A METHOD FOR COMPUTATION OF RADIATION EMITTED BY ELECTRONS AND POSITRONS IN STRAIGHT AND BENT CRYSTALS

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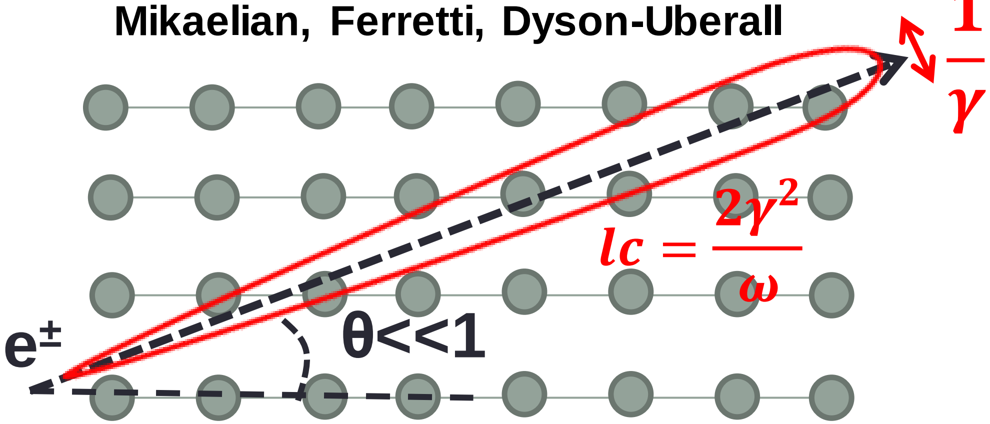
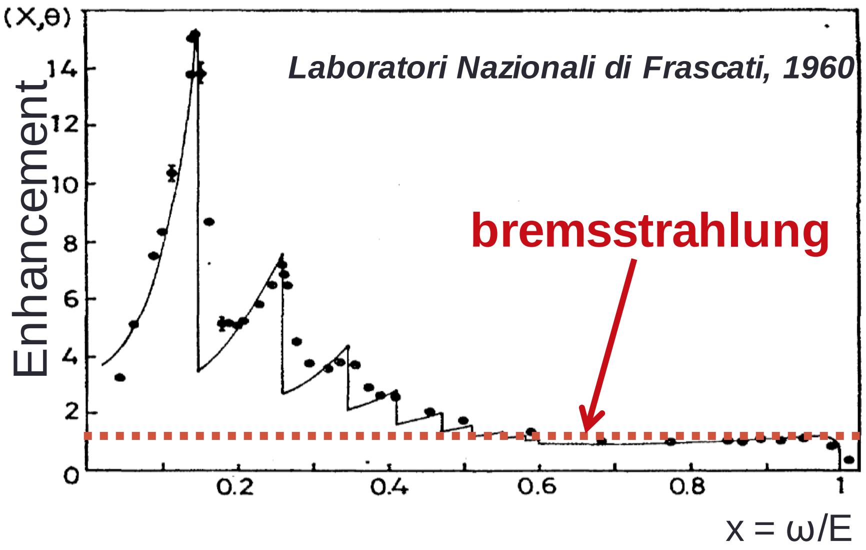
Outlook

- Introduction of radiation processes in crystals;
- Development of an algorithm to compute the radiation generation in oriented crystals based on the Baier and Katkov method;
- Comparison with experiments at intermediate energies (1 GeV $e^{\pm}$);
- Comparison with experiments at ultra-high energies (100 GeV $e^{\pm}$);

Enhancement of bremsstrahlung radiation in aligned crystals

Coherent Bremsstrahlung (1950s) Ter-Mikaelian, Ferretti, Dyson-Uberall





Enhancement of radiation in aligned crystals

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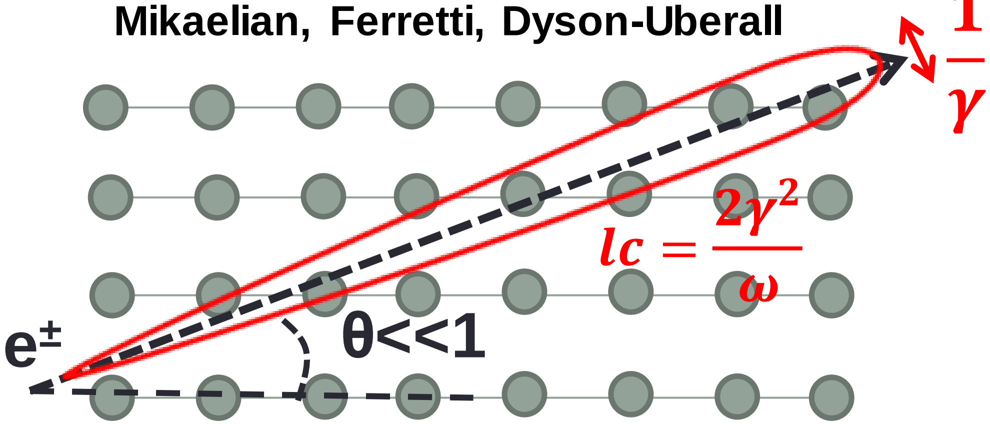
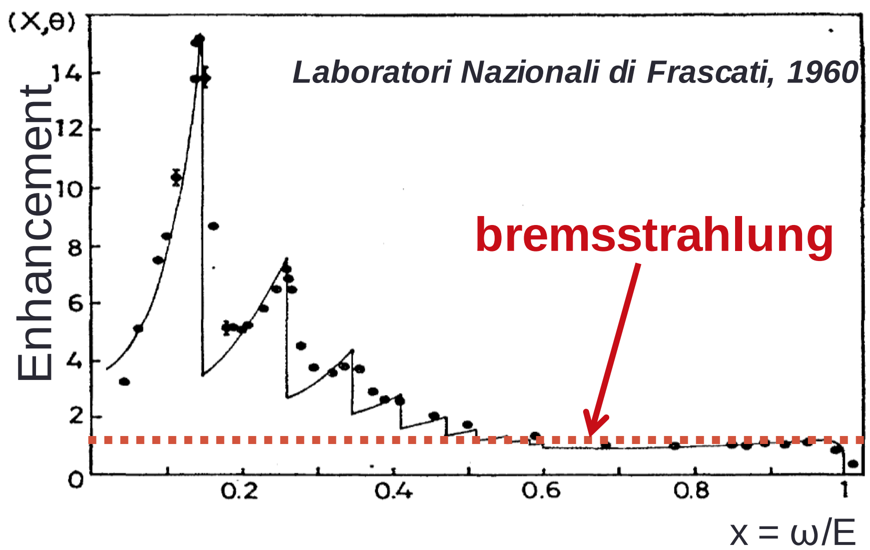
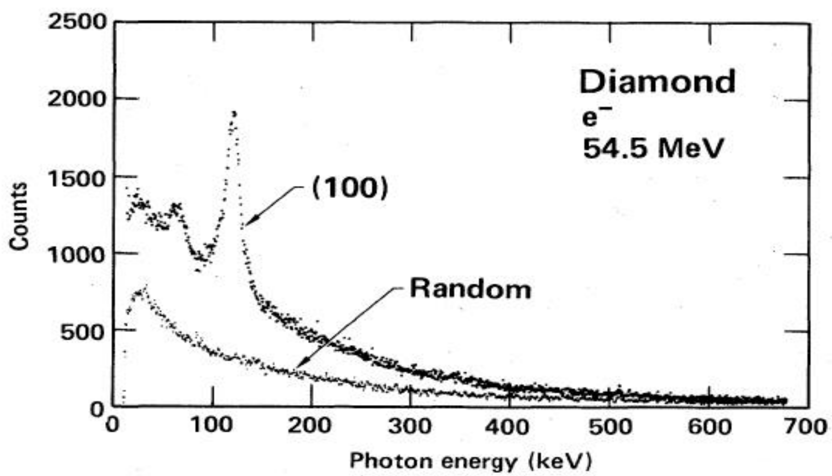
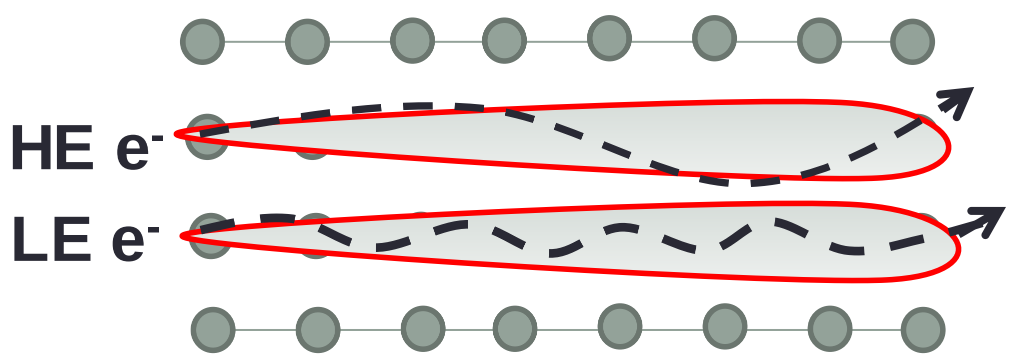






M. Kumakhov, Physics Letters A 57, 17 (1976).

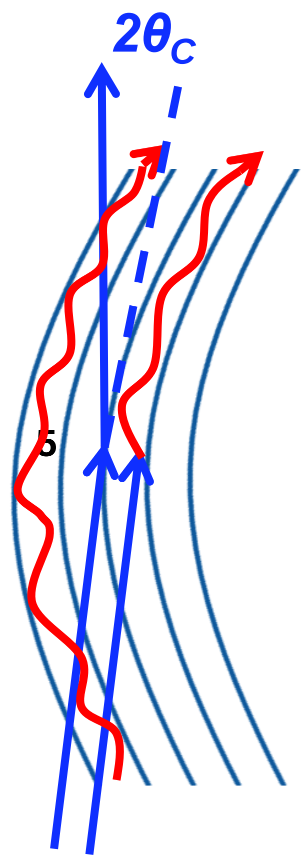


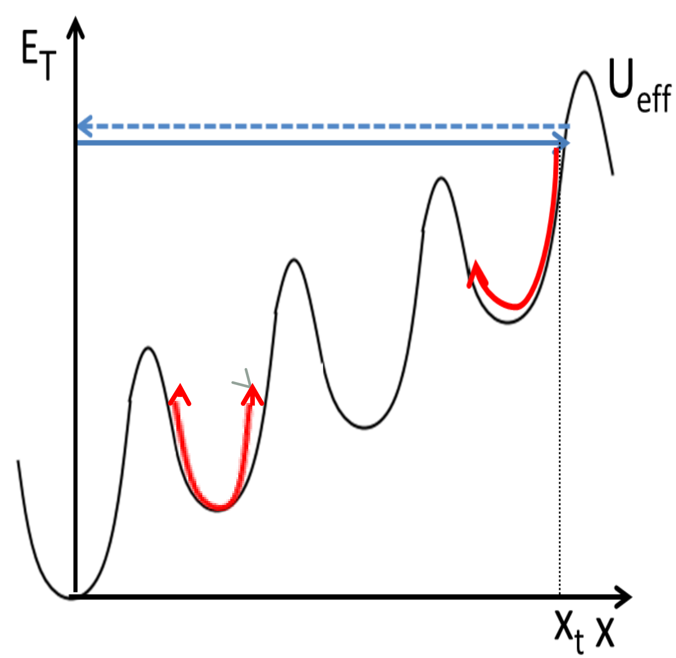
Channeling radiation (Kumakhov)

Channeling and related effects in a bent crystal

- ✓ **Tsyganov (1976):** channeling in bent crystals;
- ✓ **Taratin and Vorobiov (1987)**: Volume Reflection of over-barrier particles.

$$U_{eff}(x) = U(x) + \frac{pv}{R}x$$





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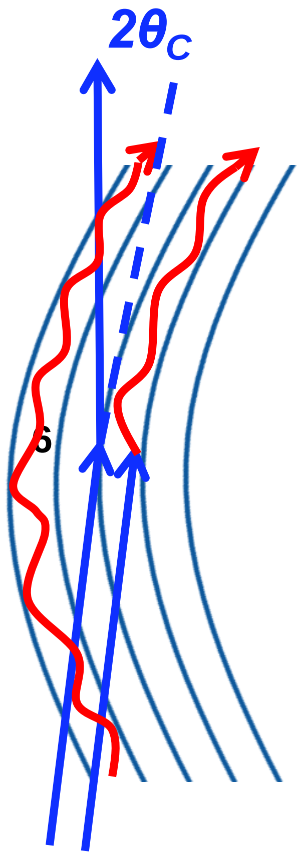
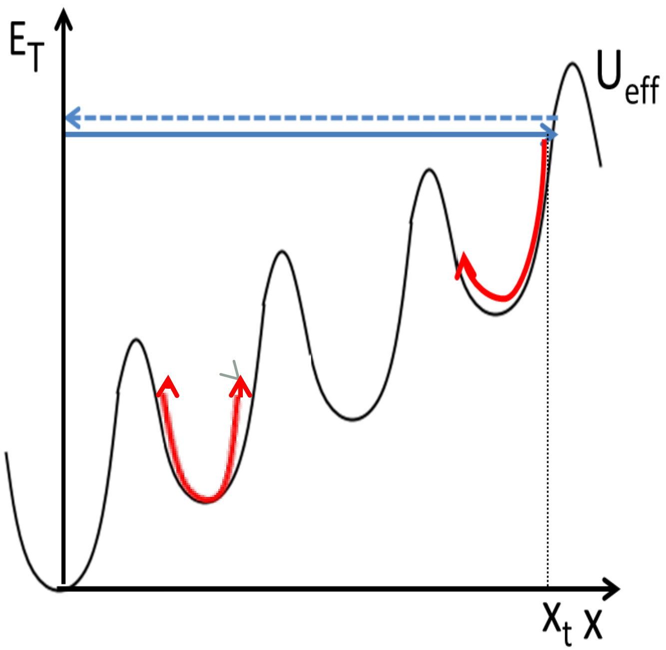
- ✓ **Taratin and Vorobiov (1987)**: <u>Volume Reflection </u>of over-barrier particles.

- Under VR **the angle between the particle trajectory and crystalline planes changes during the motion**
- **Need of a general method for radiation computation**

$$U_{eff}(x) = U(x) + \frac{pv}{R}x$$





Baier-Katkov quasiclassical operator method (1967-1968)

General method for calculation of radiation generated by $e^{\pm}$ in an external field

The electromagnetic radiated energy is evaluated with the BK formula:

$$\frac{dE}{d^3k} = \omega \frac{dN}{d^3k} \frac{\alpha}{4\pi^2} \iint dt_1 dt_2 \frac{\left[\left(E^2+E'^2\right)(v_1 v_2 - 1) + \omega^2/\gamma^2\right]}{2E'^2} e^{-ik'(x_1 - x_2)} \qquad (1)$$

where the integration is made over the classical trajectory.

The generality of the Baier-Katkov operator method permits to simulate the electromagnetic radiation emitted by e± in very different cases, e. g., straight, bent and periodically bent crystals, and for different beam energy range, from sub-GeV to TeV.

Baier-Katkov quasiclassical operator method (1967-1968)

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Why classical trajectory?

2 types of quantum effects :

- the quantization of particle motion $\sim\hbar\omega_0/E$
  In crystals: **negligible for electron/positron energy >10-100 MeV**
- the **quantum recoil** of the particle when it radiates a photon with energy $\hbar\omega \sim E$
  NOT negligible for electron/positron energy >50 GeV

An algorithm for radiation in crystals

Integration of the BK formula

SMALL ANGLE APPROXIMATION: Since the angle between particle trajectories and crystal planes or axes is small and at ultrarelativistic energies the radiation angle 1/γ is much smaller than unity the particle velocity **v** and photon momentum **k** can be represented in the form :

$$\mathbf{v}(t) \simeq \mathbf{v}_\perp(t) + \mathbf{e}_z \left[1 - 1/2\gamma^2 - v_\perp^2(t)/2\right],$$
$$\mathbf{k} = \mathbf{n}\omega \simeq \mathbf{e}_\perp \omega\theta + \mathbf{e}_z \omega \left(1 - \theta^2/2\right),$$

where the angle $\theta \ll 1$ represents the radiation angle. The formula (1) can be rewritten as:

$$\frac{dE}{d^3k} \sim \frac{\alpha}{8\pi^2}\frac{\varepsilon^2 + \varepsilon'^2}{\varepsilon'^2}\omega^2 C, \qquad (2)$$

where $$C = |\, \boldsymbol{I}_\perp \,|^2 + \gamma^{-2}\frac{\omega^2}{\varepsilon^2 + \varepsilon'^2}\, |\, J \,|^2 \qquad (3)$$

V. Guidi, L. Bandiera, V. Tikhomirov, Phys. Rev. A 86 (2012) 042903]

L. Bandiera, et al., Nucl. Instrum. Methods Phys. Res., Sect. B 355, 44 (2015).

An algorithm for radiation in crystals

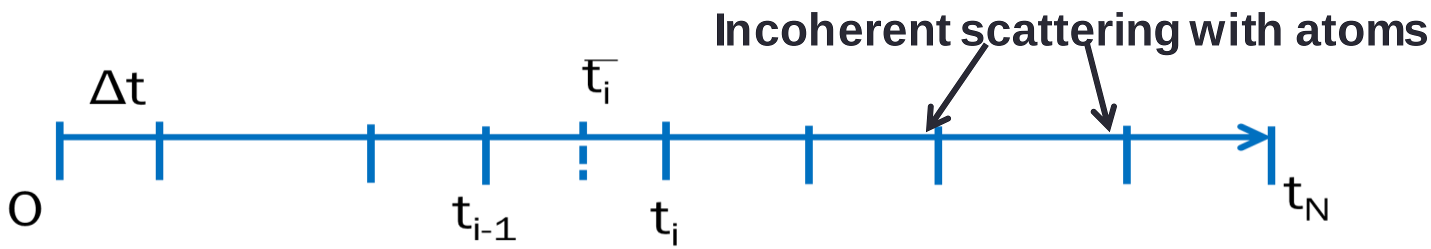
Integration of the BK formula

SMALL ANGLE APPROXIMATION: the integrals of eq. (1) can be represented as follows:

$$\begin{matrix} J \\ \boldsymbol{I}_\perp \end{matrix} = \int_{-\infty}^{+\infty} dt \begin{matrix} 1 \\ (v_\perp - \boldsymbol{\theta}) \end{matrix} e^{-i\phi(t)} \qquad (4)$$

being $\phi(t) = \frac{\omega'}{2} \int_{-\infty}^{t} dt' [\gamma^{-2} + (\boldsymbol{v}_\perp(t') - \boldsymbol{\vartheta})^2]$ and $\omega' = \omega\varepsilon/\varepsilon'$.

ACCOUNT OF INCOHERENT SCATTERING:

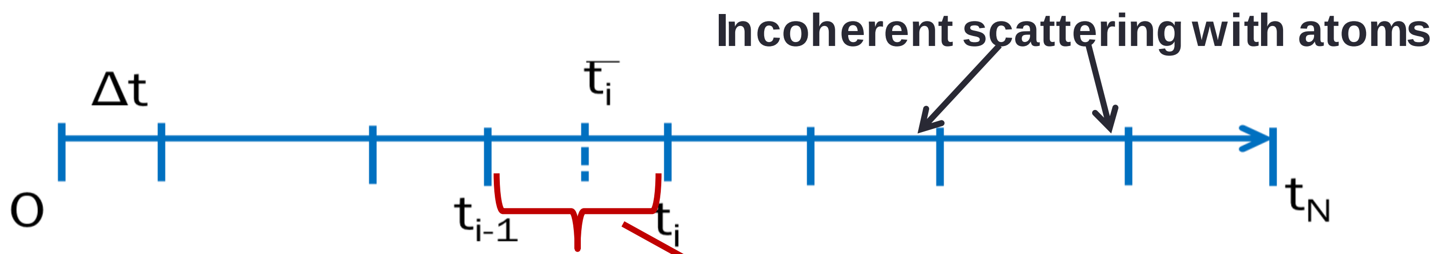


The **particle trajectory is then divided in N small steps**, within which the particle trajectory is calculated through the integration of equation of motion in the continuous potential. **At the end of each step the scattering by nuclei and electrons is sampled** and the transverse velocity for the i-step becomes

$$\mathbf{v}_{\perp,\mathbf{i}} \rightarrow \mathbf{v}_{\perp,\mathbf{i}} + \theta_{\mathbf{s},\mathbf{i}}$$

An algorithm for radiation in crystals

Integration of the BK formula



In order to improve the convergence of its integration over t and $\boldsymbol{\theta}$ (photon emission angle), the integrals of eq. 4 are computed as follows after an integration by parts:

$$J \approx i\sum_{i=1}^{N}\left\{\exp\left[i\phi(t_i)\right]\left[\frac{1}{\dot{\phi}_{t_i+0}}-\frac{1}{\dot{\phi}_{t_i-0}}\right]-\exp\left[i\phi(\bar{t}_i)\right]\left[\left.\frac{2\ddot{\phi}}{\dot{\phi}^3}\right|_{\bar{t}_i}\sin\left(\left[\phi(t_i-0)-\phi(t_{i-1}+0)\right]/2\right)\right]\right\}$$

If incoherent scattering is switched off, it is go to zero.

$$\dot{\phi}(t<t_i)=\frac{\omega'}{2}\left[1/\gamma^2+\left(\mathbf{v}_\perp(t)-\boldsymbol{\theta}\right)^2\right], \qquad \ddot{\phi}(t)=\omega'\left(\mathbf{v}_\perp(t)-\boldsymbol{\theta}\right)\dot{\mathbf{v}}_\perp(t),$$

$$\dot{\phi}(t_i+0)=\frac{\omega'}{2}\left[1/\gamma^2+\left(\mathbf{v}_\perp(t)+\theta_{\mathbf{s},\mathbf{i}}-\boldsymbol{\theta}\right)^2\right], \qquad \dot{\mathbf{v}}_\perp=-\frac{1}{\varepsilon}\frac{\partial U(\mathbf{r})}{\partial \mathbf{r}_\perp},\ U(\mathbf{r}) \text{ being the continuous potential.}$$

The contributions of the trajectory ends are not taken into account, thus neglecting the soft contribution of transition radiation.

The integration over θ leads to the radiation spectral intensity, ωdN/dω.

COMPARISON WITH EXPERIMENTS

GeV and sub-GeV energy range:

- No quantum correction;
- Thin crystals -> single photon emission.

RADCHARM++

The algorithm for direct integration of the BK formula has been included in the RADCHARM++ routine [1] , which is an expansion of the DYNECHARM++ code [2] (*see E. Bagli talk*)

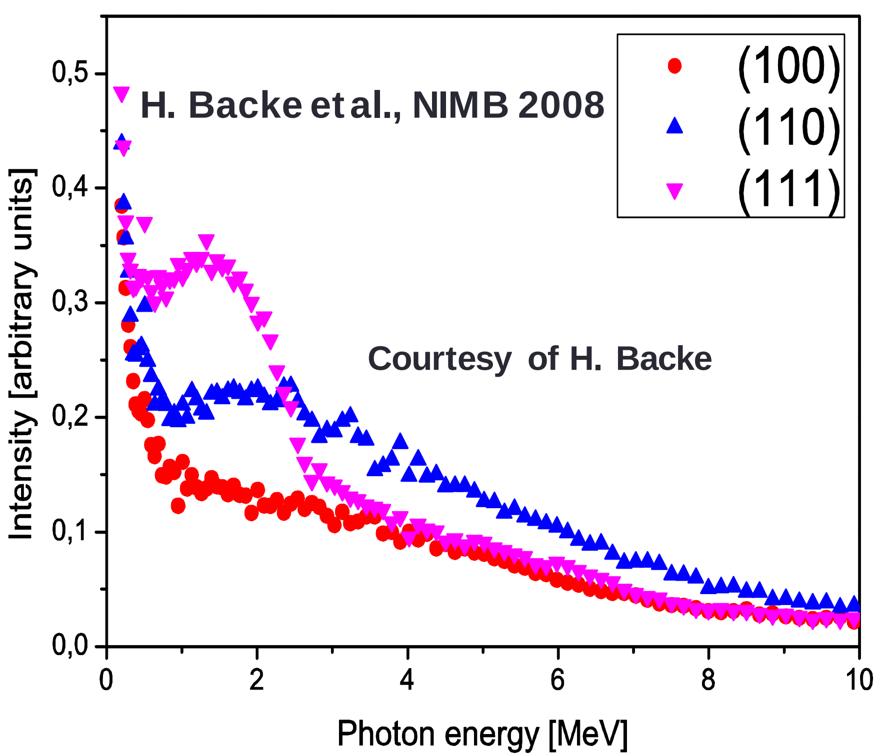
- The electrical characteristic of the crystal are evaluated by using the atomic form factors from x-ray diffraction data;
- Numerical integration of the classical equation of motion of particle trajectories under the continuum potential approximation;
- At the end of each step the multiple and single scattering by nuclei and electrons is sampled.

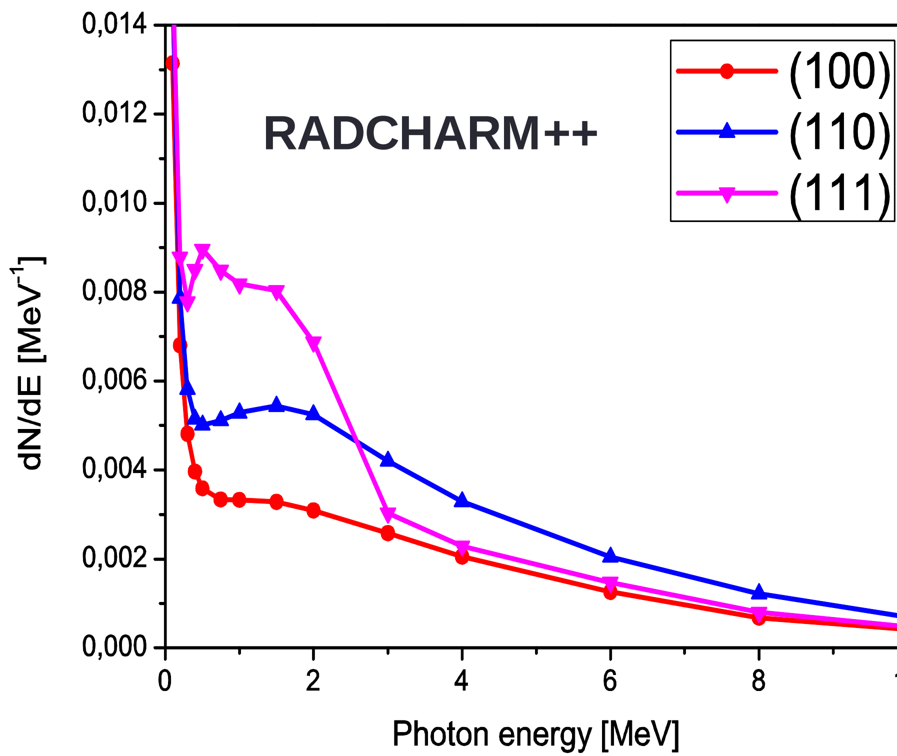
DYNECHARM++ has already been implemented in Geant4 [3]. The RADCHARM++ routine can also be implemented to include the bremsstrhalung radiation enhancement in crystals.

[1] L. Bandiera, et al., Nucl. Instrum. Methods Phys. Res., Sect. B 355, 44 (2015).
[2] E. Bagli, V. Guidi, Nucl. Instr. and Meth. in Phys. Res. Section B 309 (2013) 124
[3] E. Bagli, M. Asai, D. Brandt, et al. Eur. Phys. J. C (2014) 74: 2996.

Comparison with previous experiments: Simulation of e.m. radiation emitted by ultrarelativistic electrons in the field of any crystal plane

Comparison with past experiments performed at the Mainzer Mikrotron with 855 MeV electrons interacting with a 175 μm straight Si crystal



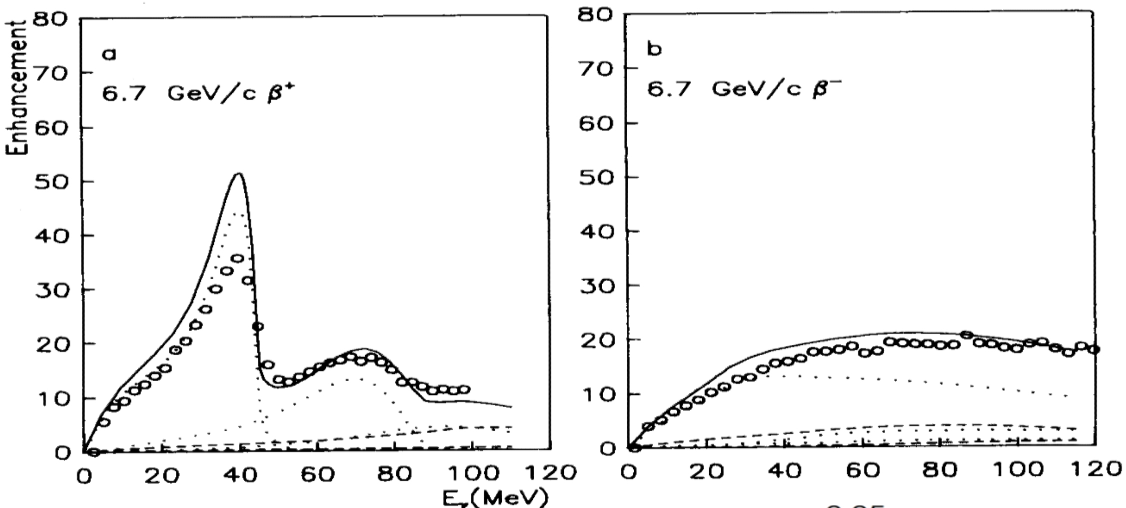


L. Bandiera, et al., Nucl. Instrum. Methods Phys. Res., Sect. B 355, 44 (2015).

Comparison with previous experiments:

Simulation of e.m. radiation emitted by both positive and negative particles

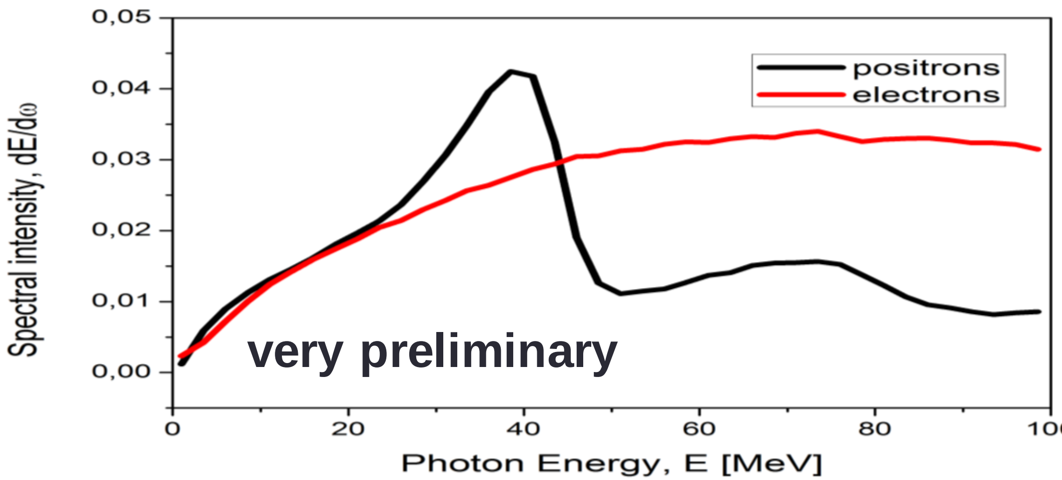
Comparison with past experiments at CERN: 6.7 GeV positrons/electrons channeling in a 0.1 mm thick Si (110)



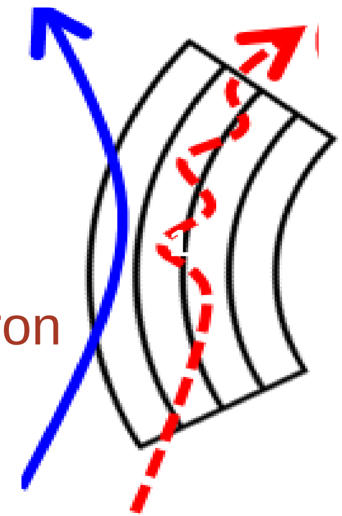
Enhancement with respect to random

J. Bak et al., Nucl.Phys. B254(1985)491.

Spectral Intensity, $dE/d\omega = \omega dN/d\omega$ (ideal case)



Experiment with bent crystals: Motivations

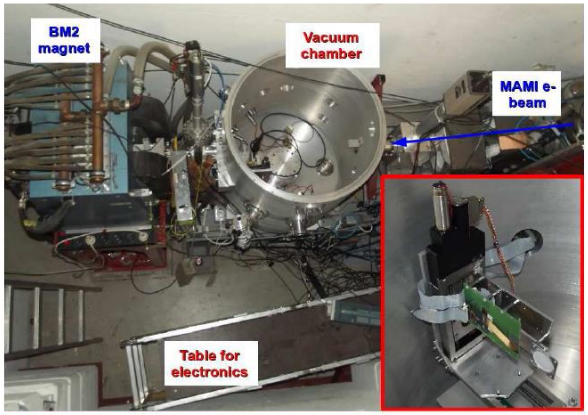


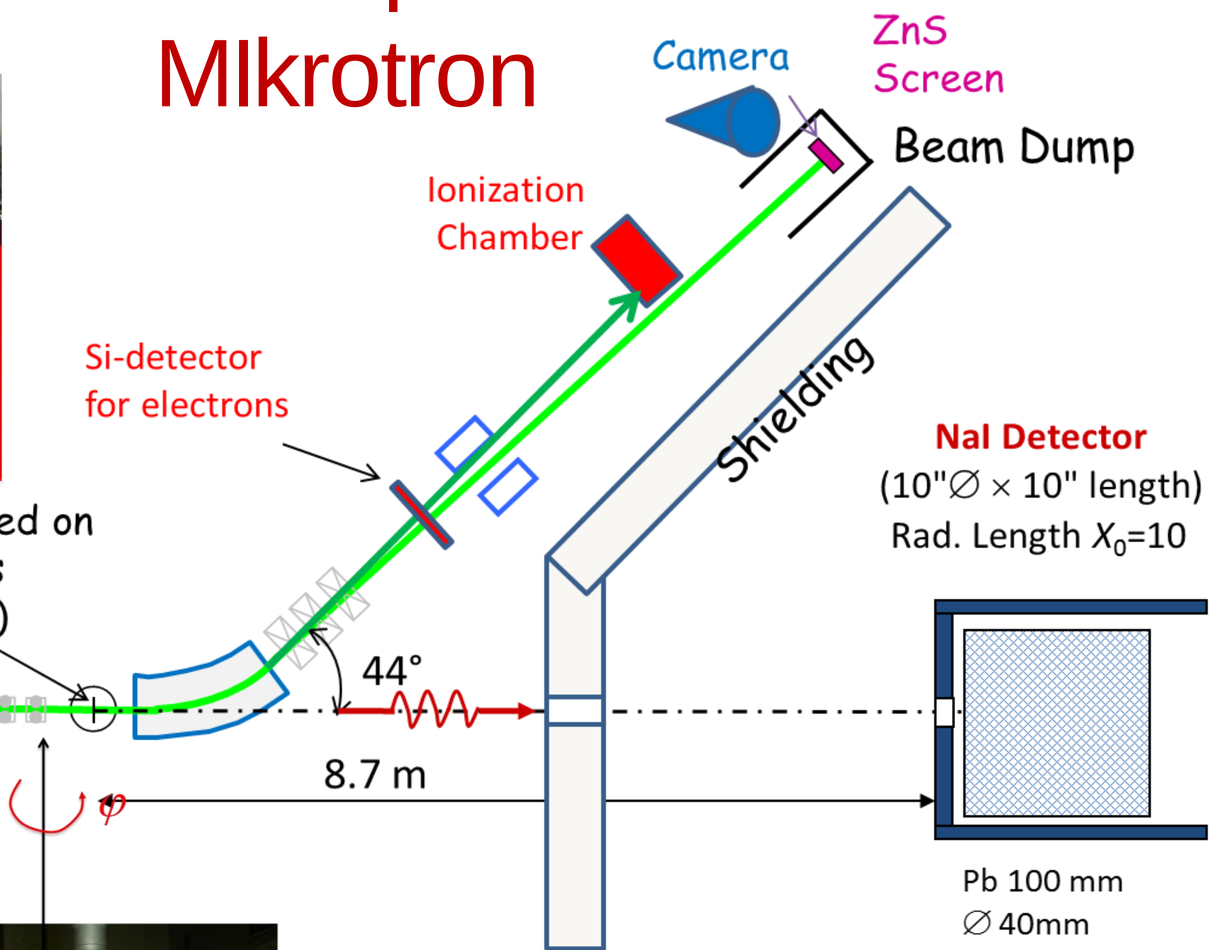
- A lot of attention is devoted to channeling effects of electron around GeV :
  - Interest for alternatives x-ray sources
  - Relatively large availability accelerators

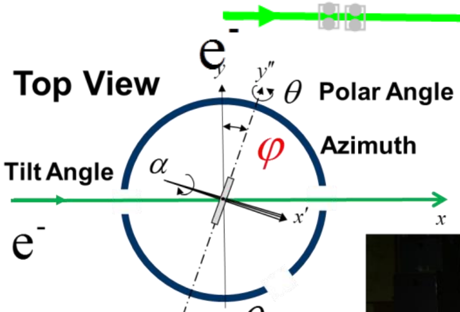
- Study of the influence of the curvature on Channeling Radiation (CR) and Coherent Bremsstrahlung (CB). This experimental knowledge may be exploited to **determine with more accuracy the CR contribution in crystalline undulators**;

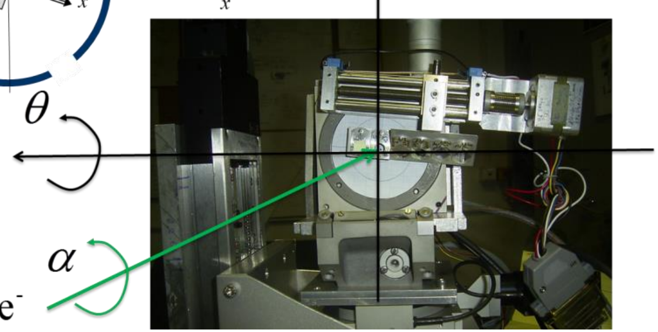
- Steering of sub-GeV electron trajectories through channeling in bent crystals was not possible before due to the lack of thin-enough bent crystals.

Experimental setup at the MAinzer MIkrotron









855 MeV electron beam characteristics:

Beam-Spot size σ_{hor} = 200 μm,

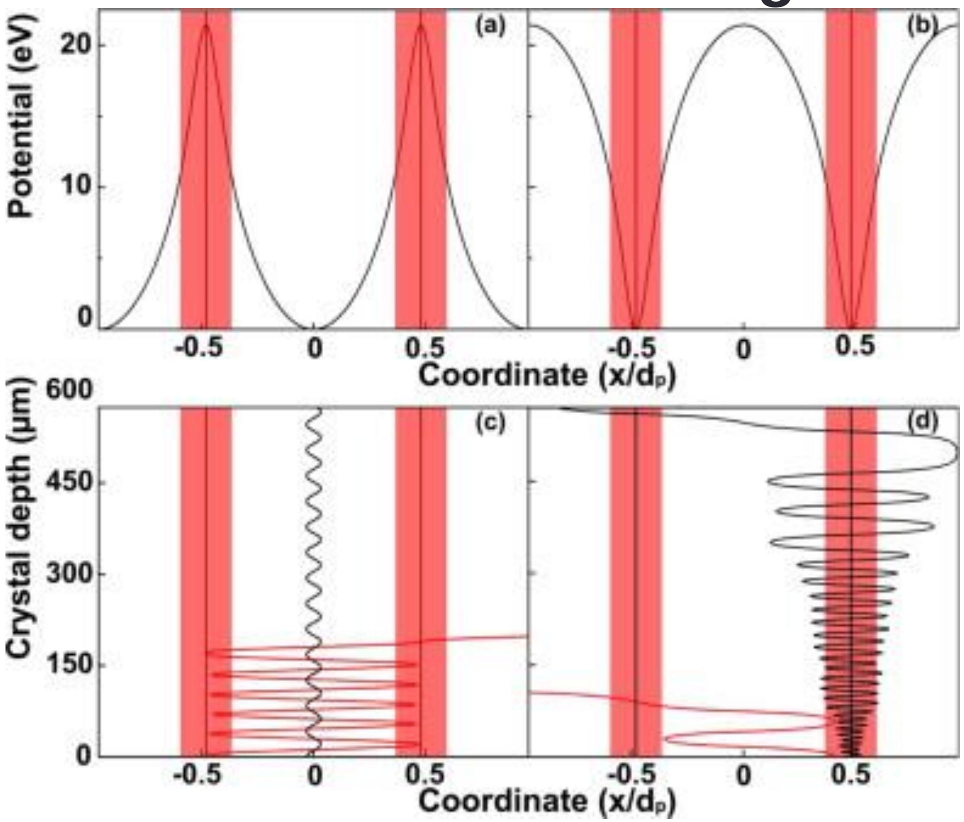
Beam Divergence σ'_{hor} = 70 μrad

Beam-Spot size σ_{vert} = 70 μm,

Beam Divergence σ'_{vert} = 30 μrad

Dechanneling of positive and negative particles

Positive vs Negative



Channeled negative particles are dechanneled faster than positive ones due to higher probability to suffer nucler incoherent scattering;

Ultra thin bent crystals are required for efficient deflection of negative particles

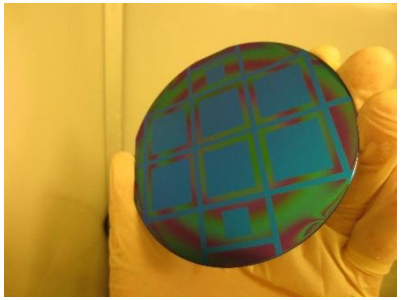
To determine how thin, one has to know the dechanneling length for negative particles!

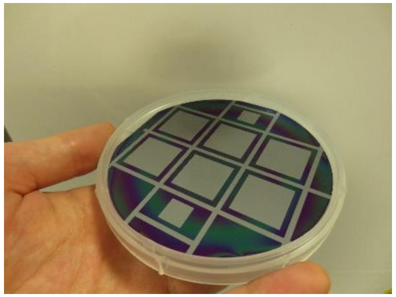
N.B. L_d decrease with energy, being some tens of microns for 1 GeV electrons in Si [1].

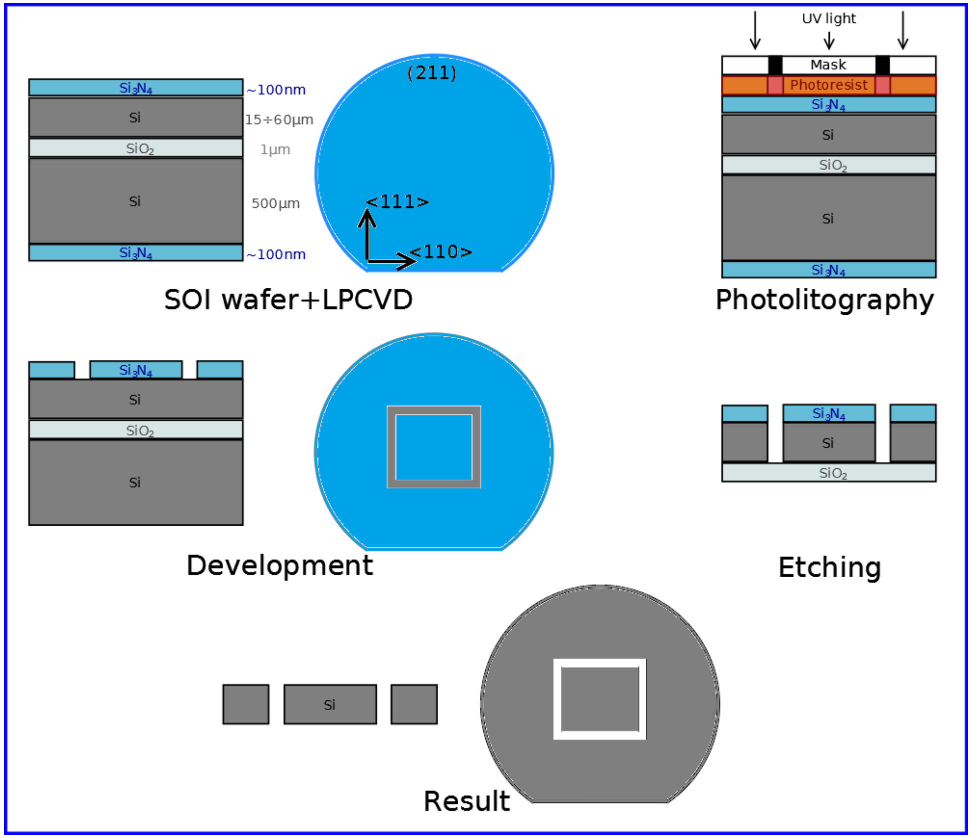
[1] W. Lauth, H. Backe, P. Kunz, A. Rueda, Int. Journal of Modern Physics A, 25, 1 136-143 (2010)

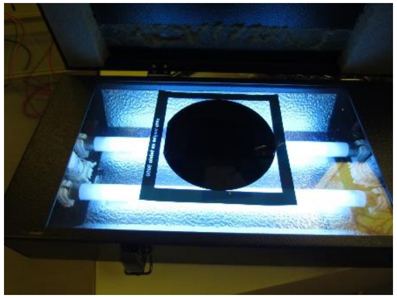
Sensor and Semiconductor Laboratory

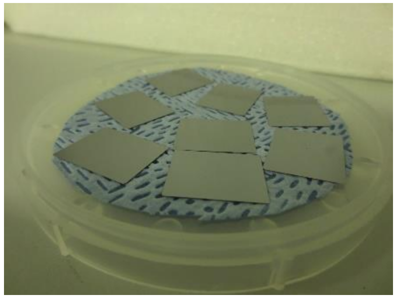
Thin crystals manufacturing







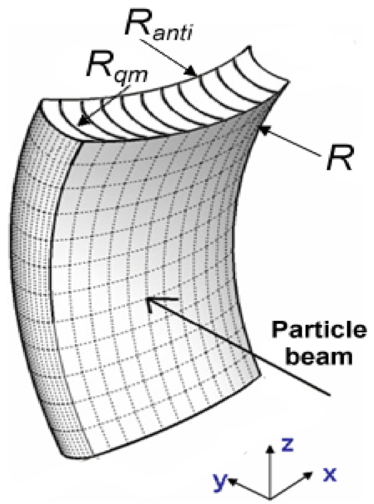


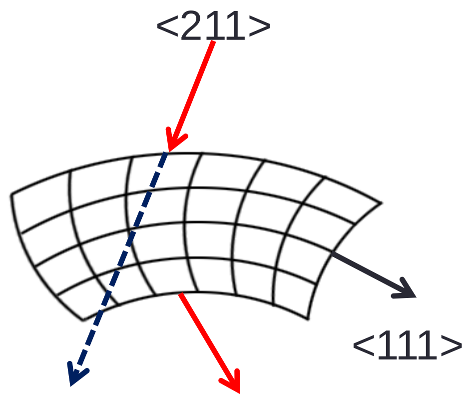


G. Germogli, A.Mazzolari et al., Nucl. Instr. Meth. B 355 (2015)

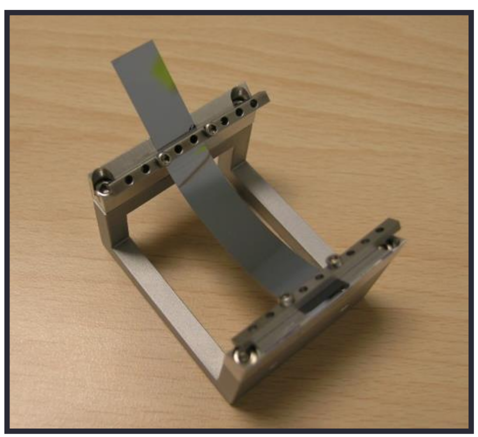
Sensor and Semiconductor Laboratory

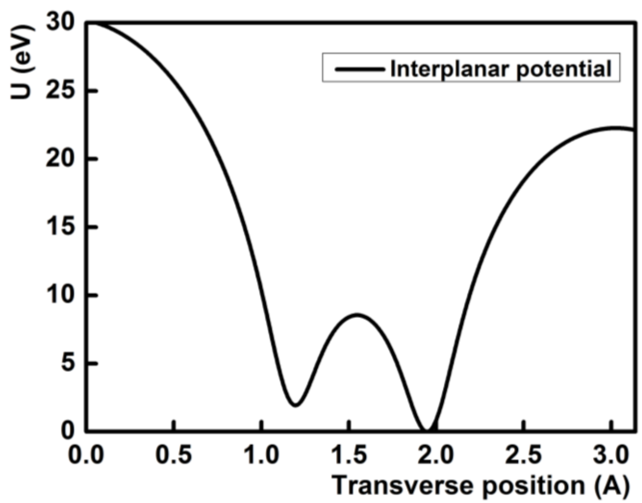
Crystal bending





Sketch of the quasi-mosaic [1] crystal highlighting the crystal crystallographic orientations. Anticlastic deformation absent due to strong bending along primary direction.





Crystal parameters

T = 30.5 µm

R_{qm} = 33.5 mm

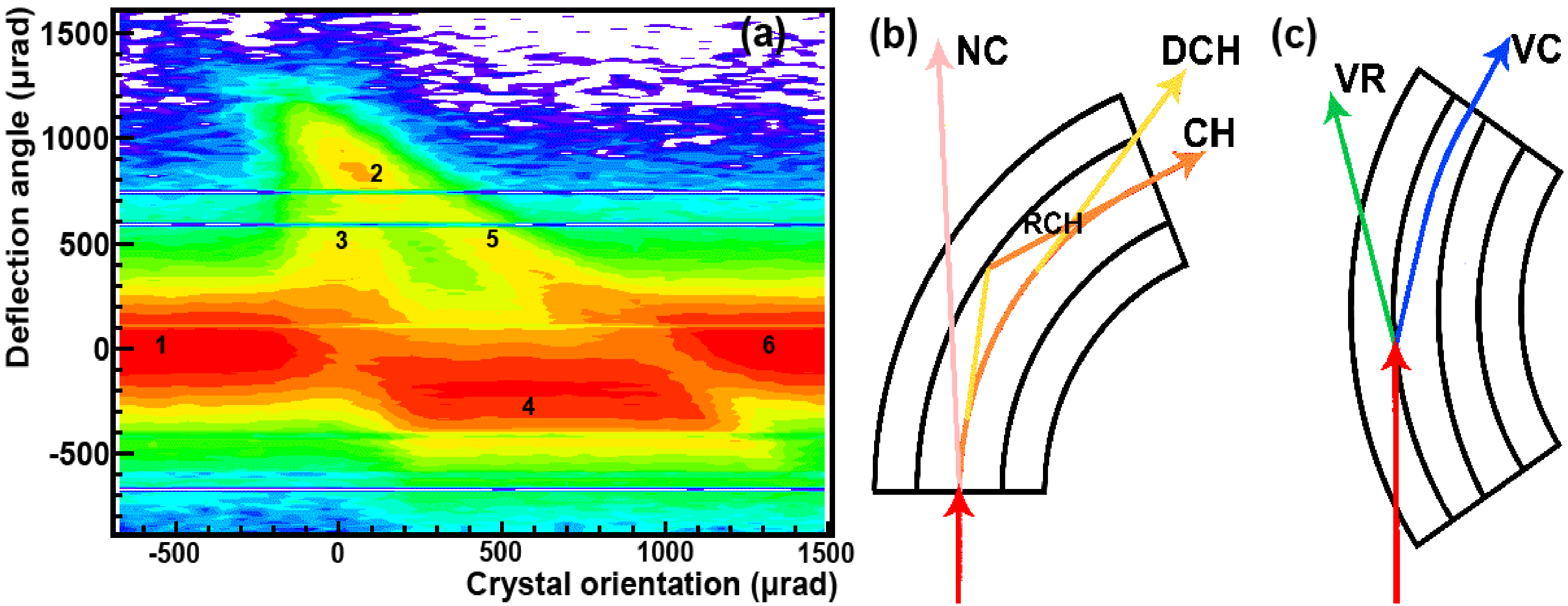
(111) Bent planes

U_0= 23 eV

θc= 220 µrad

[1] Y. M. Ivanov, A. A. Petrunin, and V. V. Skorobogatov, Jetp Letters 81,99 (2005).

Experimental results on beam steering

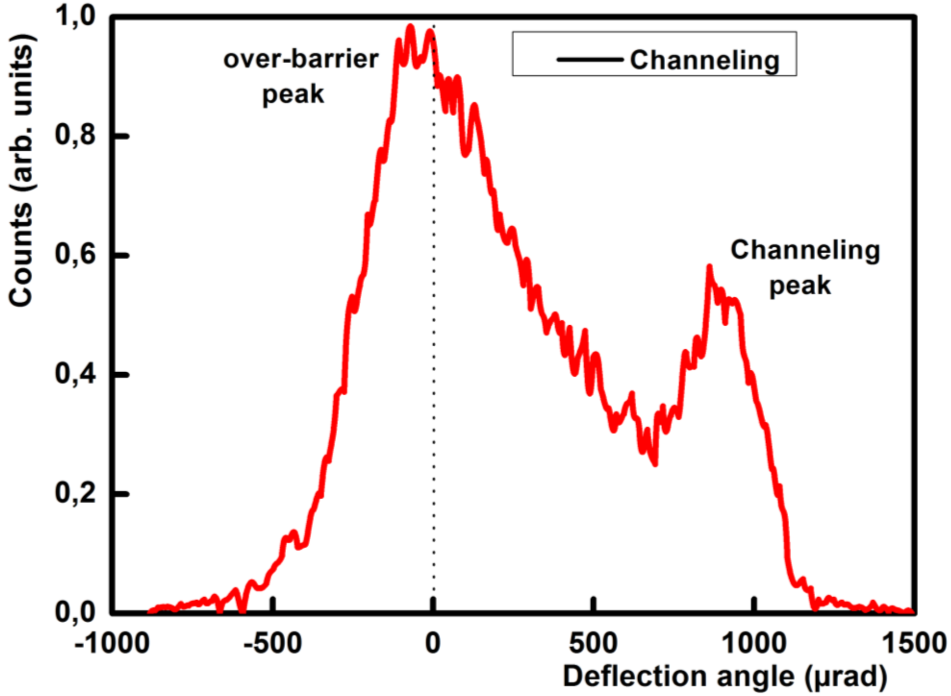


Angular scan for deflected beam distribution: (1) and (6) nonchanneling regime; (2) channeling; (3) dechanneling; (4) volume reflection; and (5) volume capture.

A. Mazzolari et al., Phys. Rev. Lett. 112 (2014) 135503

Experimental results on beam steering

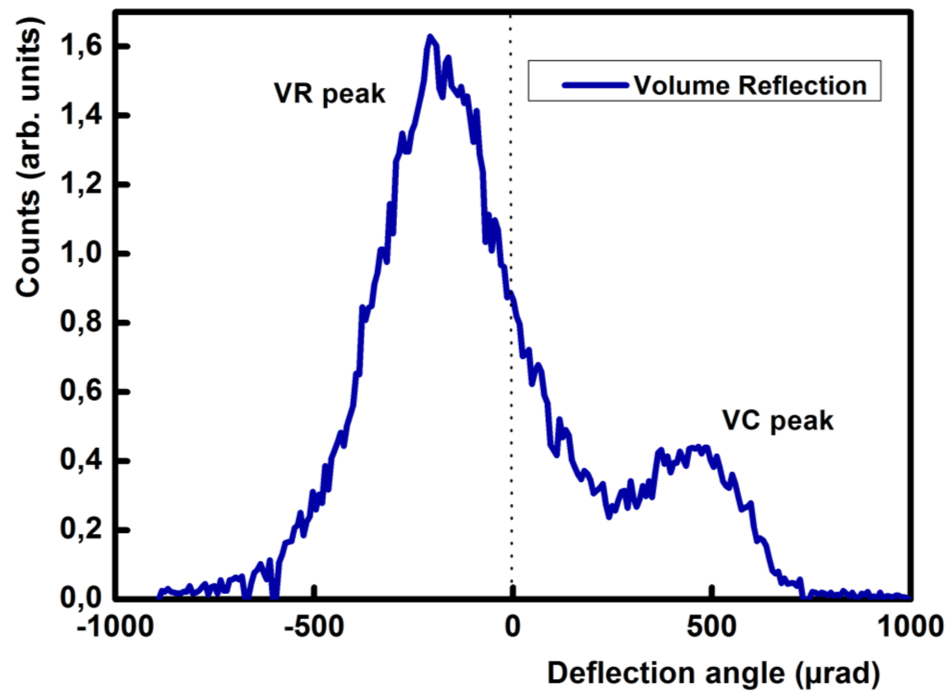
CHANNELING



Θinc ~ 0 μrad Deflection = 910±5 μrad

Fraction of channeled is 20.1±1.2%

VOLUME REFLECTION

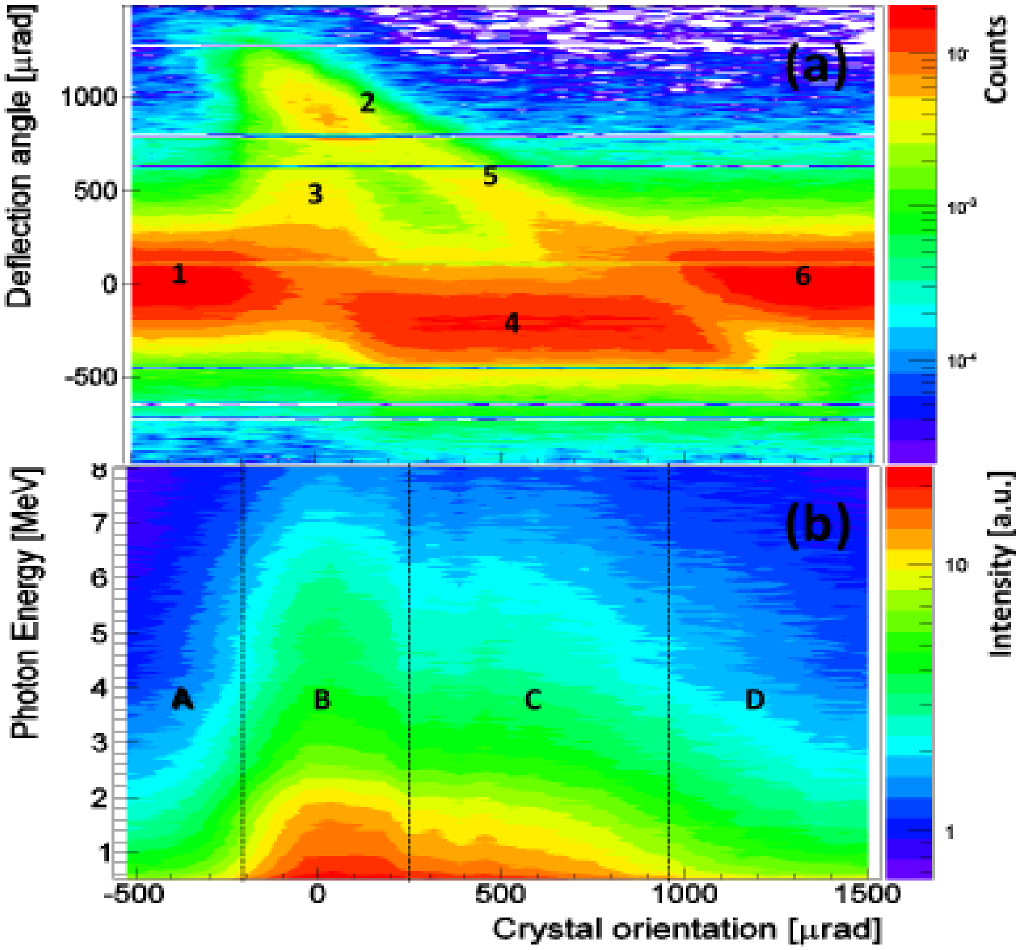


Θinc~450 μrad Deflection =191±10 μrad

Fraction of reflected is 76.7±1.1%

First experimental observation of channeling and VR of negative particles in the sub-GeV energy range

Experimental results on radiation emission

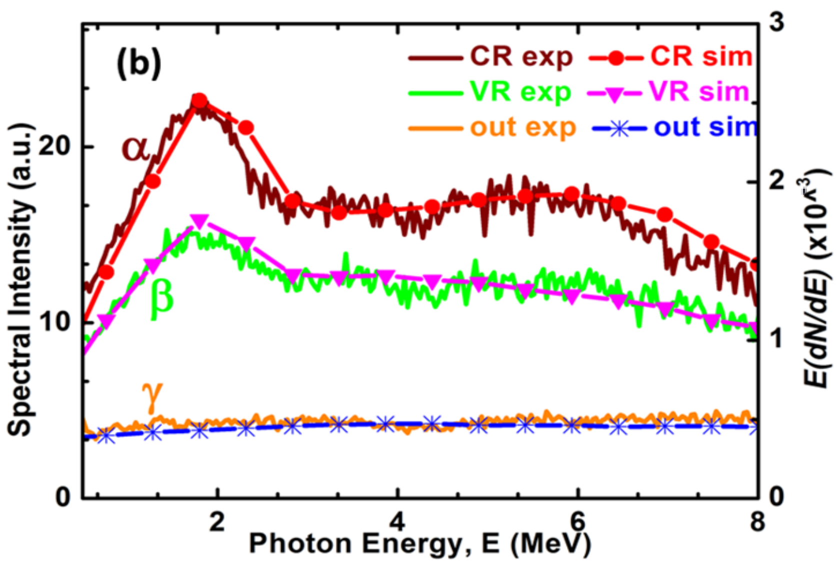


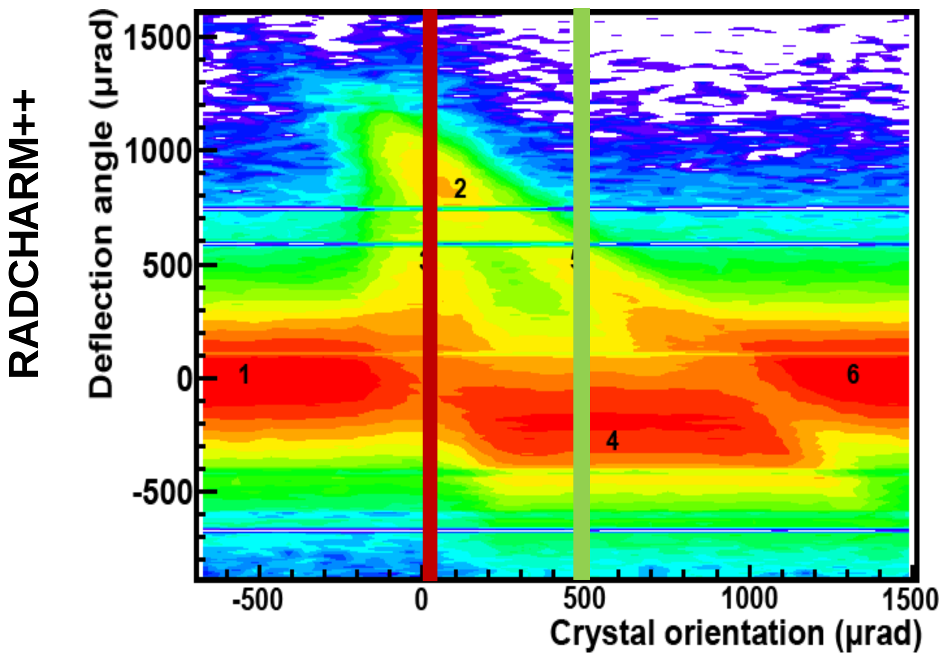
Observation: in VR orientation, emitted radiation seems to remain soft and intense as for channeling

L. Bandiera et al. Phys. Rev. Lett. 115, (2015) 025504.

Simulation of e.m. radiation emitted by ultrarelativistic electrons in a bent crystal

Comparison with experiment performed at the Mainzer Mikrotron with 855 MeV electrons interacting with a 30.5 μm bent Si crystal along the (111) planes





Channeling at <zero> position

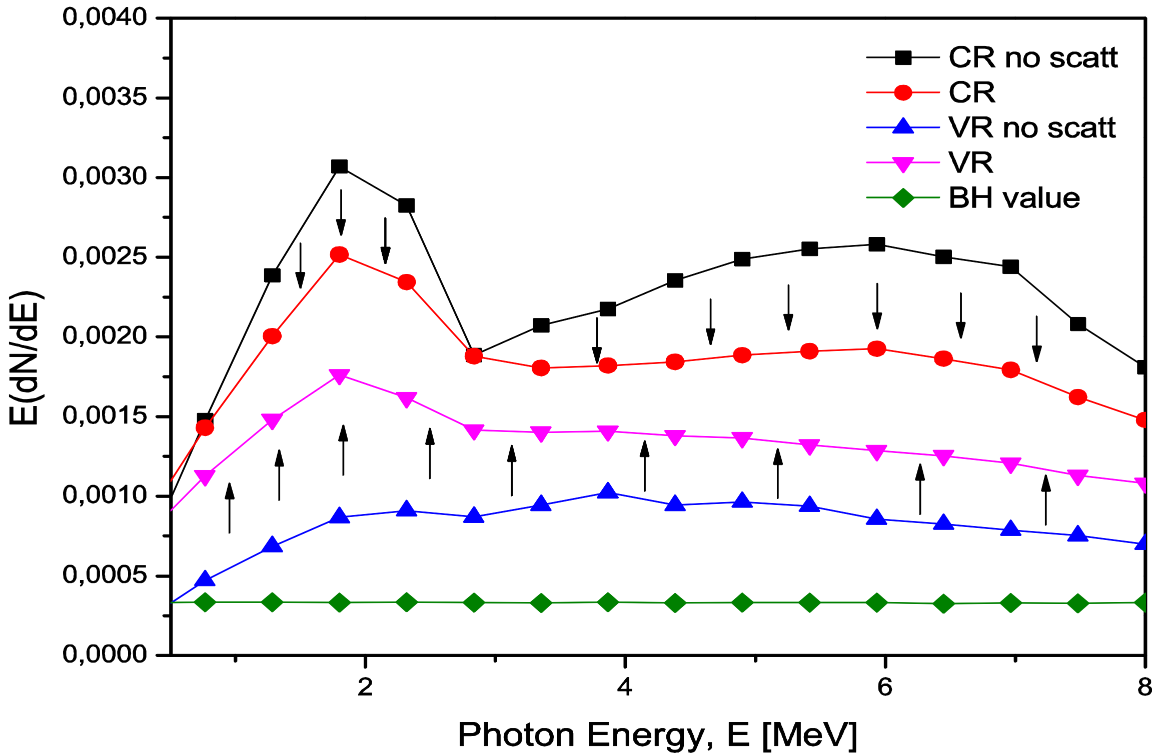
Volume reflection at +490 μrad

OUT: not aligned at 8 mrad

Channeling peak at $E_\gamma \sim$ 1.8 MeV for 855 MeV electrons in (111) Si bent planes

L. Bandiera et al., Phys. Rev. Lett. 115 (2015) 025504.

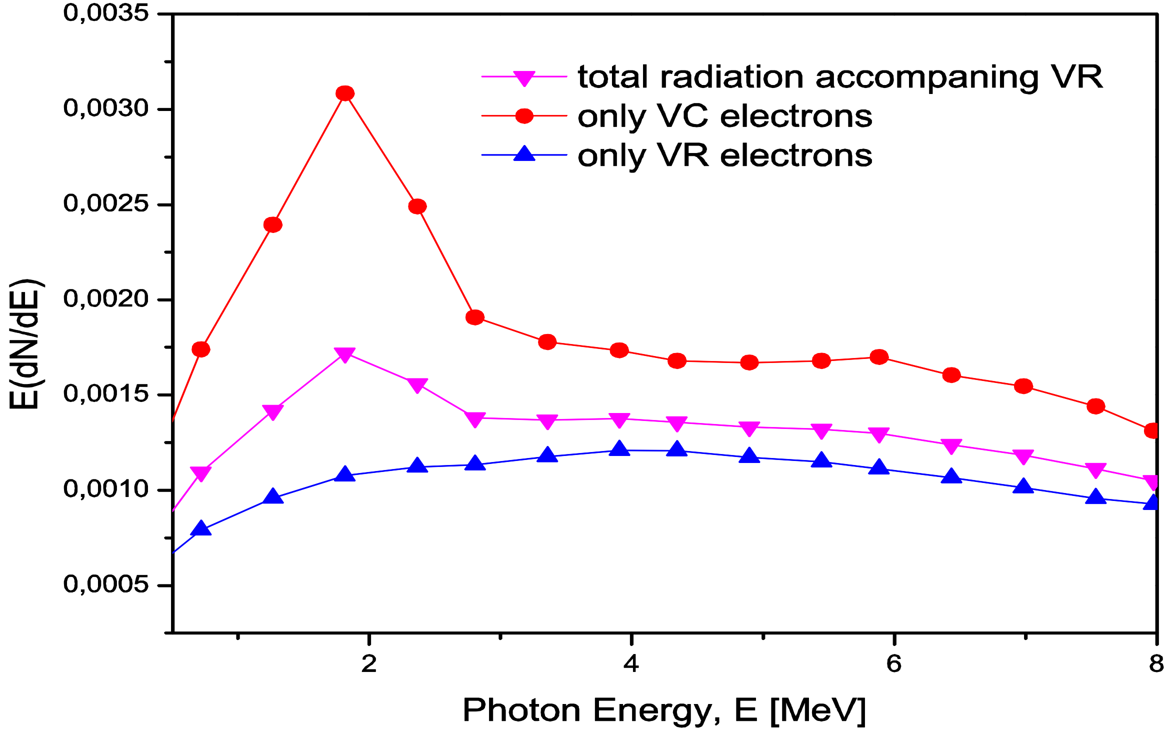
Simulation of the contribution to radiation of incoherent scattering



Simulation of the contribution of the scattering with nuclei and electrons to radiation spectral intensity [E(dN/dE)].

L. Bandiera et al., Phys. Rev. Lett. 115 (2015) 025504.

Incoherent scattering contribution to radiation accompanying VR



32% of volume-captured and 68% of pure volume-reflected electrons. The contribution of VC particles maintains the electromagnetic radiation accompanying VR close in intensity to that for CR over the whole angular acceptance

L. Bandiera et al., Phys. Rev. Lett. 115 (2015) 025504.

COMPARISON WITH EXPERIMENTS

100 GeV energy range:

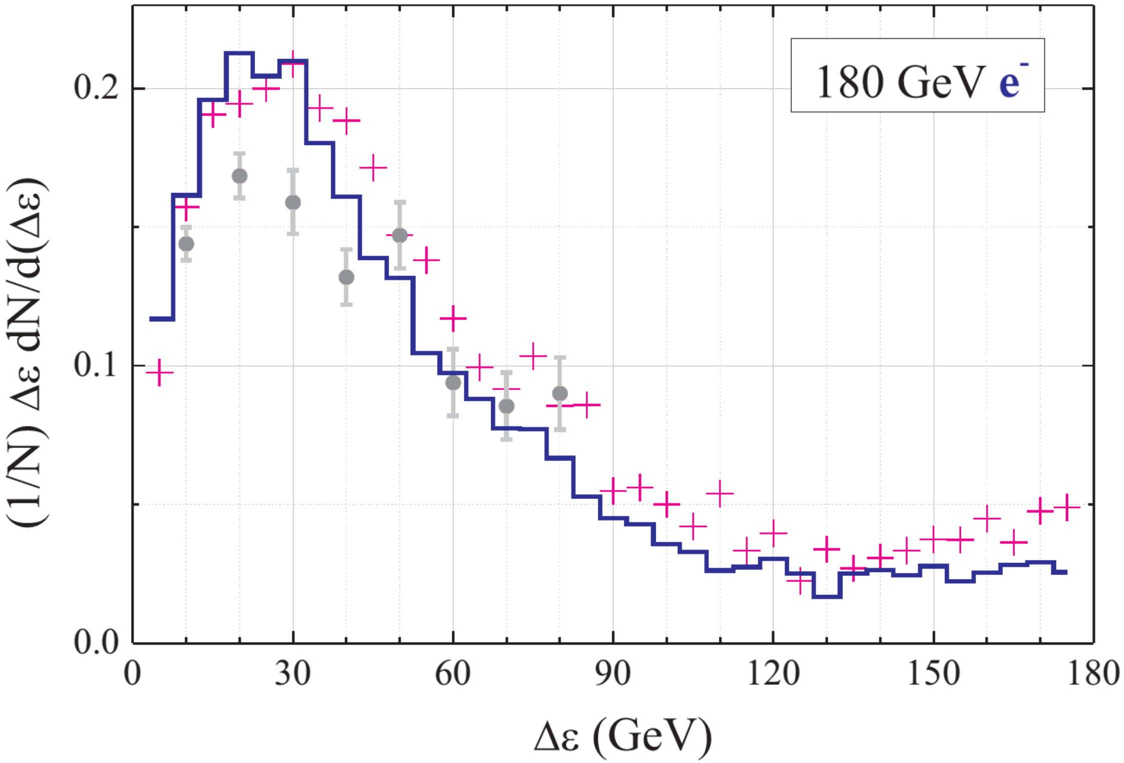
- Quantum correction;
- Multiple photon emission.

Multi photon emission

- In principle, the BK formula should be integrated along the whole particle trajectory.
- At very-high energy, the total probability of radiation may exceed unity -> **multiple photon emission!**
- **Separation of particle trajectory in intermediate lenghts** > coherence length and << typical distance between two sequential photon emission points. **Total probability of radiation on such trajectory part does not exceed 0.1**.
- The trajectory-part ends are neglected as the interference between them.

V. Guidi, L. Bandiera, V. Tikhomirov, Phys. Rev. A 86 (2012) 042903.

Simulation of PLANAR volume reflection



Si Crystal parameters:
Length = 0.84 mm;
Bending radius = 12 m;
Plane (111)
Beam divergence:
σ_x = 25 μrad and σ_y = 46 μrad
Incidence angle:
Θ_{X0} = 40 μrad

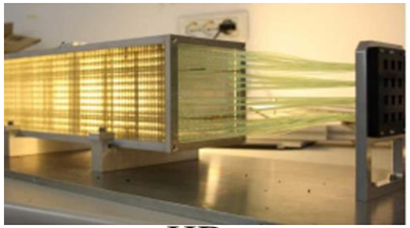
Photon energy ≥ 1 GeV has been selected.

Energy loss spectral intensities: (dn/dE)*E of 180 GeV/c volume reflected electrons

Experiment: W. Scandale, et al.,Phys.Rev.A79, 012903 (2009).
Simulation: V. Guidi, L. Bandiera, V. Tikhomirov, Phys. Rev. A 86 (2012) 042903

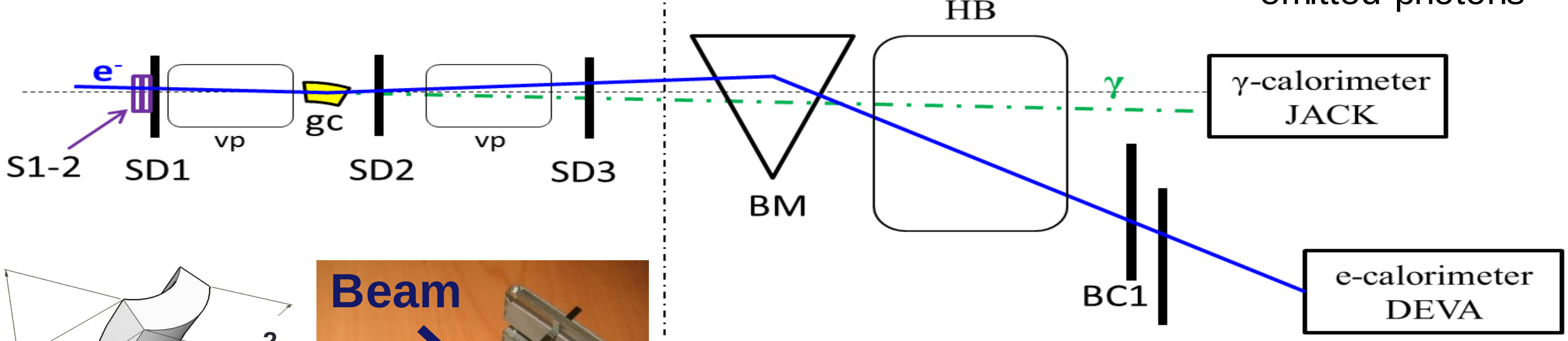
Interaction of 120 GeV/c electrons with a 2 mm long bent Si crystal

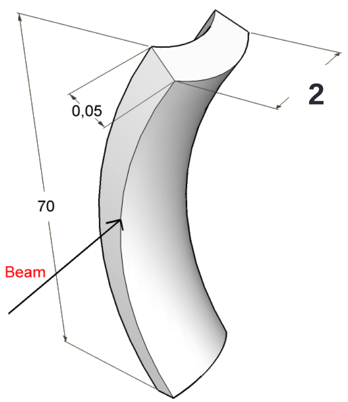
H4 extracted line from CERN SPS

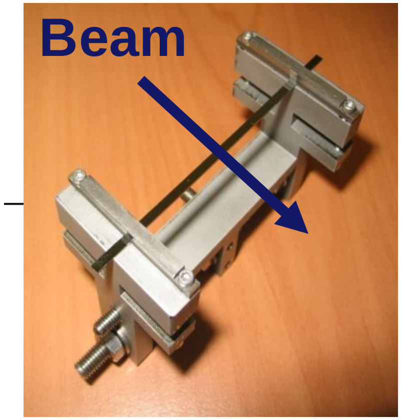


γ-beam *shashlik* calorimeter for a total of 19 X0 to measure the emitted photons

Double sided silicon detectors SDi (300μm) [5-μm spatial resolution]







(110) Bent Si planes

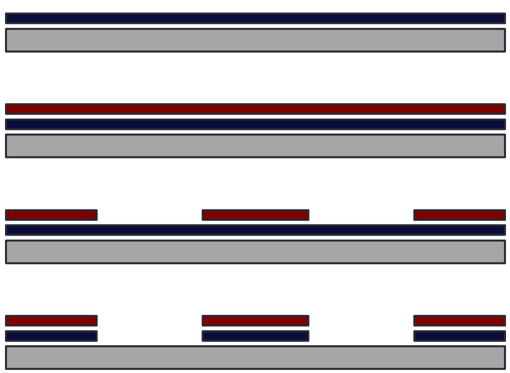
A bending magnet (BM) and a pair of two single sided silicon beam chambers, BCi, form the **SPECTROMETER** to measure the particle momentum

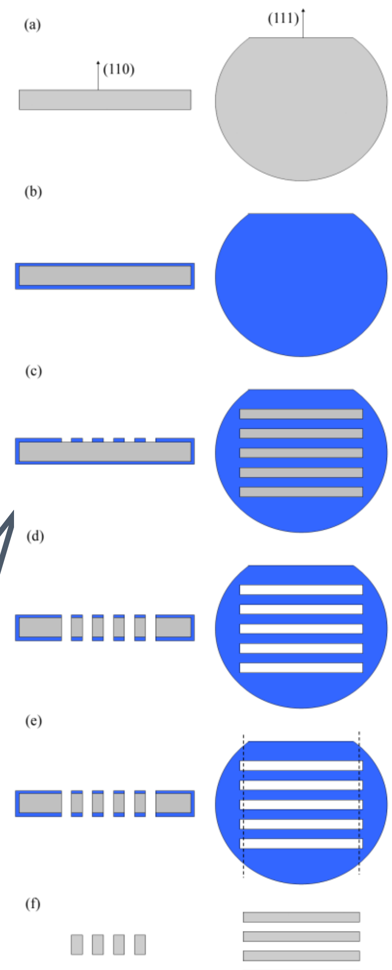
Sensor and Semiconductor Laboratory

Silicon Crystal Fabrication

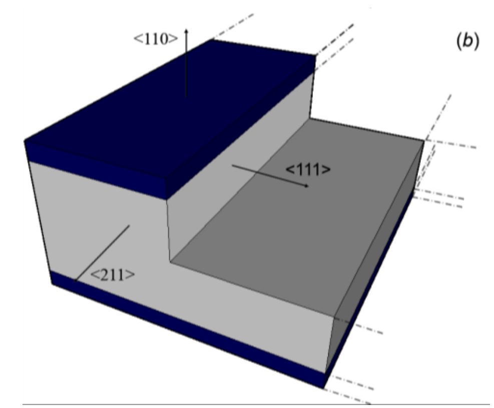
LPCVD deposition of silicon nitride thin layer

Silicon nitride patterning

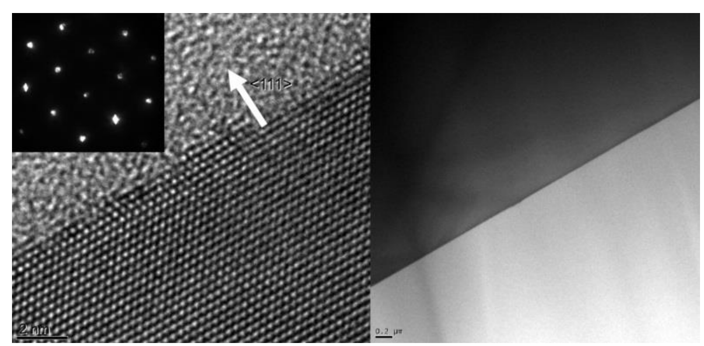




Anisotropic etching: Etching rate on different silicon planes for KOH 20% at 40 °C



Crystalline surfaces



S. Baricordi et al., Journal of Physics D: Applied Physics 41 (24), 245501
S. Baricordi et al., Applied Physics Letters 91 (6), 061908

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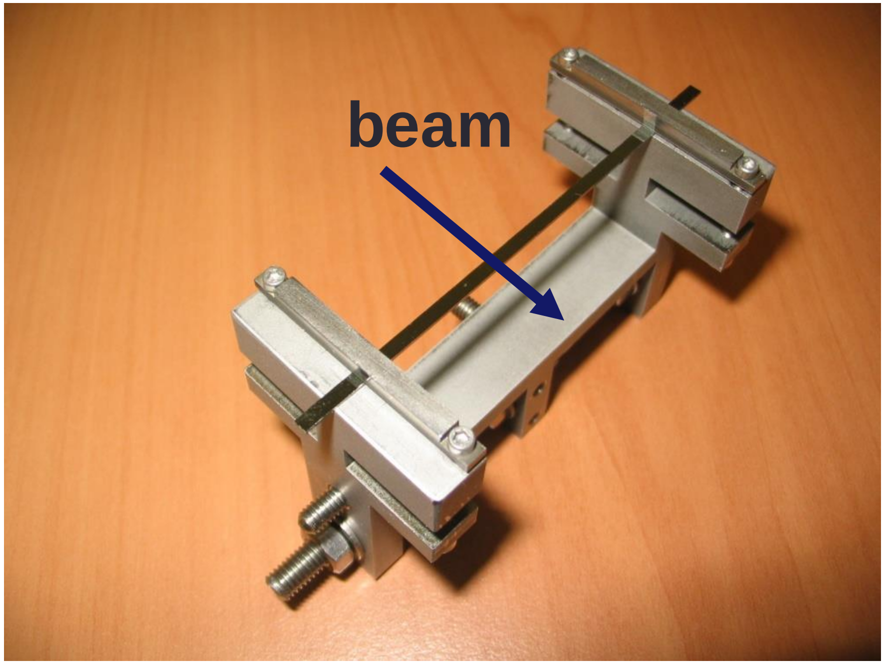
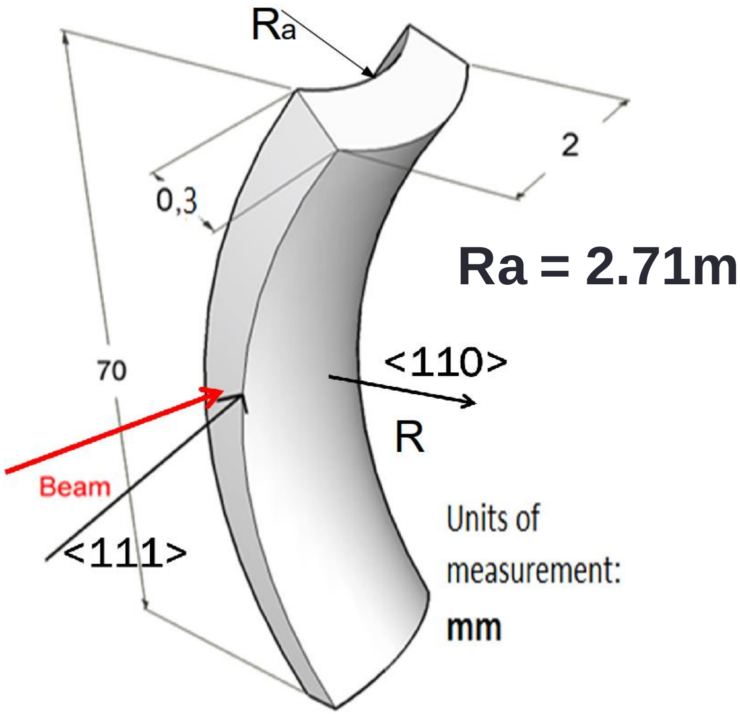
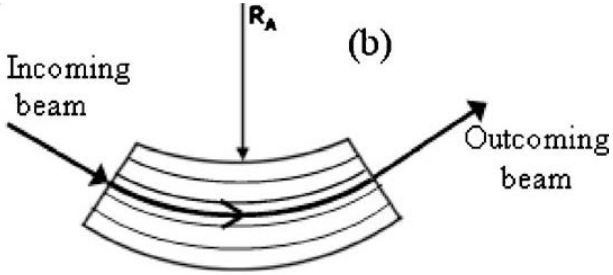
Bent Si crystal





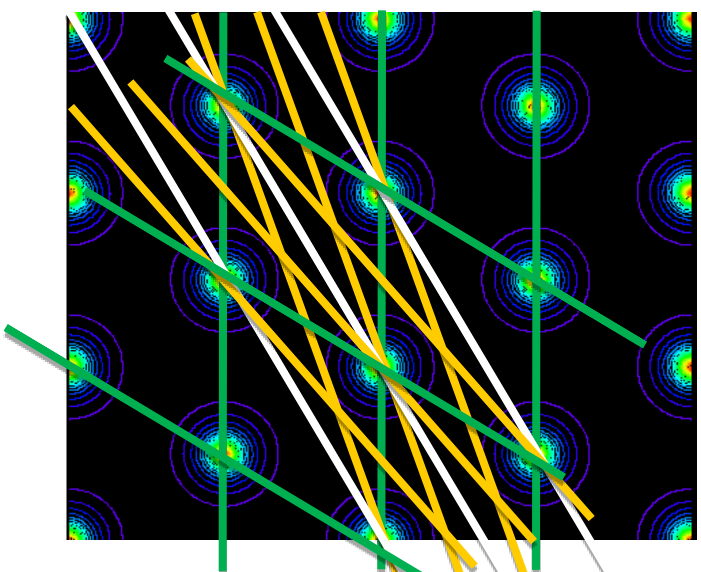


- **A primary curvature is imparted by mechanical external forces, which result in a secondary (anticlastic) curvature;**
- **The mechanical holder used to impart the primary strain can set far apart from the particles and hence it reduces any wanted interaction with the beam.**

JAP 107 (2010) 113534

Single vs Multiple Volume Reflection in One bent Crystal

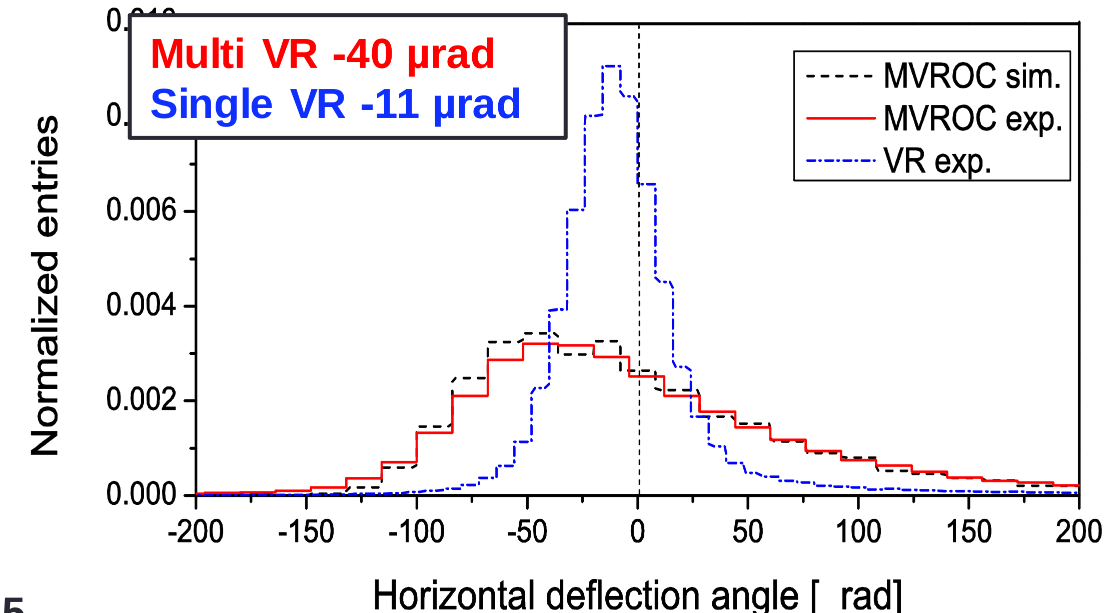
- ✓ **Single VR results in a small deflection angle, which decreases while increasing the particle energy.**
- ✓ **<u>Possible solution to increase the VR deflection angle</u>: Multi VR from different planes of a bent crystal becomes possible when particles move at a small angle with respect to a crystal axis.**



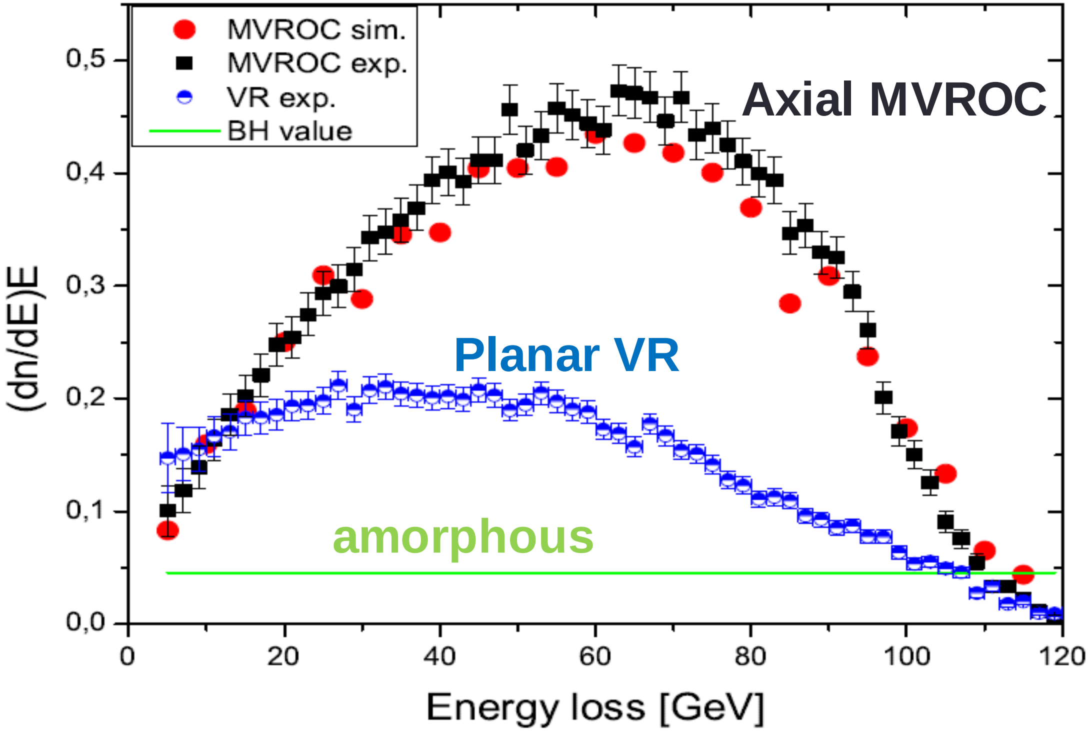
Potential along a Si crystal [111] direction

V. Tikhomirov Phys. Lett. B 655 (2007) 5,

Deflection distribution



Simulation of AXIAL multi-volume reflection



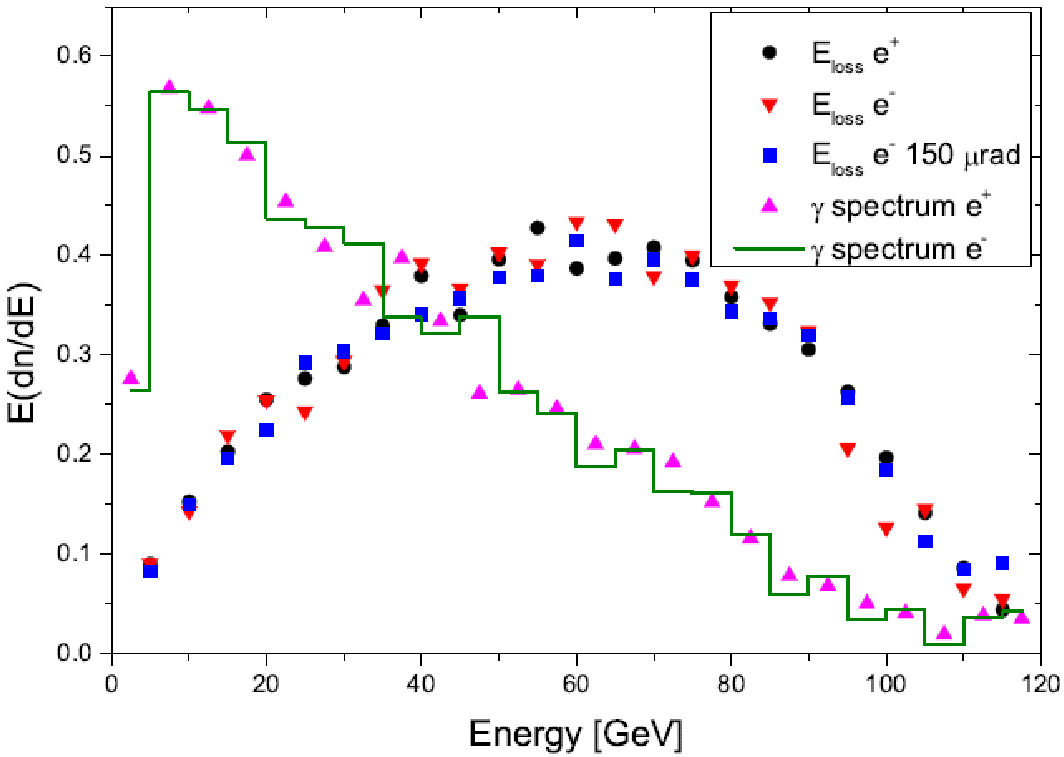
Si Crystal parameters:
Length = 2mm;
Bending radius = 2.7 m;
Plane (110)
Axis <111>
Beam divergence:
σ_x = 50 μrad and σ_y = 65 μrad
Incidence angle:
Θ_{X0} = 365 μrad ≈ half bending
Θ_{Y0} misaligned for planar VR
= 205 μrad for axial MVROC

Photon energy ≥ 1 GeV has been selected.

Energy loss spectral intensities: (dn/dE)*E of 120 GeV/c single and multi-reflected electrons

L. Bandiera et al., Phys. Rev. Lett. 111(2013) 255502

Radiation features: single photon spectra



Single photon spectrum for 120 GeV/c positrons

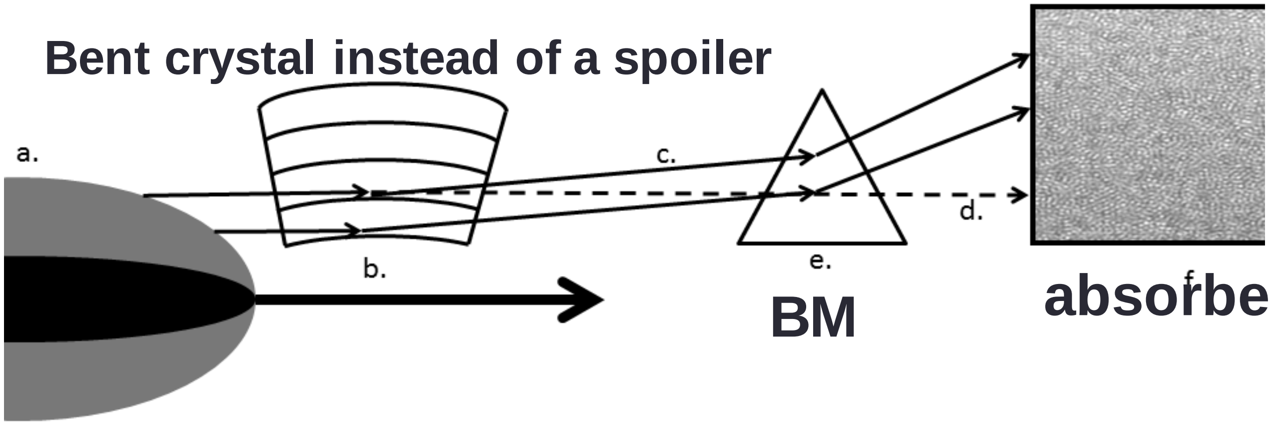
Single photon spectrum for 120 GeV/c electrons

A mean number of photons emitted by each particles (multiplicity factor) equal to 2.2/e^- for $\hbar\omega > 1$ GeV.

L. Bandiera et al., Phys. Rev. Lett. 111(2013) 255502

Possible application

Some opportunities in the collimation system of future linear electron/positron colliders, such as ILC



- **The insertion of a short Si crystal (0.02-0.05 X_0) instead of a long spoiler (0.5-1 X_0) to clean the halo particles would diminish the perturbations on the beam.**
- **The deflection** of **particles under single and multiple VR and the increase in energy loss (more than 30% in 0.02 X_0) may increase the discrimination of halo particles**, which are deflected more in the bending magnets placed downstream after the collimator.

A. Seryi, Nucl. Instrum. Methods Phys. Res., Sect. A 623, 23 (2010)

L. Bandiera et al., et al., Journal of Physics: Conference Series 517 (2014) 012043

Summarizing

- An algorithm to compute of radiation emitted by relativistic $e^{\pm}$ in crystals based on the Baier-Katkov method has been presented;
- Such algorithm has already been implemented in existing Monte Carlo codes for simulation of particle trajectories in crystals;
- Comparison with experiments show a very good agreement in a wide energy range (from 1 GeV to 100 GeV);
- Such a method can be inserted in the most general toolkits for the simulation of the passage of particles through matter, such as Geant 4, as an implementation of the radiation processes in oriented crystalline structures.