# Puzzles of excited charm meson masses 

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# Plan of Talk 

Introduction
\% Experimental status
© Chiral Lagrangian for Heavy Mesons
Meson masses in HHChPT
Results from different fits
Conclusion

## Charm mesons spectrum



Some symmetry ?

## Experimental Status

Lowest lying states, odd-parity : well-determined

In accordance with iso-spin symmetry and SU(3) predictions.

Hyperfine splittings in strange and nonstrange well-determined

Adopted by PDG

Excited states, even parity: strange sector is well-determined

Excited states, even parity: non-strange sector, poor shape

Charged non-strange sector is particularly bad.

Only 0+ has been seen by FOCUS.
Heavier than corresponding strange state!

FOCUS mass measurement of neutral $0+$ consistent with charged $0+$

CLEO has seen neutral $1+$.
Heavier than corresponding strange state!

Only BELLE does not have conflict with $\mathrm{SU}(3)$.
But sees only neutral states, both $0+$ and $1+$

## Heavy Quark Symmetry $\mathrm{m}_{\mathrm{Q}} \rightarrow \infty$

- Typical length scale of the confinement $\mathrm{R}_{\text {had }} \sim 1 / \Lambda_{\mathrm{QCD}} \sim 1 \mathrm{fm}$
- The order of energy exchanged between quarks $\sim \Lambda_{\mathrm{QCD}}$
- Since $\lambda_{Q} \ll R_{\text {had }}$ the soft gluons cannot probe the structure of Heavy quark
$\rightarrow$ Flavour-symmetry

Spin Interaction $\propto 1 / m_{Q}$
$\rightarrow$ Spin-symmetry


## Mass splitting

The first term which is independent of spin and flavor gives

$$
\frac{1}{2}<H_{Q}\left|H_{1}\right| H_{Q}>\left.\equiv \bar{\Lambda} \equiv\left(m_{H}-m_{Q}\right)\right|_{m_{Q} \rightarrow \infty}
$$

The second term is kinetic energy arising from the offshell residual motion of the heavy quark and gives

$$
\frac{1}{2}<H_{Q}\left|\frac{1}{2 m_{Q}} \bar{h}_{v}\left(i D_{\perp}\right)^{2} h_{v}\right| H_{Q}>\equiv-\frac{\lambda_{1}}{2 m_{Q}}
$$

The third term gives contribution to the chromomagnetic interaction of heavy quark with the gluon field and hence violates the spin symmetry

$$
\frac{1}{2}<H_{Q}\left|\frac{g_{\alpha}}{4 m_{Q}} \bar{h}_{\nu} \sigma_{\mu \nu} G^{\mu \nu} h_{v}\right| H_{Q}>\equiv-\frac{\lambda_{2}}{4 m_{Q}}\left(\vec{S}_{Q} \cdot \vec{S}_{l}\right)
$$

## Mass splitting...

$$
\vec{S}_{Q} \cdot \vec{S}_{l}=\frac{1}{2}\left(\vec{J}^{2}-\vec{S}_{Q}^{2}-\vec{S}_{l}^{2}\right)=\frac{1}{2}[J(J+1)-3 / 2]
$$

Here both $S_{Q}, S_{1}$ are $1 / 2$ while $]$ can be 0 or 1 depending upon whether it is pseudoscalar or vector meson. Hence in general the mass relations can be written as

$$
m_{H}=m_{Q}+\bar{\Lambda}-\frac{\lambda_{1}}{2 m_{Q}}+\frac{2[J(J+1)-3 / 2] \lambda_{2}}{2 m_{Q}}+O\left(\frac{1}{m_{Q}{ }^{2}}\right)
$$

where the hadronic parameters $\bar{\Lambda}, \lambda_{1}, \lambda_{2}$ are independent of $\mathrm{m}_{\mathrm{Q}}$, they characterize the properties of light quarks.

## Chiral symmetry

On other extreme where the masses of quarks go to zero we get chiral symmetry. The lagrangian for light quarks is

$$
L=\bar{q}(i \not \partial-m) q
$$

The presence of mass term mixes both right and left handed quarks. When the mass term goes to zero we get,

$$
L=\overline{q_{L}}(i \nexists) q_{L}+\overline{q_{R}}(i \not \partial) q_{R}
$$

It leads to (in case of $u$, $d$, s go to zero) $\mathrm{SU}(3) \times \mathrm{SU}(3)$ symmetry.

## Chiral symmetry ...

$\longrightarrow \operatorname{SU}(3) \times \operatorname{SU}(3)$ symmetry (in case of $u, d, s$ quarks)
$\longrightarrow$ eight vector and eight axial-vector currents

$$
V_{\mu}^{i}=\bar{q} \gamma_{\mu} \frac{1}{2} \lambda_{i} q, A_{\mu}^{i}=\bar{q} \gamma_{\mu} \gamma_{5} \frac{1}{2} \lambda_{i} q
$$

$\longrightarrow$ axial-symmetry is spontaneously broken giving rise to eight pseudoscalar mesons
$\longrightarrow$ The lagrangian is expressed in powers of the Goldstone fields

If we have two expansion like $A=A_{0}+A_{1}+A_{3}+\ldots$ and $B=B_{0}+B_{1}+B_{3}+\ldots$ then we can have the product as $A B=A_{0} B_{0}+A_{1} B_{0}+A_{2} B_{0}+A_{1} B_{1}+\ldots$

## Chiral lagrangian for Heavy Meson

Since the pseudoscalar and vector mesons are degenerate in the heavy Quark mass limit and the spin of the heavy quark is conserved by low Energy strong interactions, it is convenient to introduce a single field

$$
H_{a}=\frac{1+\psi}{2}\left(P_{a}^{\mu} \gamma_{\mu}-P_{a} \gamma_{5}\right)
$$

where $P_{a}$ and $P_{a}^{*}$ are the $J^{P}=0^{-}$pseudoscalar and $J^{P}=1^{-}$vector meson Fields and ' $a$ ' is an $\mathrm{SU}(3)$ index.

The leading order chiral lagrangian for heavy meson interacting with pions (Goldstone bosons) is given by

$$
L_{v}^{0}=-\operatorname{Tr} \bar{H}_{v}(i v \cdot \partial) H_{v}+\operatorname{Tr} \bar{H}_{v} H_{v}(i v \cdot V)+2 g \operatorname{Tr} \bar{H}_{v} H_{v}\left(S_{l v} \cdot A\right)
$$

## Continued...

The Goldstone octet is given by

$$
\Pi=\left[\begin{array}{ccc}
\frac{1}{\sqrt{2}} \pi^{0}+\frac{1}{\sqrt{6}} \eta & \pi^{+} & K^{+} \\
\pi^{-} & -\frac{1}{\sqrt{2}} \pi^{0}+\frac{1}{\sqrt{6}} \eta & K^{0} \\
K^{-} & \bar{K}^{0} & -\frac{2}{\sqrt{6}}
\end{array}\right]
$$

and it appears in lagrangian through the vector and axial vector Linear combinations

$$
\begin{aligned}
& V^{\mu}=\frac{1}{2}\left(\xi \partial^{\mu} \xi^{+}+\xi^{+} \partial^{\mu} \xi\right) \quad A^{\mu}=\frac{i}{2}\left(\xi \partial^{\mu} \xi^{+}-\xi^{+} \partial^{\mu} \xi\right) \\
& \text { where } \quad \xi=e^{i \Pi / f} \quad \Sigma=\xi^{2}=e^{2 i \Pi / f}
\end{aligned}
$$

## Continued...

$>$ Higher dimensional operators of the chiral lagrangian which break heavy quark spin-flavor symmetry and chiral symmetry involve factors of $1 / \mathrm{m}_{\mathrm{Q}}$ or insertion of the light quark mass matrix $m=\operatorname{diag}\left(m_{u}, m_{d}, m_{s}\right)$
> For the calculation of heavy meson masses it is useful to classify the symmetry-violating operators by the number of insertions of quark mass matrix and whether or not they violate the heavy quark spin symmetry.
$>$ There is only one other term with no insertions of the light quark mass matrix,

$$
L_{v}^{s}=-\frac{\Delta}{8} \operatorname{Tr} \bar{H}_{v} \sigma^{\mu \nu} H_{v} \sigma^{\mu \nu}=-\Delta \operatorname{Tr} \bar{H}_{v} S_{Q v}{ }^{\alpha} H_{v} S_{l v \alpha}
$$

This term violates heavy quark spin-flavor symmetry and is responsible for hyperfine ( $P^{*}-P$ ) mass splitting. The parameter $\Delta$ is function of $1 / \mathrm{m}_{\mathrm{Q}}$.

## Continued...

The terms in the chiral lagrangian which are proportional to a light quark mass and which respect the heavy quark spin symmetry are given by

$$
L^{m s}=\operatorname{aTr} \bar{H}_{v} H_{v} m_{\xi}+\sigma \operatorname{Tr} m_{\xi} \operatorname{Tr} \bar{H}_{v} H
$$

where a and $\sigma$ are functions of $1 / m_{Q}$. The term proportional to a results in $\mathrm{SU}(3)_{\mathrm{V}}$ violating mass splitting amongst the $\mathrm{P}_{\mathrm{a}}{ }^{*}$ mesons. The term proportional to $\sigma$ leads to a singlet contribution to the masses which depends linearly on the light quarks masses.

Chiral lagrangian terms with one insertion of the light quark Mass matrix violate the heavy quark spin symmetry are


These terms produce hyperfine splitting proportional to a light quark mass.

## Continued...

Introducing similar field for $\mathrm{j}=1 / 2+$

$$
S_{a}=\frac{1+\psi}{2}\left(P_{a s}^{\mu^{*}} \gamma_{\mu} \gamma_{5}-P_{a s}\right)
$$

and combining contributions from these fields we get the total lagrangian as

$$
\begin{aligned}
& L_{v}^{\operatorname{mass}}=-\frac{\Delta_{H}}{8} \operatorname{Tr}\left[\bar{H}_{a} \sigma^{\mu \nu} H_{a} \sigma_{\mu \nu}\right]+\frac{\Delta_{S}}{8} \operatorname{Tr}\left[\bar{S}_{a} \sigma^{\mu \nu} S_{a} \sigma_{\mu \nu}\right]+a_{H} \operatorname{Tr}\left[\bar{H}_{a} H_{b}\right] n_{b a}^{\xi}- \\
& a_{S} \operatorname{Tr}\left[\bar{S}_{a} S_{b}\right] n_{b a}^{\xi}-\sigma_{S} \operatorname{Tr}\left[\bar{S}_{a} S_{a}\right] n_{b b}^{\xi}+\sigma_{H} \operatorname{Tr}\left[\bar{H}_{a} H_{a}\right] n_{b b}^{\xi}-\frac{\Delta_{H}^{(a)}}{8} \operatorname{Tr}\left[\overline{H_{a}} \sigma^{\mu \nu} H_{b} \sigma_{\mu \nu}\right] n_{b a}^{\xi} \\
& +\frac{\Delta_{S}^{(a)}}{8} \operatorname{Tr}\left[\bar{S}_{a} \sigma^{\mu \nu} S_{b} \sigma_{\mu \nu}\right] n_{b a}^{\xi}-\frac{\Delta_{H}^{(\sigma)}}{8} \operatorname{Tr}\left[\overline{H_{a}} \sigma^{\mu v} H_{a} \sigma_{\mu \nu}\right] n_{b b}^{\xi}+\frac{\Delta_{S}^{(\sigma)}}{8} \operatorname{Tr}\left[\bar{S}_{a} \sigma^{\mu \nu} S_{a} \sigma_{\mu v}\right] n_{b b}^{\xi}
\end{aligned}
$$

## Continued...

At tree level we get the residual masses as

$$
\begin{aligned}
& m_{H_{a}}^{0}=\delta_{H}-\frac{3}{4} \Delta_{H}+\sigma_{H} \bar{m}+a_{H} m_{a}-\frac{3}{4} \Delta_{H}^{(\sigma)} \bar{m}-\frac{3}{4} \Delta_{H}^{(\sigma)} m_{a} \\
& m_{H_{a}^{*}}^{0}=\delta_{H}+\frac{1}{4} \Delta_{H}+\sigma_{H} \bar{m}+a_{H} m_{a}+\frac{1}{4} \Delta_{H}^{(\sigma)} \bar{m}+\frac{1}{4} \Delta_{H}^{(\sigma)} m_{a} \\
& m_{S_{a}}^{0}=\delta_{S}-\frac{3}{4} \Delta_{S}+\sigma_{S} \bar{m}+a_{S} m_{a}-\frac{3}{4} \Delta_{S}^{(\sigma)} \bar{m}-\frac{3}{4} \Delta_{S}^{(\sigma)} m_{a} \\
& m_{S_{a}}^{0}=\delta_{S}+\frac{1}{4} \Delta_{S}+\sigma_{S} \bar{m}+a_{S} m_{a}+\frac{1}{4} \Delta_{S}^{(\sigma)} \bar{m}+\frac{1}{4} \Delta_{S}^{(\sigma)} m_{a}
\end{aligned}
$$

where we define residual mass as the difference between actual mass and a reference mass.

## One loop calculations

## One loop calculations are given below

$$
\begin{aligned}
m_{H_{1}}= & m_{H_{1}}^{0}+\frac{g^{2}}{f^{2}}\left[\frac{3}{2} K_{1}\left(m_{H_{1}^{*}}^{0}-m_{H_{1}}^{0}, m_{\pi}\right)+\frac{1}{6} K_{1}\left(m_{H_{1}}^{0}-m_{H_{1}}^{0}, m_{\eta}\right)+K_{1}\left(m_{H_{3}}^{0}-m_{H_{1}}^{0}, m_{K}\right)\right] \\
& +\frac{h^{2}}{f^{2}}\left[\frac{3}{2} K_{2}\left(m_{S_{1}}^{0}-m_{H_{1}}^{0}, m_{\pi}\right)+\frac{1}{6} K_{2}\left(m_{S_{1}}^{0}-m_{H_{1}}^{0}, m_{\eta}\right)+K_{2}\left(m_{S_{3}}^{0}-m_{H_{1}}^{0}, m_{K}\right)\right] \\
m_{H_{3}}= & m_{H_{3}}^{0}+\frac{g^{2}}{f^{2}}\left[2 K_{1}\left(m_{H_{1}^{*}}^{0}-m_{H_{3}}^{0} m_{K}\right)+\frac{2}{3} K_{1}\left(m_{H_{3}^{*}}^{0}-m_{H_{3}}^{0} m_{\eta}\right)\right] \\
& \quad+\frac{h^{2}}{f^{2}}\left[2 K_{2}\left(m_{S_{1}}^{0}-m_{H_{3}}^{0}, m_{K}\right)+\frac{2}{3} K_{2}\left(m_{S_{3}}^{0}-m_{H_{3}}^{0}, m_{\eta}\right)\right]
\end{aligned}
$$

## Continued...

$$
\begin{aligned}
& m_{H_{1}^{*}}=m_{H_{1}^{*}}^{0}+\frac{q^{2}}{f^{2}} \frac{1}{3}\left[\frac{3}{2} K_{1}\left(m_{H_{1}}^{0}-m_{H_{1}^{*}}^{0} m_{\pi}\right)+\frac{1}{6} K_{1}\left(m_{H_{1}}^{0}-m_{H_{1}^{*}}^{0} m_{\eta}\right)+K_{1}\left(m_{H_{3}}^{0}-m_{H_{1}^{*}}^{0}, m_{K}\right)\right] \\
& +\frac{g^{2}}{f^{2}} \frac{2}{3}\left[\frac{3}{2} K_{1}\left(0, m_{\pi}\right)+\frac{1}{6} K_{1}\left(0, m_{\eta}\right)+K_{1}\left(m_{H_{3}^{*}}^{0}-m_{H_{1}^{\prime}}^{0}, m_{K}\right)\right] \\
& +\frac{h^{2}}{f^{2}}\left[\frac{3}{2} K_{2}\left(m_{S_{1}^{*}}^{0}-m_{H_{1}^{*}}^{0}, m_{\pi}\right)+\frac{1}{6} K_{2}\left(m_{S_{1}^{*}}^{0}-m_{H_{1}^{*}}^{0}, m_{\eta}\right)+K_{2}\left(m_{S_{3}^{*}}^{0}-m_{H_{1}^{*}}^{0}, m_{K}\right)\right] \\
& m_{H_{3}^{*}}=m_{H_{3}^{*}}^{0}+\frac{q^{2}}{f^{2}} \frac{1}{3}\left[2 K_{1}\left(m_{H_{1}}^{0}-m_{H_{3}^{\prime}}^{0}, m_{K}\right)+\frac{2}{3} K_{1}\left(m_{H_{3}}^{0}-m_{H_{3}^{*}}^{0}, m_{\eta}\right)\right] \\
& +\frac{q^{2}}{f^{2}} \frac{2}{3}\left[2 K_{1}\left(m_{H_{i}}^{0}-m_{H_{3},}^{0} m_{K}\right)+\frac{2}{3} K_{1}\left(0, m_{\eta}\right)\right] \\
& +\frac{h^{2}}{f^{2}}\left[2 K_{2}\left(m_{S_{1}^{*}}^{0}-m_{H_{3}^{\prime}}^{0}, m_{K}\right)+\frac{2}{3} K_{2}\left(m_{S_{3}^{*}}^{0}-m_{H^{*}}^{0}, m_{\eta}\right)\right] \\
& m_{S_{1}}=m_{S_{1}}^{0}+\frac{g^{\prime 2}}{f^{2}}\left[\frac{3}{2} K_{1}\left(m_{S_{1}^{*}}^{0}-m_{S_{1}}^{0}, m_{\pi}\right)+\frac{1}{6} K_{1}\left(m_{S_{1}}^{0}-m_{S_{1}}^{0}, m_{\eta}\right)+K_{1}\left(m_{S_{3}^{*}}^{0}-m_{S_{1}}^{0}, m_{K}\right)\right] \\
& +\frac{h^{2}}{f^{2}}\left[\frac{3}{2} K_{2}\left(m_{H_{1}}^{0}-m_{S_{1}}^{0}, m_{\pi}\right)+\frac{1}{6} K_{2}\left(m_{H_{1}}^{0}-m_{S_{1}}^{0}, m_{\eta}\right)+K_{2}\left(m_{H_{3}}^{0}-m_{S_{1}}^{0}, m_{K}\right)\right]
\end{aligned}
$$

## Continued...

$$
\begin{aligned}
& m_{S_{3}}=m_{S_{3}}^{0}+\frac{g^{\prime 2}}{f^{2}}\left[2 K_{1}\left(m_{S_{1}^{*}}^{0}-m_{S_{3^{\prime}}}^{0} m_{K}\right)+\frac{2}{3} K_{1}\left(m_{S_{3}^{*}}^{0}-m_{S_{3}^{\prime}}^{0}, m_{\eta}\right)\right] \\
& +\frac{h^{2}}{f^{2}}\left[2 K_{2}\left(m_{H_{1}}^{0}-m_{S_{3^{\prime}}}^{0} m_{K}\right)+\frac{2}{3} K_{2}\left(m_{H_{3}}^{0}-m_{S_{3^{\prime}}}^{0} m_{\eta}\right)\right] \\
& m_{S_{1}^{*}}=m_{S_{1}^{\prime}}^{0}+\frac{g^{\prime 2}}{f^{2}} \frac{1}{3}\left[\frac{3}{2} K_{1}\left(m_{S_{1}}^{0}-m_{S_{\mathcal{S}^{\prime}}}^{0}, m_{\pi}\right)+\frac{1}{6} K_{1}\left(m_{S_{1}}^{0}-m_{S_{1}^{\prime}}^{0}, m_{\eta}\right)+K_{1}\left(m_{S_{3}}^{0}-m_{S_{1}^{\prime}}^{0}, m_{K}\right)\right] \\
& +\frac{8^{\prime 2}}{f^{2}} \frac{2}{3}\left[\frac{3}{2} K_{1}\left(0, m_{\pi}\right)+\frac{1}{6} K_{1}\left(0, m_{\eta}\right)+K_{1}\left(m_{S_{3}^{*}}^{0}-m_{S_{1}^{*}}^{0}, m_{K}\right)\right] \\
& +\frac{h^{2}}{f^{2}}\left[\frac{3}{2} K_{2}\left(m_{H_{1}^{*}}^{0}-m_{S_{1}^{\prime}}^{0}, m_{\pi}\right)+\frac{1}{6} K_{2}\left(m_{H_{1}^{*}}^{0}-m_{S_{1}^{*},}^{0} m_{\eta}\right)+K_{2}\left(m_{H_{3}^{*}}^{0}-m_{S_{1}^{\prime}}^{0}, m_{K}\right)\right] \\
& m_{S_{3}^{\prime}}=m_{S_{3}}^{0}+\frac{g^{2}}{f^{2}} \frac{1}{3}\left[2 K_{1}\left(m_{S_{1}}^{0}-m_{S_{3^{\prime}}}^{0}, m_{K}\right)+\frac{2}{3} K_{1}\left(m_{S_{3}}^{0}-m_{S_{3^{\prime}}}^{0}, m_{\eta}\right)\right] \\
& +\frac{g^{\prime 2}}{f^{2}} \frac{2}{3}\left[2 K_{1}\left(m_{S_{1}^{\prime}}^{0}-m_{S_{3^{\prime}}^{\prime}}^{0} m_{K}\right)+\frac{2}{3} K_{1}\left(0, m_{\eta}\right)\right] \\
& +\frac{h^{2}}{f^{2}}\left[2 K_{2}\left(m_{H_{1}^{*}}^{0}-m_{S_{3}^{*}}^{0} m_{K}\right)+\frac{2}{3} K_{2}\left(m_{H_{3}^{*}}^{0}-m_{S_{3}^{*}}^{0}, m_{\eta}\right)\right]
\end{aligned}
$$

## Continued...

we have defined the functions $K_{1}$ and $K_{2}$ as below.

$$
\begin{aligned}
K_{1}(\eta, M)= & \frac{1}{16 \pi^{2}}\left[\left(-2 \eta^{3}+3 M^{2} \eta\right) \ln \left(\frac{M^{2}}{\mu^{2}}\right)\right. \\
& \left.+2 \eta\left(\eta^{2}-M^{2}\right) F\left(\frac{\eta}{M}\right)+4 \eta^{3}-5 \eta M^{2}\right] \\
K_{2}(\eta, M)= & \frac{1}{16 \pi^{2}}\left[\left(-2 \eta^{3}+M^{2} \eta\right) \ln \left(\frac{M^{2}}{\mu^{2}}\right)\right. \\
& \left.+2 \eta^{3} F\left(\frac{\eta}{M}\right)+4 \eta^{3}-\eta M^{2}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
F(x) & =2 \frac{\sqrt{1-x^{2}}}{x}\left[\frac{\pi}{2}-\operatorname{Tan}^{-1}\left(\frac{x}{\sqrt{1-x^{2}}}\right)\right] & & |x|<1 \\
& =-2 \frac{\sqrt{x^{2}-1}}{x} \ln \left(x+\sqrt{x^{2}-1}\right) & & |x|>1
\end{aligned}
$$

## Continued...

The experimentally observed residual masses are

$$
\begin{aligned}
& m_{H_{1}}=-106.1 \mathrm{MeV} \\
& m_{H_{1}^{*}}=35.4 \mathrm{MeV} \\
& m_{H_{3}}=-4.75 \mathrm{MeV} \\
& m_{H_{3}^{*}}=139.1 \mathrm{MeV} \\
& m_{S_{1}}=335.0 \mathrm{MeV} \\
& m_{S_{1}^{*}}=465.0 \mathrm{MeV} \\
& m_{S_{3}}=344.4 \mathrm{MeV} \\
& m_{S_{3}^{*}}=486.3 \mathrm{MeV}
\end{aligned}
$$

Here we have used $\left(\mathrm{m}_{\mathrm{H} 1}+3 \mathrm{~m}_{\mathrm{H} 1^{*}}\right) / 4$ as reference mass.

## Fits

- With the one loop expressions and masses we can extract various parameters like strong coupling constant ( g ), coupling of pions to heavy mesons (f), the mass and hyperfine splitting parameters etc.,
- Here we have 11 parameters, $g, g^{\prime}, h, a_{H}, a_{S}, \Delta_{H^{(a)}}{ }^{(a)} \Delta_{\mathrm{S}}{ }^{(\mathrm{a})}$
$\delta_{H}+\sigma_{H} \mathrm{~m}, \delta_{\mathrm{s}}+\sigma_{\mathrm{S}} \mathrm{m}, \Delta_{H}+\Delta_{H}{ }^{(\sigma)} \mathrm{m}$ and $\Delta_{S}+\Delta_{S}{ }^{(\sigma)} \mathrm{m}$
- We can see that there are larger \# of parameters than masses
- Unique fit is not possible
- By fitting these data to the tree level expressions we get

$$
\begin{array}{ll}
\delta_{S}+\sigma_{S} \bar{m}=431.6 \pm 26.09 \mathrm{MeV} & \\
\delta_{H}+\sigma_{H} \bar{m}=-4.8 \pm 0.65 \mathrm{MeV} & \\
\Delta_{H}+\Delta_{H}^{(\sigma)} \bar{m}=141.3 \pm 1.17 \mathrm{MeV} & \\
\Delta_{S}+\Delta_{S}^{(\sigma)} \bar{m}=129.4 \pm 49.72 \mathrm{MeV} & \\
a_{H}=1.2 \pm 0.01, & a_{S}=0.21 \pm 0.29 \\
\Delta_{H}^{(a)}=0.028 \pm 0.02 & \Delta_{S}^{(a)}=0.14 \pm 0.55
\end{array}
$$

## Fits...

Typical fits from earlier works (by Mehen and Springer) with one loop expression

- $|g|=1.15 \pm 0.06,\left|g^{\prime}\right|=0.90 \pm 0.06,|h|=2.3+$ $0.2, \delta_{H}=195 \pm 41 \mathrm{MeV}, \delta_{S}=332 \pm 31 \mathrm{MeV}$, $\Delta_{H}=465 \pm 24 \mathrm{MeV}, \quad \Delta_{S}=597 \rightleftharpoons 28 \mathrm{MeV}$, $a_{H}=7 \pm 1, \quad a_{S}=-4 \pm 1, \quad \Delta_{H}^{(a)}=-4.4 \pm 0.7$, and $\Delta_{S}^{(a)}=-10 \pm 2$.
- $|g|=0.65 \pm 0.06,\left|g^{\prime}\right|=0.89 \pm 0.08,|h|=0.2 \pm$ $0.1, \delta_{H}=117 \pm 21 \mathrm{MeV}, \delta_{S}=646 \pm 40 \mathrm{MeV}$, $\Delta_{H}=68 \pm 42 \mathrm{MeV}, \Delta_{S}=447 \geqslant 23 \mathrm{MeV}, a_{H}=$ $3.8 \pm 0.7, a_{S}=3.1 \pm 0.7, \Delta_{H}^{(a)}=-0.3 \pm 1$, and $\Delta_{s}^{(a)}=-2.8 \pm 1$.


## Fits...

Since these expressions contains more no. of free parameters the actual data we have to use some sort of minimization program to fit them. We have used "Minuit " CERN minimization software.
$\checkmark$ Fix the parameters range

- $\delta_{H}-\delta_{S} \leq 400 \mathrm{MeV} \Longrightarrow \delta_{H}, \delta_{\mathrm{S}} \sim 100-500 \mathrm{MeV}$
- $\Delta_{H}-\Delta_{S} \sim 140 \mathrm{MeV} \Longrightarrow \Delta_{\mathrm{H}}, \Delta_{\mathrm{S}} \sim 100-200 \mathrm{MeV}$
- $\mathrm{a}_{\mathrm{H}}\left(\mathrm{m}_{\mathrm{s}}-\mathrm{m}_{\mathrm{H} / \mathrm{d}}\right) \sim 100 \mathrm{MeV} \Longrightarrow \mathrm{a}_{\mathrm{H}} \mathrm{a}_{\mathrm{S}} \sim 1 \mathrm{MeV}$
- We use $m_{s}=130 \mathrm{MeV}$ and $m_{u / d}=4 \mathrm{MeV}$


## Fits...

## Our fits

- $\delta_{H}=169.2 \pm 0.5 \mathrm{MeV} \quad \delta_{S}=345.4 \pm 0.7 \mathrm{MeV}$

$$
\begin{array}{lc}
\Delta_{H}=200.3 \pm 0.5 \mathrm{MeV} & \Delta_{S}=120.4 \pm 1.1 \mathrm{MeV} \\
a_{H}=1.522 \pm 0.005 & a_{S}=0.508 \pm 0.018 \\
\Delta_{H}^{(a)}=-1.231 \pm 0.005 & \Delta_{S}^{(a)}=0.193 \pm 0.015 \\
|g|=0.66 \pm 0.01 & \left|g^{\prime}\right|=0.03 \pm 0.01 \quad|h|=0.42 \pm 0.01
\end{array}
$$

- $\delta_{H}=184.9 \pm 3.6 \mathrm{MeV} \quad \delta_{S}=368.1 \pm 2.4 \mathrm{MeV}$

$$
\Delta_{H}=196.5 \pm 3.2 \mathrm{MeV} \quad \Delta_{S}=119.4 \pm 1.4 \mathrm{MeV}
$$

$$
a_{H}=1.534 \pm 0.006 \quad a_{S}=0.453 \pm 0.039
$$

$$
\Delta_{H}^{(a)}=-1.242 \pm 0.027 \quad \Delta_{S}^{(a)}=0.189 \pm 0.022
$$

$$
|g|=0.68 \pm 0.01 \quad\left|g^{\prime}\right|=0.01 \pm 0.04 \quad|h|=0.32 \pm 0.02
$$

- $\delta_{H}=159.2 \pm 3.7 \mathrm{MeV} \quad \delta_{S}=350.6 \pm 2.7 \mathrm{MeV}$

$$
\Delta_{H}=194.4 \pm 3.2 \mathrm{MeV} \quad \Delta_{S}=145.9 \pm 1.4 \mathrm{MeV}
$$

$$
\begin{array}{lc}
a_{H}=1.524 \pm 0.008 & a_{S}=0.423 \pm 0.043 \\
\Delta_{H}^{(a)}=-1.148 \pm 0.028 & \Delta_{S}^{(a)}=-0.01 \pm 0.02 \\
|g|=0.65 \pm 0.01 \quad\left|g^{\prime}\right|=0.05 \pm 0.03 \quad|h|=0.45 \pm 0.02
\end{array}
$$

## Results

$>$ All these fits yield mass values for $0^{+} \sim 2200-2250 \mathrm{MeV}$ and $1^{+}$~ $2335-2375 \mathrm{MeV}$
$>$ Satisfies the requirement of $\left(m_{D_{0}^{*}}-m_{D}\right)-\left(m_{D_{s}{ }^{*}}-m_{D_{s}}\right) \leq 0$ given by Becirevic et.,al PLB599, 55 (2004)
$>$ We have used the recent values of $\mathrm{g}, \mathrm{g}^{\prime}, \mathrm{h}$ from the calculation of Fajfer et.,al PRD74 (2006) 074023
$>$ These values lower than the experimental observations which are $0+\sim 2308 \mathrm{MeV}$ and $1+\sim 2420 \mathrm{MeV}$
$>$ Strange counterpart masses are $0^{+} \sim 2317$ and $1^{+} \sim 2460 \mathrm{MeV}$
$>$ Other regions of parameters space
B. Ananthanarayan B, Sunanda Banerjee, K. Shivaraj, A. Upadyay (Phys. Lett.B 651,124-128, 2007)

## Conclusions

$\square$ There are puzzles about the excited charm meson states.
$\square$ Our results show that non-strange states could be lower than the experimental values
$\square$ New experiments with higher statistics may solve this puzzle
$\square$ Possibility of measuring heavy quark constants on the lattice for excited states (R Horgan)

