A Snowball’s Chance in Hell: The Hierarchy Problem in Particle Physics

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What is the Hierarchy Problem?

Figure: Magnet levitating above a superconductor.

How is it possible that an electromagnetic force coming from a cm-sized superconductor manages to overcome the gravitational pull from the \((6 \times 10^3 \text{ km radius})\) Earth?
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**A Classical Version: The Electron Self-Energy**

- Electron mass = "bare mass" + Coulomb contribution (energy cost for assembling a thin spherical shell of radius \(r_e\) and electrical charge \(e\)):

\[
(m_e c^2)_{\text{phys}} = (m_e c^2)_{\text{bare}} + \Delta E_{\text{EM}}, \quad \Delta E_{\text{EM}} \propto \frac{e^2}{r_e}.
\] (1)

- Experiments indicate that \(r_e < 10^{-17}\) cm, for which \(\Delta E_{\text{EM}} \simeq 10^4\) MeV. But \((m_e c^2)_{\text{phys}} \simeq 0.511\) MeV! \(\Rightarrow\) The bare mass needs to be fine-tuned:

\[
0.511\ \text{MeV} = -9999.489\ \text{MeV} + 10000\ \text{MeV}.
\] (2)

- Solution: add the positron and take into account quantum effects \(\rightarrow\) virtual electron-positron pairs spontaneously created in the electron’s EM field "smear out" the electron’s charge.

\[
\Delta E = \Delta E_{\text{EM}} + \Delta E_{\text{pair}} \propto \alpha m_e c^2 \log \frac{\hbar c/r_e}{m_e c^2} \Rightarrow
\]

\[
(m_e c^2)_{\text{phys}} = (m_e c^2)_{\text{bare}} \left[ 1 + \text{const.} \times \alpha \log \frac{\hbar c/r_e}{m_e c^2} \right]
\] (3)

- Morality:
  1) for \(r_e \lesssim 10^{-13}\) cm or \(\hbar c/r_e \equiv \Lambda \gtrsim m_e c^2\), classical EM no longer provides an accurate description of nature;
  2) \((m_e c^2)_{\text{phys}} \propto (m_e c^2)_{\text{bare}}\) because of chiral symmetry.
A More Precise Statement of the Hierarchy Problem

The Quantum Field Theory Version

Figure: Standard Model Particles.
Chiral symmetry and gauge symmetry protect fermion and gauge boson masses respectively (in natural units: $\hbar = c = 1$):

$$m_{\text{phys}} - m_{\text{bare}} \propto m_{\text{bare}} \log \frac{\Lambda}{m} \quad (5)$$

No symmetry protecting the Higgs mass $\rightarrow$ correction proportional to $\Lambda$ (the cutoff):

$$m_{h,\text{phys}}^2 - m_{h,\text{bare}}^2 \propto \Lambda^2 \quad (6)$$

Higgs mass measured to be $\sim 125$ GeV.

Assume the Standard Model is valid up to $\Lambda \sim M_{P1} \sim 10^{18}$ GeV $\Rightarrow$ huge fine tuning needed:

$$(125 \text{ GeV})^2 \simeq m_{h,\text{bare}}^2 + 10^{36} \text{ GeV}^2. \quad (7)$$
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Figure: Minimal Supersymmetric Standard Model (MSSM) particle content.
Some Resolutions

Supersymmetry

The top and stop (scalar top) contributions to the Higgs mass give

$$m_{h,\text{phys}}^2 - m_{h,\text{bare}}^2 \propto (m_t^2 - m_{\tilde{t}}^2) \log \frac{\Lambda}{m_{\tilde{t}}}.$$  \hspace{1cm} (8)

Mass corrections are no longer $\propto \Lambda^2 \Rightarrow$ fine-tuning not needed anymore (unless $m_t^2 - m_{\tilde{t}}^2 \gg m_h^2$)!

Figure: Top and stop (scalar top) contributions to the Higgs mass.
Composite Higgs

- Composite Higgs theories postulate that the Higgs scalar is a bound state of a new strong interaction.
- The Higgs can be thought of as a meson → analogy with the strong nuclear interaction.
- At higher energies, one does not "see" the Higgs anymore, but its constituents → the concept of Higgs mass is no longer defined at very high energies ⇒ no fine tuning!
- Prediction → other bound states (conceptually similar to resonances in nuclear physics).

Figure: Left: A $\pi^+$ meson's substructure. Right: various mesons.
Extra Dimensions

World looks 3D ⇒ extra dimensions should be microscopical.

Two classes of extra dimensions → flat or warped.

Main idea → one still has $m_{h,\text{phys}}^2 - m_{h,\text{bare}}^2 \propto \Lambda^2$, but the cutoff $\Lambda$ is expected to be of order $10^3$ GeV (the extra dimension(s) become "visible" at an energy scale close to $\Lambda$) ⇒ no fine tuning.

Gravity appears weaker because:
1) it gets “diluted” (“leaks”) into the flat extra dimension(s);
2) the high curvature (warping) of the extra dimension creates an exponential hierarchy between the weak and gravitational interactions.

Figure: Warped extra dimension.
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The Hierarchy/Fine-Tuning Problem has been a fruitful playground in particle physics: several interesting ideas were put forward as solutions to it (supersymmetry, extra dimensions etc.).

Resolution of this problem implies the appearance of new particles in the TeV range → the LHC is actively probing their existence.

Nothing new found at the LHC till now, but there is still hope!


Figure: The Large Hadron Collider (LHC).
Thank you for your attention!