
Calibration of the Top Quark Mass Parameter in Pythia 8.2

André H. Hoang

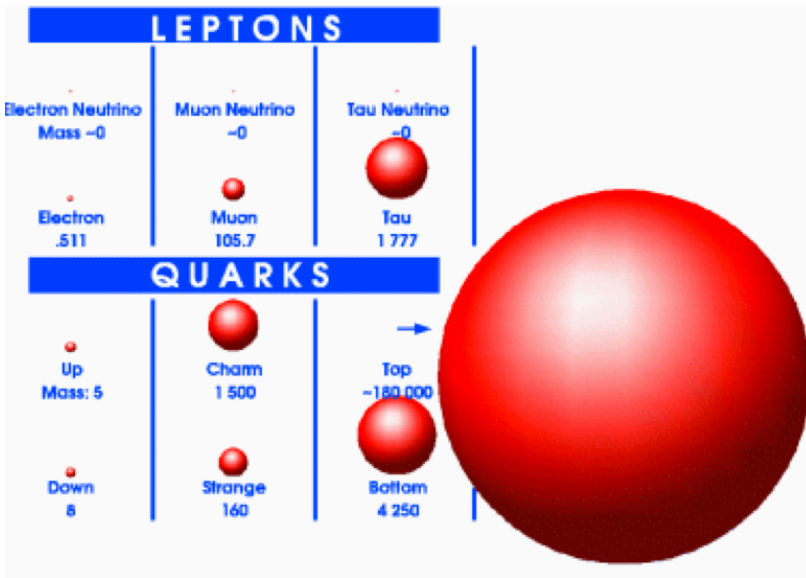
University of Vienna

∫dk **Π** Doktoratskolleg
Particles and Interactions

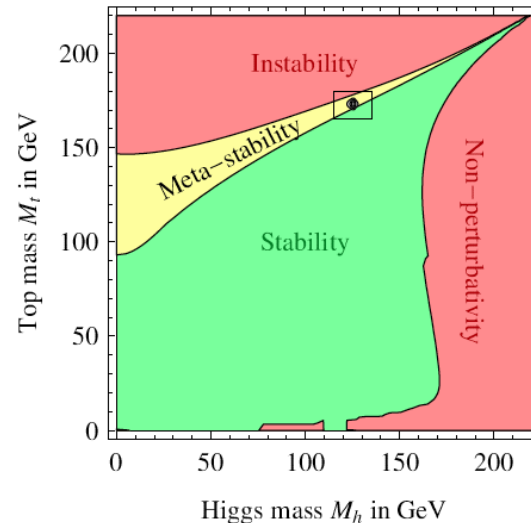
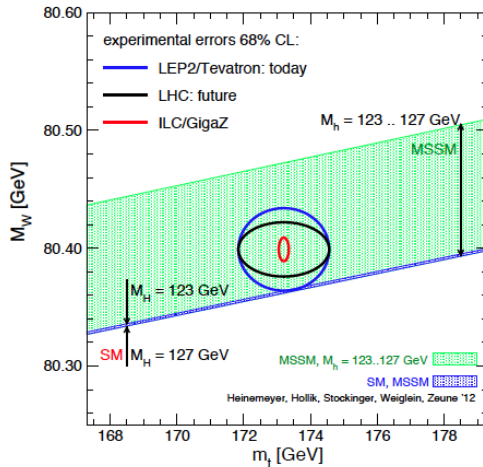


FWF
Der Wissenschaftsfonds.

Why the top quark is not just heavy



- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background in new physics searches
- One of crucial motivations for SUSY
- Excellent ground for high-precision studies of QCD and electroweak physics



Outline

- Introduction
- Monte Carlo generators and the top quark mass
- Calibration of the Monte Carlo top mass parameter
- Preliminary detailed results of first serious systematic analysis
- Summary, future plans

In collaboration with:

M. Butenschön

B. Dehnadi,

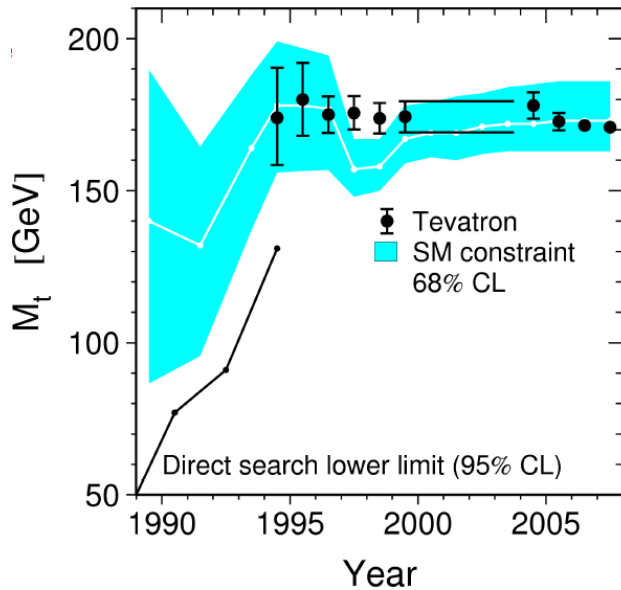
V. Mateu,

M. Preisser

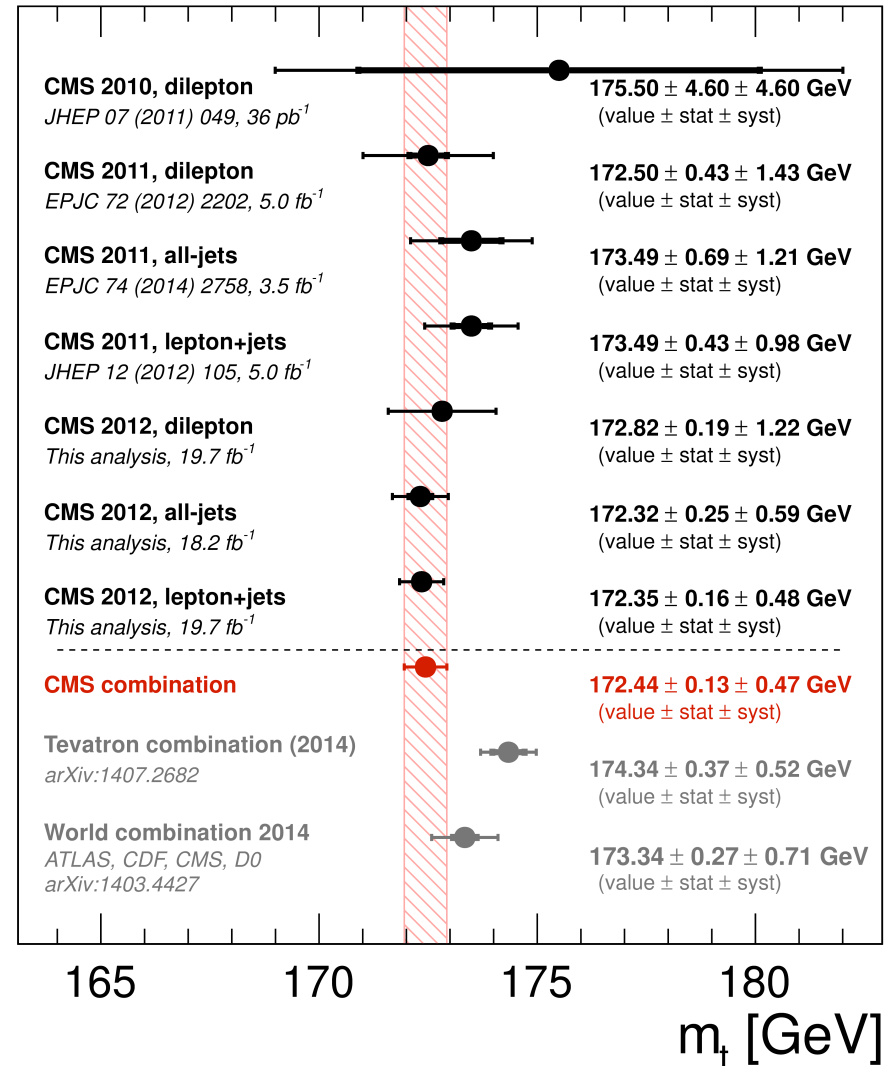
I. Stewart



A small history on top mass reconstruction



- Many individual measurements with uncertainty below 1 GeV.
- Some discrepancies between LHC and Tevatron
- Reached <500MeV range.



Main Top Mass Measurements Methods

LHC+Tevatron

Direct Reconstruction:

Kinematic Fit

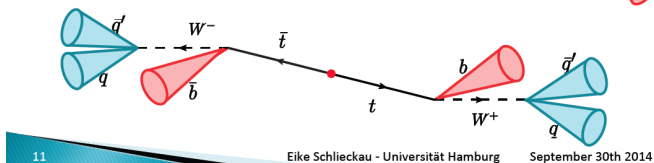
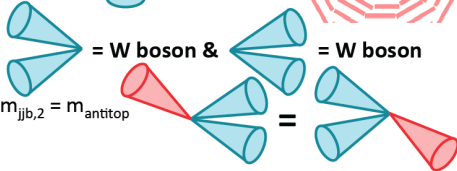
Selected objects:

- 4 untagged jets
- 2 b-tagged jets



Constraints:

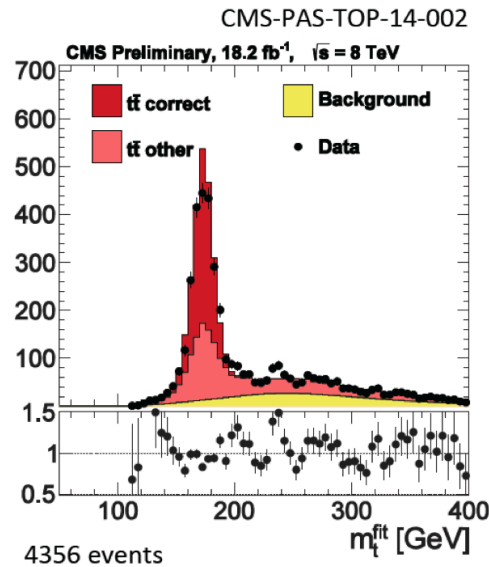
- $2 \times m_{jj} = m_W$
- $m_{top} = m_{jjb,1} = m_{jjb,2} = m_{antitop}$



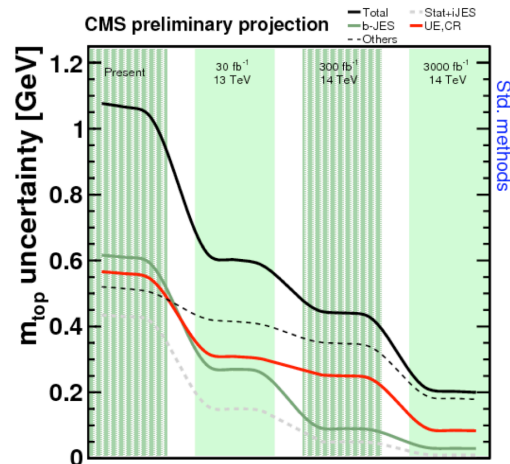
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Eike Schlieckau - Universität Hamburg September 30th 2014

kinematic mass determination



Determination of the best-fit value of the Monte-Carlo top quark mass parameter



← $\Delta m_t \sim 200 \text{ MeV}$ (projection)

⊕ High top mass sensitivity

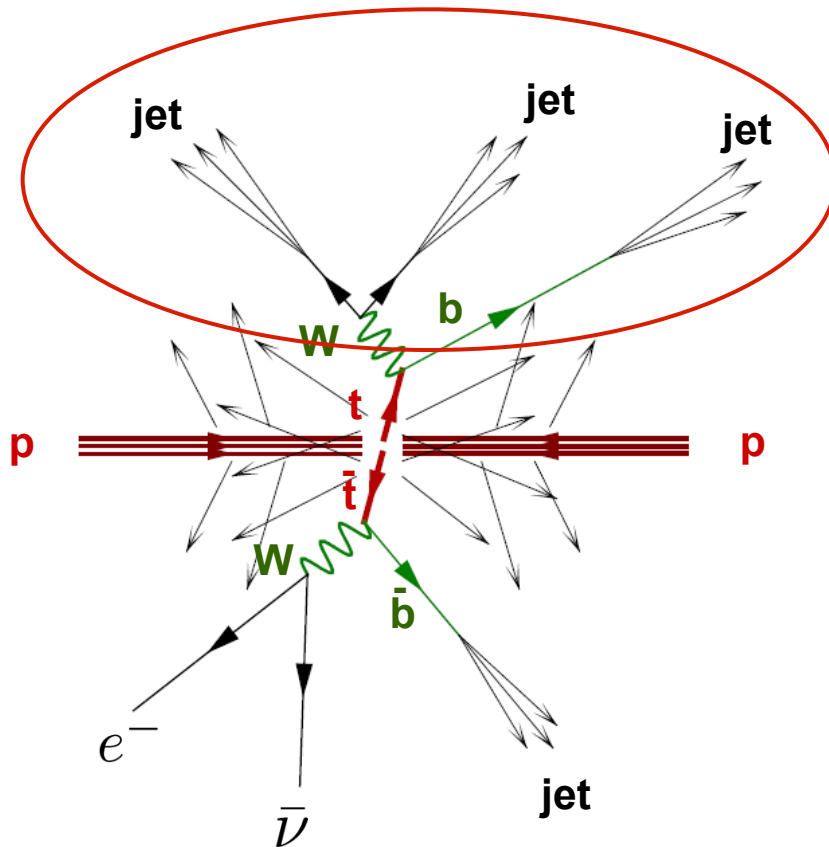
⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

$\Delta m_t \sim 0.5 \text{ GeV}$

Top quark mass reconstruction

LHC:

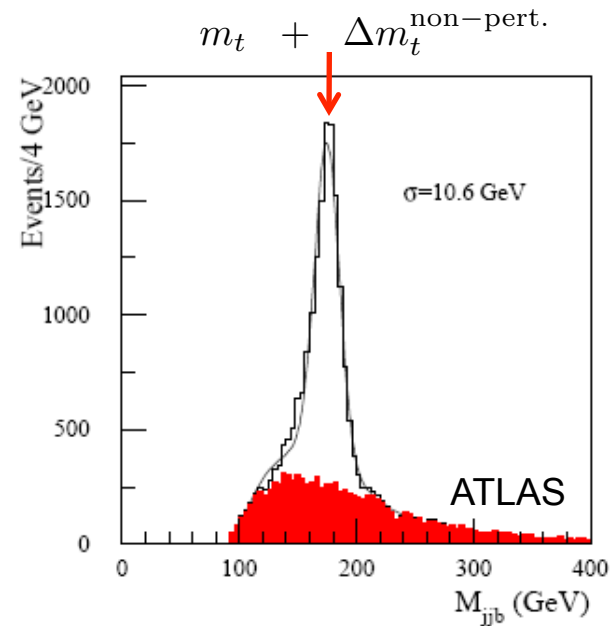


Principle of mass measurements:

Identification of the top decay products

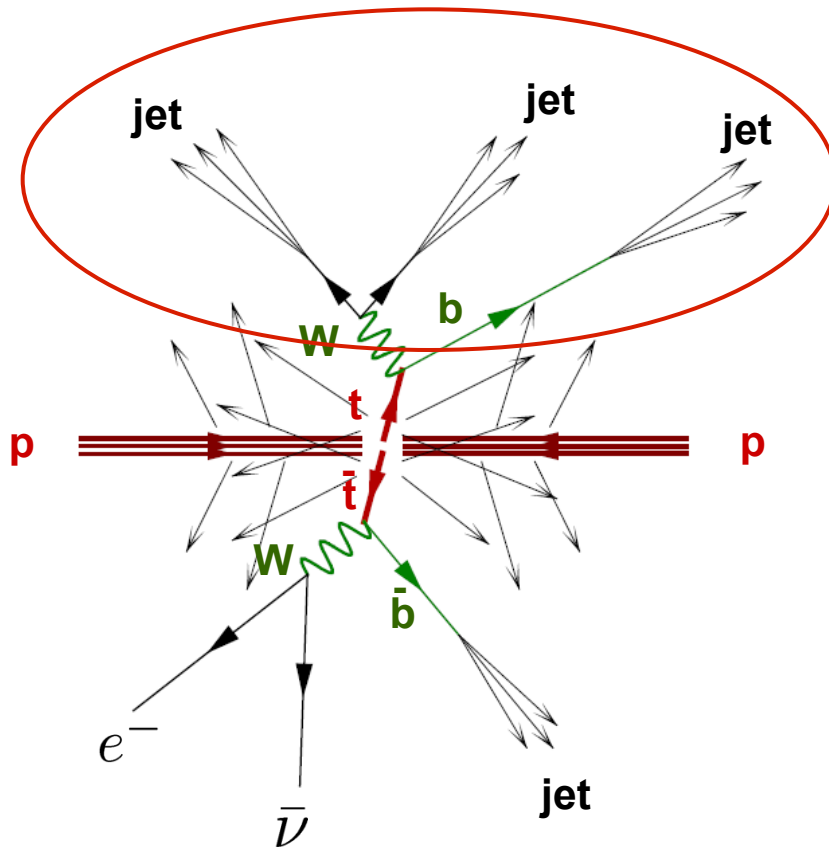
$$“ m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu \right)^2 ”$$

Invariant mass distribution



Top quark mass reconstruction

LHC:

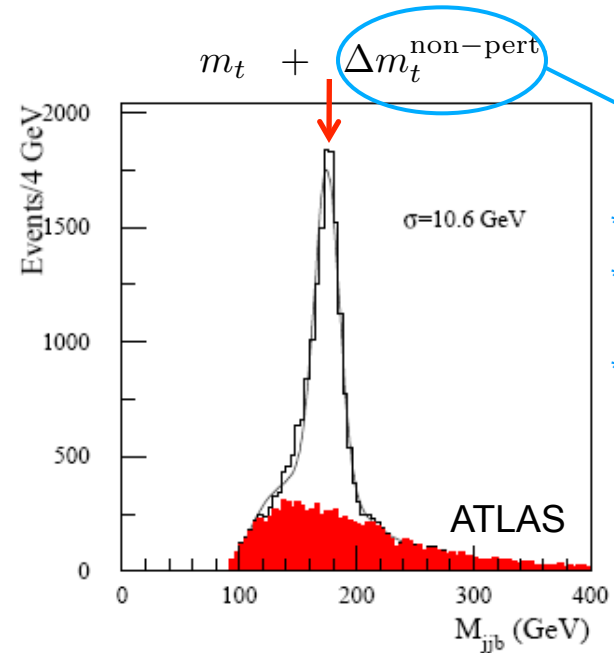


Principle of mass measurements:

Identification of the top decay products

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Invariant mass distribution

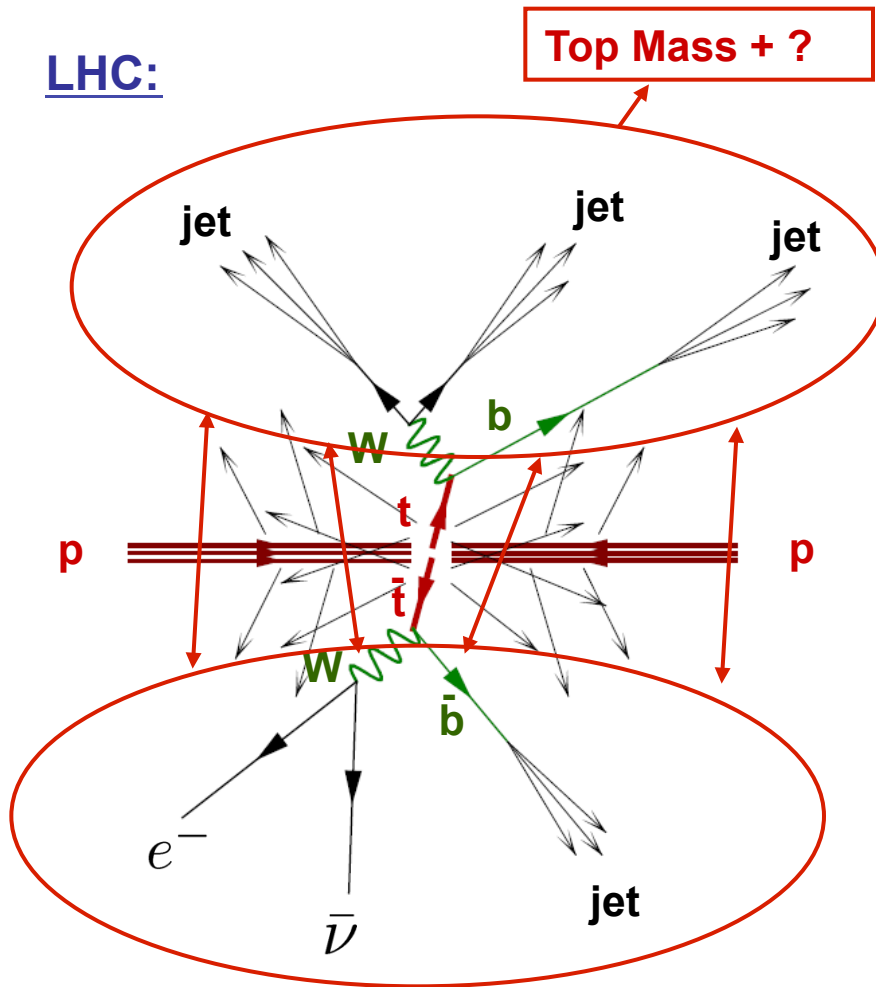


depends on:

- * jet definition
- * Soft correlation effects
- * Color reconnection

Top quark mass reconstruction

LHC:



Principle of mass measurements:

Identification of the top decay products

$$“ m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu \right)^2 ”$$

Problem is non-trivial !

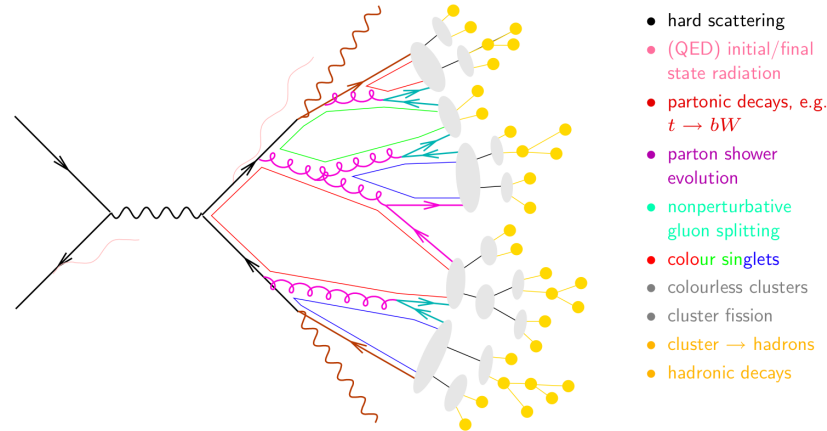
- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects !!**

The idea of a - by itself - well defined object having a well defined mass is incorrect !!

First principles QCD calculations impossible for the direct reconstruction method !

Only possible for Monte-Carlo generators.

Monte-Carlo Event Generators



- Full simulation of all processes (all experimental aspects accessible)
- QCD-inspired: partly first principles QCD \Leftrightarrow partly model (observable-dependent)
- Description power of data better than intrinsic theory accuracy.
- Top quark: treated like a real particle ($m_t^{\text{MC}} \approx m_t^{\text{pole}} + ?$).

But pole mass ambiguous by $O(1 \text{ GeV})$ due to confinement.

Better mass definition needed.

Uncertainty (a): But how precise is modelling? \rightarrow Part of exp. Analyses

Uncertainty (b): What is the meaning of MC QCD parameters? \rightarrow

Depends strictly speaking on the observable, because of model character of MCs !

Must be addressed for each type of observable (until we have better MCs).

MC Top Quark Mass

AHH, Stewart 2008
AHH, 2014

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) \sim \mathcal{O}(1 \text{ GeV})$$

- small size of $\Delta_{t,\text{MC}}$
- Renormalon-free
- little parametric dependence on other parameters

MSR Mass Definition

MS Scheme: $(\mu > \bar{m}(\bar{m}))$

$$\bar{m}(\bar{m}) - m^{\text{pole}} = -\bar{m}(\bar{m}) [0.42441 \alpha_s(\bar{m}) + 0.8345 \alpha_s^2(\bar{m}) + 2.368 \alpha_s^3(\bar{m}) + \dots]$$

MSR Scheme: $(R < \bar{m}(\bar{m}))$



$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R [0.42441 \alpha_s(R) + 0.8345 \alpha_s^2(R) + 2.368 \alpha_s^3(R) + \dots]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \bar{m}(\bar{m})$$

⇒ $m_{\text{MSR}}(R)$ Short-distance mass that smoothly interpolates all R scales
≈ “pole mass subtraction for scales larger than R”

Calibration of the MC Top Mass

Method:

- ✓ 1) Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !)
- ✓ 2) Accurate analytic hadron level QCD predictions at \geq NLL/NLO with full control over the quark mass scheme dependence.
- ✓ 3) QCD masses as function of m_t^{MC} from fits of observable.
- 4) Cross check observable independence

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(R = 1 \text{ GeV}) + \Delta_{t,\text{MC}}(R = 1 \text{ GeV})$$

$$\Delta_{t,\text{MC}}(1 \text{ GeV}) = \bar{\Delta} + \delta\Delta_{\text{MC}} + \delta\Delta_{\text{pQCD}} + \delta\Delta_{\text{param}}$$

Experimental systematics

Monte Carlo errors:

- different tunings
- parton showers
- color reconnection
- Intrinsic error, ...

QCD errors:

- perturbative error
- scale uncertainties
- electroweak effects

Parametric errors:

- strong coupling α_s
- Non-perturbative parameters

Treated in our analysis

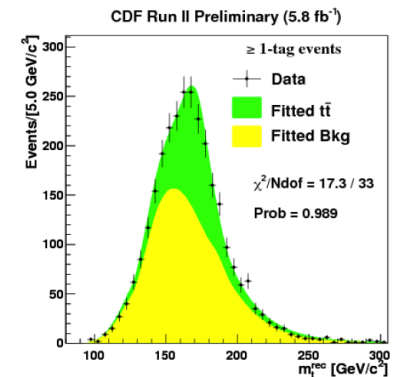
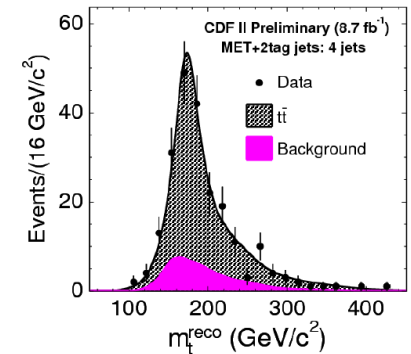
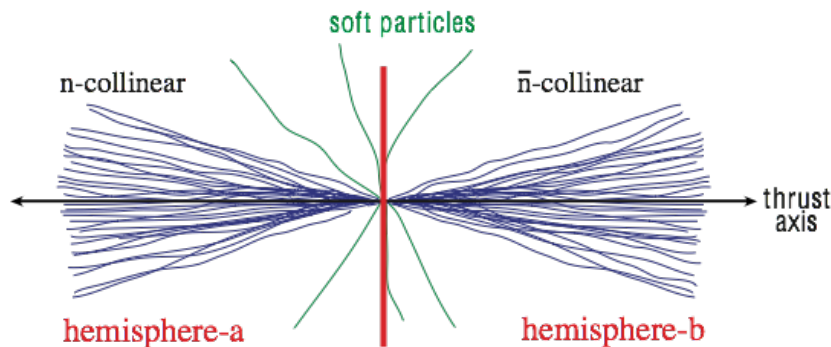
Thrust Distribution

Observable: 2-jettiness in e^+e^- for $Q \sim p_T \gg m_t$ (boosted tops)

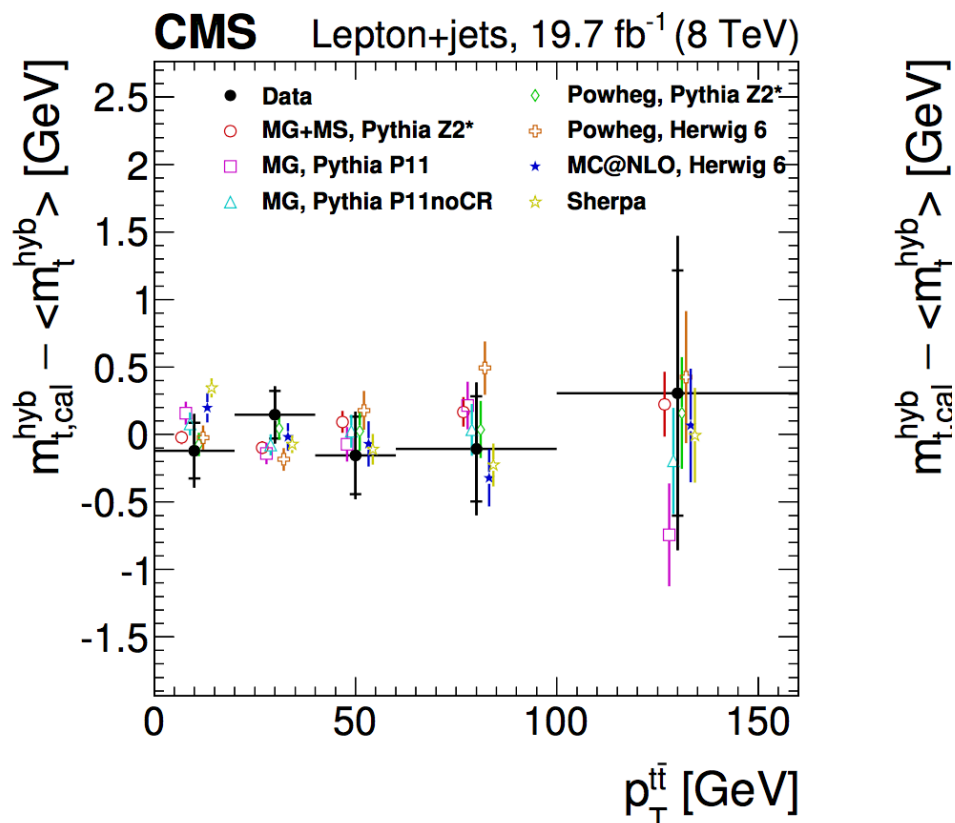
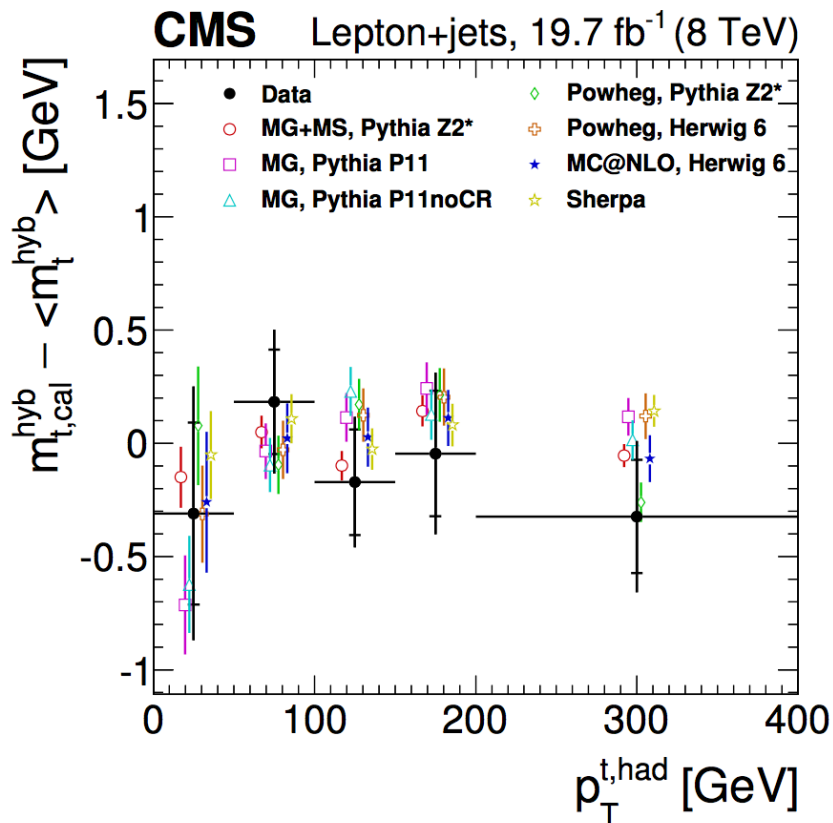
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

$$\xrightarrow{\tau \rightarrow 0} \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region of wide hemisphere jets !

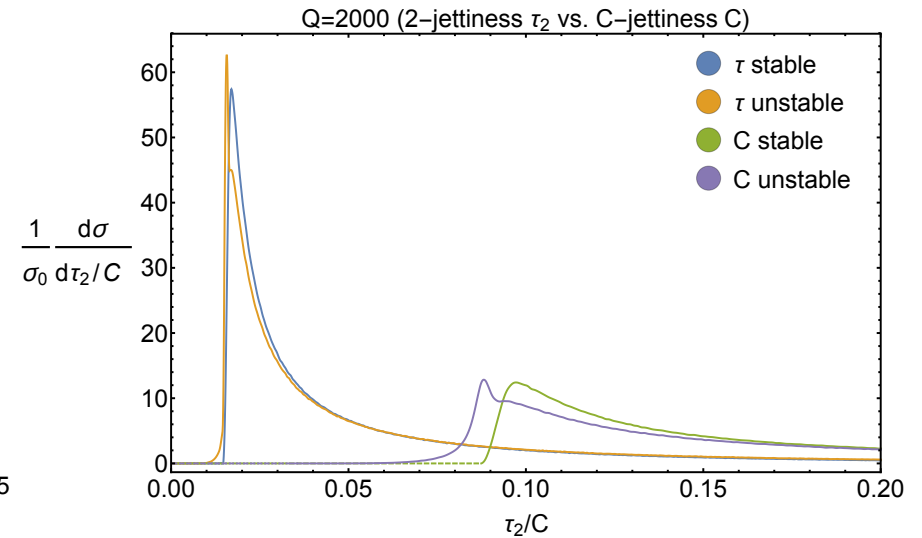
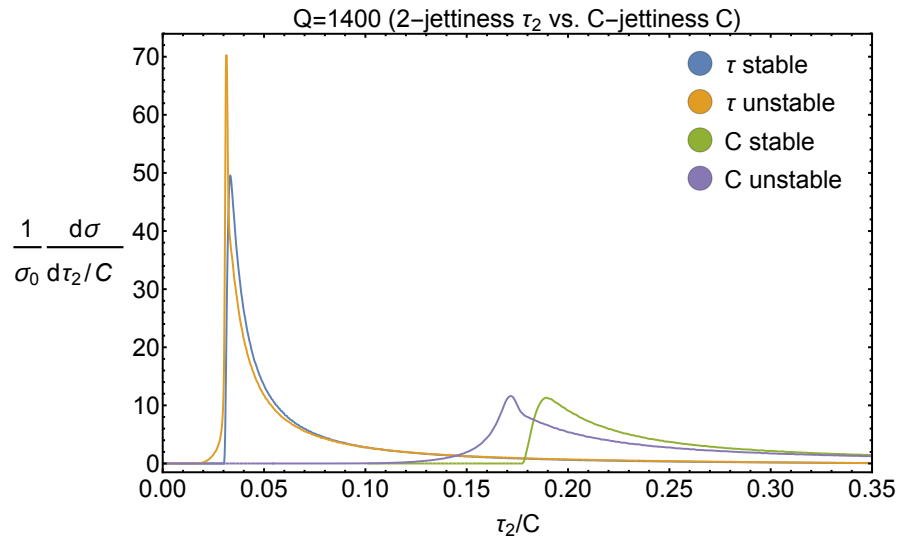
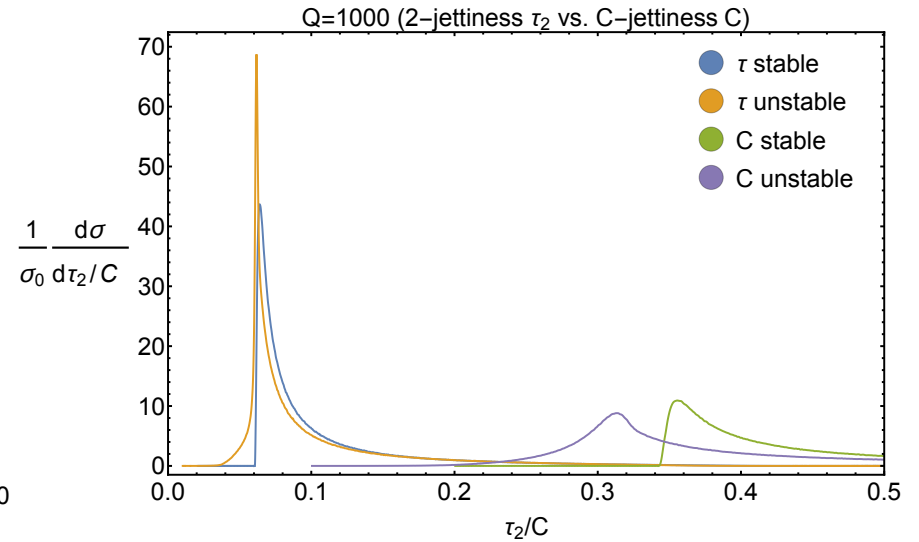
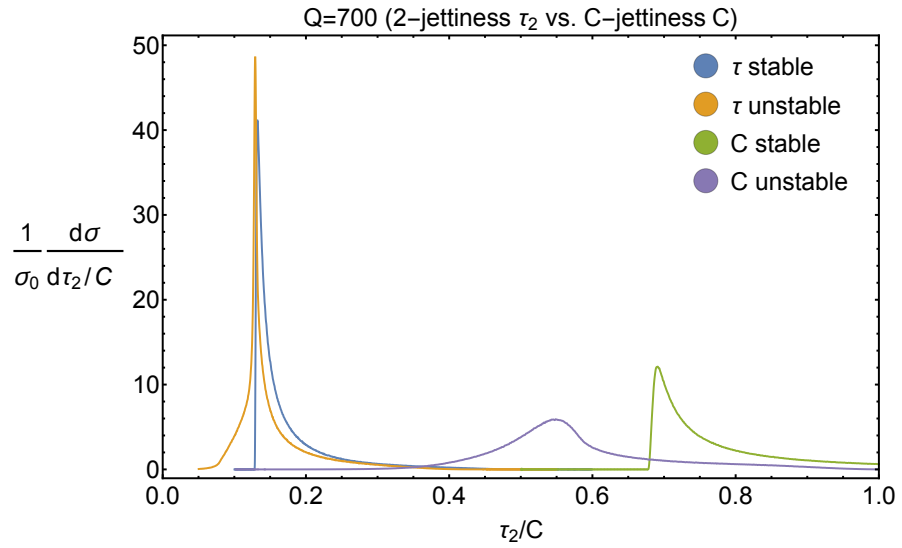


Boosted Top Mass Measurements at CMS



- Top mass from reconstruction of boosted tops consistent with low p_T results.
- More precise studies possible with more statistics from Run2.

Event Shape Distributions (Pythia 8.2)



Factorization for Event Shapes

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q, \mu) \int dl J_0(Ql, \mu) S_0(Q\tau - l, \mu)$$

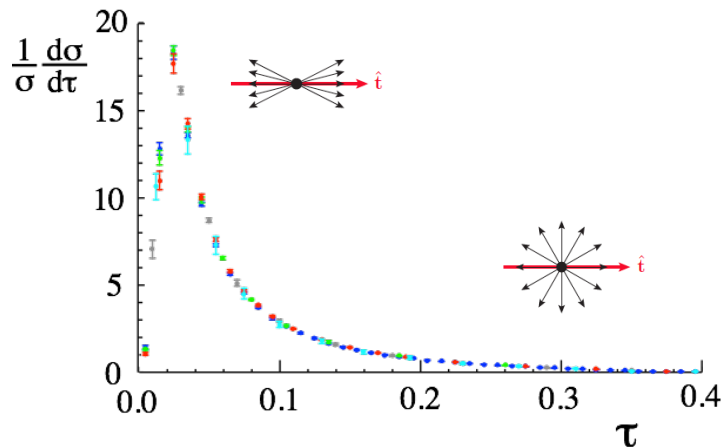
Massless quarks:

Korshemski, Sterman 1995-2000

Bauer, Fleming, Lee, Sterman (2008)

Becher, Schwartz (2008)

Abbate, AHH, Fickinger, Mateu, Stewart 2010



Extension to massive quarks:

- VFNS for final state jets (with massive quarks): log summation incl. mass
- Boosted fat top jets

Fleming, AHH, Mantry, Stewart 2007

Gritschacher, AHH, Jemos, Mateu Pietrulewicz 2013-2014

Butenschön, Dehnadi, AHH, Mateu 2016 (to appear soon)

➔ NNLL + NLO + non-singular + hadronization + renormalon-subtraction

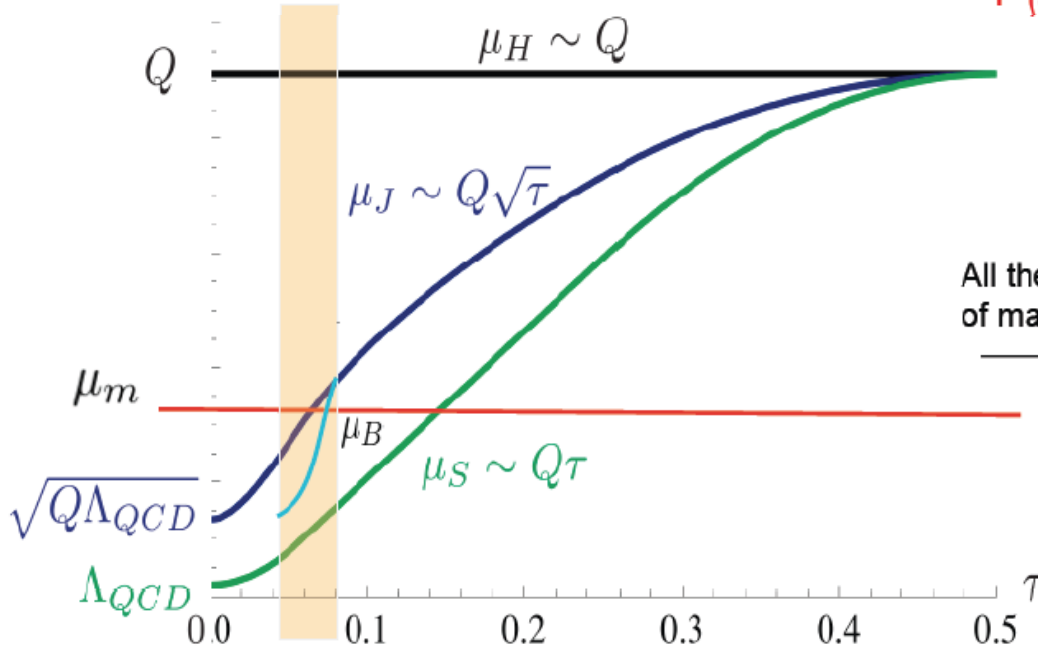
b(oosted)HQET Factorization

$$\left| \frac{1}{\sigma_0} \frac{d\hat{\sigma}(\tau)}{d\tau} \right|^{\text{bHQET}} = Q H_Q^{(n_f)}(Q, \mu_Q) U_{H_Q}^{(n_f)}(Q, \mu_Q, \mu_m) H_m^{(n_f)}(\bar{m}^{(n_f)}, \mu_m) U_{H_m}^{(n_l)}\left(\frac{Q}{\bar{m}^{(n_l)}}, \mu_m, \mu_B\right) \int ds \int dk B^{(n_l)}\left(\frac{s}{m^{(n_l)}}, \mu_B, m_J^{(n_l)}\right) U_S^{(n_l)}(k, \mu_B, \mu_S) S_{\text{part}}^{(n_l)}\left(Q\tau - Q\tau_{\text{MIN}} - \frac{s}{Q} - k, \mu_S\right)$$

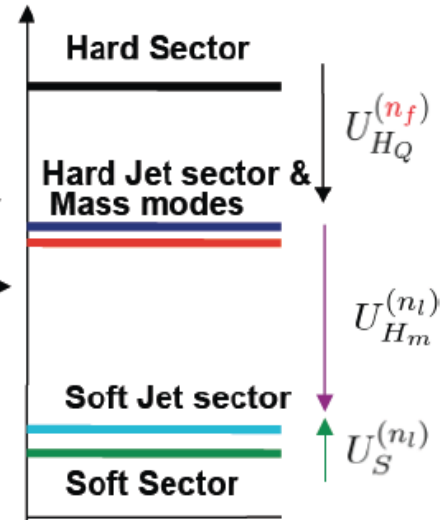
$n_f = n_l + 1$

Yukawa corrections here!

+ (SCET) Non-Singular + (QCD) Non-Singular



All the fluctuations of the order of mass are integrated out.

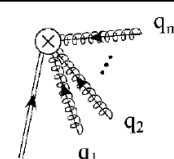


> Matching coefficient of SCET and bHQET have a large log from secondary corrections.

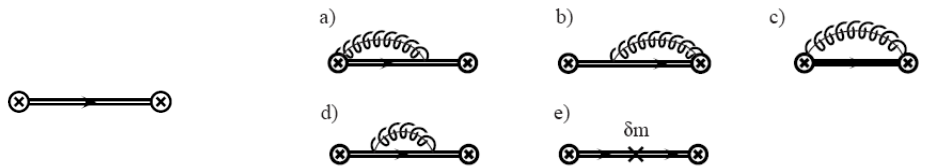
b(oosted)HQET Factorization

Jet function: $B_+(2v_+ \cdot k) = \frac{-1}{8\pi N_c m} \text{Disc} \int d^4x e^{ik \cdot x} \langle 0 | T \{ \bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x) \} | 0 \rangle$

- perturbative, any mass scheme
- depends on m_t, Γ_t
- Breit-Wigner at tree level
- Gauge-invariant off-shell top quark dynamics



$$W = \sum_{m=0}^{\infty} \sum_{\text{perms}} \frac{(-g)^m}{m!} \frac{\bar{n} \cdot A_{n,q_1}^{a_1} \cdots \bar{n} \cdot A_{n,q_m}^{a_m}}{\bar{n} \cdot q_1 \bar{n} \cdot (q_1 + q_2) \cdots \bar{n} \cdot (\sum_{i=1}^m q_i)} T^{a_m} \cdots T^{a_1}$$



$$\hat{s} = \frac{M^2 - m_t^2}{m_t}$$

$$\mathcal{B}_\pm(\hat{s}, 0, \mu, \delta m) = -\frac{1}{\pi m} \frac{1}{\hat{s} + i0} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left[4 \ln^2 \left(\frac{\mu}{-\hat{s} - i0} \right) + 4 \ln \left(\frac{\mu}{-\hat{s} - i0} \right) + 4 + \frac{5\pi^2}{6} \right] \right\} - \frac{1}{\pi m} \frac{2\delta m}{(\hat{s} + i0)^2}$$

Fleming, AHH, Mantry, Stewart 2007

b(oosted)HQET Factorization

Is the pole mass determining the top single particle pole?

NO !

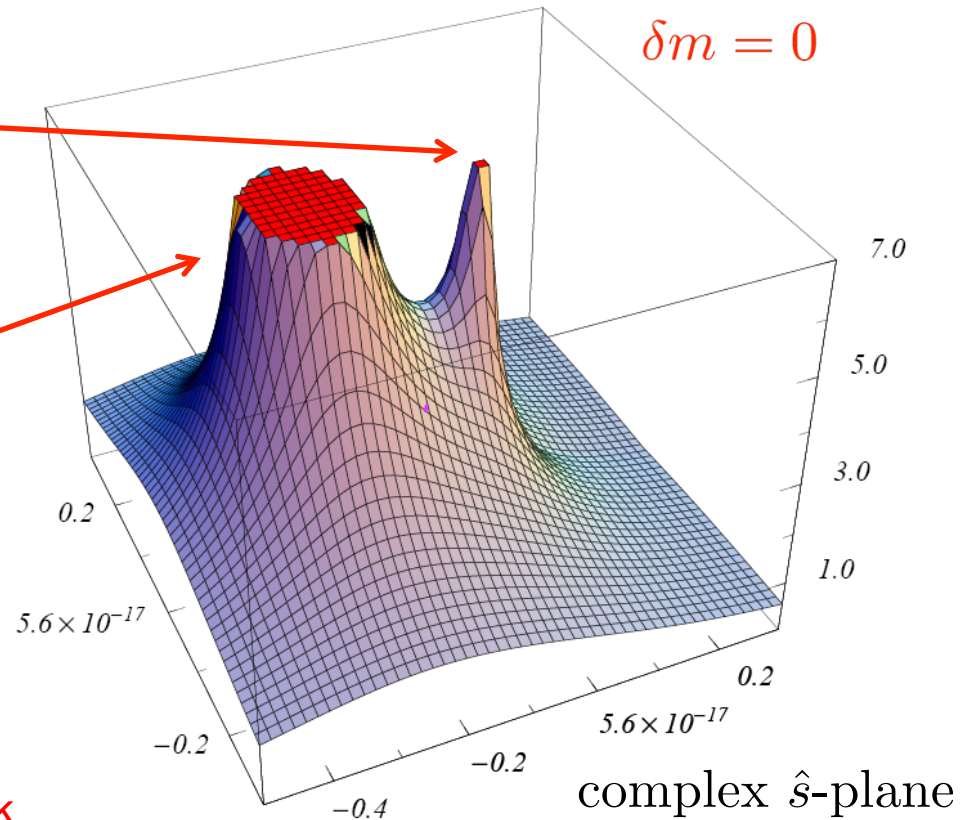
$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t}$$

$$|\mathcal{B}_{\pm}(\hat{s}, \Gamma_t, \mu)|^2$$

$$\delta m = 0$$

pole mass peak

observable peak



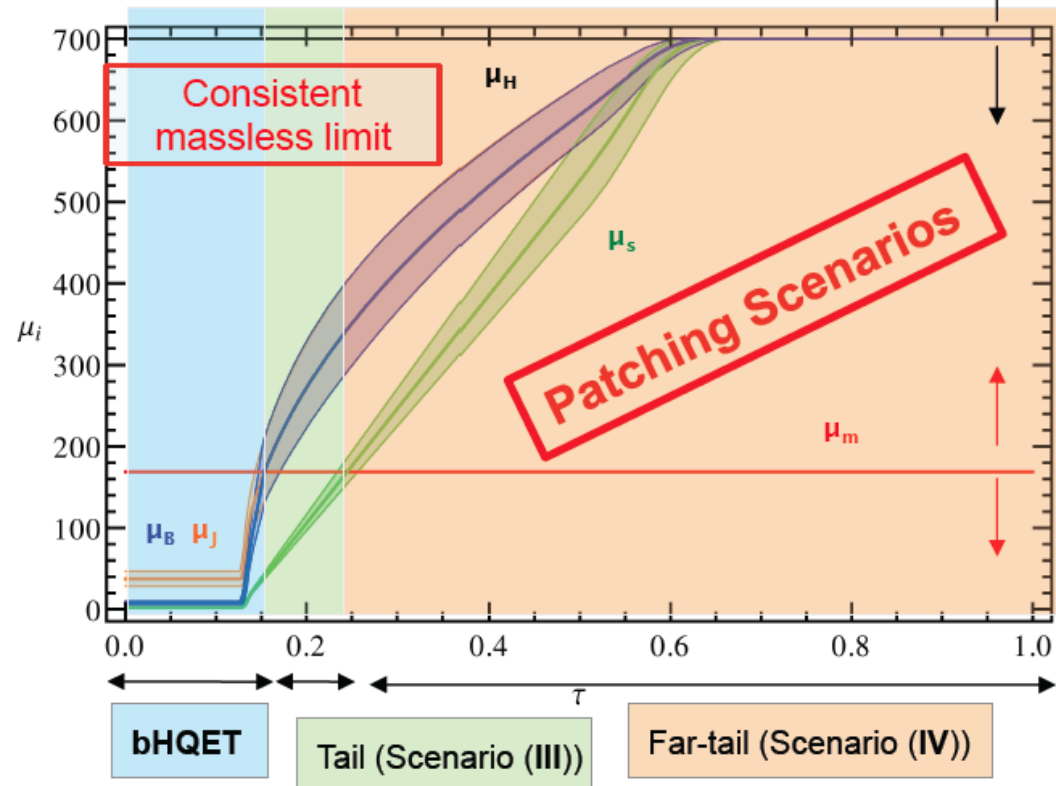
- pole mass and observable peak separated by renormalon
- pole mass peak residue decreases with order
- MSR mass close to observable peak

Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau - \tau_{\min}) + 2\Lambda_{\text{QCD}}}{\mu_s}\right)$$

$Q = 700 \text{ GeV}$



Scales Variation

- ✓ Generalized to arbitrary mass values
- ✓ Compatible with massless profiles

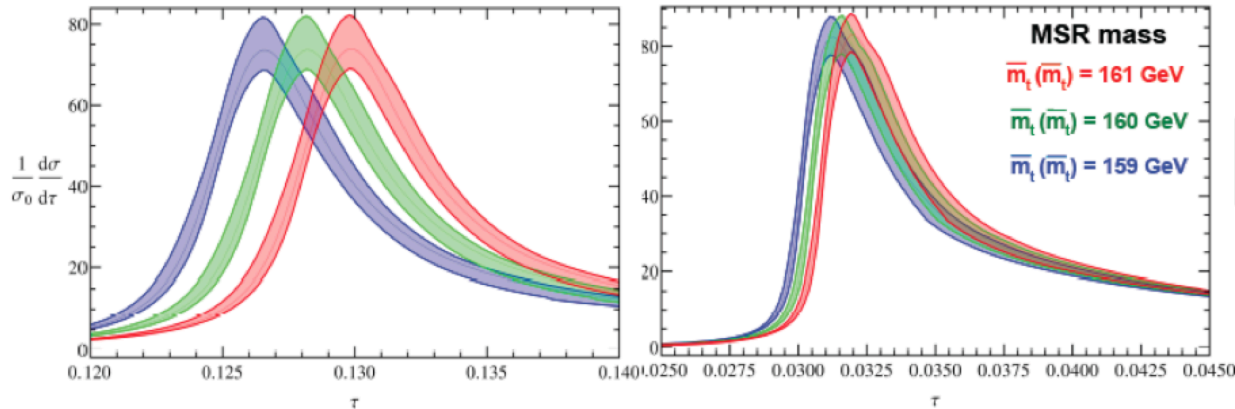
Proper scale variations are essential in reliable estimation of missing higher order terms.

2-Jettiness for Top Production (QCD)

$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m, R, \Gamma_t}_{\text{renorm. scales finite lifetime}})$$

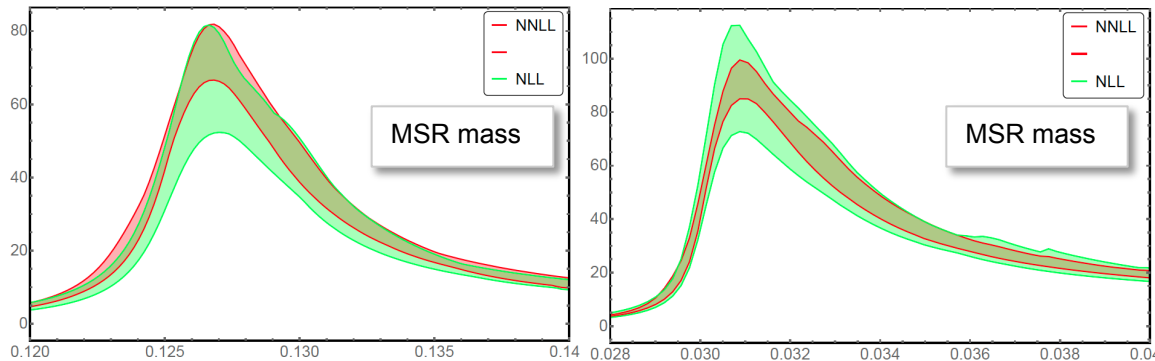
Q=700 GeV

Q=1400 GeV



Q=700 GeV

Q=1400 GeV



- Higher mass sensitivity for lower Q (p_T)
- Finite lifetime effects included
- Dependence on non-perturbative parameters
- Convergence: $\Omega_{1,2,\dots}$
- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution

Fit Procedure Details

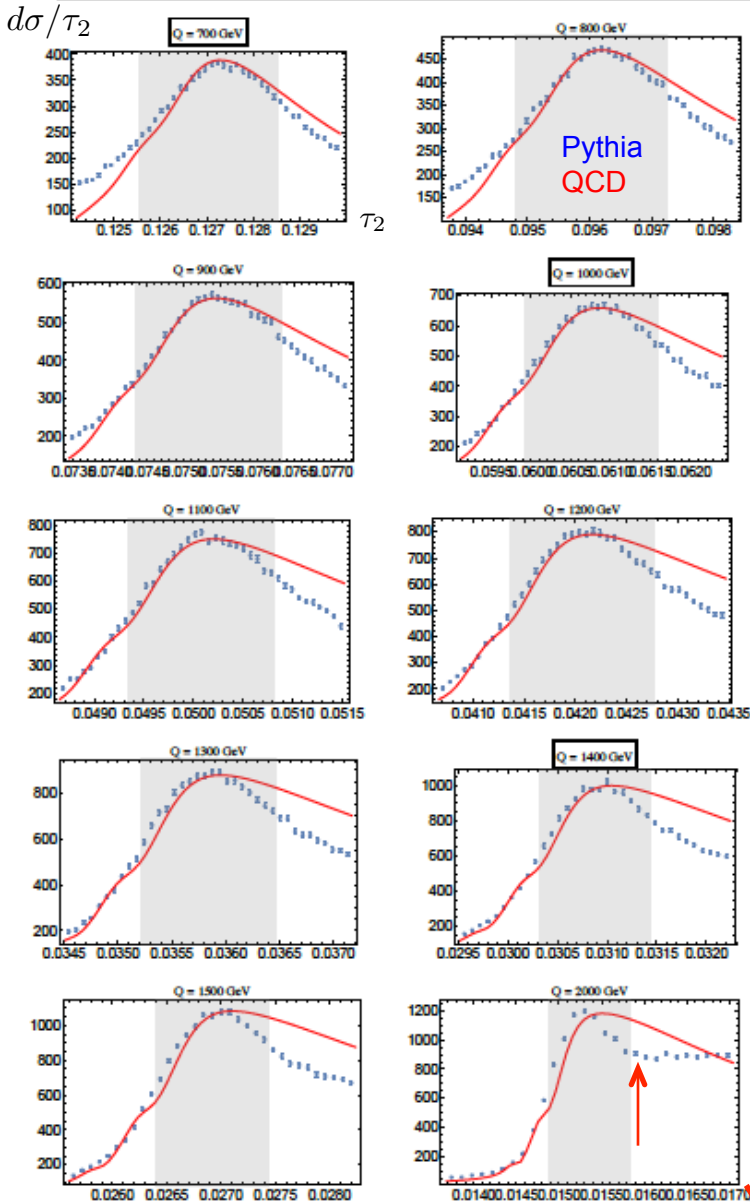
$$\frac{d\sigma}{d\tau_2} = f(\underbrace{m_t^{\text{MSR}}(R)}_{\text{any scheme possible}}, \underbrace{\alpha_s(M_Z), \Omega_1, \Omega_2, \dots}_{\text{Non-perturbative}}, \underbrace{\mu_h, \mu_j, \mu_s, \mu_m}_{\text{renorm. scales}}, \underbrace{R, \Gamma_t}_{\text{finite lifetime}})$$

QCD parameters measured from Pythia

- Fit parameters: $m_t^{\text{MSR}}(R), \alpha_s(M_Z), \Omega_1, \Omega_2, \dots,$
- Perturbative error: fits for 500 randomly picked sets of renor. scales
- Tunings: 1 (“very old”), 3 (“LEP”), 7 (“Monash”)
- Top quark width: $\Gamma_t =$ dynamical (default), 0.7, 1.4, 2.0 GeV
- External smearing (Detector effects): $\Omega_{1,\text{smear}} = 0, 0.5, \dots, 3.0, 3.5, \text{ GeV}$ (just for cross checks)
- Pythia masses: $m_t^{\text{Pythia}} = 170, \dots, 175 \text{ GeV}$
- Strong coupling: $\alpha_s(M_Z) = 0.114, 0.116, 0.118, 0.120, 0.122$
- Fit possible for any order / mass scheme (so far NLL+NNLL / MSR)

Number of fits entering the first analysis: $2.8 \cdot 10^6$

Peak Fits



Default renormalization scales; $\Gamma_t=1.4$ GeV,
 tune 3, $\Omega_{1,smear}=0$ GeV, $m_t^{Pythia}=170$ GeV,
 $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%,
 normalized to fit range

- Good agreement of Pythia 8.2 with NNLL+NLO QCD description
- Pythia statistics: 10^6 events
- Discrepancies in distribution tail and for higher energies (Pythia is less reliable where fixed-order results valid, well reliable in soft-collinear limit)
- **Pythia kink issue ?**
- Excellent sensitivity to the top quark mass.
- Tree-Level:

$$\tau_2^{\text{peak}} = 1 - \sqrt{1 - \frac{4m_t^2}{Q^2}}$$

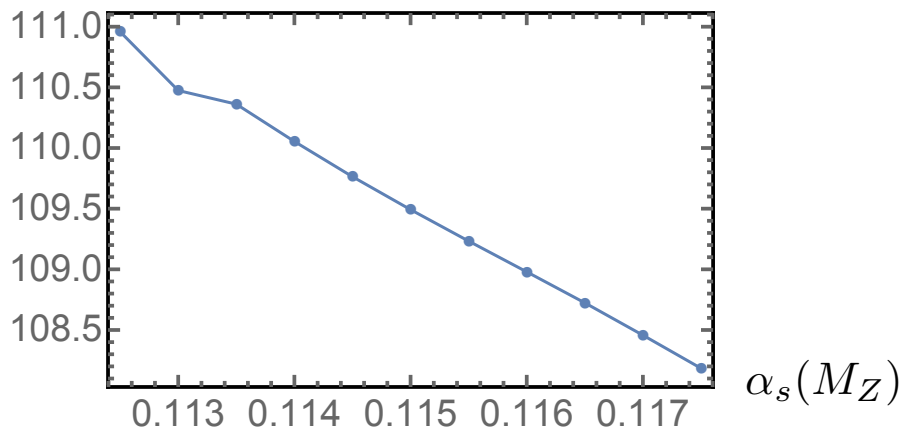
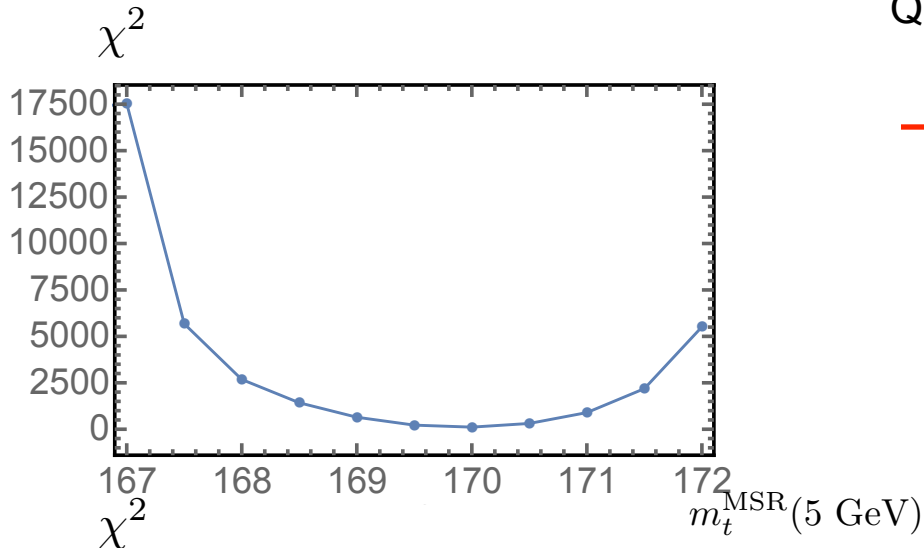
tune = 3
 $m_{MC} = 170$.
 $\Gamma_t = -1$. GeV
 $\alpha = 0.118$
 $m^{SR}(5 \text{ GeV}) = 169.138 \pm 0.099$
 $\frac{\chi^2}{\text{dof}} = 35.36$

$\Omega_1 = 0.434 \pm 0.060$ GeV
 $\Omega_2 = 0.473 \pm 0.060$ GeV
 $\Omega_3 = -0.158 \pm 0.300$ GeV
 $\Omega_4 = -2.226 \pm 1.000$ GeV

Preliminary

Peak Fits

Default renormalization scales; $\Gamma_t=1.4$ GeV, tune 7, $\Omega_{1,\text{smear}}=2.5$ GeV, $m_t^{\text{Pythia}}=171$ GeV, $Q=\{700, 1000, 1400\}$ GeV, peak fit (60/80)%

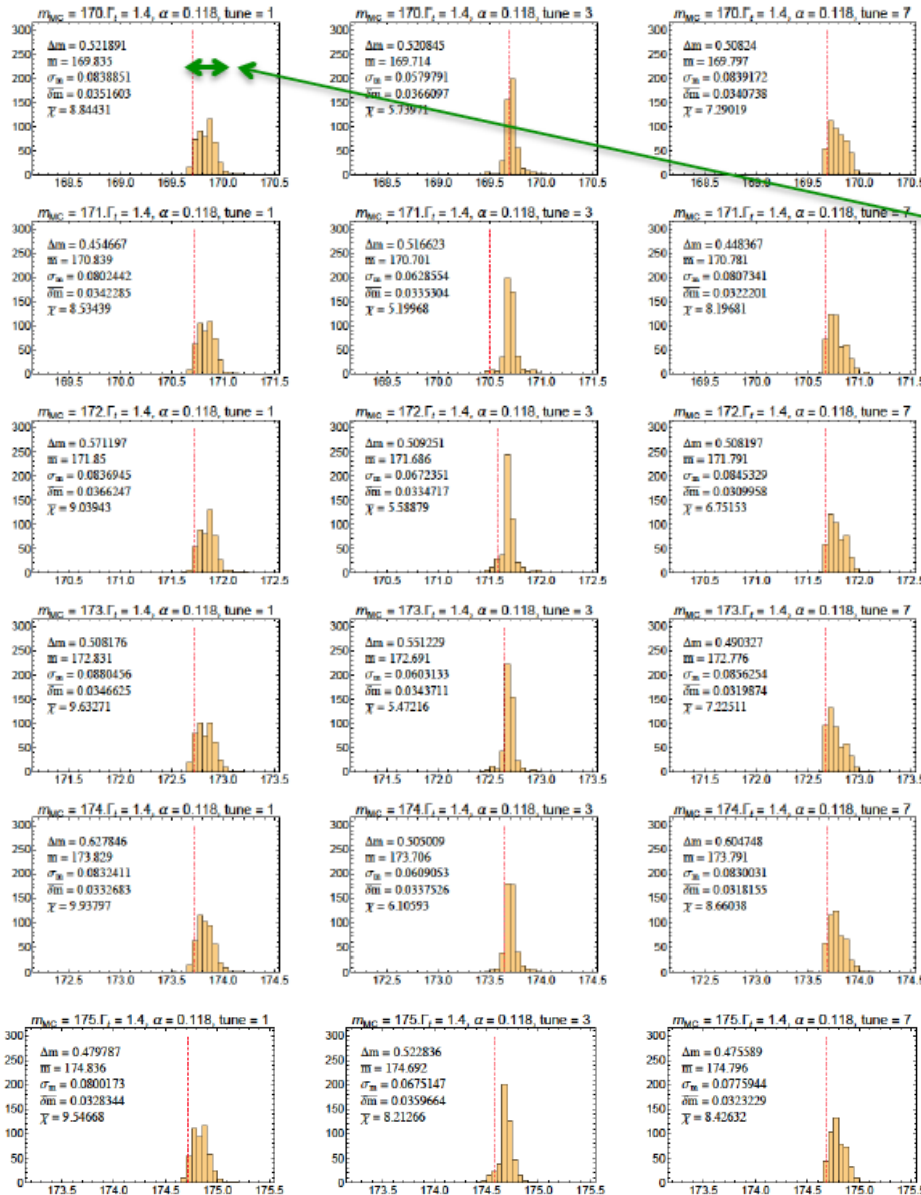


→ $\chi^2_{\text{min}} \sim O(100)$

- Very strong sensitivity to m_t
- Low sensitivity to strong coupling
- Take strong coupling as input
- χ^2_{min} and δm_t^{stat} do not have any physical meaning
- We use rescaled χ^2/dof (PDG prescription) to define “intrinsic MC compatibility uncertainty”

Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$



Scale variation error

Preliminary

- 4×10^6 fits carried out for verification of stability:
Shown:
Tune 1,3,7,
 $m_t^{\text{MC}} = 170, 171, 172, 173, 174, 175 \text{ GeV}$
 $\alpha_s(M_Z) = 0.118$
 $\Gamma_t = 1.4 \text{ GeV}$
 $Q = \{700, 1000, 1400\} \text{ GeV}$
 $n_{\text{range}} = (60/80)\%$

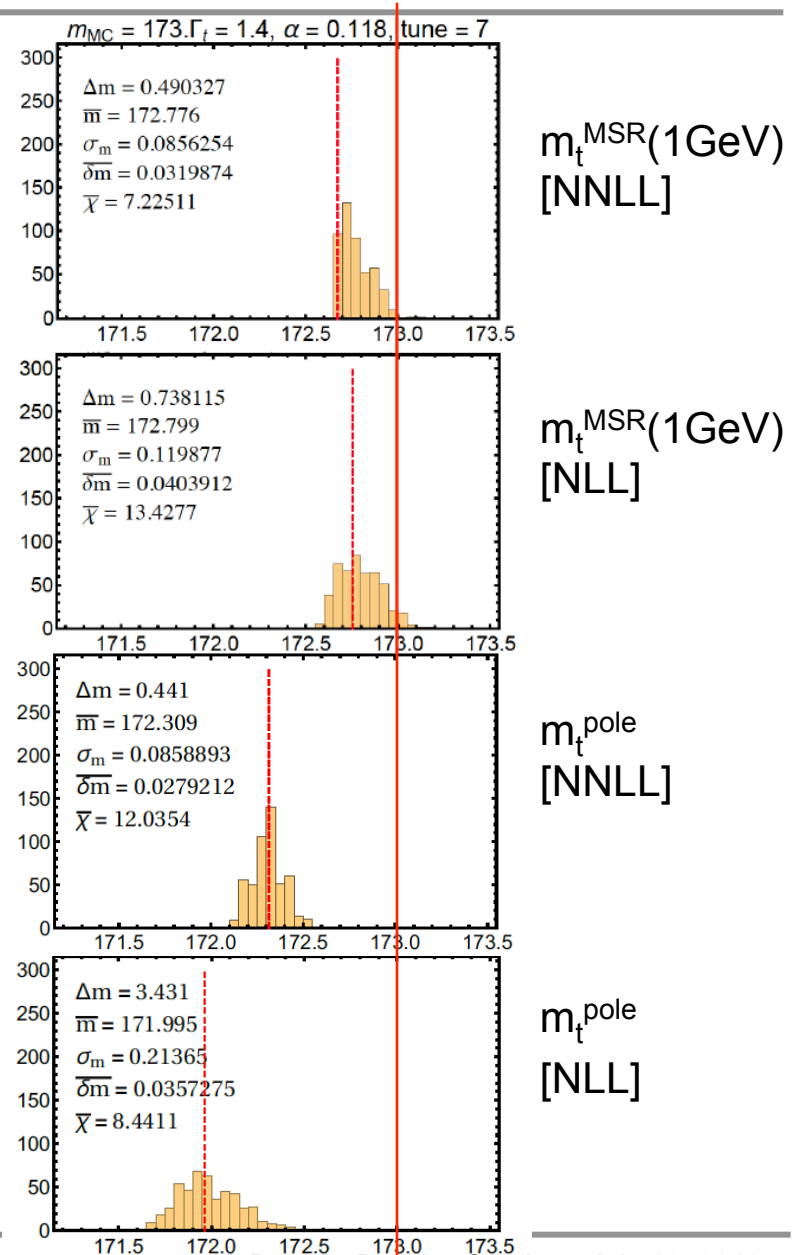
Order Behavior: MSR vs. Pole Mass

$m_{MC} = 173 \text{ GeV}$

$\Gamma_t = 1.4 \text{ GeV}$

Preliminary

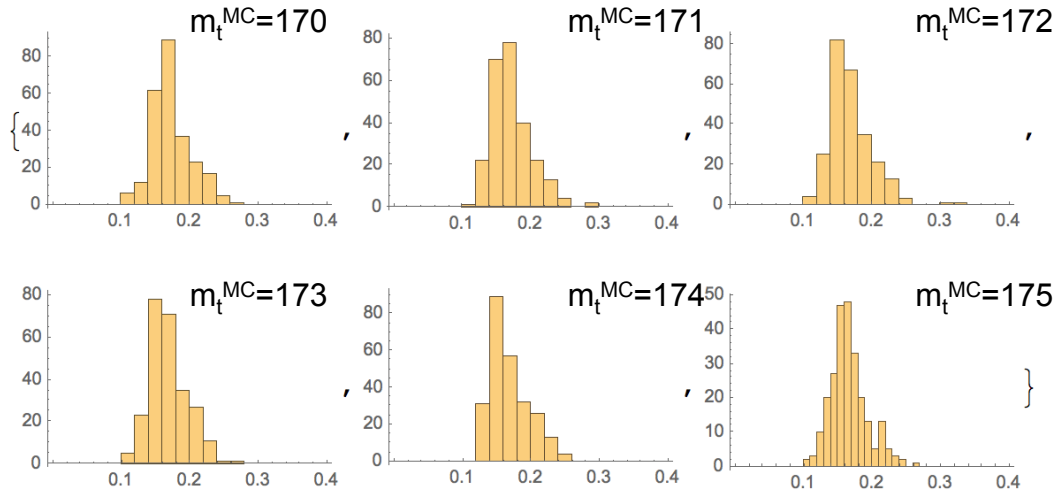
- Very good stability for **MSR mass**
- Mass $m_t^{\text{MSR}}(1\text{GeV})$ mass definition closest to the MC mass.
- **Pole mass** shows much worse convergence.
- Poles mass not close numerically to the MC mass: numbers are observable dependent and great care has to be taken to use the results as input in other calculations.
- Current world average:
 $m_{MC} = 172.44 \pm 0.49 \text{ GeV}$



Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

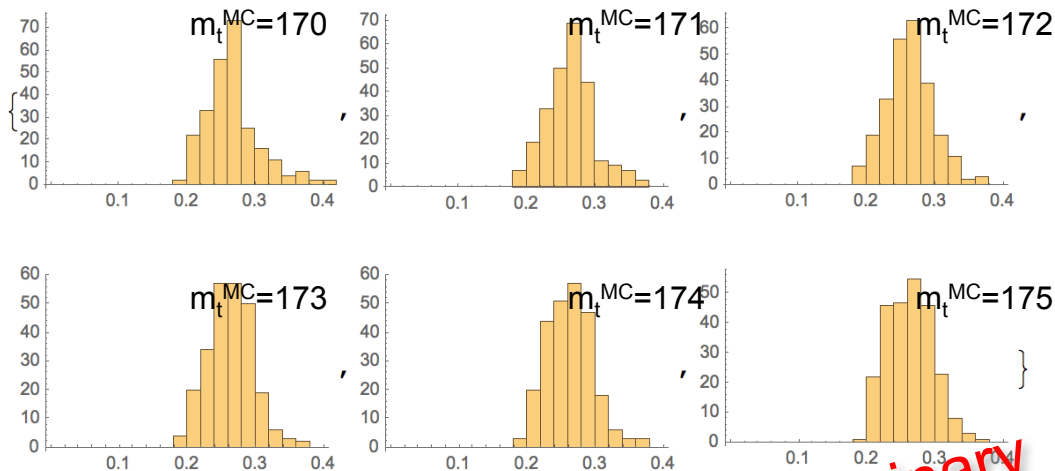
Distribution of coverage range /2: each from scan over 500 profile functions

NNLL



- Renormalization scale error
- NNLL: 150-170 MeV
- NLL: 250-300 MeV
- Good convergence!

NLL



- Histograms include $\alpha_s(M_Z)=0.114 - 0.122$ and $\Gamma_t=-1, 1.4$, and tunes 1,3,7; 7 Q sets, 2 bin fit ranges (252 combinations)

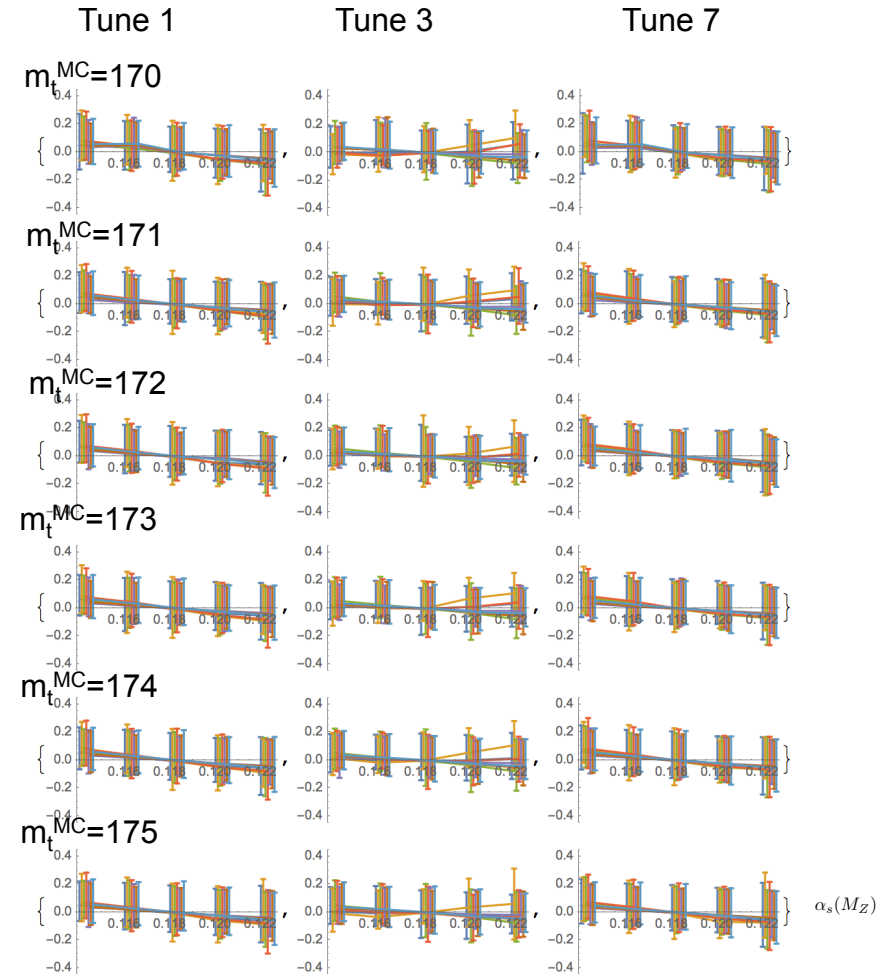
Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Parametric dependence on strong coupling

$$m_t^{\text{MSR}}[\alpha_s(M_Z)] - m_t^{\text{MSR}}[0.118]$$

- Small sensitivity of $m_t^{\text{MSR}}(1\text{GeV})$ on $\alpha_s(M_Z)$. [$\sim 50 \text{ MeV}$ error] ✓
- Error bars: envelope of best mass value distribution in 500 profile function fits



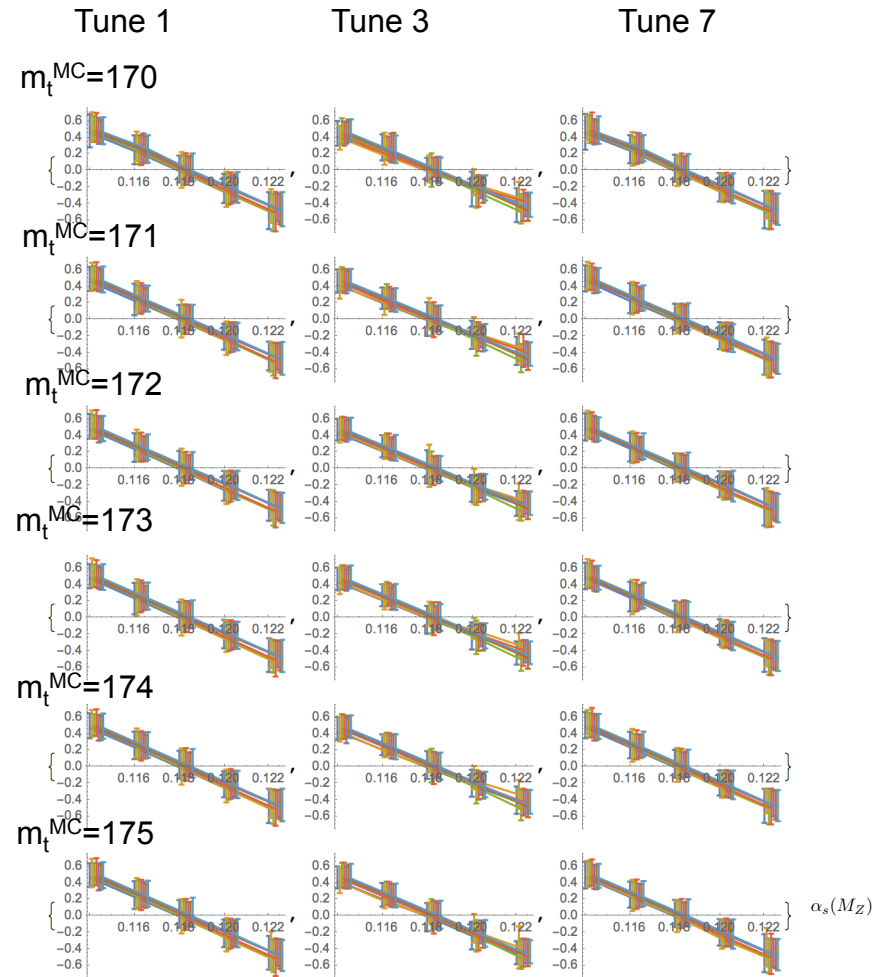
Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Parametric dependence on strong coupling

$$m_t(m_t)[\alpha_s(M_Z)] - m_t(m_t)[0.118]$$

- Large sensitivity of MSbar mass on $\alpha_s(M_Z)$. [not an error, but calculated from MSR mass] ✓
- The MC top mass IS FAR AWAY from the MSbar mass.
 - Error bars: envelope of best mass value distribution in 500 profile function fits

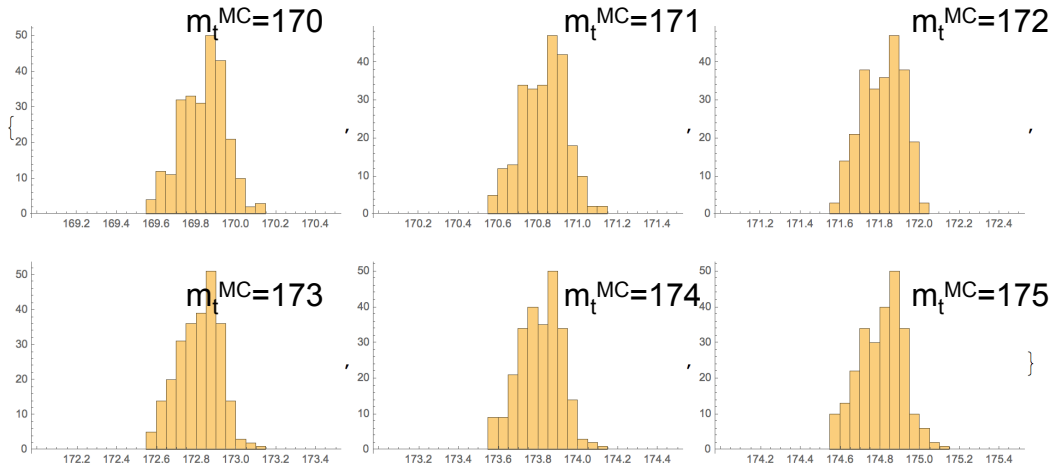


Preliminary

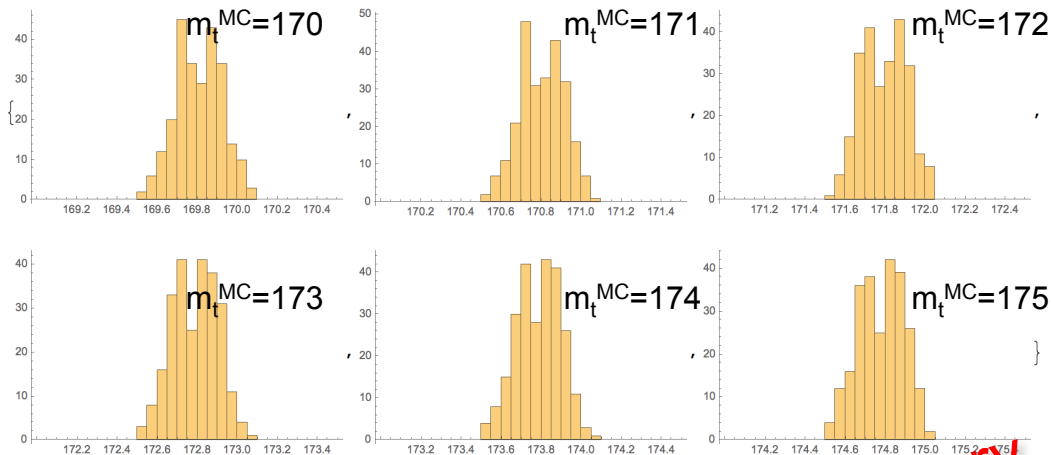
Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Intrinsic MC Compatibility Error (distribution of mean values)

NNLL



NLL

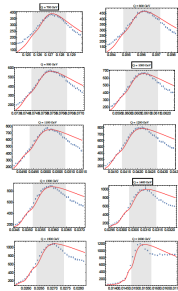


- Coverage is measure for intrinsic MC uncertainty
- NNLL: $\sim 200 \text{ MeV}$
- NLL: $\sim 200 \text{ MeV}$

- Probably never before accounted in reconstruction analyses

- **Measure for ultimate precision (MC dependent !)**

- Histograms include $\alpha_s(M_Z)=0.114 - 0.122$ and $\Gamma_t=-1, 1.4$, and tunes 1,3,7; 7 Q sets, 2 bin fit ranges (252 combinations)



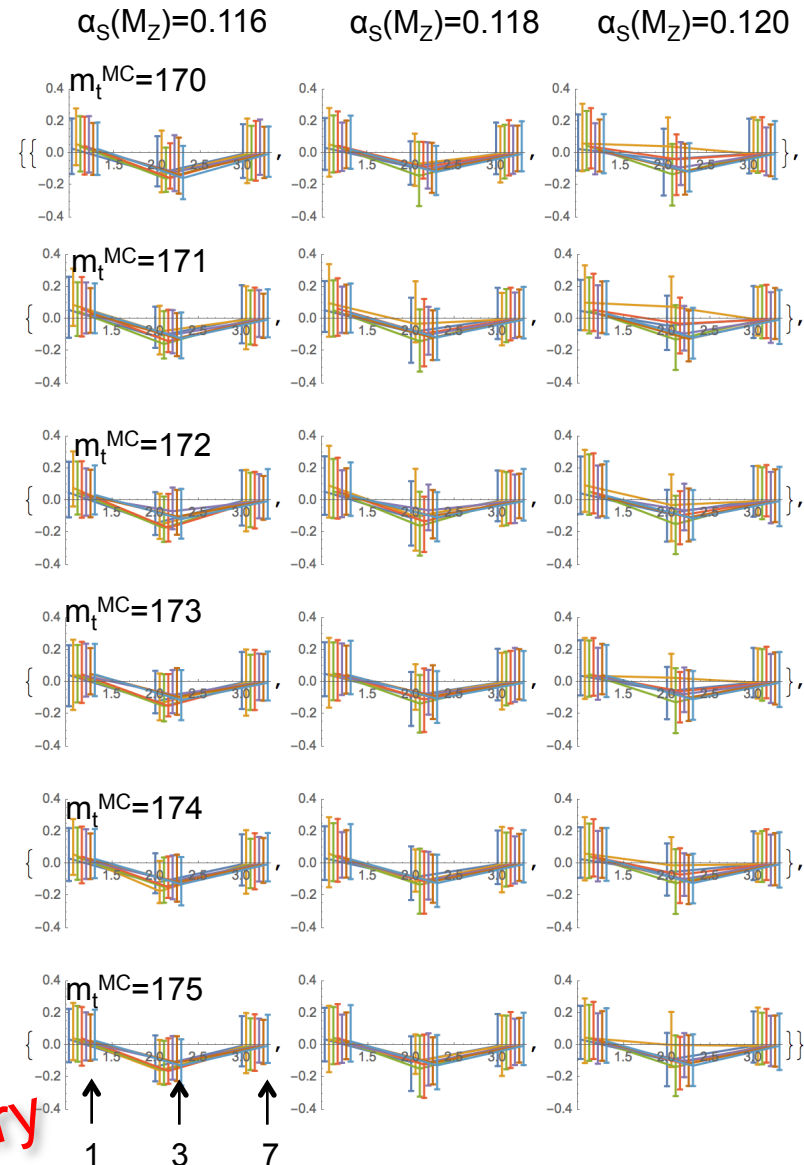
Preliminary

Peak Fits: $m_t^{\text{MSR}}(1 \text{ GeV})$

Tune dependence

$$m_t^{\text{MSR}}[\text{tune}] - m_t^{\text{MSR}}[\text{tune 7}]$$

- Clear sensitivity to tune.
- MC top mass is tune-dependent !
- Tune-dependence is not an error !
- Opposite dependence should be visible in MC top mass determinations from experimental data.
(highly nontrivial validation)
- Top widths: $\Gamma_t = -1, 1.4$
- Error bars: standard deviation of best mass value distribution in 500 profile function fits



Summary

- First serious precise MC top quark mass calibration based on e^+e^- 2-jettiness (large p_T): **closely related to observables dominating the reconstruction method**
- NNLL+NLO QCD calculations based on an extension of the SCET approach concerning massive quark effects (all large logs incl. $\ln(m)$'s summed systematically).
- The Monte Carlo top mass calibration in terms of $m_t^{\text{MSR}}(1\text{GeV})$:
 - Scale dependence (NNLL): ~ 170 MeV
 - α_s dependence ($\delta\alpha_s=0.002$): ~ 50 MeV
 - Intrinsic MC error: ~ 200 MeV
- MC top mass is tune-dependent and MC dependent !
Using MC top mass calibration might eliminate these error sources from the experimental analyses.
Confirmation of the dependence predicted by calibration provides highly non-trivial cross check concerning the universality of the calibration.

Preliminary !!!

Outlook & Plans

- Full verified error analysis @ NNLL/NLO → publication
 - Different sets of Q (p_T) values
 - Different fit ranges
 - Bug fixes
- Calibration Package for public use
 - Calibration $m_t^{MC} \rightarrow m_t^{MSR}(1\text{GeV})$
 - Code $m_t^{MSR}(1\text{GeV}) \rightarrow$ any other scheme
- Heavy jet mass, C -parameter (NNLL),
- pp-2-jettiness analysis (NLL) w.i.p.
- NNNLL+NNLO (2-jettiness for e^+e^-) w.i.p
- Mass (+ Yukawa coupling) conversions w. QCD + electroweak (Yukawa effects)

Backup Slides

Pole Mass from MSR Mass

$$\alpha_s(M_Z) = 0.118$$
$$n_f = 5$$

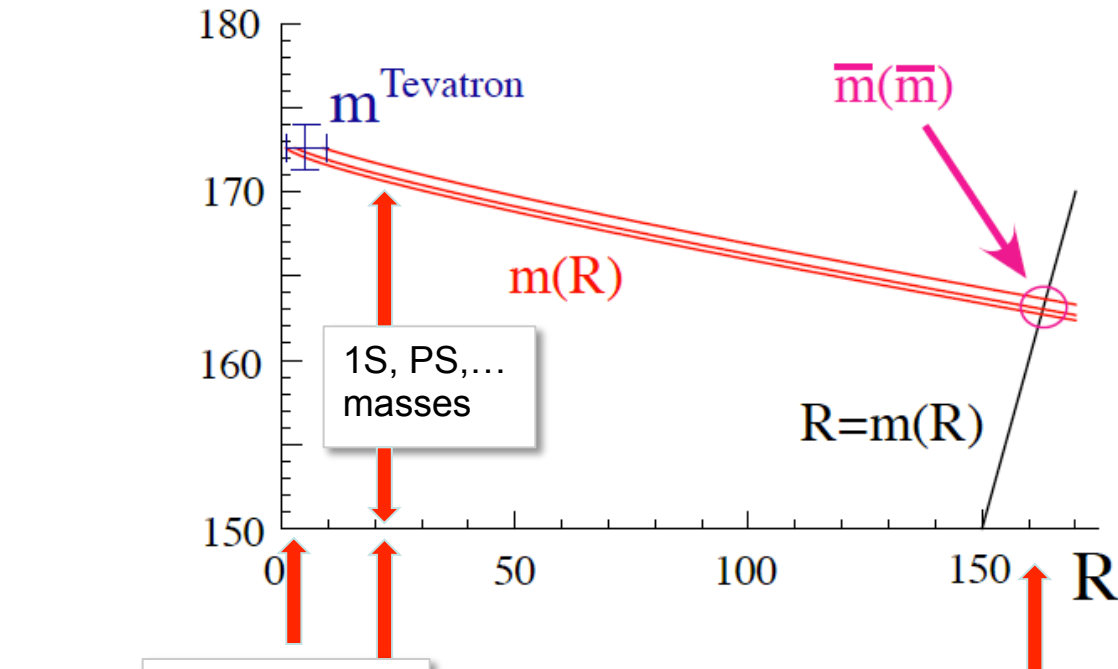
$$m_t^{\text{pole}} - m_t^{\text{MSR}}(1 \text{ GeV}) = \begin{array}{cccc} \mathcal{O}(\alpha_s) & \mathcal{O}(\alpha_s^2) & \mathcal{O}(\alpha_s^3) & \mathcal{O}(\alpha_s^4) \\ 0.173 & + 0.138 & + 0.159 & + 0.23 \text{ GeV} \leftarrow \text{calculated} \\ + 0.53 & + 1.43 & + 4.54 & + 16.6 \text{ GeV} \leftarrow \text{extrapolated} \\ + 68.6 & + 317.7 & + 1629 & + 9158 \text{ GeV} \end{array}$$

- Size of terms consistent with scale error estimate of calibration.
- No stable determination of pole mass.

MSR Mass Definition

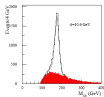
AH, Stewart: arXiv:0808.0222

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(3_{-2}^{+6} \text{ GeV}) = m_t^{\text{MSR}}(3 \text{ GeV})_{-0.3}^{+0.6}$$



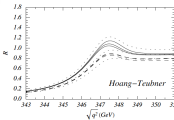
Good choice for R:

Of order of the typical scale of the observable used to measure the top mass.



Peak of invariant mass distribution, endpoints

Top-antitop threshold at the ILC



Total cross section, e.w.precision obs., Unification, MSbar mass

Masses Loop-Theorists Like to use

Total cross section (LHC/Tev):

$$m_t^{\text{MSR}}(R = m_t) = \bar{m}_t(\bar{m}_t)$$

$$M_t = M_t^{(O)} + M_t(0)\alpha_s + \dots$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

Threshold cross section (ILC):

$$m_t^{\text{MSR}}(R \sim 20 \text{ GeV}), m_t^{1S}, m_t^{\text{PS}}(R)$$

$$M_t = M_t^{(O)} + \langle p_{\text{Bohr}} \rangle \alpha_s + \dots$$

$$\langle p_{\text{Bohr}} \rangle = 20 \text{ GeV}$$

Inv. mass reconstruction (ILC/LHC):

$$m_t^{\text{MSR}}(R \sim \Gamma_t), m_t^{\text{jet}}(R)$$

$$M_t = M_t^{(O)} + \Gamma_t \alpha_s + \dots$$

$$\Gamma_t = 1.3 \text{ GeV}$$

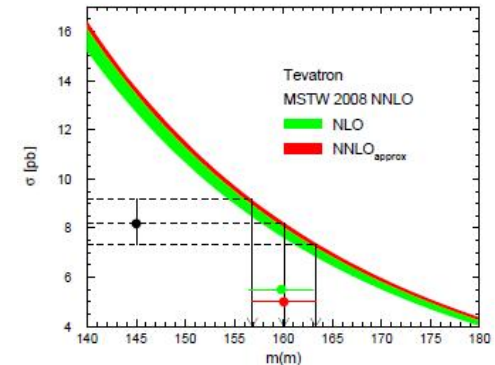
- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections



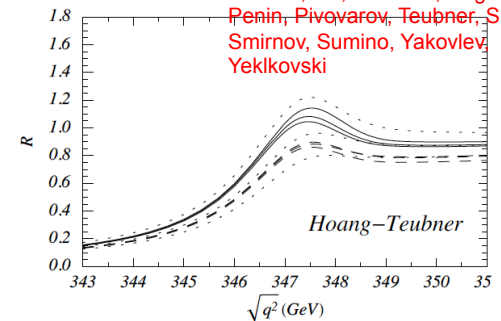
Mass schemes related to different computational methods

Relations computable in perturbation theory

Langenfeld, Moch, Uwer



Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Fleming, AH, Mantry, Stewart

