

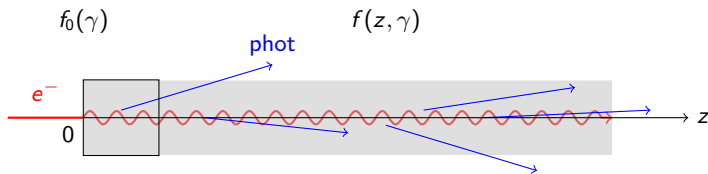
# Quantum effects in production of gammas for positrons

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# Problem general setup: ultra-relativistic electrons, quantum recoils



## problem setup

- periodic force with given envelop – undulator, laser pulse
- radiation: statistically independent photons, given spectra
- recoils decrease electrons' energy
- goal: evolution of initially given electron spectrum along the field  
special attention – front end of the field

## motivation

- ILC positron source: effect of the undulator on the beam parameters
- ILC alternative Compton positron source – performance
- lack of analytic description for small average number of photons emitted
- drawbacks of the diffusive approximation

# Kinetic equation for electron spectrum $f(z, \gamma)$

Landau 1944; Akhiezer, Shul'ga 1996; Khokonov 2004

System of units:  $m_e = c = \hbar = 1$ , initial distribution  $f_0(\gamma) \equiv f(z = 0, \gamma)$

$$\frac{\partial}{\partial z} f(z, \gamma) = \int [f(z, \gamma + \omega) W(z, \gamma + \omega, \omega) - f(z, \gamma) W(z, \gamma, \omega)] d\omega$$

where  $W(z, \gamma, \omega)$  is the probability density

**Approximation:**  $\gamma \gg 1$ ,  $\omega_{\max} \ll \gamma$ ,  $W(z, \gamma, \omega) = \psi(z) w(\gamma, \omega) \approx \psi(z) w(\omega)$

Kinetic equation casts into

$$f'_x = f \star w - f$$

with  $f \star w \equiv \int f(\gamma + \omega) w(\omega) d\omega$  cross correlation;  $x = \int_0^z \psi(z') dz'$  number of emitted photons.

# Kinetic equation: solution and moments (rigorous)

E.Bulyak, N.Shulga (2016), submitted to EPL

Fourier transform of electron spectrum evolution then the inverse

$$\hat{f} = \sum_{n=0}^{\infty} \frac{e^{-x} x^n}{n!} (\hat{f}_0 \check{w}^n)$$

$$f(x, \gamma) = \sum_{n=0}^{\infty} \frac{e^{-x} x^n}{n!} F_n(\gamma),$$

with  $F_0(\gamma) = f_0(\gamma) = f(x=0, \gamma)$  the initial distribution (spectrum),

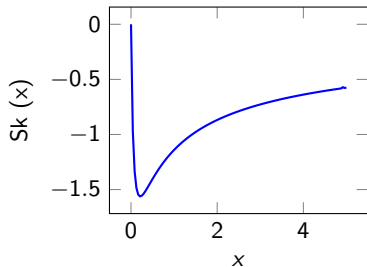
$$F_n(\gamma) = \int F_{n-1}(\gamma + \omega) w(\omega) d\omega$$

mean energy  $\bar{\gamma}$  variance  $\overline{(\gamma - \bar{\gamma})^2}$  skewness  $\overline{(\gamma - \bar{\gamma})^3} \Rightarrow$  linear of  $x$

$$\bar{\gamma}(x) = \bar{\gamma}_0 - x \bar{\omega}; \quad \text{Var}[\gamma](x) = \text{Var}[\gamma_0] + x \bar{\omega}^2; \quad \text{Sk}[\gamma](x) = \text{Sk}[\gamma_0] - x \bar{\omega}^3$$

Relation with distance along the axis:  $z \rightarrow x(z) = \int_0^z \psi(z') dz'$ .

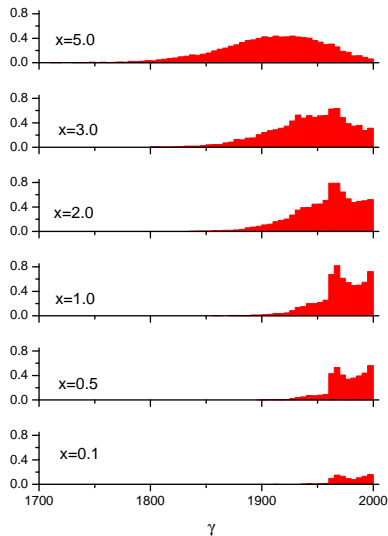
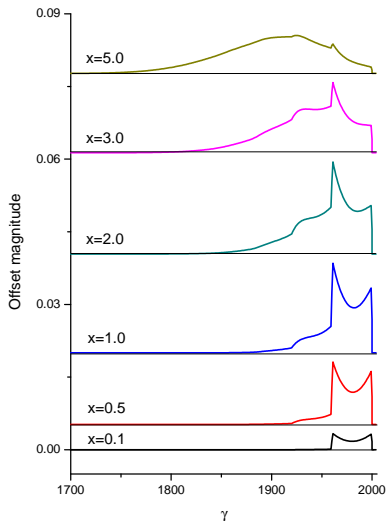
Pearson's skewness  $\frac{(\gamma - \bar{\gamma})^3}{(\gamma - \bar{\gamma})^2^{3/2}}$



- Contribution of the initial distribution,  $f_0$ , decays as  $e^{-x}$
- Mean energy decreases linearly with  $x$
- Variance (spread) increases linearly
- Tail on the left (negative skewness)
- Asymmetry  $\sim 1/\sqrt{x}$

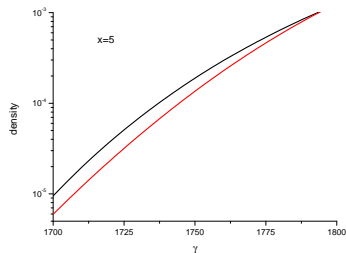
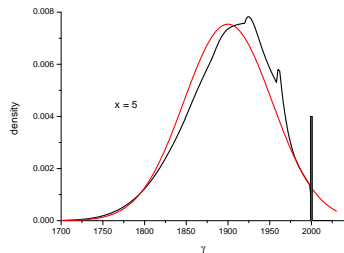
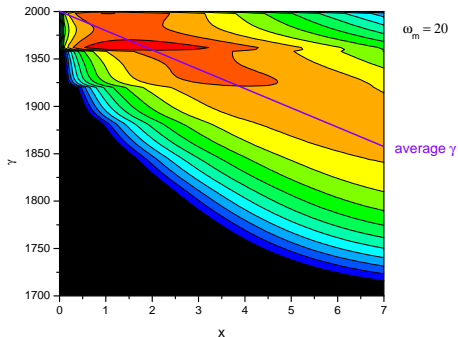
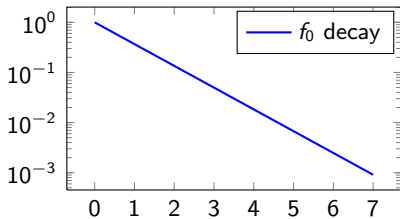
# Gamma Sources: Analytic vs. Simulation

$f_0 = \delta(\gamma - \gamma_0)$  not included



# Dipole harmonic

Compton source, 1 GeV + 1 eV

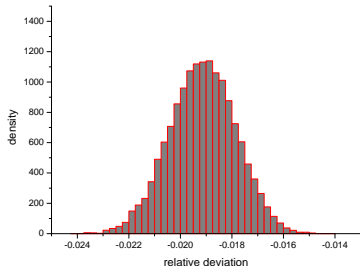


# Simulations – ILC undulator: $K = 0.92$ vs $K = 0.46$ at 150 GeV

$f_0 = \delta(\gamma - \gamma_0)$ , 15 000 particles

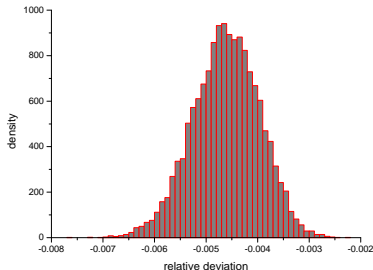
$$\text{relative deviation} \equiv \frac{\gamma - \gamma_0}{\gamma_0}$$

$K=0.92$  (300 gammas)



mean = - 0.019; st.dev = 1.29E-3;  
min = -0.024; max = -0.014  
min=mean-0.005; max=mean+0.005

$K=0.46$  (70 gammas)

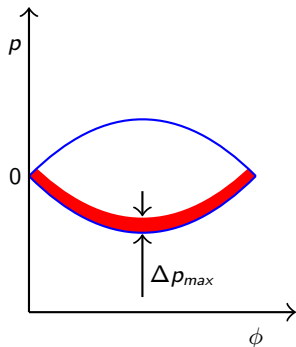


mean = - 0.0046; st.dev = 6.48E-4;  
min = -0.0077; max = -0.0023  
min=mean-0.0031; max=mean+0.0023



# Where tails matter: Quantum lifetime in Compton rings

longitudinal phase space

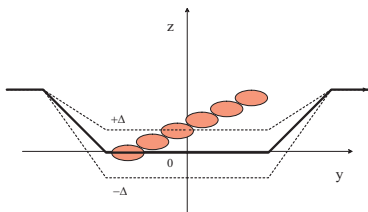


- electrons escape the separatrix downward, in the 'tail direction'  
**separatrix cuts out the tail**
- rate of losses  $\propto$  bunch density  $\times$  laser density
- **red band width**  $\propto$  tail length  $\times$  synchrotron period

We proposed to mitigate quantum losses via *the asymmetric cooling*

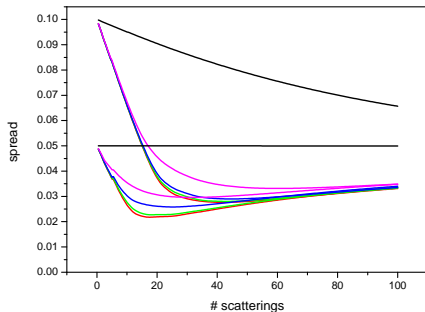
# Asymmetric (Fast) Cooling

E.Bulyak, J.Urakawa, F.Zimmermann 2011–2013



Model setup

Laser radiation field exists at  $z \geq 0$



Spread vs. # scatterings

**Table:** Results of simulation dependence on crossing angle  $\phi$  (3.0 mm dispersion at CP, 2 m beta function).

run	$\phi$ , rad	shift, $\mu\text{m}$	yield $\times 10^{-5}$	lifetime, turn	yield $\times$ l.t.
14j20p	0.0	+30	31.90	19 857	6.33
14j20	0.0	0	41.30	12 872	5.31
14j20m	0.0	-30	58.00	7 689	4.46
13j20pp	0.03	+30	1.88	2 422 370	45.54
13j20p	0.03	0	1.58	1 649 434	26.06
13j20m	0.03	-30	1.75	1 447 969	25.30
12j20p	0.13963	+30	1.35	20 582 321	277.86
12j20	0.13963	0	1.26	13 081 740	164.83
12j20m	0.13963	-30	1.28	12 248 299	156.78

## summary

- Kinetic equation for electron spectra in gamma-sources solved
- Spectrum substantially differs from the diffusion model at small number of photons, converging to that when many photons emitted
- For Compton sources account for asymmetry of the electron spectrum is important
- Diffusion model sufficient for the undulator-based gamma sources

## discussion

- Diffusion model sufficient for the undulator-based gamma sources
- For Compton sources (both ring- and ERL-based) account for asymmetry of the electron spectrum is important
- Quantum effects may reduce or limit performance of gamma- and hard x-ray sources
- Quantum recoils affect beam dynamics