ADAPTIVE MULTILEVEL SPLITTING FOR MONTE CARLO PARTICLE TRANSPORT

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PhD at CEA since October 1st 2014, directed by Cheikh DIOP (CEA) and Tony LELIÈVRE (CERMICS-ENPC) and supervised by Eric DUMONTEIL.

**Goal:** Adapt a mathematical Monte Carlo variance reduction technique to the field of particle transport. Applications to shielding simulations in the code TRIPOLI-4®.
Monte Carlo particle transport

- Monte Carlo transport simulations
- Variance reduction

Adaptive Multilevel Splitting

- The AMS algorithm
- Implementation

Results

- Implementation validation
- Comparison between AMS and TRIPOLI-4®

Conclusion
MONTE CARLO PARTICLE TRANSPORT
Goal of Monte Carlo particle transport simulations:
- Estimate a score (flux) in a volume of interest

How is it done?
- $n$ particles are simulated
- The $i$-th particle contribution to the score is stored as $\hat{\phi}_i$
- We define the **average flux** and its associated variance:

\[
\bar{\phi} = \frac{1}{n} \sum_{i=1}^{n} \hat{\phi}_i \\
\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (\hat{\phi}_i - \bar{\phi})^2
\]
If the attenuation is really strong:
- Many particles will not reach the volume of interest
- Their contributions will be null
- The variance or the computation time will explode

Variance reduction techniques:
- Modify the simulation behavior
- Reduce the variance **for a given computation time**
THE AMS METHOD:
ALGORITHM
AMS: Adaptive Multilevel Splitting

- **Theory:**
  

- **Application in molecular dynamics:**
  
AMS: Adaptive Multilevel Splitting

- **Theory:**
  

- **Application in molecular dynamics:**

AMS is a population control algorithm. It takes place at the end of the simulation.

A free parameter:

- An importance function denoted $I$:
  \[
  I : \mathbb{R}^6 \rightarrow \mathbb{R}
  \]

- Associates an importance value to any point of the phase space.
THE AMS METHOD

First step
- $n$ particles are simulated

AMS iterations:
- Each particle track is given a note according to $I$
- The less interesting particles is suppressed
- One of the remaining particles is splitted
- We get a new set of $n$ particles:
  - 1 new replica
  - $n-1$ particles that were not suppressed
Stopping criterion:
- When the less interesting particle is in the volume of interest
- The total number of iterations is denoted $N$
- The probability of reaching the volume of interest is estimated by:

$$\alpha = \left(1 - \frac{1}{n}\right)^N$$

An **unbiased** estimate of the score is computed using the last generated points and weighting the result by $\alpha$
THE AMS METHOD: IMPLEMENTATION
TRIPOLI-4®:

- 3D continuous-energy Monte Carlo particle transport code
- Dedicated to shielding, reactor physics, criticality, safety and nuclear instrumentation
- Developed at CEA Saclay since the mid-60s

Existing variance reduction technique:

- “Exponential transform”
- Also uses an importance map
- Has a module that estimates the importance from the geometry
THE AMS METHOD

Example of geometry

Associated importance map
RESULTS
Illustration of the variance reduction use:

- We want to estimate the neutron flux attenuation when traversing 3m of water and 1m of concrete.
- The neutron source is mono-directional at the entrance of a box with perfect reflectors around it.
- The flux is estimated every 20cm between the source and the detector.
RESULTS

Neutron flux and associated standard deviation obtained with **AMS**, compared to an **analog** calculation (i.e. without variance reduction)

**OBSERVATIONS:**

- **Very good accordance** with the reference near the source
- The analog simulation has **no results** deeper than 2m
- AMS is able to yield results **all the way** to the detector (and probably beyond)
Neutron flux and associated standard deviation obtained with AMS, compared to an analog calculation (i.e. without variance reduction)
RESULTS

Neutron flux and associated standard deviation obtained with AMS, compared to an analog calculation and the current variance reduction method of TRIPOLI-4®

OBSERVATIONS:

- Very good accordance between AMS and the exponential transform near the detector
- The exponential transform results away from the detector can't be trusted
- The importance map used to get results with exponential transform had to be optimized
Neutron flux and associated standard deviation obtained with AMS, compared to an analog calculation and the current variance reduction method of TRIPOLI-4®.
CONCLUSION
CONCLUSION

Implementation:
- The AMS implementation in TRIPOLI-4® is over and stable
- The method will be available in the developer's version of the code soon

Testing:
- The AMS efficiency is tested in many problems, from simple cases to full nuclear cores
THANK YOU FOR YOUR ATTENTION