FIFTY YEARS THAT CHANGED OUR PHYSICS

LAL, 21 Nov. 2016

Jean Iliopoulos

ENS, Paris
The twentieth century was the century of revolutions in Physics
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- Relativity - Special and General
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- Quantum Mechanics
- Particles and Fields
- Gauge theories and Geometry
- Each one involved new physical concepts, new mathematical tools and new champions
Some were radical, others were conservative.
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I will talk about the last two:

Particles and Fields - Gauge theories and Geometry
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They were conservative: Things changed just enough so that they could remain the same.
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Yet, they influenced profoundly our way of looking at the fundamental laws of Nature
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Yet, they influenced profoundly our way of looking at the fundamental laws of Nature

They were mostly rejected by the champions of the previous revolutions
A bit of history

- The rules of counting states in a statistical ensemble
  Boltzmann, Gibbs, Planck, Natanson, Ehrenfest, Fowler, ...

- The Bose-Einstein rule
  Bose (1924), Einstein (1924)

- The Pauli exclusion principle
  Pauli (1925)

- The Fermi-Dirac rule
  Fermi (1926), Dirac (1926)

- Applications (mostly incorrect) to various physical systems
  Einstein, Heisenberg, Dirac, Pauli, Hund, Dennison, Wigner, ...
Nuclear structure and the puzzles of $\beta$-decay
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- $\beta$-decay ($t < 1930$) : $N_1 \rightarrow N_2 + e$

  **Rule**: What comes out must be in

  $\Rightarrow$ Nuclei are made out of protons and electrons

  Measurements of: (i) electron spectra and (ii) nuclear spins, show non-conservation of energy and angular momentum. Electrons in nuclei did not obey the Pauli exclusion principle.
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- Bohr versus Pauli

  Bohr (et al) : Conservation laws may be violated in Quantum Mechanics

  Pauli (1930) : $N_1 \rightarrow N_2 + e + \nu$

  $\Rightarrow$ Nuclei are made out of protons, electrons and neutrinos
Nuclear structure and the puzzles of $\beta$-decay

- In 1932 Chadwick discovers the neutron, but

  For most people the neutron is a proton-electron bound state.

  ⇒ The discovery does not seem to solve any of the puzzles.
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▶ In 1932 Heisenberg introduces the concept of isospin.
He puts the proton and the neutron in an $SU(2)$ doublet, but

In the Bohr-Pauli controversy he sides with Bohr

He believes that a neutron decays into a proton and an electron, something incompatible with it being a fermion
"...under suitable circumstances the neutron will break up into a proton and an electron in which case the conservation laws of energy and momentum probably do not apply....The admittedly hypothetical validity of Fermi statistics for neutrons as well as the failure of the energy law in $\beta$-decay proves the inapplicability of present quantum mechanics to the structure of the neutron."
Fermi’s *Tentativo*

An English version had been submitted earlier in *Nature*, but it was rejected “because it contained speculations too remote from reality to be of interest to the reader”.

In the Bohr-Pauli controversy Fermi sides with Pauli. In the Fermi theory of $\beta$-decay the neutrino is a particle like any other.

But he goes further: he breaks with the prevailing doctrine according to which whatever comes out from a nucleus must be already in. For Fermi a particle, like a photon in a spontaneous emission, is created the moment of the decay.
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- He showed how this could actually happen.

\[
\{a_s(p), a_{s'}^\dagger(p')\} = \hbar (2\pi)^3 2\omega_p \delta^3(p - p') \delta_{ss'}
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\[
\{a_s(p), a_{s'}(p')\} = \{a_{s}^\dagger(p), a_{s'}^\dagger(p')\} = 0
\]

It is amazing how fast Fermi’s theory was universally accepted. The times were ripe. Quantum Field Theory became the language of particle physics.

Bohr continued to play with energy non-conserving theories for several years, but he was soon alone. A. Pais: “It is clear that Particles and Fields belong to the post-Bohr era.”
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- Symmetries and Current Algebras, Weak Interactions and $CP$-violation
  *Secondary subjects*

- Notice the absence of Quantum Field Theory
  *A totally marginal subject*
The analytic $S$-matrix theory

A series of (more or less) reasonable axioms formulated directly on the scattering amplitudes.

- Invariance under Poincaré and internal symmetries
- Crossing symmetry
- Unitarity

\[ S = \frac{1}{1 + iT} S^\dagger = S^\dagger S = \frac{1}{1 + 2i T} \Rightarrow 2i T = T^\dagger T \]

- Maximum analyticity
- Polynomial boundedness

Not very well defined, fuzzy rules

An important addition: Analyticity in the complex angular momentum plane (Regge)
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Some important by-products

- Cutkosky unitarity relations
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- Bootstrap
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- Cutkosky unitarity relations

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- Duality (*Dual Resonance Model*)
The Veneziano amplitude

\[ A(s, t) \sim \frac{\Gamma(-1 + s/2)\Gamma(-1 + t/2)}{\Gamma(-2 + (s + t)/2)} \]

This amplitude, appropriately generalised, was the starting point of a concept which turned out to be seminal and important:

**The string model**

Initially, it was meant to be a theory for hadronic physics and gave rise to interesting phenomenological models.

But it was soon realised that it contains a version of quantum gravity

*(more about that later)*
Symmetries and Current Algebras, Weak Int. and CPV

SYMMETRIES

The pre-history
- Space-time symmetries
- Internal symmetries (Heisenberg 1932, Kemmer 1937, Fermi 1951)
- Gauge symmetries (Gauss, Einstein 1914, Fock 1926, Klein 1937, Pauli 1953, Yang and Mills 1954)

Early history
- Higher symmetry (Gell-Mann 1961 (+ Ne'eman))
- Current Algebras (Gell-Mann 1962)


Quarks (Gell-Mann 1964 (+ Zweig))
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▶ Early history
  • Higher symmetry (Gell-Mann 1961 (+ Ne’eman)) SU(3)
  • Current Algebras (Gell-Mann 1962)

\[ [V, V] = V \quad ; \quad [V, A] = A \quad ; \quad [A, A] = V \]

• Quarks (Gell-Mann 1964 (+Zweig))
In this talk I will concentrate on very few particular subjects:

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- The renormalisation and QCD
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I. THE WEAK INTERACTIONS. PHENOMENOLOGY
Fermi 1933
The Electroweak Standard Model

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- The Fermi theory of the weak interactions was phenomenologically very successful

\[ \mathcal{L}_W = \frac{G}{\sqrt{2}} J_{(w)}^\mu (x) J_{(w)}^\dagger (x) \]
I. THE WEAK INTERACTIONS. PHENOMENOLOGY
Fermi 1933

- The Fermi theory of the weak interactions was phenomenologically very successful

\[ \mathcal{L}_W = \frac{G}{\sqrt{2}} J^\mu_{(w)}(x) J^\dagger_{(w)\mu}(x) \]

- But it was a non-renormalisable theory, Fierz 1936

\[ d\sigma(\bar{\nu} + p \rightarrow n + e^+) = \frac{G_F^2}{2\pi^2} p_{\nu}^2 d\Omega \]
\[ A \sim C_0^1(G_F\Lambda^2) + C_1^1 G_F M^2 \\
+ C_0^2(G_F\Lambda^2)^2 + C_1^2 G_F M^2(G_F\Lambda^2) + C_2^2(G_F M^2)^2 \\
+ \ldots \\
+ C_0^n(G_F\Lambda^2)^n + C_1^n G_F M^2(G_F\Lambda^2)^{n-1} + \ldots \\
+ \ldots \]

Effective coupling constant: \( \lambda = G_F\Lambda^2 \)

\[ A \sim \lambda^n + G_F M^2 \lambda^{n-1} + \ldots \]

\[ A \sim \text{“leading”} + \text{“next-to-leading”} + \ldots \]

The Theory is valid up to a scale \( \sim \Lambda \)

\( G_F\Lambda^2 \sim 1 \Rightarrow \Lambda \sim 300 \text{ GeV} \)
B.L. Joffe and E.P. Shabalin (1967)

- At leading order

  Limits on Parity and Strangeness violation in strong interactions

  \[ G_F \Lambda^2 << 1 \Rightarrow \Lambda \sim 3 \text{ GeV} \]
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- At leading order
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- At next-to-leading order
  Limits on \( K^0 \rightarrow \mu^+\mu^- \) and \( K^0 - \bar{K}^0 \) mass difference
  \[ G_F \Lambda^2 << 1 \Rightarrow \Lambda \sim 3 \text{ GeV} \]
In a purely phenomenological approach the idea was to push the value of the cut-off beyond the reach of the experiments.

Example:

- Assume the approximate invariance of the strong interactions under chiral $SU(3) \times SU(3)$

- Assume an explicit breaking via a $(3 \bar{3}, \bar{3} \bar{3})$ term.

Like a quark mass term

The leading divergences respect all the strong interaction symmetries

Cl. Bouchiat, J. I., J. Prentki 1968

Following this line attempts were made to "determine" the properties of the weak interactions, for example to calculate the value of the Cabibbo angle.

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  *Gatto, Sartori, Tonin; Cabibbo, Maiani; Gell-Mann, Goldberger, Kroll, Low*
The argument on the leading divergences can, and has been, phrased entirely in terms of currents and symmetries of the strong interactions, although the assumption of an intermediate charged vector boson was always made. The Wilson short distance expansion was not used.

\[ A \sim \frac{G}{\sqrt{2}} \int d^4 k \ e^{ikx} < a| T(J_\mu(x), J_\nu(0))|b > \frac{k^\mu k^\nu / m_W^2}{k^2 - m_W^2} \]

⇒

Only the symmetry properties of the currents are used, not their explicit expression in terms of elementary fields. The argument can be generalised to all orders in perturbation theory \((J.I.)\)
In principle, the same formalism can be used for the next-to-leading divergences, those which produce FCNC. (*G.I.M.*)

\[
d \nu_s \nu_{\mu^-} \nu_{\mu^+} W_{-} W_{+}
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At this point, however, the paradigm gradually changed from symmetries and currents to the quark model.
Intermezzo

Two seemingly disconnected contributions:
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- Spontaneous symmetry breaking in the presence of gauge interactions
  
  *Brout and Englert; Higgs; Guralnik, Hagen and Kibble* 1964
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- A model for leptons
  *Weinberg 1967; Salam 1968*
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  \textit{Brout and Englert; Higgs; Guralnik, Hagen and Kibble 1964}

- A model for leptons
  \textit{Weinberg 1967; Salam 1968}

- Both went totally unnoticed
II. THE WEAK INTERACTIONS. FIELD THEORY

*Developed in parallel, kind of a sub-culture*

Both, the phenomenological approach and the field theory approach, aimed at controlling the divergences of perturbation theory. In the first, you do not know the fields, you do not know the interactions. In the second you start from a given field theory.
Early attempts

- Use scalar intermediate bosons. Kummer, Segré 1965
- Introduce "physical" unstable particles with negative metric, but try to “confine” the violation of unitarity to very short times. Lee, Wick 1968
- The electrodynamics of charged vector bosons. ξ-limiting formalism Lee and Yang; Lee 1962
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  *ξ-limiting formalism Lee and Yang; Lee 1962*
Massive Yang-Mills; Trial and error strategy.

Veltman

Find the Feynman rules for gauge invariant theories.

Feynman; Faddeev, Popov; 't Hooft

Combine with scalar fields.

't Hooft, Veltman

Prove renormalisability

't Hooft, Veltman 1971

Then all hell broke loose!

Formal Ward Identities.

Slavnov; Taylor; Lee, Zinn-Justin

In the same family of gauges you find renormalisable gauges and unitary gauges.

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Understand why it works.

Becchi, Rouet, Stora; Tyutin
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- Understand why it works. Becchi, Rouet, Stora; Tyutin
Geometry and Dynamics

Gauge theories on a space-time lattice

The dictionary:

- A field $\Psi(x) \Rightarrow \Psi_n$
- A local term such as $\bar{\Psi}(x)\Psi(x) \Rightarrow \bar{\Psi}_n\Psi_n$
- A derivative $\partial^\mu \Psi(x) \Rightarrow (\Psi_n - \Psi_{n+\mu})$
- The kinetic energy term $\bar{\Psi}(x)\partial^\mu \Psi(x) \Rightarrow \bar{\Psi}_n\Psi_n - \bar{\Psi}_n\Psi_{n+\mu}$
- A gauge transformation $\Psi(x) \rightarrow e^{i\Theta(x)}\Psi(x) \Rightarrow \Psi_n \rightarrow e^{i\Theta_n}\Psi_n$

- All local terms of the form $\bar{\Psi}_n\Psi_n$ remain invariant
- The kinetic energy $\bar{\Psi}_n\Psi_n + \mu \rightarrow \bar{\Psi}_n e^{-i\Theta_n}e^{i\Theta_n + \mu}\Psi_n + \mu$

- Introduce $U_n$, $n + \mu \rightarrow e^{i\Theta_n}U_n$, $n + \mu e^{-i\Theta_n + \mu}$

- $\bar{\Psi}_n U_n$, $n + \mu \Psi_n + \mu$
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Geometry and Dynamics

Gauge theories on a space-time lattice

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Geometry and Dynamics

Gauge theories on a space-time lattice

Matter fields $\Psi$ live on lattice points

Gauge fields $U_n, U_{n+\mu}$ live on the oriented link joining the two neighbouring points.

The mathematicians are right when they do not call the gauge field "a field" but "a connection"

The kinetic energy of the gauge field on the lattice:

$$\text{Tr} F_{\mu \nu}(x) F_{\mu \nu}(x) \Rightarrow ???$$

A path $p_{n,m}$: $P(p)(n,m) = \prod U_{n,n+\mu} ... U_{m-\nu,m}$

For a closed path $c = p_{n,n}$ the quantity $\text{Tr} P(c)$ is gauge invariant.

⇒ "a curvature"
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First conclusion

The 1960’s was an extraordinary decade....

although no one at the time had realised that a revolution was taking place!
The renormalisation group and QCD

Contrary to what you may think, the study (rather the re-birth) of the renormalisation group was not initially motivated by the SLAC results on DIS.

A short history

• The RG equation was first written down by Stückelberg and Petermann in 1953

\[ [M \frac{\partial}{\partial M} + \beta \frac{\partial}{\partial \lambda} + \gamma_m m \frac{\partial}{\partial m} - n\gamma] \Gamma^{(2n)}(p_1, ..., p_{2n}; m, \lambda; M) = 0 \]

It was meant to clarify the meaning of the subtraction in the renormalisation procedure

• Gell-Mann and Low in 1954 observed that it can be used to study the asymptotic behaviour of the theory, but, in the late sixties, the emphasis was to use the equation \( \beta = 0 \) for QED as an eigenvalue equation to determine \( \alpha \)
The renormalisation group and QCD

• In the very late sixties Callan and Symanzik wrote an independent equation, which was \textit{the broken scale invariance Ward identity}

\[
\left[ m_R \frac{\partial}{\partial m_R} + \beta \frac{\partial}{\partial \lambda_R} + n\gamma \right] \Gamma_R^{(2n)} = m_R^2 \delta \Gamma_{\phi^2R}^{(2n)}
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• These two equations, which have a totally different physical content, share a common property: *they both describe the response of the system under the change of a dimensionfull parameter* ⇒ They can be used to study the asymptotic behaviour of the theory.
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- Two physical applications:
  (i) Phase transitions and critical phenomena (Kadanoff, Fischer, Wilson)
  (ii) Scaling properties in DIS ⇒ Asymptotic freedom and QCD (Gross, Politzer, Wilcek)
The renormalisation group and QCD

DIS phenomena were described by:

- The parton model: Simple intuitive picture, no mathematical justification
- QCD: Field theory foundation, no simple picture
- The synthesis: The DGLAP equations - The best of two worlds
The renormalisation group and QCD

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In the non-perturbative region
THE STANDARD MODEL

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- Gauge theories describe \textit{ALL} interactions among elementary particles (?)

- Dynamics=Geometry

"Let no one ignorant of geometry enter under this roof", 
\textit{Platon}
THE STANDARD MODEL and anomalies

An obscure higher order effect determines the structure of the world.
The mathematical consistency of a gauge field theory is based on the strict respect of the underlying Ward identities. This can be roughly translated into saying that the corresponding currents should be conserved.
THE STANDARD MODEL and anomalies

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- The weak currents have a vector and an axial part. We know that, in general, we cannot enforce the conservation of both.

\[ \partial_\mu j^{(5)}_\mu (x) = \frac{e^2}{8\pi^2} \epsilon_{\nu\rho\sigma\tau} F^{\nu \rho}(x) F^{\sigma \tau}(x) \]
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Anomaly cancellation condition \( A = \sum_i Q_i = 0 \)
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- Anomalies should be cancelled at all levels
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- The discovery of a very special anomaly cancellation in string theories, established the super-string theory as the only viable candidate for a quantum gauge theory of all interactions (Green and Schwarz, 1983)
Imagine we integrate over all degrees of freedom heavier than a scale $M$

$M$ does not have to correspond to a physical threshold, although it could!

$\Rightarrow$

We obtain an effective theory in terms of the light, $< M$, degrees of freedom:

$$\mathcal{L}_{\text{eff}} = \sum_{i=0}^{\infty} C_i \mathcal{O}_i$$

By dimensional analysis: $C_i \sim M^{4-d_i}$

$\Rightarrow$

The only dominant operator in the SM is the scalar mass term $\phi^2$
This is not "The end of History"
▶ This is not "The end of History"

▶ It is not even the end of the story!
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- We are looking forward to the next chapter