Calibration and Beam Profile Fitting

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Visibility of a Strong Point Source

The observed visibility of a strong point source with flux S_c is

$$\begin{aligned} V_{ij} &= g_i g_j^* A_i(\hat{\boldsymbol{n}}_0) A_j^*(\hat{\boldsymbol{n}}_0) S_c e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot (\boldsymbol{u}_i - \boldsymbol{u}_j)} \\ &= S_c \cdot g_i A_i(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_i} \cdot (g_j A_j(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_j})^* \\ &= S_c \cdot G_i G_j^*, \end{aligned}$$

where $G_i = g_i A_i(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_i}$.
In the presence of outliers and noise, we have

$$\frac{V_{ij}}{S_c} = G_i G_j^* + S_{ij} + n_{ij}.$$

Written in matrix form, it is

$$\mathbf{V}' = \frac{\mathbf{V}}{S_c} = \mathbf{V}_0 + \mathbf{S} + \mathbf{N},$$

where $\mathbf{V}_0 = \mathbf{G}\mathbf{G}^H$ is a rank 1 matrix represents the visibilities of the strong point source, $\mathbf{S}$ is a sparse matrix whose elements are outliers or misssing values, and $\mathbf{N}$ is a matrix with dense small elements represents the noise.

Matrix Decomposition

Now denote V' as V, by solving the optimization problem

$$\min_{V_0,S} \frac{1}{2}\|V_0 + S - V\|_F^2 + \lambda\|S\|_0$$

or its relaxation

$$\min_{V_0,S} \frac{1}{2}\|V_0 + S - V\|_F^2 + \lambda\|S\|_1$$

we can get V_0, S, and $N = V - V_0 - S$.
Where

$$\text{Frobenius norm}: \quad \|M\|_F = \sqrt{\sum_{ij} M_{ij}^2}$$

$$l_0\text{-norm}: \quad \|M\|_0 = \text{number of non-zero elemets of } M$$

$$l_1\text{-norm}: \quad \|M\|_1 = \sum_{ij} |M_{ij}|$$

Method to solve G

We have $V_0 = GG^H$, where $G_i = g_i A_i(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_i}$.

To sovle G, we Eigen-decompose V_0 as

$$V_0 = U \Lambda U^H = \lambda u u^H,$$

where, as rank$(V_0) = 1$, Λ has only one non-zero eigen-value λ, and u is the corresponding eigen-vector.

We see

$$G = \sqrt{\lambda} u$$

Method to Solve Gain g_i

From $G_i = g_i E_i$, where $E_i = A_i(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_i}$, suppose the gain g_i is stable during a period of time, we can get g_i as

$$g_i = \frac{E_i^\dagger \cdot G_i}{E_i^\dagger \cdot E_i}.$$

We get the calibrated visibility as

$$V_{ij}^{\text{cal}} = V_{ij}/(g_i g_j^*).$$

Calibration uncertainties may due to:

- beam profile: $A_i(\hat{\boldsymbol{n}}_0)$;
- time accuracy: $\hat{\boldsymbol{n}}_0$;
- position of feeds: $\boldsymbol{u}_i$.

Beam Profile Fitting

From $G_i = g_i A_i(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_i}$, we see

$$|G_i| = |g_i| \, A_i(\hat{\boldsymbol{n}}_0).$$

suppose the gain g_i is stable during a period of time, we have

$$|G_i| \propto A_i(\hat{\boldsymbol{n}}_0).$$

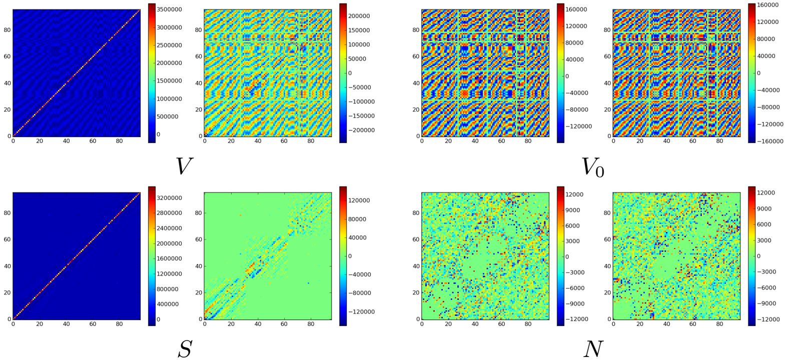
For a calibrator which has a DEC close to the array's latitude, such as Cygnus A, we have

$$(A_i)_{t_0} \approx 1,$$
$$|(g_i)_{t_0}| = |(G_i)_{t_0}|,$$

at transit time t_0.

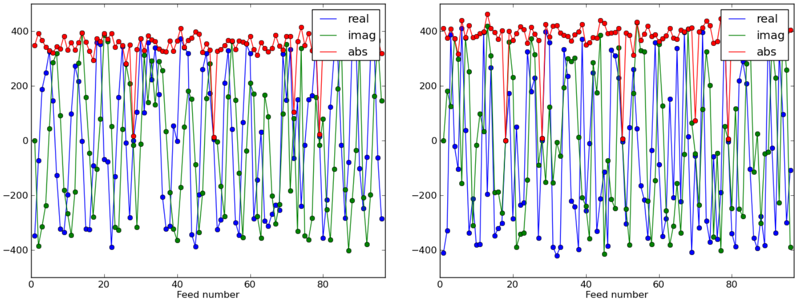
Visibility Matrix Decomposition

$$V = V_0 + S + N$$



G at transit time of Cygnus A

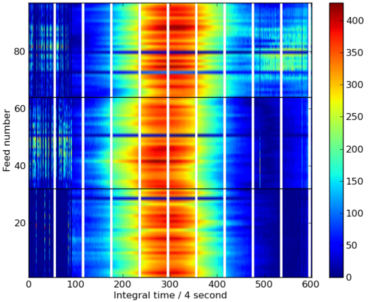
$$G_i = g_i A_i(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_i}$$



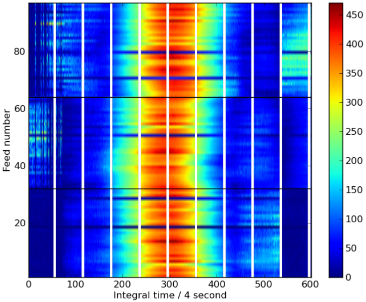
East-West pol   North-South pol

G during a period of time

$$G_i = g_i A_i(\hat{\boldsymbol{n}}_0) e^{2\pi i \hat{\boldsymbol{n}}_0 \cdot \boldsymbol{u}_i}$$

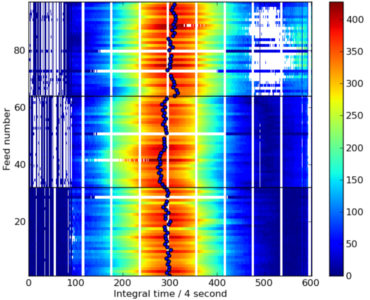


East-West pol

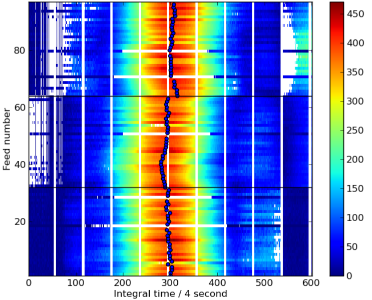


North-South pol

Center Finding by Sinc / Gaussian Fitting

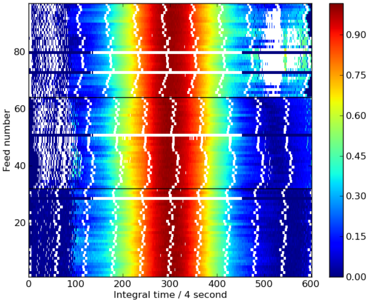


East-West pol

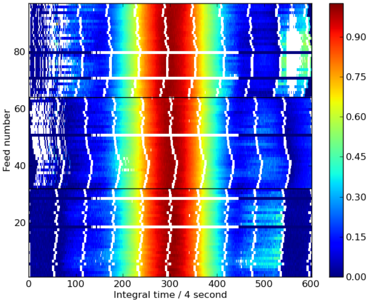


North-South pol

Align and Normalize

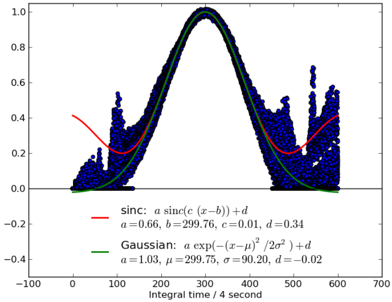


East-West pol

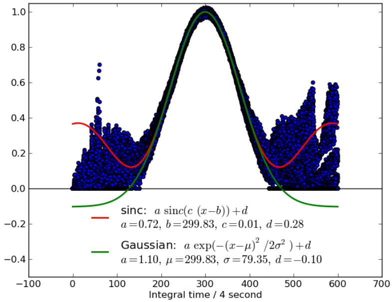


North-South pol

Beam Profile Fitting



East-West pol, FWHM $= 3.7^\circ$



North-South pol, FWHM $= 3.13^\circ$