerge?
- How do some technological standards manage to dominate the market?
- How do GDP, employment and inflation move together along economic cycles?
- Why real and financial economy do not correlate across time but across episodes?
Economics and Complexity: from Disorder to Order

Micro Level

- Firms competing in turbulent markets
- Undertaking strategic decisions (output, investments, marketing, R&D, innovation, etc.)
A system is typically defined to be complex if it exhibits the following two properties:

- The system is composed of interacting units.
- The system exhibits emergent properties, that is, properties arising from the interactions of the units that are not properties of the individual units themselves.

(Flake, 1998; Tesfatsion and Judd, 2006)
Complex Systems - Firms R&D alliances
Complex Systems in Social Sciences: an Example

- Schelling segregation model (Schelling, 1971)
  - reds and blues live in a grid
  - they are happy if enough neighbours of same color, unhappy if not
  - at each period, one agent is randomly chosen:
    - if unhappy, moves in another place where she is happy
    - if happy, stays there
  - process repeats until everybody is happy or no more movements are possible
Schelling’s model - very tolerant people

to be happy: $10\%$ of neighbours of the same color
Schelling’s model - moderately tolerant people

- to be happy: 50% of neighbours of the same color
Schelling’s model - moderately intolerant people

- to be happy: 70% of neighbours of the same color
Schelling’s model - very intolerant people

- to be happy: 90% of neighbours of the same color
Schelling’s model - very intolerant people

- to be happy: 90% of neighbours of the same color
Simple Lesson from Schelling

Micro Properties ⇏ Macro Properties

- for moderate level of tolerance, segregation appears robustly
- for extreme level of (in)tolerance, segregation absent
The economy, both in broad and strict sense, is a complex system!
Features of (Social) Complex Systems

- Many micro entities
  - relatively simple and routinised behaviour

- People decisions might be affected by
  - Inherent difficulty in dealing with uncertainty and probability (risk)
  - Framing and Context matters
  - Adaptive (Trial & Error) and Simple Behavioral Rules
  - Problem decomposition (Rubik’s Cube)
Features of (Social) Complex Systems

- People exchange *locally* information, knowledge, goods

- Interaction Structures as *non-trivial networks*
  - Who owns who, boards of directors, …
  - Patent citations, collaboration citations, …
  - R&D joint-ventures, knowledge spillovers, …
  - Banks’ liabilities

- Persistently heterogeneous economic agents
Agent Based Models

An Agent Based Model (ABM) is a computational tool used to study the behaviour of complex systems composed by multiple agents that are:

▶ possibly heterogenous in all their characteristics
▶ boundedly rational (especially in economic applications)
▶ interacting among each other
How to model complex systems

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Agent Based Models

Target System

Agent based model

Entities

Agents

Interaction between entities

Interactions between agents
Why ABM?

Economics focus

Agents of change

Conventional economic models failed to foresee the financial crisis. Could agent-based modelling do better?
Agent Based Models: an Agent

Variables
- Employees (t)
- Capital (t)
- Sales (t)
- Cost
- Structure (t)
- [...]

Rules
- Production (t)
- Employment (t)
- Investment (t)
- Pricing (t)
- Marketing (t)
- [...]

Micro Pars
- Mark up
- Investment
- R&D
- Propensity
- [...]

Interactions
- Agent (t)
- Agent (t)
- [...]
- Strengths, etc.
Agent Based Models: an Economy

Time $t-1$
- Micro Vars

Macro Pars

Time $t$
- F1
- F2
- F3
- F4
- F5
- F6

Time $t+1$
- Time $t$ Micro Vars
- Macro Pars

Time $t-1$
- Macro Vars

Time $t$
- Time $t$ Macro Vars
Some Macro-oriented ABM

1. **Schumpeter meeting Keynes model** - Pisa Group

2. **EURACE** - Bielefeld/Genoa Groups

3. **CATS** - Milan/Ancona Group

4. **Housing Market Model** - Axtell et al.

5. **ENGAGE** - Dartmouth/Pisa Groups

6. **Macro-Finance model** - Brown Group
Schumpeter meeting Keynes (K+S)

- **objective**: study growth and business cycles dynamics
- **number of agents**: $\geq 500$
- **number of parameters**: $\geq 30$
- **time scale**: quarters

```
<table>
<thead>
<tr>
<th>Industry/Market</th>
<th>Number of Firms</th>
<th>Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine-tool Industry</td>
<td>$j=1,...,F_1$</td>
<td>- Perform R&amp;D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Produce heterogeneous machines</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use labor only to produce</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Each firm produces only a machine</td>
</tr>
<tr>
<td>Consumption-Good Industry</td>
<td>$i=1,...,F_2$</td>
<td>- Buy machines from MT industry</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use machine and labor to produce</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Sell products to consumers</td>
</tr>
<tr>
<td>Consumers/Workers</td>
<td>$n=1,...,N$</td>
<td>- Inelastically sell labor to firms</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Fully consume their income</td>
</tr>
<tr>
<td>Banking Sector</td>
<td></td>
<td>- Uses firm savings as deposits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Provide credit to consumption-good firms for investment and production</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Uses different rules to set credit supply</td>
</tr>
<tr>
<td>Public Sector</td>
<td></td>
<td>- Imposes taxes on firms’ profits</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Gives unemployment benefits</td>
</tr>
</tbody>
</table>
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Discrete-Time: $t=0,1,2,...$
objective: study of business cycles dynamics of EU economy (with spatial structure)

number of agents: >1600

number of parameters: >50

time scale: months

<table>
<thead>
<tr>
<th>Agent</th>
<th>Context</th>
<th>Role</th>
<th>Messages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household</td>
<td>Consumption goods market</td>
<td>Buyer</td>
<td>units demanded</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Worker</td>
<td>application, accept/reject job</td>
</tr>
<tr>
<td></td>
<td>Labour market</td>
<td>Depositor</td>
<td>cash holdings</td>
</tr>
<tr>
<td></td>
<td>Credit market</td>
<td>Investor</td>
<td>index share orders</td>
</tr>
<tr>
<td></td>
<td>Financial market</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm</td>
<td>Investment goods market</td>
<td>Buyer</td>
<td>units demanded</td>
</tr>
<tr>
<td></td>
<td>Consumption goods market</td>
<td>Seller</td>
<td>price, quality</td>
</tr>
<tr>
<td></td>
<td>Labour market</td>
<td>Employer</td>
<td>vacancy, job offer</td>
</tr>
<tr>
<td></td>
<td>Credit market</td>
<td>Borrower</td>
<td>loan request</td>
</tr>
<tr>
<td>Investment Goods Firm</td>
<td>Investment goods market</td>
<td>Seller</td>
<td>price, productivity</td>
</tr>
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<td></td>
<td>Labour market</td>
<td>Employer</td>
<td>vacancy, job offer</td>
</tr>
<tr>
<td>Bank</td>
<td>Credit market</td>
<td>Lender</td>
<td>credit conditions</td>
</tr>
<tr>
<td>Government</td>
<td>Public sector</td>
<td></td>
<td>tax payments</td>
</tr>
<tr>
<td>Central Bank</td>
<td>Credit market</td>
<td>Regulator</td>
<td>base interest</td>
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</table>
ENGAGE

- **objective**: study the transition towards a “green” economy and emissions paths
- **number of agents**: $>600$
- **number of parameters**: $>40$
- **time scale**: years
Simulation time

- Models are usually stochastic
- Monte Carlo runs of size at least 50 are typically required

- Simulation time for a complete MC exercise vary from:
  - few seconds
  - more then a week
Challenges with ABM

- computational time
- calibration/estimation
- validation
Calibration

- **Calibration** ≈ find a parameter vector that minimize some distance between real data and simulation output

“Even in our extremely simple model, with one parameter only, simulation time accounts for more than 50% of all estimation (calibration) time.”

Grazzini et al. (2015)
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> "Even in our extremely simple model, with one parameter only, simulation time accounts for more than 50% of all estimation (calibration) time."

Grazzini et al. (2015)

- the time required to estimate the model in Grazzini et al. (2015) is about **800 hours** on a 36 cores machine
Computational Time

- If a model has to be used by policy makers or regulators
  - ECB, FED
  - United States Securities and Exchange Commission
Computational Time

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- it has to provide *timely insight* into the problem
Computational Time

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- it has to provide **timely insight** into the problem

- Models that take **too long** to run and produce data that is **too large** are of limited interest for such users
Our issue: Behaviour of Prices
Price Behaviours - some questions for an economist

- Can we model a pricing system such that returns show some of the observed behaviours?
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- We are going to win the Nobel prize
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- We are going to win the Nobel prize or make a lot of money
Prices and Returns: basics

- Let $p(t)$ be the **price** of an asset at time $t$, then

- $r_\tau(t) = \frac{p(t + \tau) - p(t)}{p(t)} \approx \ln p(t + \tau) - \ln p(t)$ is the **return** over the period $\tau$

- $r_\tau(t) - m\tau$, where $m\tau$ is the mean return at scale $\tau$, is the **normalized return** over the period $\tau$. 
Behaviour of Price is Complex

- linear growth of variance with time scale:
  \[ \langle [r_\tau(t) - m\tau]^2 \rangle \simeq \sigma^2 \tau \]

- distribution of returns has power law tails:
  \[ |r|^{-1-\mu} \]

(Ibex35 data at different time scales); y-axis in log
Behaviour of Price is Complex

- volatility clustering

(absolute value of SP500 returns for 100, 10, 1 year)
Source: Borland et al. (2005)

- and a lot of others features (multifractality, leverage effects...)
Price Behaviour

- Random Walks and Brownian Motion (Bachelier, 1900)
  - returns are i.i.d.; the underlying distribution is normal
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- Eugene Fama
  - Efficient Markets
    - (strong form) all information is reflected by prices
    - implicit rationality of traders
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- Eugene Fama
  - Efficient Markets
    - (strong form) all information is reflected by prices
    - implicit rationality of traders
  - The joint hypothesis
    - market equilibrium hypothesis ⇔ market efficiency
    - a challenge for the price formation mechanism!
A simple asset pricing model

The Brock and Hommes model

Key references:


Basic structure and time-line of events

- 1 risky asset, 1 risk-free asset, $N$ traders of different type

1. history of prices and dividends is observed
2. agents form their expectation on next period prices
3. each agent submit her sell/buy orders
4. market clears and asset prices are determined in equilibrium
5. dividends are paid to stockholders
The BH model - trader types

- trend followers
The BH model - trader types

▶ trend contrarians
The BH model - trader types

- both types might have a bias towards some value
Agents’ trading strategy is determined by a function $f_h(\cdot)$

- Rational: $f_{Rt} = x_{t+1}$
- All other types: $f_{ht} = g_h x_{t-1} + b_h$
  - Trend chasers $g_h > 0$
  - Trend contrarians $g_h < 0$
  - Fundamentalists $g_h = b_h = 0$
The BH model

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- Agents might switch their type according to accumulated past profits and a switching parameter
  - $n_{ht} = \exp[\beta U_{h,t-1}] / \sum_h \exp[\beta U_{h,t-1}]$
The BH model - I

- agents’ wealth evolves according to
  - \( W_{t+1} = RW_t + (p_{t+1} + y_{t+1} - Rp_t)z_t \)
The BH model - I

▶ agents’ wealth evolves according to
  \[ W_{t+1} = RW_t + (p_{t+1} + y_{t+1} - Rp_t)z_t \]

▶ Equilibrium of demand and supply implies (no supply of external shares)
  \[ Rp_t = \sum n_{ht}E_{ht}(p_{t+1} + y_{t+1}), \text{ where} \]

▶ agent of type \( h \) forms expectations on future price and dividend
  \[ E_{ht}(p_{t+1} + y_{t+1}) = E_t(p^*_{t+1}) + f_h(x_{t-1}, \ldots, x_{t-L}), \text{ where} \]
  \[ p^* \text{ denotes the fundamental price} \]
  \[ f_h(\cdot) \text{ is a deterministic function depending on the agent’s type} \]
  \[ x_t = p_t - p^*_t \text{ denotes the price deviation from the fundamental} \]
The BH model - returns dynamics
Can we calibrate the model in a way that it resembles real-world return dynamics?
Our target: S&P 500 - long run dynamics
Our target: distribution of last year returns
Distribution of returns: “calibrated” model vs. real data
MC runs of “calibrated” model
Challenge

Can we do better?

Can we calibrate avoiding/reducing the computational burden of simulations?
THANKS !!!