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Quantifying Tensions in Cosmology

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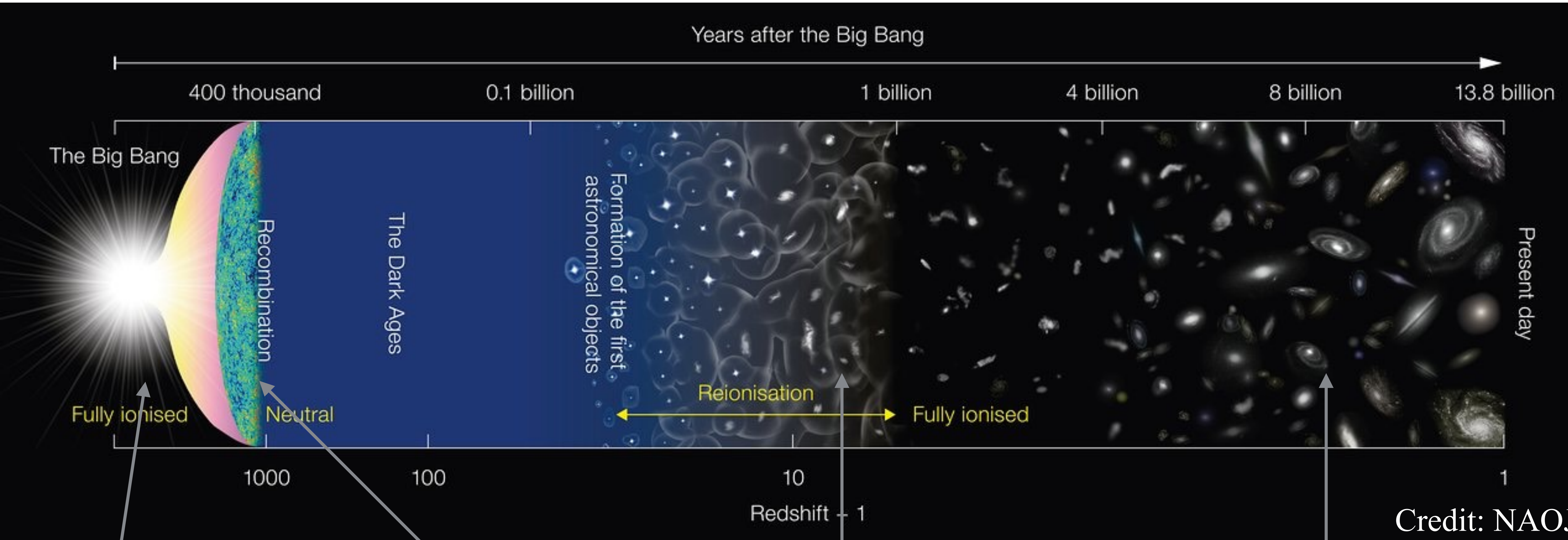
Introduction

History of the Universe: Expansion and Formation of Structures



Credit: NAOJ

hot & dense plasma

Last Scattering Surface (Universe becomes transparent)

Formation of Stars and Galaxies

observable Large Scale Structure

Introduction

Evolution of the Universe described by Standard Model of Cosmology

⇒ expanding spacetime described by metric

⇒ filled with different components:

- photon, neutrinos, …
- ordinary matter ("baryons")
- missing, non-luminous matter ("Dark Matter")
- something driving late-time accelerated expansion ("Dark Energy")

⇒slight initial perturbations of the metric (gravitational potentials)

Interaction and Gravity lead to
Expansion and Formation of Structure

Introduction

Universe described by finite set of cosmological parameters

- present-day densities of components: $\Omega_\gamma, \Omega_\nu, \Omega_\mathrm{b}, \Omega_\mathrm{cdm}, \Omega_\Lambda, \Omega_\mathrm{k}$
  - photons (Ω_γ), neutrinos (Ω_ν), baryons (Ω_b), DM (Ω_cdm), DE (Ω_Λ), curvature (Ω_k)
- present-day expansion rate (sets physical scale of the Universe): H_0
- prescription for initial fluctuations: $n_\mathrm{S}, A_\mathrm{S}$ alternatively σ_8
  - $n_\mathrm{S}, A_\mathrm{S}$: spectral index and amplitude of primordial scalar fluctuations
  - σ_8: root-mean square of present-day matter fluctuations at 8 Mpc/h
- and others

$\Downarrow$

Predictions for:
- power spectra of different components
- distances as function of redshift

Observational Cosmology infers these parameters from observations (Cosmological probes)

Cosmological Probes

Cosmic Microwave Background (CMB)

While expanding/cooling, the early Universe turned from optically thick to transparent: **Last Scattering Surface**

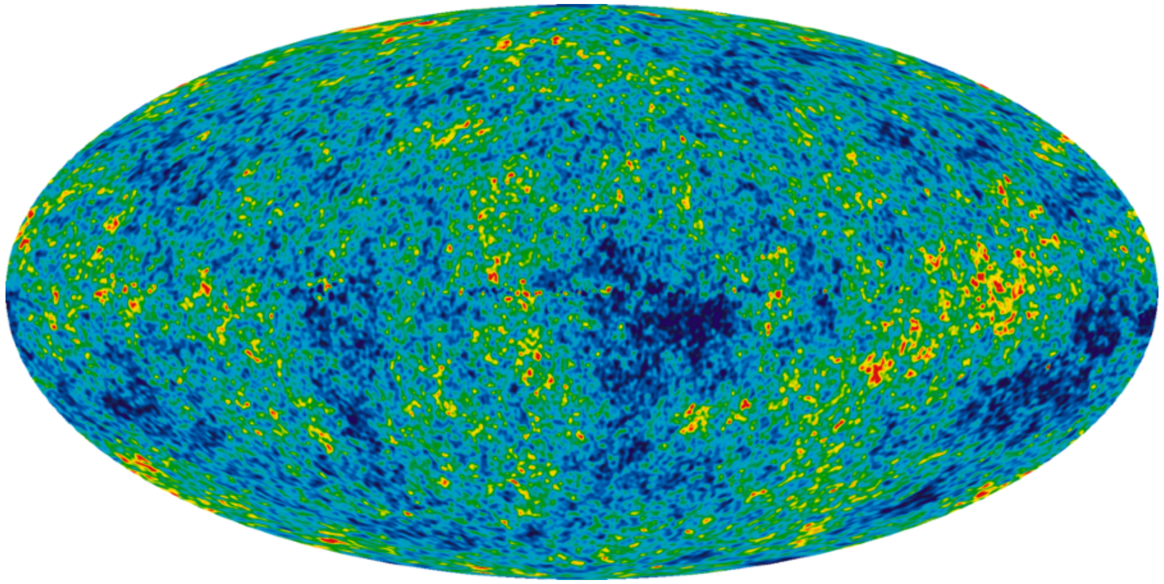
⇕

Since LSS photons travel freely ⟹ “Echo” from the primordial Universe

⇕

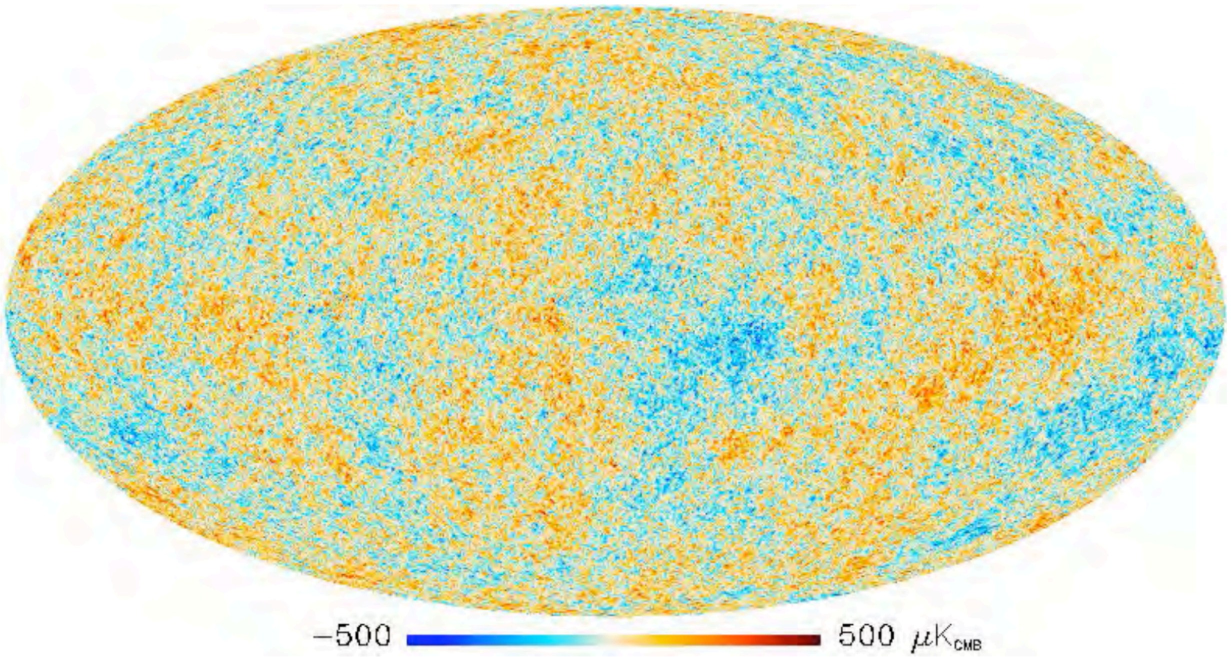
Photon Background in Microwaves with **tiny temperature fluctuation**

WMAP



Bennett et al. 2013

Planck

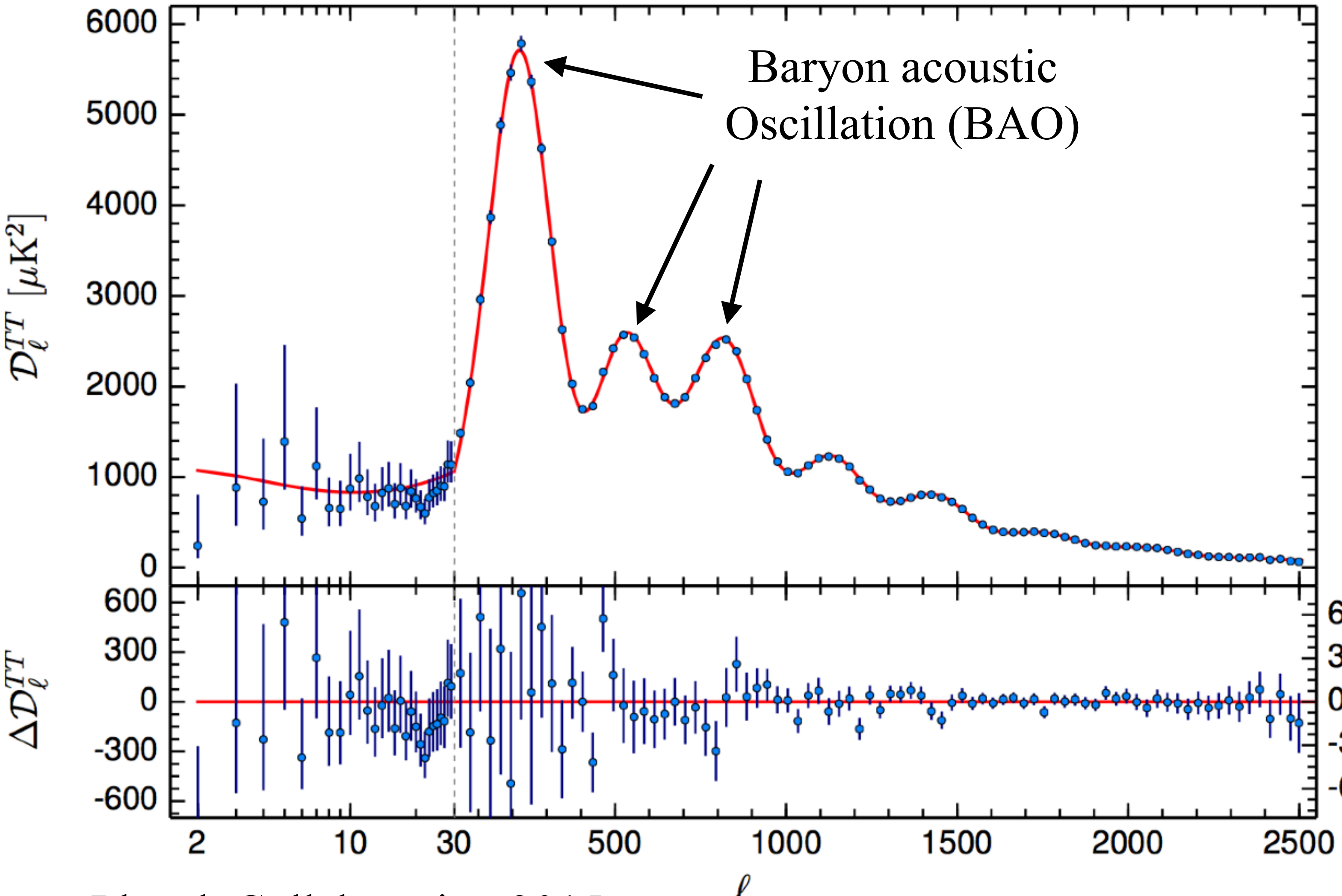


Planck Collaboration 2014

Cosmological Probes

CMB perfect Blackbody $\Rightarrow$ mean Temperature of CMB fixes Ω_γ

angular power spectrum of temperature fluctuation



Planck Collaboration 2015

complex observable
(many features)

$\Downarrow$

very informative

$\Downarrow$

(sub-)percent constraints on some cosmological parameters

$\Omega_\mathrm{b}h^2 \quad \Omega_\mathrm{cdm}h^2 \quad n_\mathrm{S} \quad A_\mathrm{S}$

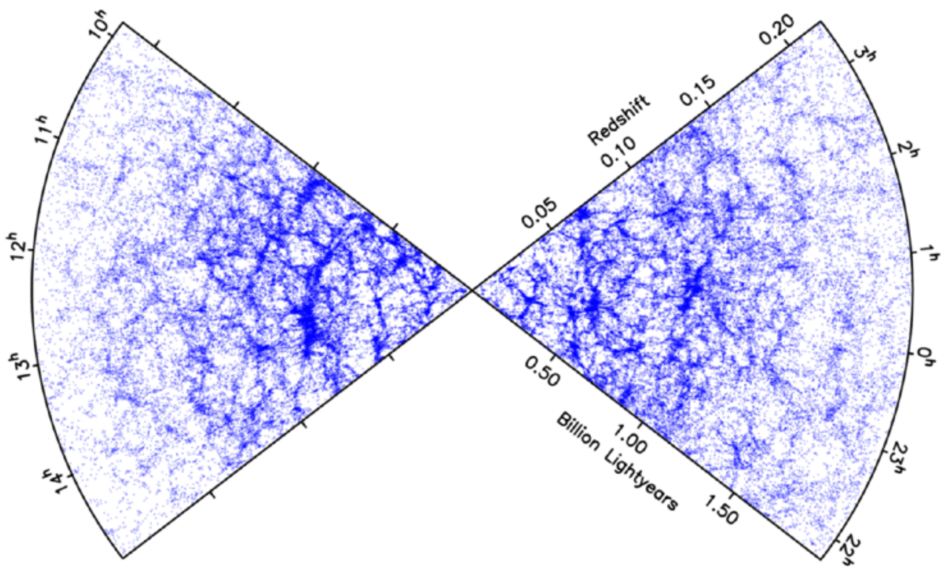
acoustic waves in primordial plasma imprint peaks θ_BAO

angular scale

<u>extra information:</u> fluctuation of CMB polarisation

Cosmological Probes

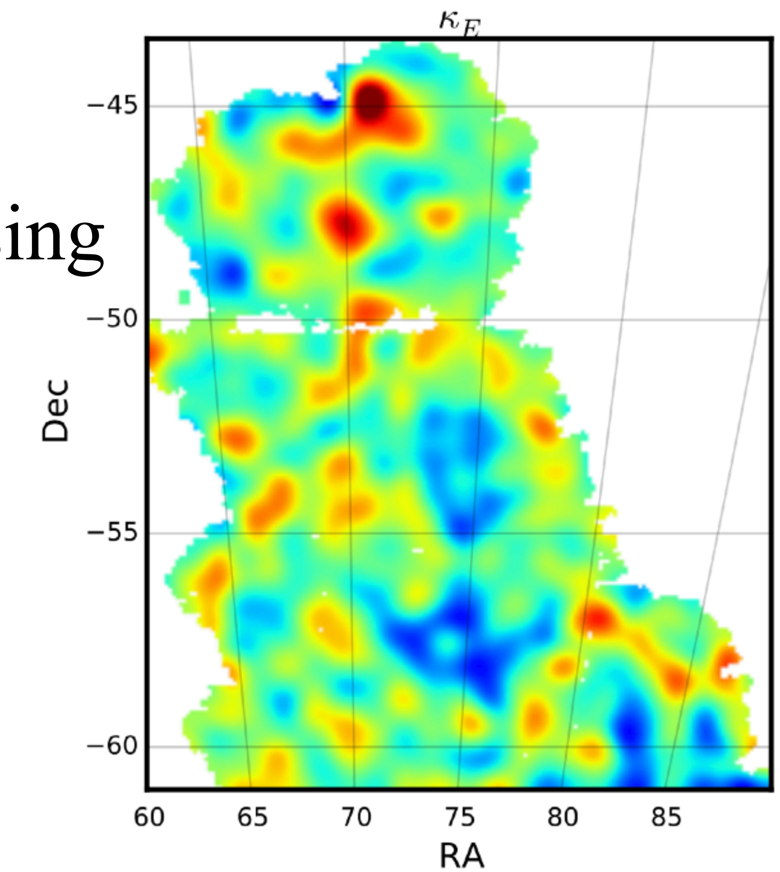
Measurements of present day distribution of matter



2dF Galaxy Survey

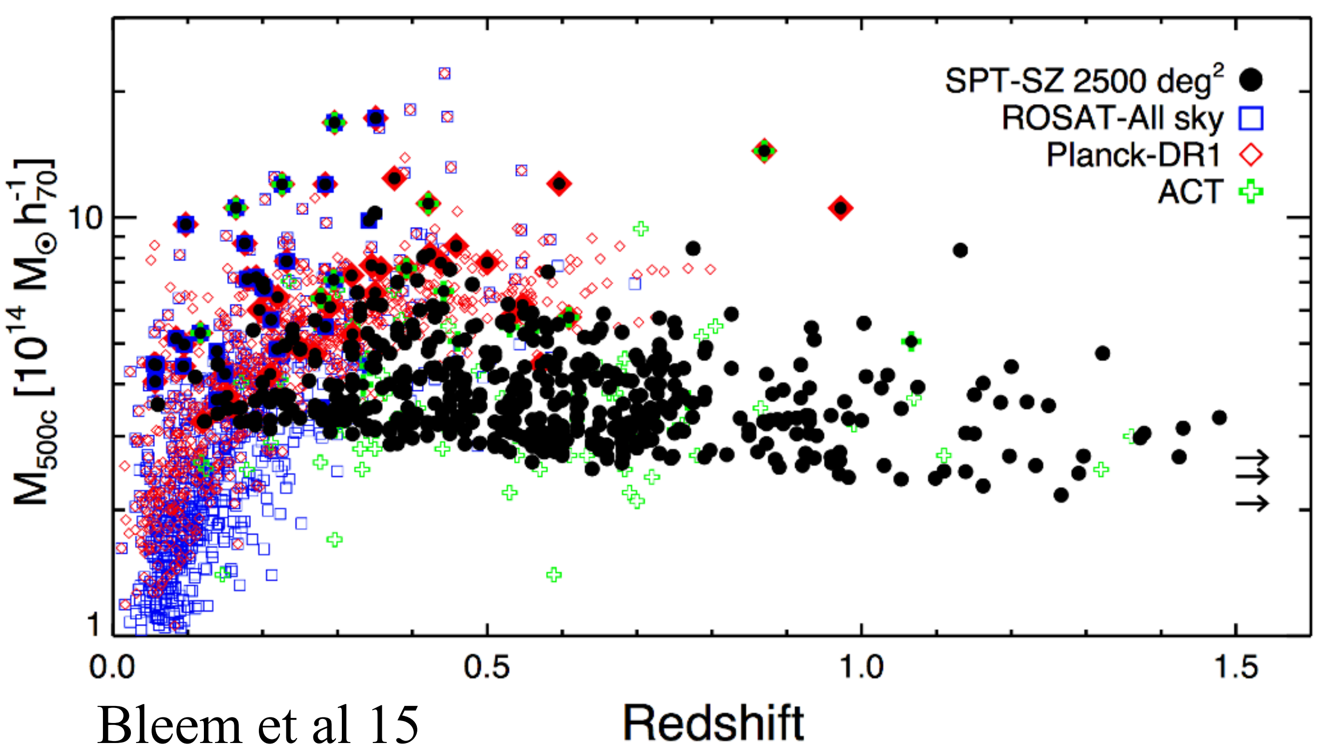
spectroscopic
galaxy surveys

gravitational lensing
of galaxy shapes



DES Collaboration 2015

abundance of galaxy clusters



Bleem et al 15

Observation of
amplitude of fluctuations
at low redshift

$\Rightarrow$ constraints on Ω_M and σ_8

$\Rightarrow$ possible clues about DE

Cosmological Probes

Measurements of cosmological distances

BAOs can be found in distribution of galaxies

⇓

standard ruler

object of constant size

⇓

angular size of standard ruler is distance measure

Supernovae Type Ia (SNe) have constant luminosity

⇓

standard candle

object of constant luminosity

⇓

flux of standard candle is distance measure

Calibration

BAOs imprinted on CMB

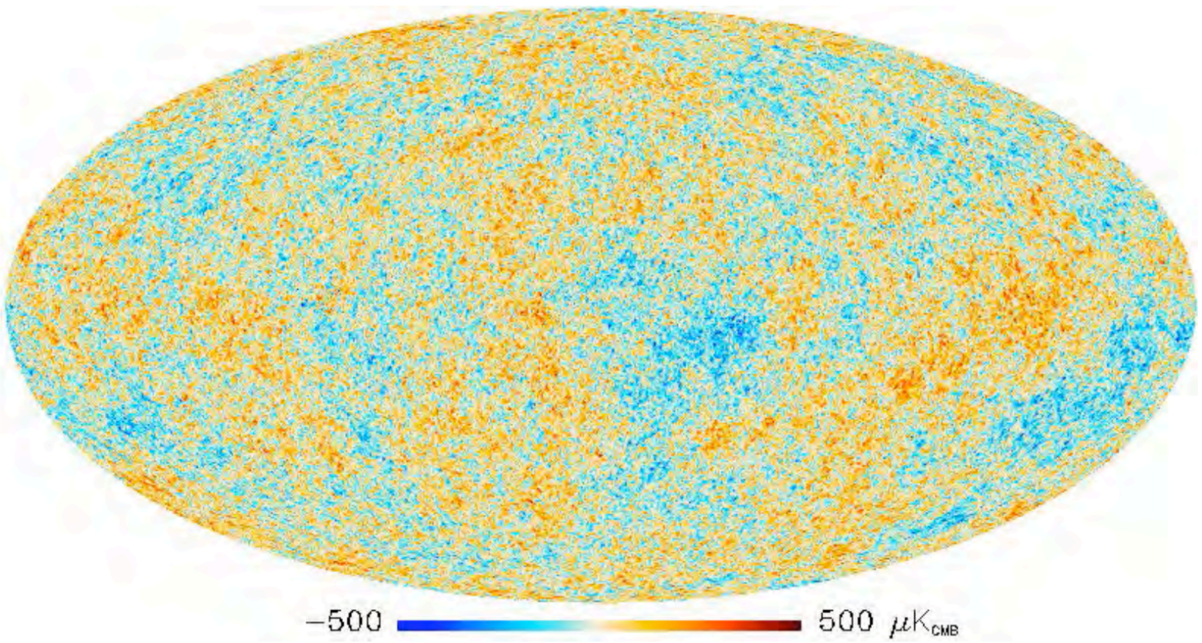
"Distance ladder"

- parallax measurements to Cepheid stars (variable)
- Cepheids used to determine intrinsic luminosity of SNe

Cosmological Probes

Cosmology is blessed with a variety of (almost) independent measurements

Cosmic Microwave Background (CMB)

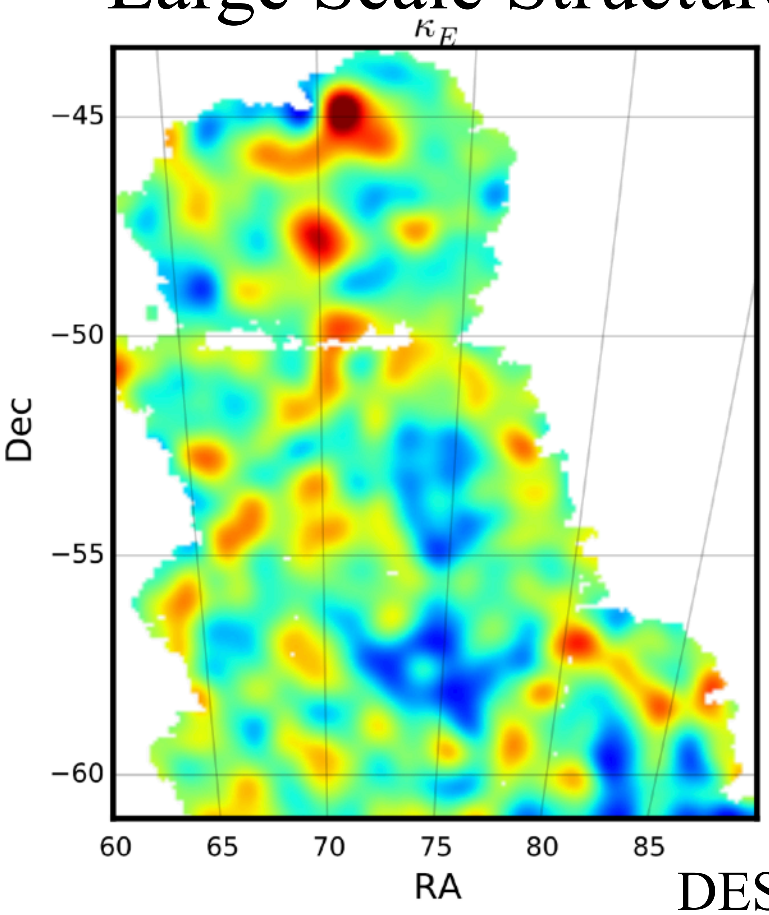


Planck Collaboration 2014

Distance Measurements

Hubble Space Telescope, European Space Agency

Large Scale Structure



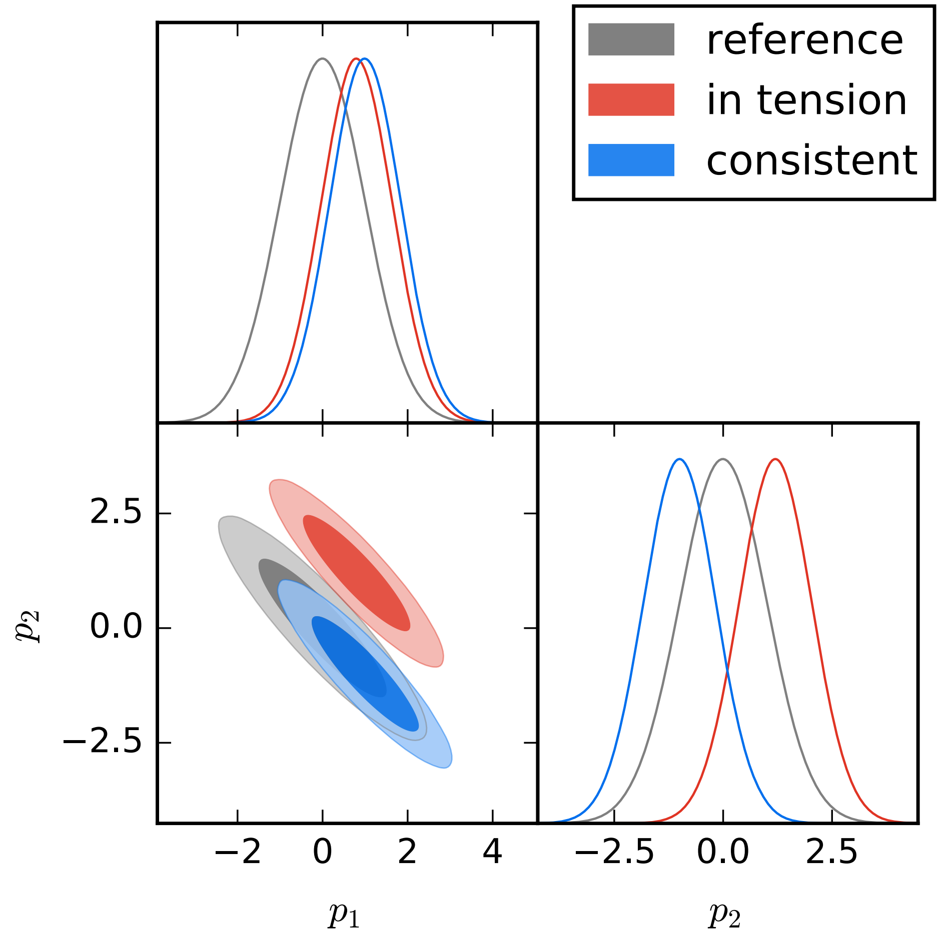
DES Collaboration 2015

Very different datasets put constraint on the same model

Introduction

Given large **variety of datasets** constraining the same model $\Rightarrow$ Need to measure **mutual consistency** of different datasets

Different datasets $\Rightarrow$ Different observables $\Rightarrow$ Comparison of posterior distribution in model parameter space



Comparing marginal contours might be misleading

$\Rightarrow$ Projection effects due to correlations between parameters

$\Rightarrow$ Qualitative

Relative Entropy

Let D_1 be the reference dataset, and θ the model parameters

Bayesian Inference:

prior	Bayesian Update →	posterior
$p(\theta \| D_1)$		$p(\theta \| D_2,\ D_1)$
degree of belief in θ given D_1		degree of belief in θ given D_1 and D_2

Change in degree of belief

⇕

negative Entropy ⟺ **Information** gained by update

$$-H = \sum_i p_i \ln p_i$$

Shannon 1950

⇕ Kullback & Leibler 1951

relative Entropy

$$KL = \int \mathrm{d}^n\theta \, p(\theta|\, D_2, D_1) \ln \left(\frac{p(\theta|\, D_2,\ D_1)}{p(\theta|\, D_1)} \right)$$

also Kullback-Leibler (KL) divergence

Relative Entropy

- Relative Entropy measures the difference between distributions, i.e. the **Information Gain**
- Relative Entropy is **invariant** under (invertible) transformation in parameter space $\theta' = \psi(\theta)$
- Relative Entropy is a **function of the data** $KL[D_2|\, D_1]$

Given D_1, we expect that D_2 is distributed $P(D_2|\, D_1)$

Seehars et al. 14, 15
Grandis et al. 2016a

$\langle KL \rangle_{D_2|D_1}$
expected information

$S = KL - \langle KL \rangle_{D_2|D_1}$
"Surprise", i.e. excess information
$\Updownarrow$
Measure of consistency $\Rightarrow$ Significance of Tension

$\mathrm{Var}(KL)$
expected variance of the information
$\Downarrow$
Significance of Tension

The Surprise

Positive Surprise $S > 0$ $\Leftrightarrow$ More Information than expected $\Leftrightarrow$ D_2 in Tension with D_1

For Gaussian prior, and linear Gaussian Likelihood:

Seehars et al. 14, 15

Information Gain

$$KL = \frac{1}{2}\Delta\mu^T\Pi^{-1}\Delta\mu + \frac{1}{2}\mathrm{tr}\left(\Sigma\Pi^{-1} - \mathbb{I}\right) + \frac{1}{2}\ln\left(\frac{\det\Pi}{\det\Sigma}\right)$$

Shift of central values

Change in volume of credibility contours

Surprise

$$S = \frac{1}{2}\Delta\mu^T\Pi^{-1}\Delta\mu + \frac{1}{2}\mathrm{tr}\left(\Sigma\Pi^{-1} - \mathbb{I}\right)$$

Variance

$$\sigma^2(KL) = \frac{1}{2}\mathrm{tr}\left((\Sigma\Pi^{-1} - \mathbb{I})^2\right)$$

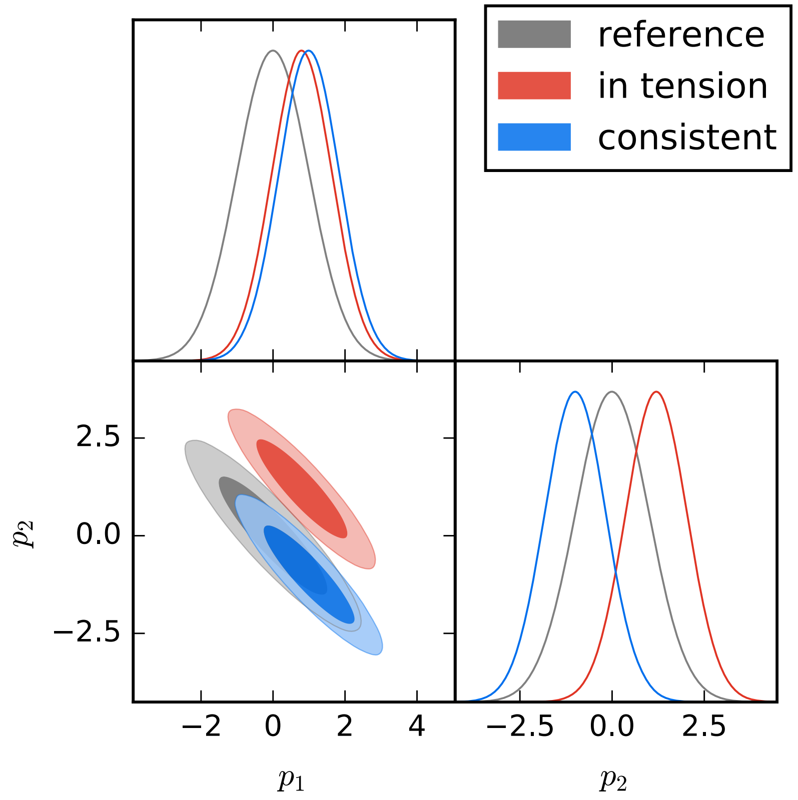
Π priors covariance

Σ posteriors covariance

$\mathbb{I}$ identity matrix

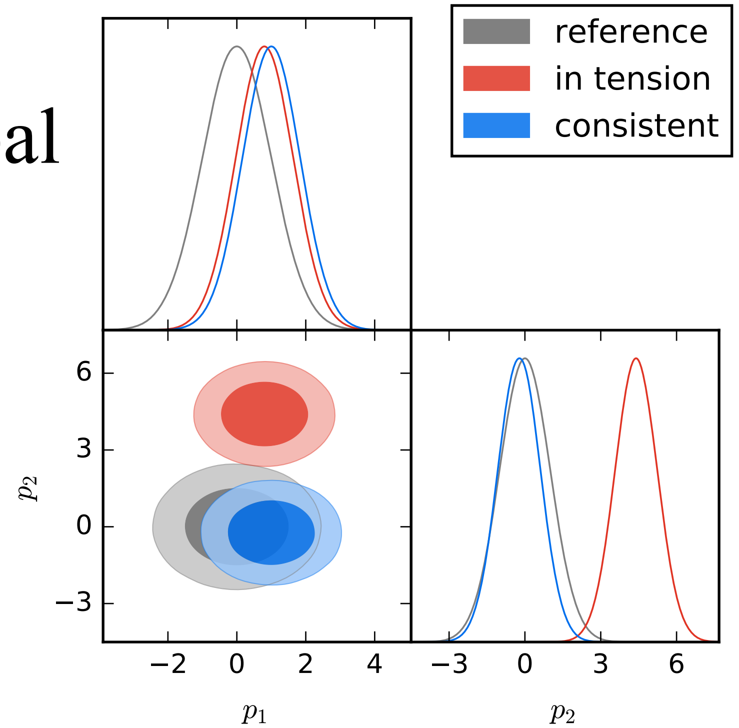
$\Delta\mu$ difference in means

The Surprise



	KL	$\langle KL \rangle$	S	$\sigma(KL)$
red	14.5	0.51	14.0	0.43
blue	0.84	0.51	0.32	0.43

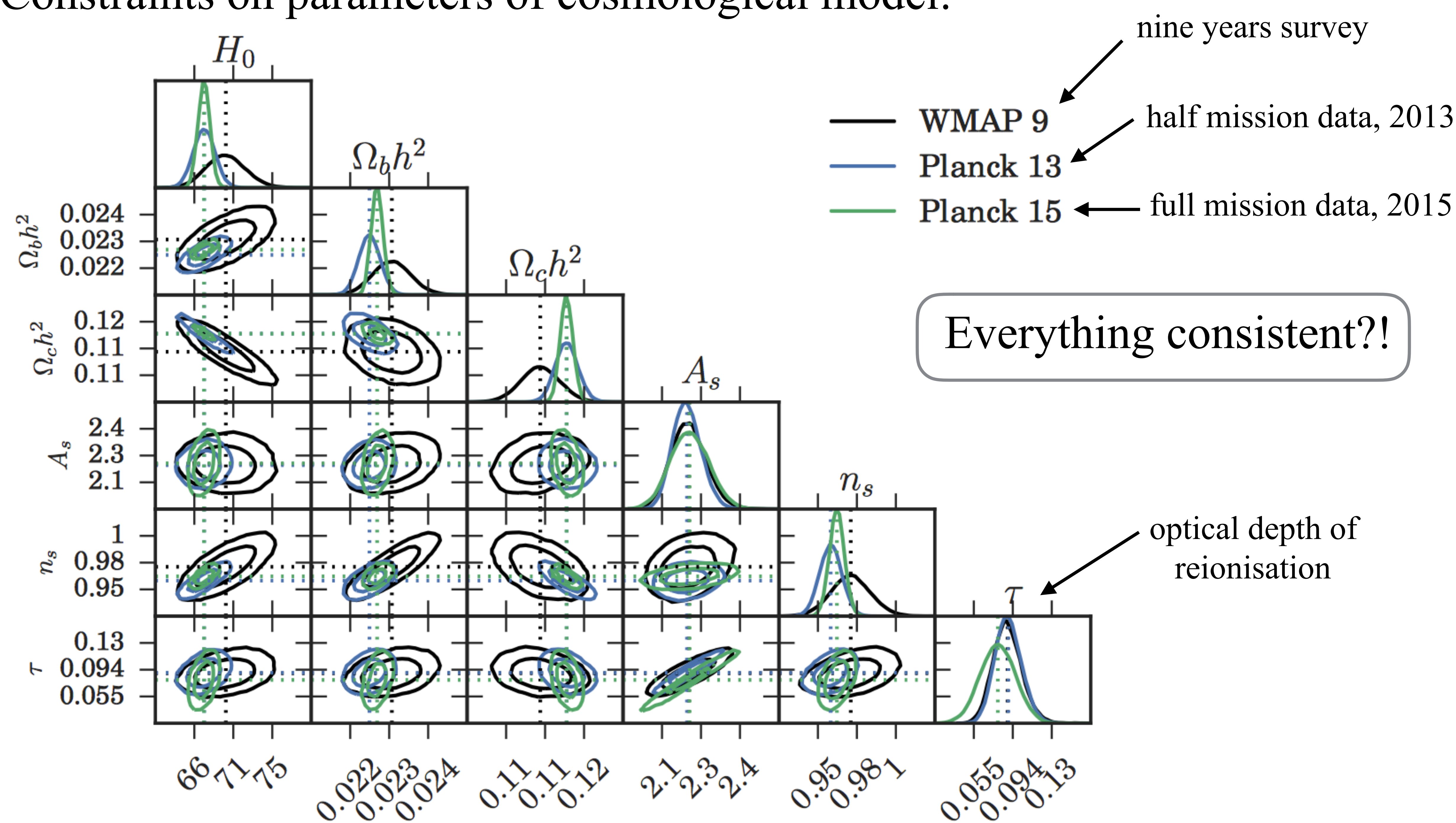
Mapping into Principal Components of prior



Surprise spots "hidden" Tension

Quantifying Tensions I: CMB

Constraints on parameters of cosmological model:

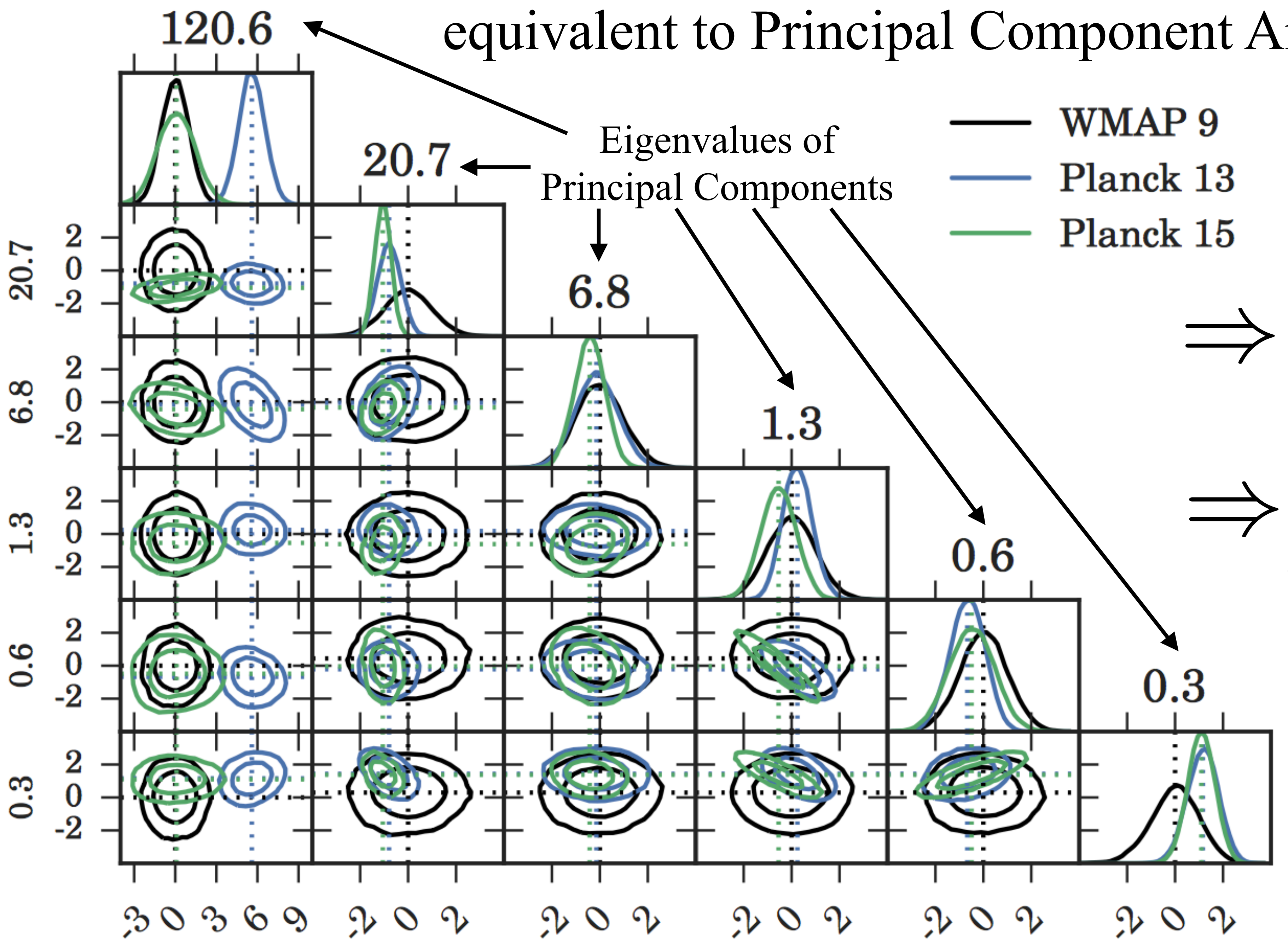


Everything consistent?!

Seehars, SG, et al. 2015

Quantifying Tensions I: CMB

Transformation, such that WMAP constraints are uncorrelated
equivalent to Principal Component Analysis



Seehars, SG, et al. 2015

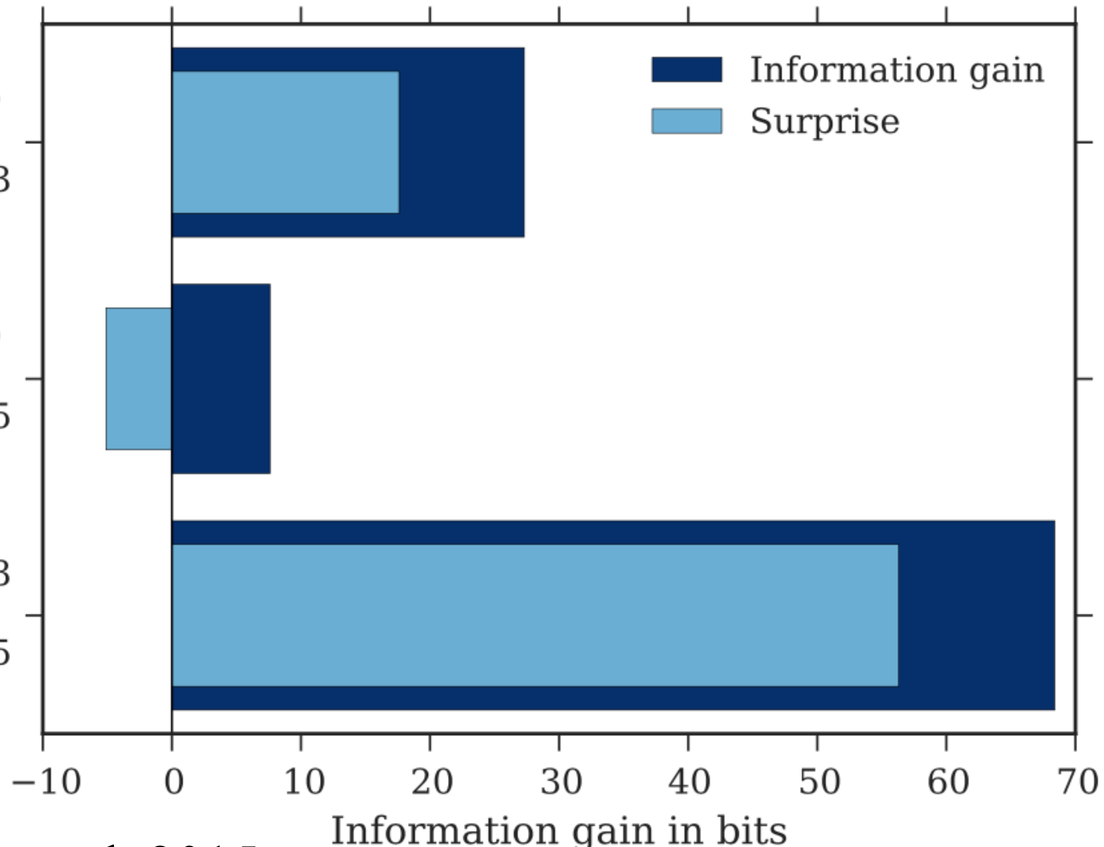
⇒ Planck 13 in Tension with WMAP 9

⇒ Planck 15 consistent with WMAP 9

⇓

model is adequate

Quantifying Tensions I: CMB



Seehars et al. 2015

- Surprise spots "hidden" Tension
- negative Surprise

$\Updownarrow$

better agreement than expected

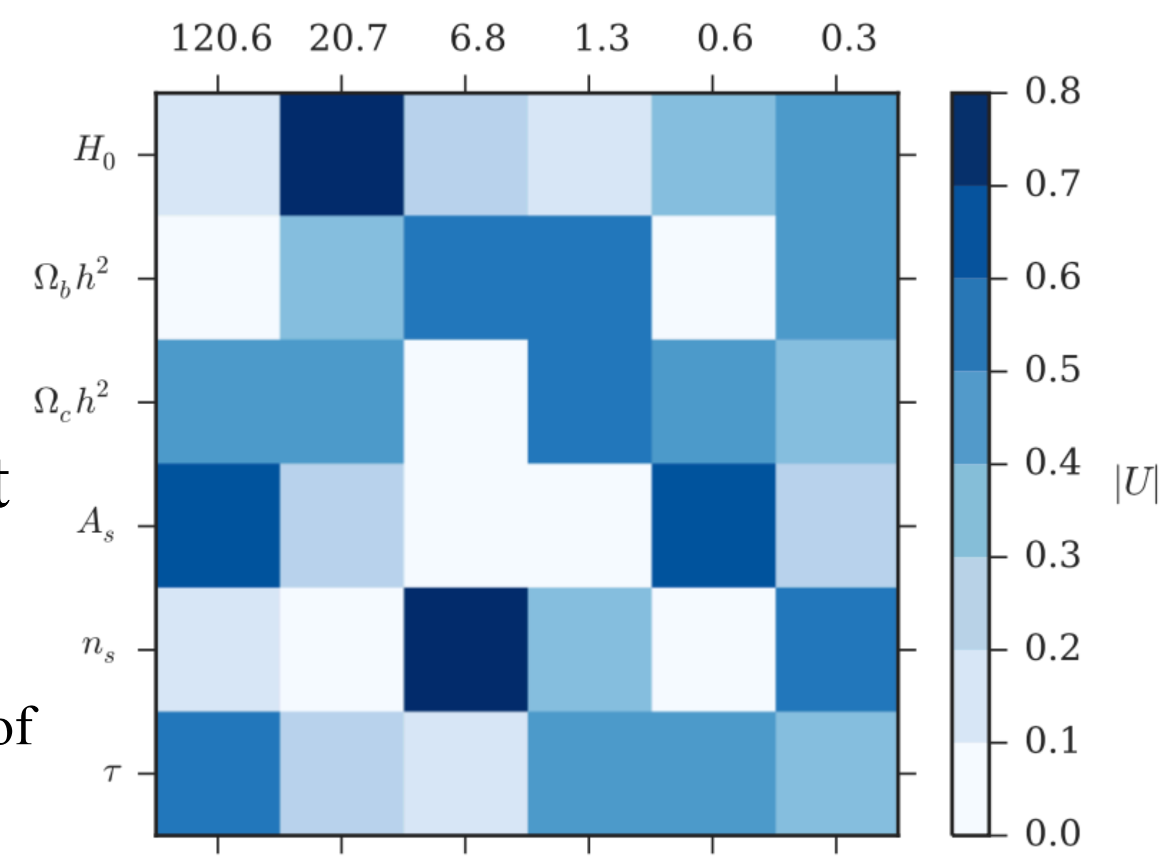
in this case: statistically not significant

What happened?

major shift in strongest Principal component

from rotation matrix: shifts in A_S and τ

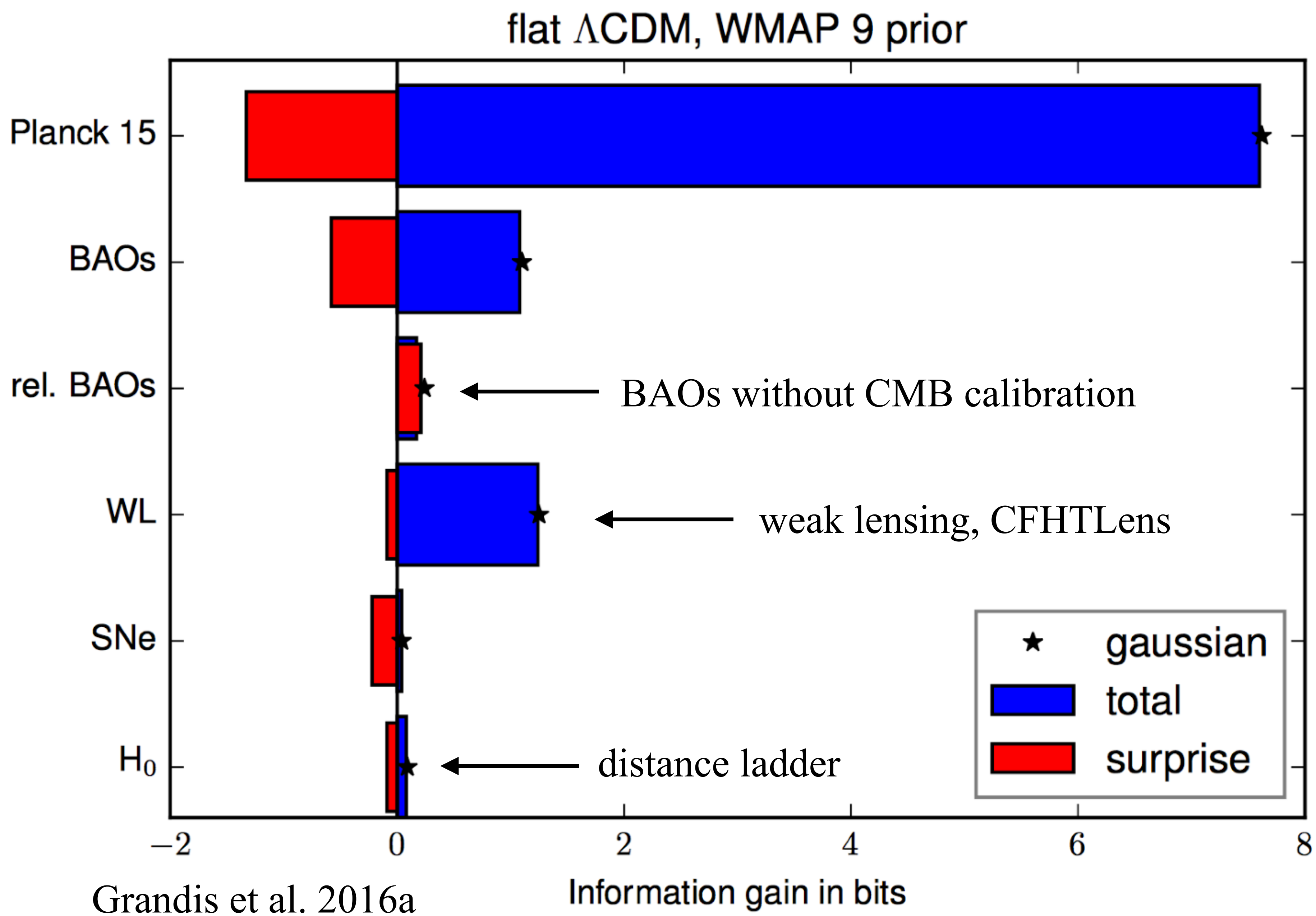
optical depth of reionisation



Seehars, SG, et al. 2015

Quantifying Tensions I, b

Information gain (blue) and Surprise (red) when combining different probes with WMAP9



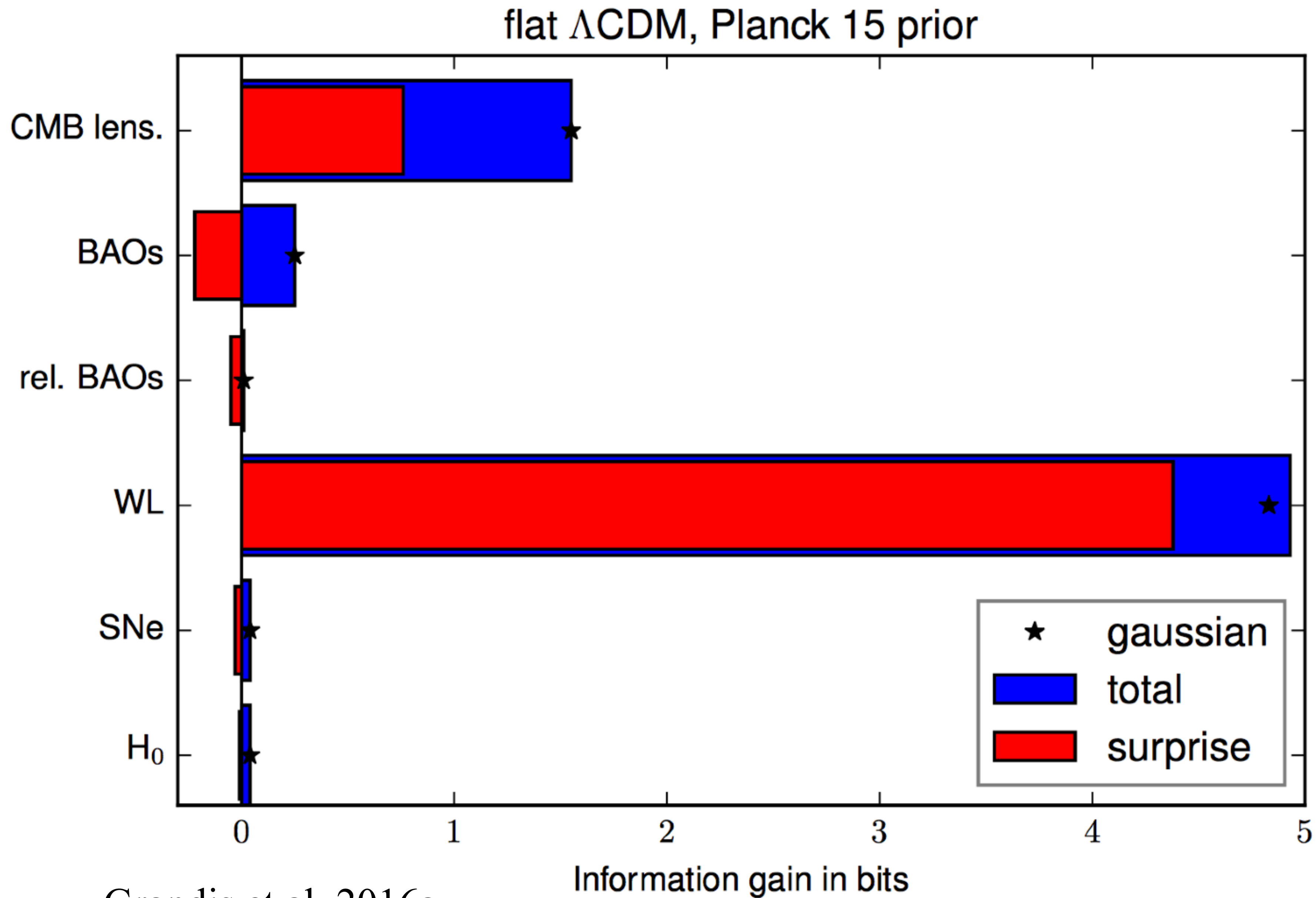
Grandis et al. 2016a

- everything consistent when using WMAP as a prior
- Planck much more informative than other probes

in flat ΛCDM

sigma8 Problem

Information gain (blue) and Surprise (red) when combining different probes with Planck 15 (full mission)



Grandis et al. 2016a

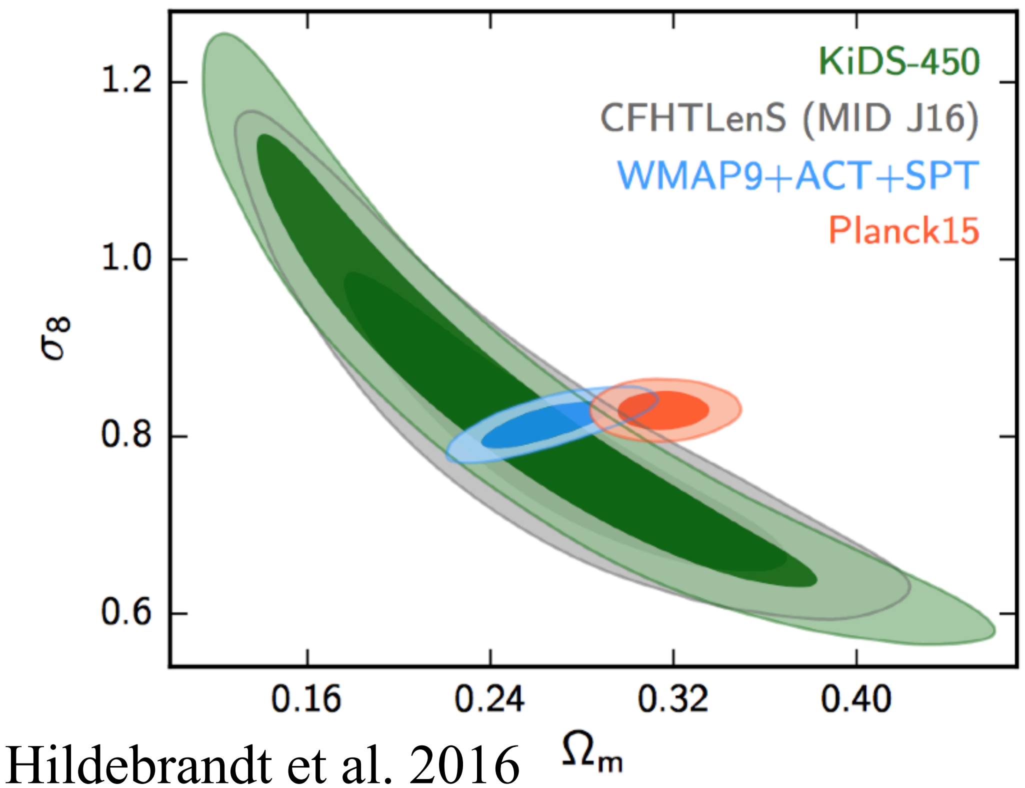
- small gains by adding other probes
- large Surprise when adding WL to Planck (8σ significance)

$\Downarrow$

σ_8 problem

sigma8 Problem

WL and galaxy clusters want lower amplitudes than CMB



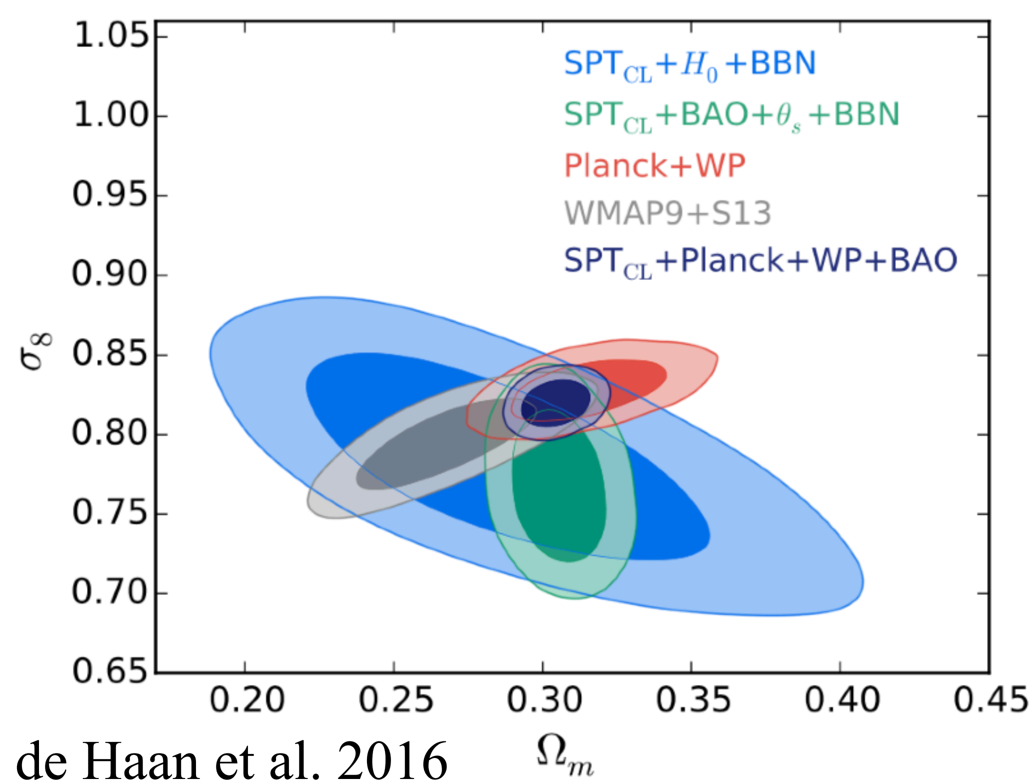
Hildebrandt et al. 2016

Systematic effects:

- CMB: τ and foregrounds
- WL: photo-zs, non linearities, …
  Maccrann et al. 2015, Joudaki et al. 2016
- Clusters: mass calibration
  Planck Collaboration, SZ clusters, 2014, 2015

Possible physical effects:

- massive neutrinos
  Maccrann et al. 2015, Joudaki et al. 2016
- Interactions DM-Dγ or DM-Dgluons
  Lesgourgues et al. 2015
- modified gravity (less gravity)



de Haan et al. 2016

The Surprise

PROS

- quantitative method
- unaffected by projection effects
- invariant under transformations

CONS

- only applicable to Gaussian constraints
- $\langle \cdot \rangle_{D_2|D_1}$ very hard to compute
- other methods already exist (what is the gain?)

Remark:

Surprise is not symmetric

choice of reference data set, i.e. priors, matters

Other Measures of Tension

Bayes Theorem

Posterior · Prior · Likelihood

$$P(\theta|\,D) = \frac{P(\theta)\,L(D|\,\theta)}{E(D)}$$

← Evidence

Prob. that D is described by the model

Evidence Ratio

Joint Evidence over Product of individual Evidences

$$R(D_1,\ D_2) = \frac{E(D_1, D_2)}{E(D_1)\,E(D_2)}$$

Marshall et al 06

$$= \frac{\text{Prob. that D1, D2 are described by same set of parameters}}{\text{Prob. that D1, D2 are described by different sets of parameters}}$$

Tension between Data Sets $\Leftrightarrow R < 1$

Other Measures of Tension

Grandis et al. 2016b, Appendix B

For linear, normal model $$L(D_i|\boldsymbol{\theta}) = \exp\Big(-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu}_i)^T\Sigma_i^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu_i})\Big)$$

Then $$\ln R = \boxed{-\frac{1}{2}\boldsymbol{\Delta\mu}^T(\Sigma_1+\Sigma_2)^{-1}\boldsymbol{\Delta\mu}} - \boxed{\frac{n}{2}\ln 2\pi - \frac{1}{2}\ln\det(\Sigma_1+\Sigma_2)}$$

- quadratic form of difference in means
- immune to projection effects
- always < 0

- offset depending on precision of data

$\Downarrow$

$R < 1$ not generally applicable

Need to calibrate by expected value:

calibrated Evidence Ratio

$$\ln R - \langle \ln R\rangle = -\frac{1}{2}\boldsymbol{\Delta\mu}^T(\Sigma_1+\Sigma_2)^{-1}\boldsymbol{\Delta\mu} + \frac{n}{2}$$

Furthermore, the scale of significance is set by $\mathrm{Var}[\ln R] = \frac{n}{2}$

Gaussianisation

any measure of Tension needs to be calibrated

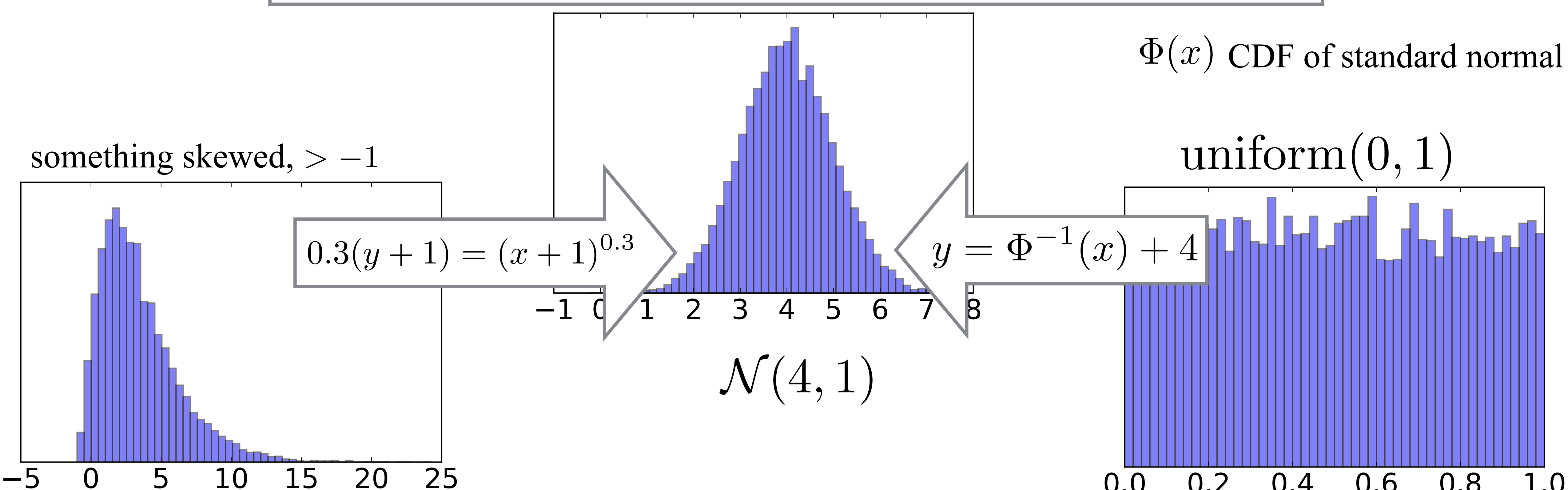
$\Updownarrow$

need to compute $\langle \cdot \rangle_{D_2|D_1}$ or $\langle \cdot \rangle_{D_1,D_2}$

Average over expected distribution of data

analytic for Gaussians, hard in general

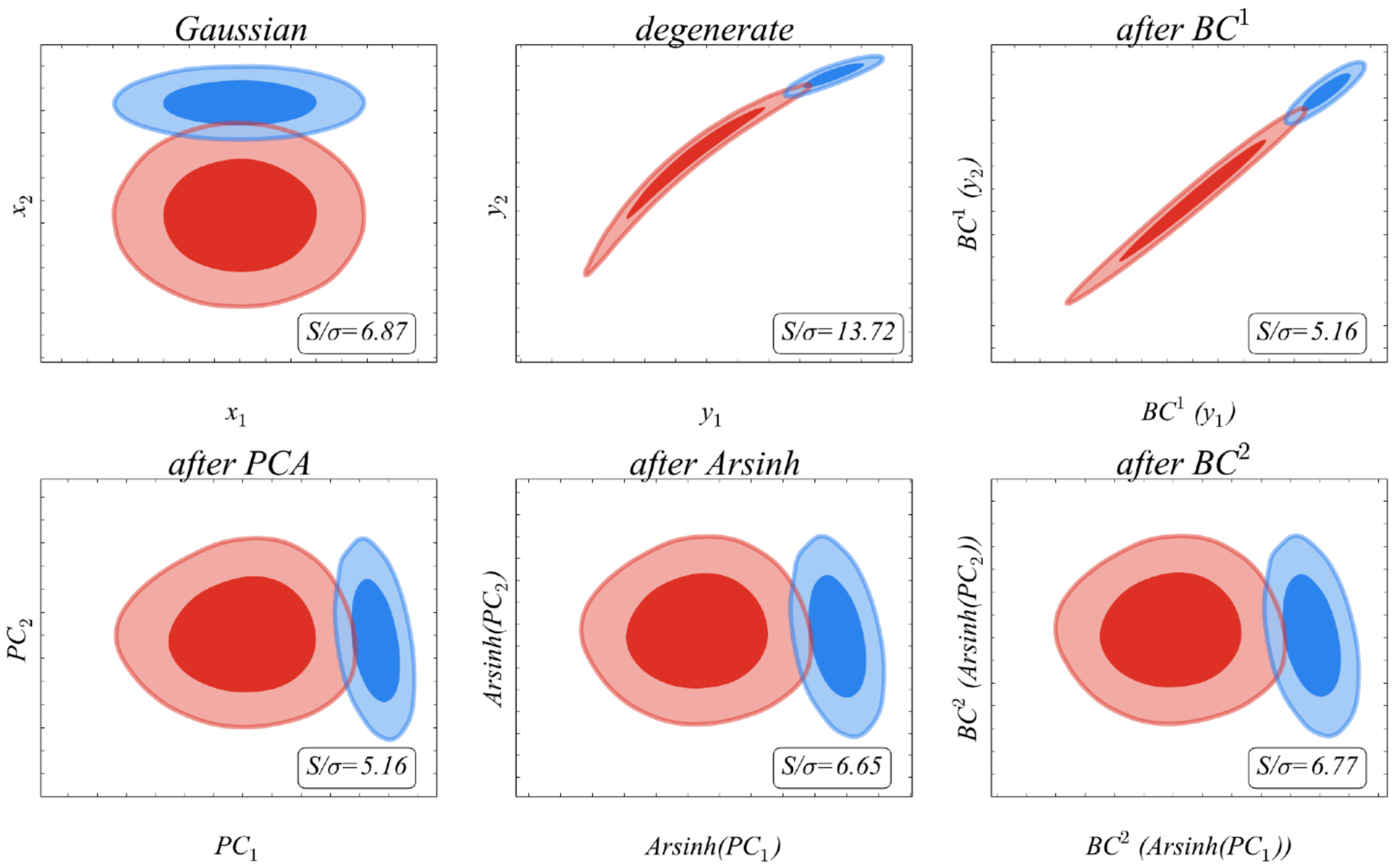
Remark: Information Gain and Evidence are **invariant** under transformation in parameter space $\theta' = \psi(\theta)$



Gaussianisation

Optimise **subsequent transformations** of parameter space, such that the **distribution becomes Gaussian** upon application of the transformations

for one distribution: Schuhmann et al. 2016
for two distribution: Grandis et al. 2016b



Grandis et al. 2016b, Appendix A

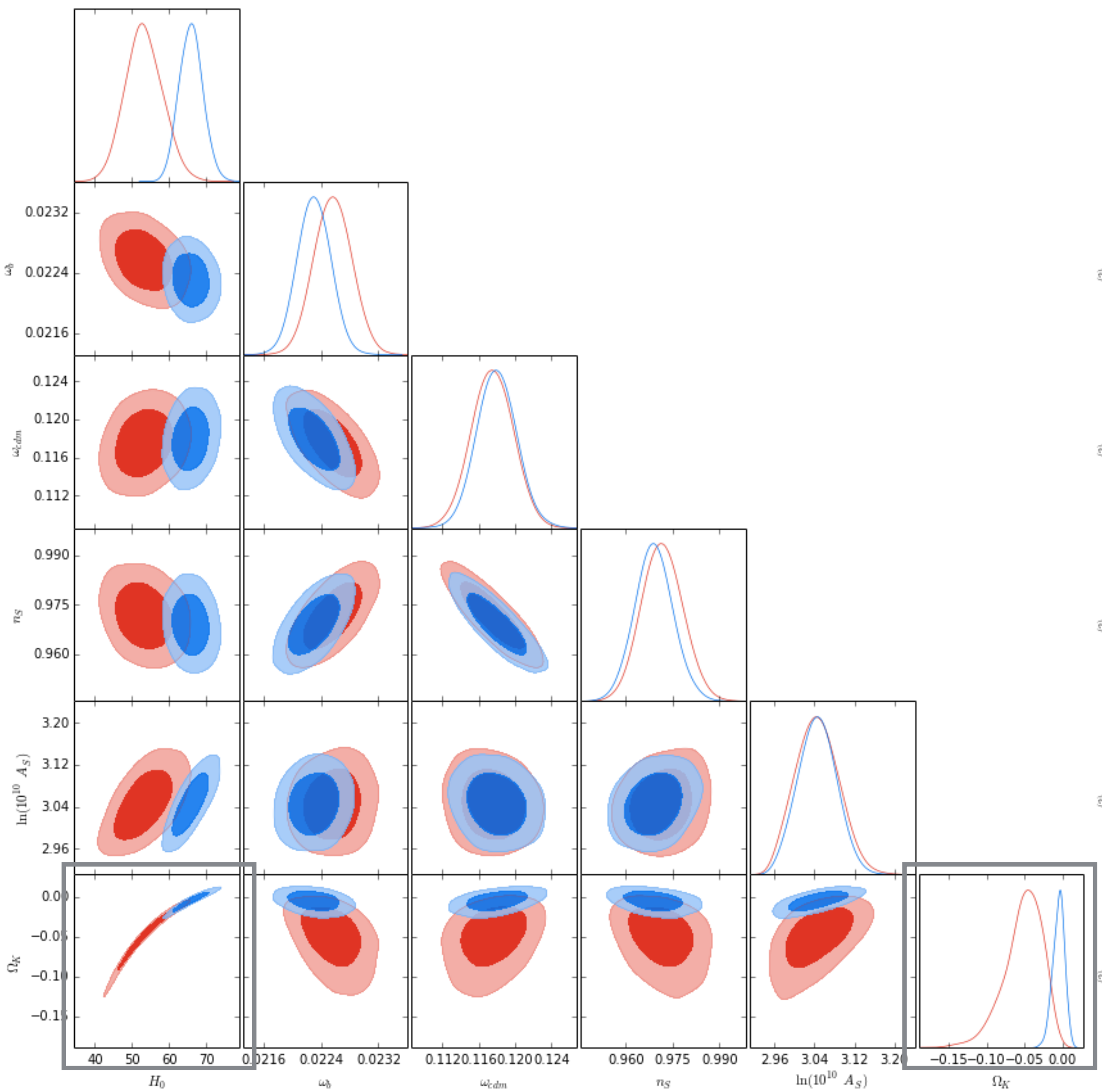
BC: Box Cox Transformation
PCA: Principal Component Rotation

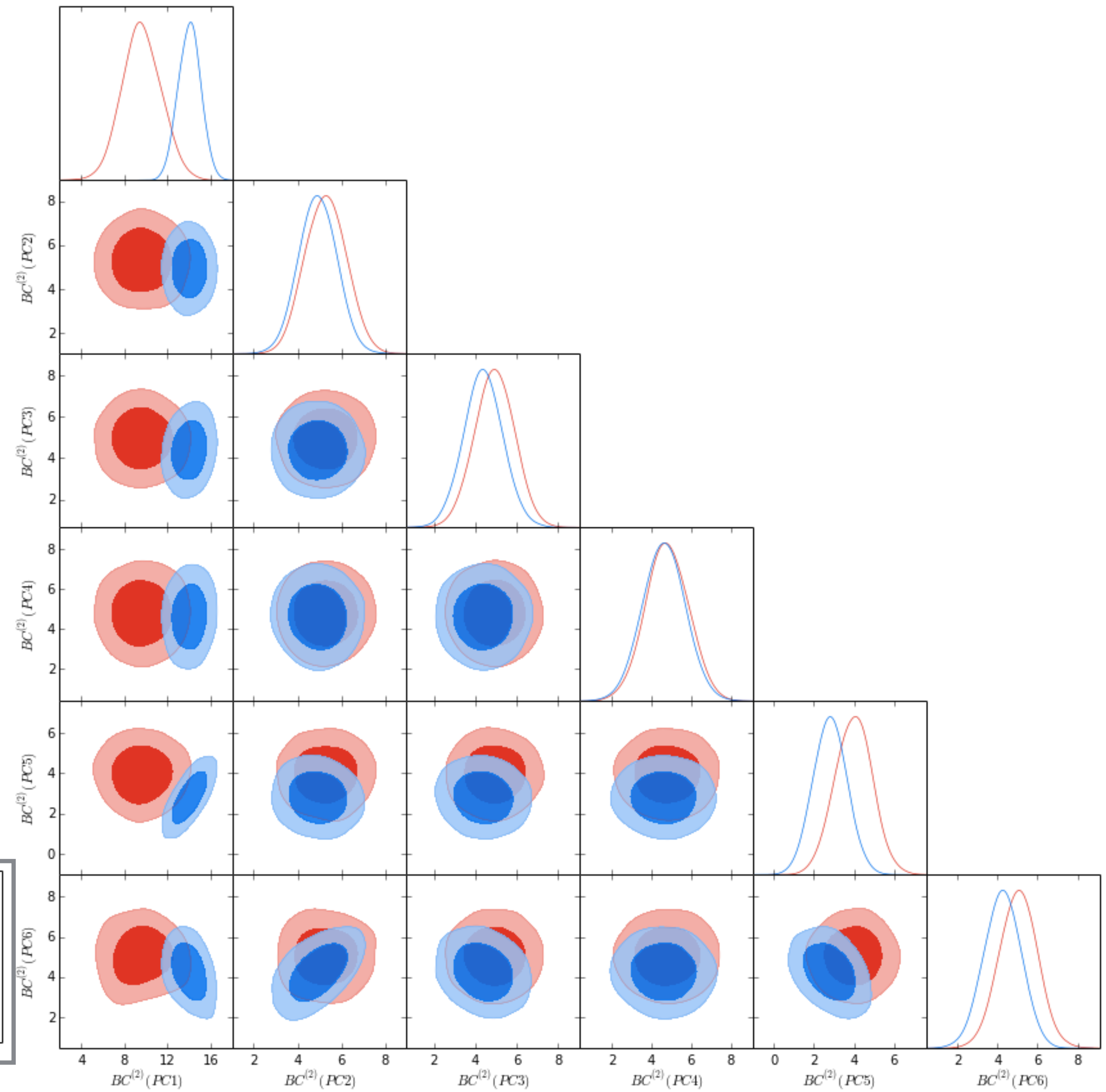
Accuracy of Surprise estimation

⇕

Accuracy of Gaussianisation
+
Uncertainty due to finite samples

Gaussianisation





Checked explicitly that the Gaussianisation is good enough

Grandis et al. 2016b, Appendix A

Quantifying Tensions II

SG et al. 2016b

The *Planck* power spectra give the constraint

$$\Omega_K = -0.052^{+0.049}_{-0.055} \quad (95\%, \textit{Planck}\ \text{TT+lowP}). \qquad (47)$$

Taken at face value, Eq. (47) represents a detection of positive curvature at just over $2\,\sigma$, largely via the impact of lensing on the power spectra. One might wonder whether this is mainly a parameter volume effect, but that is not the case, since the best fit closed model has $\Delta\chi^2 \approx 6$ relative to base ΛCDM, and the fit is improved over almost all the posterior volume, with the mean chi-squared improving by $\langle\Delta\chi^2\rangle \approx 5$ (very similar to the phenomenological case of ΛCDM+A_L). Addition of the *Planck* polarization spectra shifts Ω_K towards zero by $\Delta\Omega_K \approx 0.015$:

$$\Omega_K = -0.040^{+0.038}_{-0.041} \quad (95\%, \textit{Planck}\ \text{TT,TE,EE+lowP}), \qquad (48)$$

but Ω_K remains negative at just over $2\,\sigma$.

Planck Collaboration 2015, cosmological parameters, pag. 38

Quantifying Tensions II

Considered 3 models: flat ΛCDM curved ΛCDM flat $\Lambda\mathrm{CDM} + A_{\mathrm{L}}$

- flat: universe with flat spatial geometry
- Λ: Dark Energy "just" a constant
- CDM: and "cold Dark Matter"
- curved: allows for more general geometry (flat, 3d sphere, 3d hyperboloid)
- A_{L}: parameter expressing systematic uncertainties in lensing of CMB photons

Considered 6 datasets in addition to primary Planck 15 CMB

- prior: Planck 15 temperature and large scale polarisation
- BAO: compilation of BAO measurements used by Planck 15
- SNe: binned version presented by Betoule et al. 13
- CMB lensing: constraints presented from Planck 15
- H_0: latest results from Riess et al. 16
- Lyα BAO: BAO feature in Lyman alpha forest (absorption from distant quasars) Delubac et al. 15
- TEEE: small scale polarisation from Planck 15

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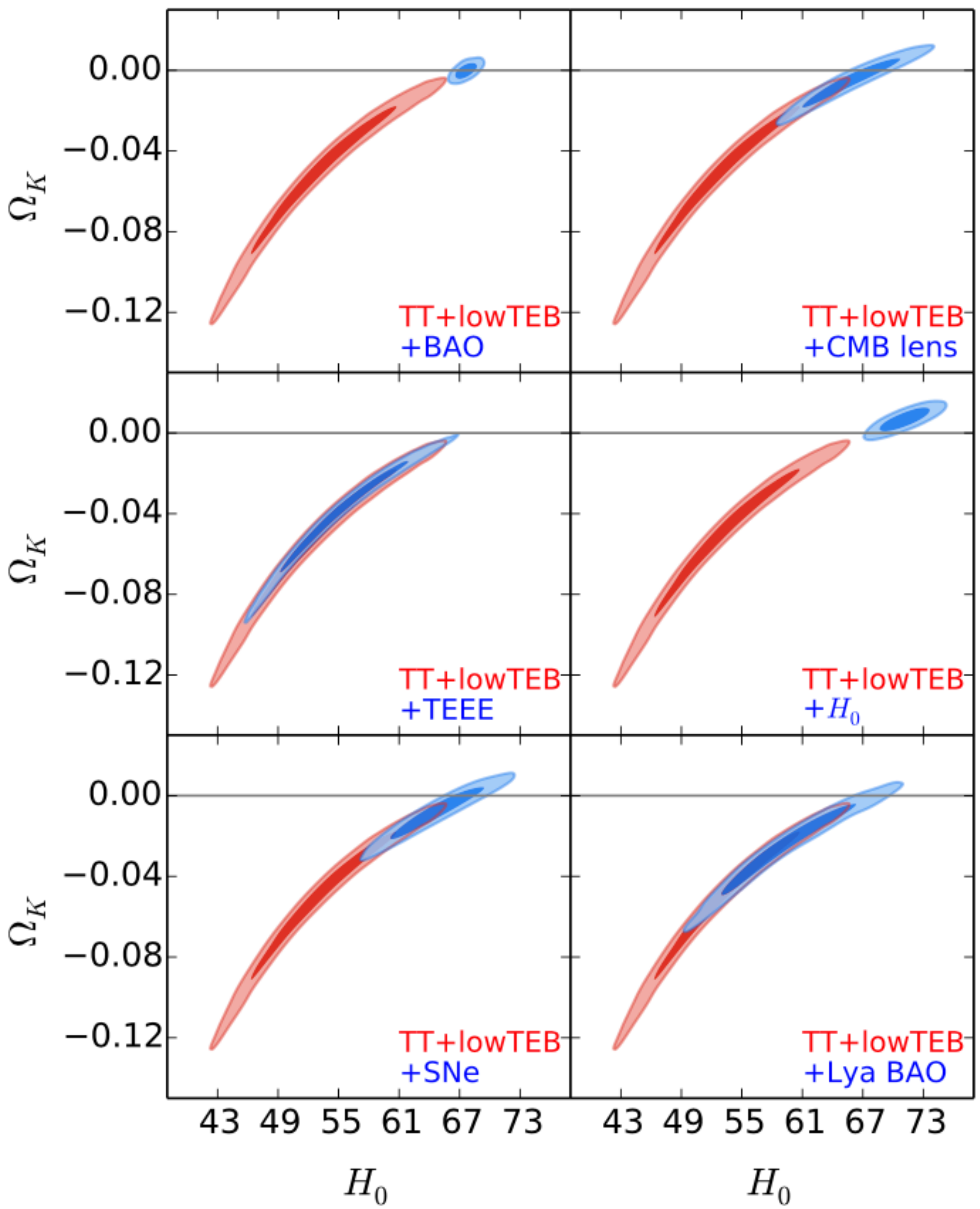
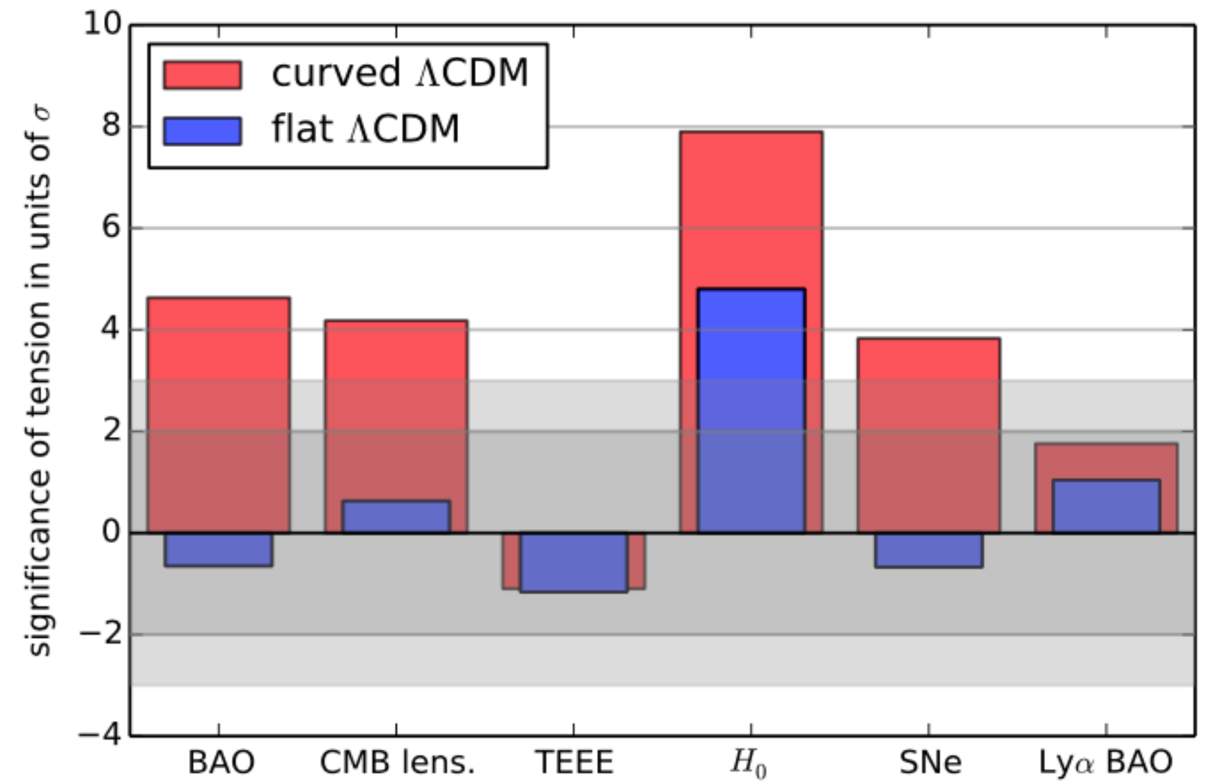
Quantifying Tensions II



Grandis et al. 2016b

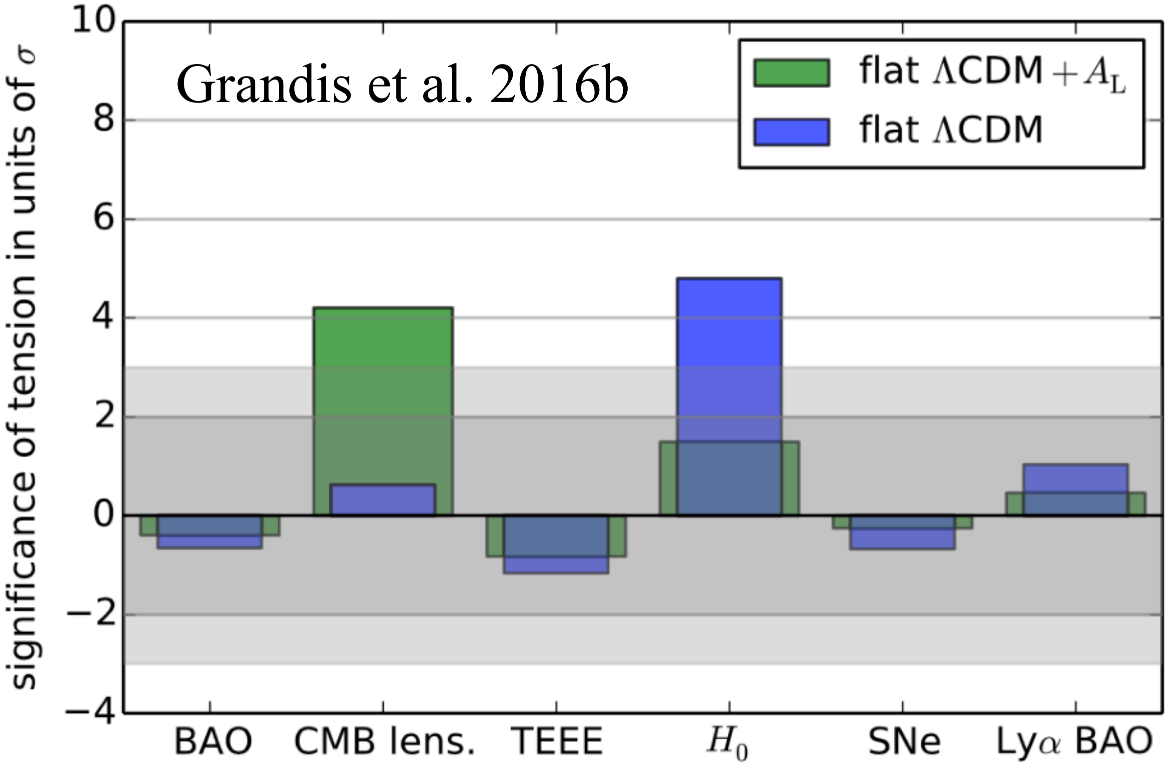


Grandis et al. 2016b

in curved ΛCDM:

- Significant tensions between CMB and distance measures, especially distance ladder
- Significant tension between CMB and CMB lensing

Quantifying Tensions II



for base ΛCDM is $A_L = 1$. The results of such an analysis for models with variable A_L is shown in Fig. 12. The marginalized constraint on A_L is

$$A_L = 1.22 \pm 0.10 \quad (68\%, \textit{Planck}\ \mathrm{TT+lowP}). \qquad (22)$$

This is very similar to the result from the 2013 *Planck* data reported in PCP13. The persistent preference for $A_L > 1$ is discussed in detail there. For the 2015 data, we find that $\Delta\chi^2 = -6.4$ between the best-fitting ΛCDM+A_L model and the best-fitting base ΛCDM model.

Planck Collaboration 2015, cosmological parameters

<u>in flat ΛCDM + A_L:</u>

- no tensions between CMB and distance measures
- agreement between CMB and H_0, contrary to flat ΛCDM

$\Downarrow$

H_0 problem

- tension between CMB and CMB lensing (only other probe sensitive to A_L)

$\Downarrow$

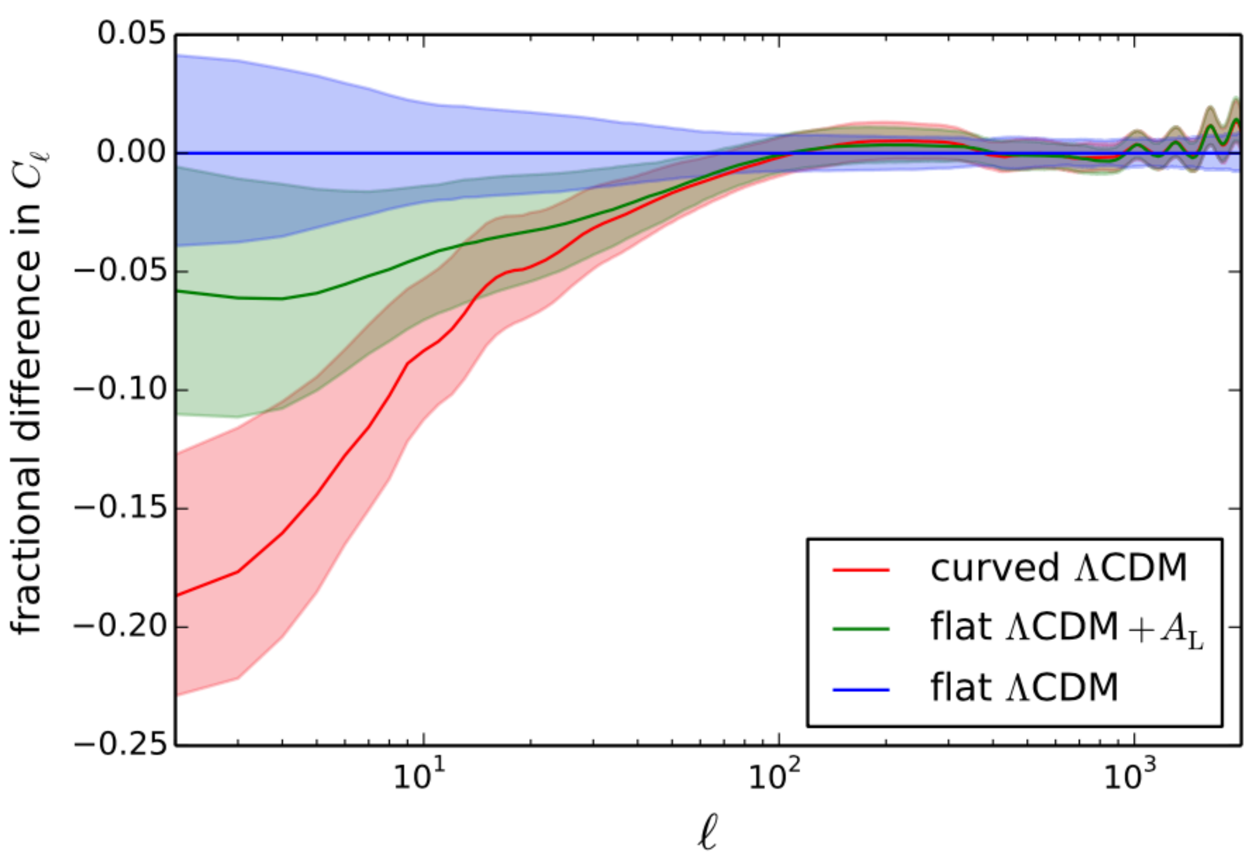
A_L problem

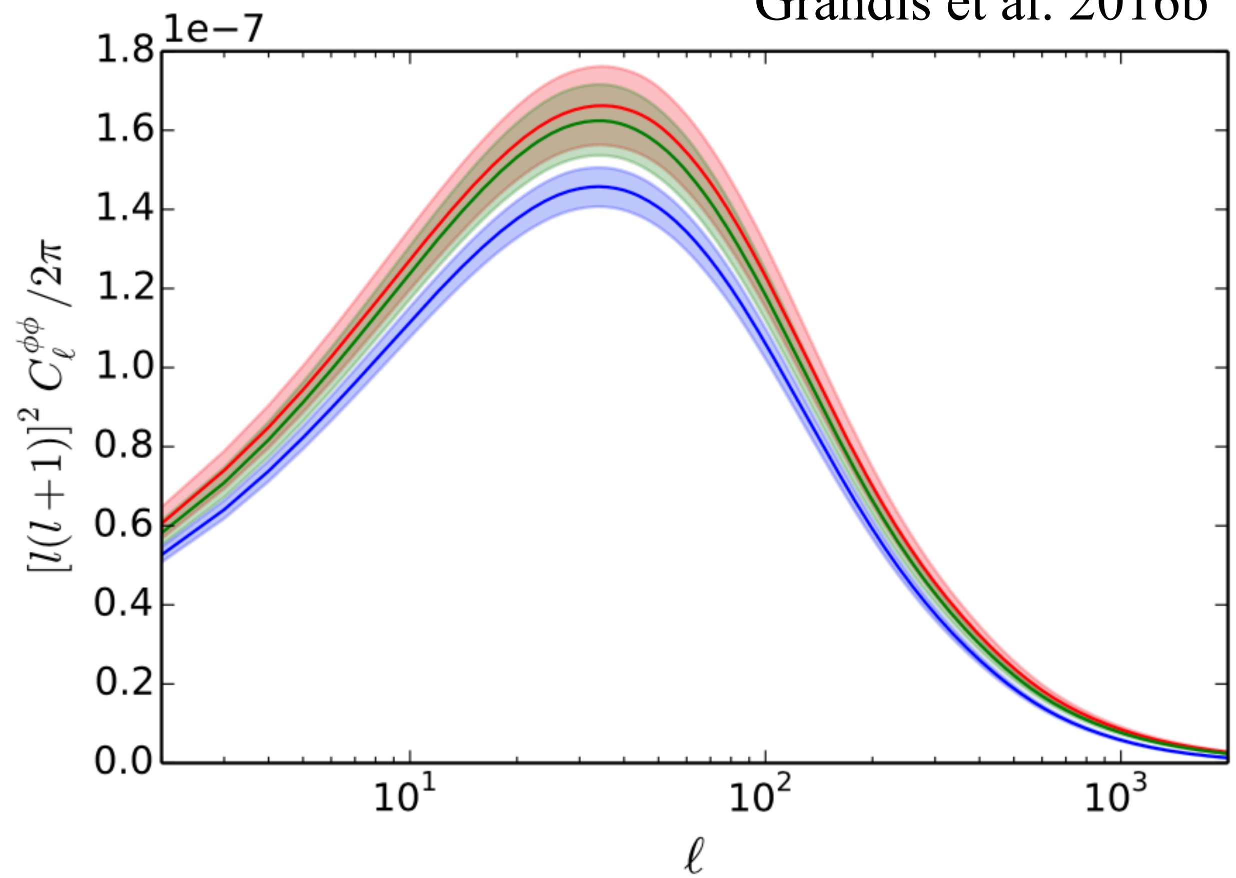
Quantifying Tensions II

What drives $\Omega_K < 0$ and $A_L > 1$?

Check constraints put by Planck 15 temperature data on CMB power spectrum and lensing potential in these models

Grandis et al. 2016b

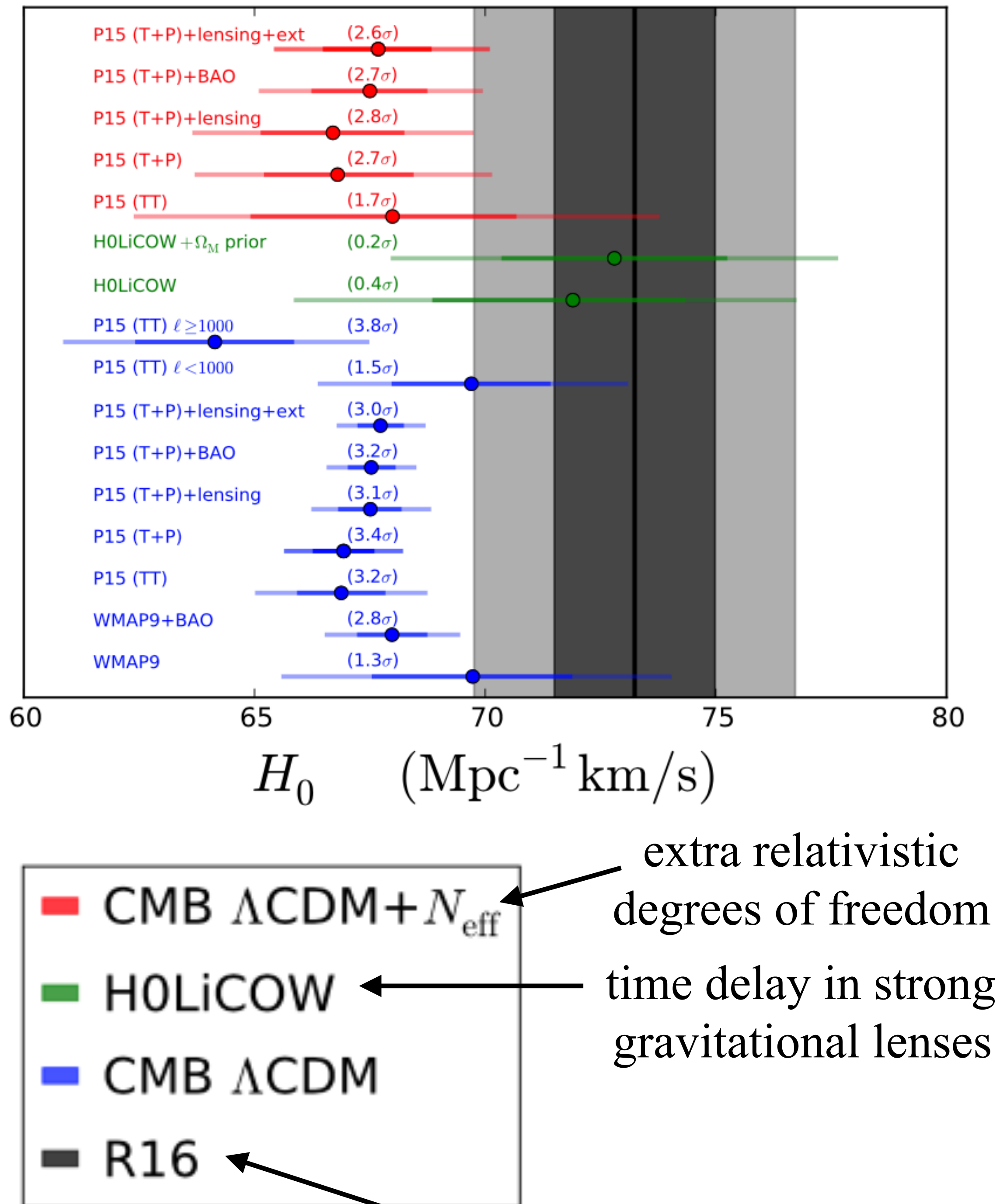




- less power on large scales preferred
- larger lensing potential

<u>Possibilities:</u> modified gravity, large scale anomaly, systematic effect, …

The H_0 Problem



Bernal et al 16

Tension between local measurements and CMB measurements of H_0

- variety of physical explanations attempted, e.g. N_{eff} e.g. Riess et al 16
- systematic effects in Cepheids calibration proposed e.g. Efstathiou et al 14
- exacerbated by free curvature
- solved by $A_{\mathrm{L}} > 1$

  $\Longrightarrow$ unphysical in Standard Model

  $\Longrightarrow$ creates A_{L} problem

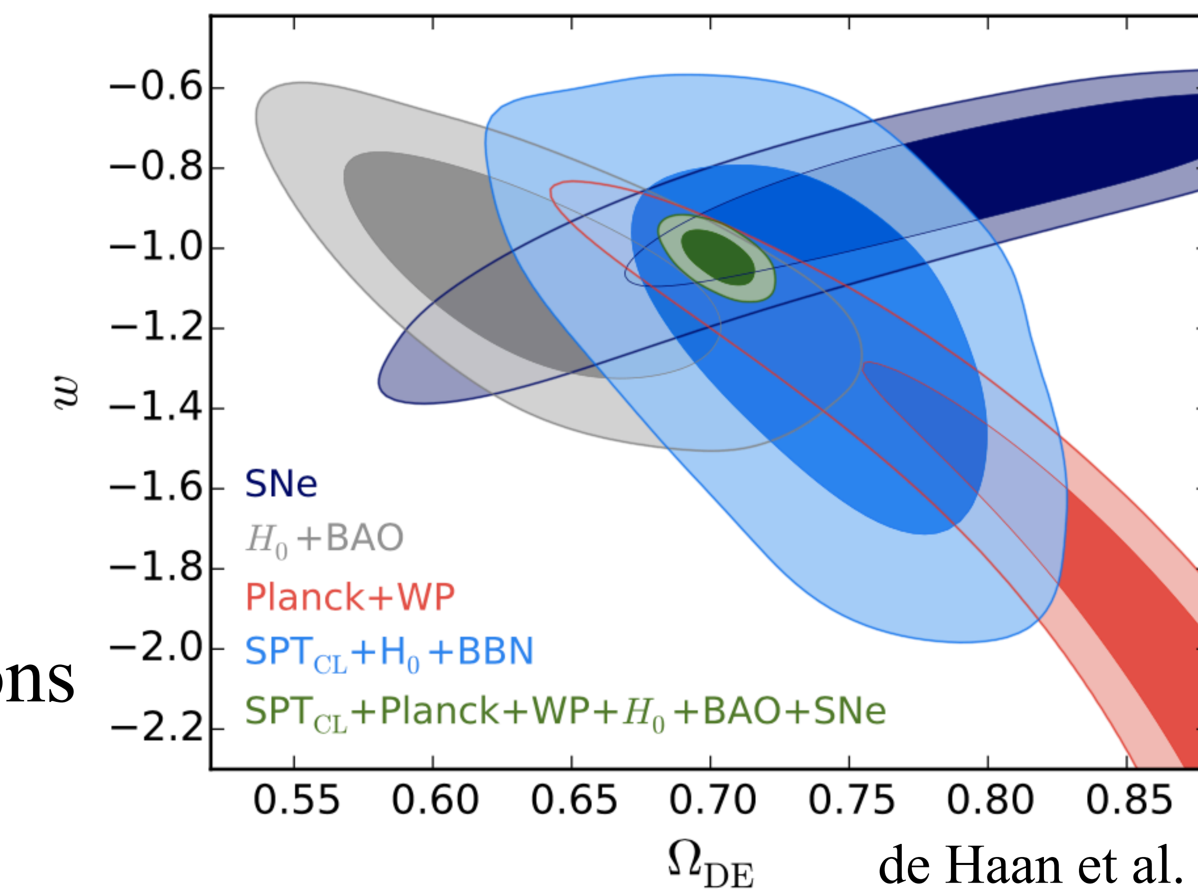
Conclusions

- The Surprise is a quantitative, information theoretically motivated measure of the agreement of datasets
- The Surprise can only be estimated for Gaussian constraints

  due to need of calibration, also true for other measures
  Seehars et al. 2015, SG et al. 2016b
- Some distributions can be "made" Gaussian with appropriate transformation
- current Gaussianising transformation have issues with hard cuts, flat distributions, especially if correlated

  ⇓

  need more flexible transformations



de Haan et al. 2016

Conclusions

A_{L}, σ_8, and H_0 problem persist

$\Rightarrow$ new physical models necessary?

- impact of new models larger when prediction code is provided, e.g. CAMB, Class

Ideally with python wrapper :)

$\Rightarrow$ unresolved systematic effects

$\Rightarrow$ large amounts of new data from ongoing and planned surveys

Growing necessity to check quantitatively for possible tensions in different models

Thank you for your attention!!

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Joël Akereth, Giulia Chirivì,
the anonymous reviewers

Bibliography

Seehars, SG, et al. 2015: http://adsabs.harvard.edu/abs/2016PhRvD..93j3507S

Grandis et al. 2016a: http://adsabs.harvard.edu/abs/2016JCAP...05..034G

Grandis at al. 2016b: http://adsabs.harvard.edu/doi/10.1093/mnras/stw2028

and references therein

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