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Spontaneous symmetry breaking in particle physics: some history and a look to the future

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Higgs hunting, July 24, 2017

Abstract

In this talk I will review my collaboration with Yoichiro Nambu on the first model of fermion mass generation based on the idea of spontaneous symmetry breaking (SSB) and providing a non-trivial example of the associated zero mass boson. I will then consider symmetry breaking in non-equilibrium states pointing out features that may be of interest in connection with the matter-antimatter asymmetry in the universe.

Spontaneous (dynamical) symmetry breaking

L. Euler, Memoires de l'Academie des Sciences de Berlin, **13**, *252 (1759)*

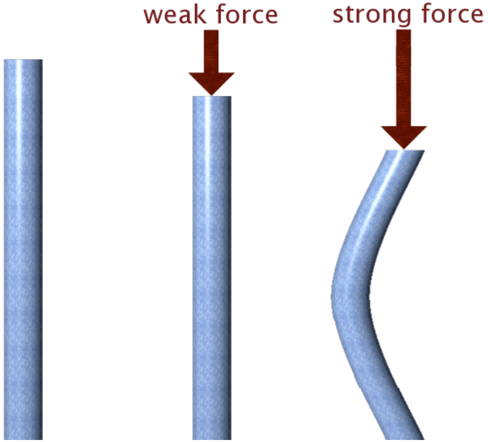


Figure: Elastic rod compressed by a force of increasing strength

Other examples

physical system	broken symmetry
ferromagnets	rotational invariance
crystals	translational invariance
superconductors	local gauge invariance
superfluid ^4He	global gauge invariance

When spontaneous symmetry breaking takes place, the ground state of the system is degenerate

Heisenberg 1959 - 1960

W. Heisenberg, W. H. Dürr, H. Mitter, S. Schlieder, K. Yamazaki, Zeit. f. Naturf. **14**, *441 (1959); W. Heisenberg, Proceedings of the 1960 Rochester Conference*

> *It has been emphasized that non-conservation of isospin in the electromagnetic forces must be due to an asymmetry of the ground state "world". The "world" possesses a very big total isospin, and this ground state is therefore highly degenerate.*
>
> *.... here we have come to a very general mathematical problem, and we should now get accustomed to those problems in which the lowest state is degenerate. There are many different problems in physics that belong to this class. If on develops a new mathematical theory to deal with this class of problems, in the near future we may make great progress in the theory of elementary particles.*

Quasi-particles in superconductivity

Electrons near the Fermi surface are described by the following equation

$$\begin{aligned} E\psi_{p,+} &= \epsilon_p \psi_{p,+} + \phi \psi^{\dagger}_{-p,-} \\ E\psi^{\dagger}_{-p,-} &= -\epsilon_p \psi^{\dagger}_{-p,-} + \phi \psi_{p,+} \end{aligned}$$

with eigenvalues

$$E = \pm\sqrt{\epsilon_p^2 + \phi^2}$$

Here, $\psi_{p,+}$ and $\psi^{\dagger}_{-p,-}$ are the wavefunctions for an electron and a hole of momentum p and spin $+$

Analogy with the Dirac equation

In the Weyl representation, the Dirac equations reads

$$\begin{aligned} E\psi_1 &= \boldsymbol{\sigma} \cdot \boldsymbol{p}\psi_1 + m\psi_2 \\ E\psi_2 &= -\boldsymbol{\sigma} \cdot \boldsymbol{p}\psi_2 + m\psi_1 \end{aligned}$$

with eigenvalues

$$E = \pm\sqrt{p^2 + m^2}$$

Here, ψ_1 and ψ_2 are the eigenstates of the chirality operator γ_5

Zero mass boson in superconductivity

Y. Nambu, Phys. Rev. **117***, 648 (1960)*

Approximate expressions for the charge density and the current associated to a quasi-particle in a BCS superconductor

$$\begin{aligned} \rho(x,t) &\simeq \rho_0 + \frac{1}{\alpha^2}\partial_t f \\ \boldsymbol{j}(x,t) &\simeq \boldsymbol{j}_0 - \boldsymbol{\nabla} f \end{aligned}$$

where $\rho_0 = e\Psi^\dagger\sigma_3 Z\Psi$ and $\boldsymbol{j}_0 = e\Psi^\dagger(\boldsymbol{p}/m)Y\Psi$ with Y, Z and α constants and f satisfies the wave equation

$$\left(\nabla^2 - \frac{1}{\alpha^2}{\partial_t}^2\right) f \simeq -2e\Psi^\dagger\sigma_2\phi\Psi$$

Here, $\Psi^\dagger = (\psi_1^\dagger, \psi_2)$

The Goldstone theorem

J. Goldstone, Nuovo Cimento **19**, 154 (1961)

Whenever the original Lagrangian has a continuous symmetry group, which does not leave the ground state invariant, massless bosons appear in the spectrum of the theory.

physical system	broken symmetry	massless bosons
ferromagnets	rotational invariance	spin waves
crystals	translational invariance	phonons

Plasmons

The Fourier transform of the wave equation for f gives

$$\tilde{f} \propto \frac{1}{q_0^2 - \alpha^2 q^2}$$

The pole at $q_0^2 = \alpha^2 q^2$ describes the excitation spectrum of the zero-mass boson.

A better approximation reveals that, due to the Coulomb force, this spectrum is shifted to the plasma frequency $e^2 n$, where n is the number of electrons per unit volume. In this way the field f acquires a mass.

The axial vector current

Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960)

Electromagnetic current		Axial current
	$\Longleftrightarrow$	
$\bar{\psi}\gamma_\mu\psi$		$\bar{\psi}\gamma_5\gamma_\mu\psi$

The axial current is the analog of the electromagnetic current in BCS theory. In the hypothesis of exact conservation, the matrix elements of the axial current between nucleon states of four-momentum p and p' have the form

$$\Gamma_\mu^A(p', p) = \left(i\gamma_5\gamma_\mu - 2m\gamma_5 q_\mu/q^2\right) F(q^2) \qquad q = p' - p$$

Conservation is compatible with a finite nucleon mass m provided there exists a massless pseudoscalar particle, the Nambu-Goldstone boson.

In Nature, the axial current is only approximately conserved. Nambu's hypothesis was that the small violation of axial current conservation gives a mass to the N-G boson, which is then identified with the π meson. Under this hypothesis, one can write

$$\Gamma^A_\mu(p', p) \simeq \left(i\gamma_5\gamma_\mu - \frac{2m\gamma_5 q_\mu}{q^2 + m_\pi^2} \right) F(q^2) \qquad q = p' - p$$

This expression implies a relationship between the pion nucleon coupling constant G_π, the pion decay coupling g_π and the axial current β-decay constant g_A

$$2mg_A \simeq \sqrt{2} G_\pi g_\pi$$

This is the Goldberger–Treiman relation

An encouraging calculation

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961), Appendix

It was experimentally known that the ratio between the axial vector and vector β-decay constants $R = g_A/g_V$ was slightly greater than 1 and about 1.25. The following two hypotheses were then natural:

1. under strict axial current conservation there is no renormalization of g_A;
2. the violation of the conservation gives rise to the finite pion mass as well as to the ratio $R > 1$ so that there is some relation between these quantities.

Under these assumptions a perturbative calculation gave a value of R close to the experimental one. More important, the renormalization effect due to a positive pion mass went in the right direction.

The Nambu–Jona-Lasinio (NJL) model

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, *345 (1961)*

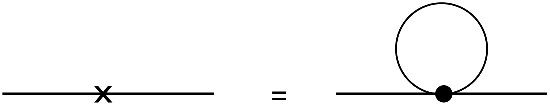
The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi + g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

It is invariant under ordinary and γ_5 gauge transformations

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$
$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

Mean field approximation



$$m = -\frac{g_0 m i}{2\pi^4} \int \frac{d^4 p}{p^2 - m^2 - i\varepsilon} \; F(p, \Lambda)$$

The spectrum of the NJL model

Mass equation

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{m^2}{\Lambda^2} \ln\left(1 + \frac{\Lambda^2}{m^2}\right)$$

where Λ is the invariant cut-off

Spectrum of bound states

nucleon number	mass μ	spin-parity	spectroscopic notation
0	0	0^-	${}^1S_0$
0	$2m$	0^+	${}^3P_0$
0	$\mu^2 > \frac{8}{3}m^2$	1^-	${}^3P_1$
± 2	$\mu^2 > 2m^2$	0^+	${}^1S_0$

The effective action

G. Jona-Lasinio, Nuovo Cimento **34**, *1790 (1964)*

Let $G[J]$ be the generator of the vacuum expectation values (in statistical mechanics G is the free energy in the presence of an external field J)

$$\frac{\delta G}{\delta J} = \langle \Phi \rangle = \phi$$

The effective action is the dual functional $\Gamma[\phi]$ defined by the Legendre transformation

$$\frac{\delta \Gamma}{\delta \phi} = -J$$

The vacuum of the theory is defined by the variational principle

$$\frac{\delta\Gamma}{\delta\phi} = 0$$

$\Gamma[\phi]$ is the generator of the one-particle irreducible amplitudes and can be constructed by simple diagrammatic rules. Its general form is

$$\Gamma[\phi] = L_{\mathsf{cl}}[\phi] + \hbar Q[\phi]$$

Broken symmetry and the mass of gauge vector mesons

P. W. Anderson, Phys. Rev. **130**, 439 (1963)
F. Englert, R. Brout, Phys. Rev. Lett. **13**, 321 (1964)
P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964)

A simple example (Englert, Brout). Consider a complex scalar field $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$ interacting with an abelian gauge field A_μ

$$H_{\text{int}} = ieA_\mu \varphi^\dagger \overset{\leftrightarrow}{\partial}_\mu \varphi - e^2 \varphi^\dagger \varphi A_\mu A_\mu$$

If the vacuum expectation value of φ is $\neq 0$, e.g. $\langle\varphi\rangle = \langle\varphi_1\rangle/\sqrt{2}$, the polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory is

$$\Pi_{\mu\nu}(q) = (2\pi)^4 ie^2 \langle\varphi_1\rangle^2 \left[g_{\mu\nu} - \left(q_\mu q_\nu / q^2\right)\right]$$

Therefore the A_μ field acquires a mass $\mu^2 = e^2\langle\varphi_1\rangle^2$ and gauge invariance is preserved, $q_\mu \Pi_{\mu\nu} = 0$.

Electroweak unification

S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interaction. What could be more natural than to unite these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and the electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum.

The NJL model as a low-energy effective theory of QCD

e.g. T. Hatsuda, T. Kunihiro, Phys. Rep. **247**, 221 (1994)

The NJL model has been reinterpreted in terms of quark variables. One is interested in the low energy degrees of freedom on a scale smaller than some cut-off $\Lambda \sim 1$ Gev. The short distance dynamics above Λ is dictated by perturbative QCD and is treated as a small perturbation. Confinement is also treated as a small perturbation. The total Lagrangian is then

$$L_{\mathsf{QCD}} \simeq L_{\mathsf{NJL}} + L_{\mathsf{KMT}} + \varepsilon \left(L_{\mathsf{conf}} + L_{\mathsf{OGE}} \right)$$

where the Kobayashi–Maskawa–'t Hooft term

$$L_{\mathsf{KMT}} = g_D \det_{i,j} \left[\bar{q}_i (1 - \gamma_5) q_j + \text{h.c.} \right]$$

mimics the axial anomaly and L_{OGE} is the one gluon exchange potential.

Other examples of BCS type SSB

- ^{3}He superfluidity
- Nuleon pairing in nuclei
- Fermion mass generation in the electro-weak sector of the standard model?

Nambu used to call the last entry *my biased opinion*

The mass hierarchy problem

Y. Nambu, *Masses as a problem and as a clue*, May 2004

- Unlike the internal quantum numbers like charge and spin, mass is not quantized in regular manner
- Mass receives contributions from interactions. In other words, it is dynamical.
- The masses form hierarchies. Hierarchical structure is an outstanding feature of the universe in terms of size as well of mass. Elementary particles are no exception.

Hierarchical spontaneous symmetry breaking

Y. Nambu, *Masses as a problem and as a clue*, May 2004

The BCS mechanism is most relevant to the mass problem because introduces an energy (mass) gap for fermions, and the Goldstone and Higgs modes as low-lying bosonic states. An interesting feature of the SSB is the possibility of hierarchical SSB or "tumbling". Namely an SSB can be a cause for another SSB at lower energy scale.

... [examples are]

1. the chain crystal–phonon–superconductivity. ... Its NG mode is the phonon which then induces the Cooper pairing of electrons to cause superconductivity.

2. the chain QCD–chiral SSB of quarks and hadrons–π and σ mesons–nuclei formation and nucleon pairing–nuclear π and σ modes–nuclear collective modes.

Symmetry breaking in non-equilibrium

SSB has been studied so far mainly as an equilibrium phenomenon typical of systems with infinitely many degrees of freedom. It was discovered however some time ago that out of equilibrium SSB can take place through mechanisms not available in equilibrium: currents are flowing through the system and their dynamics is crucial.

Stationary states are the obvious generalization of equilibrium states but the conditions under which SSB takes place in nonequilibrium are different from equilibrium. In stationary nonequilibrium states SSB may be possible even when it is not permitted in equilibrium.

A toy model

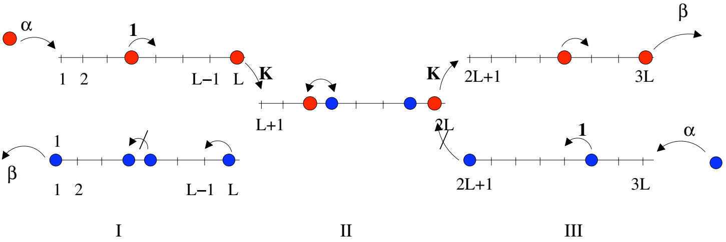


Figure 1. The bridge model with two junctions. Positively (negatively) charged particles hop to the right (left). The model is invariant with respect to left–right reflection and charge inversion. Section II is the bridge. It contains positive and negative particles and holes. Sections I and III comprise parallel segments each containing pluses and holes or minuses and holes.

Summarizing the dynamics, during a time interval dt three types of exchange events can take place between two adjacent sites

$$+0 \to 0+ \, , \quad 0- \to -0 \, , \quad +- \to -+ \, , \tag{1}$$

with probability dt. The last one takes place only on the bridge. At the left of the access lane of plus particles we have

$$0 \to + \, , \tag{2}$$

with probability αdt. At the right end of the exit lane of plus particles

$$+ \to 0 \, , \tag{3}$$

with probability βdt, and similarly for minus particles after reflection.

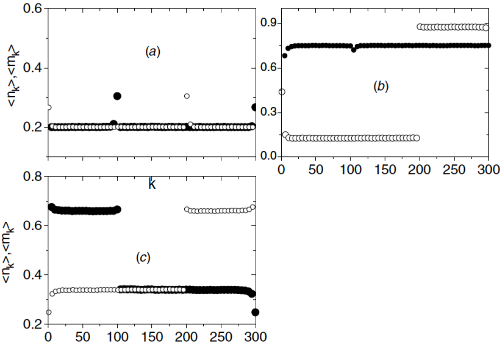


Figure 3. Average density profiles for pluses and minuses, from Monte Carlo simulations, in the LDS1 phase (panel (*a*)), in the SSB phase with pluses majority (panel (*b*)) and in the LDS2 phase (panel (*c*)). Pluses (minuses) correspond to closed (open) circles. The system of 300 sites was equilibrated and then, averaging over 10^6 Monte Carlo steps was done. Parameters: (*a*) $\alpha = 0.2$, $\beta = 0.4$; (*b*) $\alpha = 0.2$, $\beta = 0.25$; (*c*) $a = 0.8$, $\beta = 0.9$.

The existence of two SSB steady states can be easily established in mean field approximation for appropriate values of the rates $\alpha > \beta$. The SSB states are connected by the CP operation.

When the size L is finite the system flips between the two states. The flipping time τ_{flip} can be estimated

$$\tau_{flip} \simeq \exp \kappa L \quad \kappa = 2 \log \frac{\alpha(1-\alpha)}{\beta(1-\beta)}$$

Nambu's reaction

"..... The traffic jam problem is interesting. Recently I actually thought of the traffic problem while driving on Chicago Expressways. ...."

Why is nonequilibrium SSB interesting?

G. Jona-Lasinio, Progr. Theor. Phys. **124**, *731 (2010)*

There are facts in the world around us that so far have eluded a really satisfactory explanation.

At the planetary scale we know that in living matter left-handed chiral molecules are the rule.

At the cosmic scale matter is widespread and we do not see regions with antimatter.

Explanations have been proposed in both cases invoking initial small fluctuations which are amplified over a long nonequilibrium evolution to reach the present state.

Biological homochirality

M. Mauksch, S. B. Tsogoeva, *Life's Single Chirality: Origin of Symmetry Breaking in Biomolecules*, in *Biomimetic Organic Synthesis*, eds E. Poupon and B. Nay, Wiley, 2011.

> *...several competing theories are vying to explain the origin of bio- logical homochirality, which appears so central to life in the forms we are familiar with. The race between these different approaches is far from decided, as more theories and observations are reported. While some theories for the endogenous origin of homochirality stress a thermodynamic origin of enantioenrichment in the solution phase, others put more weight on mirror symmetry breaking kinetic mechanisms of asymmetric amplification building up upon initial imbalances in the enantiomeric compositions in homogeneous ensembles of chiral molecules..... We have apparently not reached yet a level of understanding of prebiotic chemistry that allows us to decide between alternative explanations.*

Baryogenesis

M. Shaposhnikov, J. Phys.: Conf. Ser. 171 (2009), 012005.

Baryogenesis gives a possible answer to the following question: Why there is no antimatter in the Universe? A (qualitative) solution to this problem is known already for quite some time: the Universe is charge asymmetric because it is expanding (the existence of arrow of time, in Sakharov's wording), baryon number is not conserved and the discrete CP-symmetry is broken. If all these three conditions are satisfied, it is guaranteed that some excess of baryons over anti-baryons will be generated in the course of the Universe evolution. However, to get the sign and the magnitude of the baryon asymmetry of the Universe (BAU) one has to understand the precise mechanism of baryon (B) and lepton (L) number non-conservation, to know exactly how the arrow of time is realized and what is the relevant source of CP-violation.

A non-equilibrium scenario

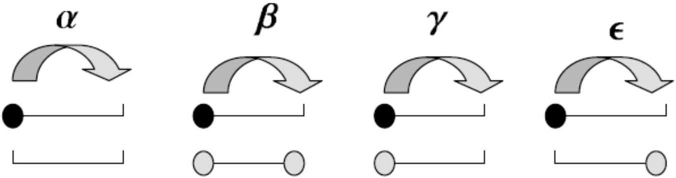
The idea is to view biological homochirality and/or baryogenesis as the outcome of a nonequilibrium phase transition.

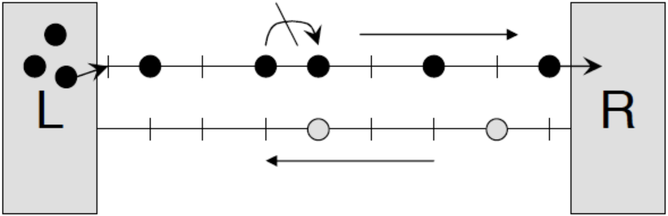
This means that if the nonequibrium conditions are such that the stable phase is the SSB phase very small perturbations can drive the system of interest to the stable state avoiding the difficulties of reconstructing a history with many uncertainties.

In the case of the Ising model, below the critical point a single oriented spin is sufficient to drive a macroscopic system to the corresponding ferromagnetic state.

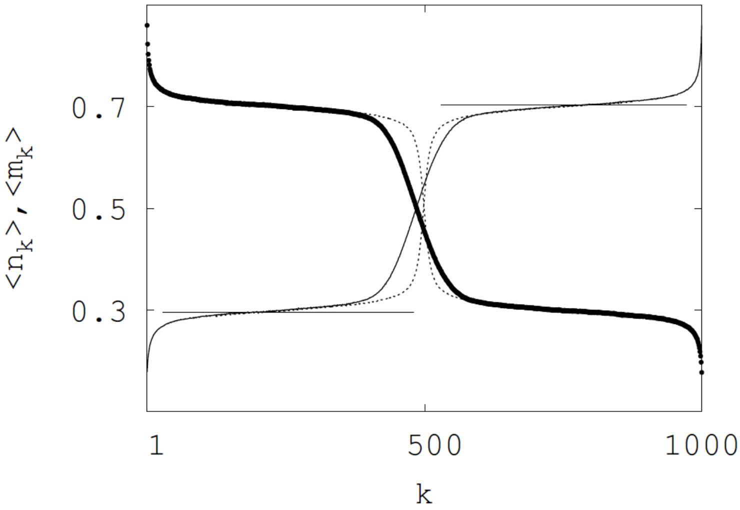
An alternative non-equilibrium scenario

The following model has been studied. The dynamics describes particles with repulsive hard-core interaction which hop unidirectionally along two chains of L sites: One chain for right-hopping particles and another chain for left-hopping particles. At each instant of time the system is fully described by occupation numbers n_k (for the right movers) and m_k (for the left-movers). A right-moving particle at site k can hop to its neighbouring site $k+1$ provided it is empty, with a rate that depends on the occupancies at sites $k, k+1$ on the adjacent chain.


α
β
γ
ϵ


L
R

For values of the interaction in a certain range one observes something very unusual and different. The bulk density profile becomes inhomogeneous and consists of two plateaux with an interface in the middle. The profiles of the two species are left-right symmetric but in each plateau the densities ρ_1, ρ_2 of the left and right movers are different. As the interaction becomes stronger, the difference $\rho_1 - \rho_2$ grows and reaches the maximum $\rho_1 - \rho_2 = 0.5$. Note that the asymmetry of the profile is not a result of a spontaneous symmetry breaking since the profiles are left-right symmetric and the stationary currents of both species remain equal.



0.7
0.5
0.3
$\langle n_k \rangle$, $\langle m_k \rangle$
1
500
1000
k

Some references on symmetry breaking in non-equilibrium

M. R. Evans, D. P. Foster, C. Godreche, D. Mukamel, Phys. Rev. Lett. **74**, 208 (1995)

C. Godreche, J. M. Luck, M. R. Evans, D. Mukamel, S. Sandow, E. R. Speer, J. Phys. A **28**, 6039 (1995)

V. Popkov, M. R. Evans, D. Mukamel, J. Phys. A **41**, 432002 (2008)

S. Gupta, D. Mukamel, G. M. Schütz, J. of Phys. A 42 (2009), 485002.

V. Popkov, G. M. Schütz, Phys. Rev. E 86, 031139 (2012).

V. Popkov, Eur. Phys. J. Special Topics 216, p. 139 (2013).