GRAVITATIONAL WAVES
and
COSMOLOGY

Danièle Steer
APC
Paris Diderot

Gravitational wave observations can tell us about cosmology

Individual sources
at cosmological distances
e.g. binary black holes,
binary neutron stars…

late-time universe
$z \lesssim 10$

– structure and kinematics of universe
– formation of structures
– H_0, Hubble constant
– dark energy and dark matter
– modified gravity…..

Stochastic background
of GWs of cosmological origin

Very early universe
$t \gtrsim t_{Pl}$

– quantum processes during inflation
– Phase transitions in Early universe
– topological defects, eg cosmic strings
– …..

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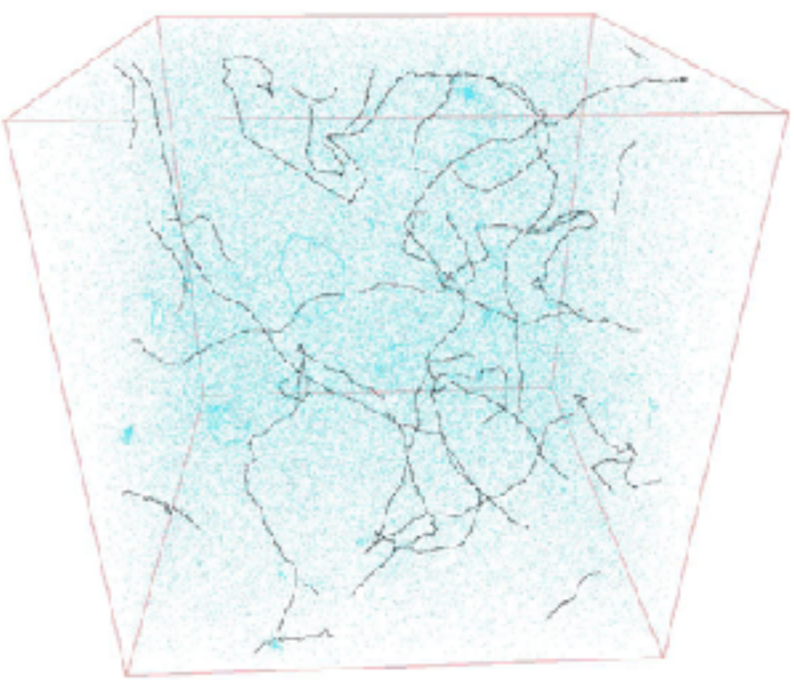

Stochastic background
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Very early universe
$t \gtrsim t_{Pl}$

– quantum processes during inflation
– Phase transitions in Early universe
– topological defects, e.g. *cosmic strings*
– …..

<u>cosmic strings</u>: at cosmological distances can detect individual bursts;
but also a stochastic bkgd of GWs ranging over many decades in frequency.

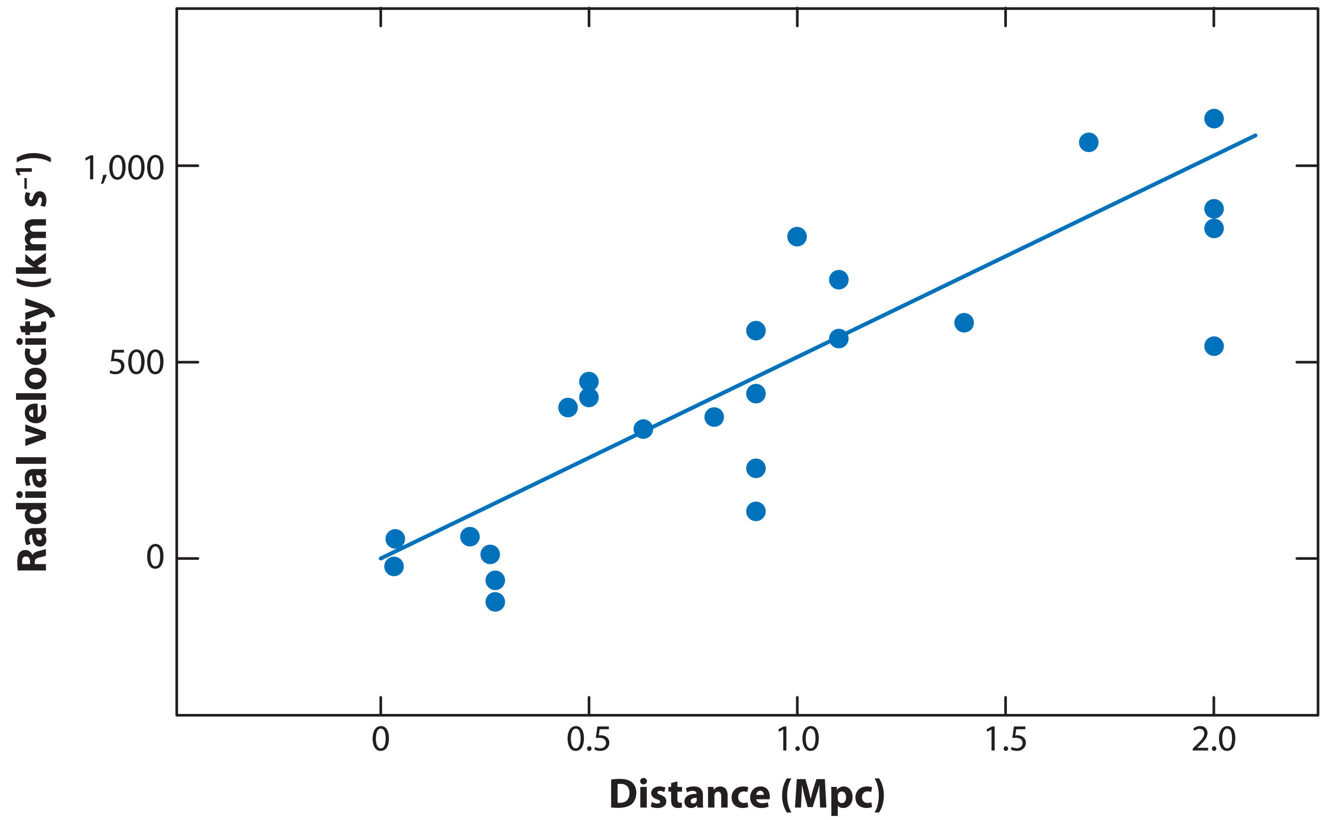
Why is measuring H_0 interesting/important?

- In a FRWL universe: $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$
- Redshift: $1 + z = \frac{a(t_0)}{a}$
- Hubble parameter: $H(t) = \frac{\dot{a}(t)}{a(t)}$
- **Hubble constant:** $H_0 = H(t_0)$

  is a *fundamental* quantity in cosmology:
  - age of universe;
  - defines observable size of universe
  - …

$$t_o = \int_0^\infty \frac{dz}{(1+z)H(z)} = H_o^{-1} \int_0^\infty \frac{dz}{(1+z)[\Omega_{matter}(1+z)^3 + \Omega_{DE}(1+z)^{3(1+w)}]^{1/2}}.$$

- Its value? $H_0 = 100 h_0 \text{ km/s/Mpc}$
  - Hubble *[1929]*: $h_0 = 5$



Using photographic data obtained at the 100-in Hooker telescope situated at Mount Wilson, California, Hubble measured the distances to six galaxies in the Local Group using the period-luminosity relation (hereafter, the Leavitt Law) for Cepheid variables. He then extended the sample to an additional 18 galaxies reaching as far as the Virgo cluster, assuming a constant upper limit to the brightest blue stars (HII regions) in these galaxies. Combining these distances with published radial velocity measurements (corrected for solar motion), Hubble constructed **Figure 1**. The slope of the velocity versus distance relation yields the Hubble constant, which parameterizes the current expansion rate of the Universe.

[W.Freedman, 1706.02739]

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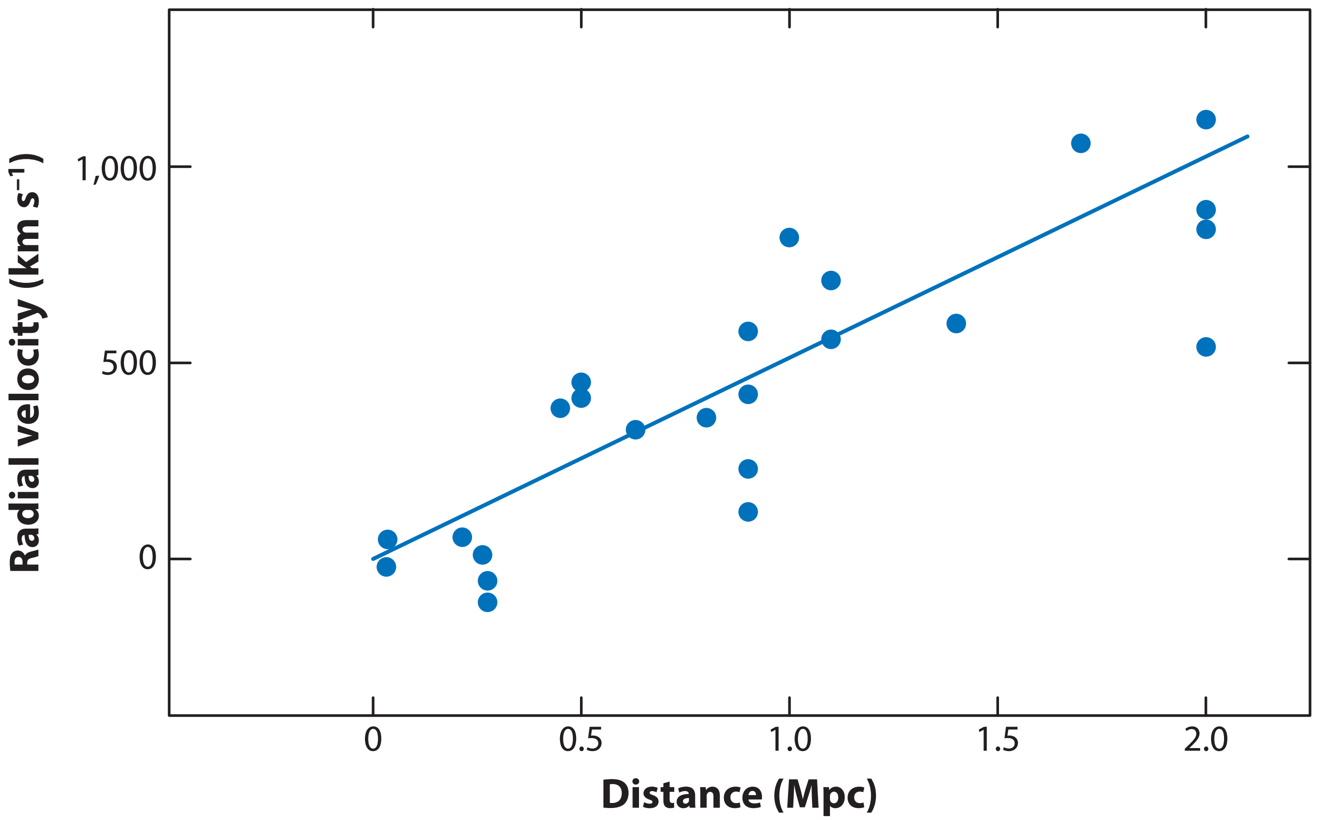
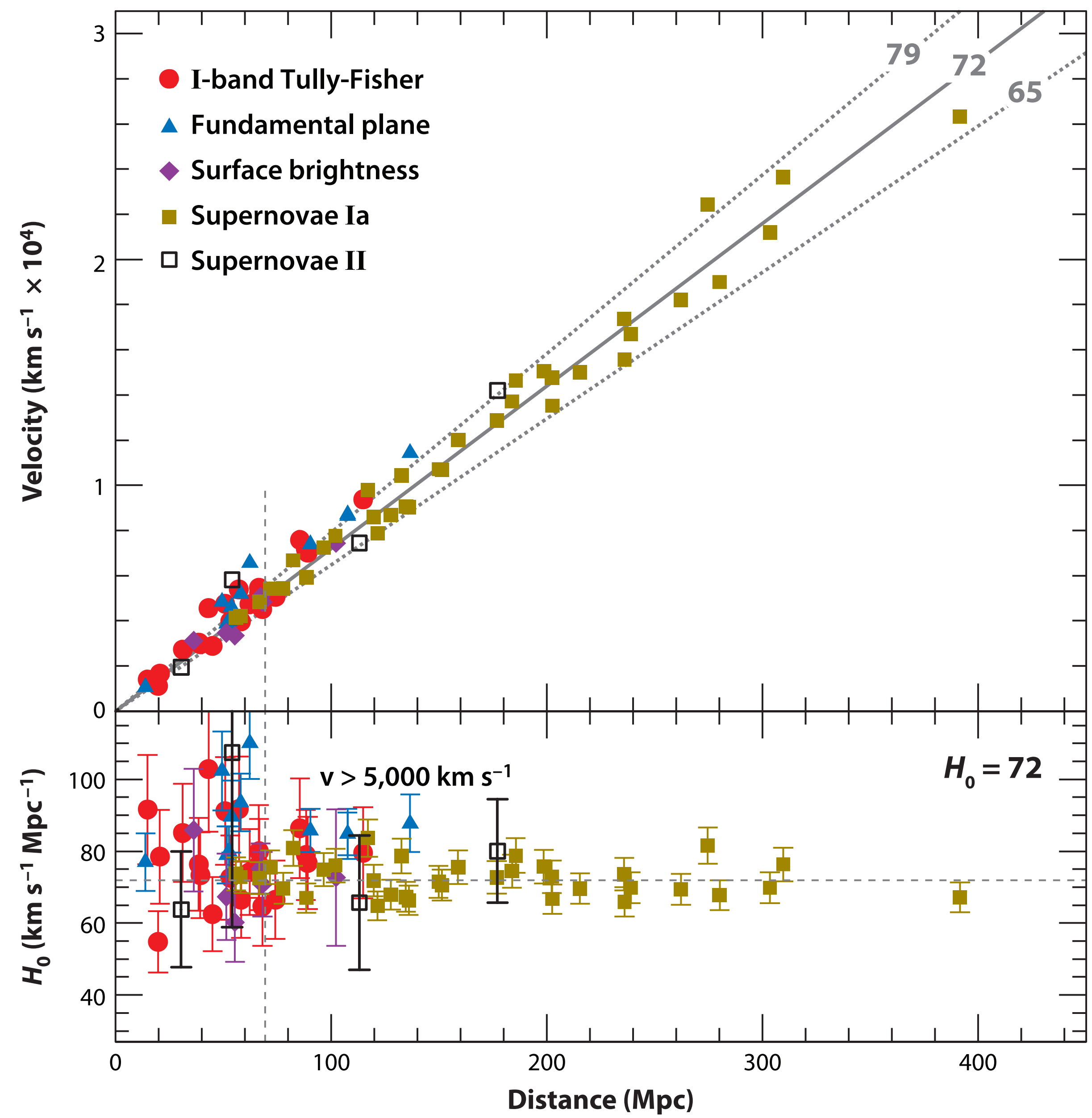




Graphical results of the *Hubble Space Telescope* Key Project (Freedman et al. 2001). (*Top*) The Hubble diagram of distance versus velocity for secondary distance indicators calibrated by Cepheids. Velocities are corrected using the nearby flow model of Mould et al. (2000). Dark yellow squares, Type Ia supernovae; filled red circles, Tully-Fisher (TF) clusters (I-band observations); blue triangles, fundamental plane clusters; purple diamonds, surface brightness fluctuation galaxies; open black squares, Type II supernovae. A slope of $H_o = 72 \pm 7$ km s^{-1} Mpc^{-1} is shown (*solid and dotted gray lines*). Beyond 5,000 km s^{-1} (*vertical dashed line*), both numerical simulations and observations suggest that the effects of peculiar motions are small. The Type Ia supernovae extend to about 30,000 km s^{-1}, and the TF and fundamental plane clusters extend to velocities of about 9,000 and 15,000 km s^{-1}, respectively. However, the current limit for surface brightness fluctuations is about 5,000 km s^{-1}. (*Bottom*) The galaxy-by-galaxy values of H_o as a function of distance.

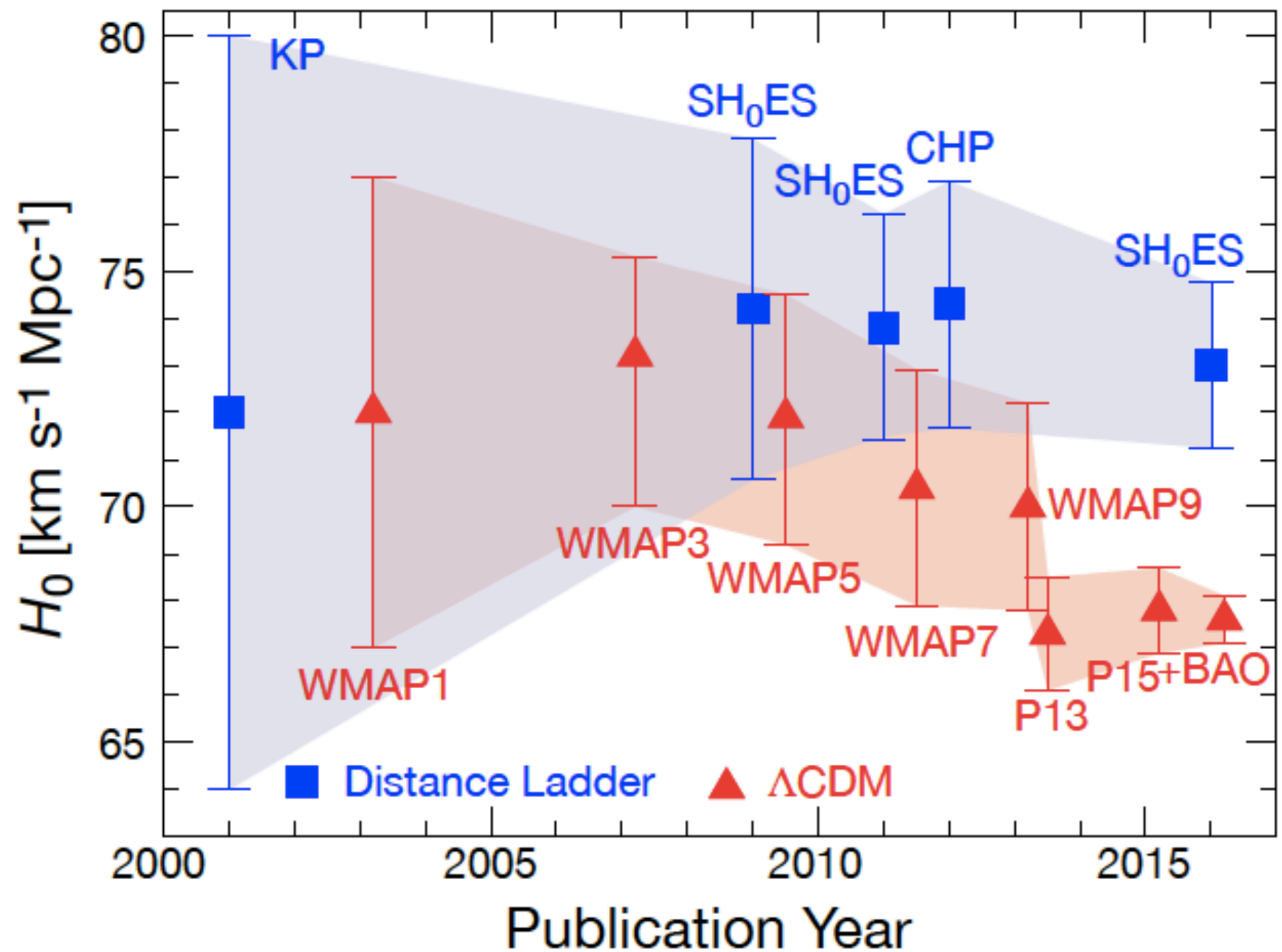
- Blue: determined from nearby universe with a calibration based on the Cepheid distance scale - distance ladder. (SNIa measurements)

- Red: from early universe CMB physics

- shaded: evolution of uncertainties

- Obvious tension between CMB and SNIa measurements: disagreement at greater than 3σ

- on other hand, remarkable: measurements/results entirely independent of each other; and universe evolved 13.8 billion years since surface of last scattering of CMB and present day



- Is this a real discrepancy or unknown systematic errors? Does it point to physics beyond the current standard model: e.g.:
– evolving dark energy?; modified gravity?

[W.Freedman, 1706.02739]

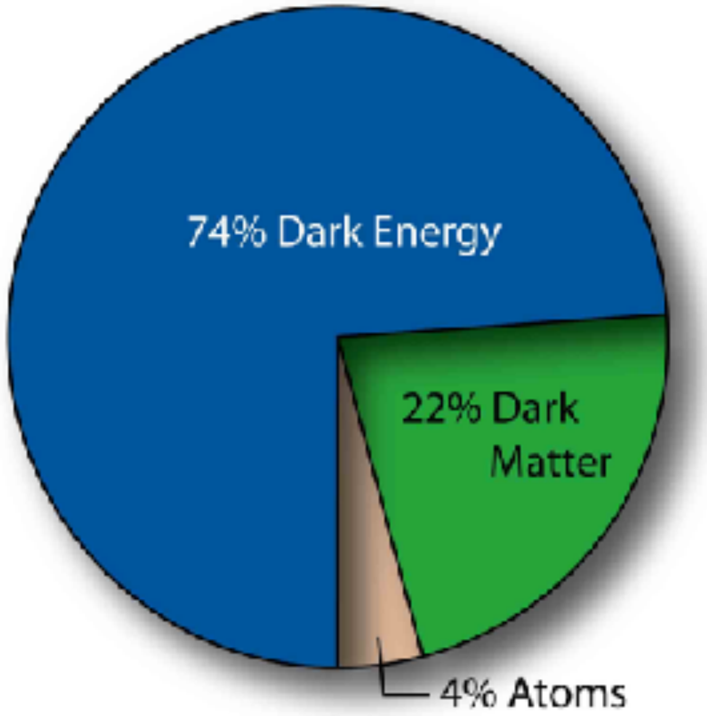
• Gravitational waves from **individual sources at cosmological distances** (e.g. binary black holes, binary neutron stars…) have the potential to give a ***totally independent measurement*** of H_0

[B.Schultz, Nature, 1986] standard sirens

• LIGO-Virgo: sensitive to small z, $z \lesssim 0.1$

• LISA: probe expansion of universe up to $z \lesssim 8$

Not only H_0 , but also much information about
– cosmological parameters; $\Omega_M, \Omega_\Lambda, w, k$
– type of dark energy (cosmological constant, quintessence..),
– matter content of universe
– modified gravity…



Outline

1. ***Standard sirens***

2. ***Stochastic background*** from early universe sources

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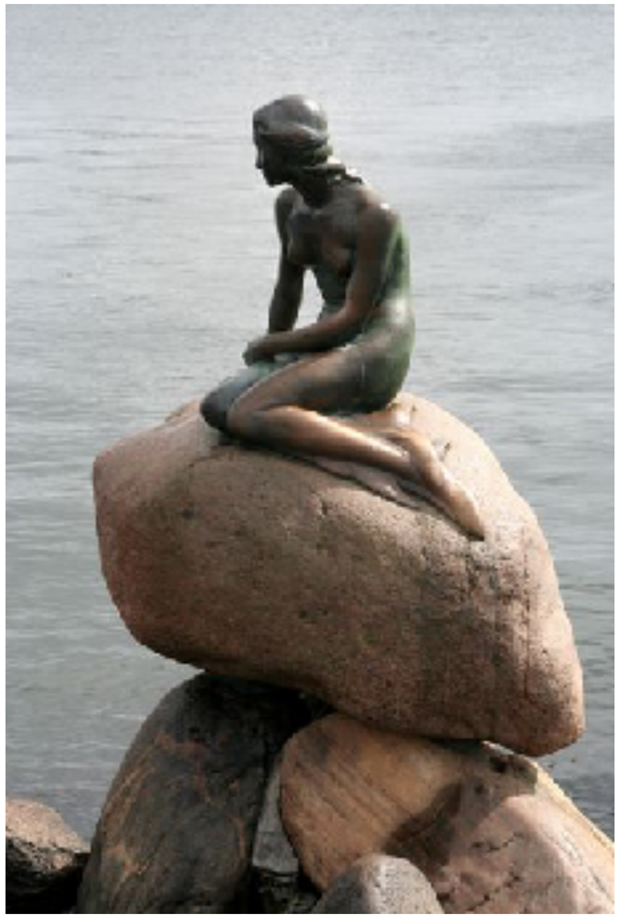
1) Probing late-time cosmology through GW from binaries of Black Holes or Neutron Stars or..

STANDARD SIRENS *[B.Schutz, 1986]*



1) Probing late-time cosmology through GW from binaries of Black Holes or Neutron Stars or..

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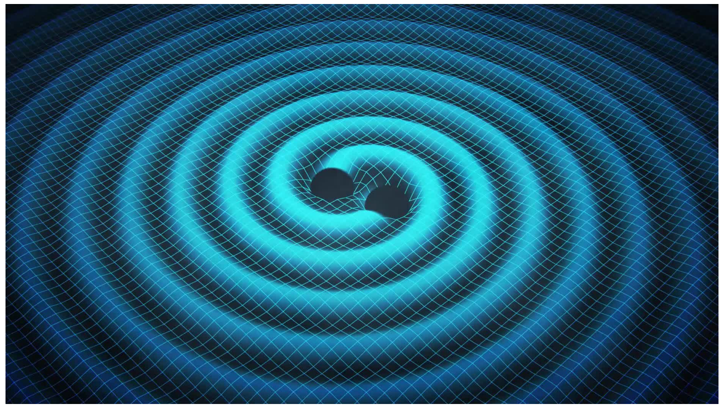
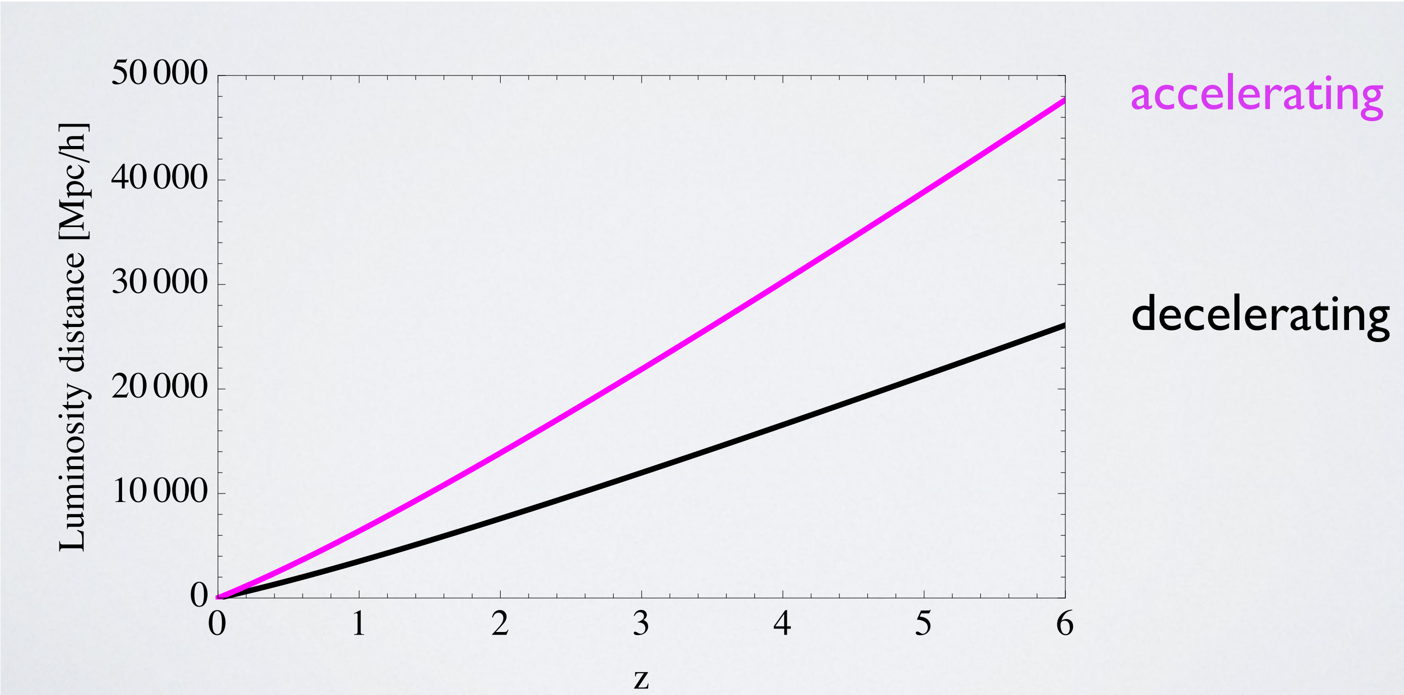


- Detect GWs emitted by coalescing binaries
- From the waveform, measure directly the luminosity distance $d_L(z)$
- If, in addition, can determine the redshift z of the source, then have a point on curve $d_L(z)$



i.e. direct probe of cosmology,

$$D_{\rm L} = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{\left[\Omega_{\rm M}(1+z')^3 + \Omega_\Lambda(1+z')^{3(1+w)}\right]^{1/2}}.$$

• For low redshift, $z \ll 1$, the relationship reduces simply to the Hubble law:

$$c\,z = H_0 \times d_L$$

redshift

Hubble constant

Luminosity distance

• three quantities: pick any two and infer the third.

• With standard sirens:

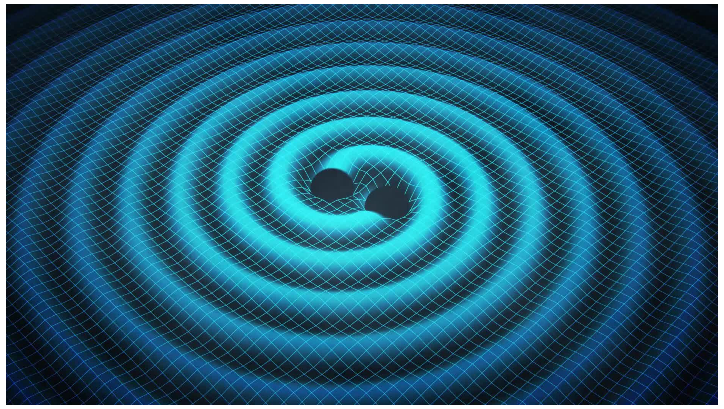
d_L from GW measurements;

z from, e.g. electromagnetic measurements (if have an optical counterpart, and know the host galaxy, can determine z).

=> independent measure of H_0

1) Not trivial as galaxies are moving wrt Hubble flow: need to take into account bulk flows, virial velocities, …

Standard sirens vs Standard candles

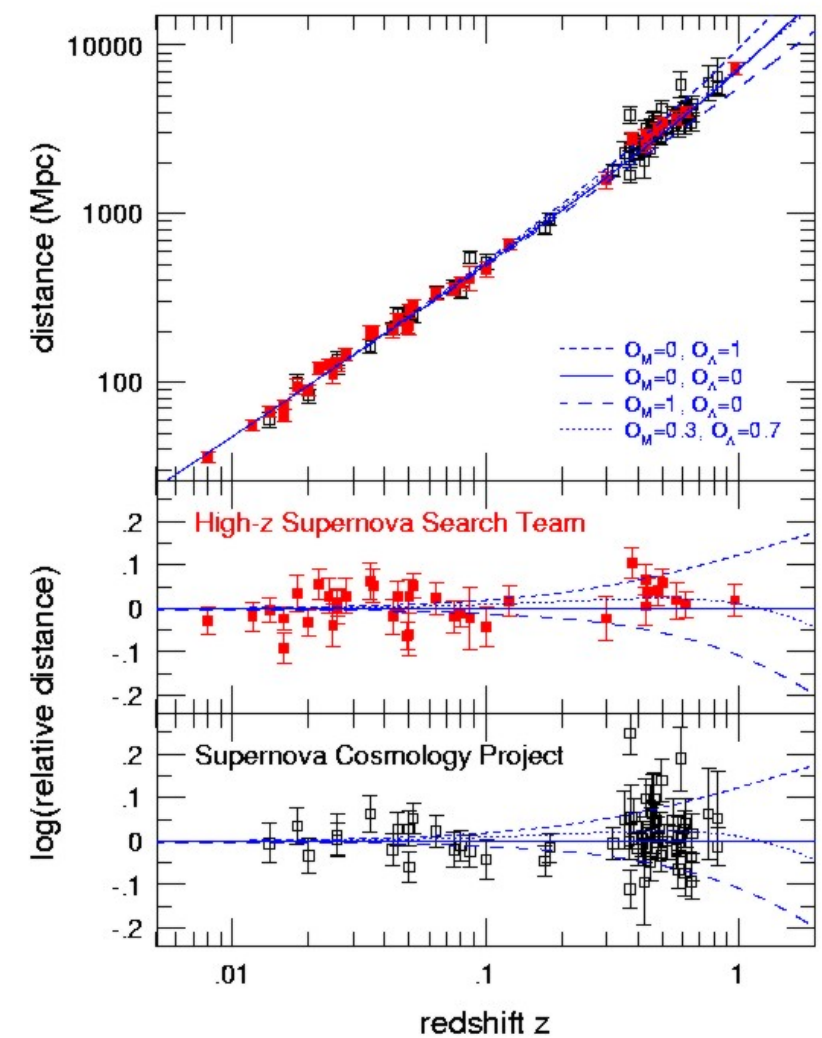


vs

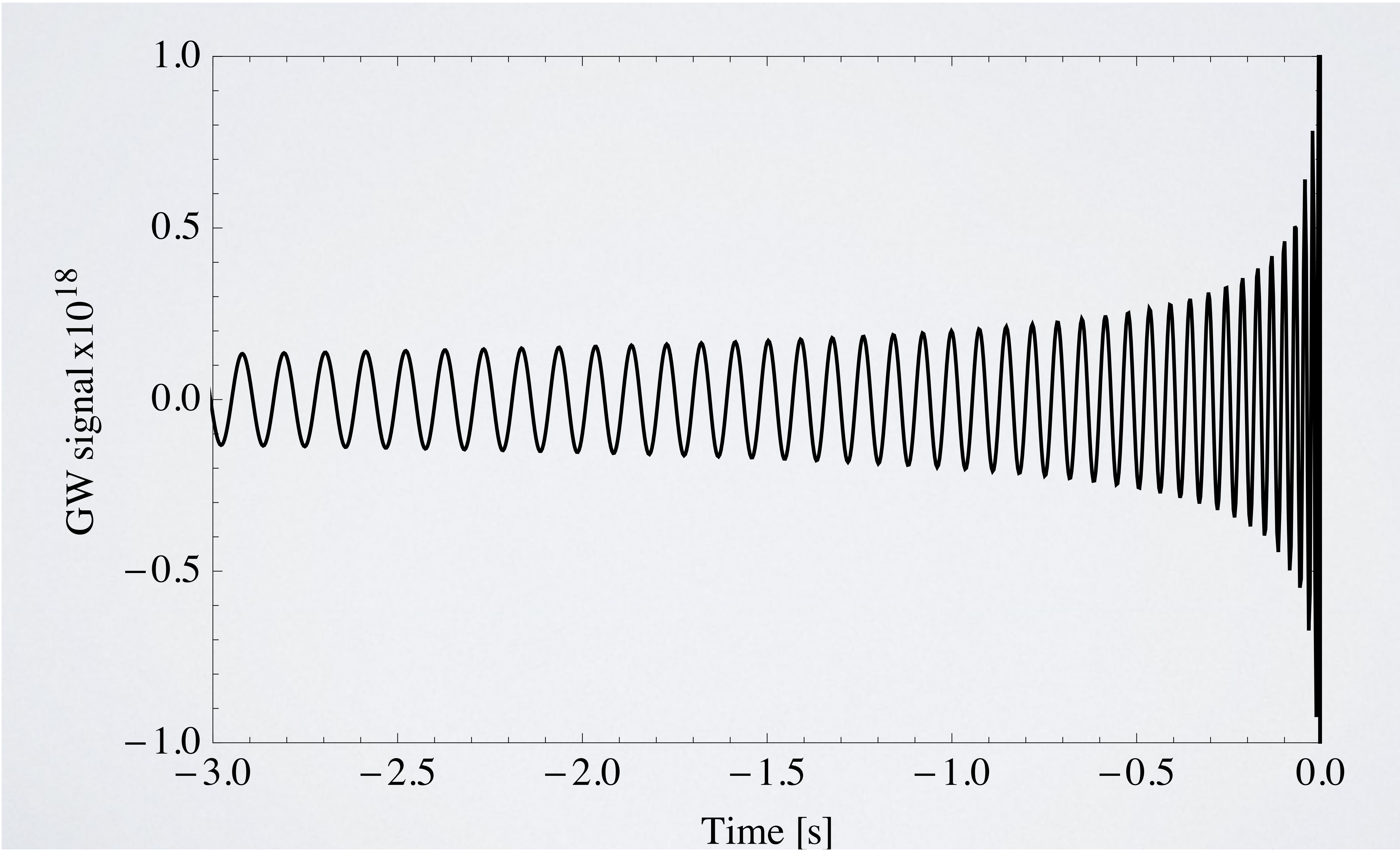
"*standard candles*" = objects the emit same luminosity (energy)

z measured directly

d_L harder: inferred from luminosity and observed flux



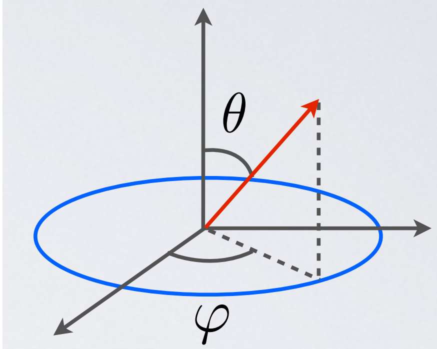
- "*standard sirens*" do NOT emit the same energy: waveform depends on the system.



1.0
0.5
0.0
−0.5
−1.0
GW signal x10^18
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
Time [s]

A) **First step: ignore expansion the universe**

• the standard calculation
(*point particles of mass m1 and m2*; no tidal effects, no spins,…, assuming circular orbit; and on using quadrupole formula)



$$f_{\mathrm{GW}} = \frac{\omega_{\mathrm{GW}}}{2\pi} = \frac{2\omega_B}{2\pi}$$

$$h_+(t,\theta,\varphi) = \frac{4}{r}(GM_c)^{5/3}\big(\pi f_{\mathrm{GW}}(\tau)\big)^{2/3}\frac{1+\cos^2\theta}{2}\cos\big(\phi(\tau)\big)$$

$$h_\times(\tau,\theta,\varphi) = \frac{4}{r}(GM_c)^{5/3}\big(\pi f_{\mathrm{GW}}(\tau)\big)^{2/3}\cos\theta\sin\big(\phi(\tau)\big)$$

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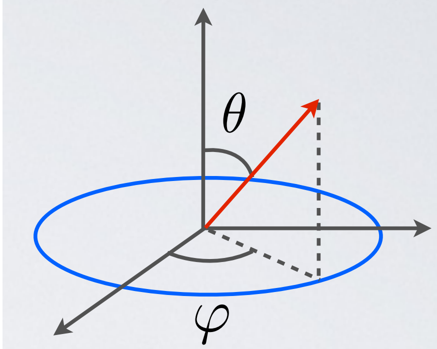

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distance to source

chirp mass

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

GW frequency as a function of

$$\tau = t - t_c$$

Phase:

$$\phi(t) = 2\pi\int_{t_c}^{t} dt'\, f_{\rm GW}(t')$$

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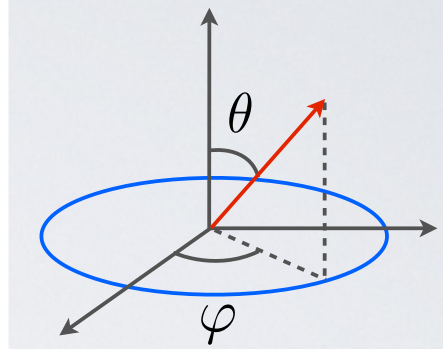

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Phase:

$$\phi(t) = 2\pi\int_{t_c}^{t} dt' f_{\rm GW}(t')$$

- The frequency is time dependent; power is radiated in GW:

$$\dot{f}_{\rm GW} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_{\rm GW}^{11/3}$$

with solution

$$f_{\rm GW} = \frac{1}{\pi}(GM_c)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8}$$

- so

$$\phi(\tau) = -2\left(\frac{\tau}{5GM_c}\right)^{5/8} + \phi_c$$

B) **Now include expansion: GW source at cosmological distance**

- Perturbed Friedmann-Robertson-Walker-Lemaitre metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$|h_{ij}| \ll 1$$

$$h_i^i = \partial_j h^j{}_i = 0$$

- wave equation: $\ddot{h}_{ij} + 3H\dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$

source: tensor anisotropic stress

- Close to the source, expansion should be negligible: use previous solution, with replacement

$$r \to r_{\rm phys} = a_S r$$

$$h_+(\tau_S) = \frac{4}{\boxed{a_S r}}(GM_c)^{5/3}\left(\pi f_S(\tau_S)\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos\left(\phi_S(\tau_S)\right)$$

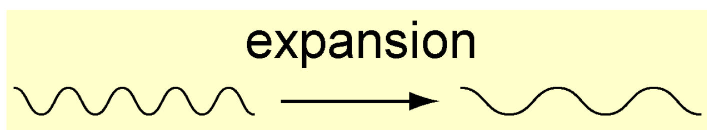
$$h_\times(\tau_S) = \frac{4}{\boxed{a_S r}}(GM_c)^{5/3}\left(\pi f_S(\tau_S)\right)^{2/3}\cos\theta\sin\left(\phi_S(\tau)\right)$$

$$\frac{df_S}{dt_S} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3}f_S^{11/3}$$

• In going <u>from the source to the observer</u>, need to take into account that the wave propagates in an expanding universe:

$$\Box \bar{h}_{\mu\nu} = 0 \qquad\qquad \frac{1}{r} \to \frac{1}{ar}$$

• wavelength *stretched* by the expansion of the universe



$$f_O = \frac{f_S}{1+z}$$

• redshift: $$1+z = \frac{a_O}{a_S}$$

(note: in the real universe, inhomogeneities+ anisotropies induce perturbations in the redshift, and hence modify this expression)

• Time intervals also affected by the expansion, so that the rate of change of frequency of the GW

$$dt_O = (1+z)dt_S$$

$$\frac{df_S}{dt_S} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_S^{11/3}$$

$$(1+z)\frac{d\left[f_O(1+z)\right]}{dt_O} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_O^{11/3}(1+z)^{11/3}$$

- **IF** we assume that z is constant during the time of observation (NOT necessarily a good approximation for LISA)

$$(1+z)\frac{d[f_O(1+z)]}{dt_O} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_O^{11/3}(1+z)^{11/3}$$

$$\frac{df_O}{dt_O} = \frac{96}{5}\pi^{8/3}(G\mathcal{M}_c(z))^{5/3} f_O^{11/3}$$

$$\mathcal{M}_c = (1+z)M_c$$

$$f_O(\tau_O) = \frac{1}{\pi}(G\mathcal{M}_c)^{-5/8}\left(\frac{5}{256\,\tau_O}\right)^{3/8}$$

Redshift absorbed in a shift in the chirp mass

- How about the phase at the observer?

$$\phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{5/8} + \phi_c$$

- Note that $\phi_O(\tau_O) = \phi_S(\tau_S)$ reflecting the fact that the phase is constant along null geodesics

- Collecting everything together:

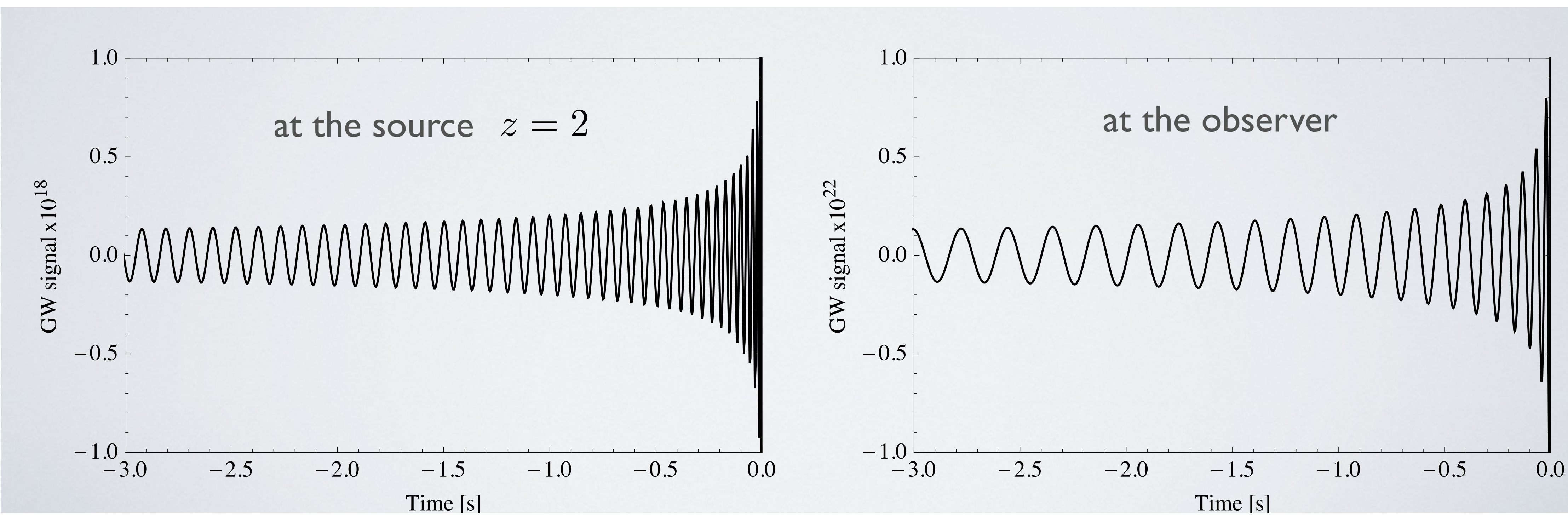
luminosity distance

$$h_+(\tau_O) = \frac{4}{a_O r(1+z)} (G\mathcal{M}_c)^{5/3} \left(\pi f_O(\tau_O)\right)^{2/3} \frac{1+\cos^2\theta}{2} \cos\left(\phi_O(\tau_O)\right)$$

$$h_\times(\tau_O) = \frac{4}{a_O r(1+z)} (G\mathcal{M}_c)^{5/3} \left(\pi f_O(\tau_O)\right)^{2/3} \cos\theta \sin\left(\phi_O(\tau_O)\right)$$

$$d_L = a_O r(1+z) = \sqrt{\frac{L}{4\pi F}}$$

- Impact of expansion: dilute the amplitude with d_L, and redshift the phase.



- **Degeneracy**: binary with parameters (m, S) at redshift z has same phase evolution as binary with parameters $((1+z)m, (1+z)^2 S)$ at redshift 0.

Information

[Slide courtesy of C.Bonvin]

- What can we **learn** if we measure h_+ and $h_\times$?

- We measure $$\frac{df_O}{dt_O} = \frac{96}{5}\pi^{8/3}(G\mathcal{M}_c(z))^{5/3} f_O^{11/3}$$

→ measurement of the redshifted **chirp mass** $\mathcal{M}_c$

- Ratio of the amplitude $$\frac{A_+}{A_\times} = \frac{1+\cos^2\theta}{2\cos\theta}$$

→ measurement of the **orientation** of the binary

We can measure directly the **luminosity distance**

$$h_+(\tau_O) = \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}\left(\pi f_O(\tau_O)\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos\left(\phi_O(\tau_O)\right)$$

Have the distance…but not the redshift!

- Ways to break z degeneracy:

1) Assume cosmology. Use GW-determined distance to infer redshift.

This is how redshift and restframe parameters are inferred for the GW events that have been announced so far.

2) Measure "electromagnetic" counterpart : optical, radio, X-rays, gamma-rays…
Independently determine z (EM) and luminosity distance (GW) — determine Hubble parameter and other cosmological parameters.

WHAT IS THE MOST PROMISING ELECTROMAGNETIC COUNTERPART OF A NEUTRON STAR BINARY MERGER?

B. D. Metzger[1,3] and E. Berger[2]
[1] Department of Astrophysical Sciences, Peyton Hall, Princeton University, Princeton, NJ 08544, USA
[2] Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA
Received 2011 August 30; accepted 2011 November 10; published 2012 January 24

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ABSTRACT

The final inspiral of double neutron star and neutron-star–black-hole binaries are likely to be detected by advanced networks of ground-based gravitational wave (GW) interferometers. Maximizing the science returns from such a discovery will require the identification of an electromagnetic counterpart. Here we critically evaluate and compare several possible counterparts, including short-duration gamma-ray bursts (SGRBs), "orphan" optical and radio afterglows, and day-long optical transients powered by the radioactive decay of heavy nuclei synthesized in the merger ejecta ("kilonovae"). We assess the promise of each counterpart in terms of four "Cardinal Virtues": detectability, high fraction, identifiability, and positional accuracy. Taking into account the search strategy for typical error regions of tens of square degrees, we conclude that SGRBs are the most useful to confirm the cosmic origin of a few GW events, and to test the association with neutron star mergers. However, for the more ambitious goal of localizing and obtaining redshifts for a large sample of GW events, kilonovae are instead preferred. Off-axis optical afterglows are detectable for at most tens of percent of events, while radio afterglows are promising only for energetic relativistic ejecta in a high-density medium. Our main recommendations are: (1) an all-sky gamma-ray satellite is essential for temporal coincidence detections, and for GW searches of gamma-ray-triggered events; (2) the Large Synoptic Survey Telescope should adopt a one-day cadence follow-up strategy, ideally with 0.5 hr per pointing to cover GW error regions; and (3) radio searches should focus on the relativistic case, which requires observations for a few months.

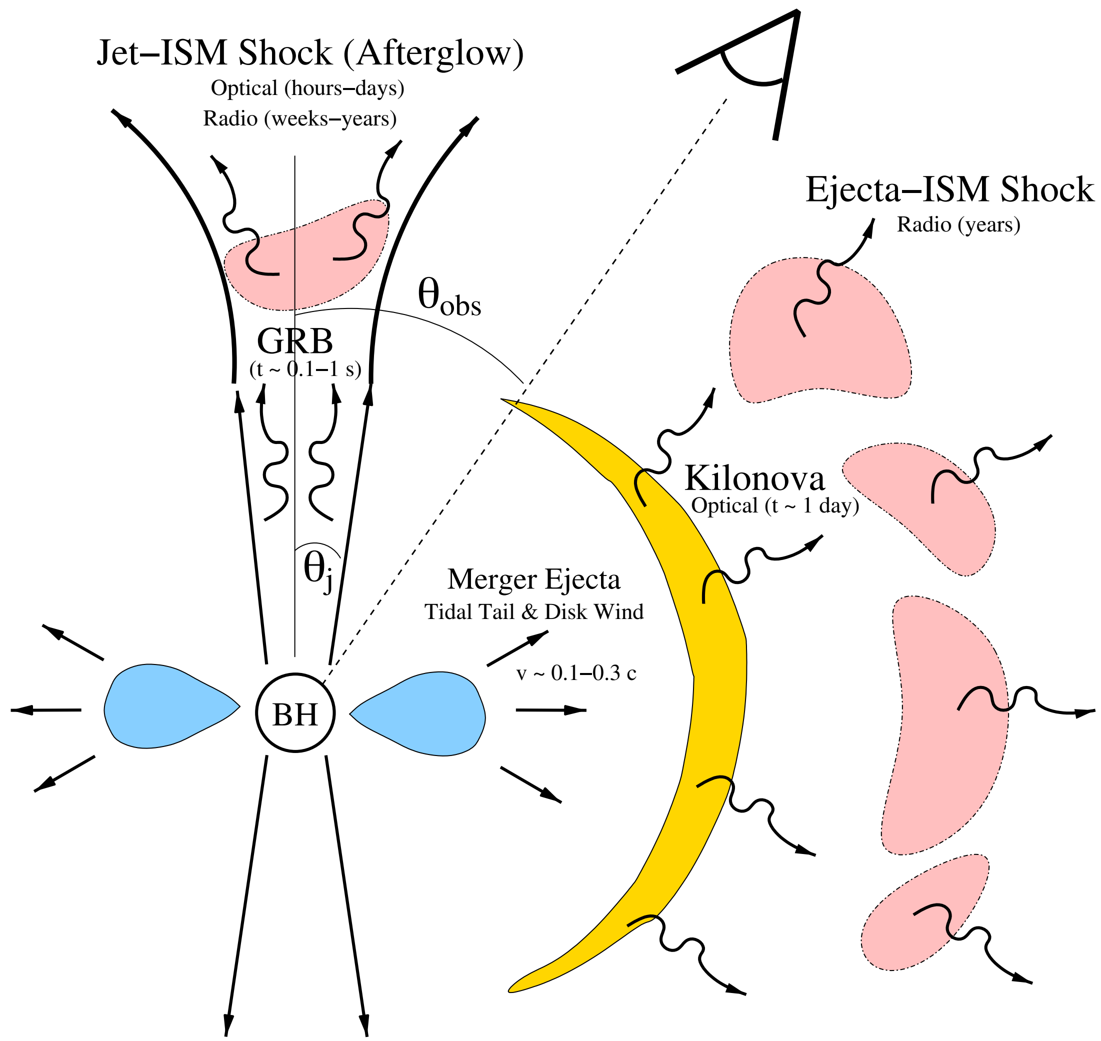


Figure 1. Summary of potential electromagnetic counterparts of NS–NS/NS–BH mergers discussed in this paper, as a function of the observer angle, $\theta_{\rm obs}$. Following the merger a centrifugally supported disk (blue) remains around the central compact object (usually a BH). Rapid accretion lasting $\lesssim$1 s powers a collimated relativistic jet, which produces a short-duration gamma-ray burst (Section 2). Due to relativistic beaming, the gamma-ray emission is restricted to observers with $\theta_{\rm obs} \lesssim \theta_j$, the half-opening angle of the jet. Non-thermal afterglow emission results from the interaction of the jet with the surrounding circumburst medium (pink). Optical afterglow emission is observable on timescales up to $\sim$ days–weeks by observers with viewing angles of $\theta_{\rm obs} \lesssim 2\theta_j$ (Section 3.1). Radio afterglow emission is observable from all viewing angles (isotropic) once the jet decelerates to mildly relativistic speeds on a timescale of weeks–months, and can also be produced on timescales of years from sub-relativistic ejecta (Section 3.2). Short-lived isotropic optical emission lasting ~few days (kilonova; yellow) can also accompany the merger, powered by the radioactive decay of heavy elements synthesized in the ejecta (Section 4).

• Other methods also proposed to determine z:

– if one knows the intrinsic mass function of neutron stars, and/or their equation of state, then by comparing with the observed - redshifted - mass, can extract the redshift of the source. *[Messenger+Reid, Taylor et al]*

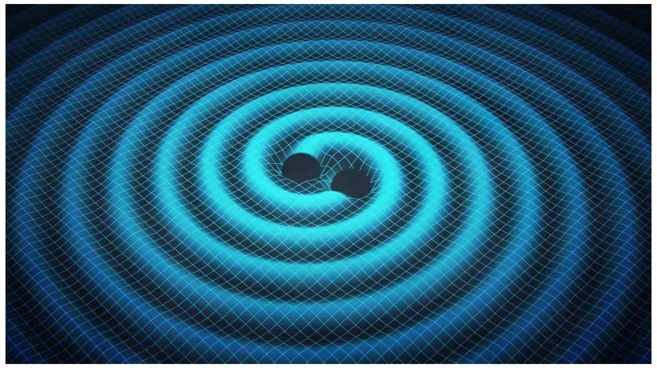
– If the source can be sufficiently well located, which should be the case with LISA, then can use galaxy catalogues to see what is in that portion of the sky. Using statistical methods then….

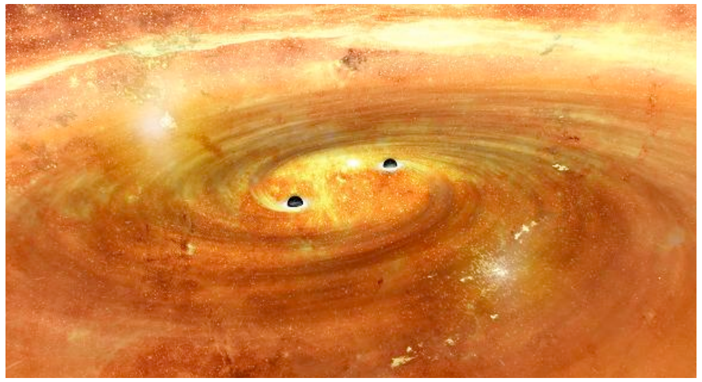
Inference of the cosmological parameters from gravitational waves: application to second generation interferometers

Walter Del Pozzo[1,2] 2011

I show that combining the results from few tens of observations from a network of advanced interferometers will constrain the Hubble constant H_0 to an accuracy of $\sim 4-5\%$ at 95% confidence.

Standard sirens for LISA





- How many standard sirens will be detected by LISA?
- What type of sources can be used?
- For how many will it be possible to observe a counterpart?

Possible standard sirens sources for LISA:

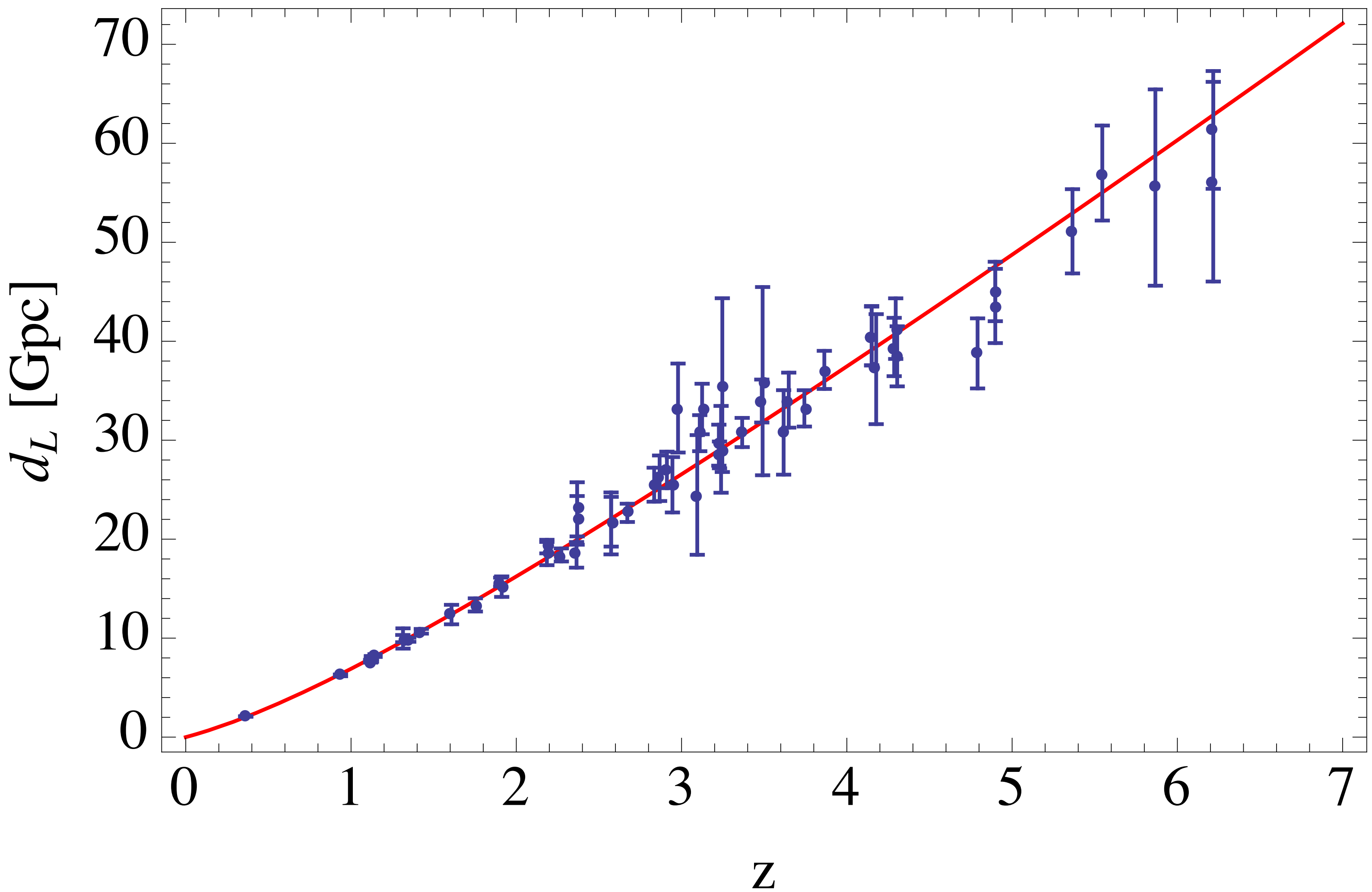
- $\boxed{\text{MBHBs } (10^4 - 10^7\ M_\odot)}$
- LIGO-like BHBs ($10 - 100\ M_\odot$)
- EMRIs

Advantages of MBHB mergers:

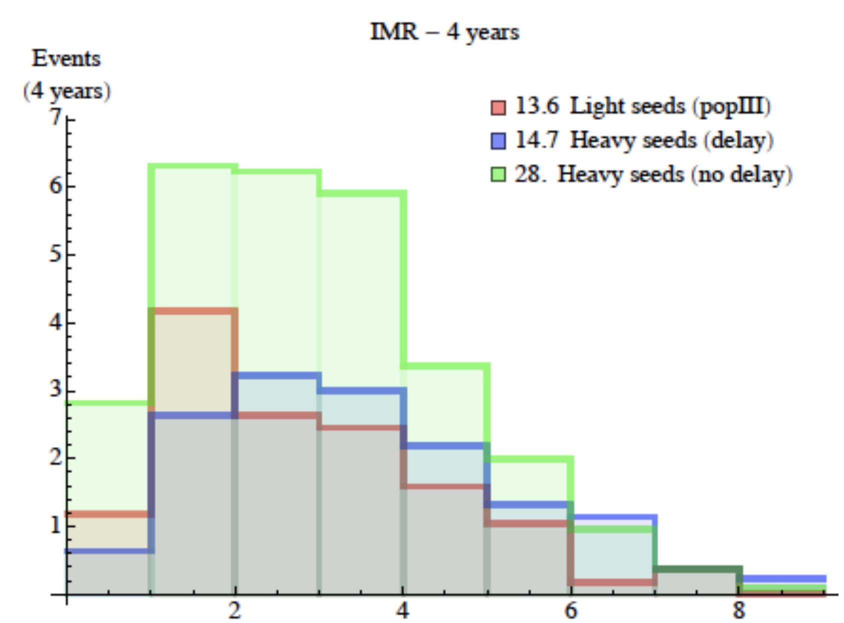
- High SNR
- High redshifts (up to ∼10-15)
- Merger within LISA band ↴
- Gas rich environment → *EM counterparts!*

LISA cosmological forecasts: MBHB standard sirens rate

[Tamanini et al, 1601.07112]



Simulated data with EM counterparts (SKA+EELT) and weak lensing errors



• few events at low redshift: bad for Dark energy, but can use other GW sources.

• LISA will be able to map expansions at very high z < 8.

=> can test expansion at high redshift.

[Slide courtesy of Tamanini, Caprini]

Standard sirens with LISA

[Tamanini et al, 1601.07112]

$\boxed{\Lambda\mathrm{CDM}}$

most optimistic scenario for BBH formation gives an independent measurement of the Hubble parameter to 1%

LISA alone :

$$\Omega_M = 0.3 \pm [0.05,\ 0.03]$$
$$h = 0.67 \pm [0.02,\ 0.01]$$

Planck alone :

$$\Omega_M = 0.308 \pm 0.0012$$
$$h = 0.678 \pm 0.009$$

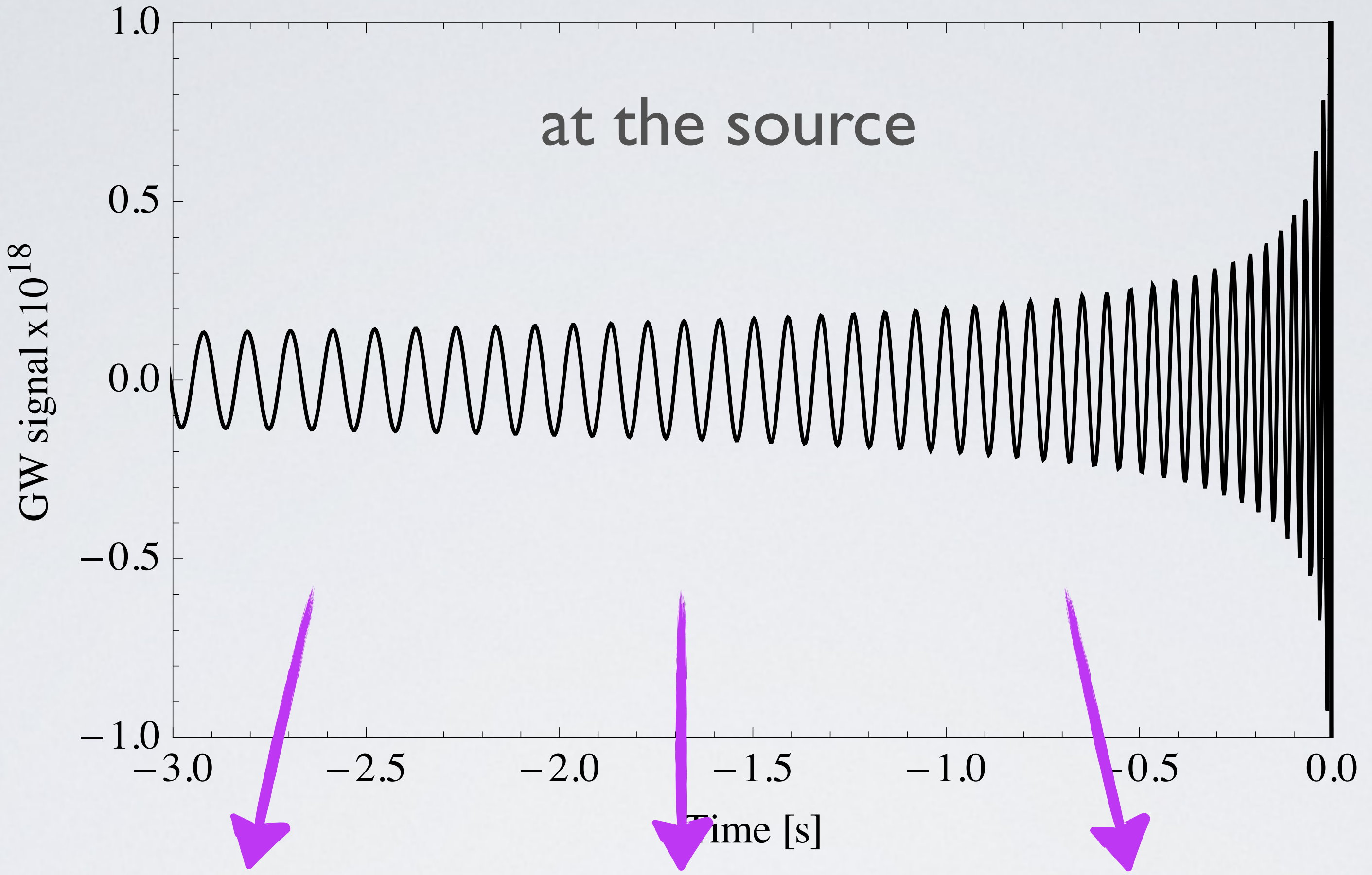
LISA fixing Ω_M :

$$h = 0.67 \pm [0.006,\ 0.004]$$

- fully independent constraint
- 0.6% in best case

[Slide courtesy of Tamanini, Caprini]

Evolving redshift



Different stretches at different times

→ **distortion** in the signal

Binary at cosmological distance

time variation of the redshift changes the evolution of the frequency with time as the binary chirps:

1. the background expansion of the universe varies during the time of observation of the binary

2. the redshift perturbations due to the distribution of matter between the GW source and the observer vary in time during the time of observation of the binary

main effect: peculiar acceleration of the binary centre of mass

[C Bonvin et al arXiv:1609.08093]

[Slide courtesy of Caprini]

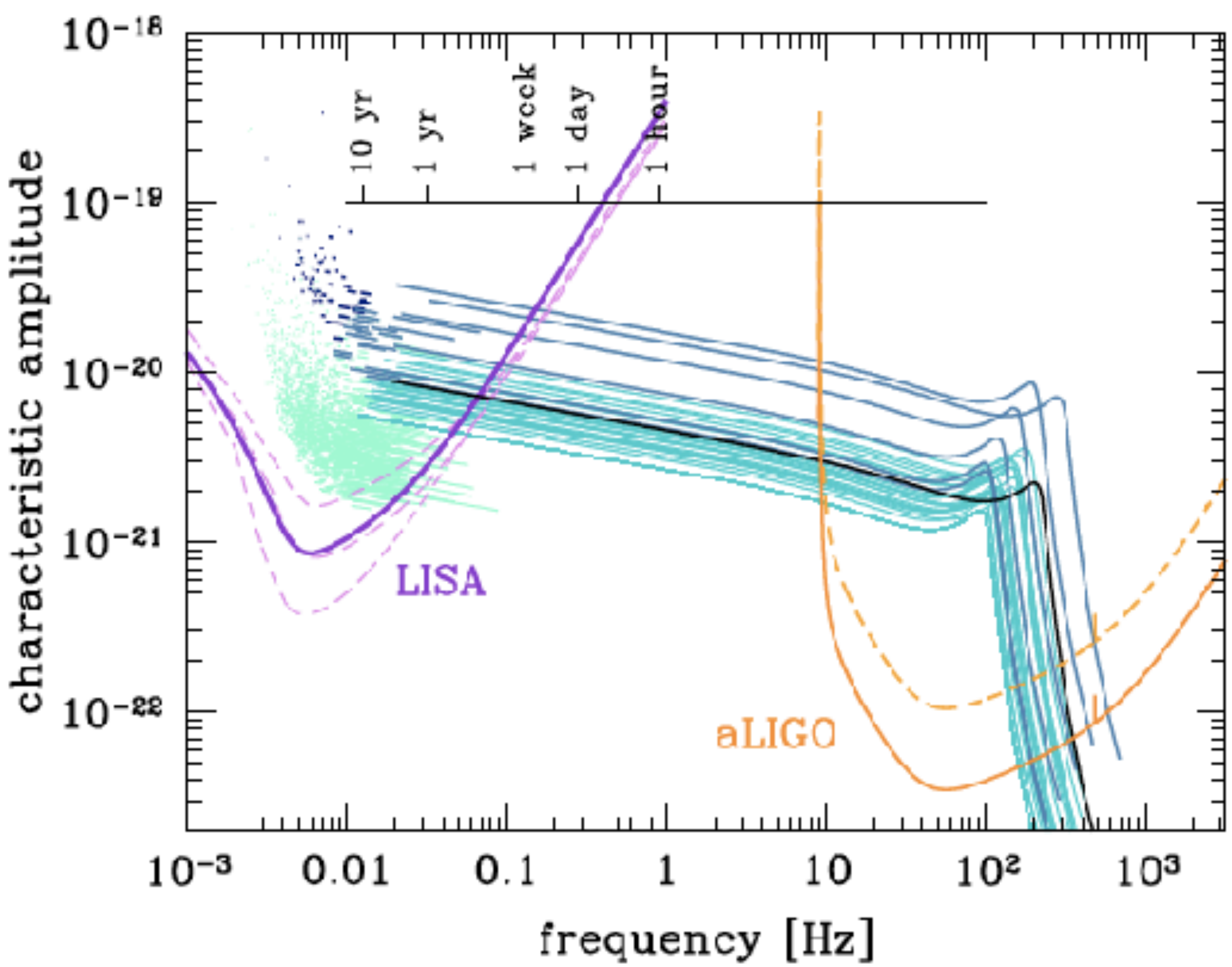
Extra term in the waveform phase

- Earth based interferometers are not sensitive to this effect: they do not follow the GW source for enough time

[Inayoshi et al 1702.06529]

- but this effect is <u>relevant for LISA</u>: binaries which stay in band for enough time, with *low chirp-mass, that enter the detector around ten mHz* and go to the LIGO band after ~5 years

- if not accounted for, it can introduce a *bias on the binary parameters* (time to coalescence,..)



[A Sesana arXiv:1702.04356]

[Slide courtesy of Caprini]

The stochastic GW background from early universe sources

The stochastic GW background from early universe sources

1) Basics
2) Importance for cosmology
3) Characteristic frequency for causal sources.
4) Existing experimental bounds.
5) Early universe sources, potentially detectable by LIGO/Virgo, LISA…
    - Inflation
    - Bubble collisions from Electroweak phase transition.
    - Cosmic strings

1) Basics. What is it?

• Stochastic background: superposition of GWs arriving at random times and from random directions —> overlapping so much that individual waves not detectable

• Assume that there are so many sources (astrophysical or cosmological): individual ones can't be distinguished.

• Appears in detectors as noise which, by central limit theorem —> Gaussian

• Competes with instrumental noise

• detectable by single detector if > instrument noise; if weaker then by ≥ 2 detectors looking for a correlated component of their noise

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}}{df},$$

$d\rho_{GW}$ energy density of GWs in frequency range f to f+df

$\rho_c = 3H_0^2/8\pi G$ critical energy density of universe.

• Can range over many decades in frequency—> probed/constrained by many different experiments, from pulsars at nHz to LIGO at ~Hz.

2) Importance?

- To do with *weak* interactions of GWs

  – reminder: particles that decouple from primordial plasma at $t \sim t_{dec}$ $T \sim T_{dec}$ give shapshot of state of universe at that time. Before, they are coupled and interactions obliterate all information.

  – The weaker the interactions, the earlier the particles decouple, and higher the energy scale when they drop out of thermal equilibrium.

- In thermal equilibrium

rate of process maintaining thermal equilibrium ← $$\Gamma \sim n\sigma|v| > H = \frac{\dot{a}}{a}$$

n: Number density of particles

σ: X-section for interaction

$|v|$: typical velocity

- Particles drop out of equilibrium when $\Gamma \sim H$

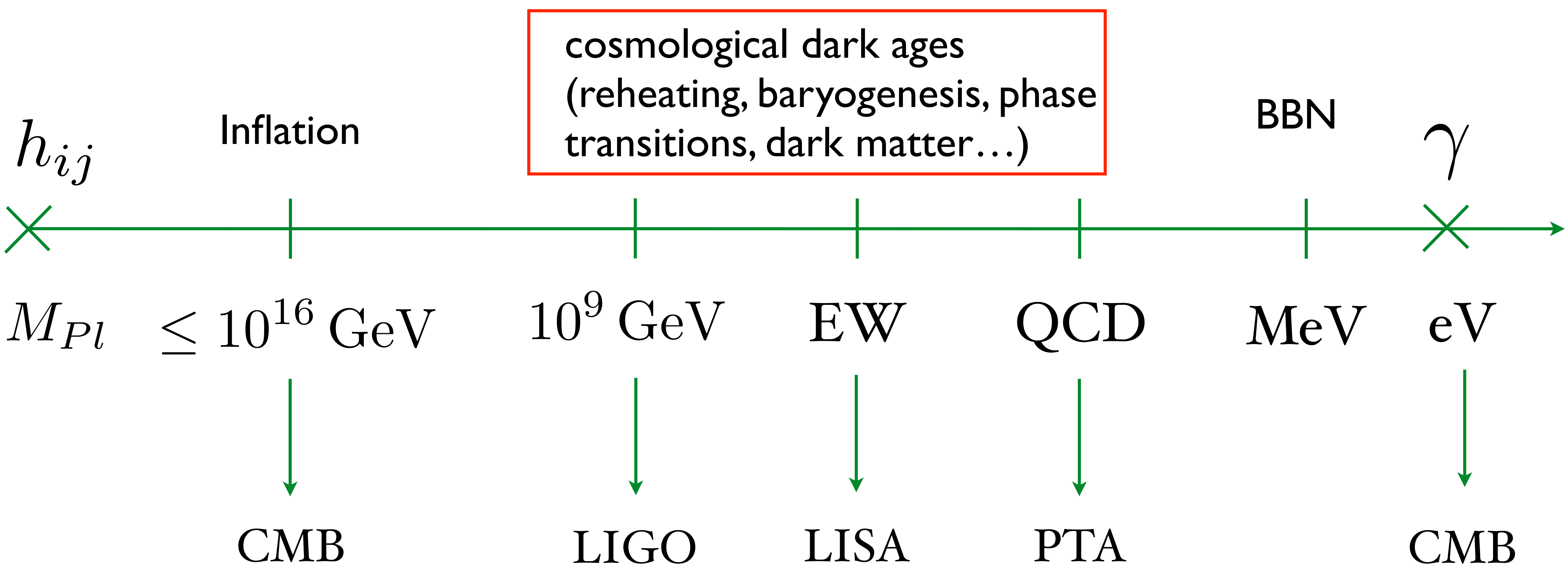
- <u>Neutrinos:</u> $\sigma \sim G_F^2 T^2$ $$\left(\frac{\Gamma}{H}\right)_{\rm neutrino} \sim \left(\frac{T}{1\mathrm{MeV}}\right)^3$$

For light/massless particles at temperature T

$n \sim T^3$, $v \sim 1$, $H^2 \sim T^4 M_{\rm Pl}^{-2}$

- **Gravitons:** $\sigma \sim G_N^2 T^2 = T^2 M_{\rm Pl}^{-2}$ $\qquad \left(\frac{\Gamma}{H}\right)_{\rm graviton} \sim \left(\frac{T}{M_{\rm Pl}}\right)^3$

– gravitons decoupled below Planck scale:
– do not loose memory of conditions when produced
– retain spectrum/shape/typical frequency & intensity of physics at corresponding high energy scales.



3) Characteristic frequency for *causal sources?*

- Depends on:

– production mechanism (model-dependent)
– kinematical (depending on the redshift from the production era)

- GWs produced with frequency $f_\star$ at $t = t_\star$ have characteristic frequency today of

$$f_c = f_\star \left(\frac{a_\star}{a_0}\right) = 2 \times 10^{-5} \left(\frac{f_\star}{H_\star}\right) \left(\frac{T_\star}{1\mathrm{TeV}}\right)$$

(assuming standard thermal history and radiation era)

- What about $f_\star$? Dynamics enters, but clearly $H(T_\star)$ is the relevant parameter.

Dynamics must be on time scales < $H(T_\star)$

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

$\epsilon_* \leq 1$ parameter depending on the dynamics of the source

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

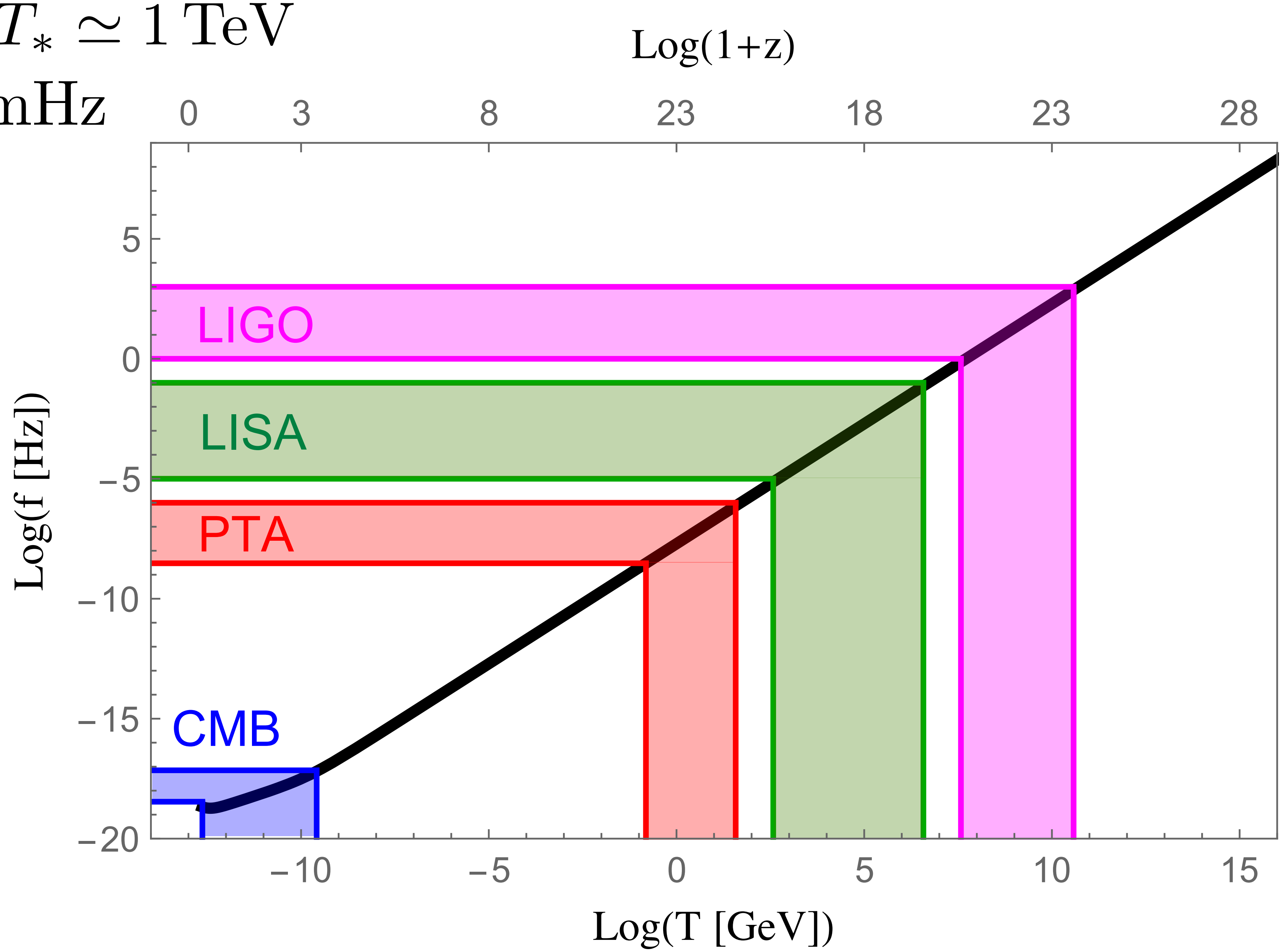
characteristic frequency today

temperature (energy density) of the universe at the source time

e.g., for electroweak scales $T_* \simeq 1\,\mathrm{TeV}$

with $\epsilon_* \simeq 10^{-2} \longrightarrow f_c \simeq \mathrm{mHz}$

LISA frequency



Can GWs probe the very high energy regime?

$$f_c = f_* \frac{a_*}{a_0} = \frac{2 \cdot 10^{-5}}{\epsilon_*} \frac{T_*}{1\,\mathrm{TeV}}\,\mathrm{Hz}$$

~GUT scales; inflationary scale $T_\star \sim 10^{16}\mathrm{GeV} \Longrightarrow f_c \sim 2\mathrm{GHz}$

~ Planck scales (q-gravity?) $T_\star \sim 10^{18}\mathrm{GeV} \Longrightarrow f_c \sim 100\mathrm{GHz}$

- seem totally inaccessible to ground based interferometers and LISA

- BUT: think again! True that

$$\Omega(f > f_c) \to 0$$

but spectrum for **lower** frequencies is **not** fixed by these arguments:
– *if* for e.g. it is flat when $f < f_c$ and with a sufficiently high amplitude, then it *could* be seen at lower frequencies

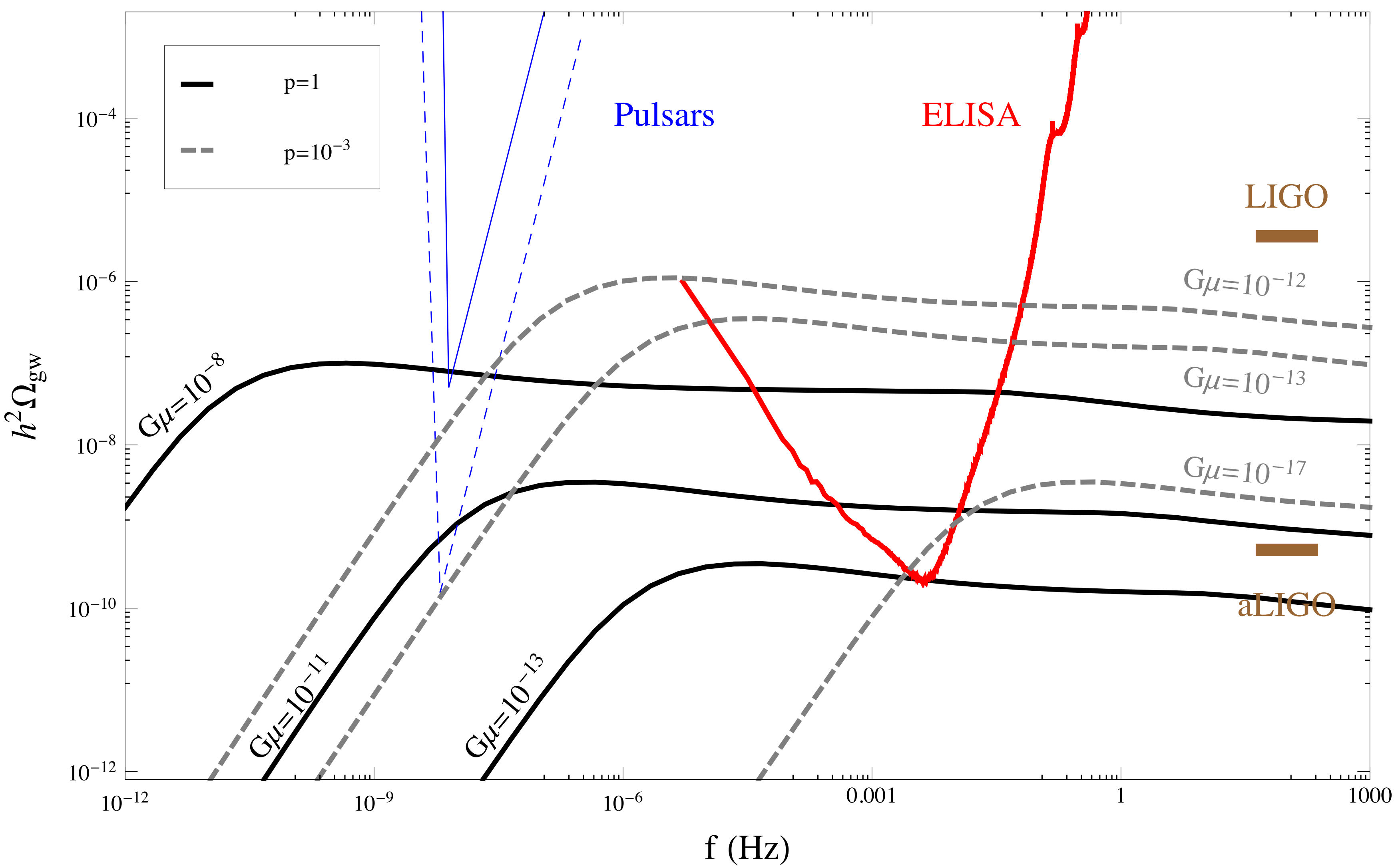
- As our examples will show, in many cases the spectrum is (nearly) flat over a large range of frequencies

4) Experimental bounds on $h^2\Omega_{\rm GW}(f)$

• Often freq dependence of $\Omega_{\rm GW}(f)$ determined by dynamics, but overall **amplitude** depends on parameters of model/cosmological mechanism producing the GWs

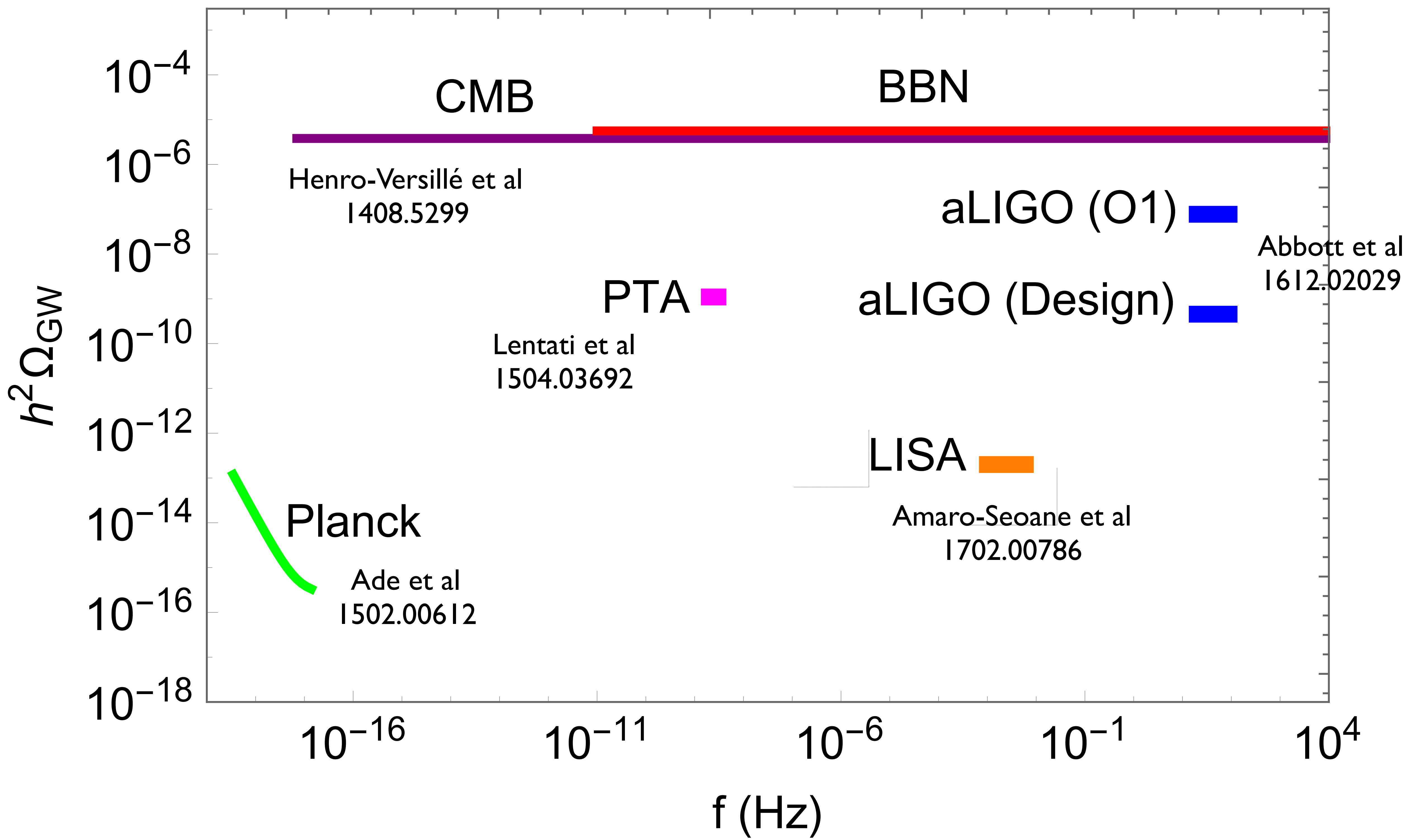
e.g. μ, the energy/unit length of a cosmic string;
n_T , for inflation.

• So experimental bounds at different frequencies constrain params of model.



Predicted spectrum for cosmic strings

[Bohé et al, 2012]



CMB
BBN
Henro-Versillé et al
1408.5299
aLIGO (O1)
Abbott et al
1612.02029
aLIGO (Design)
PTA
Lentati et al
1504.03692
LISA
Amaro-Seoane et al
1702.00786
Planck
Ade et al
1502.00612
$h^2\Omega_{GW}$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
f (Hz)

Nucleosynthesis bound $(T \sim \text{Mev},\ t \sim 1\text{sec})$

• *idea*: if have large $\Omega_{\rm GW}(f)$ at nucleosynthesis, then larger H

—> Feeds into a larger freeze out temperature, and higher ratio of neutrons to protons,
—> finally into over production of Helium 4

• Bound which applies to GWs generated before BBN

$$h_0^2 \Omega_{\rm GW}(f) \lesssim 5 \times 10^{-6}$$

Planck/CMB bound

• *idea*: stochastic bkgd of GW —> fluctuations in CMB temperature: GWs stretch and compress space, in which the decoupled CMB photons travel

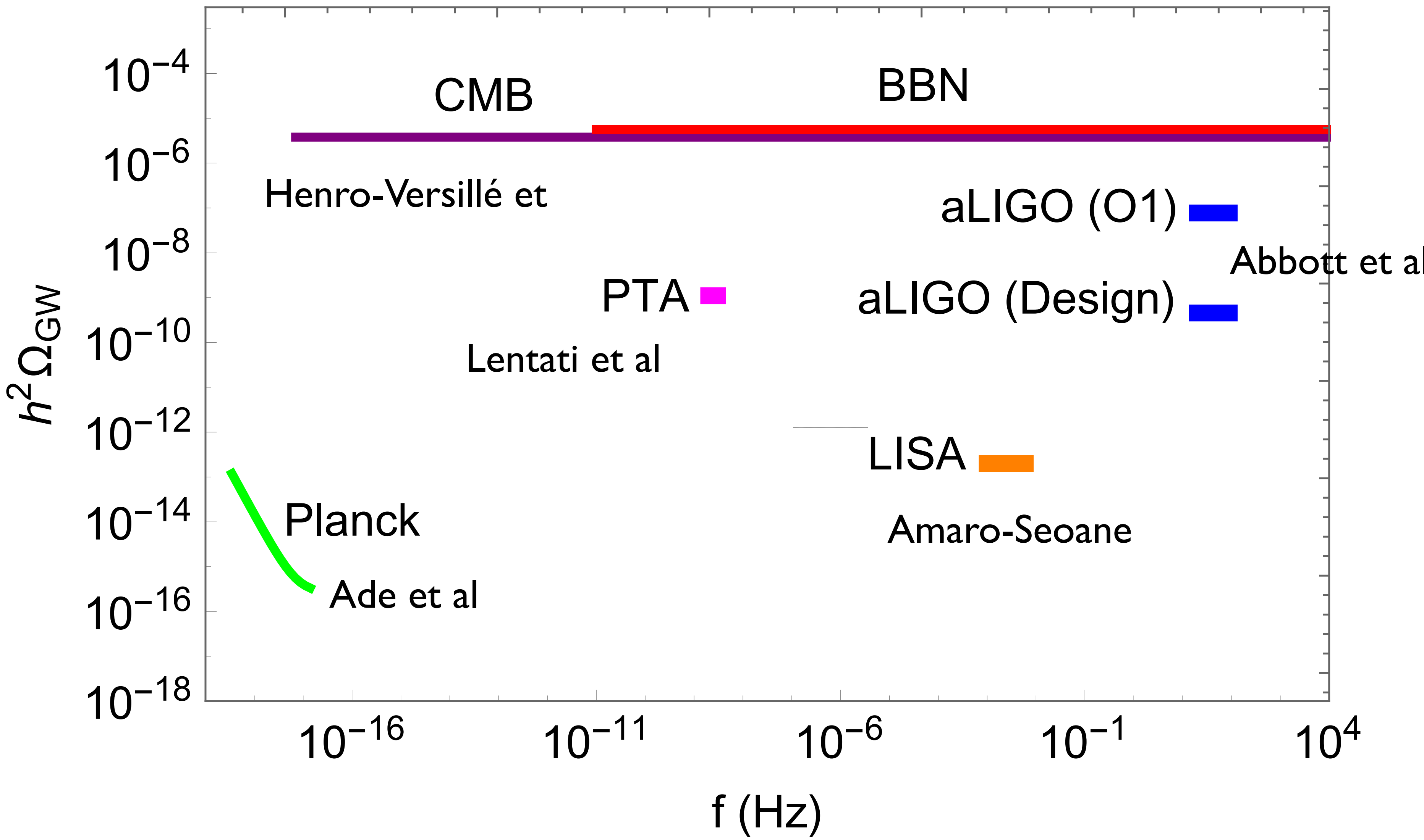
$$\frac{\Delta T}{T} = -\int_i^f \dot{h}_{ij} n^i n^j d\lambda$$

• Bound applies to GWs generated before photon decoupling, and in practice only relevant for generation mechanisms that produce spectra with significant amplitude on super horizon scales (eg inflation)

$$h_0^2 \Omega_{\rm GW}(f) \lesssim 7 \times 10^{-11} \left(\frac{H_0}{f}\right)^2 \qquad \frac{H_0}{2\pi} < f < \frac{H_{dec}}{2\pi}\frac{a_{dec}}{a_0}$$

• Tightest constraint is for large f

$$h_0^2 \Omega_{GW} \lesssim 10^{-16}$$



<u>Implications?</u>

• Often models (e.g. inflation) predict spectra ranging in frequency from $f \sim H_0 \sim 10^{-17} \longrightarrow f_c$

• In order for these spectra to be observable in the 1Hz-1kHz range, they must grow enough —> sizeable value at these frequencies!

Signal from a *simple slow roll inflation model* :

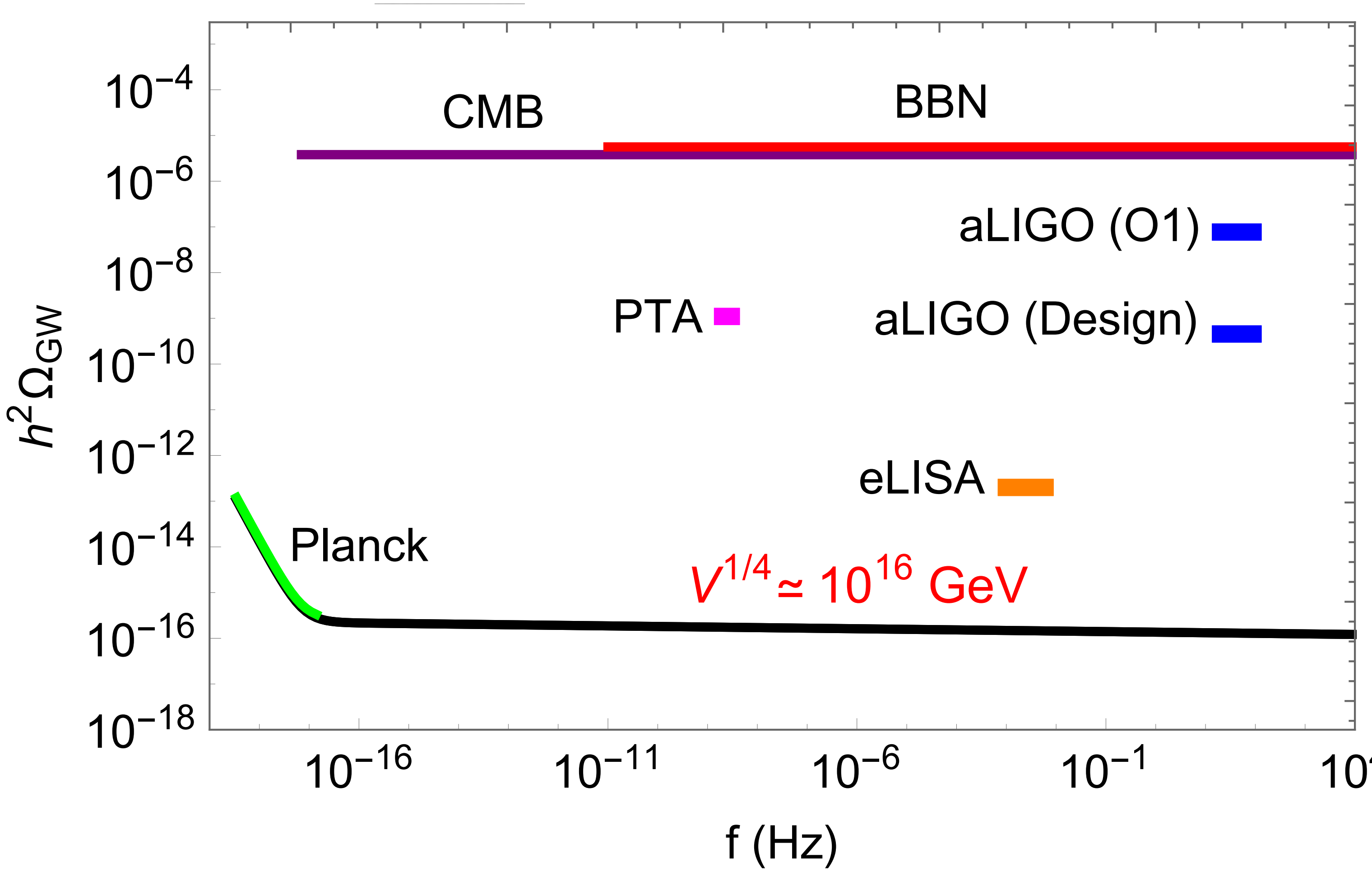
• amplification of vacuum fluctuations during inflation: $\ddot{h}_{ij} + 3H\dot{h}_{ij} + k^2 h_{ij} = 0$

• In de Sitter space (zeroth order in slow roll): scale invariant spectrum.
Hence **well beyond the reach of direct detection!**

• To first order in slow roll, even worse! $\Omega_{GW}(f) \sim f^{n_T}$ with $n_T = -r/8$
where, from CMB, $r \lesssim 0.1$ so $n_T < 0, \quad n_T \ll 1$

• To have an observational signal from inflation, will need to go to non-standard models

• Of course, though, GWs leave an imprint on CMB through polarisation pattern of B-modes which is a primary probe for its detection today.



PTA = Pulsar bound

• Pulsars = rapidly rotating+highly magnetized neutron stars

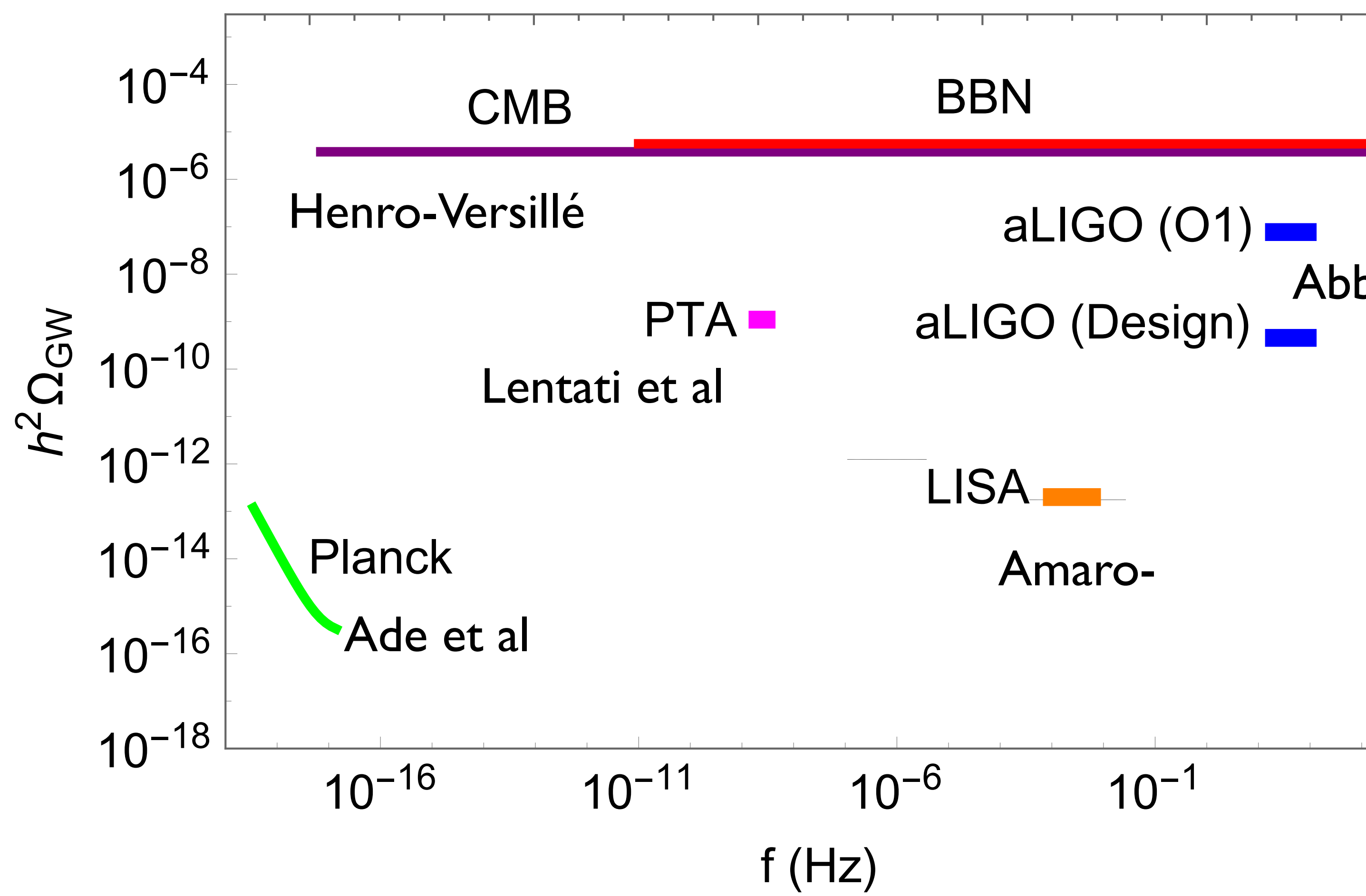
• Cosmic lighthouses: emit beam of EM radiation in direction of rotating magnetic axis.
=>Regular train of pulsed radiation reaching the earth each time the beam crosses observers line of sight.

• Arrival times predicted very accurately over long time scales —> stable clocks.

• Can be used as direct detectors of GWs, through fluctuation generated in the time of arrival of the pulse due to the GWs. *Pulsar timing.*

• Observations on time scales of a year

$$h_0^2\Omega_{GW}^{PTA}(f = 2.8\times 10^{-9}\mathrm{Hz}) < 2.3\times 10^{-10}$$



5) Other possible sources of GW in the early universe more promising for direct detection ?

(with future interferometers or PTA)

mechanisms that produce a non-zero tensor anisotropic stress

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

<u>considerable amount of energy (in some anisotropic form) is needed to generate a detectable signal</u>

Example: amplitude for detection with LISA:

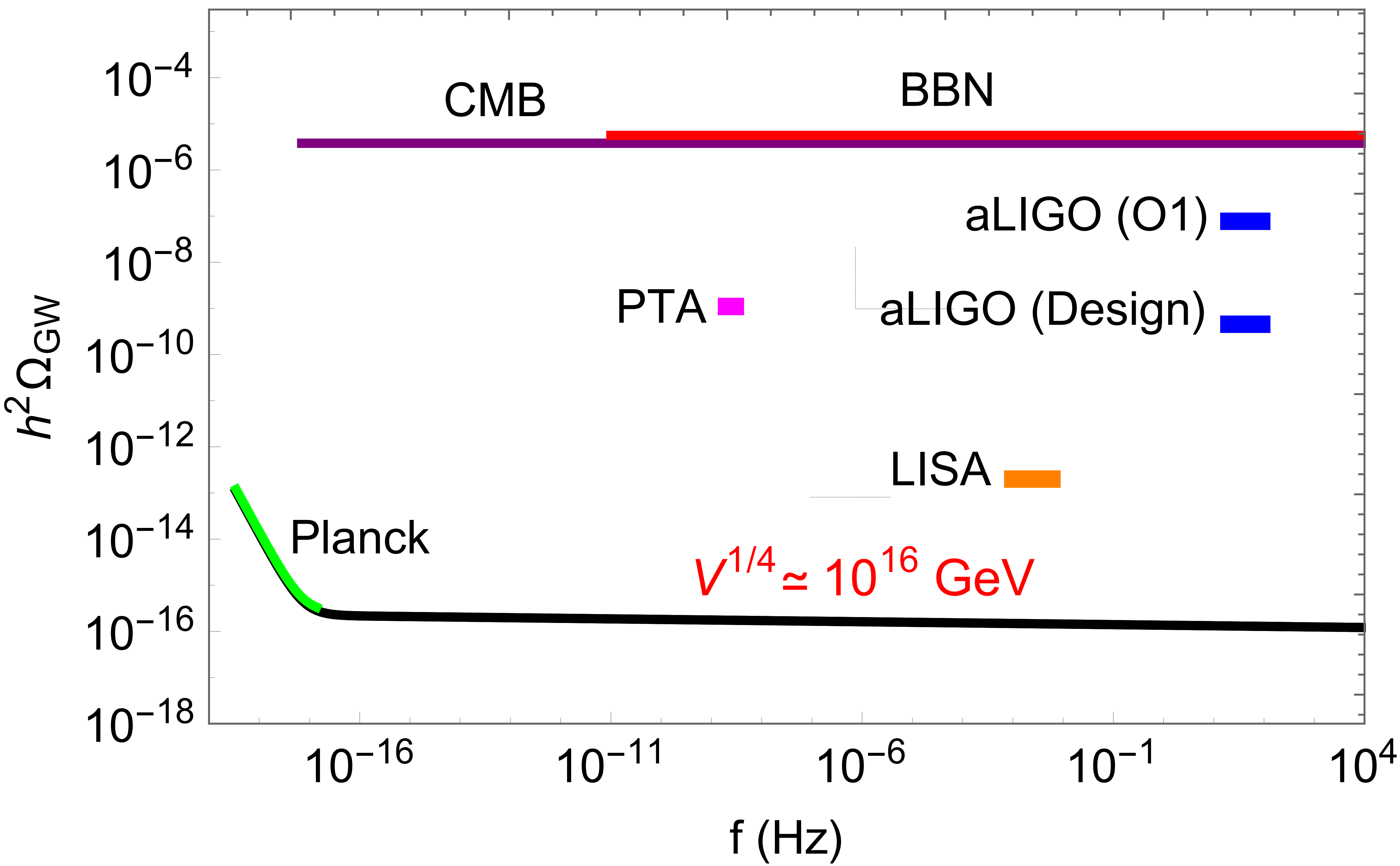
$$\boxed{\rho_{\rm s}^* \simeq 0.1\,\rho_{\rm tot}^*}$$

Possible GW sources in the early universe

- "non-standard" inflation
  - particle production during inflation
  - fluid stiffer than radiation after inflation
  - preheating after inflation
  - phase transitions at the end or during inflation
  - ...
- first order phase transitions
- cosmic strings
- other topological defects e.g. domain walls
- primordial black holes
- scalar field self-ordering
- …

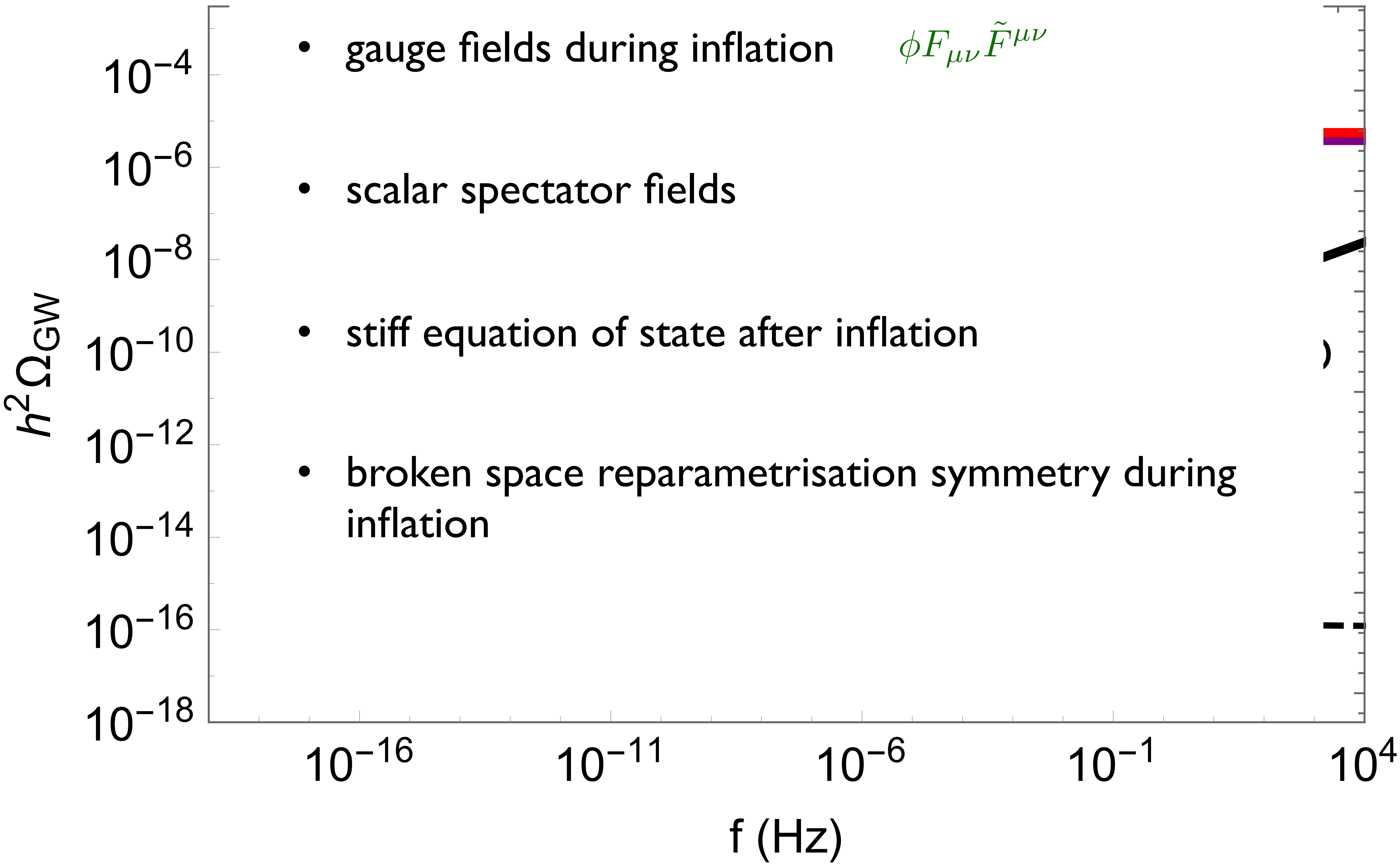
“Non-standard inflation”

- aim: get a blue tilted spectrum



“Non-standard inflation”

[N. Bartolo et al, 1610.06481]



- gauge fields during inflation $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$
- scalar spectator fields
- stiff equation of state after inflation
- broken space reparametrisation symmetry during inflation

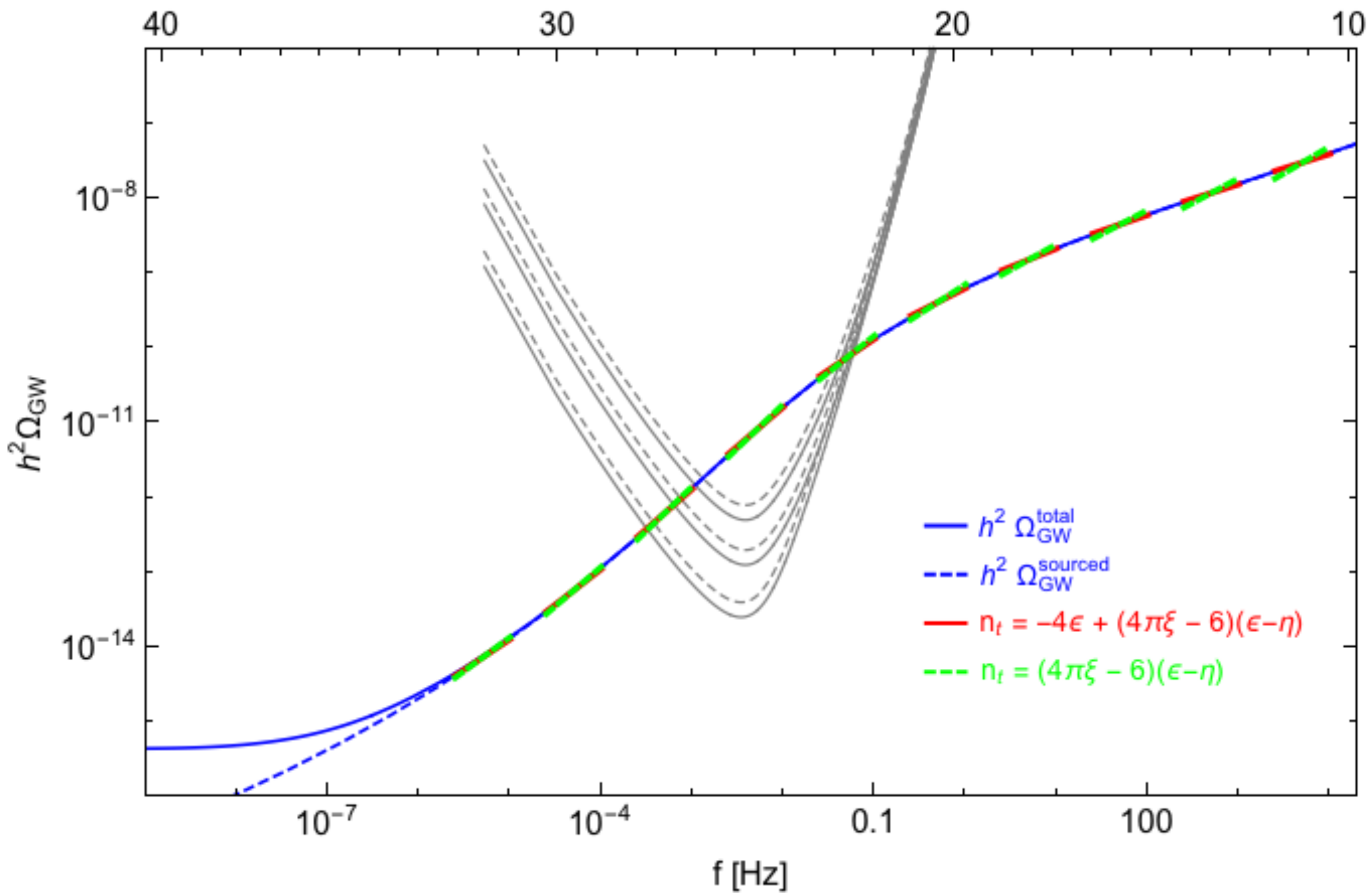


Figure 4. Spectrum of GWs today $h^2\Omega_{\rm GW}$ obtained from a numerical integration of the dynamical equations of motion (for a model of quadratic inflaton potential, with inflaton - gauge field coupling $f = M_{\rm Pl}/35$), versus the local parametrization $h^2\Omega_{\rm GW} \propto (f/f_*)^{n_T}$, evaluated at various pivot frequencies f_* and with the spectral tilt n_T obtained from successive approximations to the analytic expression (3.13).

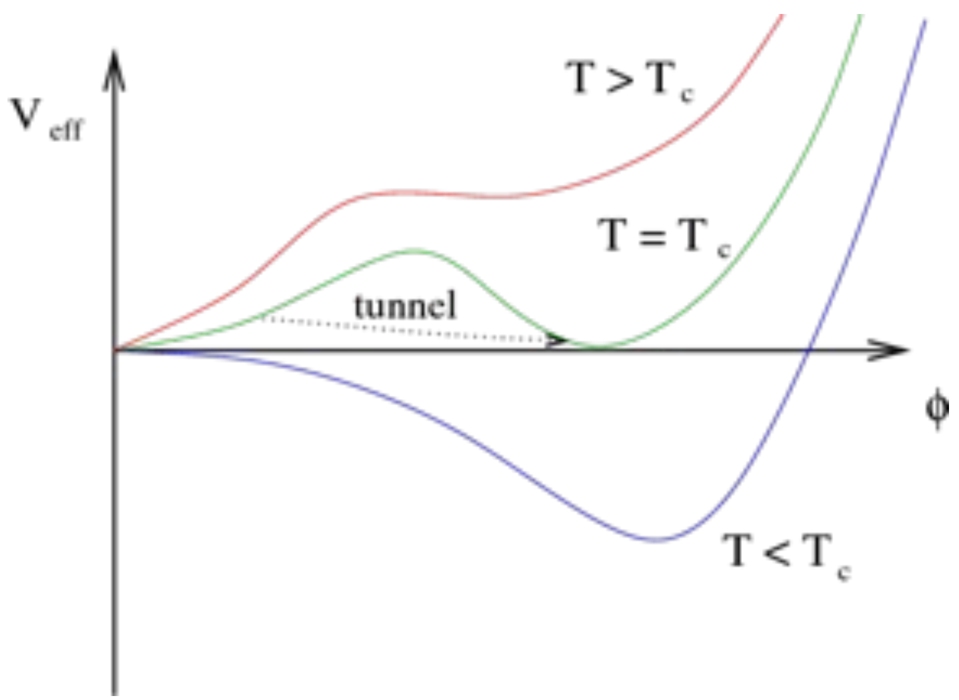
[arXiv: 1610.06481]

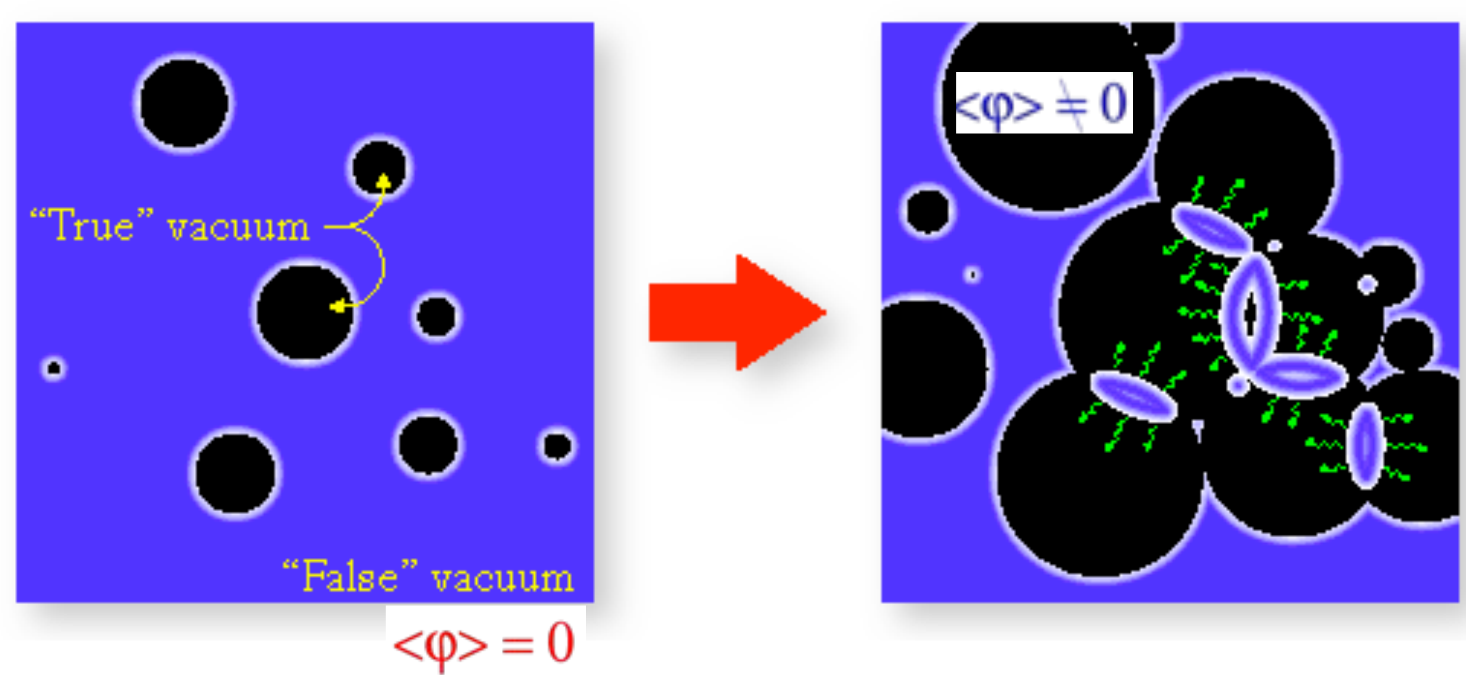
First order phase transitions

• universe expands and temperature decreases : Phase transitions; if 1st lead to GW

• potential barrier separates true and false vacua

quantum tunneling across the barrier : nucleation of bubbles of true vacuum





- collisions of bubble walls
- sound waves and turbulence in the fluid
- primordial magnetic fields (MHD turbulence)

source: Π_{ij} tensor
anisotropic stress

First order phase transitions

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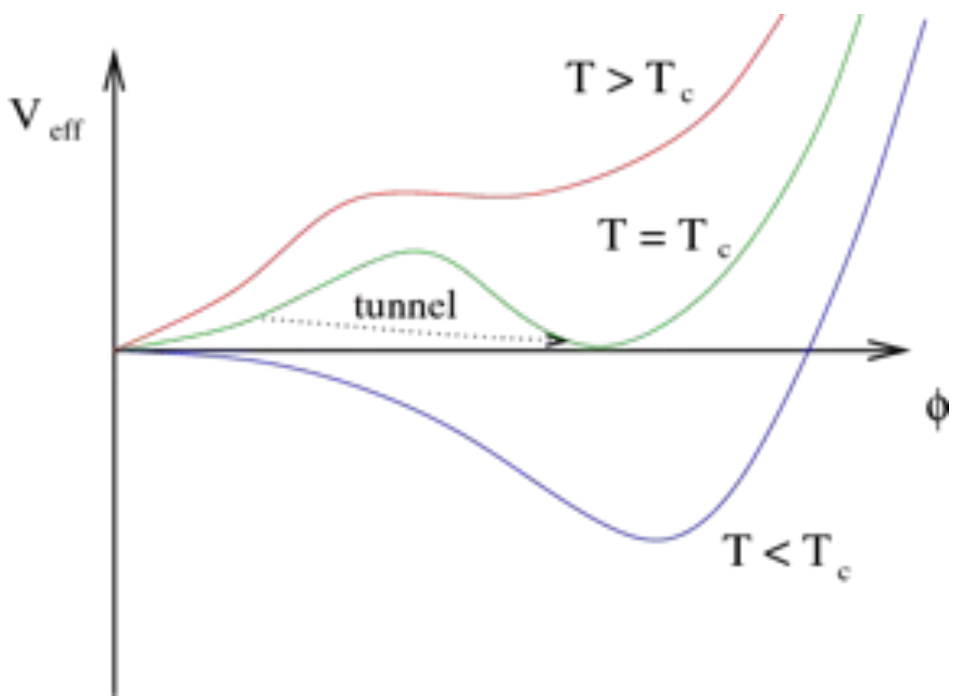
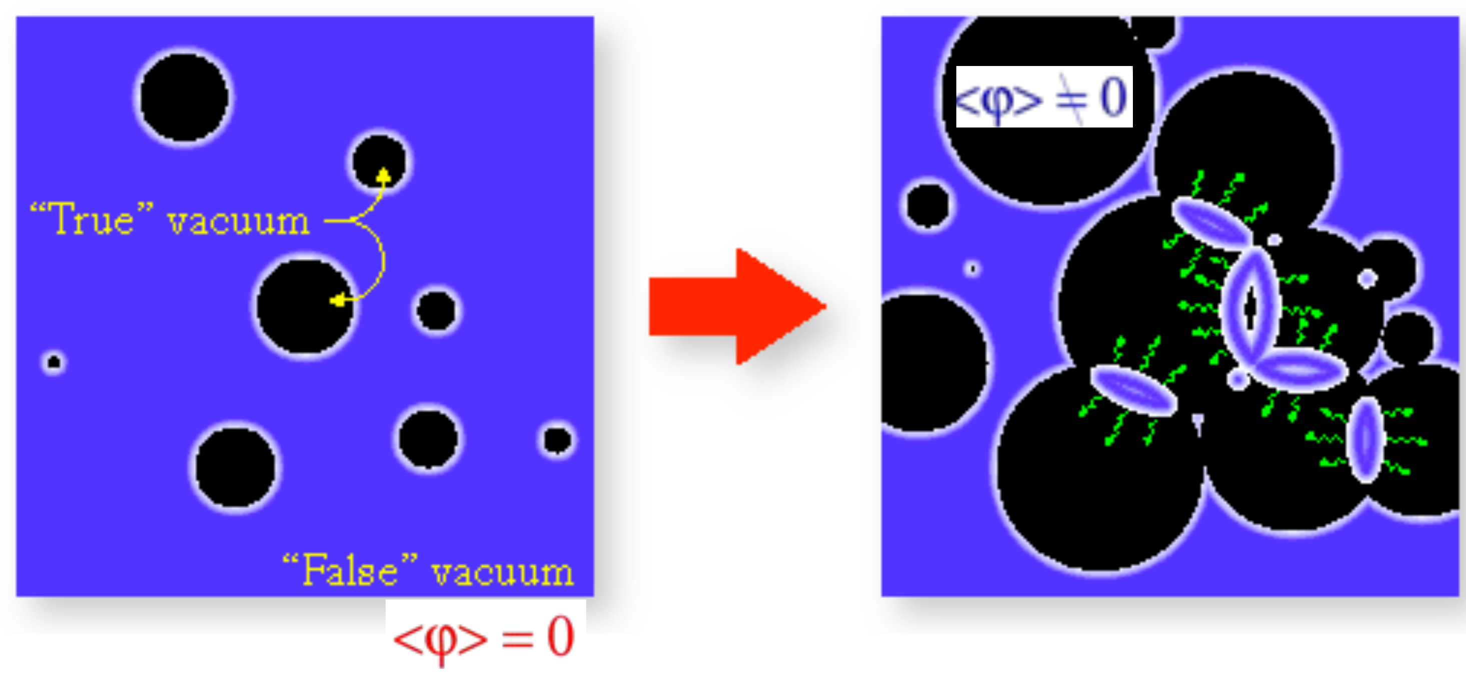




$$\Pi_{ij} \sim \partial_i\phi\,\partial_j\phi$$

source: Π_{ij} tensor
anisotropic stress

$$\Pi_{ij} \sim \gamma^2(\rho + p)\, v_i v_j$$

$$\Pi_{ij} \sim (E^2 + B^2)\frac{\delta_{ij}}{3} - E_iE_j - B_iB_j$$

$$h^2\Omega_{\rm GW} \approx h^2\Omega_\phi + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$$

Relevant parameters:

$$T_*$$

temperature
of the PT

$$\frac{\beta}{H_*}$$

inverse duration
of the PT with respect to
Hubble time

$$(\epsilon_*)$$

$$\alpha = \frac{\rho_{\rm vac}}{\rho^*_{\rm rad}}$$

strength
of the PT

$$v_w$$

bubble
wall speed

Putting it all together

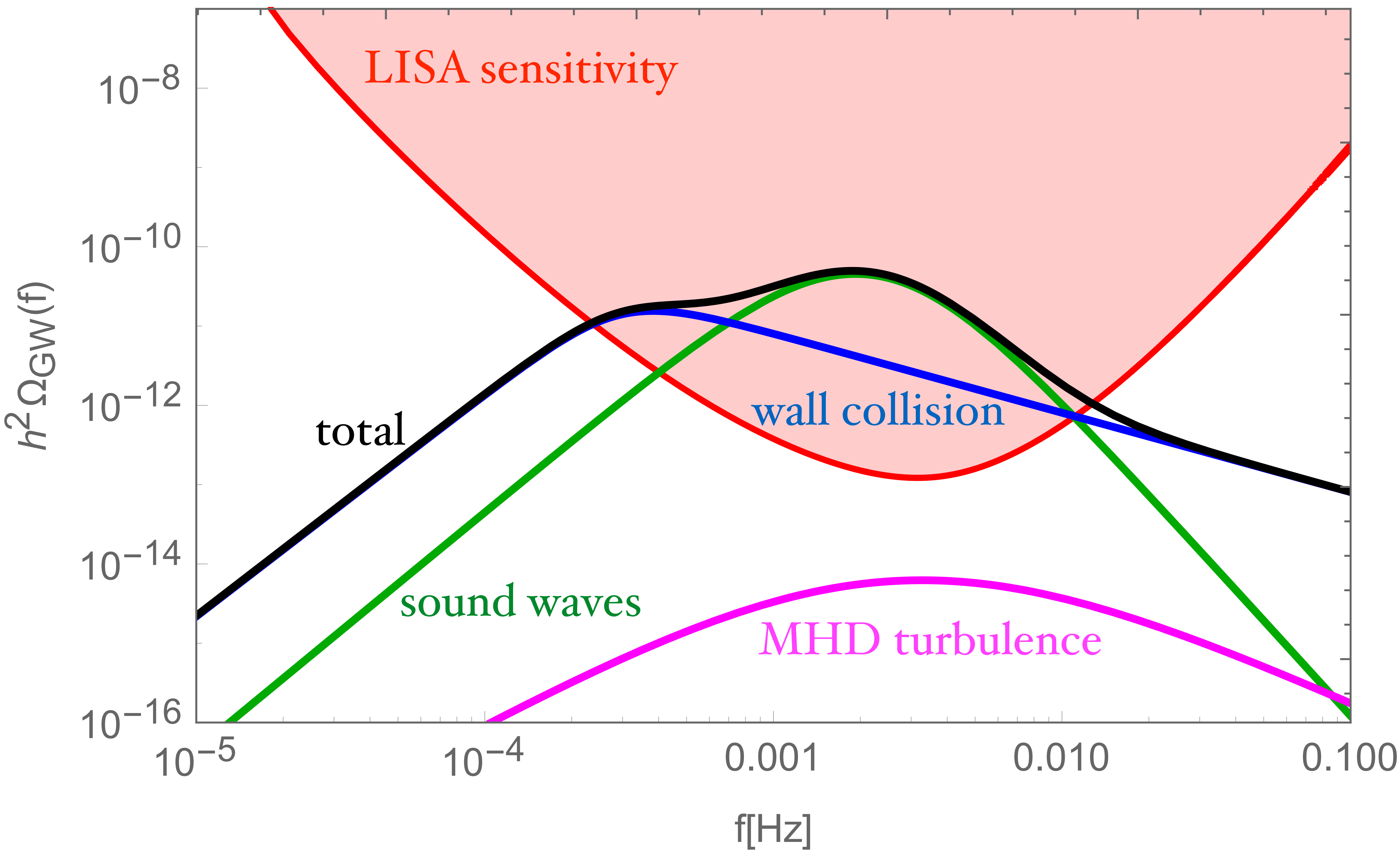
[1512.06239]

- three sources $\approx h^2\Omega_\phi,\, h^2\Omega_{\rm sw},\, h^2\Omega_{\rm turb}$
- know their dependence on the 4 parameters *[Espinosa, Konstantin,No, Servant, Caprini…]*
- predict the signal.

$$(\text{example, } T_* = 100\text{GeV},\, \alpha_{T_*} = 0.5,\, v_{\rm w} = 0.95,\, \beta/H_* = 10)$$

Example of signal

(example, $T_* = 100\text{GeV}$, $\alpha_{T_*} = 0.5$, $v_w = 0.95$, $\beta/H_* = 10$)

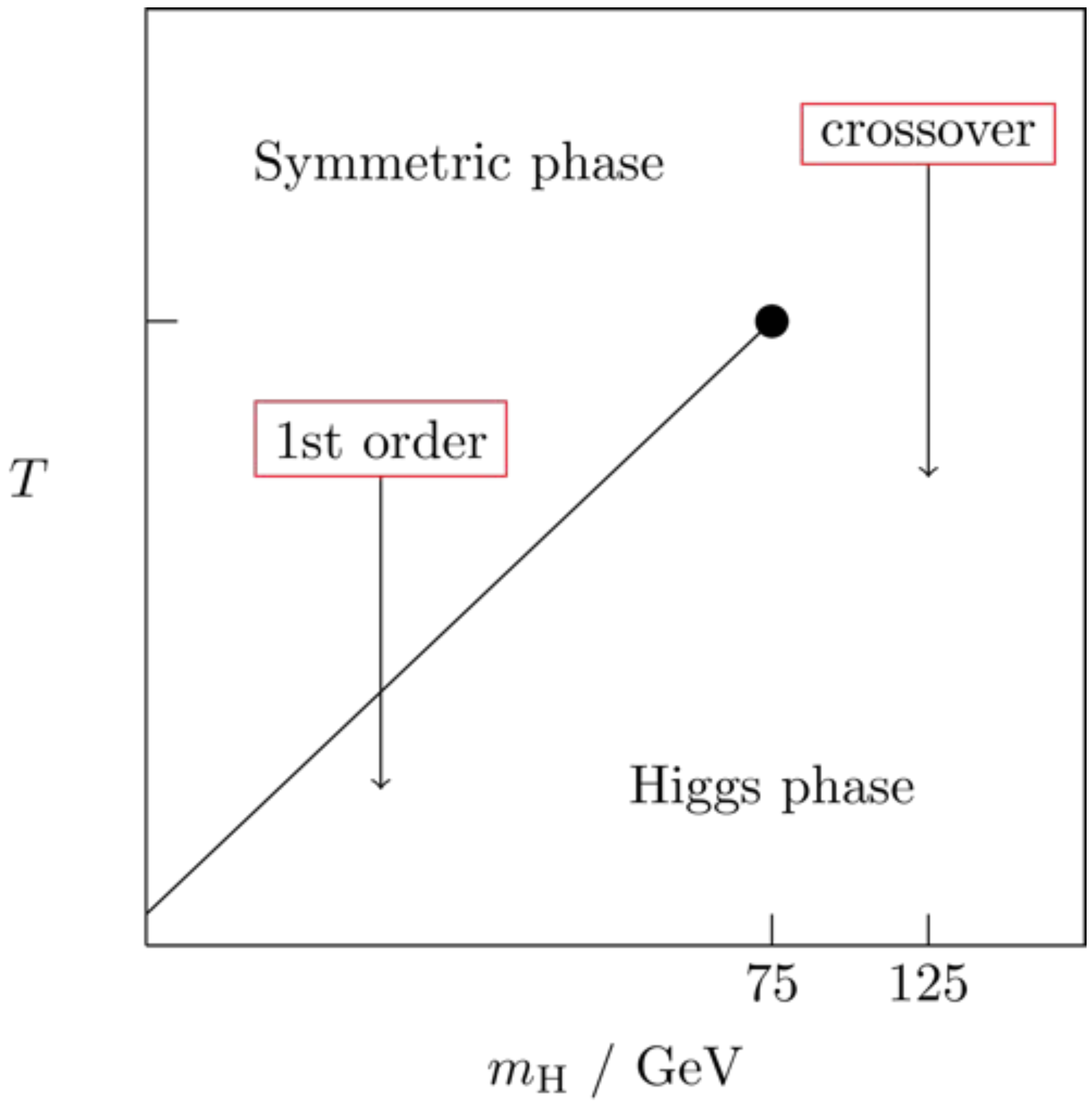


[Caprini et al, arXiv:1512.06239]

EW phase transition?

• Phase diagram for the Standard Model was found in the 1990's

[Kajantie et al, Gurtler et al, Csikor et al]



• With a Higgs mass at 125GeV, the EW PT is a crossover, and NOT first order!

• No significant departure from thermal equilibrium =>no significant GW production or baryogenesis.

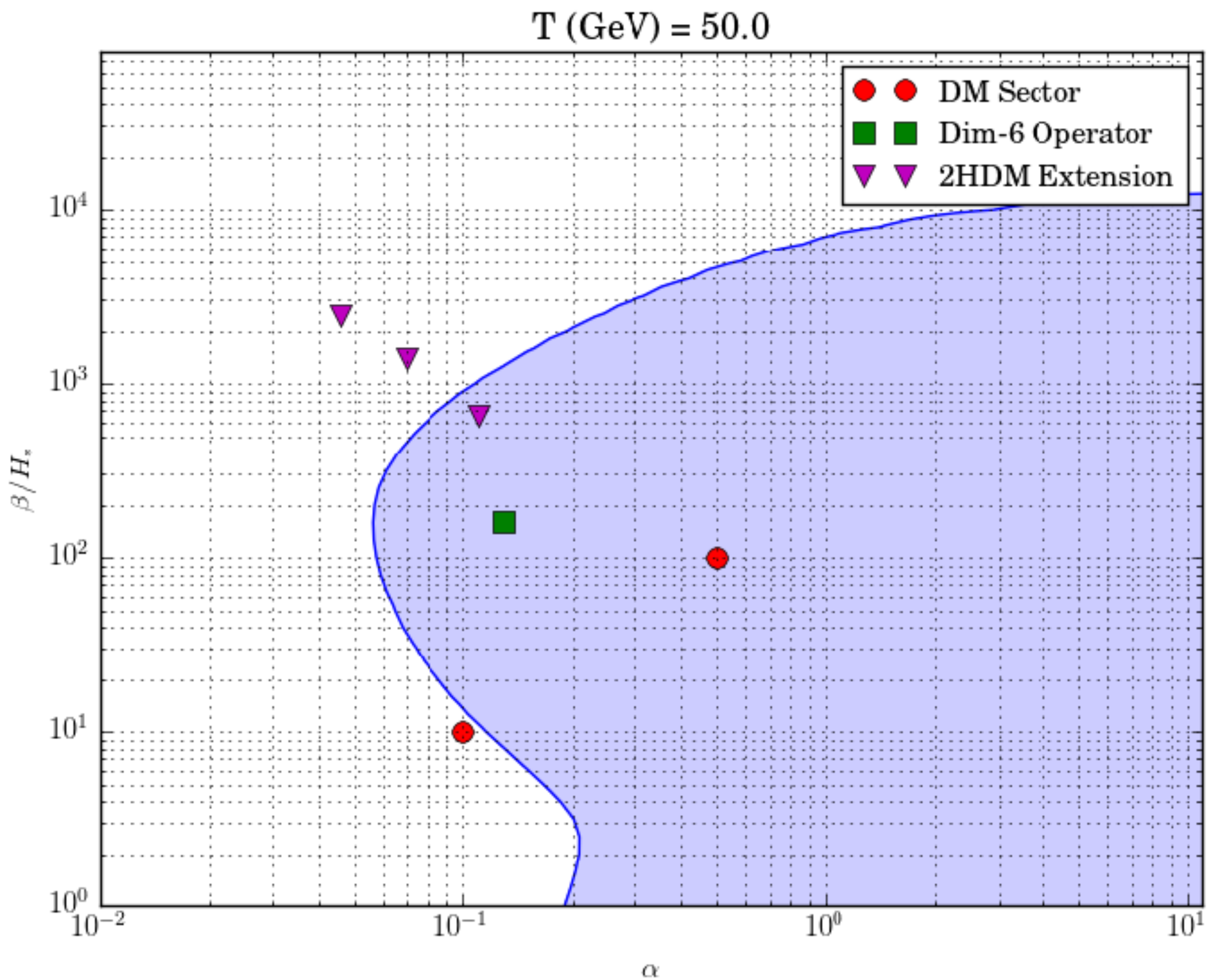
• So why bother??????!

• EW PT can be first order in many extensions of the SM.

- singlet extensions of MSSM (Huber et al 2015)
- direct coupling of Higgs sector with scalars (Kozackuz et al 2013)
- SM plus dimension six operator (Grojean et al 2004)

Model $\longrightarrow (T_*, \alpha_{T_*}, v_\mathrm{w}, \beta) \longrightarrow$ this plot

[C.Caprini et al, 1512.06239]



... which tells you if it is detectable by LISA (see 1512.06239)

GW background from cosmic strings

Cosmic strings: some basics

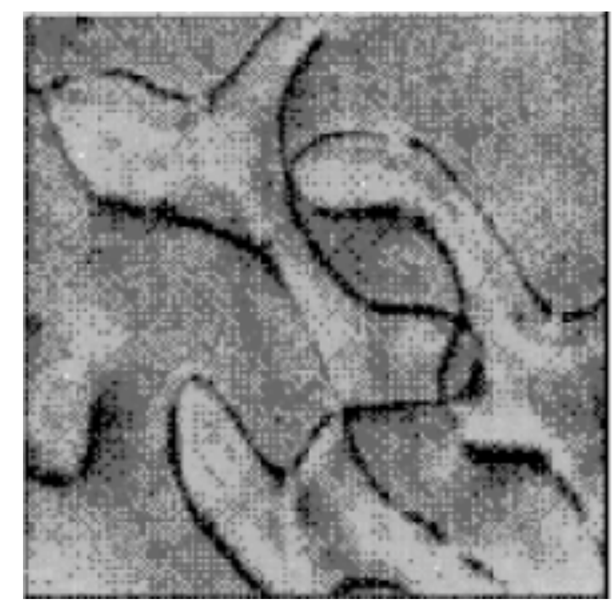
[Kibble '76]

– <u>line-like</u> *topological defects*, formed in a symmetry breaking phase transition $G \rightarrow H$
provided the vacuum manifold contains non-contractible loops $\Pi_1(G/H) = \Pi_1(\mathcal{M}) \neq 1$

– A lot of input/interplay with other branches of physics:

- difficult to see cosmic strings in the sky
- "easier" to see strings in the lab (vortex loops in He4, He3, superconductors, strings in NLC...)

– Generically formed at the end of hybrid-like inflation *[Jeannerot et al 03]*
or in brane inflation (cosmic super-strings) *[Jones et al, Sarangi and Tye]*

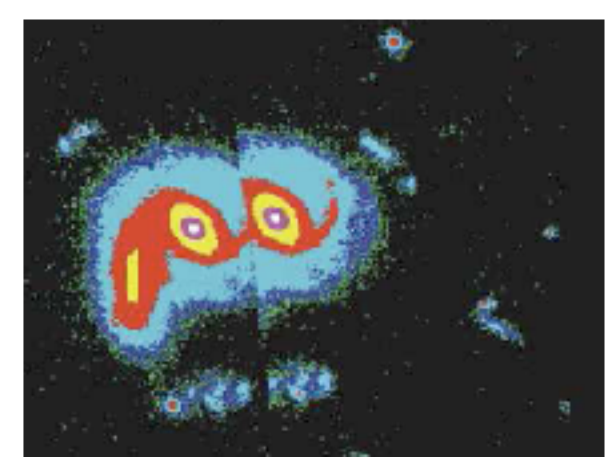


– if formed, should still exist today

– Numerous potentially observable signatures:
Gravitational wave emission;
CMB anisotropies & B-modes;
lensing,....
particle emission
electromagnetic radiation

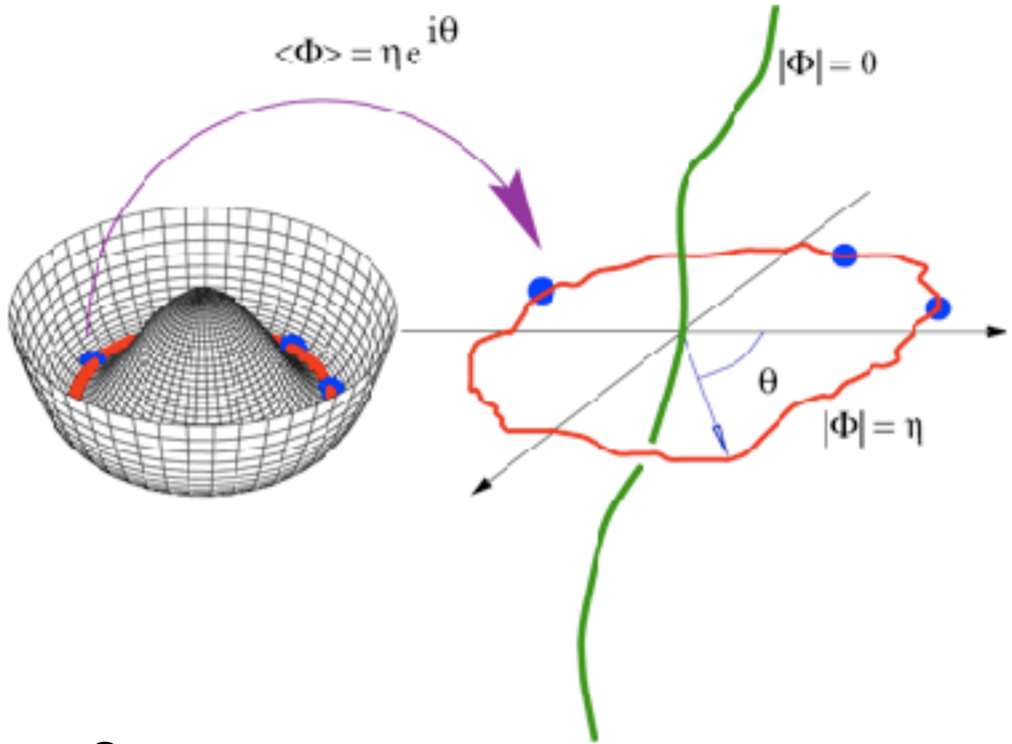
$$\boxed{G\mu \leq \text{few} \times 10^{-7}}$$

[Planck paper XXV]



– Typical example: strings in the Abelian Higgs model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - \frac{\lambda}{4}(|\phi|^2 - \eta^2)$$



- Degenerate vacuum/ground state with $\langle\phi\rangle = \eta e^{i\alpha}$
- U(1) invariance $\phi \rightarrow \phi e^{i\alpha}$ broken by choice of phase
- String/vortex is a linear defect around which α changes by $2n\pi$

infinite, or *closed loops*

(n = winding number)

$G = U(1) \quad \mathcal{M} = S^1 \quad \Pi_1(\mathcal{M}) = Z$

- Energy/unit length of string: μ_n

• Scales: $r \sim M^{-1}$ $G\mu \sim G\eta^2 \sim GM^2$

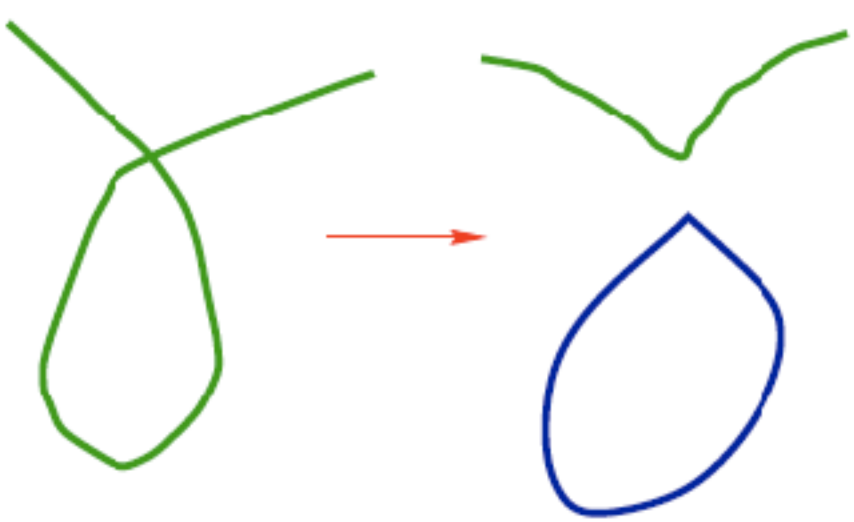
GUT: $\sim 10^{-32}\text{cm}$ $\sim 10^{-7}$

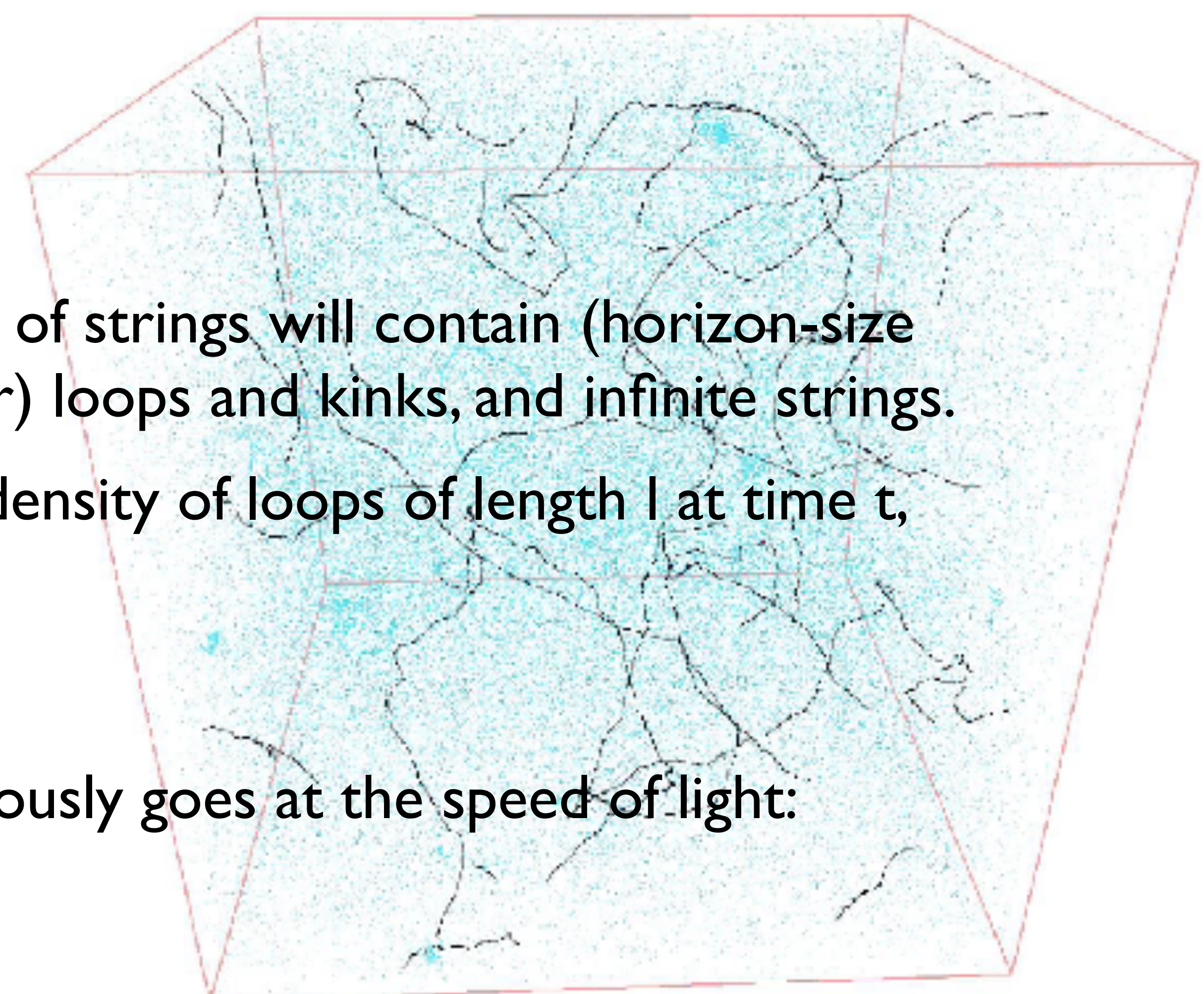
• Prototypical model of infinitely thin strings: Nambu-Goto strings $$S = -\mu \int d^2\sigma \sqrt{-\det(\gamma_{ab})}$$

– Approx. dynamics of relativistic string: action = area of world-sheet

• <u>only one free parameter</u> $G\mu$

• intercommutation: 4 **kinks** (discontinuity in tangent vector of string)





• Network of strings will contain (horizon-size and smaller) loops and kinks, and infinite strings.

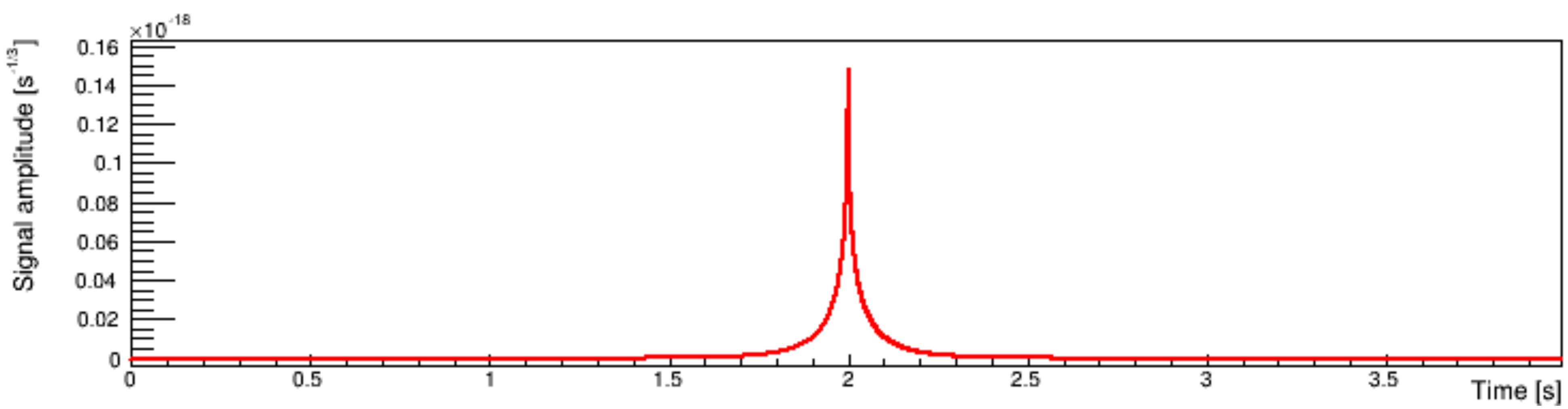
• number density of loops of length l at time t,

• <u>cusps</u>: points at which the string itself instantaneously goes at the speed of light:

- Cosmic strings produce 2 types of GW signals

1) sharp, non-gaussian **bursts** of gravitational waves from kinks and cusps, of a characteristic form which may be directly detected by LIGO and Virgo.
[Waveform known; use match-filtering techniques]

[Damour+Vilenkin 2001]



Predicted gravitational waveform produced by a cosmic string cusp.

• Cosmic strings produce 2 types of GW signals

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[Waveform known; use match-filtering techniques]

[Damour+Vilenkin 2001]

2) A **stochastic GW background** ranging over many decades in frequency

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}}{df},$$

(which can be probed by e.g. pulsar timing at nHz frequencies, LIGO/Virgo…)

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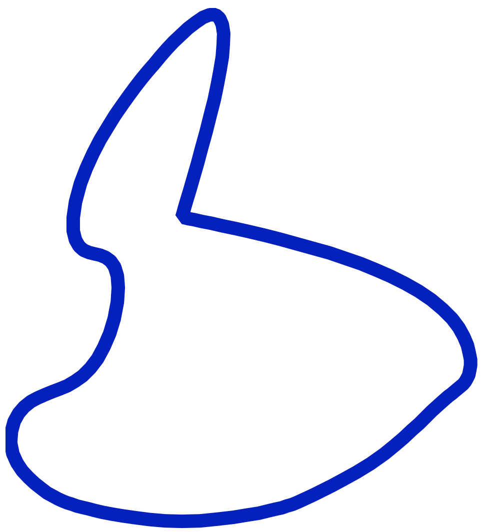
[Waveform known; use match-filtering techniques]

[Damour+Vilenkin 2001]

2) A **stochastic GW background** ranging over many decades in frequency

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}}{df},$$

(which can be probed by e.g. pulsar timing at nHz frequencies, LIGO/Virgo…)



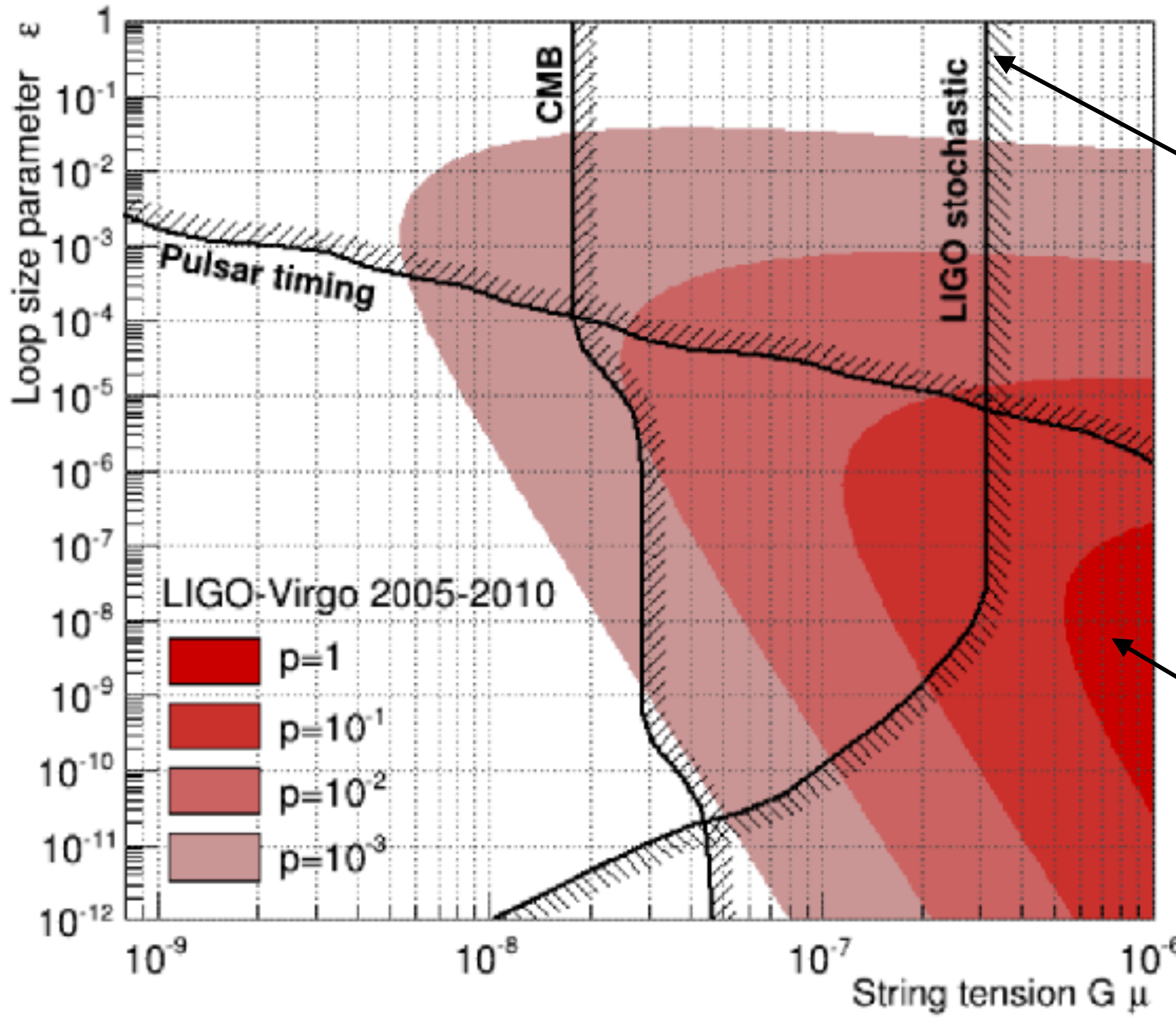
• Both are sourced by oscillating closed loops of string

• Crucial quantity: $n(\ell, t)d\ell$ number density of loops with length between $\ell \to \ell + d\ell$ at time t

• finally understood and agreed on, as of very recently…and not…what was used in first LIGO paper

Constraints on Cosmic Strings from the LIGO-Virgo Gravitational-Wave Detectors.

PRL 112 (2014) 131101



Stochastic bkgd at LIGO frequencies

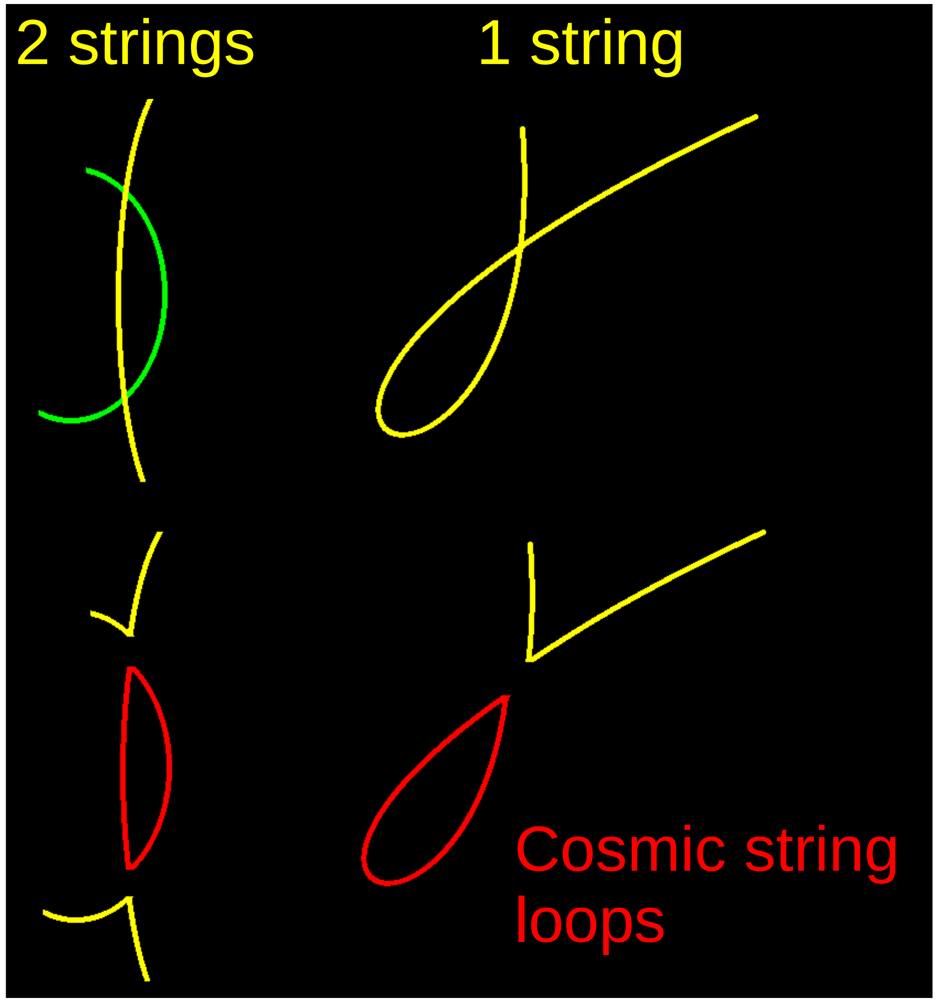
$p = 10^{-3}$

Bursts not observed: Rejected at 90% confidence level

$$\ell_f = \epsilon(\Gamma G\mu)t$$

– loops assumed to be formed with tiny size (fraction of horizon size), decay in a Hubble time

– Now most loops thought to be large, of order 0.1 horizon size: new constraints.

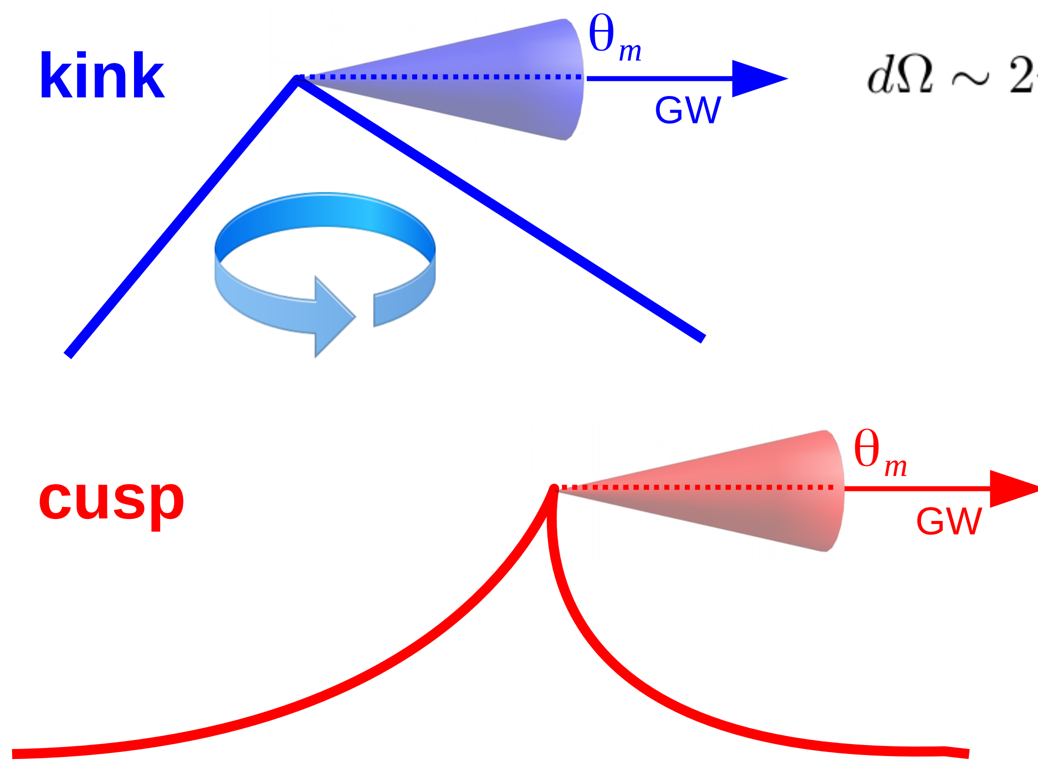


Loop formation

Loop oscillation → cusps and kinks

$$\theta_m(\ell, z, f) = (g_2 f (1+z)\ell)^{-1/3}$$

$$\theta_m < 1$$



$$d\Omega \sim 2\pi\theta_m$$

$$d\Omega \sim \pi\theta_m^2$$

GW waveform:

$$h(\ell, z, f) = A_q(\ell, z) f^{-q} \Theta(f_h - f)$$

$$A_q(\ell, z) = g_1 \frac{G\mu \ell^{2-q}}{(1+z)^{q-1} r(z)}$$

q = 4/3 for cusps, q = 5/3 for kinks

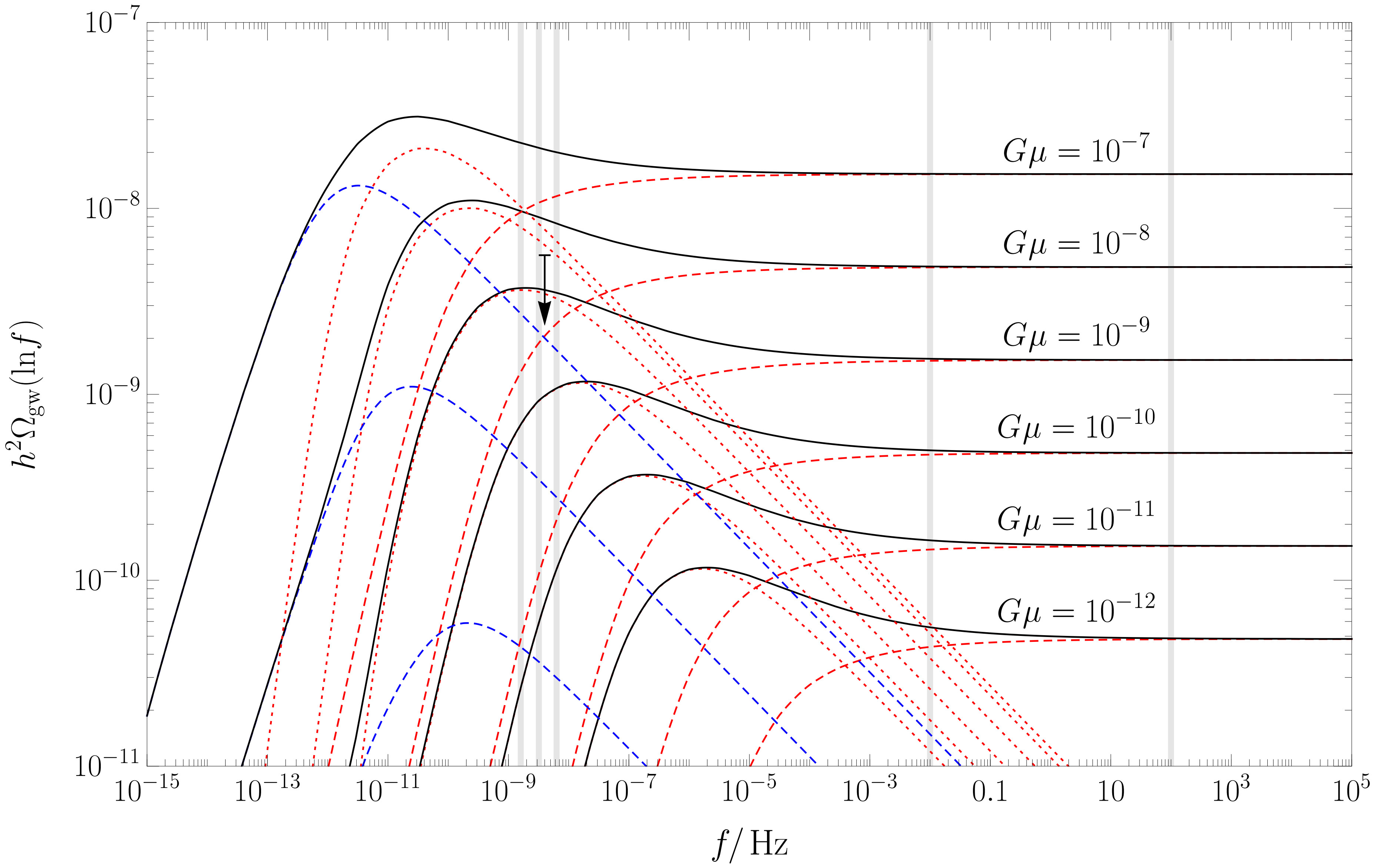


FIG. 13. The normalized spectrum of gravitational waves for various values of string tension. The red dashed lines show the contribution from loops radiating during the radiation era, the red dotted lines represent the contribution from loops produced during the radiation era, but radiating during the matter era, and the blue dashed lines represent loops produced during the matter era. In light gray from left to right, the 20-, 10-, and 5-year PTA, eLISA [12, 65] and LIGO [66] peak sensitivity frequencies are shown.

[Blanco-Pillado et al; arXiv:1309.6637]

$$G\mu \leq 2.8 \times 10^{-9},$$

Conclusion

• aLIGO/Virgo detection opens the era of GW astronomy and cosmology : we have a new, independent "messenger" to be added to EM emission

• GW could be a powerful means to probe the early universe (and consequently high energy physics) and the cosmological expansion: detection is difficult but great payoff

• Not mentioned: tests of GR, modified gravity,…

Gravitational wave observations can tell us about cosmology

Individual sources
at cosmological distances
e.g. binary black holes,
binary neutron stars…

late-time universe
$z \lesssim 10$

- structure and kinematics of universe
- formation of structures
- H_0, Hubble constant
- dark energy and dark matter
- modified gravity…..

Stochastic background
of GWs of cosmological origin

Very early universe
$t \gtrsim t_{Pl}$

- quantum processes during inflation
- Phase transitions in Early universe
- topological defects, eg cosmic strings
- …..