### Parton distributions for the LHC Run-2

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Based on studies with CTEQ-TEA and PDF4LHC working groups

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http://bit.ly/LAL160419

### "Simple" questions for the experienced audience

- PDFs are basic blocks of theoretical predictions for hadronic scattering in perturbative QCD. They cannot be computed, but their accuracy must match accuracy of hard-scattering cross sections.
- Soon PDFs and  $\alpha_s$  will drive main theoretical uncertainties in various LHC measurements.
- 1. Which PDFs should be used?
- 2. How should the PDF uncertainties be estimated?
- 3. Are there hidden PDF uncertainties?
- 4. How can the LHC constrain the PDFs better?

Many backup slides

# 0. Parton distribution functions in perturbative QCD

A 5-minute introduction

### The inner world of a hadron



The structure of the hadron drastically changes as the resolution of the "microscope" (scattering process) increases

### The inner world of a hadron



Unpolarized collinear parton distributions  $f_{a/p}(x, Q)$  are associated with probabilities for finding a parton *a* with the "+" momentum component  $xp^+$  in a proton with the "+" component  $p^+$ , at a resolution scale *Q*, for  $p^+ \to \infty$ 



#### Parton distribution functions $f_{a/p}(x, Q)$ ...



... the best-known nonperturbative functions introduced in QCD

... can be obtained from most general Wigner distribution functions  $W_a(x^{\alpha}, p^{\beta}, s^{\gamma})$ 

#### **Example: total cross section for** $gg \rightarrow Higgs \rightarrow \nu\nu$



Cross section  $\sigma_{pp \to H \to \gamma\gamma}$  for production and decay of *H*, e.g., via  $g + g \to H$ :

$$\sigma_{pp \to H \to \gamma\gamma X}(Q) = \sum_{a,b=g,q,\bar{q}} \int_0^1 d\xi_a \int_0^1 d\xi_b \hat{\sigma}_{ab \to H \to \gamma\gamma} \left(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}, \frac{Q}{\mu_R}, \frac{Q}{\mu_F}; \alpha_s(\mu_R)\right) \\ \times f_a(\xi_a, \mu_F) f_b(\xi_b, \mu_F) + O\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$$

- $\hat{\sigma}_{ab \to H \to \gamma\gamma}$  is the cross section for scattering of two partons, *a* and *b*; can be computed as a **perturbative** series in  $\alpha_s(\mu_R)$ , at a renormalization scale  $\mu_R \gg \Lambda_{QCD}$
- $f_{a/p}(\xi, \mu_F)$  is the **nonperturbative** PDF for finding a parton *a* with the momentum fraction  $\xi$  in the proton *p*, at a factorization scale  $\mu_F \gg \Lambda_{QCD}$

#### Hard-scattering cross sections for $gg \rightarrow H \rightarrow \gamma \gamma$



NNLO for differential distributions



Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 1503.06056 N3LO corrections are of the order of +2.2%. The total scale variation at N3LO is 3% 4/19/2016 P. Nadols

Anastasiou, Melnikov, Petriello, hep-ph/0409088, 0501130

P. Nadolsky, LAL Orsay

#### Recent CT14 PDFs

(S. Dulat et al., arXiv:1506.07443)



#### Q= 2 GeV

Q= 100 GeV

Phenomenological parametrizations of PDFs are provided with estimated uncertainties of multiple origins (**uncertainties of measurement, theoretical model, parametrization form, statistical analysis**, ...)

The shape of PDFs is optimized w.r.t. hundreds of **nuisance** parameters

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## Perturbative QCD loop revolution



Since 2005, generalized unitarity and related methods dramatically advanced the computations of **perturbative** NLO/NNLO/N3LO hard cross sections.

To make use of it, PDF accuracy must keep up

#### Given numerous PDF sets, what is the PDF uncertainty in my analysis?



The procedure for computing the PDF uncertainty must vary depending on the goals. The options may include

a) Using one individual set out of several similar ones (e.g., CT, MMHT, or NNPDF)

b) Using an envelope of all sets, including the outlier sets

#### c) 2015 recommendation by the PDF4LHC working group (arXiv:1510.03865):

- 1. Several procedures spelled out for computation of PDF uncertainties, depending on the context
- 2. Estimation of PDF uncertainties is streamlined in many cases by using combined PDF4LHC15 sets based on CT14, MMHT14, and NNPDF3.0 11

### Why PDF4LHC recommendation is necessary

Estimates of PDF uncertainties may vary drastically depending on the method. An overly conservative estimate greatly reduces sensitivity to BSM physics.



Gluon-Fusion Higgs production, LHC 13 TeV

### Why PDF4LHC recommendation is needed



### 1. Status of the PDFs in spring 2016

## 1. General-purpose NNLO PDFs



For general-purpose **nucleon** PDFs, the goal is to determine very precise parametrizations for PQCD calculations up to (N)(N)NLO in  $\alpha_s$ . The focus is on the high-Q data that is not sensitive to higher twists, small-x, nuclear, and target mass corrections

## 1. General-purpose NNLO PDFs



2016 updates include new combined HERA1+2 data and 8 TeV LHC data

#### Expect mild changes in the PDFs and uncertainties

Compare CT14 and CT10 quark PDFs



## CT14HERA2: effect of increasing the statistical weight of HERAI+II data from w=1 to w=6



With w = 6, CT14HERA2 PDFs for g, u, and d are more similar to HERA2.0, not to ABM12 in a different theory framework with  $\alpha_s = 0.113$ 

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For sea PDFs, similarity is less pronounced (not as sensitive to HERA DIS)

#### The ratios of W<sup>+</sup> to W<sup>-</sup> and (W<sup>+</sup>+W<sup>-)</sup> to Z cross sections CT14HERA2 vs. CT14



 $p_T^l > 25 \; GeV \;, \quad |\eta_l| < 2.5 \;, \quad 66 < m_{ll} < 116 \; GeV$ 

 $p_T^l > 25 \; GeV \;, \quad p_T^{\nu} > 25 \; GeV \;, \quad |\eta_l| < 2.5 \;, \quad m_T > 50 \; GeV$ 

PRELIMINARY

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# 2012→2015: Agreement between global NNLO PDFs greatly improved



Note in particular the changes in the gg luminosity, especially important in the Higgs mass region

LHC data has been added for all 3 new PDFs, but most changes are due to benchmarking of formalisms

Figure 1: Comparison of the  $q\bar{q}$  (left) and gg (right) PDF luminosities at the LHC 8 TeV for CT10, MSTW2008 and NNPDF2.3. Results are shown normalized to the central value of CT10.



### Other new sets published as well



behavior for HERAPDF2.0 and ABM12 somewhat different

HERAPDF2.0 uncertainties tend to be larger

Figure 5: Comparison of the gluon-gluon (upper plots) and quark-antiquark (lower plots) PDF luminosities from the CT14, MMHT14 and NNNPDF3.0 NNLO sets (left plots) and from the NNPDF3.0, ABM12 and HERAPDF2.0 NNLO sets (right plots), for a center-of-mass energy of 13 TeV, as a function of the invariant mass of the final state  $M_X$ .



Figure 6: Same as Fig. 5 for the quark-quark (upper plots) and the quark-gluon (lower plots) PDF luminosities. P. Nadolsky, LAL Orsay 23

## 2. Specialized PDFs at NLO and NNLO



Are obtained under special assumptions or for special goals. May or may not be suitable for general physics

- 1. CJ15: NLO PDFs with large-*x*/low DIS data
- 2. Many groups, e.g. ABM: PDFs with up to 3, 4 active flavors
- **3. CT, NNPDF, MSTW:** QCD+QED PDFs
- 4. CT, NNPDF: PDFs with intrinsic charm
- 5. NNPDF: PDFs for threshold resummation

6. ...

See backup slides

#### **2.** Choosing the estimator for the PDF+ $\alpha_s$ uncertainty



#### PDF4LHC recommendations for LHC Run II

Jon Butterworth<sup>1</sup>, Stefano Carrazza<sup>2</sup>, Amanda Cooper-Sarkar<sup>3</sup>, Albert De Roeck<sup>4,5</sup>, Joël Feltesse<sup>6</sup>, Stefano Forte<sup>2</sup>, Jun Gao<sup>7</sup>, Sasha Glazov<sup>8</sup>, Joey Huston<sup>9</sup>, Zahari Kassabov<sup>2,10</sup>, Ronan McNulty<sup>11</sup>, Andreas Morsch<sup>4</sup>, Pavel Nadolsky<sup>12</sup>, Voica Radescu<sup>13</sup>, Juan Rojo<sup>14</sup> and Robert Thorne<sup>1</sup>.

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### PDF4LHC publication, topics

1. Review of updates on PDFs from various groups

**NNLO Global PDF sets:** CT14, MMHT'14, NNPDF3

PDFs using other methodologies: ABM'12, CJ15, HERAPDF2.0

2. Average PDF sets by PDF4LHC group: PDF4LHC15\_30, \_100, \_MC

Criteria for combination

 $\alpha_{\rm s}(M_7) = 0.1180 \pm 0.0015$  at 68% c.l.

3. Recommendation on selecting PDF sets for various LHC applications

- B. Precision tests of SM and PDFs A. New physics searches
- C. Monte-Carlo simulations

D. Acceptance estimates

Average PDF sets can be used for bulk of applications in A, C, D 27

### Now on LHAPDF:

# NLO, NNLO, varied $\alpha_s$ sets $N_f = 5$ and 4 (upcoming)

LHAPDF6 grid	Pert order	ErrorType	$N_{ m mem}$	$\alpha_s(m_Z^2)$
PDF4LHC15_nnlo_mc	NNLO	replicas	100	0.118
PDF4LHC15_nnlo_100	NNLO	symmhessian	100	0.118
PDF4LHC15_nnlo_30	NNLO	symmhessian	30	0.118
PDF4LHC15_nnlo_mc_pdfas	NNLO	replicas+as	102	mem $0{:}100 \rightarrow 0{.}118$
				mem 101 $\rightarrow 0.1165$
				mem 102 $\rightarrow 0.1195$
PDF4LHC15_nnlo_100_pdfas	NNLO	symmhessian+as	102	mem $0:100 \rightarrow 0.118$
				mem 101 $\rightarrow 0.1165$
				mem $102 \rightarrow 0.1195$
PDF4LHC15_nnlo_30_pdfas	NNLO	symmhessian+as	32	mem $0:30 \rightarrow 0.118$
				mem $31 \rightarrow 0.1165$
				mem $32 \rightarrow 0.1195$
PDF4LHC15_nnlo_asvar	NNLO	-	1	mem $0 \rightarrow 0.1165$
				mem $1 \rightarrow 0.1195$

Table 5: Summary of the combined NNLO PDF4LHC15 sets with  $n_f^{\text{max}} = 5$  that are available from LHAPDF6. The corresponding NLO sets are also available. Members 0 and 1 of PDF4LHC15\_nnlo\_asvar coincide with members 101 and 102 (31 and 32) of PDF4LHC15\_nnlo\_mc\_pdfas and PDF4LHC15\_nnlo\_100\_pdfas (PDF4LHC15\_nnlo\_30\_pdfas). Recall that in LHAPDF6 there is always a zeroth member, so that the total number of PDF members in a given set is always  $N_{\text{mem}} + 1$ . See text for more details.

## 2015 Les Houches contributions on usage of PDF4LHC distributions

Aim to address a few questions not covered in the main document of 2015 PDF4LHC recommendation (arXiv:1510.03865), and provide illustrations

# **1.** Phenomenological applications of PDF4LHC distributions

*J. Gao, T.-J. Hou, J. Huston, P. N., B. Wang, K. Xie, …* Physics issues, predictions for typical QCD cross sections

# 2. On the accuracy and Gaussianity of the PDF4LHC15 combined sets of parton distributions

S.Carrazza, S. Forte, Z. Kassabov, J. Rojo

Comparisons of PDF4LHC ensembles, non-Gaussian

#### effects

#### Choosing the right PDF set for an LHC application

#### 6.1 Delivery and guidelines

The PDF4LHC15 combined PDFs are based on an underlying Monte Carlo combination of CT14, MMHT14 and NNPDF3.0, denoted by MC900, which is made publicly available in three different reduced delivery forms:

- PDF4LHC15\_mc: a Monte Carlo PDF set with N<sub>rep</sub> 100 replicas.
- PDF4LHC15\_30: a symmetric Hessian PDF set with Note = 30 eigenvectors.
- PDF4LHC15\_100: a symmetric Hessian PDF set with N<sub>sig</sub> 100 eigenvectors.

In the three cases, combined sets are available at NLO and at NNLO, for the central value of  $\alpha_s(m_Z^2) = 0.118$ . In addition, we provide additional sets which contain the central values for  $\alpha_s(m_Z^2) = 0.1165$  and  $\alpha_s(m_Z^2) = 0.1195$ , and that can be used for the computation of the combined PDF+ $\alpha_s$  uncertainties, as explained in Sect. 6.2. Finally, for ease of usage, the combined sets for  $\alpha_s(m_Z^2) = 0.118$  are also presented bundled with the  $\alpha_s$ -varying sets in dedicated grid files. The specifications of each of the combined NNLO PDF4LHC15 sets that are available from LHAPDF6 are summarized in Table 5; note that the corresponding NLO sets are also available.

Usage of the PDF4LHC15 sets. As illustrated in Sect. 5, the three delivery options provide a reasonably accurate representation of the original prior combination. However, each of these methods has its own advantages and disadvantages, which make them more suited in different specific contexts. We now attempt to provide some general guidance about which of the three PDF4LHC15 combined sets should be used in specific phenomenological applications.

1. Comparisons between data and theory for Standard Model measurements

Recommendations: Use individual PDF sets, and, in particular, as many of the modern PDF sets [5–11] as possible.

Rationale: Measurements such as jet production, vector-boson single and pair production, or top-quark pair production, have the power to constrain PDFs, and this is best utilized and illustrated by comparing with many individual sets.

As a rule of thumb, any measurement that potentially can be included in PDF fits falls in this category.

The same recommendation applies to the extraction of precision SM parameters, such as the strong coupling  $\alpha_s(m_Z^2)$  [75, 124], the W mass  $M_W$  [125], and the top quark mass  $m_e$  [126] which are directly correlated to the PDFs used in the extraction.

#### 2. Searches for Beyond the Standard Model phenomena

Recommendations: Use the PDF4LHC15\_mc sets.

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Rationale: BSM searches, in particular for new massive particles in the TeV scale, often require the knowledge of PDFs in regions where available experimental constraints are limited, notably close to the hadronic threshold where  $x \rightarrow 1$  [127]. In these extreme kinematical regions the PDF uncertainties are large, the Monte Carlo combination of PDF sets is likely to be non-Gaussian. c.f. Figs. 10 and 11. The PDF4LHC document contains detailed guidelines to help decide which individual or combined PDFs to use depending on the circumstances

To assist in choosing the best PDF(s), demonstrative comparisons were generated of typical LHC cross sections for recent PDFs

#### 1. MC2H gallery of LHC cross sections: ApplGrid, typical experimental cuts

www.hep.ucl.ac.uk/pdf4lhc/mc2h-gallery/

#### 2. SMU gallery of LHC cross sections: ApplGrid or full calculations, minimal cuts

www.physics.smu.edu/botingw/2016\_pdf4lhc/

## Three main uses of PDFs at the LHC

- 1. Assessment of the total uncertainty on a cross section based on the available knowledge of PDFs, *e.q.*, when computing the cross section for a process that has not been measured yet (such as supersymmetric particle production cross-sections), or for estimating acceptance corrections on a given observable. This is also the case of the measurements that aim to verify overall, but not detailed, consistency with Standard Model expectations, such as when comparing theory with Higgs measurements.
- 2. Assessment of the accuracy of the *PDF sets themselves* or of related Standard Model parameters, typically done by comparing theoretical predictions using individual PDF sets to the most precise data available.
- 3. Input to the *Monte Carlo event generators* used to generate large MC samples for LHC data analysis.

#### For 2), compute cross sections with individual PDF sets.

For 1) or 3), the PDF uncertainty based on the totality of available PDF sets must be estimated. Estimate the combined PDF error using an average of various PDF sets. P. Nadolsky, LAL Orsay 31

## Averaging of PDF ensembles

The 2012 recommendation estimated the combined uncertainty as an envelope of **cross sections** for 3 PDF sets; the envelope was overly sensitive to outliers

By 2015, several methods for combination (averaging) of **PDFs** (before computing cross sections) were developed. Criteria allowing the combination were outlined.

#### **Combination workflow:**

 Generate 900 MC replicas from all input ensembles (currently CT14, MMHT14, NNPDF3.0) using Thorne-Watt procedure

Other PDF sets can be added in the future if they satisfy the listed criteria

 Reduce the number of final replicas from 900 to 100 or 30 by keeping most relevant PDF combinations

### Reduced sets

- 900 error PDFs are too much for general use
- 3 reduction techniques have been developed
  - Compressed Monte Carlo PDFs (PDF4LHC15\_nnlo(nlo)\_mc)
    - 100 PDF error sets; preserve non-Gaussian errors
  - META Hessian PDFs (PDF4LHC15\_nnlo(nlo)\_30
    - 30 PDF error sets using METAPDF technique; Gaussian (symmetric) errors
  - MCH Hessian PDFs (PDF4lhc15\_nnlo(nlo)\_100
    - 100 PDF error sets using MCH technique; Gaussian (symmetric errors)
- The META technique is able to more efficiently reproduce the uncertainties when using a limited number (30) of error PDFs
- The MCH technique best reproduces the uncertainties of the 900 MC set prior

# Comparisons of ensembles with 900, 100, 30 replicas



Three PDF4LHC sets reproduce well the 900replica prior. Keep in mind that the uncertainty of the prior has an uncertainty of its own. By their construction, the lowest Hessian eigenvector sets are known the best, the highest sets are known with less confidence.

The 30-member ensemble keeps the lowest, best known sets and thus provides a lower estimate for the \_900 prior uncertainty, known with higher confidence. When this estimate is not sufficient, or non-Gaussianities are important, use the 100 and MC sets

#### Ranges with differences between input PDFs, prior, and reduced sets



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#### Predictions for LHC observables based on PDF4LHC15 PDFs

1. p + p → Z + X PDF4LHC15 PDFs (NLO) PDF4LHC15 PDFs (NNLO) PDF4LHC15\_nnlo\_100, HERA, ABM PDFs PDF4LHC15\_nnlo\_100, CT14, MMTH14, NNPDF3.0 PDFs

 $2. p + p \rightarrow W^+ + X$ 

PDF4LHC15 PDFs (NLO) PDF4LHC15 PDFs (NNLO) PDF4LHC15\_nnlo\_100, HERA, ABM PDFs PDF4LHC15\_nnlo\_100, CT14, MMTH14, NNPDF3.0 PDFs SMU gallery for basic processes at 7, 8, 13 TeV

Developed by Bo Ting Wang and Keping Xie

#### $p + p \rightarrow Z + X$

#### PDF4LHC15 PDFs (NLO)


#### **Gallery of phenomenological comparisons for LHC**

Process	Order	Type of calculation
•p + p $\rightarrow$ Z + X	NLO	aMCFast/Appgrid
•p + p $\rightarrow$ W <sup>+</sup> + X	NLO	aMCFast/Appgrid
•p + p $\rightarrow$ W <sup>-</sup> + X	NLO	aMCFast/Appgrid
•p + p $\rightarrow t\bar{t} + X$	NLO	aMCFast/Appgrid
•p + p $\rightarrow t\bar{t}$ + X	NLO	aMCFast/Appgrid
•p + p $\rightarrow t\bar{t}\gamma\gamma$ +X	NLO	aMCFast/Applgrid
•ATLAS inclusive jets	NLO	NLOJET++/Appgrid
•ATLAS inclusive dijet	s NLO	NLOJET++/Appgrid
•P + p $\rightarrow$ W <sup>+</sup> c + X	NLO	aMCFast/Appgrid
$\bullet P + p \to W^{-} c + X$	NLO	aMCFast/Appgrid
$\bullet P + p \to H + X$	LO,NLO	MCFM
•P + p $\rightarrow$ H+ jet + X	LO, NLO	MCFM

Compared PDFs: PDF4LHC15\_100, \_30, \_MC, ABM'12, CT14, HERA2.0, MMHT14, NN3.0

Both full (MCFM) and fast (ApplGrid) calculations. AppGrlids are generated with minimal cuts and can be downloaded.

## Cross sections and their relative differences due to PDFs are shown for most processes.



Relative differences tend to be larger when cross sections are small. Often they vary little across much of the range, can be eliminated by applying a constant rescaling factor (e.g., 1.05) to the PDF error.

#### **MCFM: compare PDF and Monte-Carlo integration errors**

## Differences of PDF4LHC PDFs matter only when MC errors are negligible



 $gg \rightarrow H$  at LO. Simple process.  $10^6$  events, PDF reweighting, ~30 min per each PDF family

MC fluctuations in PDF errors are of the same order as the primordial differences

#### **MCFM: compare PDF and Monte-Carlo integration errors**

#### Differences of PDF4LHC PDFs matter only when MC errors are negligible In progress



 $gg \rightarrow H$  at NLO. 10<sup>6</sup> events, >24 hours per each PDF family Central values of all 3 PDFs get different fluctuations! Even though they are just distinct parametrizations of the same central PDF of the prior <sup>40</sup>

#### **MCFM: compare PDF and Monte-Carlo integration errors**



 $gg \rightarrow H + j$  at LO. 10<sup>6</sup> events, ~1 hour per each PDF family

## Fluctuations in the central values at highest $p_T$

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MC fluctuations can be suppressed by increasing the number of events or enlarging bin sizes. This example also touches on broader questions.

- There are several ways for "averaging" the input PDF sets, e.g., because they use different evolution codes or round-offs
- 2. Without PDF reweighting, MC fluctuations are sensitive independently to every replica. They vary by the process, order of the calculation, etc.
- 3. Procedural adjustments made to reduce MC errors tend to wash out disparities due to PDF reduction methods

## Higgs eigenvector set

1.10 For a given class of • Normalized to central prediction observables, the \_30 set can Gluon fusion at LHC 8 TeV 1.05 be diagonalized to reproduce  $d\sigma/dy_H$ the bulk of the uncertainties 1.00 and correlations with ~6 0.95 eigenvector sets 0.90 0.0 0.5 1.10 1.10 Normalized to central prediction VBF at LHC 14 TeV 1.05 1.05  $\mathrm{d}\sigma/\mathrm{d}\mathbf{y}_H$ 1.00 1.00

0.95

0.90

0.0

0.5



												2				0	0			(, LO		
process	$\sigma_{cen.}$	$\delta_{Full}$	$\delta_{Diag.}$	$\sigma_{0.116}^{\alpha_s}$	$\sigma^{lpha_s}_{0.12}$		NNLO	V, NLO	V, NLO	V, LO	V, LO	8 TeV,	Q	2	, NNLO	eV, NL	eV, NL	eV, LO	eV, LO	14 Te\	2	P
$aa \rightarrow H$ [pb]	18.77	$^{+0.48}_{-0.46}$	$^{+0.48}_{-0.44}$	18.11	19.4		8 TeV,	c., 8 Te	c., 8 Te	c., 8 Te	c., 8 Te	l mass,	3 TeV, I	8 TeV,	14 TeV	c., 14 T	c., 14 T	c., 14 Te	c., 14 T	l mass,	l4 TeV,	14 TeV
<i>99</i> · 11 [P.5]	43.12	$^{+1.13}_{-1.07}$	$^{+1.13}_{-1.04}$	41.68	44.6		H inc.,	iH 0j ex	H 1j ex	H 2j in	iH 2j ex	iH 2j ful	Finc., 8	F exc.,	H inc.,	H Oj ex	iH 1j ex	iH 2j in	iH 2j ex	H 2j ful	Finc., 1	F exc.,
VBF [fb]	302.5	$^{+7.8}_{-6.7}$	$+7.6 \\ -6.7$	303.1	301.4		ං -0.43	g -0.49	-0.3	0.09	0.09	ලී 0.06	8 0.92	8 0.92	-0.39	-0.42	ලි -0.33	8 0.02	8 0.02	0.	8 <u>&gt;</u>	R
	878.2	$^{+19.7}_{-17.9}$	$^{+19.2}_{-17.3}$	877.3	878.	VBF exc., 14 TeV, LO	- <b>0.44</b>	- <b>0.5</b>	-0.33	0.09	0.09	0.09	0.93	0.93	-0.4	- <b>0.44</b>	-0.35	0.02	0.02	<i>0</i> .	1.	$\square$
HZ [fb]	396.3	+8.4 -7.3	+8.1 -7.4	393.0	399.	VBF inc., 14 TeV, LO	-0.45 -0.44	-0.40	-0.33	0.09	0.00	0.00	0.92	0.93	-0.4	- 0.42	-0.35	0.02	0.02	0. 0.	$\square$	
	814.3	$^{+14.8}_{-13.2}$	$^{+13.8}_{-13.0}$	806.5	823.	GGH 2j full mass, 14 TeV, LO	0.45 0.42	0.25 0.22	0.72	0.90 0.98	0.90 0.98	0.90 0.98	-0.04 -0.05	-0.04 -0.05	0.31 0.28	0.08 0.05	0.47 0.46	0.99 0.99	0.99 0.99	$\angle$		
	703.0	+14.4 -14.4	+14.3 -14.1	697.4	708.	GGH 2j exc., 14 TeV, LO	0.43 <b>0.44</b>	0.22 0.23	0.71 0.72	0.97 0.98	0.97 0.98	0.97 0.98	-0.01 - <b>0.02</b>	-0.01 - <b>0.02</b>	0.29 0.29	0.07 0.07	0.46 <b>0.48</b>	0.99 <b>0.99</b>	$\square$			
HW = [ID]	1381	$^{+28}_{-22}$	$^{+26}_{-22}$	1368	1398	GGH 2j inc., 14 TeV, LO	0.43 <b>0.44</b>	0.22 0.23	0.71 0.72	0.97 <b>0.98</b>	0.97 <b>0.98</b>	0.97 <b>0.98</b>	-0.01 - <b>0.02</b>	-0.01 - <b>0.02</b>	0.29 <b>0.29</b>	0.07 <b>0.07</b>	0.46 0.48					
	7.81	+0.33 -0.30	+0.33 -0.30	7.50	8.10	GGH 1j exc., 14 TeV, NLO	0.98 <b>0.98</b>	0.94 <b>0.94</b>	0.93 <b>0.94</b>	0.3 <b>0.33</b>	0.3 <b>0.33</b>	0.3 <b>0.33</b>	-0.34 - <b>0.34</b>	-0.34 - <b>0.34</b>	0.97 <b>0.97</b>	0.89 <b>0.9</b>						
	27.35	+0.78 -0.72	+0.78 -0.68	26.48	28.2	GGH 0j exc., 14 TeV, NLO	0.91 <i>0.92</i>	0.96 <i>0.97</i>	0.7 <b>0.73</b>	-0.07 - <b>0.08</b>	-0.07 - <b>0.08</b>	-0.07 - <b>0.08</b>	-0.4 - <b>0.4</b>	-0.4 - <b>0.4</b>	0.97 <i>0.97</i>							
$t\bar{t}$ [pb]	248.4	+9.1 -8.2	+9.2 -8.1	237.1	259.	GGH inc., 14 TeV, NNLO	0.97 <b>0.97</b>	0.97 <b>0.98</b>	0.84 <i>0.87</i>	0.14 <i>0.14</i>	0.14 <b>0.14</b>	0.14 <i>0.14</i>	-0.38 - <b>0.39</b>	-0.38 - <b>0.39</b>								
	816.9	+21.4 -19.6	+21.4 -18.4	785.5	848.	VBF exc., 8 TeV, LO	-0.41 - <b>0.41</b>	-0.44 - <b>0.45</b>	-0.31 - <b>0.33</b>	0.06 <i>0.05</i>	0.06 <i>0.05</i>	0.04 <i>0.05</i>	1. <i>0.99</i>									
$Z/\gamma^*(l^+l^-)$ [nb]	1.129	+0.025 -0.023	+0.024 -0.023	1.113	1.14	VBF inc., 8 TeV, LO	-0.41 - <b>0.41</b>	-0.44 - <b>0.45</b>	-0.31 - <b>0.33</b>	0.06 <b>0.05</b>	0.06 <i>0.05</i>	0.04 <i>0.05</i>		rrelatio	on tab	le for	Higgs	cross	secti	ons		
	1.925	+0.043 -0.041	+0.040 -0.037	1.897	1.95	GGH 2j full mass, 8 TeV, LO	0.27 <b>0.29</b>	0.06 <i>0.08</i>	0.57 <i>0.6</i>	0.99 <b>0.99</b>	0.99 <i>0.99</i>			F	led inc	licates	s  cos	( <b>ø</b> ) >0	.7			
$W^{\pm}(l^{\pm}\omega)$ [mb]	7.13	+0.14 -0.14	+0.14 -0.13	7.03	7.25	GGH 2j exc., 8 TeV, LO	0.27 <b>0.29</b>	0.06 <i>0.08</i>	0.57 <i>0.6</i>	0.99 <i>0.99</i>	Mu	mbers	s in Ita	alic-bo	old (pla META	ain) fo	r 6 eig 1 0 (1	jenve HCH	cotrs	(full se	et 50 €	ig.)
$W + (l + \nu)$ [nb]	11.64	+0.24 -0.23	+0.22 -0.21	11.46	11.8	GGH 2j inc., 8 TeV, LO	0.27 <b>0.29</b>	0.06 <i>0.08</i>	0.57 <i>0.6</i>			v	BF—lik	e cut	applie	d for	2 or m	ore je	, ets fin	al stat	es	
	4.99	+0.12 -0.12	+0.12 -0.11	4.92	5.08	GGH 1j exc., 8 TeV, NLO	0.93 <i>0.93</i>	0.83 <i>0.83</i>				jet (	anti- <i>k</i>	<sub>7</sub> , 0.4	) selec	ction v	vith  y	<4.5	and p	<sub>7</sub> >30	GeV	
$W$ $(l \ \nu)$ [nb]	8.59	$^{+0.21}_{-0.20}$	$+0.19 \\ -0.18$	8.46	8.74	GGH 0j exc., 8 TeV, NLO	0.97 <b>0.97</b>								includ	ling $\alpha_s$	unce	rtainty				
$W^+W^-$ [-1]	4.14	$+0.08 \\ -0.08$	+0.08 -0.07	4.04	4.20	GGH inc., 8 TeV, NNLO	3.3% <b>3.3</b> %	3.2% <b>3.2%</b>	3.6% <b>3.5</b> %	6.9% <b>6.8%</b>	6.9% <b>6.8%</b>	7.% <b>6.8%</b>	2.4% <b>2.4%</b>	2.4% <b>2.4%</b>	3.3% <b>3.3%</b>	3.2% <b>3.2%</b>	3.4% <b>3.4%</b>	5.7% <b>5.7%</b>	5.7% <b>5.7%</b>	5.8% <b>5.8%</b>	2.1% <b>2.</b> %	2.1% <b>2.</b> %
w w [pb]	7.54	+0.15 -0.14	+0.14 -0.12	7.39	7.57	l	NNLO	', NLO	', NLO	∮V, LO	∮V, LO	¢V, LO	øV, LO	¢V, LO	NNLO	', NLO	', NLO	₀V, LO	¢V, LO	¢, LO	¢, LO	oV, LO
ZZ [pb]	0.703	+0.016 -0.014	+0.015 -0.014	0.695	0.71		8 TeV,	, 8 TeV	, 8 TeV	c., 8 Te	с., 8 Те	ss, 8 Te	с., 8 Те	с., 8 Те	4 TeV,	14 TeV	14 TeV	., 14 Te	., 14 Te	s, 14 Te	., 14 Te	., 14 Te
	1.261	$+0.026 \\ -0.024$	$+0.024 \\ -0.022$	1.256	1.27	1	H inc.,	0j exc	1j exc	àH 2j ir	H 2j ex	full ma	VBF in	VBF ex	linc., 1	oj exc.,	lj exc.,	H 2j inc	H 2j exo	ull mas	/BF inc	BF exc
$W^+Z$ [pb]	1.045	$+0.019 \\ -0.018$	+0.019 -0.017	1.039	1.06		99	GGH	GGH	б	99	3GH 2j			GGF	GGH	GGH	GGI	GGF	3H 2j f	-	>
	1.871	$+0.033 \\ -0.031$	+0.029 -0.027	1.850	1.89	l						9								ğ		
W-7 [ph]	0.788	$+0.020 \\ -0.019$	+0.019 -0.018	0.780	0.79																44	
W Z [pb]	1.522	$^{+0.034}_{-0.032}$	$^{+0.033}_{-0.031}$	1.509	1.54	FIG. 7: Same	e as l	Fig. 3	5, wit	h $\alpha_s$	unce	ertain	ities i	ncluo	ded b	y ado	ding	in qu	adra	ture.		

## Fighting PDF uncertainties

# Why global NNLO PDFs are in better agreement now than ever

Since 2012, PDF analysis groups carried out a series of benchmarking exercises for key processes of DIS and jet production in PDF fits

Methodologies of all groups were cross-validated and improved. P. Nadolsky, LAL Orsay

#### **Theoretical accuracy**

#### A variety of comparisons was accomplished to benchmark NNLO theoretical calculations for key scattering processes

1.J. Gao et al., MEKS: a program for computation of inclusive jet cross sections at hadron colliders , arXiv:1207.0513

2. R. Ball et al., Parton Distribution benchmarking with LHC data, arXiv:1211.5142

3. S. Alekhin et al., ABM11 PDFs and the cross section benchmarks in NNLO, arXiv:1302.1516; The ABM parton distributions tuned to LHC data; arXiv:1310.3059

4. A.Cooper-Sarkar et al., PDF dependence of the Higgs production cross section in gluon fusion from HERA data, 2013 Les Houches Proceedings, arXiv:1405.1067, p. 37

5. S. Forte and J. Rojo, Dataset sensitivity of the gg->H cross-section in the NNPDF analysis, arXiv:1405.1067, p. 56

#### Verifying statistical methods

#### Parametric/Hessian methodology (CT, MMHT) and nonparametric/Monte-Carlo methodology (NNPDF) result in comparable global fits and PDF uncertainties

Advanced PDF parametrizations are employed by CT and MMHT for efficient, minimally biased, extraction of PDFs from global data

## Hessian PDFs can be converted into MC PDFs, and back

#### CT14 replicas Hou. Study of asymmetric uncertaities.



Also plot of  $\chi^2$  distribution for 1000 replicas with 28 eigenvectors and tolerance ~ 40 for one sigma. Extremely similar to NNPDF from completely different approach.



X

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## Example: $gg \rightarrow H_{SM}^0$ at the LHC

LHC 8 TeV - iHixs 1.3 NNLO - PDF+as uncertainties



For example,  $\delta_{PDF}$  on Higgs cross sections based on 3 **latest** global fits has reduced from 7% to within 3%, i.e., the PDF uncertainty is now of order of N3LO QCD scale uncertainty

This improvement is due to benchmarking of general-mass factorization schemes; but can there be hidden sources of uncertainties on  $\sigma(gg \rightarrow H)$ , e.g., associated with massive charm DIS contributions, cf. arXiv:1603.08906?

#### NLO= $O(\alpha_s)$ : GM-VFN predictions for DIS have large dependence on matching scales



The gluon PDF depends on the factorization scheme used to fit HERA DIS data

Besides the physical mass  $m_c$ , general-mass (GM-VFN) schemes used by global fits introduce matching energy scales of order  $m_c$ 

At NLO, uncertainty due to matching parameters is large; each scheme prefers an "optimal"  $m_c$  that brings  $\chi^2$  to comparable levels (cf. the figure)

#### NNLO= $O(\alpha_s^2)$ : dependence on matching parameters is suppressed, GM-VFN schemes are more similar

LH PDFs Q=2 GeV, mc=1.41 GeV



#### **GM-VFN schemes are more predictive at NNLO**

LH PDFs Q=2 GeV S-ACOT



At  $O(\alpha_s^2)$  and approximate  $O(\alpha_s^3)$ , constraints on  $m_c(m_c)$  have been first obtained from combined HERA-I data in the FFN scheme (1212.2355). Constraints on both  $m_c^{pole}$  or  $m_c(m_c)$  in GM-VFNS have been also obtained by CT, MMHT, and NNPDF under varied assumptions. They are comparable with FFNS and the PDG value for  $m_c(m_c)$ .

#### The resulting uncertainty on $\sigma(gg \rightarrow H)$ is <2-3%



 $\sigma_{tot}$  for  $m_c(m_c) = 1 - 1.36$  GeV and matching parameter  $\lambda$  varied independently,  $Q_0 = 1$  GeV. Black boxes are for  $m_c(m_c) = 1.28$  GeV (close to world average), for the explored  $\lambda$ . The error ellipse is for nominal 90% C.L. @  $Q_0 = 1.3$  GeV. Inclusive ggHiggs production @ N3LO, LHC 13 TeV



Intrinsic charm only reduces correlated dependence on  $m_c$ ,  $Q_0$ , and matching **J. Rojo's talk** 

**Bottom line:** GM-VFN schemes agree well at NNLO because of perturbative convergence, not because of  $m_c$  tuning

## Looking into the future

#### **High-luminosity LHC**

- New (N)NNLO calculations likely to be completed
- Measurements of Higgs cross sections/couplings become limited by PDFs in the HL-LHC era
- Searches for non-resonant production in TeV mass range will demand accurate predictions for sea PDFs at x > 0.1

#### Projected Experimental Uncertainties



P. Newman, DIS'2016

### Toward proton PDFs at 1% accuracy

#### Theory:

1. Development of efficient techniques to estimate PDF dependence at (N)NNLO

- a) Interfaces for fast (N)NLO computations (Applgrid, FastNLO, aMCFast)
- b) Combination at the PDF level (META, CMC), reduced PDFs for classes of processes

2. Inclusion of subleading effects (NLO EM corrections, photon PDFs, off-shell resonant production...) and theoretical uncertainties (scale dependence, heavy-quark schemes, ...)

3. Advanced statistical methods (MC, reweighting...)

4. Special-purpose PDFs: for parton showering programs, with intrinsic charm, for resummations, ...

### Toward proton PDFs at 1% accuracy

#### Experiment:

- Finding new, highly sensitive measurements for constraining PDFs
  - a) Less inclusive, yet clean, processes (e.g.  $Z p_T$  at NNLO...)
  - b) Better constraints at x>0.3
  - c) Reliable flavor separation
- 2. Cross calibration of systematic uncertainties between the measurements
- 3. Smaller bin sizes, with some loss in statistics  $\Rightarrow$  better resolution on PDF x dependence

## Thank you for your attention!

## Backup slides

### CT14 and CT14HERA 2 sets

#### Map of experiments as a function of x and Q

For nucleon PDFs, experimental measurements are selected so as to reduce dependence on theoretical input beyond the leading power in perturbative QCD



#### CT14:

only DIS data with  $Q^2 > 4 \ GeV^2$ ,  $W^2 > 12.25 \ GeV^2$  (above the red line) are accepted to ensure stable perturbative predictions

Include LHC *W* asymmetry and jet production data

Still using data from DIS and DY on nuclear targets. CT14H2 does not use NMC DIS on deuteron, will be replaced by comparable future LHC/Tevatron measurements on the proton

#### Experiments in the CT14 analysis

#### 33 **experiments**; $\chi^2/N_{pt} = 3252/2947 = 1.10$

Experimental data set	$N_{pt}$	$\chi_e^2/N_{pt}$		Experimental data set	$N_{pt}$	$\chi_e^2/N_{pt}$
BCDMS $F_2^p$		1007 1		Ecor D	119	0.98
$\frac{1}{\text{BCDMS } F_2^d}$ Red arrow	IS I	Indi	cat	e new data sets [25]	15	0.87
$\frac{2}{\text{NMC} E^{\underline{d}}/E^{\underline{p}}}$ [14]	193	1.08		E866 Drell-Yan process [25]	184	1.37
	120	1.00		CDF Run-1 electron $A_{ch}$ [26]	11	0.81
NMC $\sigma_{red}^{\nu}$ [14]	201	1.85		CDF Run-2 electron $A_{ch}$ [27]	11	1.24
$CDHSW F_2^p $ [15]	85	0.85		D0 Run-2 muon $A_{ch}$ [29]	9	0.92
CDHSW $F_3^p$ [15]	96	0.83		LHCb 7 TeV 35 pb <sup>-1</sup> $W/Z d\sigma/dy_{\ell}$	14	0.7
$CCFR F_2^p $ [16]	<b>6</b> 9	1.02		LHCb 7 TeV 35 pb <sup>-1</sup> $A_{ch}, p_{T\ell} > 20 \text{ GeV}$ [01]	5	1.19
$CCFR \ xF_3^p $ [17]	86	0.36		D0 Run 2 $Z$ rapidity [32]	28	0.59
NuTeV $\nu\mu\mu$ SIDIS [18]	38	0.62		CDF Run 2 Z rapidity [33]	29	1.64
NuTeV $\bar{\nu}\mu\mu$ SIDIS [18]	33	1.18		CMS 7 TeV 4.7 fb <sup>-1</sup> , muon $A_{ch}$	11	0.8
CCFR $\nu\mu\mu$ SIDIS [19]	40	0.72		CMS 7 TeV 840 pb <sup>-1</sup> , electron $A_{ch}$	11	0.87
CCER inter SIDIS [10]	20	0.52		ATLAS 7 TeV 35 pb <sup>-1</sup> $W/Z$ cross sections and $A_{\Delta_{11}}$	41	1.11
	00	0.00		D0 Run-2 electron $A_{ch}$ (9.7 $fb^{-1}$ )	13	1.79
H1 $\sigma_r^o$ [20]	10	0.68		CDF Run-2 inclusive jet production [40]	72	1.45
HERA charm production	47	1.26		D0 Run-2 inclusive jet production [41]	110	1.09
HERA1 Combined NC and CC DIS [22]	579	1.02	Nadolo	ATLAS 7 TeV 35 pb <sup>-1</sup> incl. jet production	90	0.55
H1 $F_L$ [23]	9	1.92	11000	CMS 7 TeV 5 fb <sup>-1</sup> incl. jet production [43]	133	1.33



The combined HERA1+2 data are included in HERA2.0, **CT14H2**, MMHT, and NNPDF3.1 analyses

 $\chi^2/d.o.f. \sim 1.2$  for HERA1+2 tends to be elevated across all analyses, compared to  $\chi^2/d.o.f. < 1.1$  for combined HERA1 data

- ⇒ This tension may arise from several sources
- Higher-twist corrections to  $F_L(x, Q)$ A. Cooper-Sarkar, R. Thorne
- Small-x/saturation
   A. Luszszak
- Experimental systematics (?)

<sup>25</sup> The impact on global PDFs is mild, changes in PDFs do not exceed uncertainties P. Nadolsky, LAL Orsay

# MMHT refit including combined HERA I+II data. Under refitting in global fit NLO – $\chi^2 = 1533/1185 = 1.29$ per point. NNLO – $\chi^2 = 1457/1185 = 1.23$ per point.



HERA II modified PDFs very well within MMHT2014 uncertainties. PDFs from HERA II data only fit in some ways similar to HERAPDF2.0.

## CT14HERA2: effect of increasing the statistical weight of HERAI+II data from w=1 to w=6



With w = 6, CT14HERA2 PDFs for g, u, and d are more similar to <sup>4/19/2016</sup> HERA2.0, not to ABM12 in a different theory framework with  $\alpha_s = 0.113$ 

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For sea PDFs, similarity is less pronounced (not as sensitive to HERA DIS)

#### Modifications to the HERAPDF2.0 fit called HHT By I.Abt, A.M.Cooper-Sarkar, B.Foster, V.Myronenko, K.Wichmann, M.Wing





PDFs – and hence high Q<sup>2</sup> physics - not changed



Tendency to overshoot some of the highest y points at low x and  $Q^2$ .

Try modification  $F_L \rightarrow (1 + A/Q^2)F_L$  for x < 0.01. Refit and leaving A as a free parameter  $\rightarrow \Delta \chi^2 = -24$  for  $Q_{\min}^2 = 2 \text{ GeV}^2$ .  $A \approx 4$ . Further modifications help little. Alternatives not as good.



Just about all evidence of a fall of  $\chi^2$  per point with  $Q_{\min}^2$  eliminated.

## Specialized PDF sets

#### CJ15: DIS data for $Q^2 > 1.3 \ GeV^2$ , $W^2 > 3 \ GeV^2$



### CJ15 vs. others





FIG. 10: Comparison of various NLO photon PDFs at the scale Q = 3.2 GeV: CT14QED with  $p_0^{\gamma} = 0\%$  (green), CT14QED with  $p_0^{\gamma} = 0.14\%$  (black), MRST2004QED0 using current quark masses (orange), MRST2004QED1 using constituent quark masses (brown), and NNPDF2.3QED with  $\alpha_s = 0.118$  and average photon (blue).



FIG. 11: Comparison of various NLO photon PDFs at the scales Q = 85 GeV (left) and Q = 1 TeV (right): CT14QED with  $p_0^{\gamma} = 0\%$  (green), CT14QED with  $p_0^{\gamma} = 0.14\%$  (black), MRST2004QED0 using current quark masses (orange), MRST2004QED1 using constituent quark masses (brown), and NNPDE2 30ED with  $\alpha = 0.118$  and average photon (blue) P. Nadolsky, LAL Orsay 72


## Photon-Photon Luminosity





FIG. 4: Photon-photon luminosity for an invariant mass of 20 GeV to 500 GeV for 13 TeV collider energy

FIG. 5: Photon-photon luminosity for and invariant mass of 500 GeV to 6000 GeV for 13 TeV collider energy

CTEQ

### High mass Drell-Yan: results and comparison to theory II/II

- The measured cross-sections are compared to theoretical predictions using a selection of recent PDFs.
- Theory uncertainties are larger than measurement uncertainties => potential for PDF constraints.
- Photon induced contribution reaches 15%.







## Compare CMS Data to various photon PDFs



 $\mathbf{C}$ 

## Intrinsic Charm PDFs from CTEQ-TEA Global Analysis

S. Dulat et al., 1309.0025; PoS DIS2015 (2015) 166





3. 3: Charm quark distribution x c(x, Q) from the BHPS1 and BHPS2 PDFs (which have 0.57% and 2%  $\langle x \rangle_{\rm IC}$ ); from SEA1 and SEA2 PDFs (which have 0.57% and 1.5%  $\langle x \rangle_{\rm IC}$ ); and from CT10.

#### 4/19/2016

P. Nadolsky, LAL Orsay

## News from **NPDF**(II)

- First determination of the fitted charm PDF in the NNPDF framework
- Non-perturbative charm can account for up to 0.8% of the proton momentum (68% CL)
- The EMC charm structure function data can be satisfactorily described
- Fitting the charm PDF stabilises the mc dependence of high-scale cross-sections



More in the talks from Juan Rojo (Tue) and Luca Rottoli (Wed)

## Origin of differences between PDF sets

- 1. Corrections of wrong or outdated assumptions
- lead to significant differences between new (post-2014) and old (pre-2010) PDF sets
- benchmarking of NNLO heavy-quark hard scattering contributions
- CT10 and MSTW'2008 PDFs implement complete heavy-quark
- treatment; previous PDFs are obsolete without it
- relaxation of ad hoc constraints on PDF parametrizations
- improved numerical approximations

## **PDF4LHC** recommendation

# (N+1)-dim. perspective eliminates wrong N-dim. solutions

