New Ideas on the Hierarchy Problem

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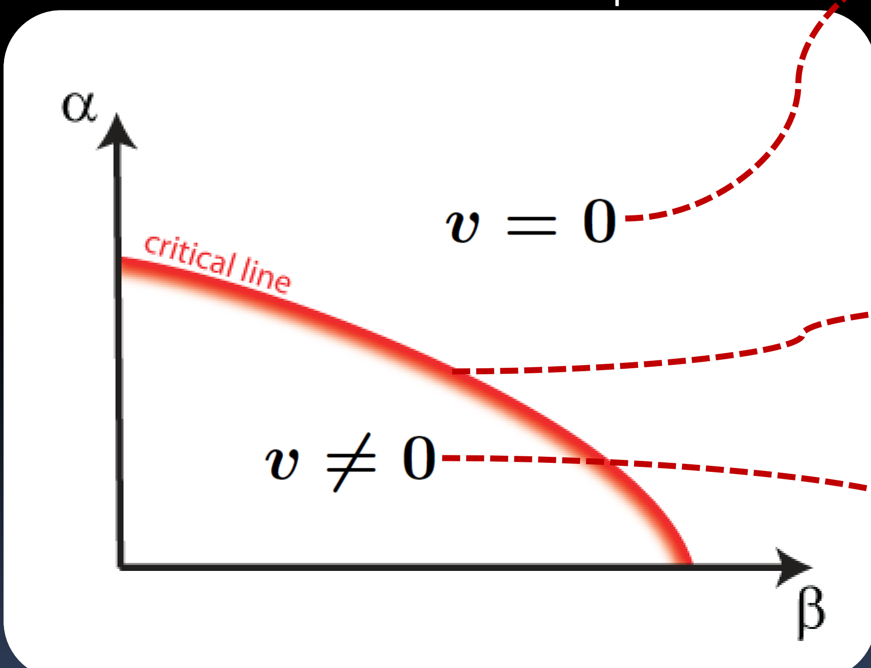
DESY

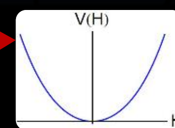
The Hierarchy Problem

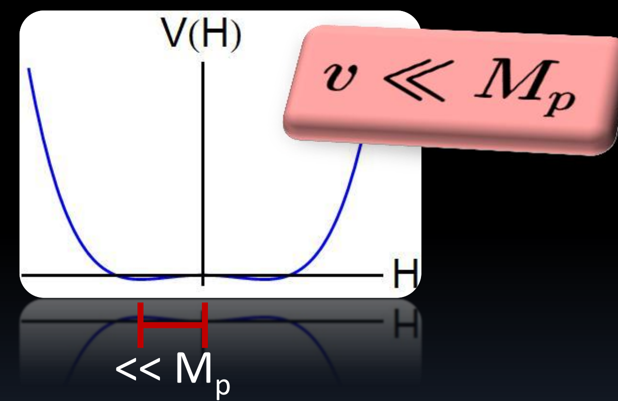
Think about the Standard Model (SM) as an EFT with a cut-off at M_p:

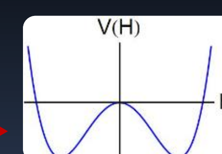
$$V(H) = m_H^2(\alpha,\beta)H^2 + \lambda h^4 + \mathcal{O}(1/M_p^2) \qquad \langle H \rangle = v$$

The only mass scale is M_p!









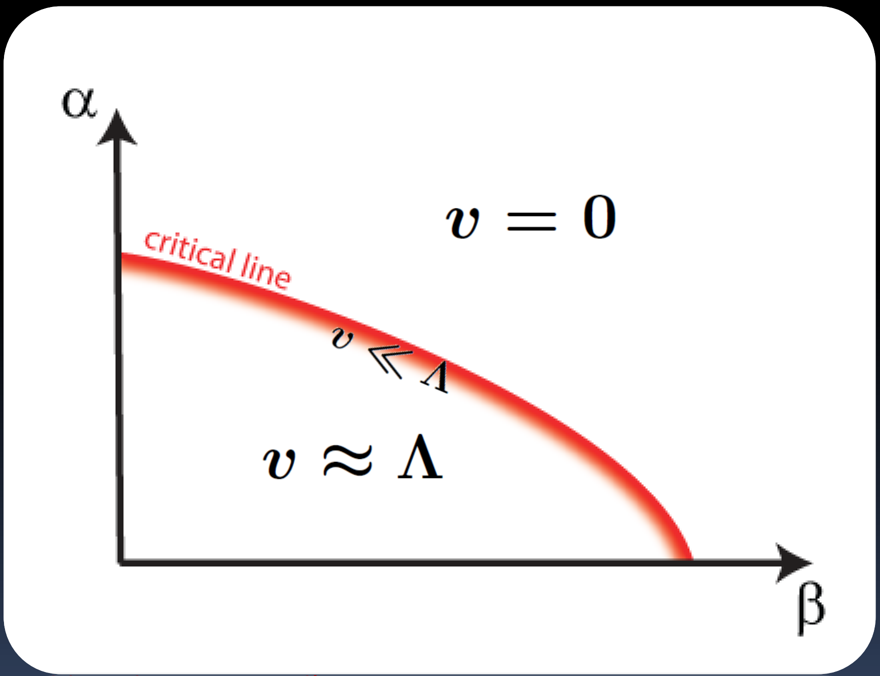
(inspired by Alex Pomarol)

Solving the Hierarchy Problem

Question: how come we live so close to the line?

Two answers: (1) Some symmetry forces it! (SUSY)

(2) The cut-off Λ, is not really M_p. In fact $\Lambda << M_p$ and $\Lambda \sim 1$ TeV
(Composite Models, Extra Dimensions et al.)



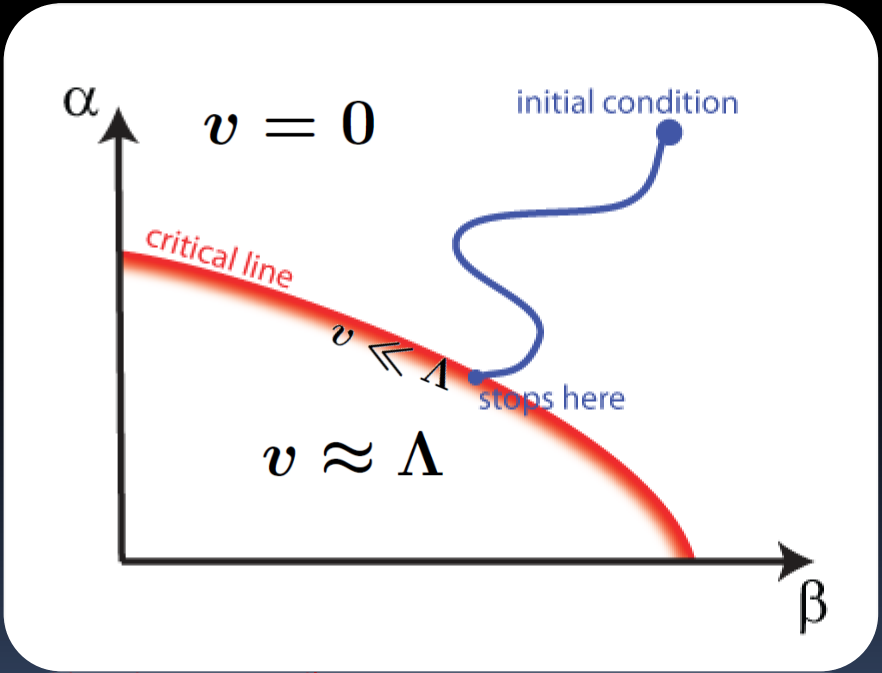
Both DEMAND new physics @ ~TeV

(inspired by Alex Pomarol)

Solving the Hierarchy Problem

Question: how come we live so close to the line?

The Third Way: (3) History! Make α and β dynamical (fields in fact)



But how does the evolution stop?

(inspired by Alex Pomarol)

Graham et al.
arXiv: 1504.07551
Espinosa et al.
arXiv: 1506.09217

The Relaxion

The minimal model:

$$V(\phi, H) = \Lambda^3 g\phi - \frac{1}{2}\Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right) H^2 + \epsilon\Lambda_c^2 H^2 \cos(\phi/f)$$

Linear slope for ϕ

½ m_H^2

Local minima in ϕ

Both g and ε break shift symmetries (more about that later) and can be naturally small !

Λ is the cut-off for the SM

Λ_c is the scale at which the periodic potential is generated

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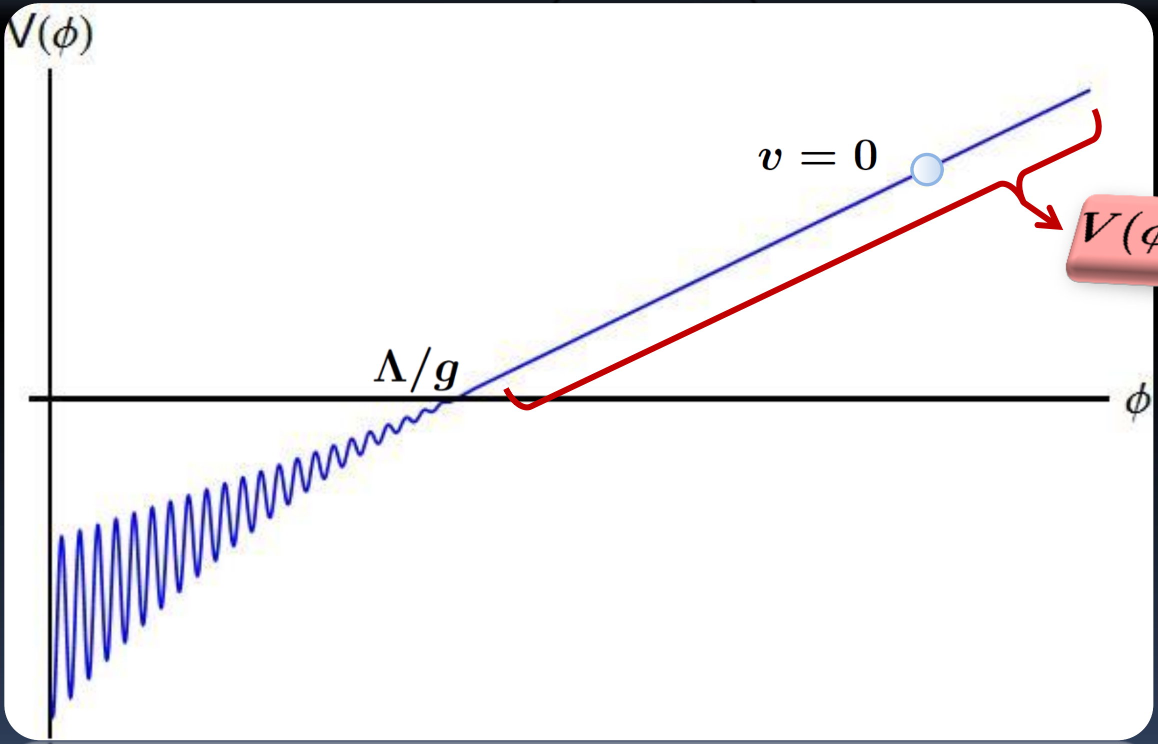

$V(\phi, H = 0) = \Lambda^3 g\phi$

The Relaxion

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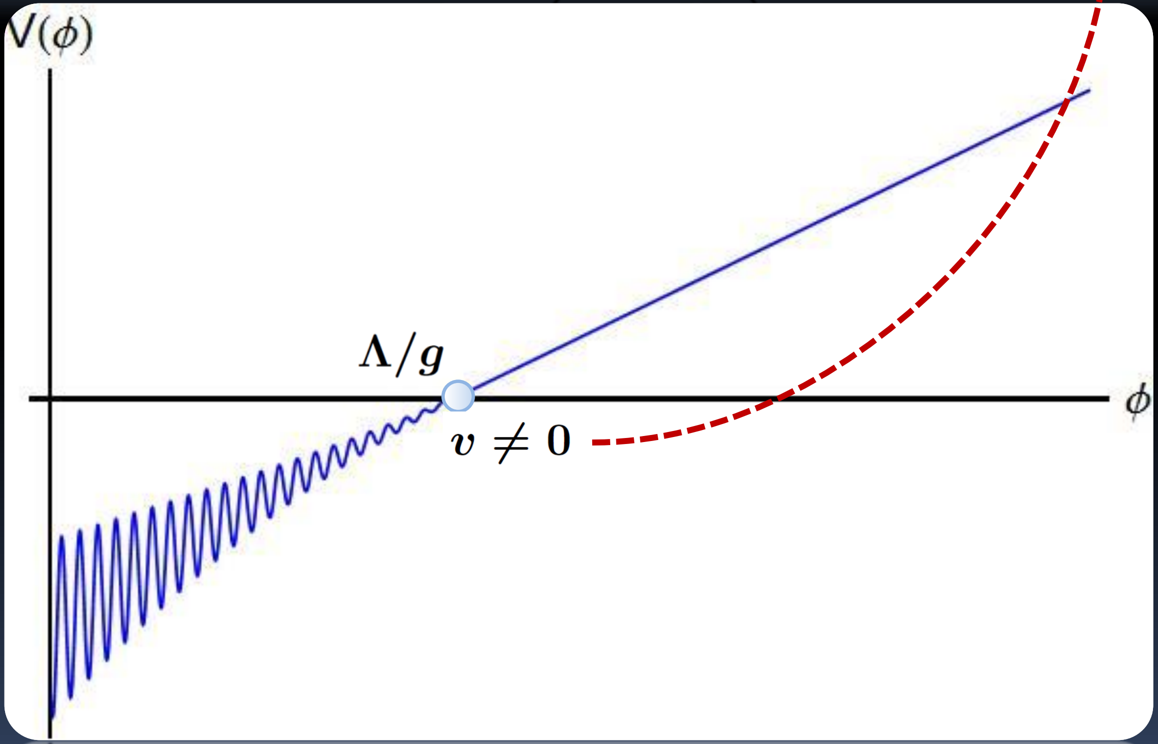

Becomes more important as *v* grows

Graham et al.
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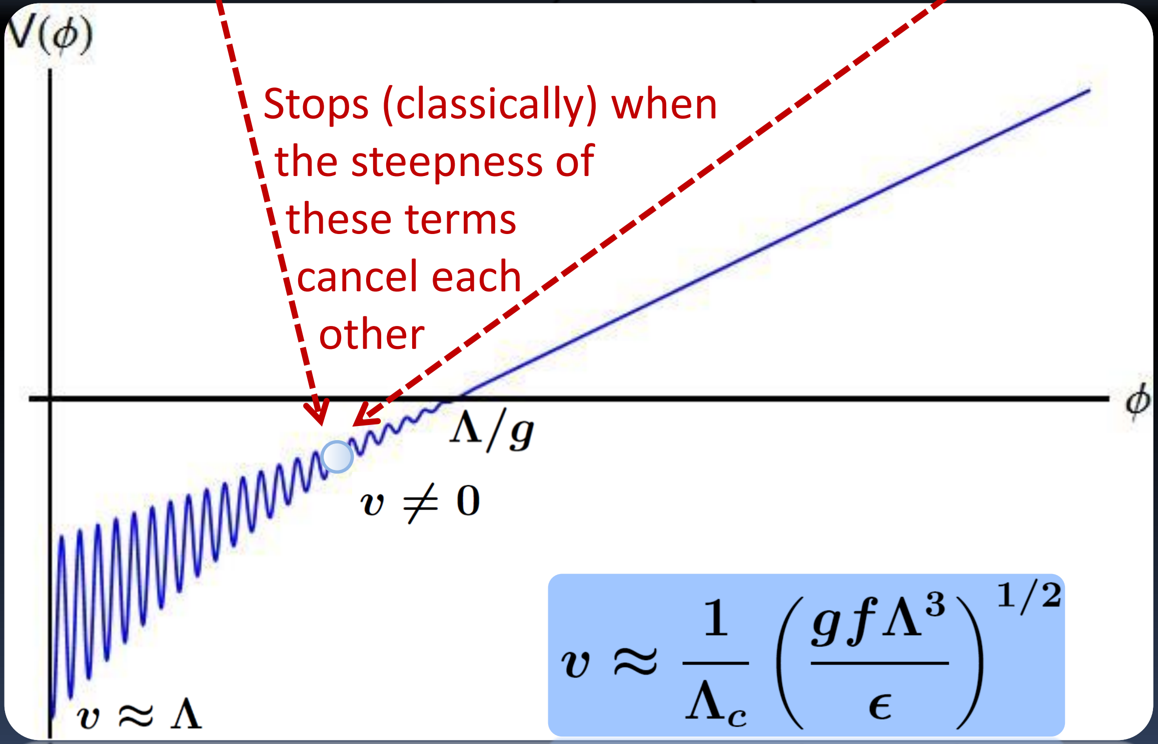

The overall slope is controlled by *g*.

$v \ll \Lambda$

⇕

$g \ll 1$

Technically Natural!

NO NEW PHYSICS

close to *v*

The Relaxion

Graham et al.
arXiv: 1504.07551
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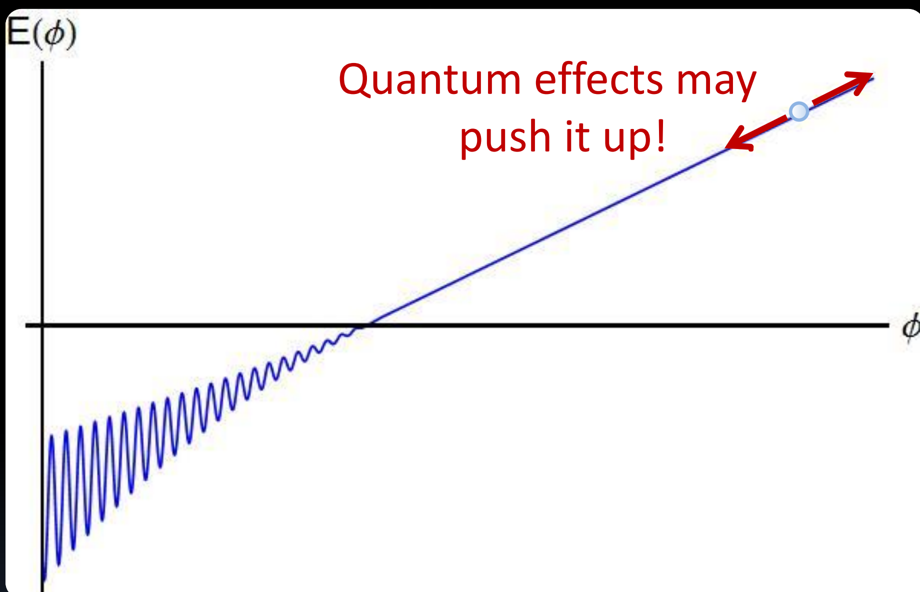
Do we need to start close to ϕ_c? NO, if slow rolling (during an inflationary epoch).

Limitations: Inflation ⇨ de Sitter space ⇨ Temperature (from Horizon)



$$\Delta\phi_{\rm class} \sim \frac{V'(\phi)}{H_I^2} = \frac{g\Lambda^3}{H_I^2}$$

$$\Delta\phi_{\rm quant} \sim H_I$$

$$\Delta\phi_{\rm class} > \Delta\phi_{\rm quant}$$

$$g > (H_I/\Lambda)^3$$

$$g > (\Lambda/M_p)^3$$

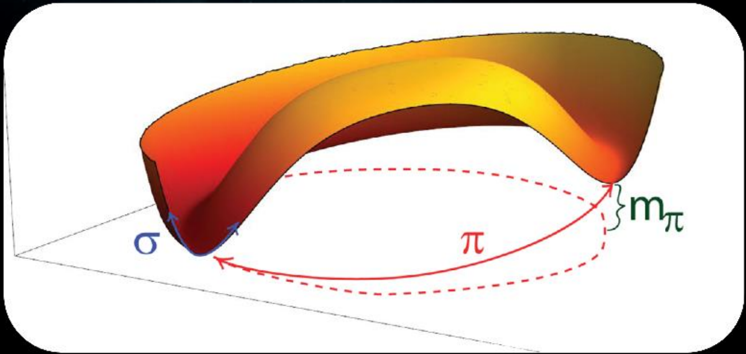
ϕ is not the inflaton

$$V(\phi \sim \Lambda/g) \approx \Lambda^4$$

$$V(\phi) < V_I \approx H_I^2 M_p^2$$

$$H_I > \frac{\Lambda^2}{M_p}$$

Symmetries



pNGB ⇨ $m_\pi < m_\sigma$

Effective theory below m_σ: non-linear sigma model

$$\Sigma = e^{i\frac{T^a \pi^a}{f}} = \cos\left(\frac{\pi}{f}\right) + i\frac{T^a \pi^a}{\pi} \sin\left(\frac{\pi}{f}\right)$$

$$\pi = \sqrt{\pi^a \pi^a}$$

What about *g ≠ 0*? (non-periodic terms) $-\Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2$

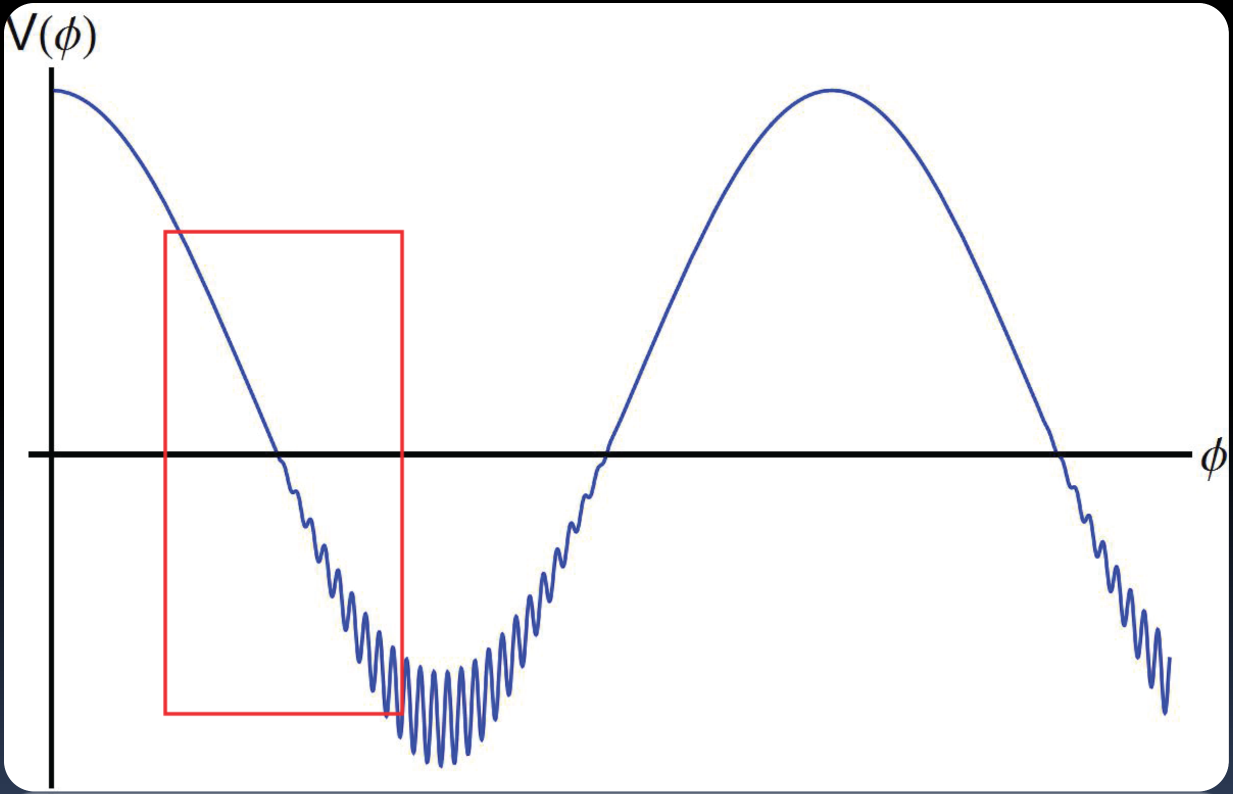
→ Makes the field space non-compact

→ The discrete shift symmetry cannot be broken by local operators
(it is a redundancy in the description, a gauge symmetry)

Symmetries

$$V(\pi, H) \sim \kappa_1(H^2)\cos\left(\frac{\pi}{F}\right) + \kappa_2(H^2)\cos\left(\frac{\pi}{f}\right)$$

$F \gg f$



But how can we get the same pNGB to have two very different periods (compact field spaces) ?

Clockwork Relaxion

A Clockwork Axion
Kaplan, Rattazzi,
arXiv:1511.01827

Key element: many pNGBs with the same decay constant f:

$$\mathcal{L}_{pNGB} = f^2 \sum_{j=0}^{N} \partial_\mu U_j^\dagger \partial^\mu U_j + \left(\epsilon f^4 \sum_{j=0}^{N-1} U_j^\dagger U_{j+1}^3 + h.c. \right) + \cdots$$

$U_j \equiv e^{i\pi_j/(\sqrt{2}f)}$

$U(1)^{N+1}$ $U(1)^{N+1} \rightarrow U(1)$

$Q_{j+1} = Q_j/3$

$$\mathcal{L}_{pNGB} = \frac{1}{2} \sum_{j=0}^{N} \partial_\mu \pi_j \partial^\mu \pi_j + \epsilon f^4 \sum_{j=0}^{N-1} e^{i(3\pi_{j+1} - \pi_j)/(\sqrt{2}f)} + h.c. + \cdots$$

$$V^{(2)} = \frac{1}{2} \epsilon f^2 \sum_{j=0}^{N} (q\pi_{j+1} - \pi_j)^2$$

$$\pi^{(0)} \sim \left(\pi_0 + \frac{1}{3}\pi_1 + \frac{1}{9}\pi_2 + \ldots + \frac{1}{3^N}\pi_N \right)$$

$$V(\pi^{(0)}) \sim \Lambda_N^4 \cos(\pi^{(0)}/F) + \Lambda_0^4 \cos(\pi^{(0)}/f)$$

$$F = 3^N f$$

Nayara Fonseca, Leonardo de Lima, Camila S. Machado, R.D.M.
arXiv: 1601.07183

N-Relaxion

Kaplan-Rattazzi clockwork axion:

$$\epsilon f^4 \sum_{j=0}^{N-1} U_j^\dagger U_{j+1}^3$$

$$U_j \equiv e^{i\pi_j/(\sqrt{2}f)}$$

$$Q_{j+1} = Q_j/3$$

$$F = 3^N f$$

In principle, it could be interpreted as an extra dimension at large N, however, there's no continuum limit!

Goals:

- Find a model closer to a dimensional deconstruction that: (i) has a relaxion and (ii) provides a effective scale *F* much greater than *f*.
- Generalize to non-abelian symmetries

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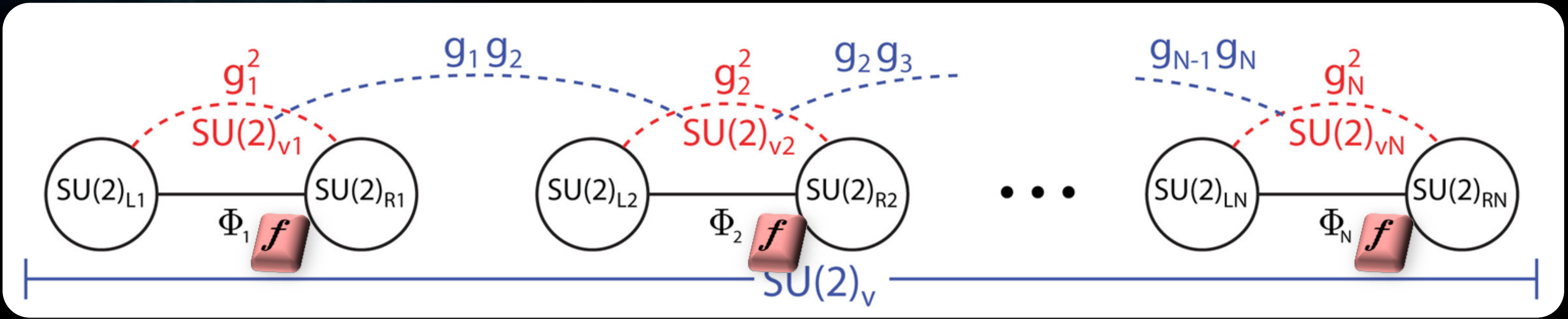
N-Relaxion



$$\sum_{j=1}^{N} \mathrm{Tr}\left[\partial_\mu \Phi_j^\dagger \partial^\mu \Phi_j + \frac{f^3}{2}(2-\delta_{j,1}-\delta_{j,N}) g_j^2 \left(\Phi_j + \Phi_j^\dagger\right)\right] - \frac{f^2}{2}\sum_{j=1}^{N-1} g_j g_{j+1} \mathrm{Tr}\left[(\Phi_j - \Phi_j^\dagger)(\Phi_{j+1} - \Phi_{j+1}^\dagger)\right]$$

Small symmetry breaking parameters

$$g_j \to q^j, \quad 0 < q < 1$$

$$q = \frac{g_{j+1}}{g_j}$$

$$\sum_{j=1}^{N}\left[\frac{1}{2}\partial_\mu \vec{\pi}_j \cdot \partial^\mu \vec{\pi}_j + f^4 (2-\delta_{j,1}-\delta_{j,N}) g_j^2 \cos\left(\frac{\pi_j}{f}\right)\right] + f^4 \sum_{j=1}^{N-1} g_j g_{j+1} \frac{\vec{\pi}_j \cdot \vec{\pi}_{j+1}}{\pi_j \pi_{j+1}} \sin\left(\frac{\pi_j}{f}\right) \sin\left(\frac{\pi_{j+1}}{f}\right)$$

Quadratic (mass) terms everywhere, diagonalization needed

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N-Relaxion

$$M_\pi^2 = f^2 \begin{pmatrix} q^2 & -q^3 & 0 & \dots & 0 & 0 \\ -q^3 & 2q^4 & -q^5 & \dots & 0 & 0 \\ 0 & -q^5 & 2q^6 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2q^{2(N-1)} & -q^{2N-1} \\ 0 & 0 & 0 & \dots & -q^{2N-1} & q^{2N} \end{pmatrix}$$

$$\vec{\eta}_0 = \sum_{j=1}^{N} \frac{q^{N-j}}{\sqrt{\sum_{k=1}^{N} q^{2(k-1)}}} \vec{\pi}_j$$

(massless at tree level, loops induce: $m = f^2 q^{2N}$)

Same as the Wilson Line in deconstructed AdS_5!

$$\mathcal{L}_\eta = \sum_{j=1}^{N} \left[\frac{1}{2} \partial_\mu \vec{\eta}_0 \cdot \partial^\mu \vec{\eta}_0 + f^4 (2 - \delta_{j,1} - \delta_{j,N}) q^{2j} \cos \frac{\eta_0}{f_j} \right] + \sum_{j=1}^{N-1} f^4 q^{2j+1} \sin \frac{\eta_0}{f_j} \sin \frac{\eta_0}{f_{j+1}}$$

Amplitudes are also controlled by *q*

Bigger frequencies ↔ smaller amplitudes

(only the first few really matter)

$$f_j \equiv f q^{j-N} \mathcal{C}_N$$

$\mathcal{C}_N \approx 1$

→ $f_N \approx f$

→ $F = f_1 \approx f / q^{N-1}$

$V(\eta_0)$ gets flat for $q \ll 1$

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N-Relaxion

Interaction with the Higgs:

$$\mathcal{L}_{\eta,H} = \left(1 + \frac{|H|^2}{\Lambda^2}\right)\mathcal{L}_\eta + |D_\mu H|^2 + \frac{\Lambda^2}{2}|H|^2 - \frac{\lambda_H}{4}|H|^4 + \epsilon\frac{\Lambda_c}{16\pi}\mathrm{Tr}[\Phi_N + \Phi_N^\dagger]|H|^2$$

Most general thing you can do

V_H^{SM}

New explicit breaking at site N

Generates the linear terms

$$-\Lambda^3 g\phi - \frac{1}{2}\Lambda g\phi H^2$$

$$\epsilon f^2 |H|^2 \cos\frac{\eta_0}{f_N}$$

Generates high frequency oscillations once *v ≠ 0*

Also generates dangerous barriers by closing the H loop. A possible solution is to adopt the double scanner of Espinosa et.al (1506.09217)

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Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon} q^{N+1}$

$$q^{N+1} < \epsilon < 1$$

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$q^{N+1} < \epsilon < 1$

Constraints:

"not the inflaton" ⇨ $H_I M_p > \Lambda^2$

"classical rolling vs quantum fluctuations" ⇨ $q^{N+1} > H_I^3 / f^3$

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Constraints:

"not the inflaton"

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$$q^{N+1} > \frac{\Lambda^6}{f^3 M_p^3}$$

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Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon} q^{N+1}$

$q^{N+1} < \epsilon < 1$

Constraints:

"not the inflaton"

"classical rolling vs quantum fluctuations"

$$q^{N+1} > \frac{\Lambda^6}{f^3 M_p^3}$$

"suppressing terms like ε *Cos^2*" $\epsilon < v^2/f^2$

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Solving for the classical stopping of the rolling: $v^2 \sim \frac{f^2}{\epsilon} q^{N+1}$

$q^{N+1} < \epsilon < 1$

$$\frac{\Lambda^6}{f^3 M_p^3} \lesssim q^{N+1} \lesssim \frac{v^4}{f^4}$$

Constraints:

$$q^{N+1} > \frac{\Lambda^6}{f^3 M_p^3} \Rightarrow q \lesssim 10^{-23/(N+1)}$$

$$\epsilon < v^2/f^2 \Rightarrow f \lesssim 10^8 \text{ GeV}$$

Conclusions

- The relaxation models are a proof of concept. If we come to the conclusion that they are self-consistent, then the hierarchy problem ceases to be an argument for new physics at the TeV scale.
- We manage to build an N-site relaxion model with a well defined continuum limit. Some improvements are needed and/or interesting:
    - To build the double scanner sector (or another solution to the high frequency oscillations induced by the Higgs)
    - Relaxion models require a low Inflation scale and a very large number of e-folds. It would be very interesting to find other sources of friction, e.g. particle production (see Hook, Tavares 1607.01786)
    - What about the continuum limit? What theory do we get in AdS_5? (In preparation)

Thank You!

Deconstructing AdS$_5$

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$S_5^A = \int d^4x \int_0^{\pi R} dy \sqrt{-g}\left\{-\frac{1}{2g_5^2}\mathrm{Tr}\left[F_{MN}^2\right]\right\}$$

$$= \int d^4x \int_0^{\pi R} dy \left\{-\frac{1}{2g_5^2}\,\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{g_5^2}e^{-2ky}\,\mathrm{Tr}\left[(\partial_5 A_\mu - \partial_\mu A_5)^2\right]\right\}.$$

$$\int_0^{\pi R} dy \to \sum_{j=0}^{N} a,$$

$$\partial_5 A_\mu \to \frac{A_{\mu,j} - A_{\mu,j-1}}{a}$$

$$S_5^A = \frac{a}{g_5^2}\int d^4x \left\{-\frac{1}{2}\sum_{j=0}^{N}\mathrm{Tr}\left[F_{\mu\nu,j}F_j^{\mu\nu}\right] + \sum_{j=1}^{N}\frac{e^{-2kaj}}{a^2}\mathrm{Tr}\left[\left(A_{\mu,j} - A_{\mu,j-1} - a\partial_\mu A_{5,j}\right)^2\right]\right\}.$$

Which is the same as the gauged pNGB to quadratic level:

$$S_4^A = \frac{1}{g^2}\int d^4x \left\{-\frac{1}{2}\sum_{j=0}^{N}\mathrm{Tr}\left[F_{\mu\nu,j}\,F_j^{\mu\nu}\right] + \sum_{j=1}^{N} f^2 g^2 q^{2j}\,\mathrm{Tr}\left[\left(A_{\mu,j} - A_{\mu,j-1} - \partial_\mu \frac{\pi_j}{f_j}\right)^2\right]\right\},$$

$$\frac{g_5^2}{a} \leftrightarrow g^2,$$

$$f \leftrightarrow \frac{1}{\sqrt{a}g_5} = \frac{1}{ag},$$

$$q \leftrightarrow e^{-ka},$$

$$U_j = e^{i\pi_j/f_j}$$

$$\leftrightarrow$$

$$\exp\left[i\int_{aj}^{a(j+1)} dy\, A_5 e^{-2ky}\right]$$