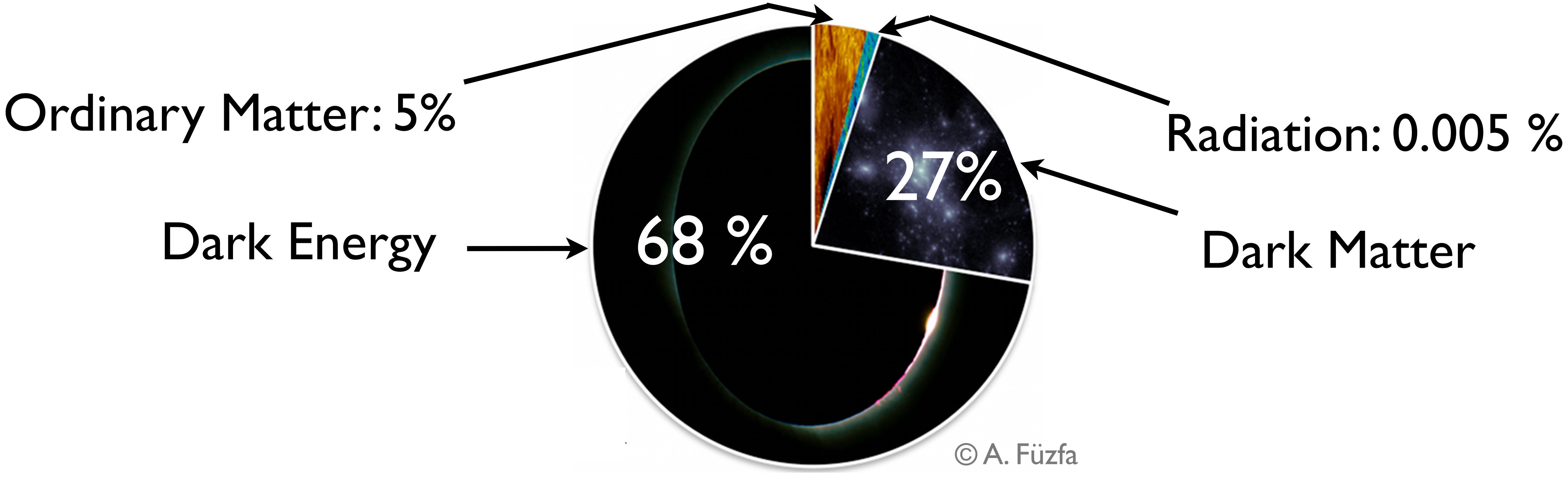
Tests expérimentaux de la Relativité Générale :

sur Terre, dans notre Système Solaire et même plus loin si nécessaire

C. Le Poncin-Lafitte
SYRTE, Observatoire de Paris, PSL Research University, Sorbonne Universités, CNRS, UPMC Univ. 06, LNE

Motivations to test GR

- Quantum theory of gravity:
    - GR: classic theory (not a quantum theory)
    - at high energy: quantum effects should appear
    - useful to study black holes and the Planck Era
- Unification of all fundamental interactions: unify Standard model of particles with gravitation
- Cosmological and galactic observations required Dark Matter and Dark Energy: not directly observed so far ⇒ hints of a deviation from GR ?



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Where to test gravity ?

Where to test gravity ?

Since Galileo Galilei, here…

Where to test gravity ?

Since Galileo Galilei, here…

and with the beginning of Deep Space exploration in the 60th

weak field tests

Where to test gravity ?

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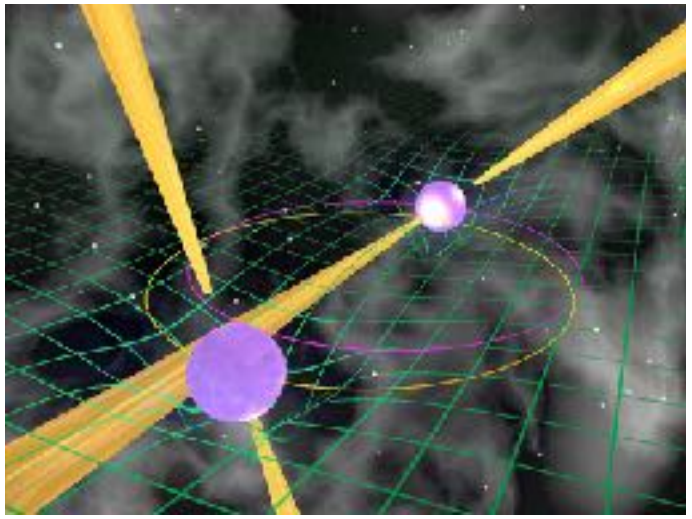
and with the beginning of Deep Space exploration in the 60th

weak field tests

1967 : discovery of pulsar.
=> first strong field tests



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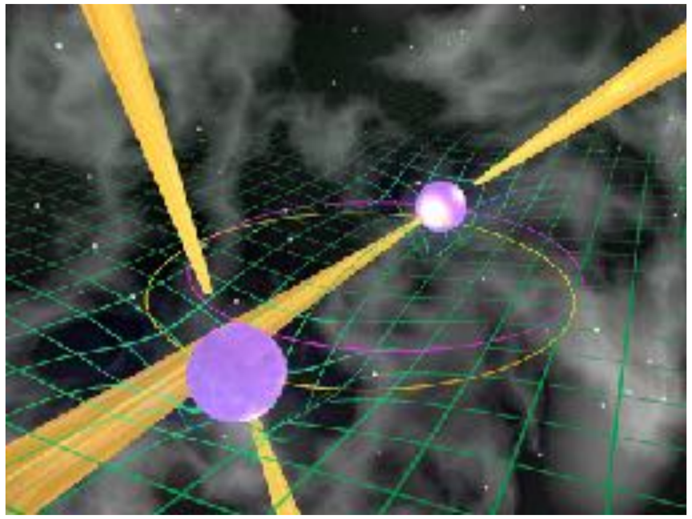
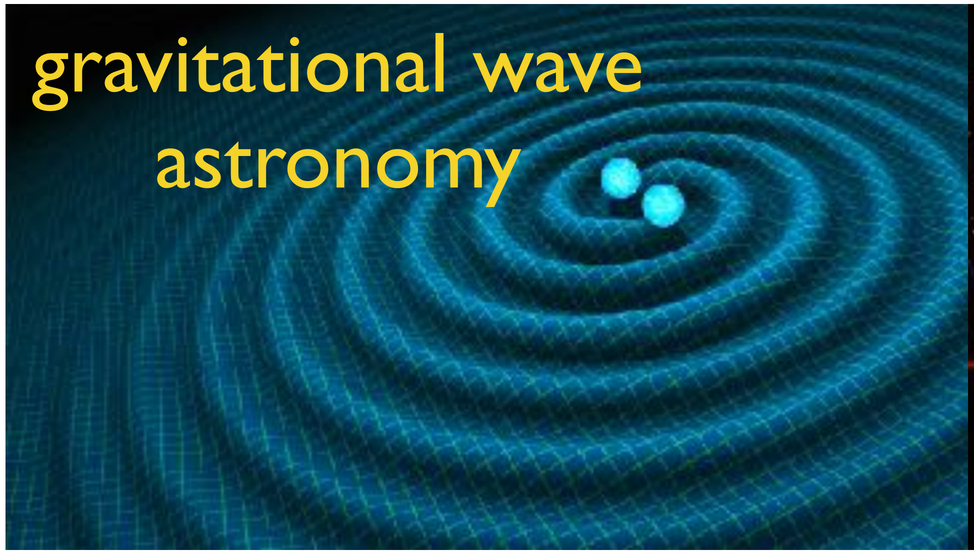


and tomorrow (even already today) :



gravitational wave astronomy

Galactic Center

Basic principles of GR

1) Equivalence Principle:

- 3 facets: Universality of free fall, Local Position/Lorentz Invariance
- very well tested (10^{-13} with Eöt-wash experiments and Lunar Laser Ranging ; 10^{-4} with grav. redshift ; no variation of constants)[1]
- more accurate measurement needed: alternative (string) theories predict violation smaller[2] → MICROSCOPE accuracy 10^{-15}

- Gravitation ⇔ space-time curvature (described by a metric $g_{\mu\nu}$)
- free-falling masses follow geodesics of this metric and ideal clocks measure proper time

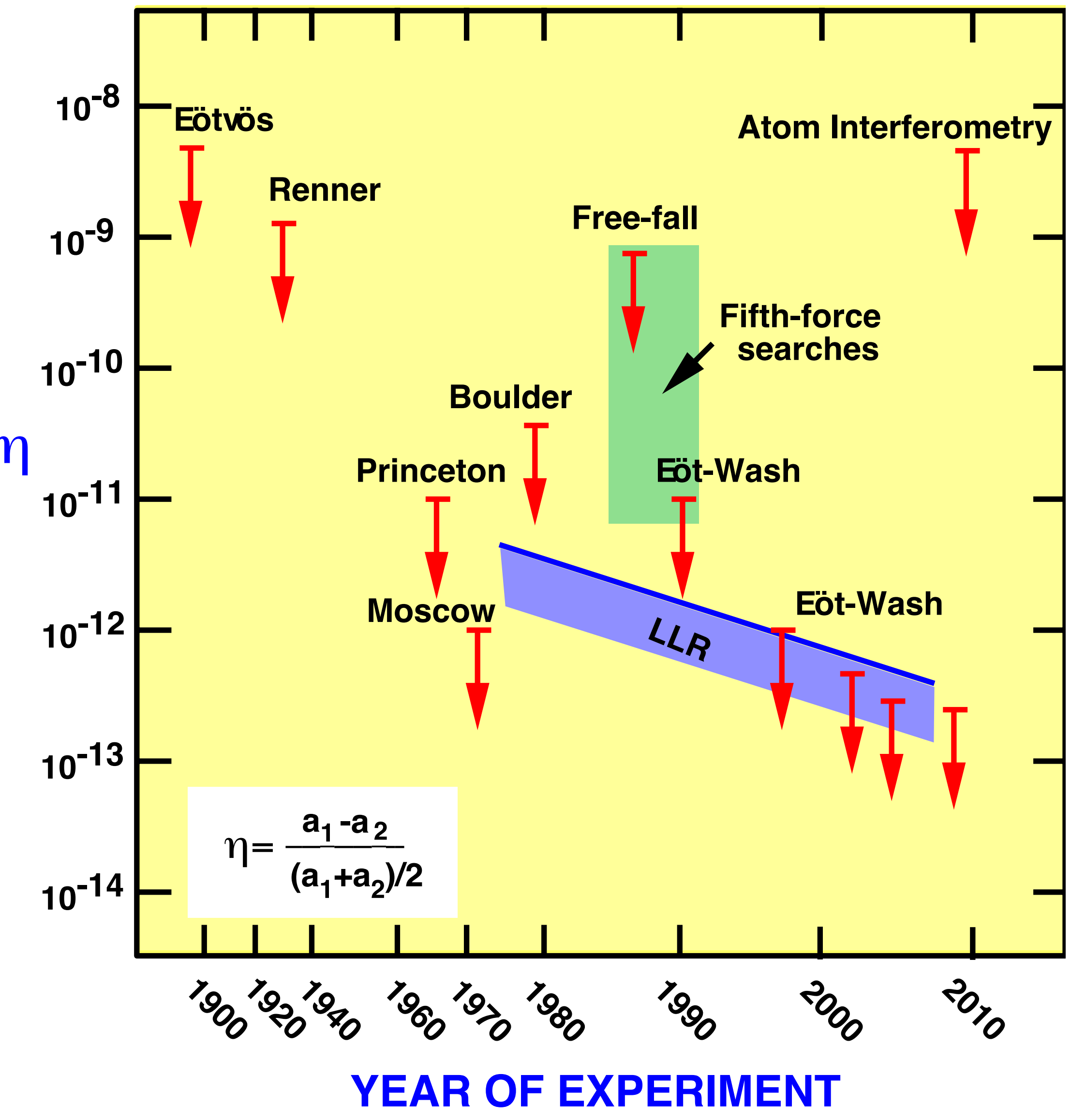
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

[1] C. Will, LRR, 9, 2006

[2] T. Damour, CQG, 29-184001, 2012

Free Fall Experiments

TESTS OF THE
WEAK EQUIVALENCE PRINCIPLE



400 CE Ioannes Philiponus: *"…let fall from the same height two weights of which one is many times as heavy as the other …. the difference in time is a very small one"*

1553 Giambattista Benedetti
proposed equality

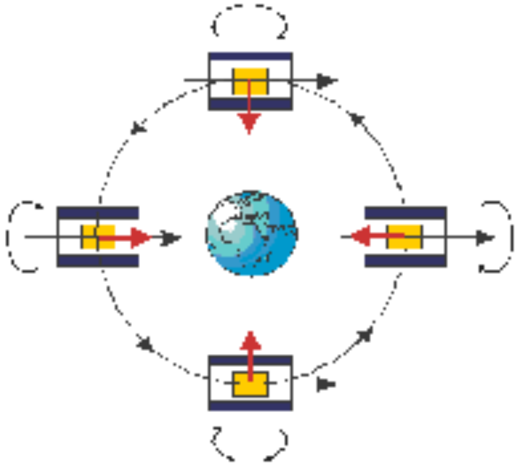
1586 Simon Stevin
experiments

1589-92 Galileo Galilei
Leaning Tower of Pisa?

1670-87 Newton
pendulum experiments

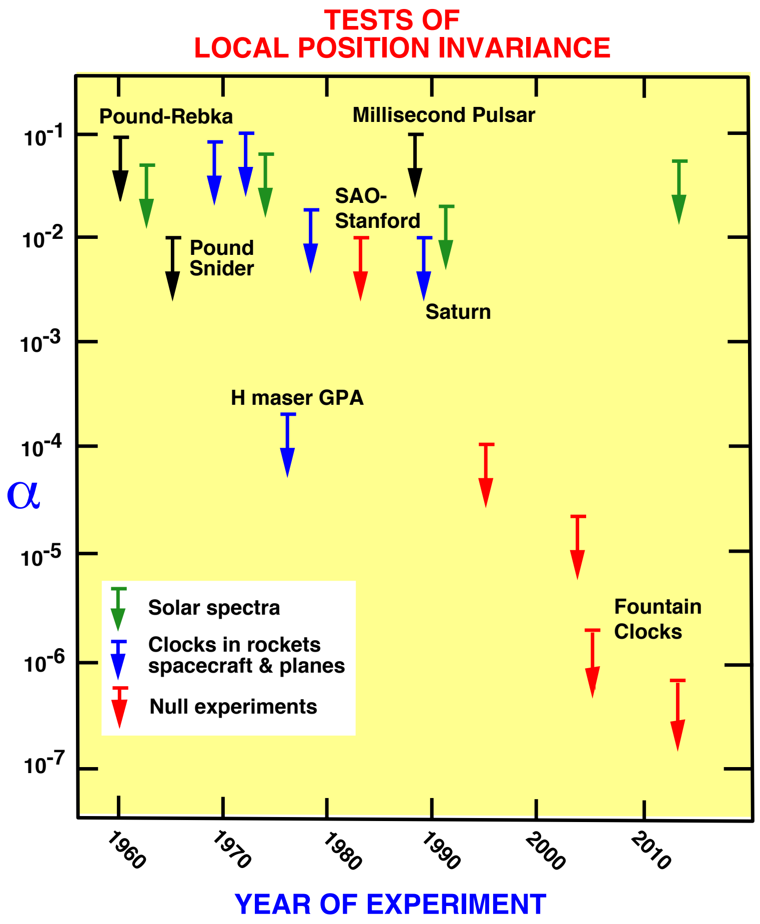
1889, 1908 Baron R. von Eötvös
torsion balance experiments (10^{-9})

1990s UW (Eöt-Wash) *10^{-13}*



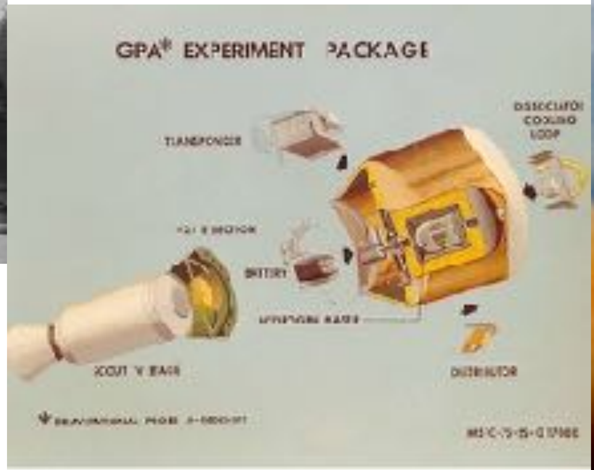
CNES Microscope Mission : 10^{-15}

Local Position Invariance : redshift



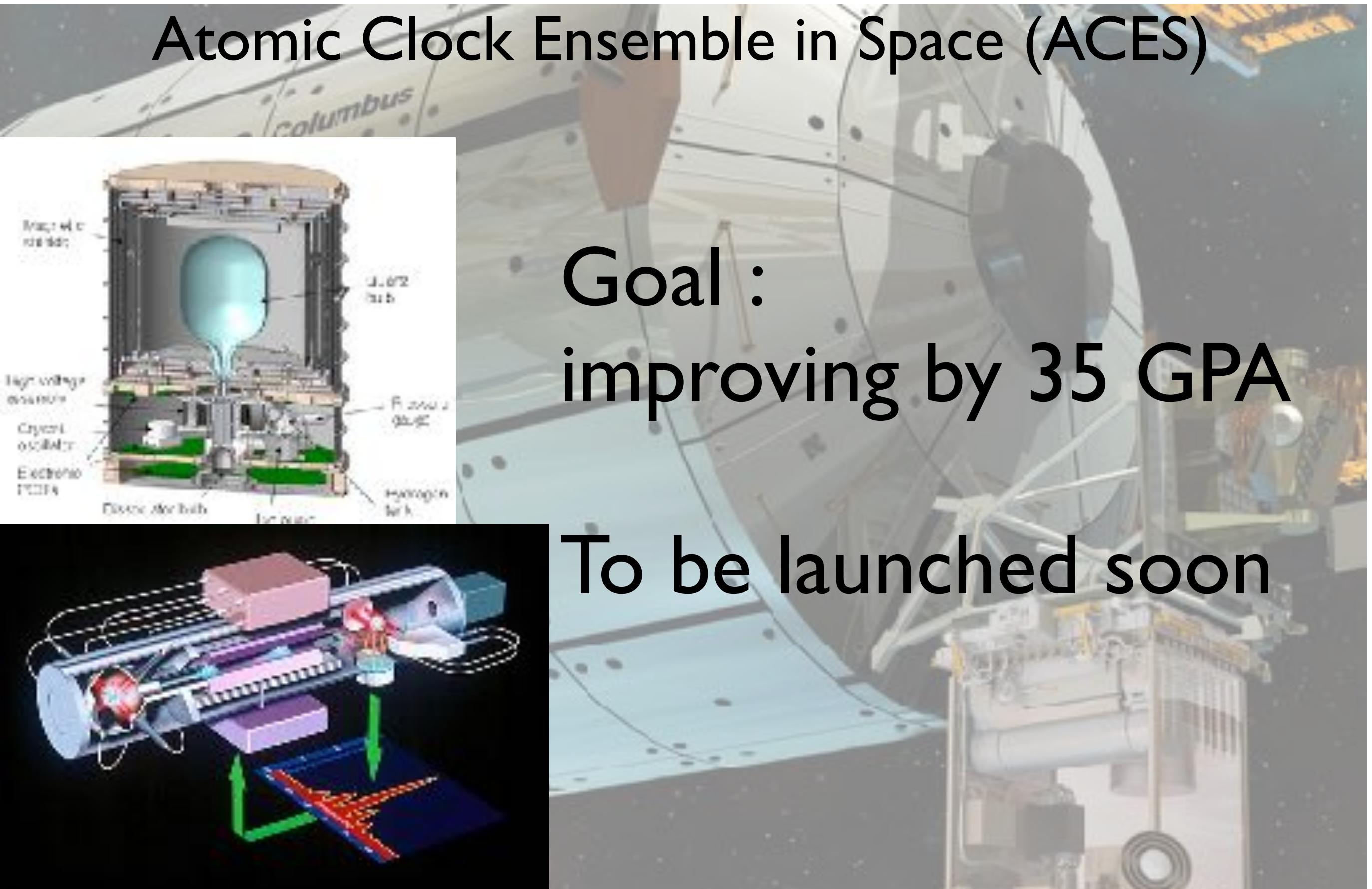
$\Delta\nu/\nu = (1+\alpha)\Delta U/c^2$

1959 : Pound & Rebka (10%)



1980 : Gravity Probe A
Vessot (0.01%)

Launch : 1976 with Scout rocket
duration : 1h55mn
where : Wallops Island

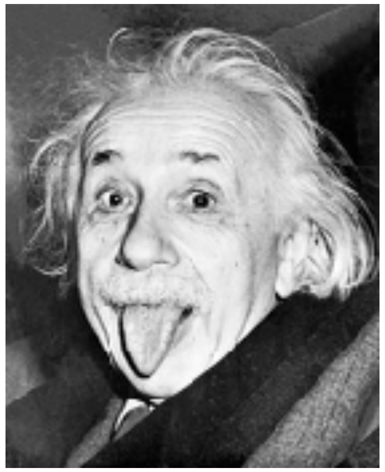


Atomic Clock Ensemble in Space (ACES)

Goal :
improving by 35 GPA

To be launched soon

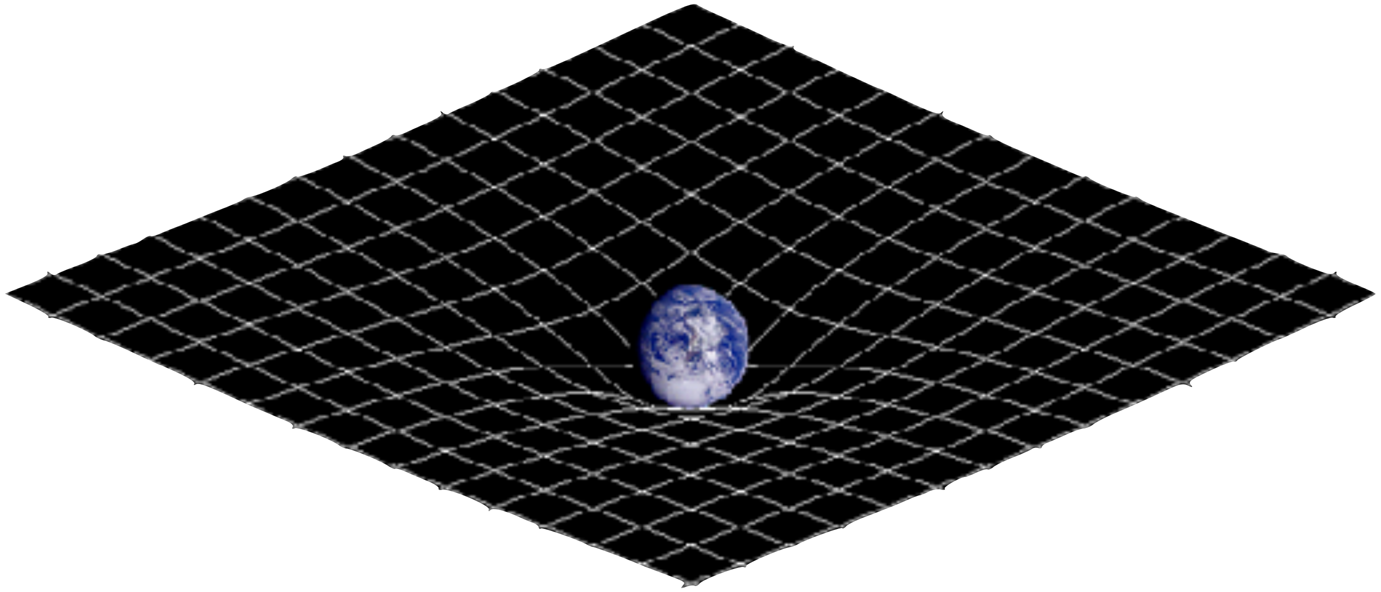
Basic principles of GR



II) Field equations (determination of the metric):

- Einstein Equations: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

space-time curvature (metric) ⇔ matter-energy content



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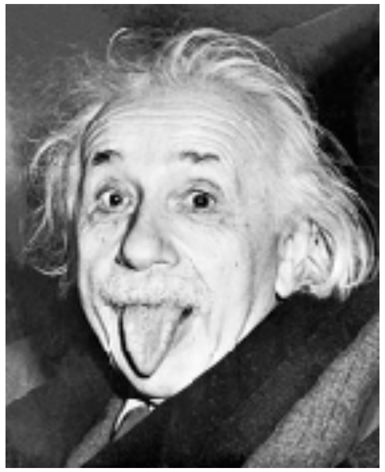

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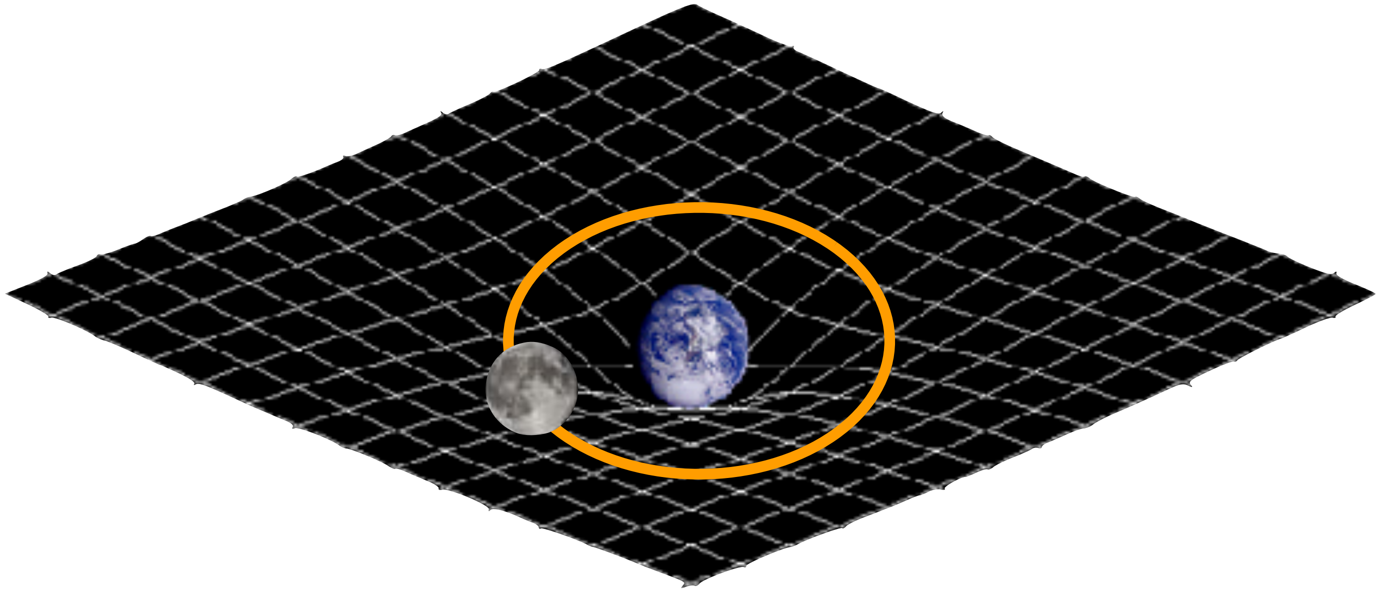

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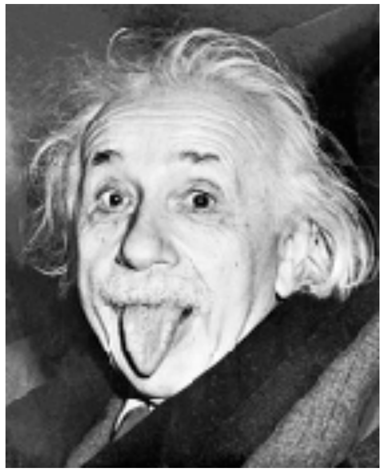

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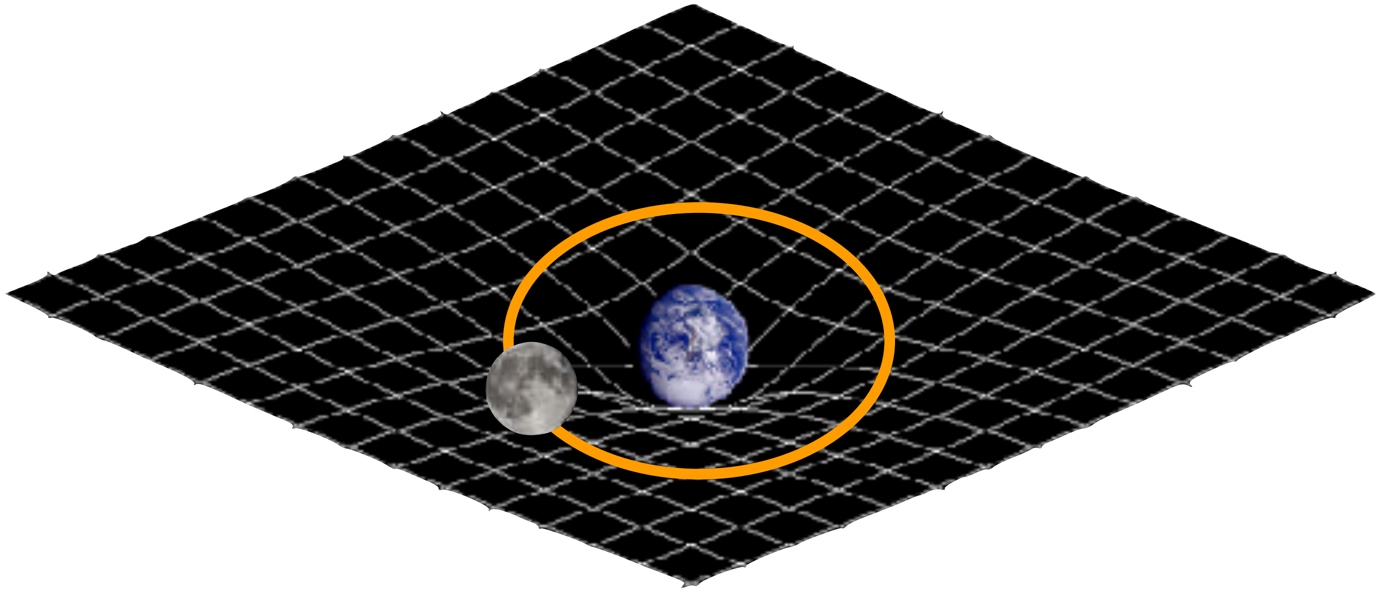

- important effects for space-mission:
  - dynamics ≠ from Newton (ex.: advance of the perihelion)
  - proper time (measured by ideal clocks) ≠ coordinate time
  - coordinate time delay for light propagation (Range/Doppler)
  - light deflection (VLBI, astrometry)

Solar System observations

Solar System observations

Ground & space geodesy accuracy is increasing:

Solar System observations

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LLR & SLR → From cm to mm

GALILEO

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Gravity Probe A to ACES/Pharao → factor 80 on Grav. Redshift

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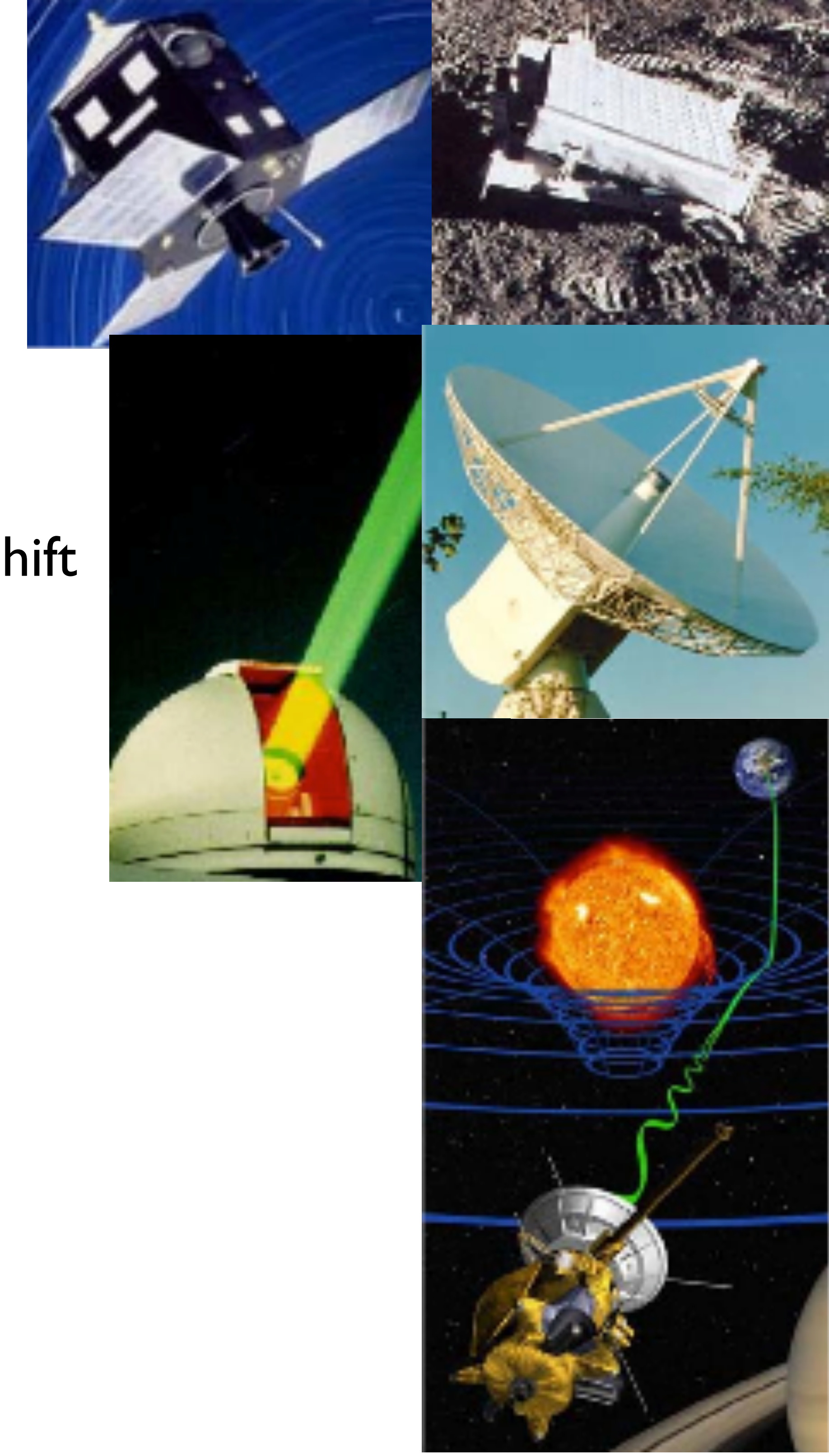
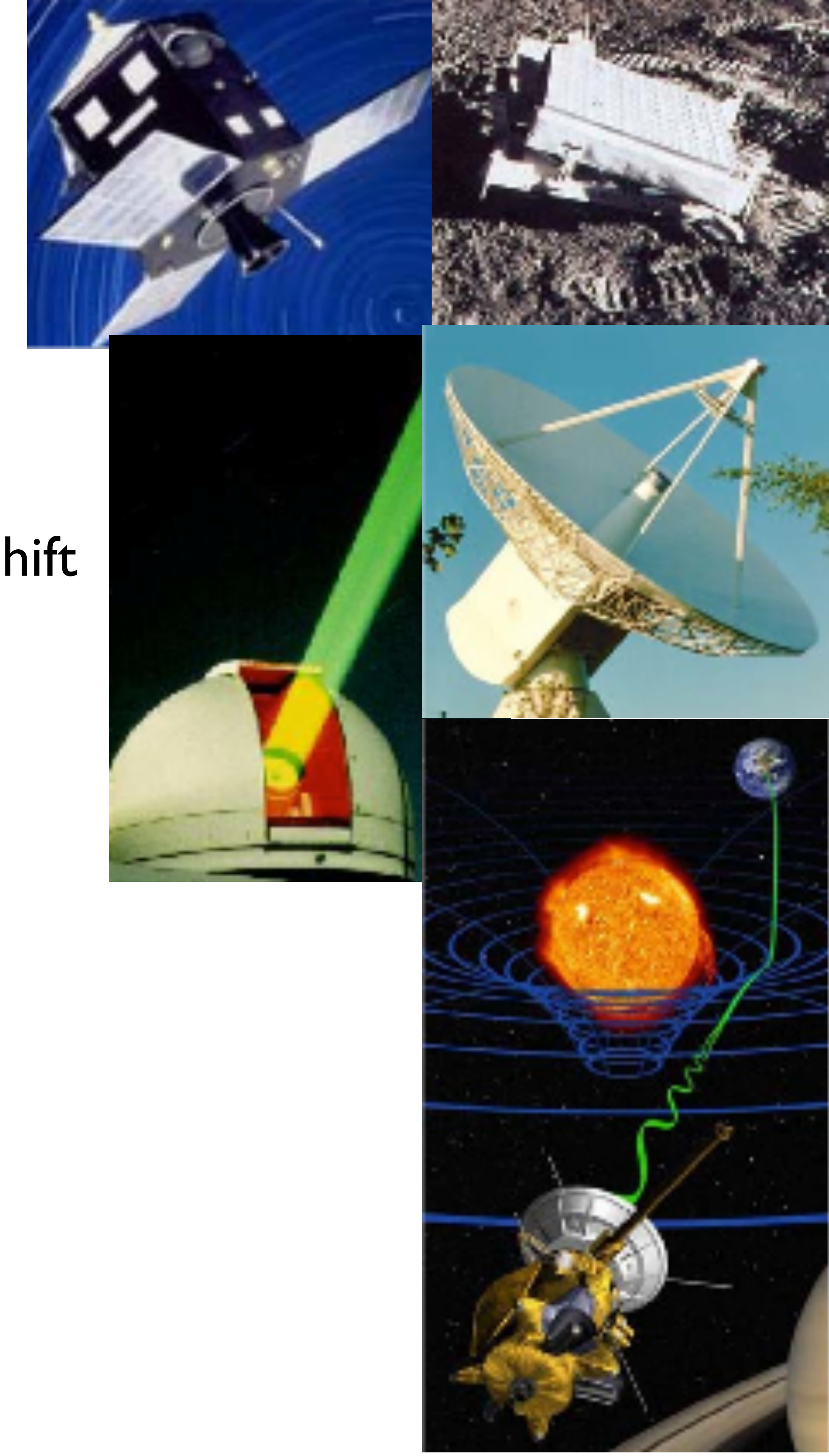
Ground & space astrometry:

Gaia, Gravity → from milli to micro-arcsecond



Solar System observations



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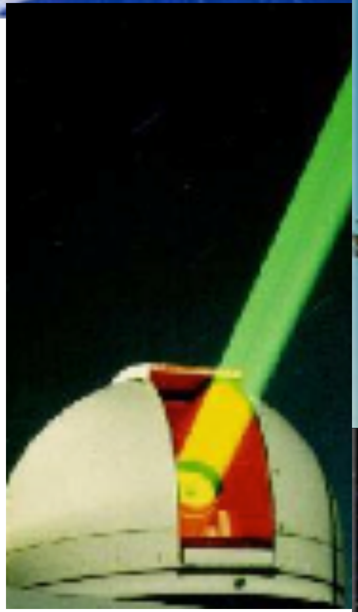
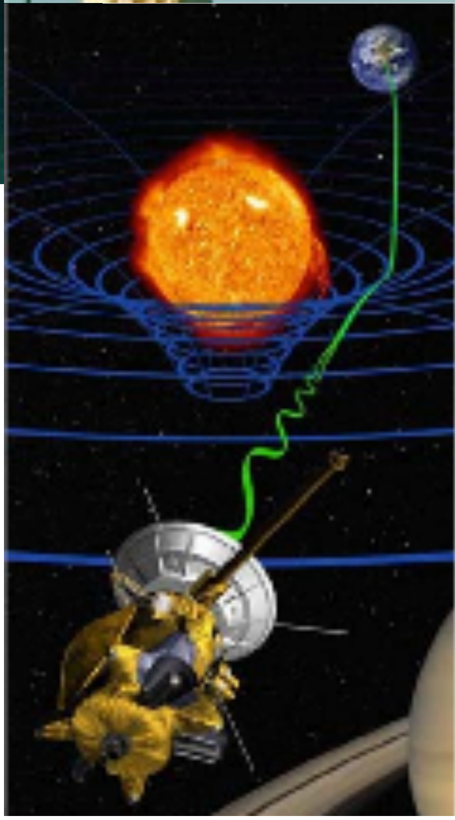
Navigation of interplanetary probes :

Solar System observations





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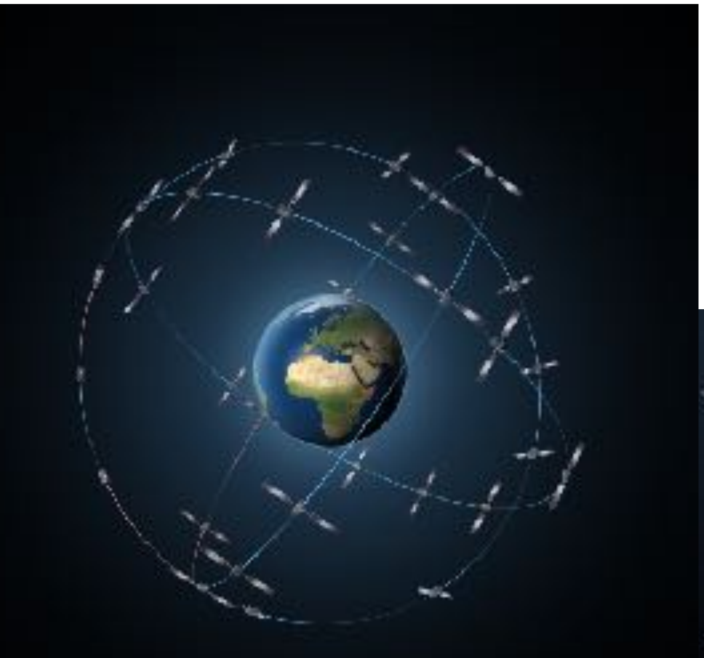
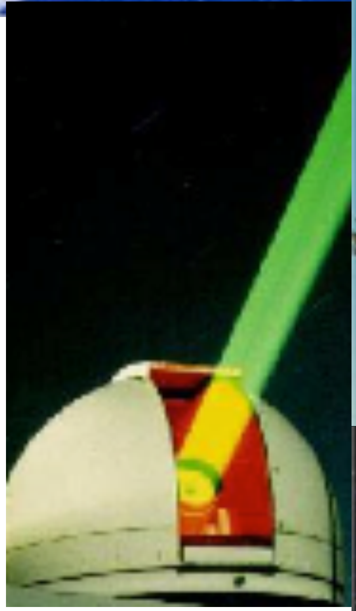
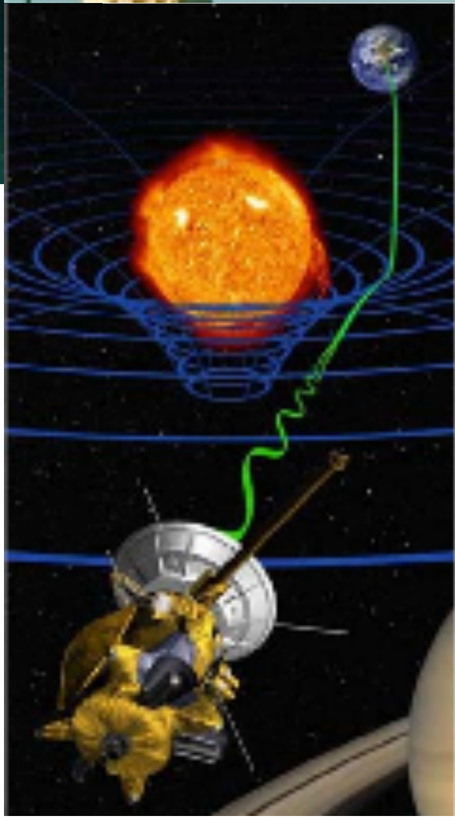
Cassini Experiment, use of Ka Band
MORE Experiment on BepiColombo → factor 10 on Doppler
JUNO Experiment 2016, JUICE towards 2030



Solar System observations







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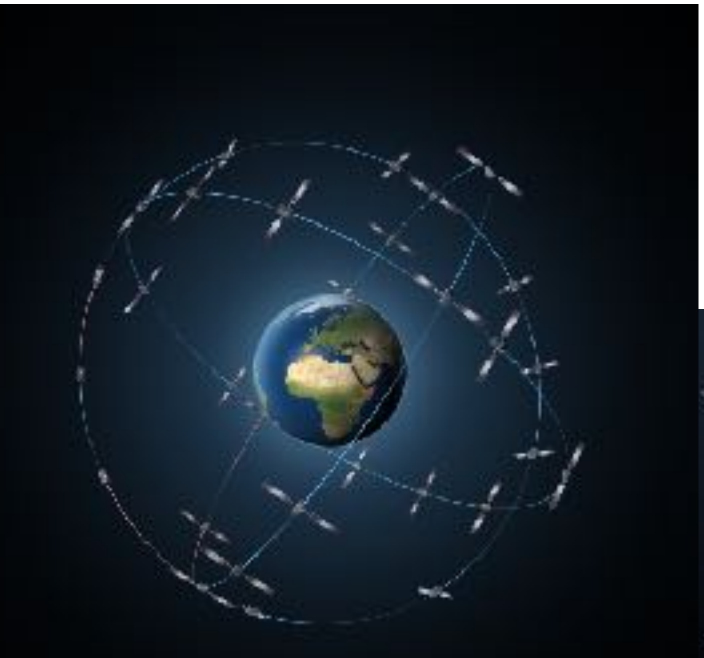
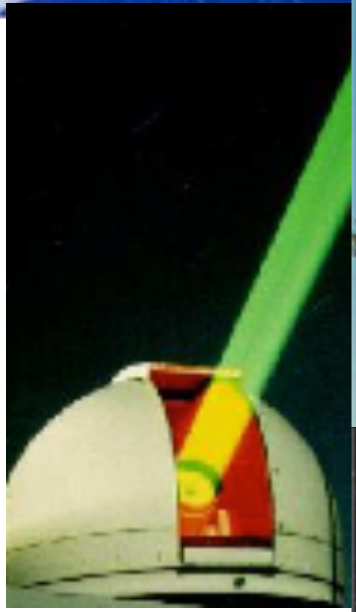
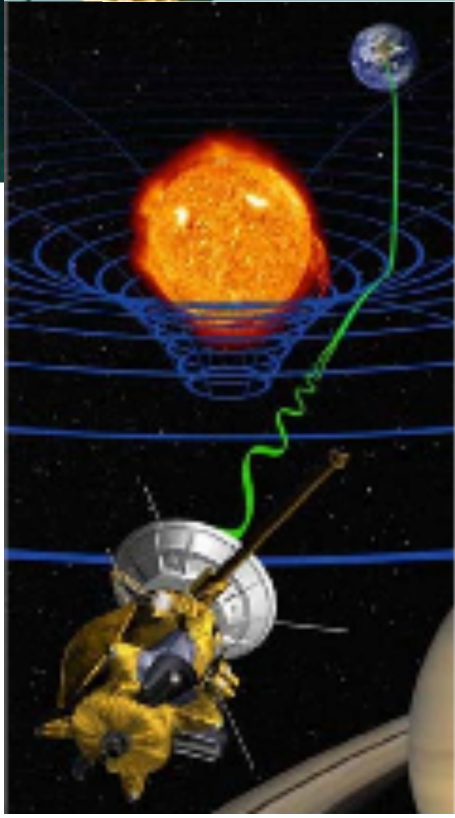



Need to describe light propagation and dynamics in relativistic framework

Solar System observations







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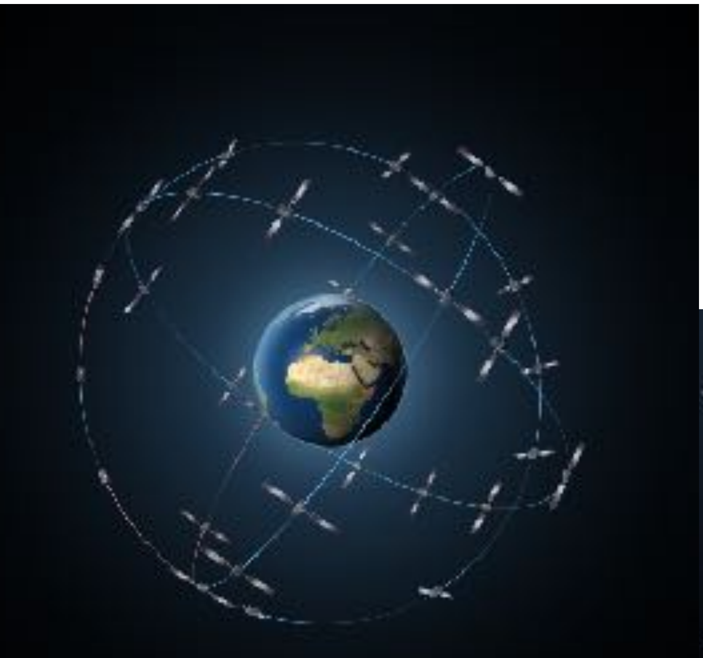
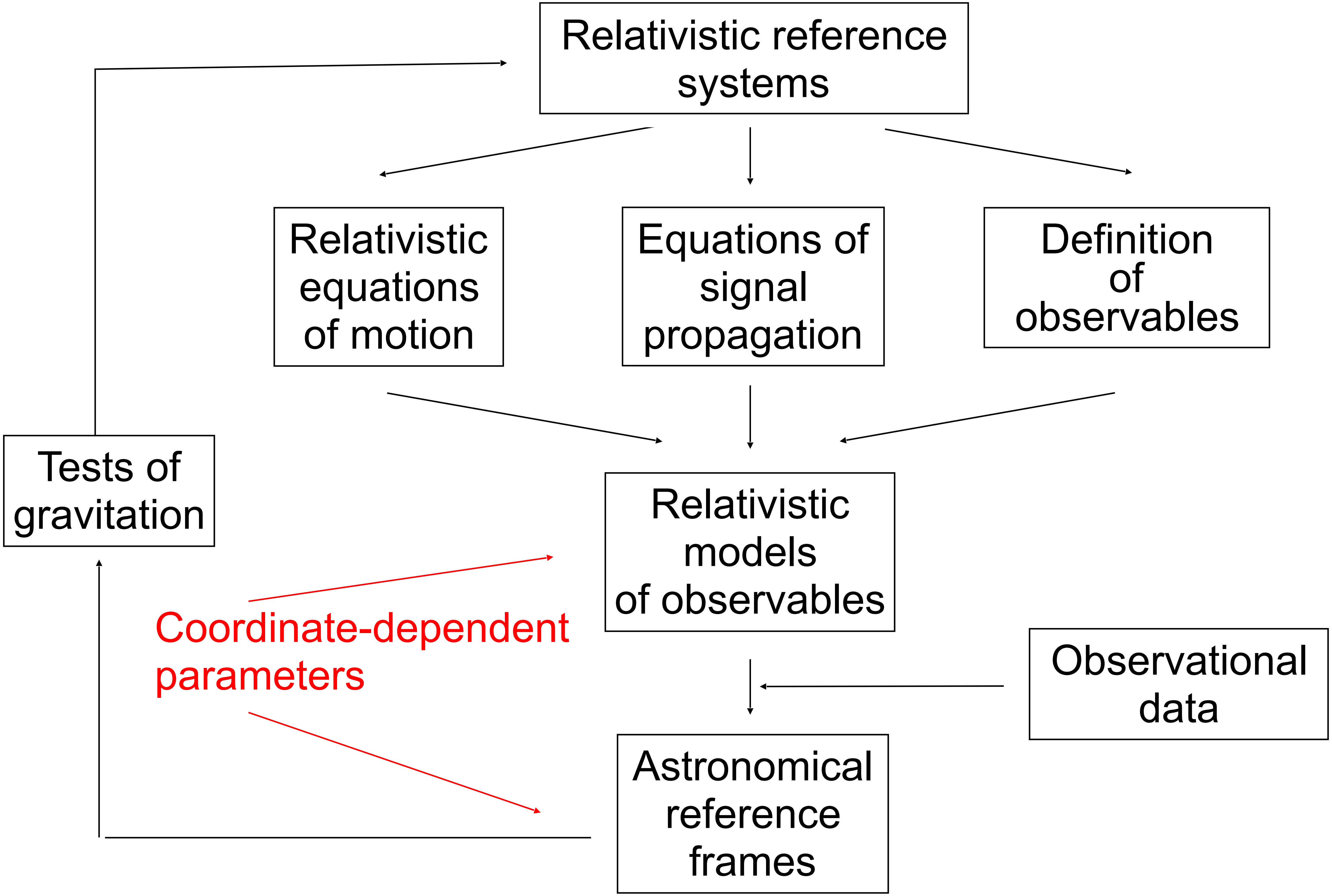



Need to describe light propagation and dynamics in relativistic framework

- one can solve geodesic
- one can introduce new tools

and define properly the observables !

Relativistic Astronomy : some basics



Relativistic reference systems

Relativistic equations of motion

Equations of signal propagation

Definition of observables

Tests of gravitation

Relativistic models of observables

Coordinate-dependent parameters

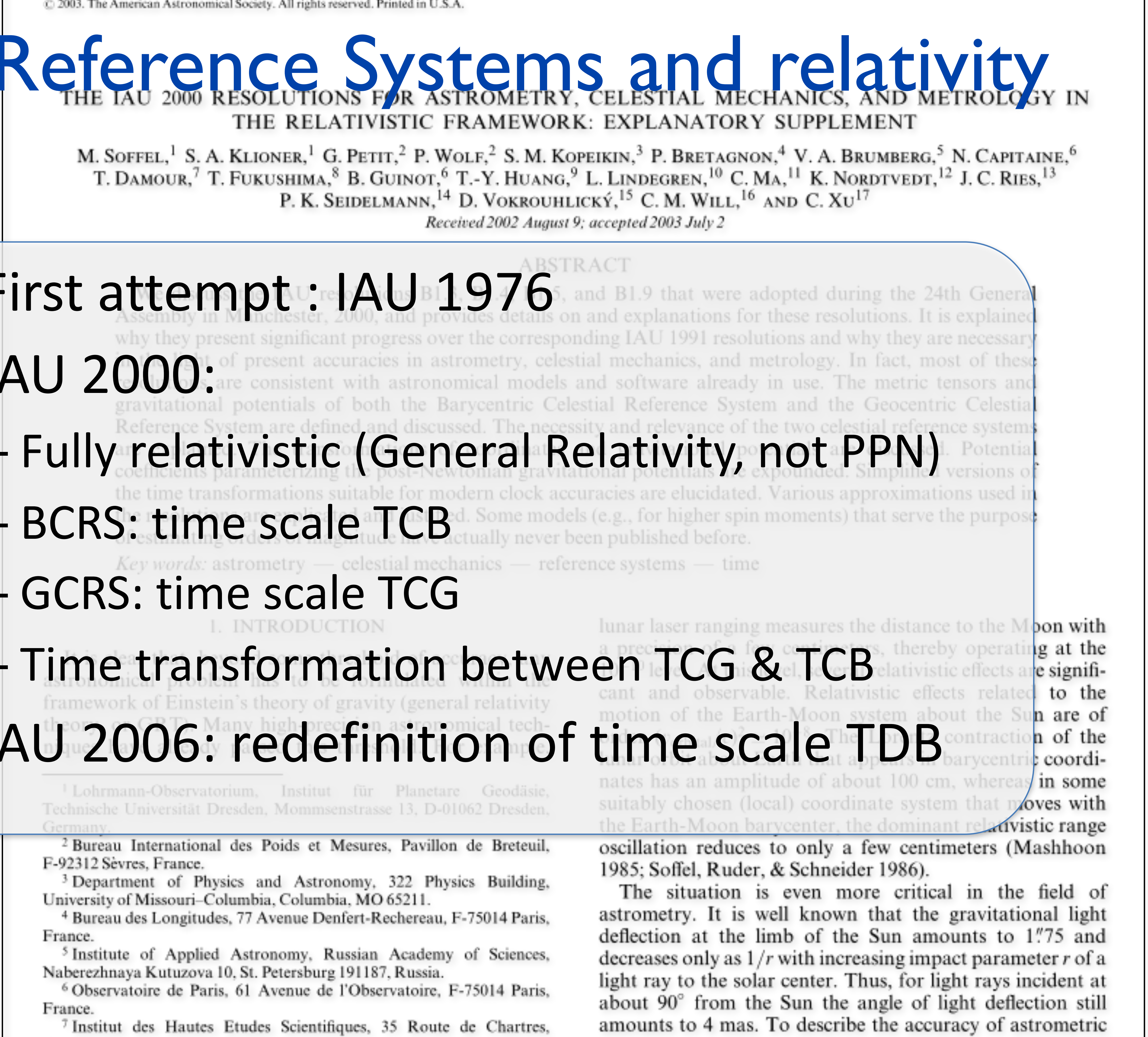
Observational data

Astronomical reference frames

Klioner, 2003

IAU Reference Systems and relativity

- First attempt : IAU 1976
- IAU 2000:
  - – Fully relativistic (General Relativity, not PPN)
  - – BCRS: time scale TCB
  - – GCRS: time scale TCG
  - – Time transformation between TCG & TCB
- IAU 2006: redefinition of time scale TDB



THE IAU 2000 RESOLUTIONS FOR ASTROMETRY, CELESTIAL MECHANICS, AND METROLOGY IN THE RELATIVISTIC FRAMEWORK: EXPLANATORY SUPPLEMENT

M. Soffel,[1] S. A. Klioner,[1] G. Petit,[2] P. Wolf,[2] S. M. Kopeikin,[3] P. Bretagnon,[4] V. A. Brumberg,[5] N. Capitaine,[6] T. Damour,[7] T. Fukushima,[8] B. Guinot,[6] T.-Y. Huang,[9] L. Lindegren,[10] C. Ma,[11] K. Nordtvedt,[12] J. C. Ries,[13] P. K. Seidelmann,[14] D. Vokrouhlický,[15] C. M. Will,[16] and C. Xu[17]

Received 2002 August 9; accepted 2003 July 2

ABSTRACT

Key words: astrometry — celestial mechanics — reference systems — time

1. INTRODUCTION

lunar laser ranging measures the distance to the Moon with a precision of a few centimeters, thereby operating at the

cant and observable. Relativistic effects related to the motion of the Earth-Moon system about the Sun are of

nates has an amplitude of about 100 cm, whereas in some suitably chosen (local) coordinate system that moves with the Earth-Moon barycenter, the dominant relativistic range oscillation reduces to only a few centimeters (Mashhoon 1985; Soffel, Ruder, & Schneider 1986).

The situation is even more critical in the field of astrometry. It is well known that the gravitational light deflection at the limb of the Sun amounts to 1.″75 and decreases only as $1/r$ with increasing impact parameter r of a light ray to the solar center. Thus, for light rays incident at about 90° from the Sun the angle of light deflection still amounts to 4 mas. To describe the accuracy of astrometric

---

[1] Lohrmann-Observatorium, Institut für Planetare Geodäsie, Technische Universität Dresden, Mommsenstrasse 13, D-01062 Dresden, Germany.

[2] Bureau International des Poids et Mesures, Pavillon de Breteuil, F-92312 Sèvres, France.

[3] Department of Physics and Astronomy, 322 Physics Building, University of Missouri–Columbia, Columbia, MO 65211.

[4] Bureau des Longitudes, 77 Avenue Denfert-Rochereau, F-75014 Paris, France.

[5] Institute of Applied Astronomy, Russian Academy of Sciences, Naberezhnaya Kutuzova 10, St. Petersburg 191187, Russia.

[6] Observatoire de Paris, 61 Avenue de l'Observatoire, F-75014 Paris, France.

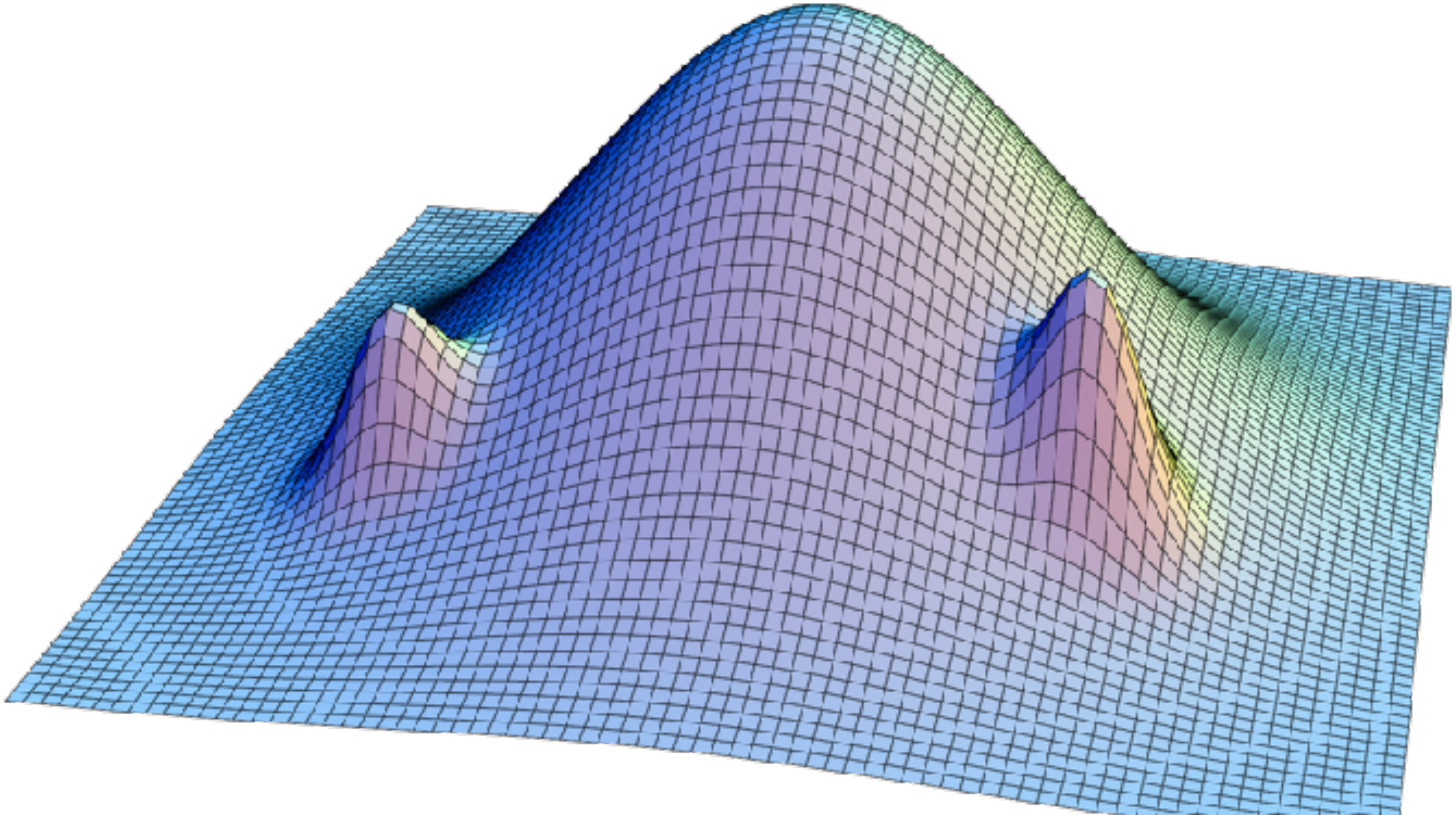
[7] Institut des Hautes Etudes Scientifiques, 35 Route de Chartres,

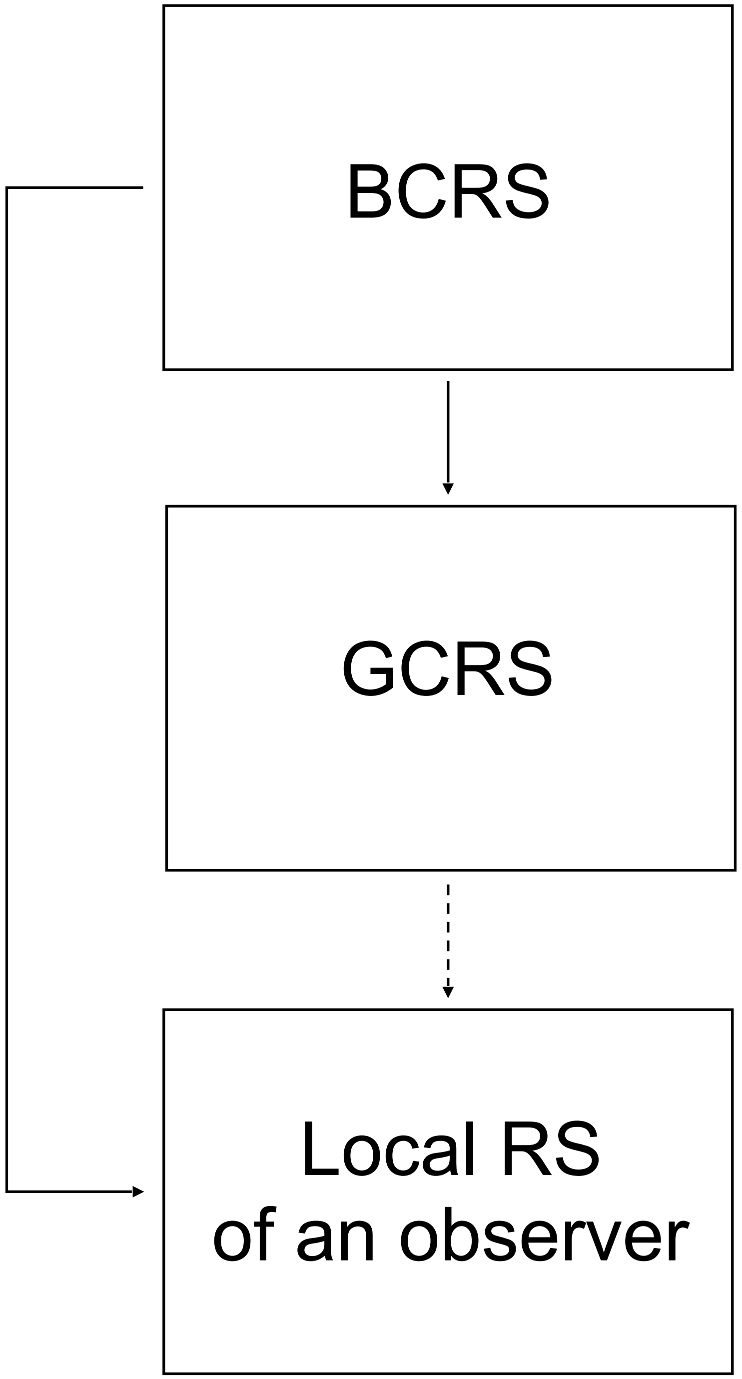
Reference systems theory

- In relativistic astronomy the
  - BCRS (Barycentric Celestial Reference System)
  - GCRS (Geocentric Celestial Reference System)
  - Local reference system of an observer

  play an important role.

- All these reference systems are defined by the form of the corresponding metric tensor.





BCRS

GCRS

Local RS of an observer

Bini, 2003
Klioner, 2004

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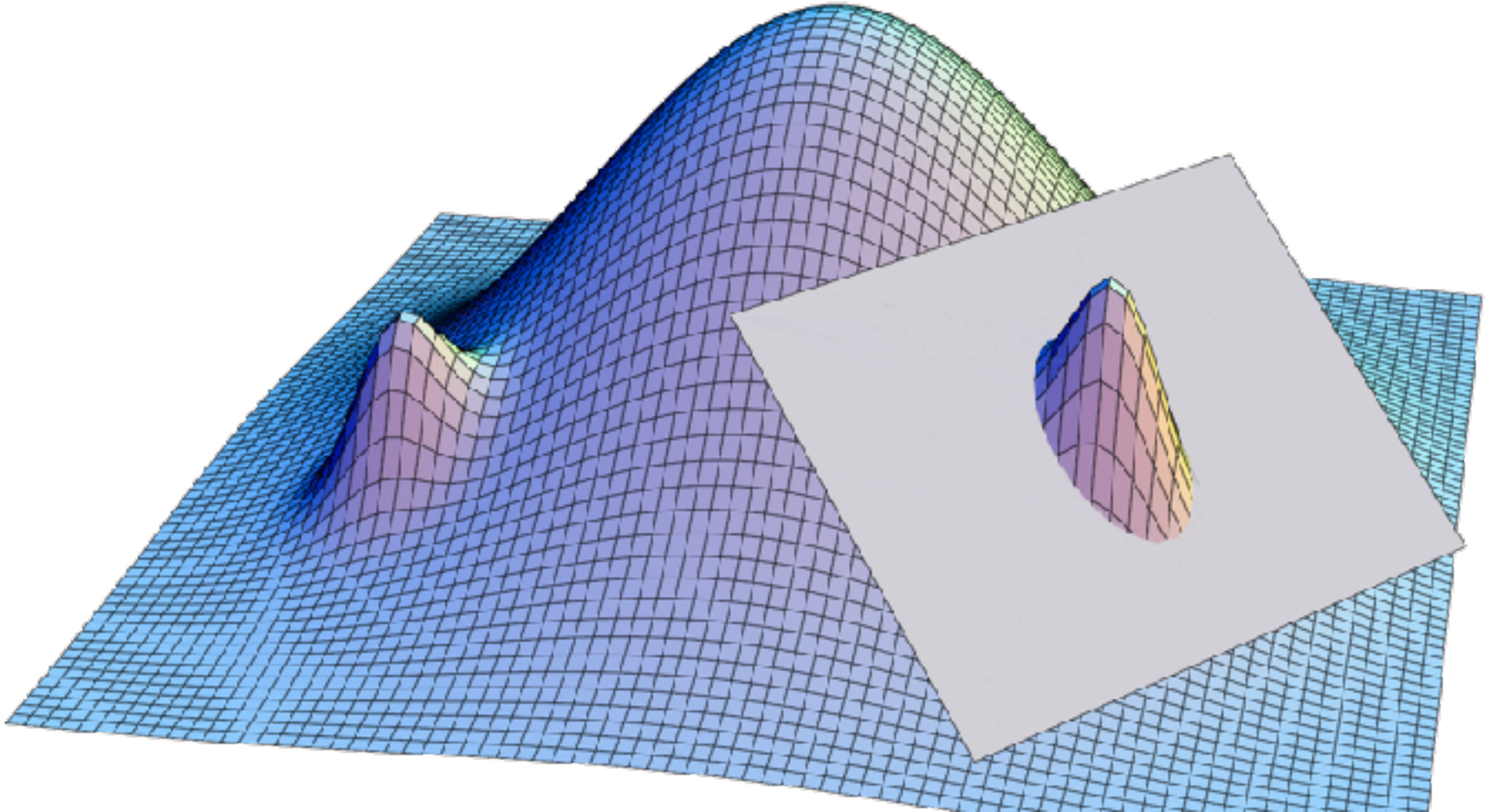

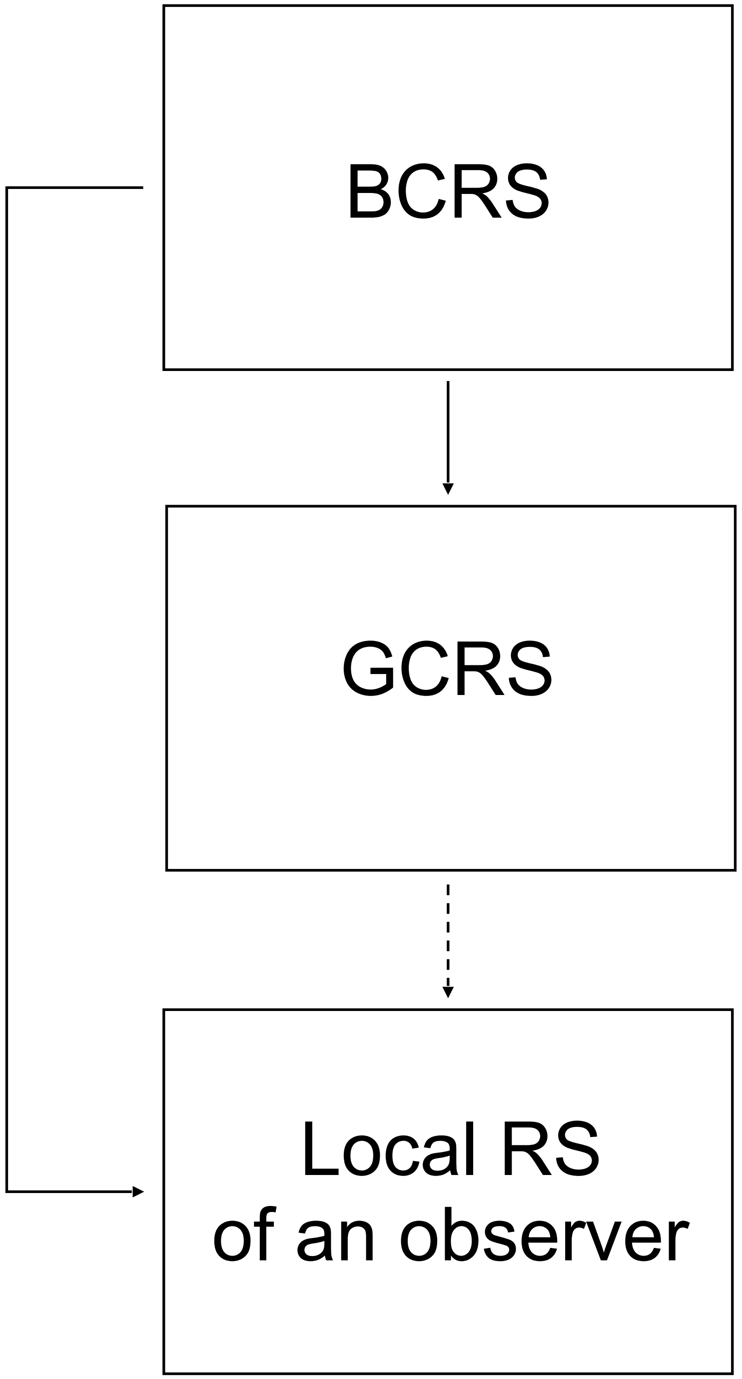

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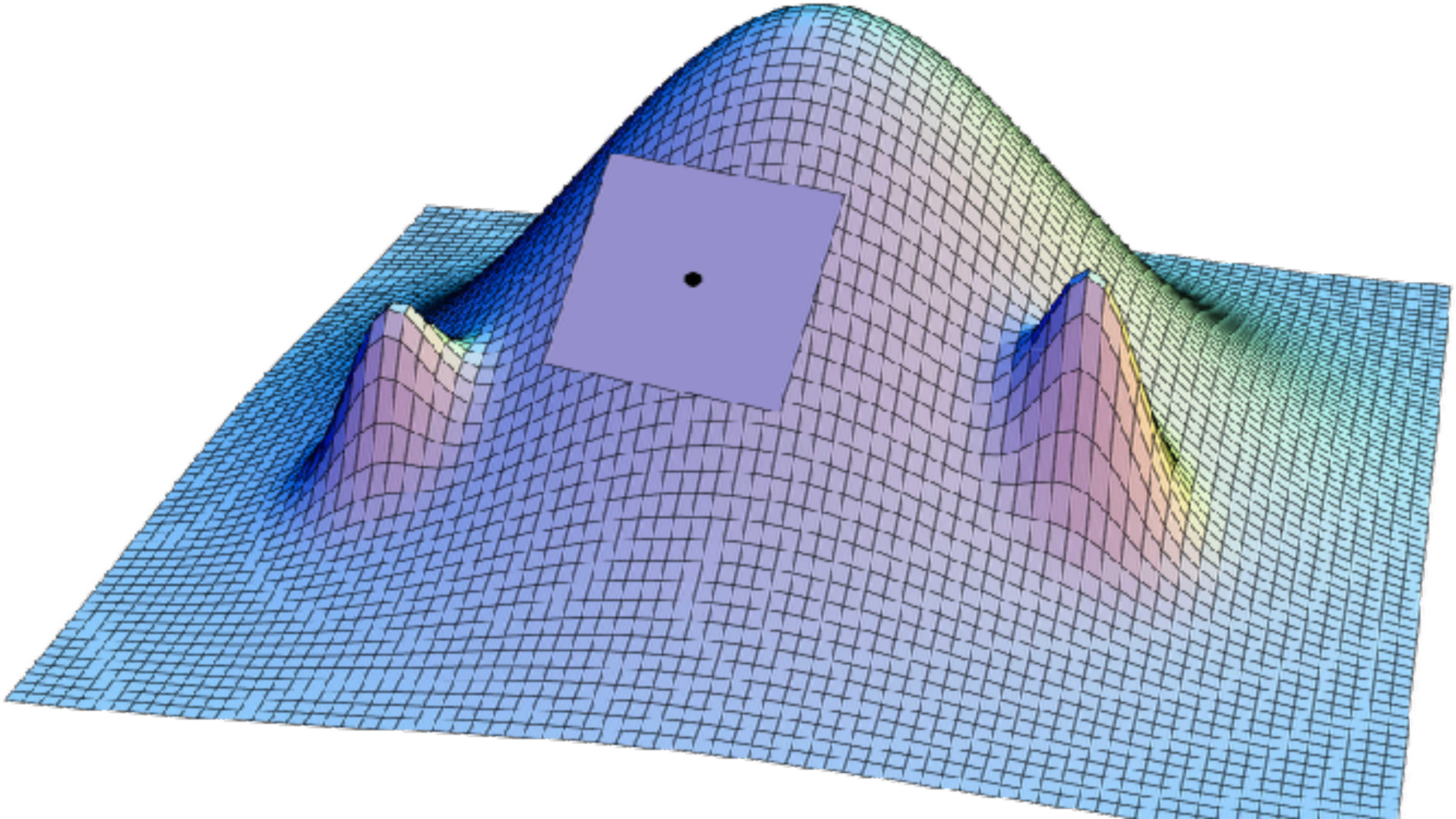

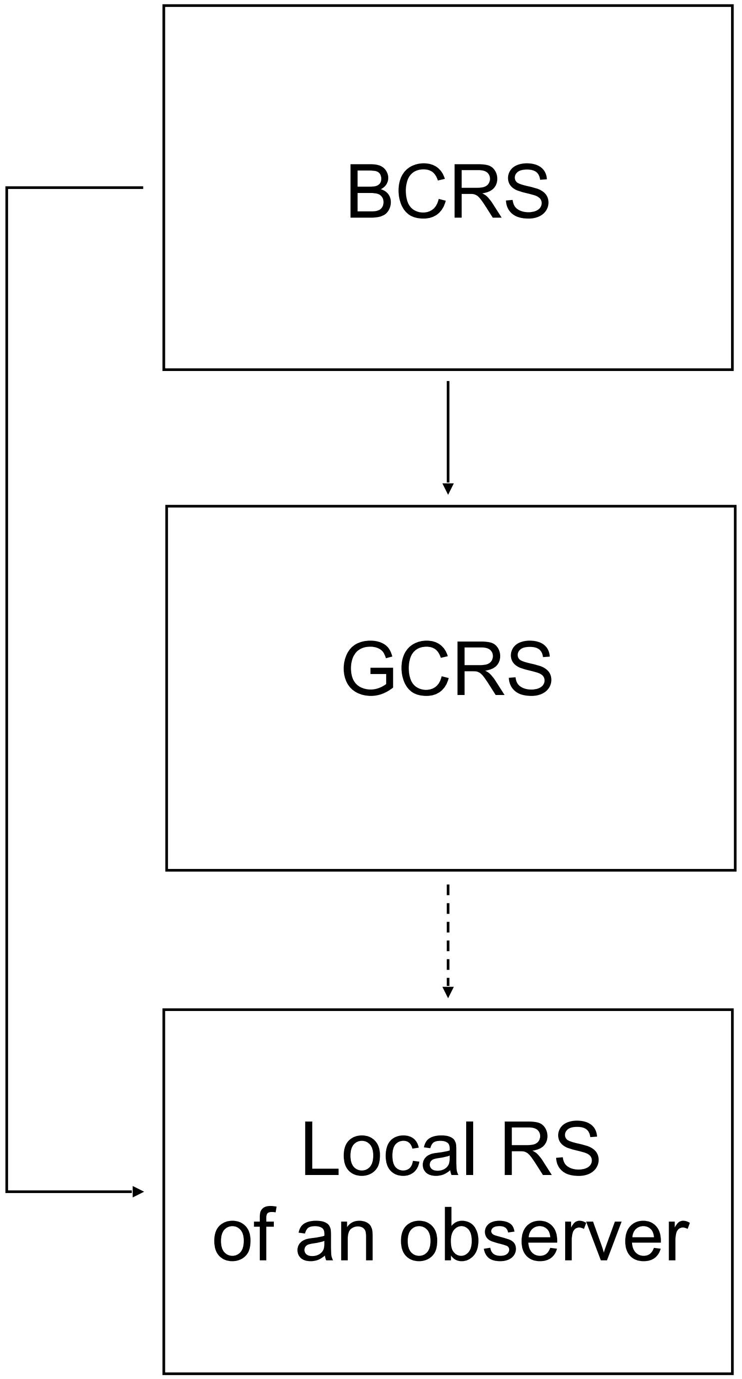

BCRS

GCRS

Local RS
of an observer

Bini, 2003
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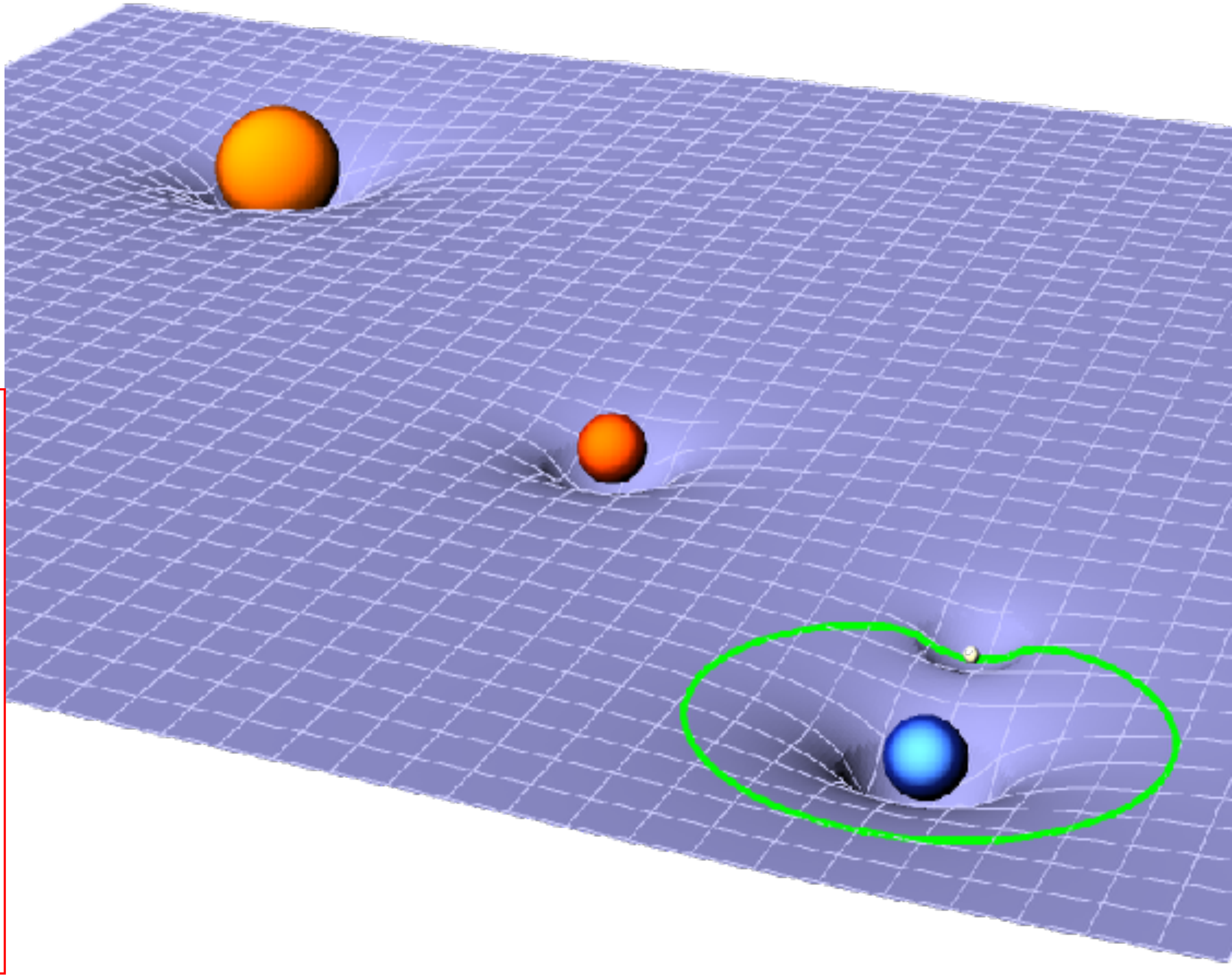
Barycentric Celestial Reference System

The BCRS is a particular reference system in the curved space-time of the Solar system

- One can use any
- but one should fix one :

ICRF by VLBI



$$g_{00} = -1 + \frac{2}{c^2} w(t, \boldsymbol{x}) - \frac{2}{c^4} w^2(t, \boldsymbol{x}) \, ,$$

$$g_{0i} = -\frac{4}{c^3} w^i(t, \boldsymbol{x}) \, ,$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2}{c^2} w(t, \boldsymbol{x}) \right) .$$

Used to describe motion of celestial body and description of light propagation

Ephemeride

Astrometry

Tests of the gravitational dynamics

- How to test the form of the metric/the Einstein field equations ? Two frameworks widely used so far:

Tests of the gravitational dynamics

- How to test the form of the metric/the Einstein field equations ? Two frameworks widely used so far:

1) Parametrized Post-Newtonian Formalism[1]

  - powerful phenomenology making an interface between theoretical development and experiments
  - metric parametrized by 10 dimensionless coefficients
  - γ and β whose values are1 in GR

$$ds^2 = (1 + 2\phi_N + 2\beta\phi_N^2 + \dots)dt^2 - (1 - 2\gamma\phi_N + \dots)d\vec{x}^2$$

---

[1] C. Will, LRR, 9, 2006
"Theory and Experiment in Grav. Physics", C. Will, 1993

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II) Fifth force formalism[2]

- modification of Newton potential of the form of a Yukawa potential

$$\phi(r) = \frac{GM}{c^2 r}\left(1 + \alpha e^{-r/\lambda}\right)$$

---

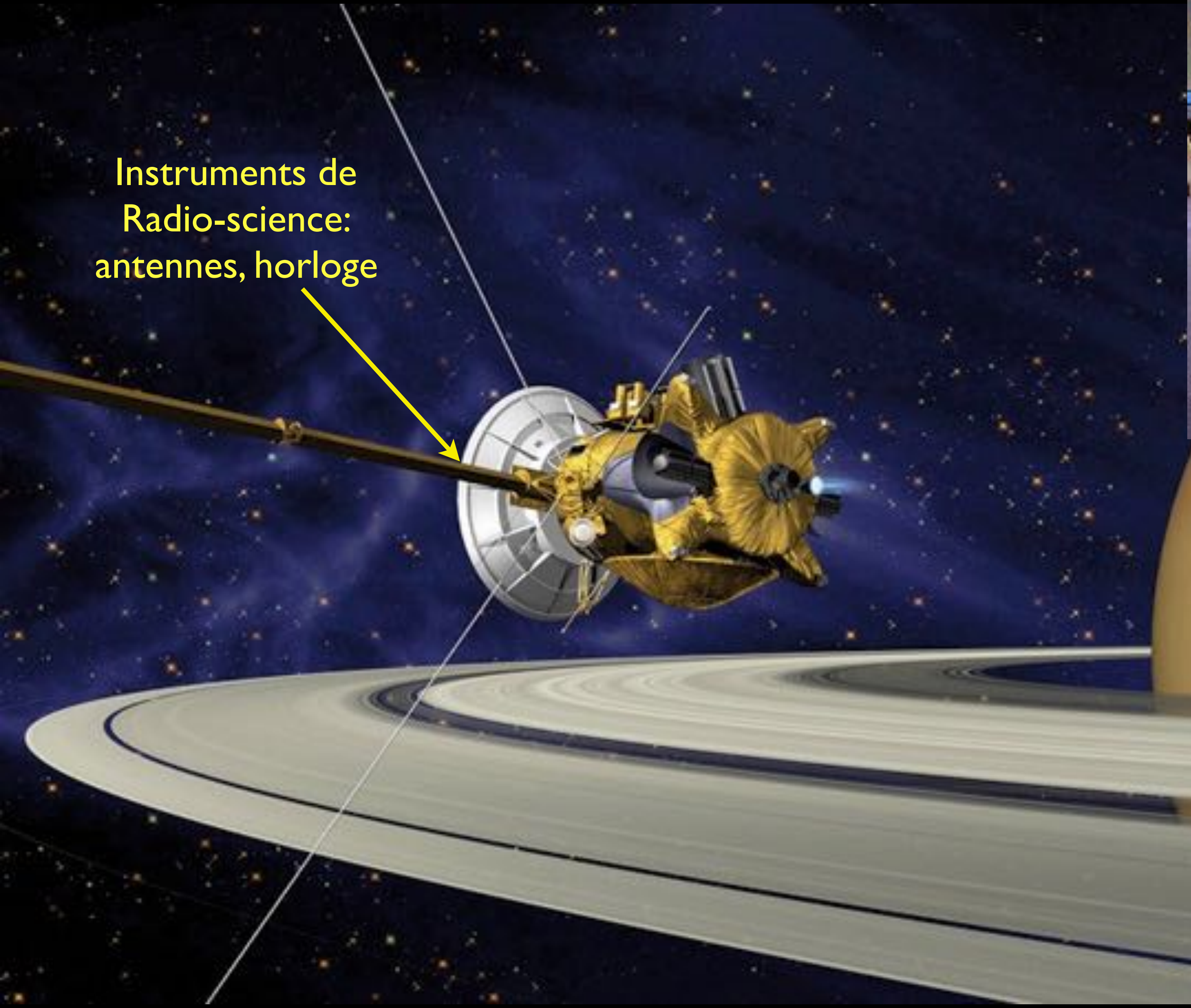
[1] C. Will, LRR, 9, 2006
"Theory and Experiment in Grav. Physics", C. Will, 1993

[2] E.G. Adelberger, Progress in Part. and Nucl. Phys., 62/102, 2009
"The Search for Non-Newtonian gravity", E. Fischbach, C. Talmadge, 1998

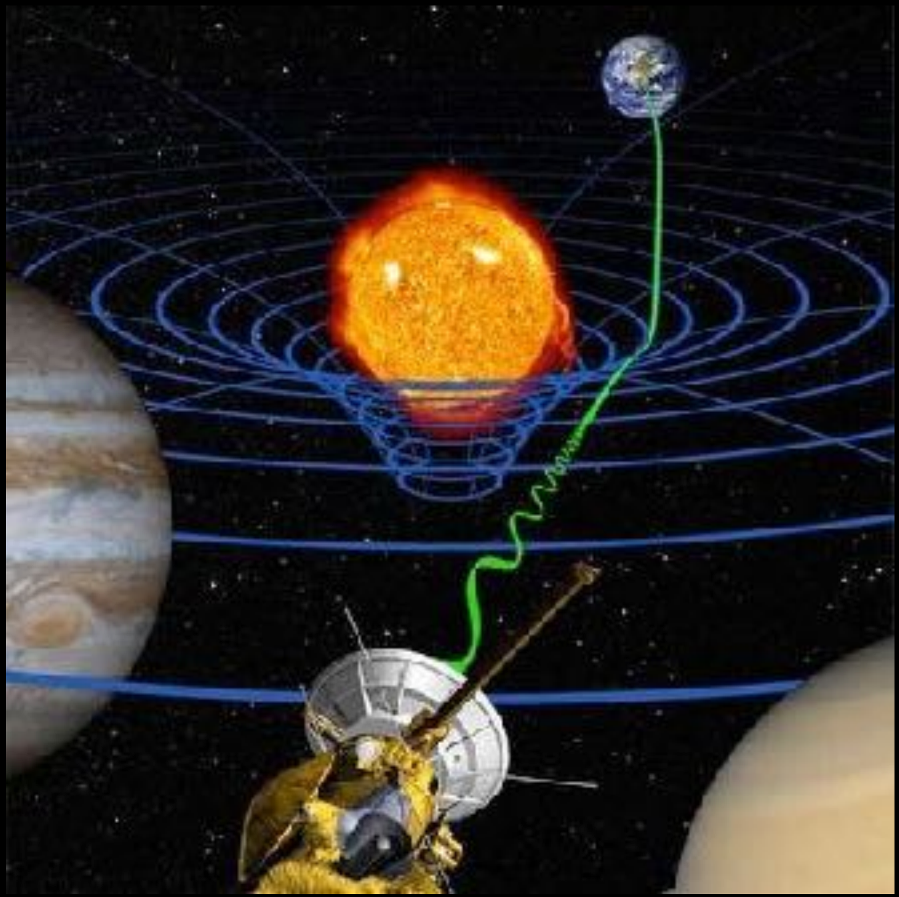
PPN parameters and their significance

Parameter	What it measures, relative to general relativity	Value in GR	Value in scalar tensor theory	Value in semi-conservative theories
γ	How much space curvature produced by unit mass?	1	$(1+\omega)/(2+\omega)$	γ
β	How "nonlinear" is gravity?	1	$1 + \Lambda$	β
ξ	Preferred-location effects?	0	0	ξ
$\alpha 1$	Preferred-frame effects?	0	0	$\alpha 1$
$\alpha 2$		0	0	$\alpha 2$
$\alpha 3$		0	0	0
$\zeta 1$	Is momentum conserved?	0	0	0
$\zeta 2$		0	0	0
$\zeta 3$		0	0	0
$\zeta 4$		0	0	0

Expériences de Radio-Science



Instruments de Radio-science: antennes, horloge



Conjonction entre la Terre et Cassini en Septembre 2003 ↔ Maximum de déflexion relativiste !

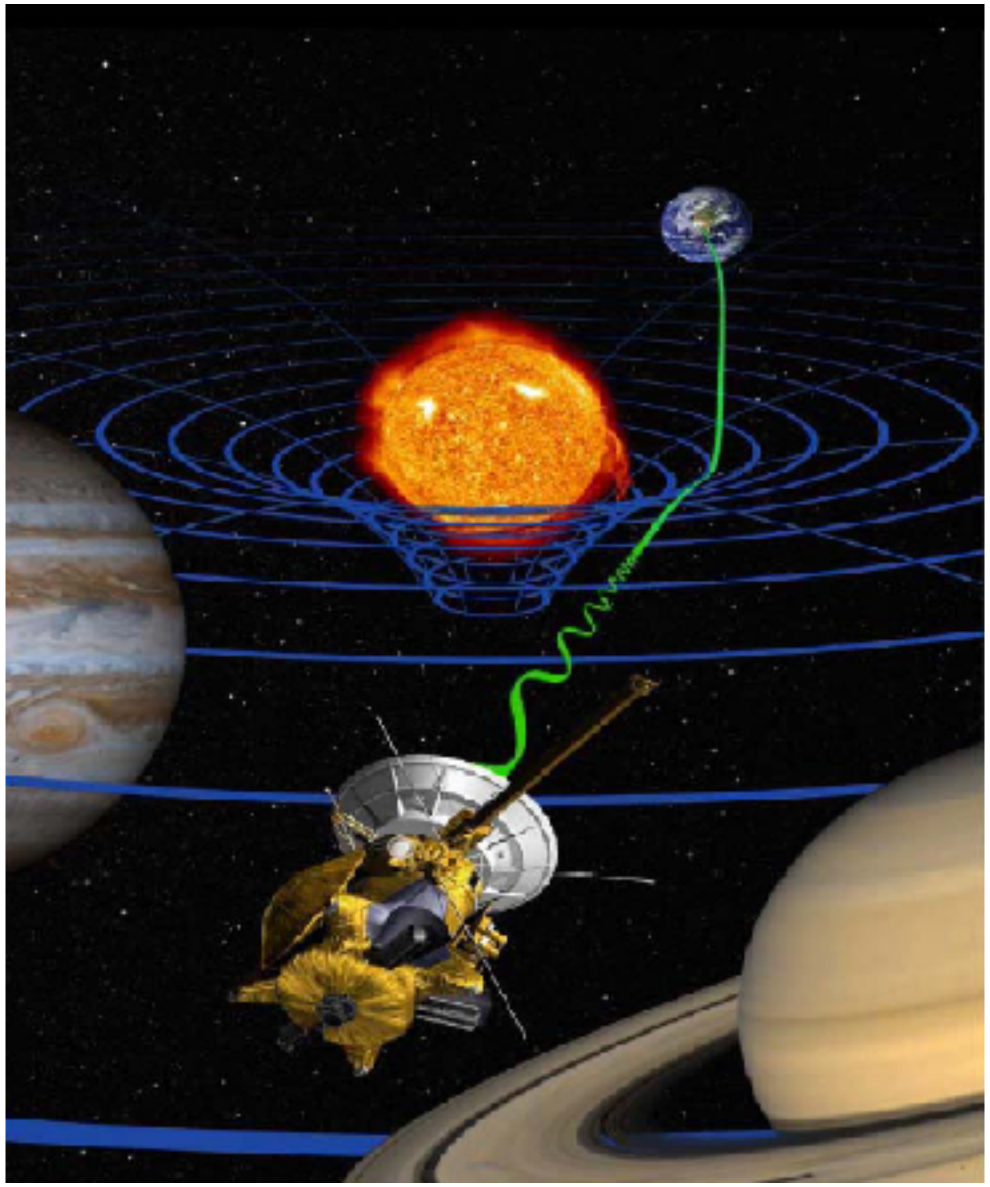
Une équipe italienne mesure le changement de la fréquence du signal avec une précision de quelques 10^{-14} de fraction de fréquence.

La Relativité serait correcte à 0.002% près

Bertotti *et al.* 2003, *Nature,* **425,** 374

Shapiro effect with Viking Probe

$$\Delta t \approx \frac{2GM(1+\gamma)}{c^3}\left[\ln\left(\frac{4r_A r_B}{r_0^{\ 2}}\right)+1\right]$$



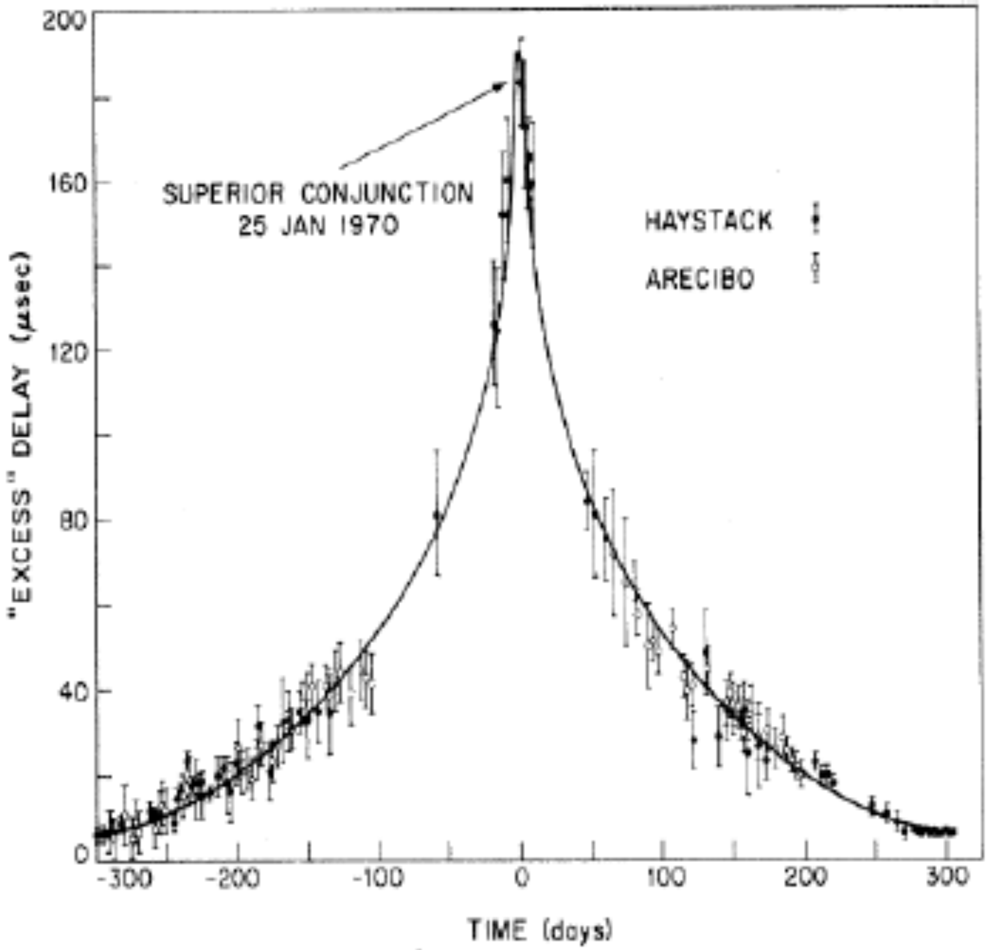
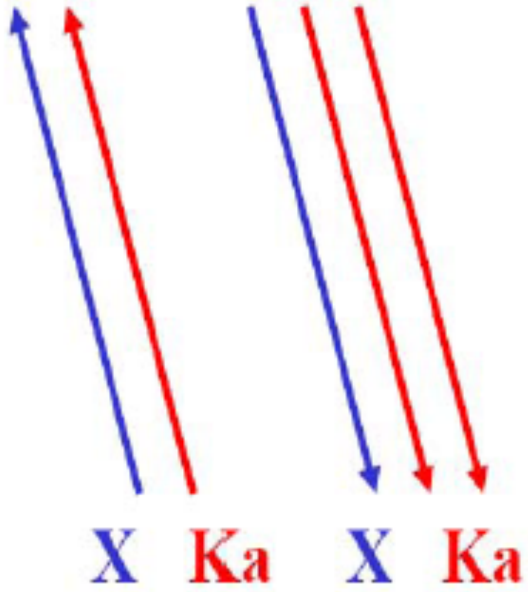
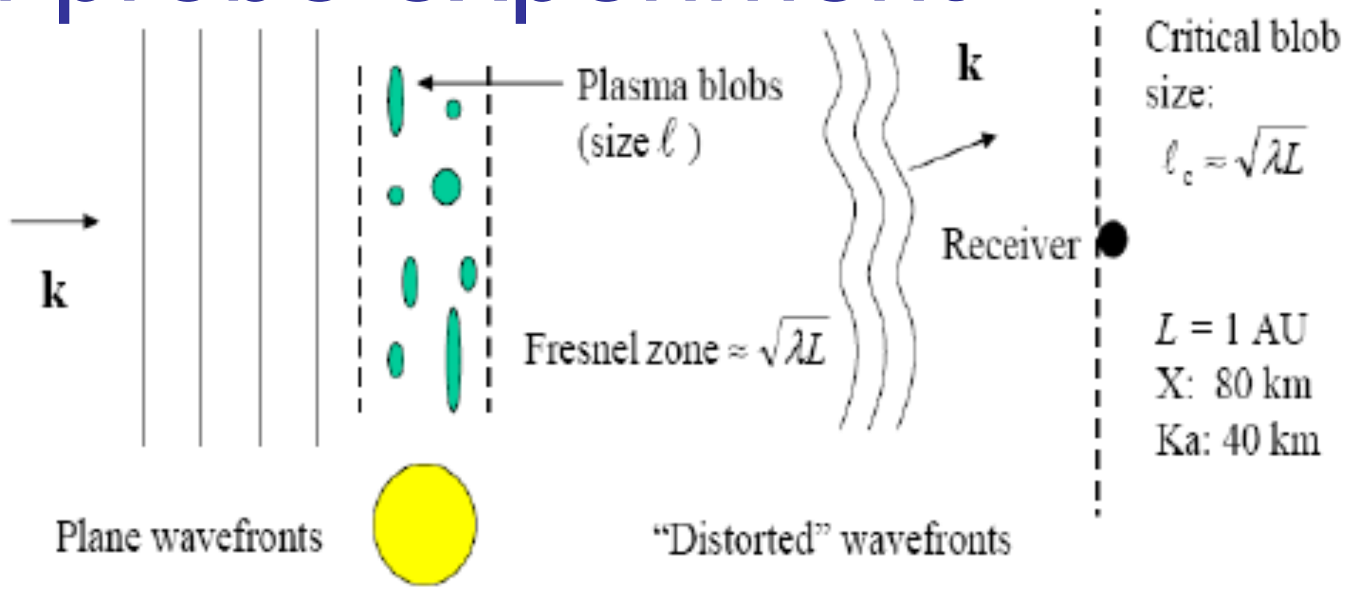


FIG. 1. Typical sample of post-fit residuals for Earth-Venus time-delay measurements, displayed relative to the "excess" delays predicted by general relativity. Corrections were made for known topographic trends on Venus. The bars represent the original estimates of the measurement standard errors. Note the dramatic increase in accuracy that was obtained with the radar-system improvements incorporated at Haystack just prior to the inferior conjunction of November 1970.

Shapiro, I.I. et al, Phys.Rev.Lett, 26, 1132 (1971)

Cassini probe experiment

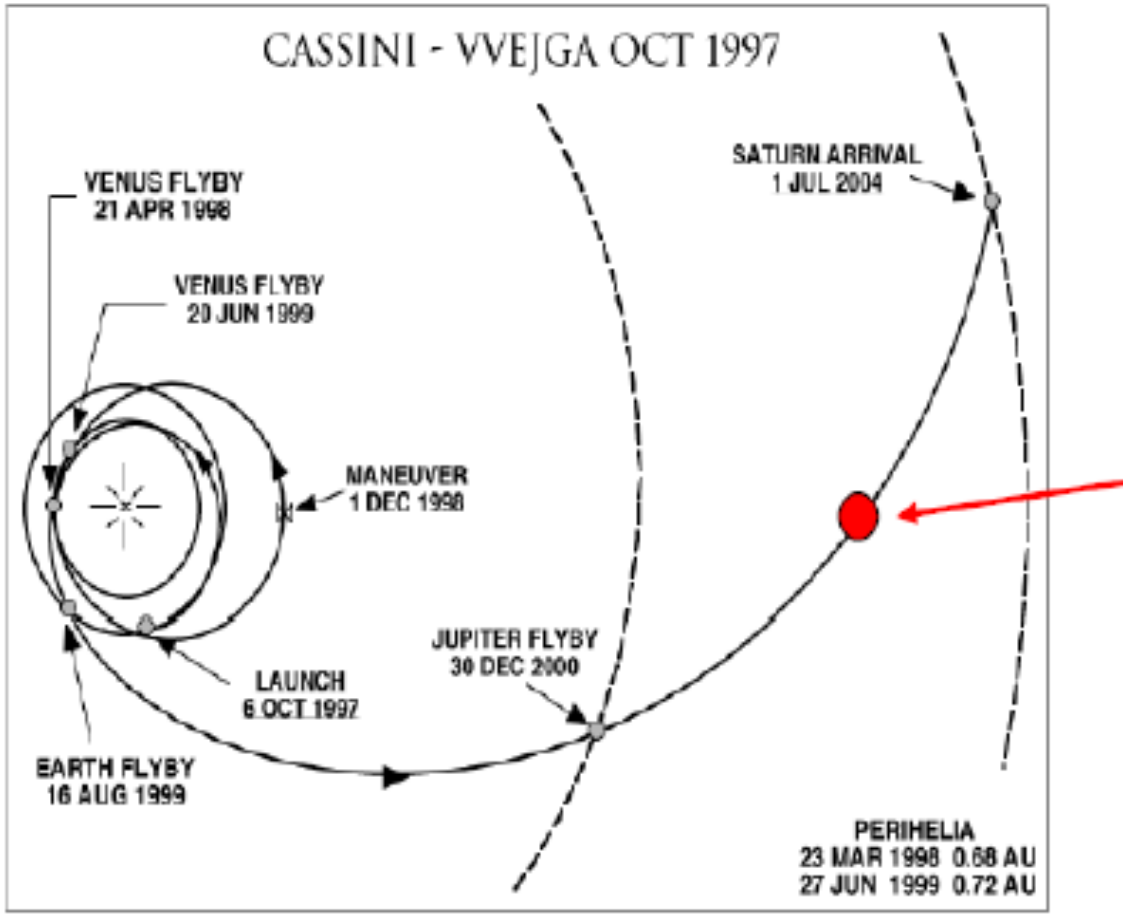




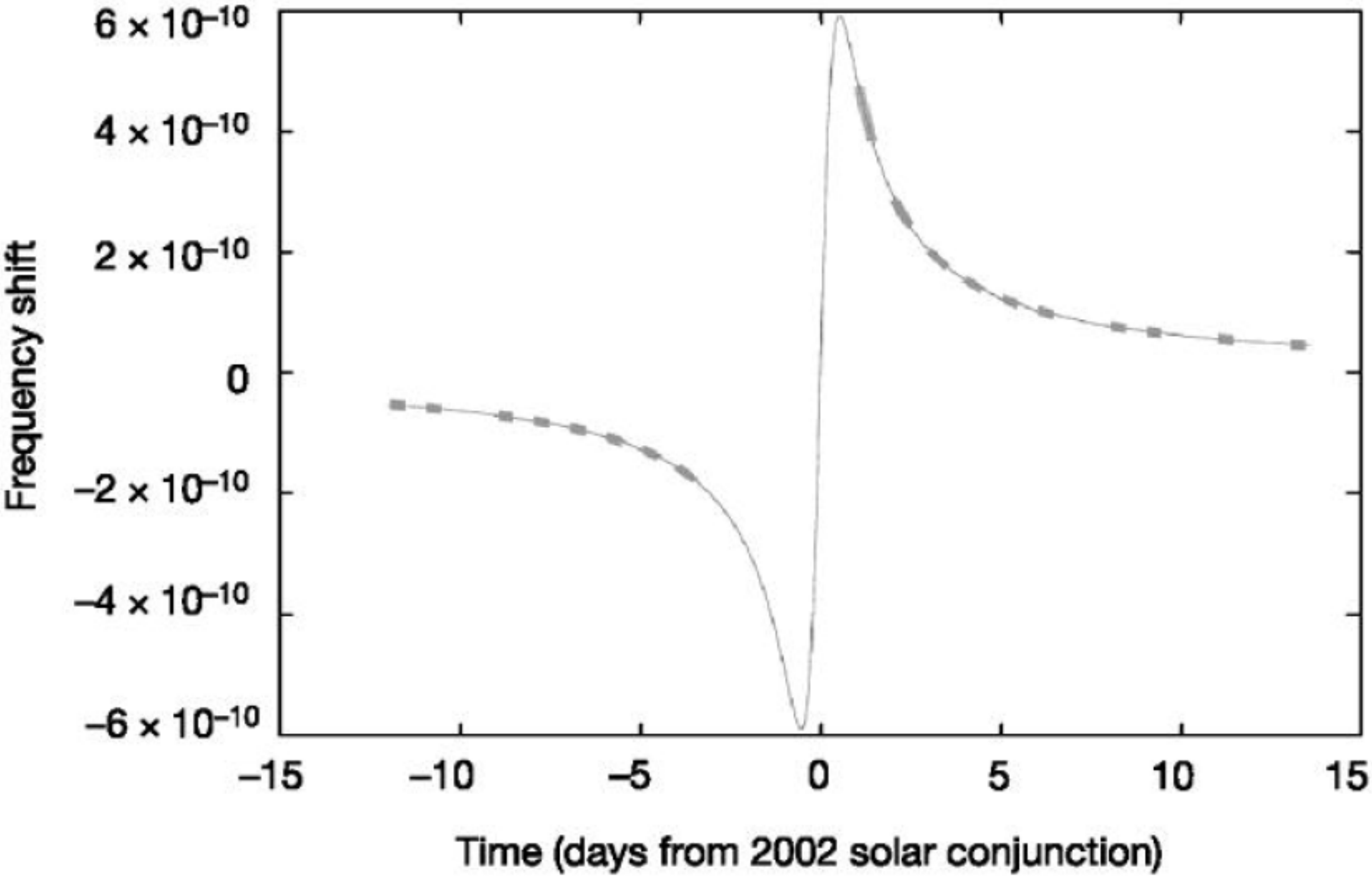
DSS 25 -
Goldstone

Doppler effect

$$\frac{\Delta\nu}{\nu} \approx 4(1+\gamma)\frac{M_{Sol}}{b}\frac{db}{dt}$$



$$\delta\left(\frac{f_r - f_e}{f_0}\right) \approx \frac{d}{dt}\left[2(1+\gamma)\frac{GM}{c^3}\ln\left(\frac{4r_S r_G}{b^2}\right)\right] \approx -4(1+\gamma)\frac{GM}{c^3 b}\frac{db}{dt}$$



$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

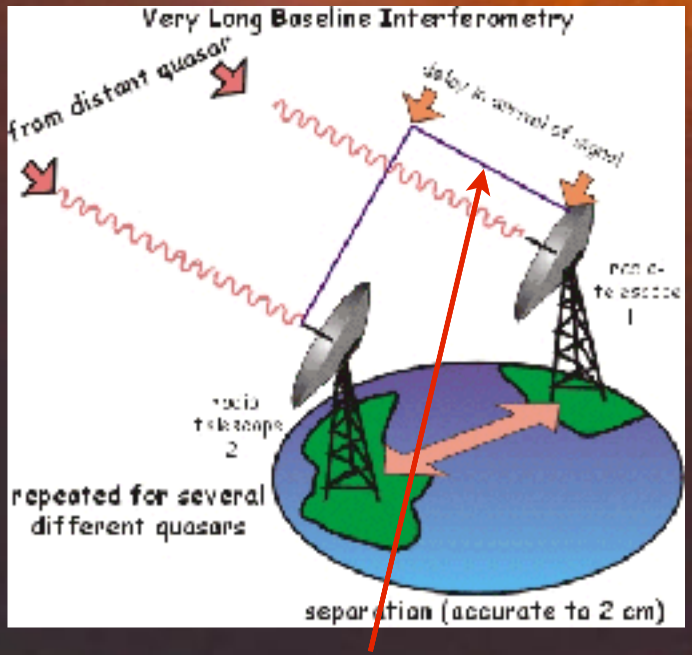
Bertotti, B. et al, Nature, 425, 374 (2003)

Interférométrie à Très Longue Base: le VLBI

On observe les objets les plus lointains:

1. Ne bougent pas, en première approximation,
2. Avec 2 radio-télescopes, on réalise donc une triangulation !

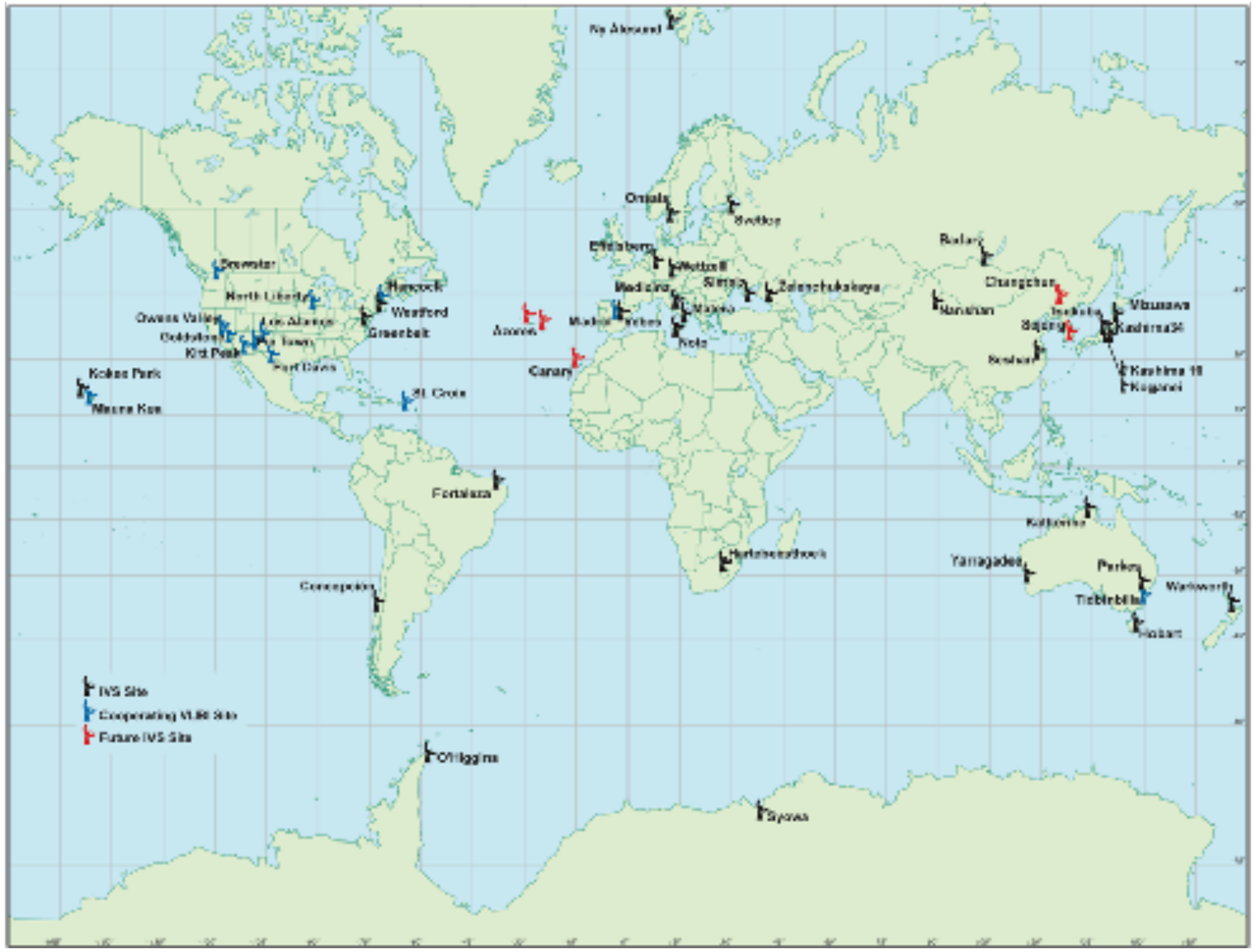
Position cinématique de la Terre dans l'espace



Délai temporel entre les réceptions des signaux: mesure temporelle !

Qui dit un laps de temps... dit une distance !

International VLBI service (IVS)



primary goals :

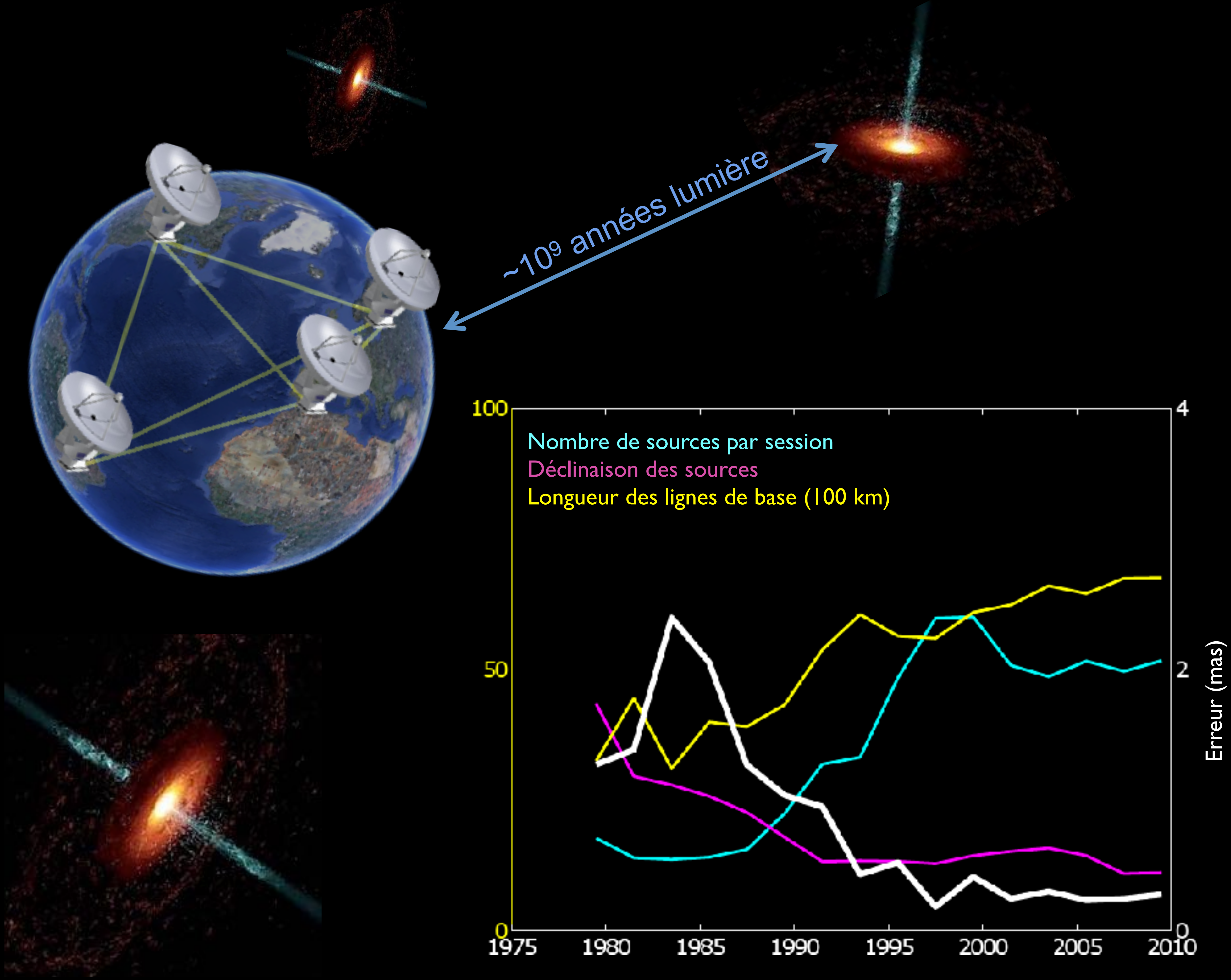
- monitoring the Earth's rotation
- determining reference frames

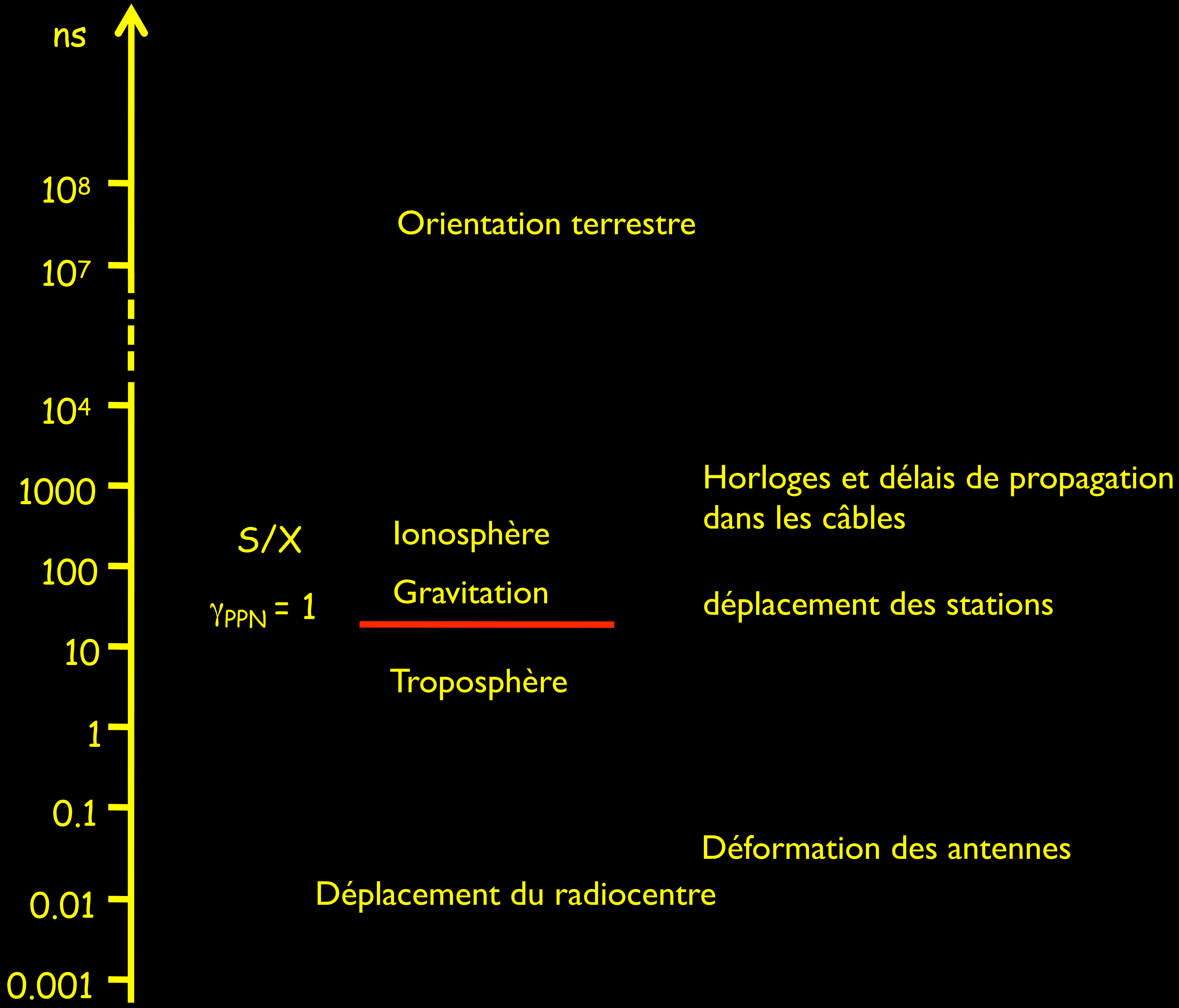
5 data centers and 29 analysis centers

IVS-OPAR @ SYRTE/Obs. Paris
lead : S. Lambert

Use Mark-5 VLBI Analysis Software Calc/Solve.
109 programs, 3680 modules
1.02 million lines of source code written mainly in Fortran-95

Observation time span : From August 1979 to mid-2016
almost 6000 VLBI 24-hr sessions
(correspondingly 10 million delays)


~10^9 années lumière
Nombre de sources par session
Déclinaison des sources
Longueur des lignes de base (100 km)
100
50
0
4
2
0
Erreur (mas)
1975
1980
1985
1990
1995
2000
2005
2010



Orientation terrestre

Horloges et délais de propagation dans les câbles

S/X

Ionosphère

$\gamma_{PPN} = 1$

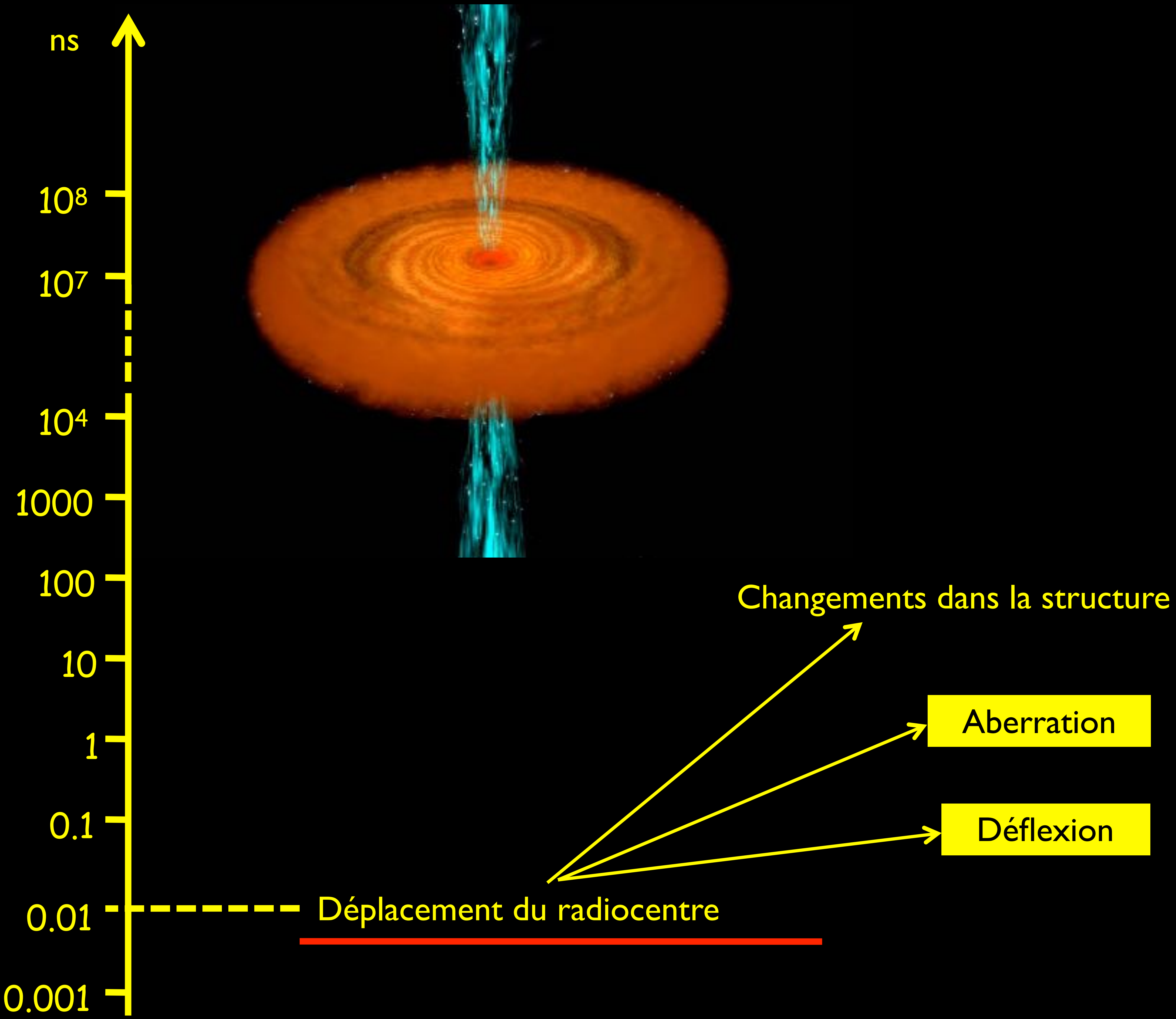
Gravitation

déplacement des stations

Troposphère

Déformation des antennes

Déplacement du radiocentre

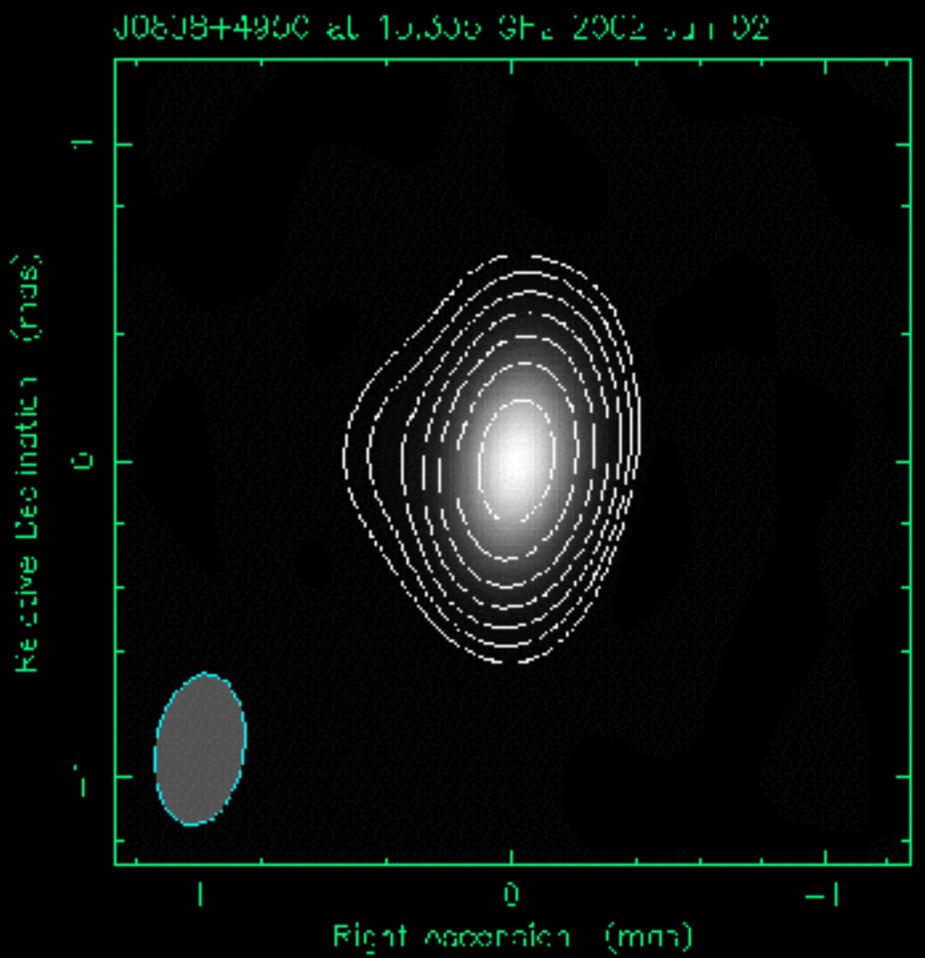


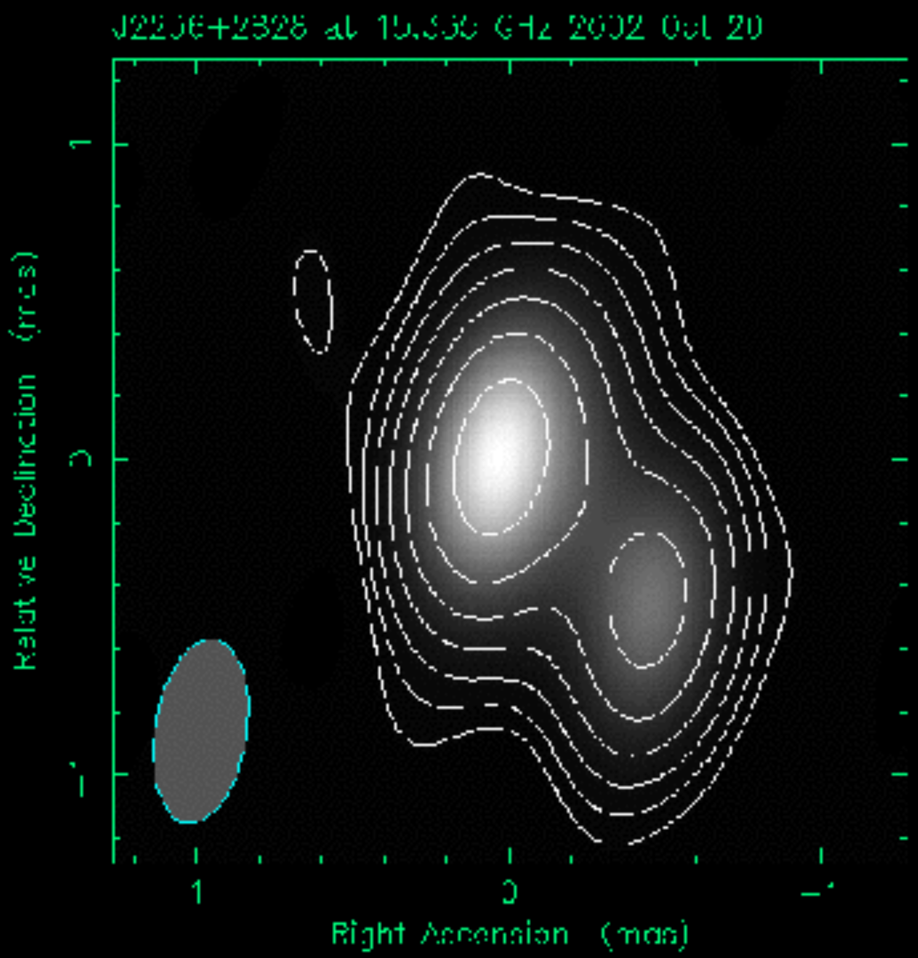
Changements dans la structure

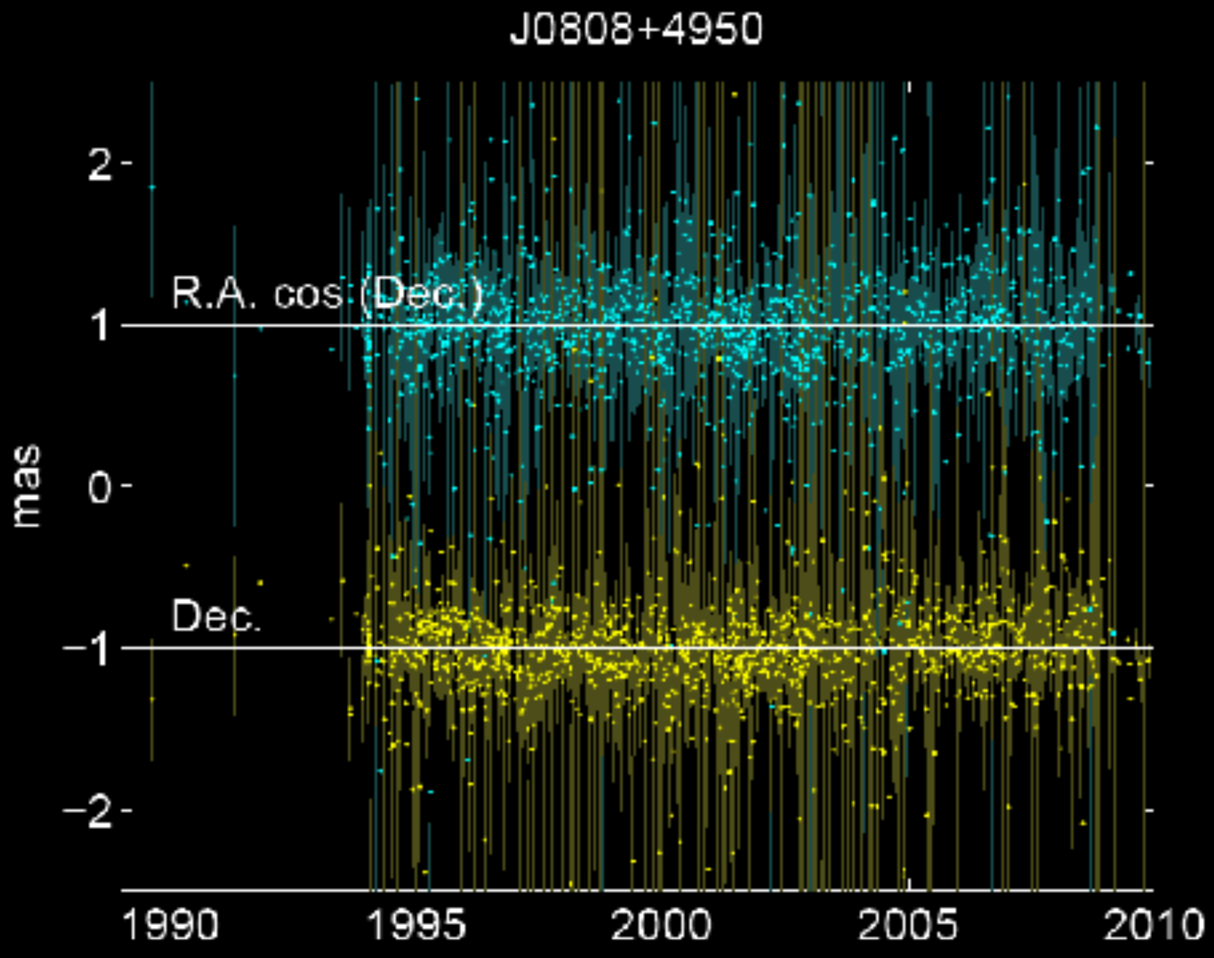
Aberration

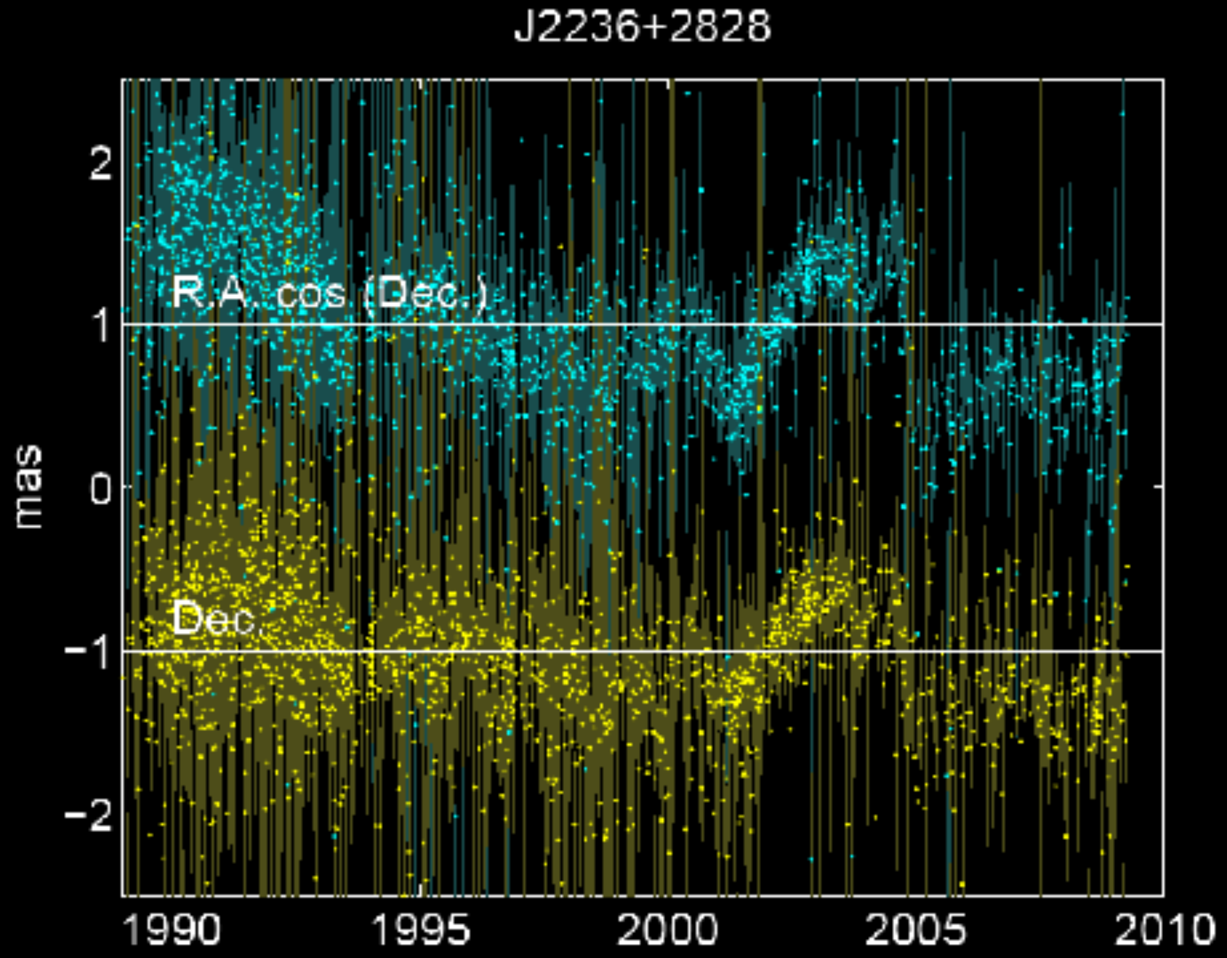
Déflexion

Déplacement du radiocentre


J0808+4950 at 15.356 GHz 2002 Jun 02
Relative Declination (mas)
1
0
-1
Right Ascension (mas)
1
0
-1


J2236+2828 at 15.356 GHz 2002 Oct 20
Relative Declination (mas)
1
0
-1
Right Ascension (mas)
1
0
-1


J0808+4950
mas
2
1
0
-1
-2
R.A. cos (Dec.)
Dec.
1990
1995
2000
2005
2010


J2236+2828
mas
2
1
0
-1
-2
R.A. cos (Dec.)
Dec.
1990
1995
2000
2005
2010

Configuration d'analyse VLBI

Modélisation du retard imparfaite

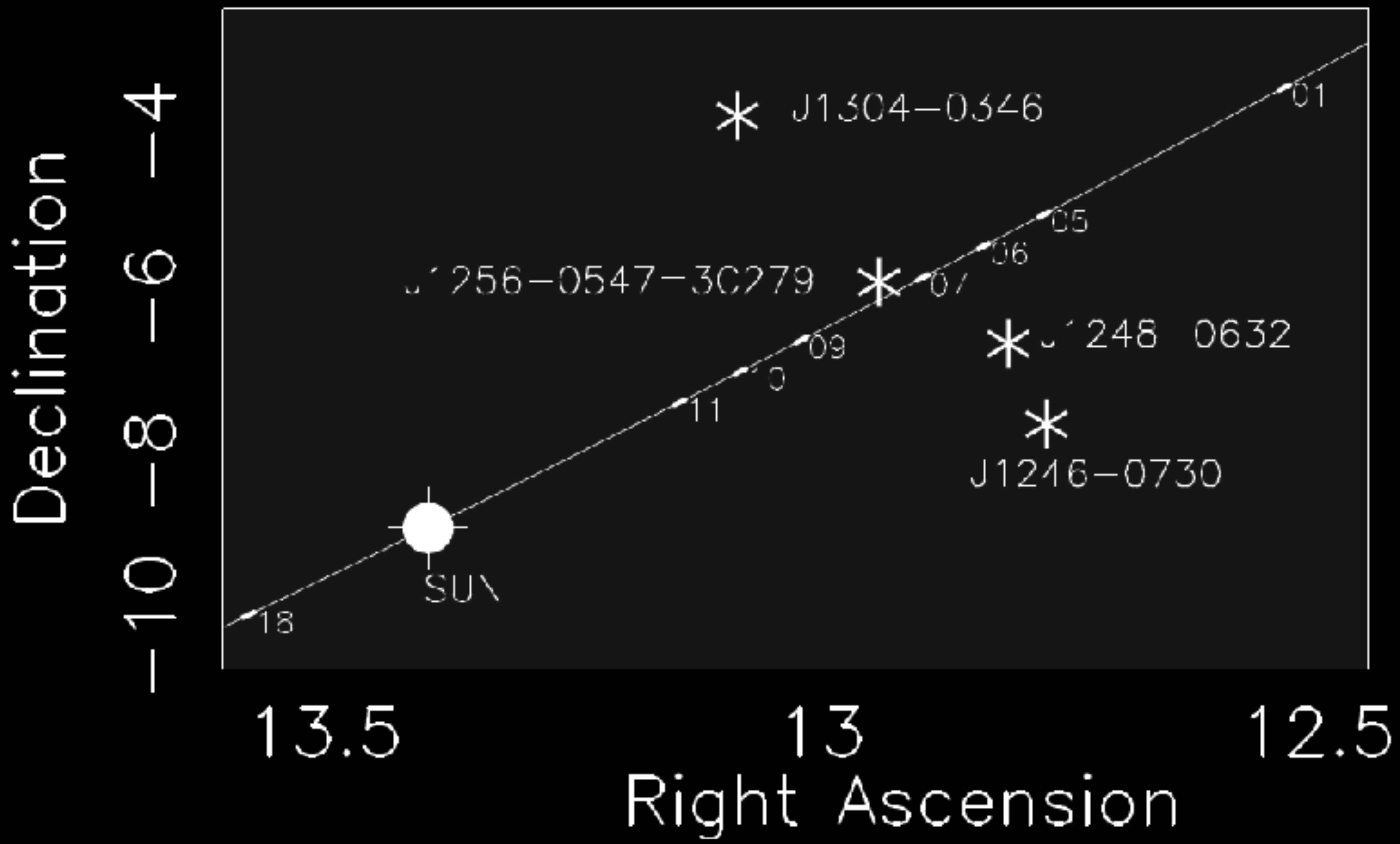
→ Estimer les écarts aux modèles

- EOP
- Déplacements des sources et stations
- ZTD et allongement troposphérique
- …

→ Contraindre

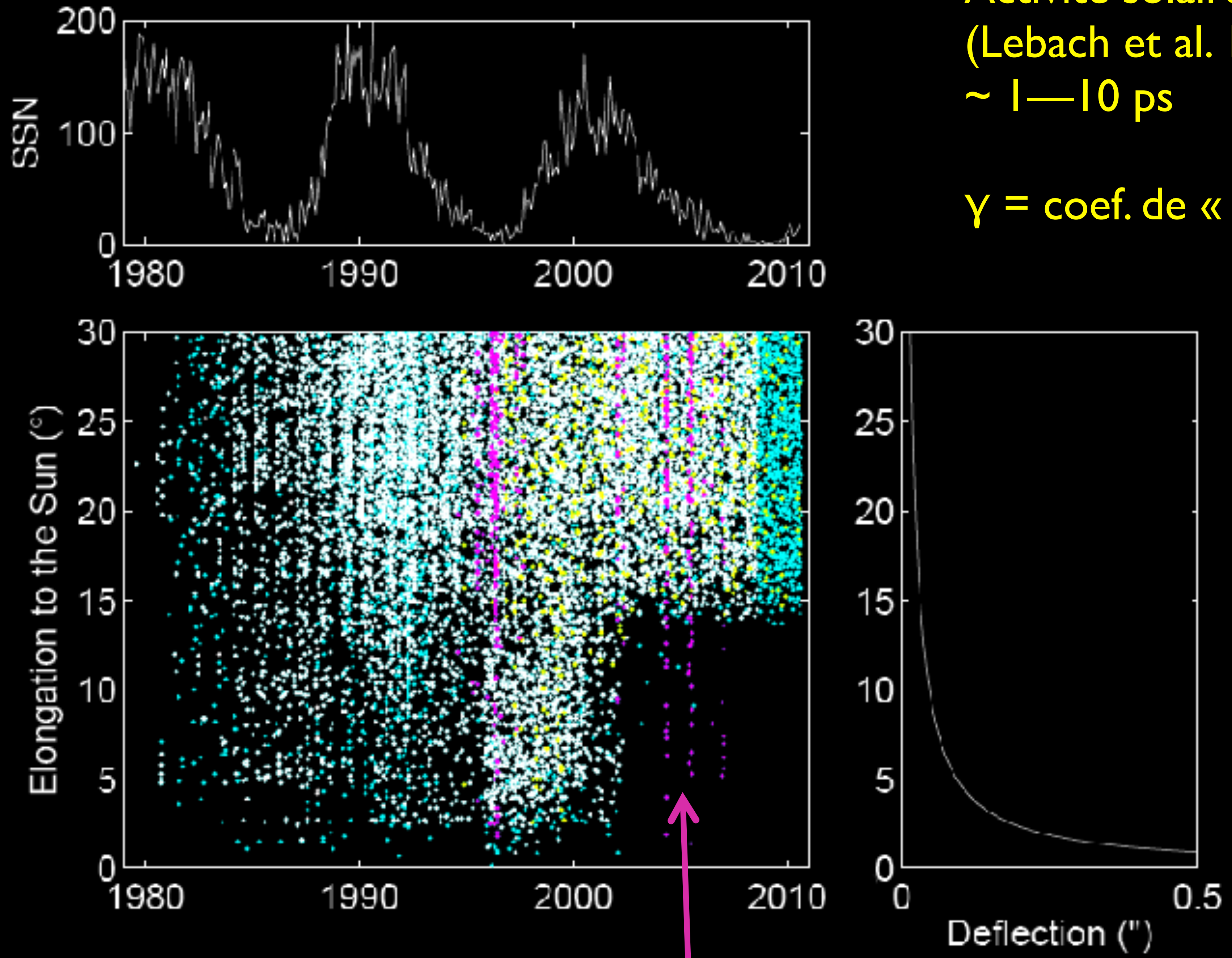
- Rattachement aux RF
- Risque de contamination entre paramètres (ex : quasars / nutations)

Fomalont et al. 2009



$\gamma - 1 = 2 \pm 3 \times 10^{-4}$

Lambert & Le Poncin-Lafitte 2009, 2010 : utilisation de la base de données VLBI complète



Activité solaire = déflexion plus forte (Lebach et al. 1995)
~ 1—10 ps

γ = coef. de « déflexion »

Sans le VLBA :
$\gamma - 1 = 0.4 \pm 1.4 \times 10^{-4}$

+ RDV :
$\gamma - 1 = -0.7 \pm 1.3 \times 10^{-4}$

+ VCS :
$\gamma - 1 = -0.8 \pm 1.2 \times 10^{-4}$

VLBA ~ 3% des sessions ~ 30% des observations

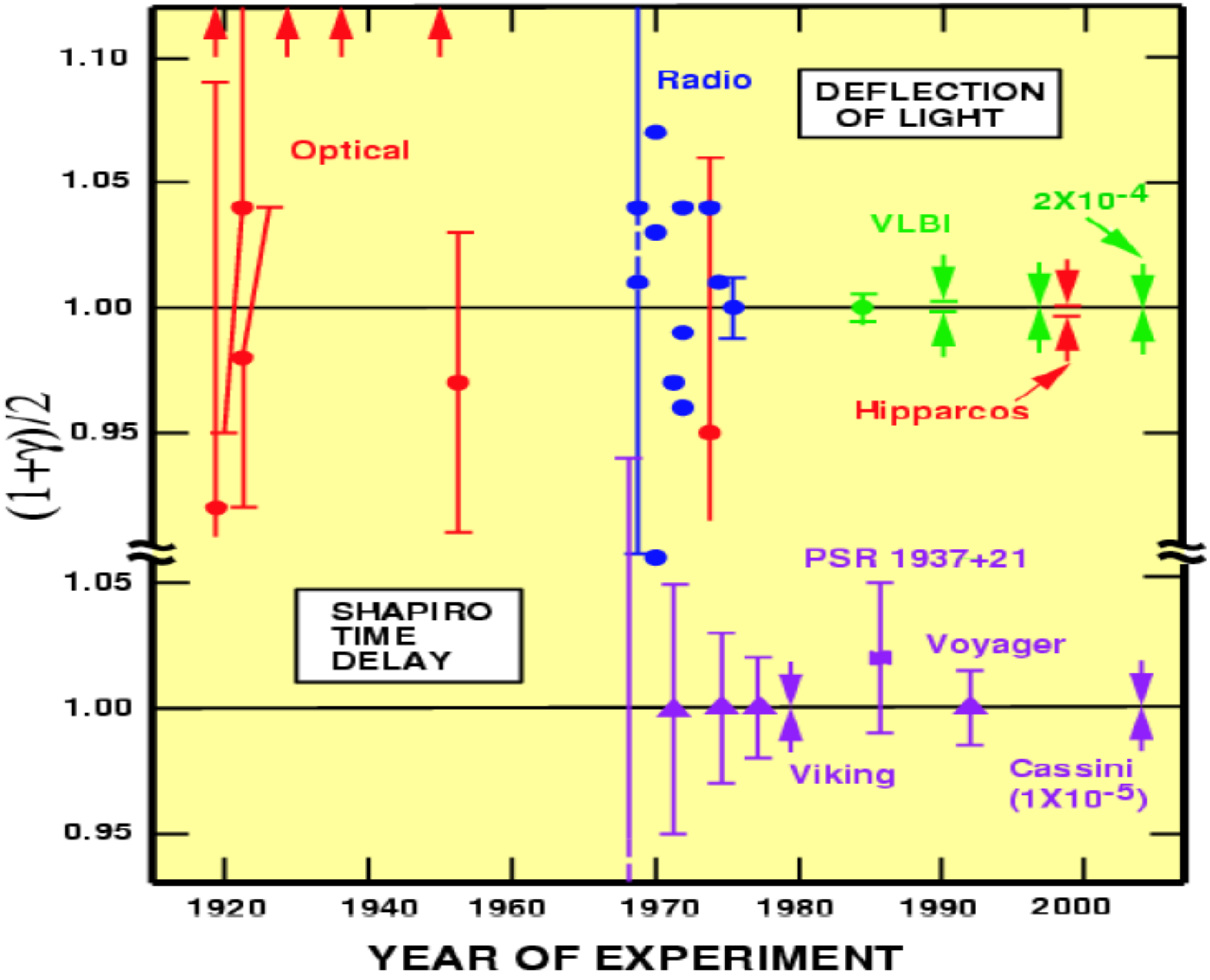
- Le VLBI mesure γ à 1.2 x 10^{-4} :
  - Effets coronaux difficiles à supprimer
  - Précision limitée par troposphère + structure de source

- Apport substantiel du VLBA

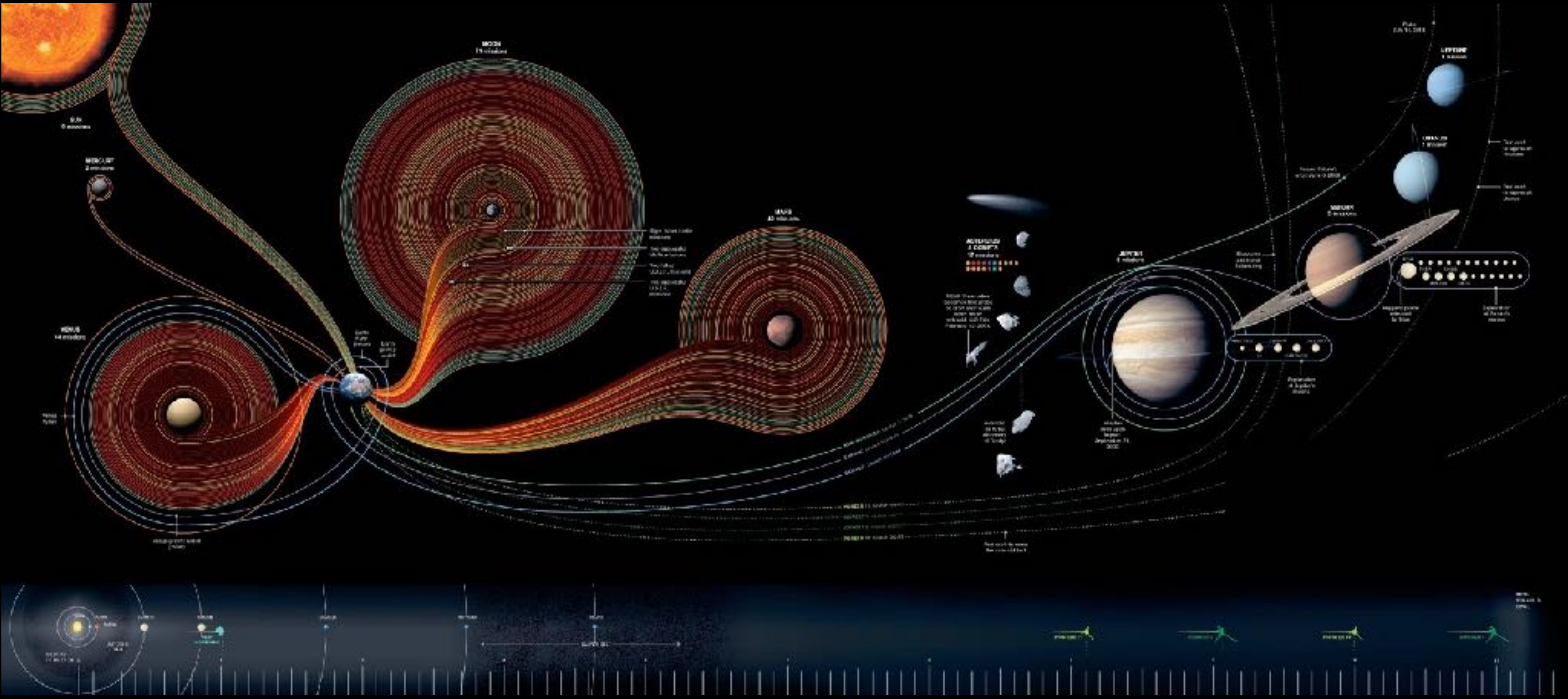
- Dommage que l'IVS « bloque » à 15° du soleil pour les sessions routinières

- Programmer des sources proches du soleil dans le futur…

THE PARAMETER $(1+\gamma)/2$



Exploration spatiale du Système solaire
Analyse du mouvement des corps



Nombreuses données de navigation →

1. Amélioration des éphémérides
2. Amélioration des tests de la gravitation

Les éphémérides planétaires et satellitaires.

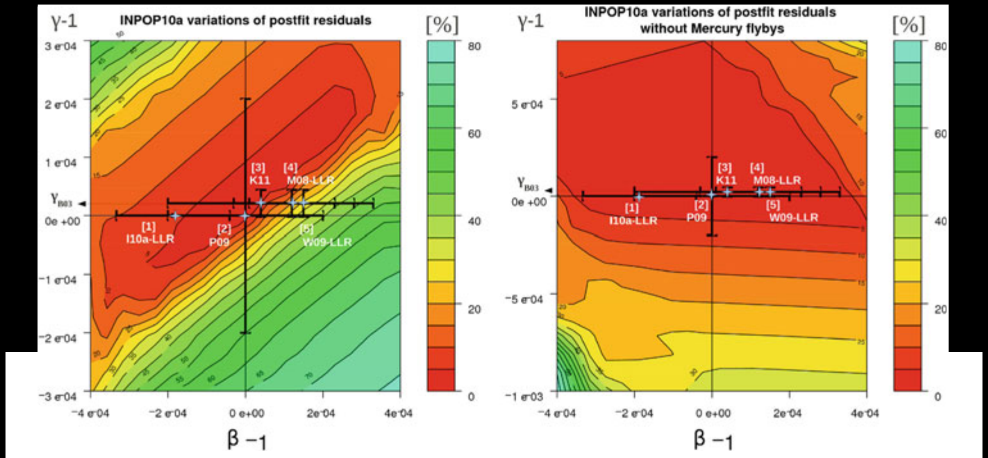


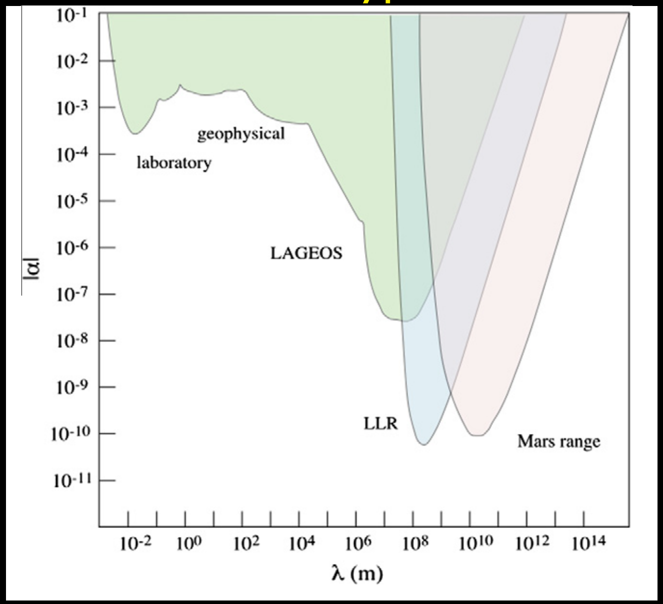
Fig. 7 Variations of postfit residuals obtained for different values of PPN β (x-axis) and γ (y-axis). [1] stands for a PPN β value obtained by Manche et al. (2010) using LLR observations with $\gamma = 0$, [2] stands for Pitjeva (2010) by a global fit of EPM planetary ephemerides. K11 stands for Konopliv et al. (2011) determinations based mainly on Mars data analysis. M08 for Müller et al. (2008) and W09 for Williams et al. (2009) give values deduced from LLR for a fixed value of γ, B03 stands for Bertotti et al. (2003) determination of γ by solar conjunction during the Cassini mission

Fienga et al. 2011

Ephémérides planétaires:

1. DE du JPL, américaine
2. VSOP - INPOP, françaises !
3. EPM, russe

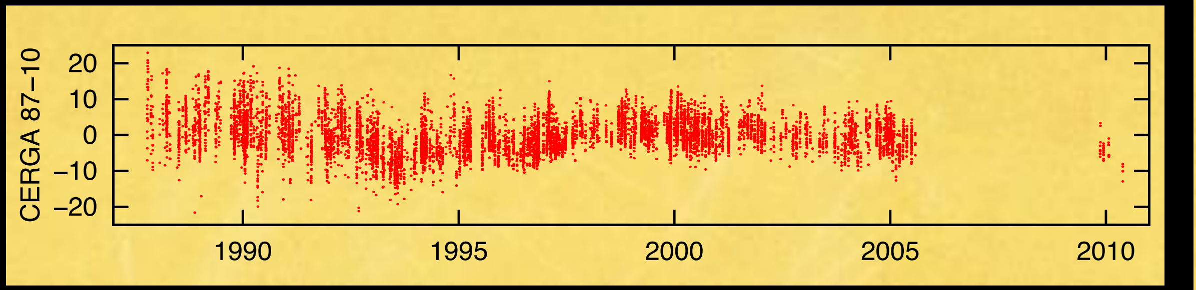
Analyse pour Mars
Déviation de type Yukawa

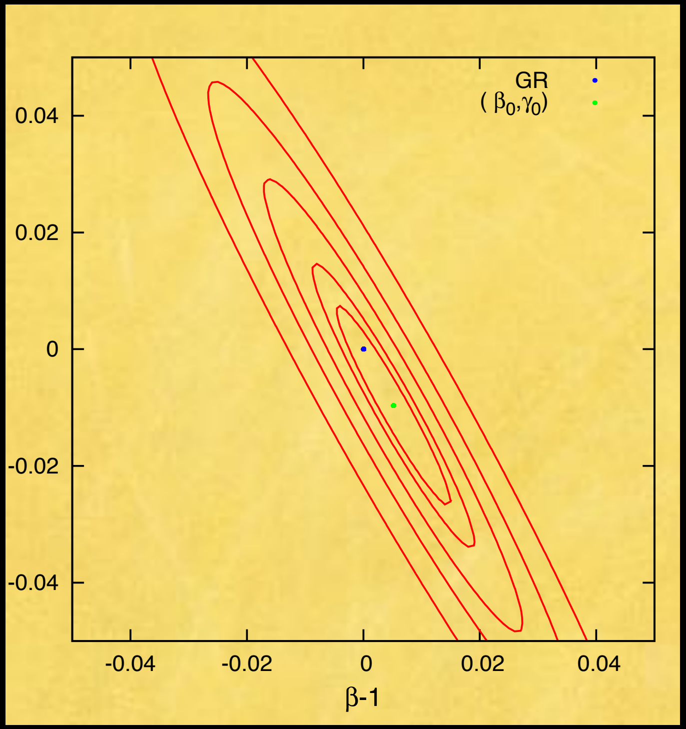


Konopliv et al. 2010

Ephémérides de satellite:

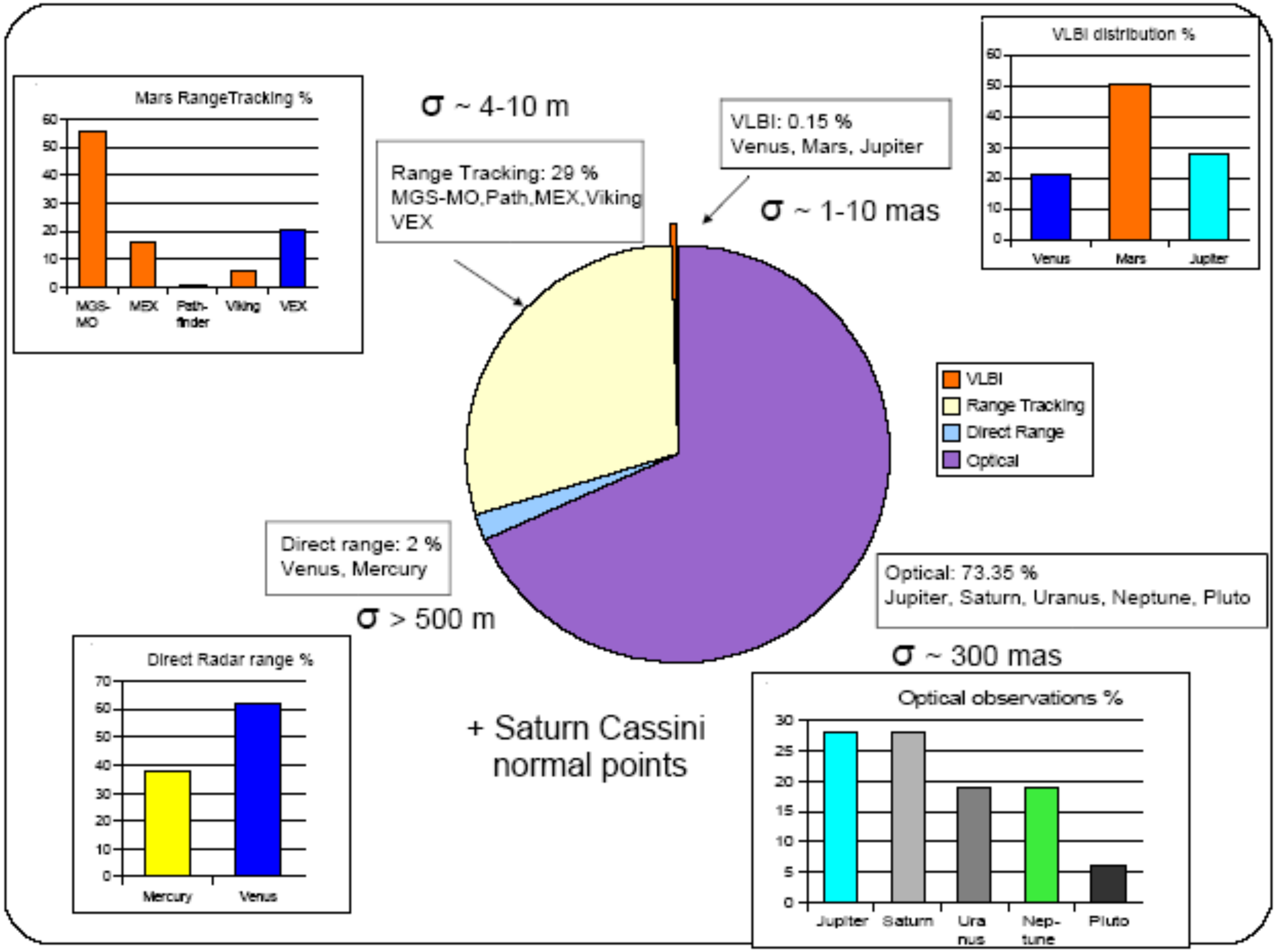
1. le mouvement de la lune (Laser-Lune)
2. les orbiteurs autour de Mars





Résidus Laser-Lune
Manche et al. 2010

Ephemerides



σ ~ 4-10 m

Range Tracking: 29 %
MGS-MO,Path,MEX,Viking
VEX

VLBI: 0.15 %
Venus, Mars, Jupiter

σ ~ 1-10 mas

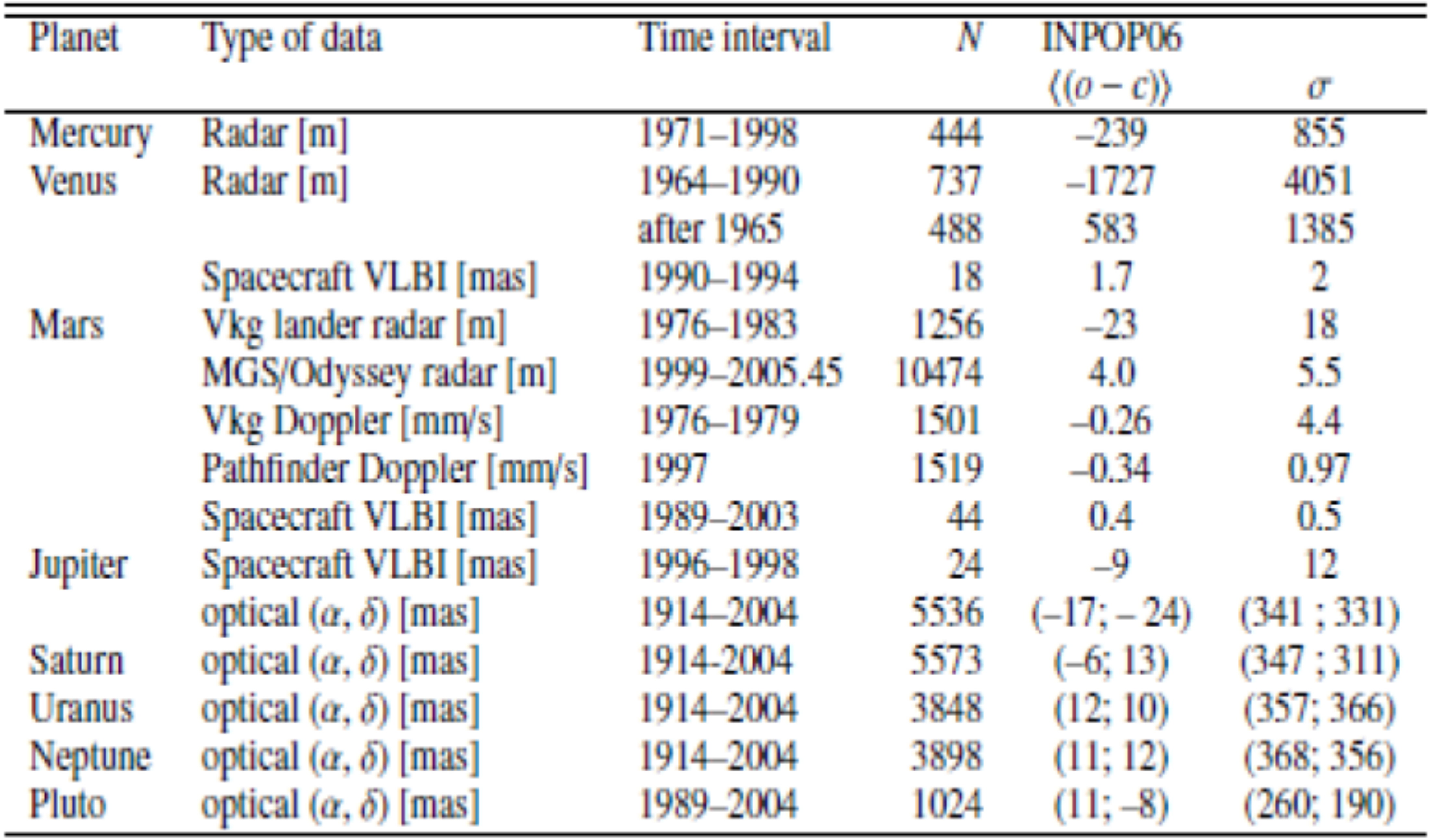
Direct range: 2 %
Venus, Mercury

σ > 500 m

Optical: 73.35 %
Jupiter, Saturn, Uranus, Neptune, Pluto

σ ~ 300 mas

+ Saturn Cassini
normal points



Planet	Type of data	Time interval	N	INPOP06	
				$\langle(o-c)\rangle$	σ
Mercury	Radar [m]	1971–1998	444	–239	855
Venus	Radar [m]	1964–1990	737	–1727	4051
		after 1965	488	583	1385
	Spacecraft VLBI [mas]	1990–1994	18	1.7	2
Mars	Vkg lander radar [m]	1976–1983	1256	–23	18
	MGS/Odyssey radar [m]	1999–2005.45	10474	4.0	5.5
	Vkg Doppler [mm/s]	1976–1979	1501	–0.26	4.4
	Pathfinder Doppler [mm/s]	1997	1519	–0.34	0.97
	Spacecraft VLBI [mas]	1989–2003	44	0.4	0.5
Jupiter	Spacecraft VLBI [mas]	1996–1998	24	–9	12
	optical (α, δ) [mas]	1914–2004	5536	(–17; – 24)	(341 ; 331)
Saturn	optical (α, δ) [mas]	1914-2004	5573	(–6; 13)	(347 ; 311)
Uranus	optical (α, δ) [mas]	1914–2004	3848	(12; 10)	(357; 366)
Neptune	optical (α, δ) [mas]	1914–2004	3898	(11; 12)	(368; 356)
Pluto	optical (α, δ) [mas]	1989–2004	1024	(11; –8)	(260; 190)

A.Fienga et al., A&A 477, 315 (2008)

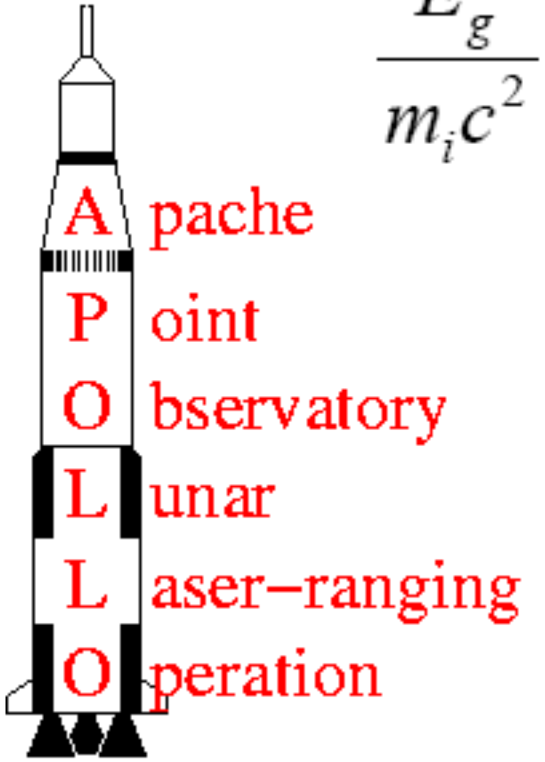
- 33 fitted GMs
- 3 bulk densities ρ for $\approx$ 270 asteroids
- tests of $\rho = \rho_0 + \rho_1 . r[\mathrm{km}]$
- ring $\approx$ 25000 small objetcs (Kuchynka et al.)
- others: EMRAT, Sun J2, AU and PPN β
- INPOP08b: AU$_{IERS03}$ and GMs modified (GM$_{\odot}$)

	Unit	DE421	INPOP06	INPOP08
Sun J2	10^{-7}	2.0	2.46 ± 0.40	1.82 ± 0.47
EMRAT		81.300569	NE	81.300540 ± 0.00005
AU-$\mathrm{AU}_{\mathrm{IERS03}}$	m	8.62	NE	8.22 ± 0.11
$\|1-\beta\|$		NE	$< 10^{-4}$	$<= 5.10^{-4}$

Lunar Laser Ranging and Nordtvedt effect

$$\frac{m_g}{m_i} = 1 - \eta_N \frac{E_g}{m_i c^2} = 1 + \frac{6}{5} \eta_N \frac{G m_i c^2}{R}$$

with $\eta_N = 4\,\beta - \gamma - 3$



A pache
P oint
O bservatory
L unar
L aser–ranging
O peration

$\frac{E_g}{m_i c^2}$ < 10^{-27} for lab experiment

= 3.6 10^{-6} Sun

10^{-8} Jupiter

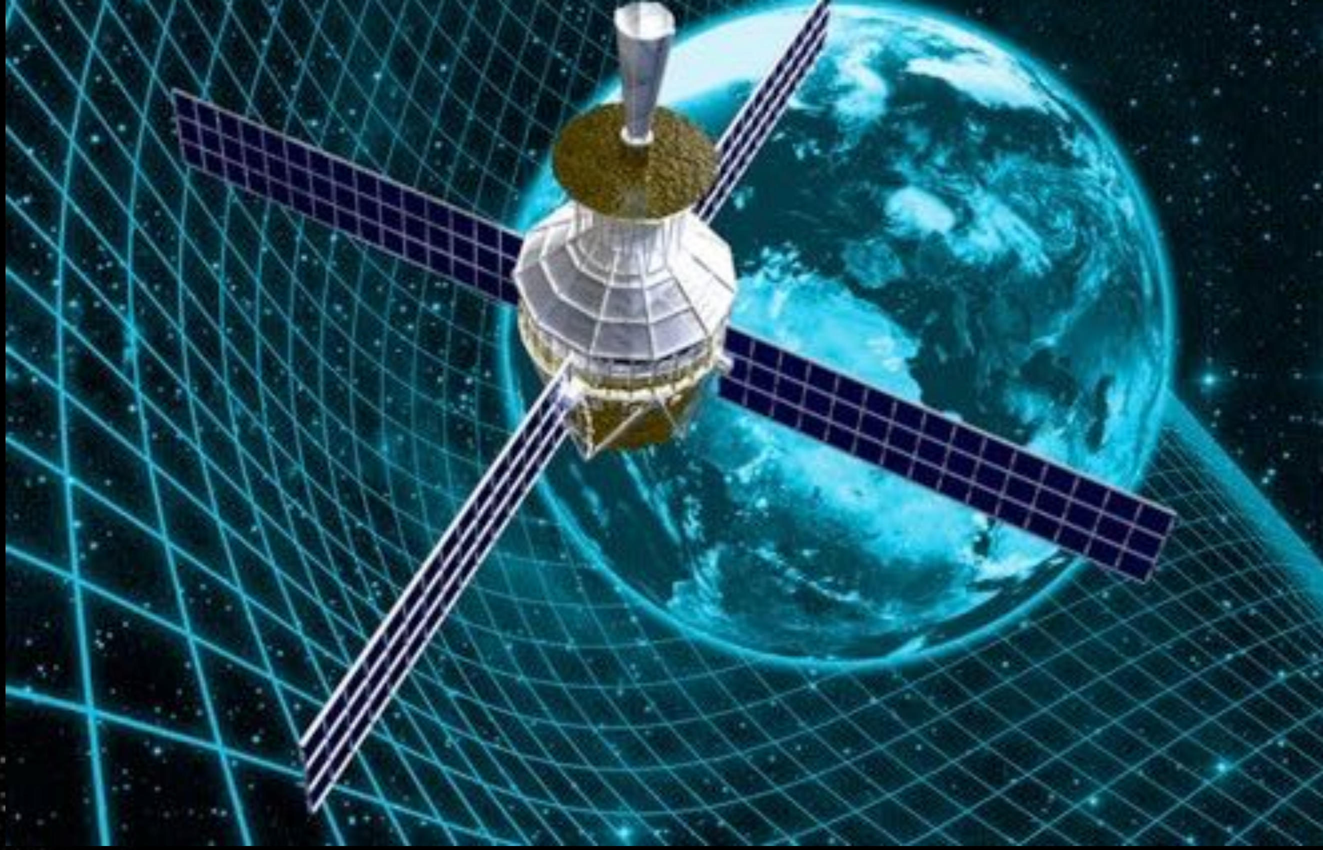
4.6 10^{-10} Earth

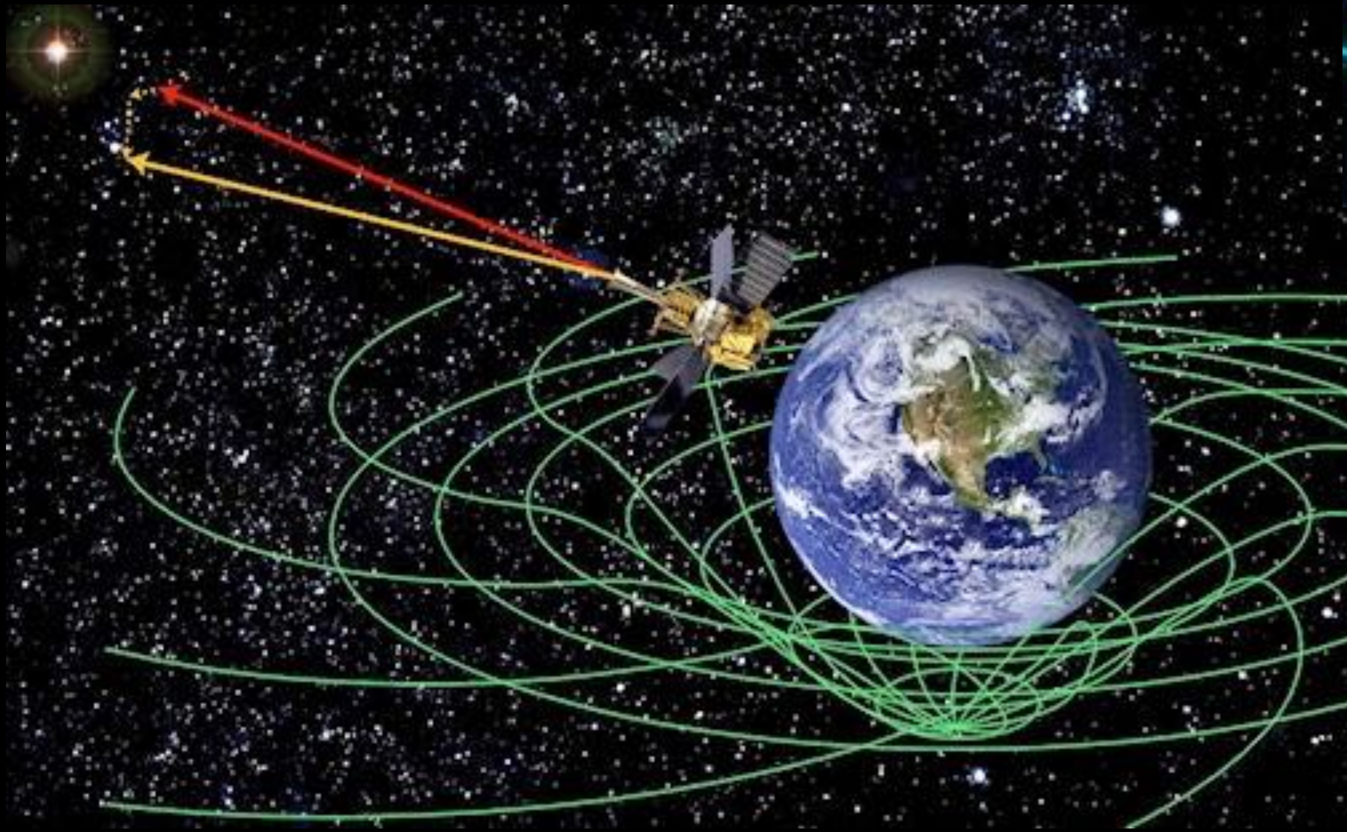
2 10^{-11} Moon

$$\eta_N = (4.4 \pm 4.5) 10^{-4}$$

précession d'accord !
mais pas que du périhélie...

l'expérience Gravity Probe B



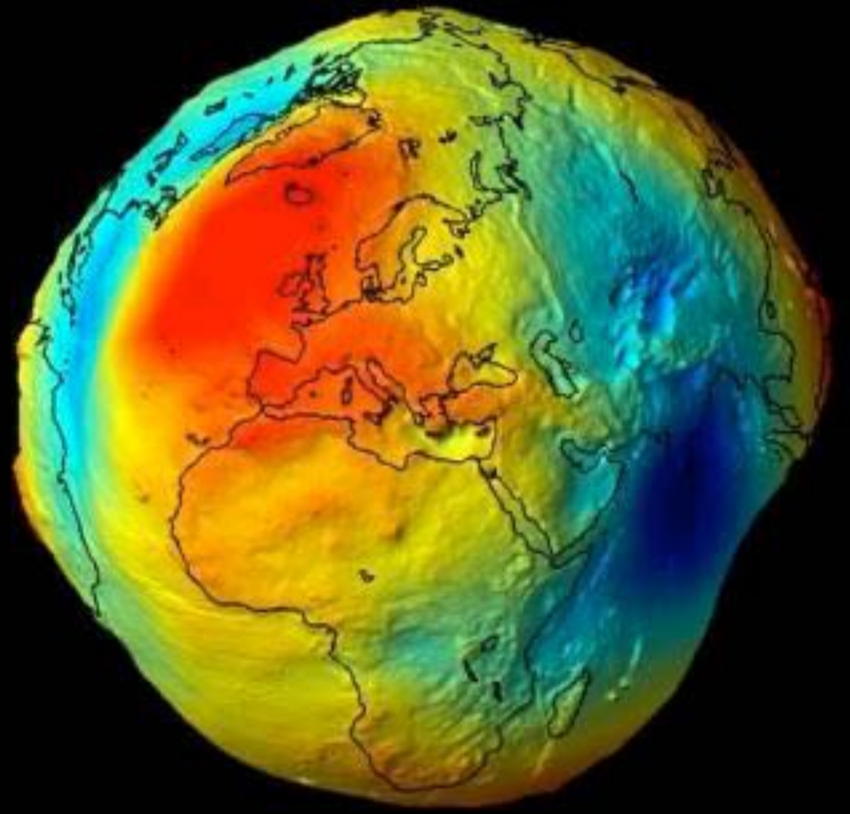


But : mesurer la précession de Lense Thirring

gyroscope de GPB

Projet le plus long de l'histoire spatiale :
idée proposée dans les années 50... Lancement le 20 Avril 2004 !
Malheureusement, les gyroscopes n'ont pas été performants...

L'apport de la géodésie spatiale



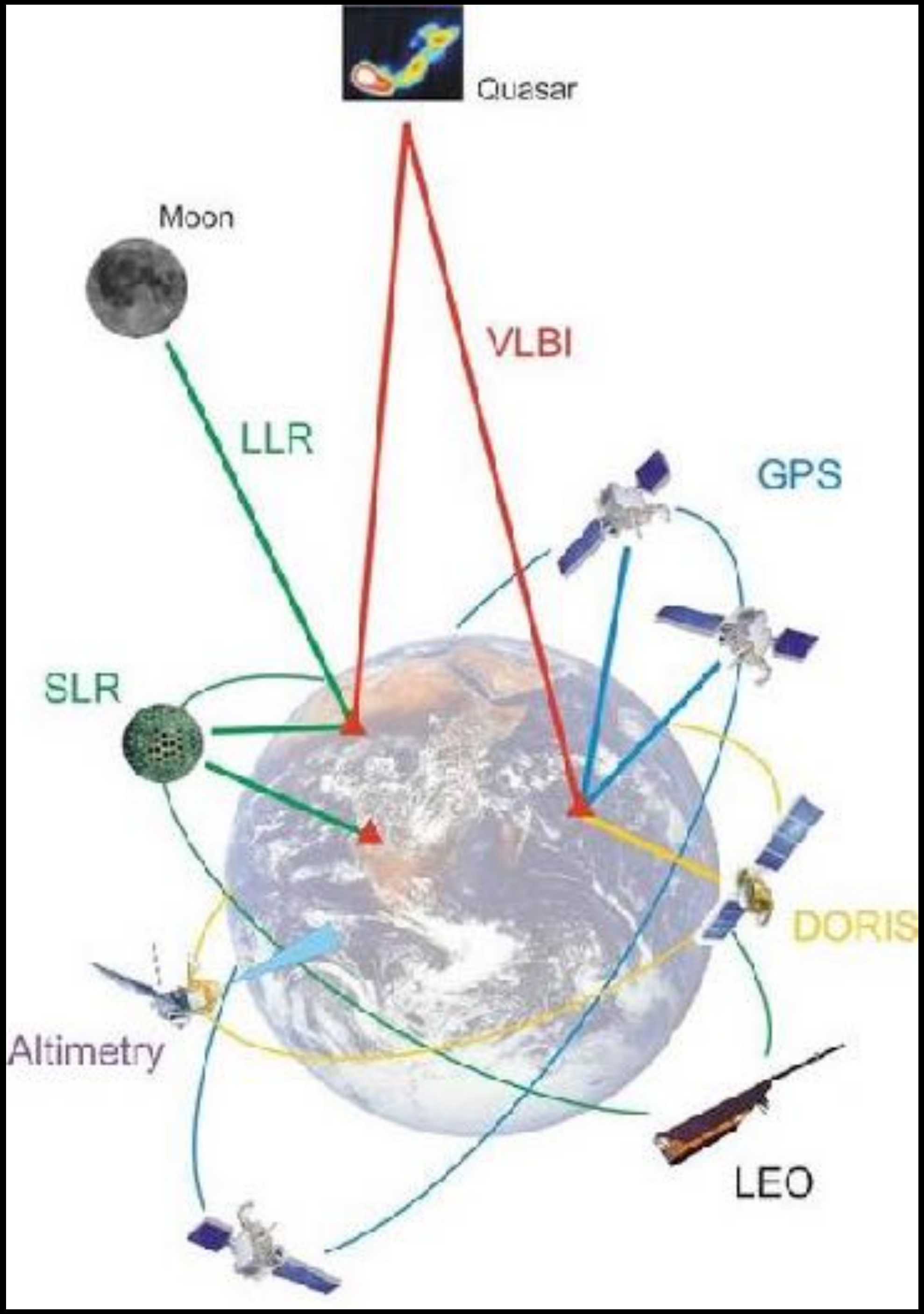
Multiples techniques

Comprendre la dynamique interne
de la Terre

Déterminer le champ de gravité

Se positionner correctement

Effet Lense-Thirring grâce à LAGEOS
Ciufolini & Pavlis,Nature 2004



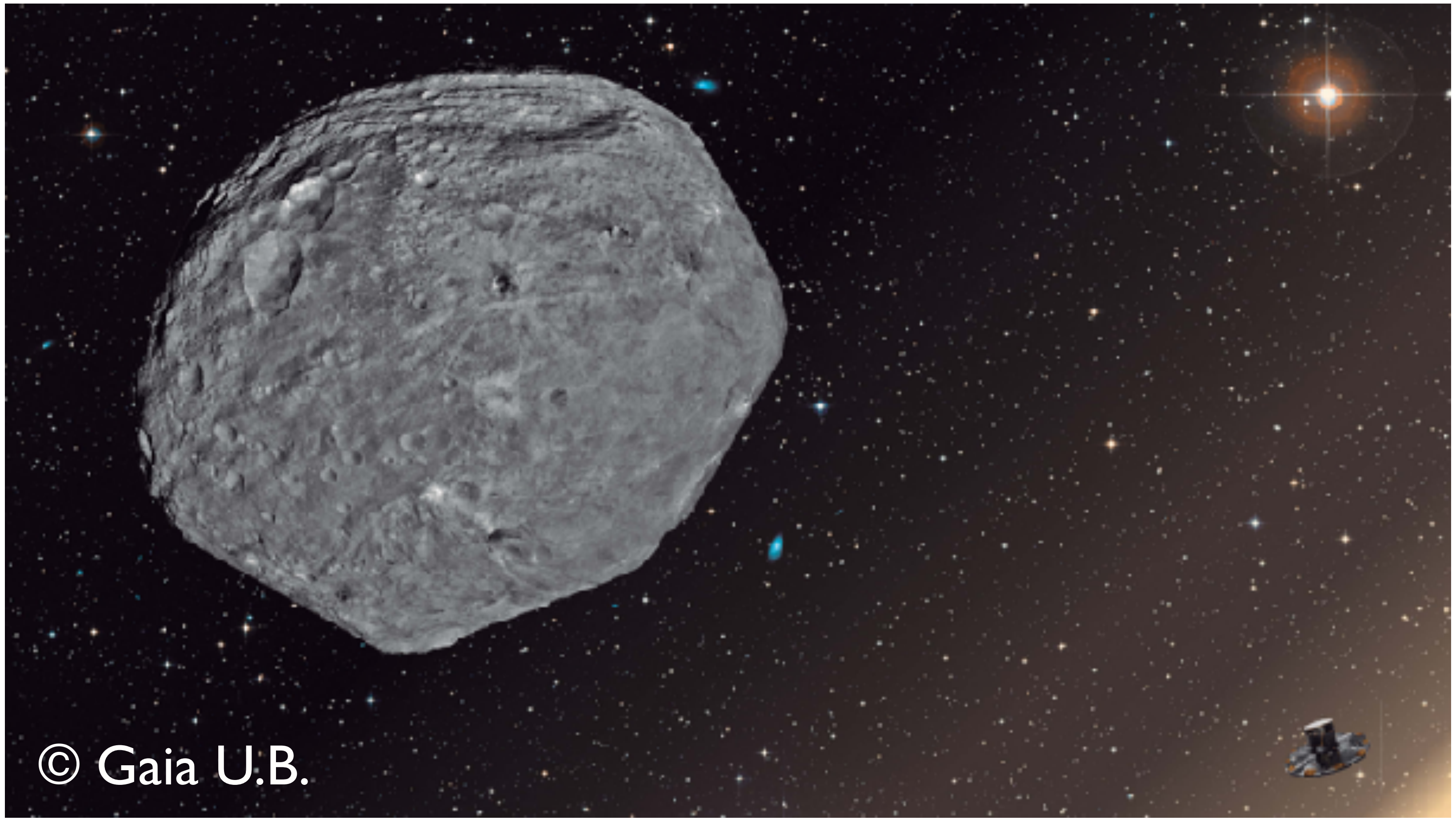
Gaia will offer two ways to test GR

measurement of the light deflection by the Sun & planets

probing space-time with massless particles: spatial and temporal part of metric

orbital dynamics of Solar System Objects

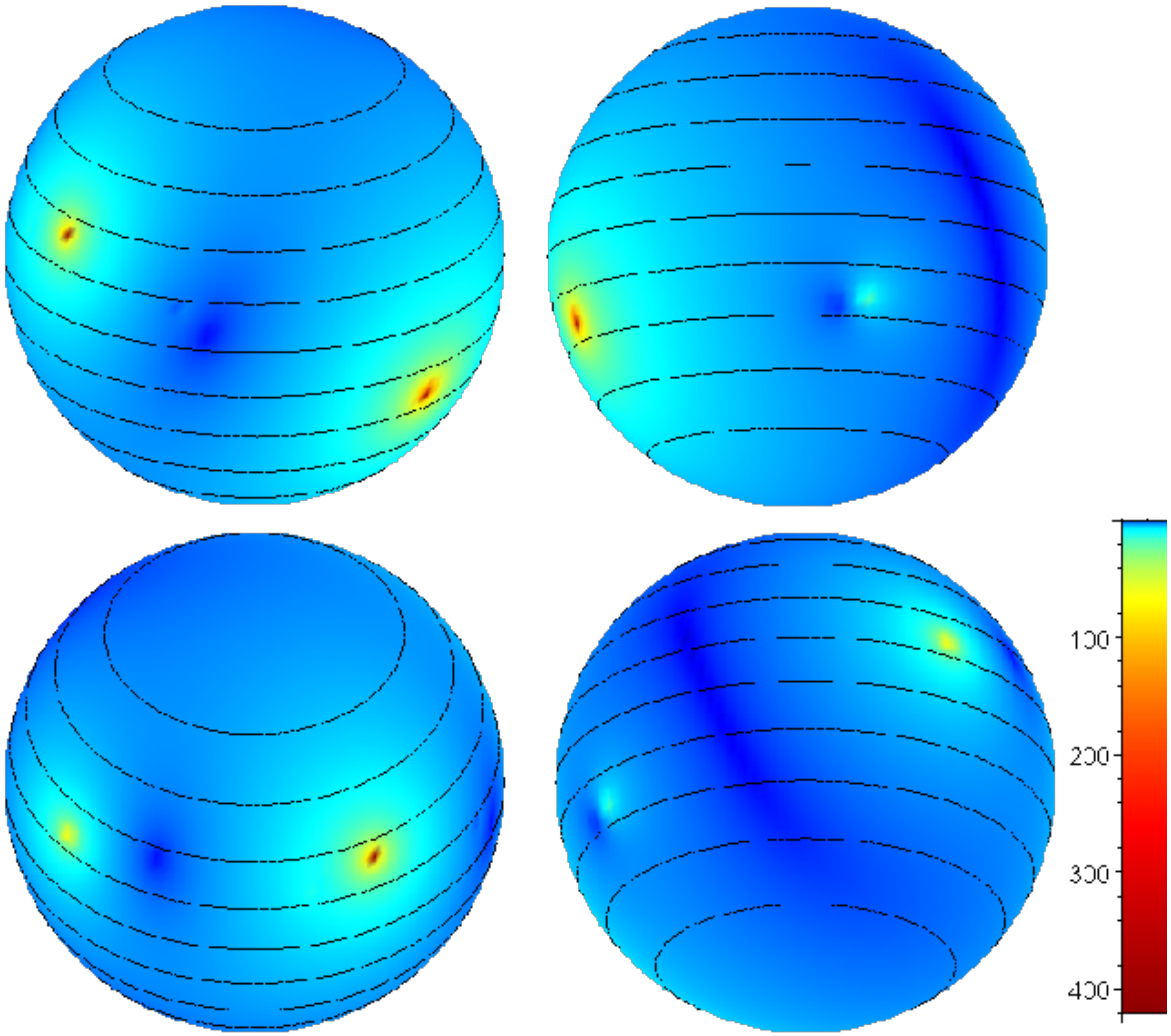
probing space-time with massive test bodies: mainly spatial part of metric



© Gaia U.B.

Light deflection : how much ?

- Monopole light deflection: distribution over the sky on 25.01.2006 at 16:45 equatorial coordinates



body	(μas)	>1μas
Sun	1.75″	180 °
Mercury	83	9 ′
Venus	493	4.5 °
Earth	574	125 °
Moon	26	5 °
Mars	116	25 ′
Jupiter	16270	90 °
Saturn	5780	17 °
Uranus	2080	71 ′
Neptune	2533	51 ′

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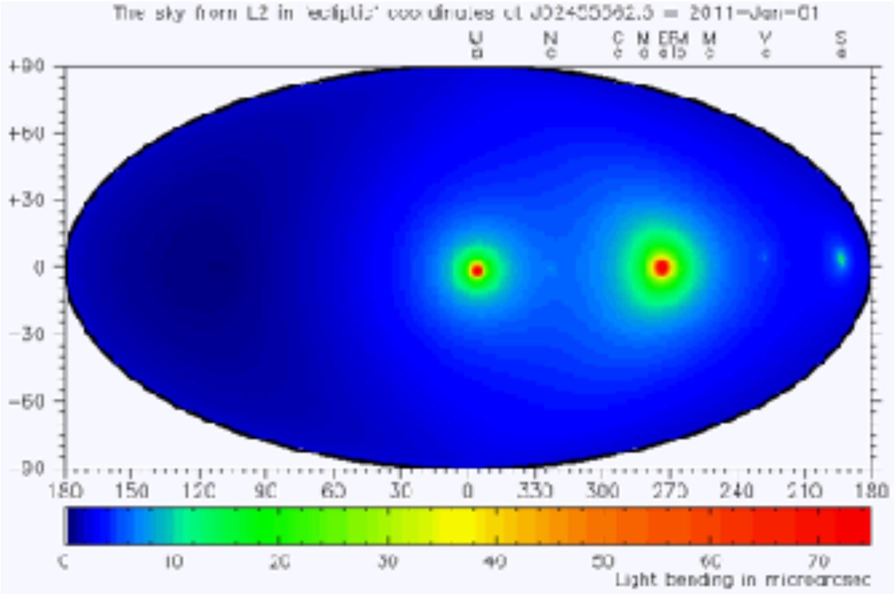
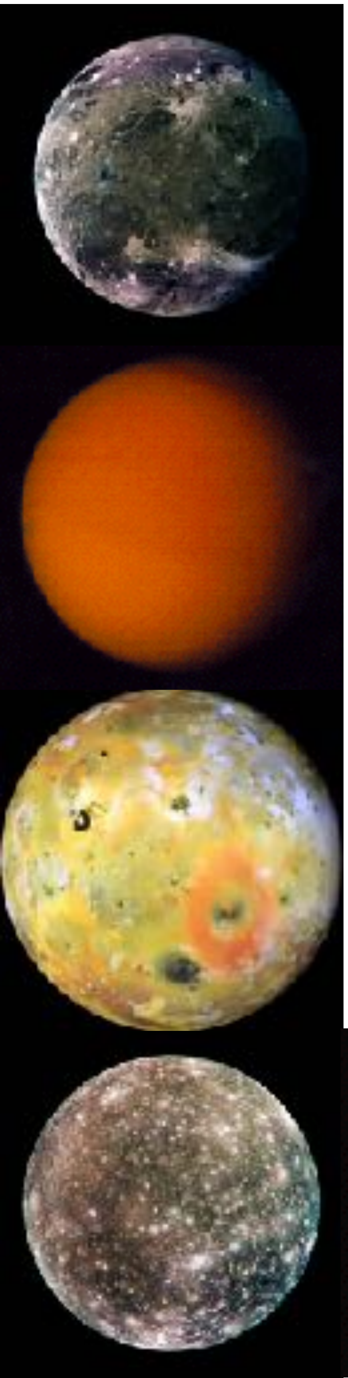
Neptune	2533	51°

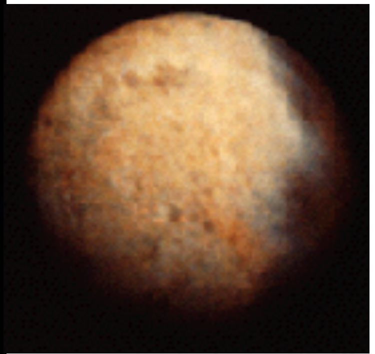
Order of magnitude for monopole light deflection.

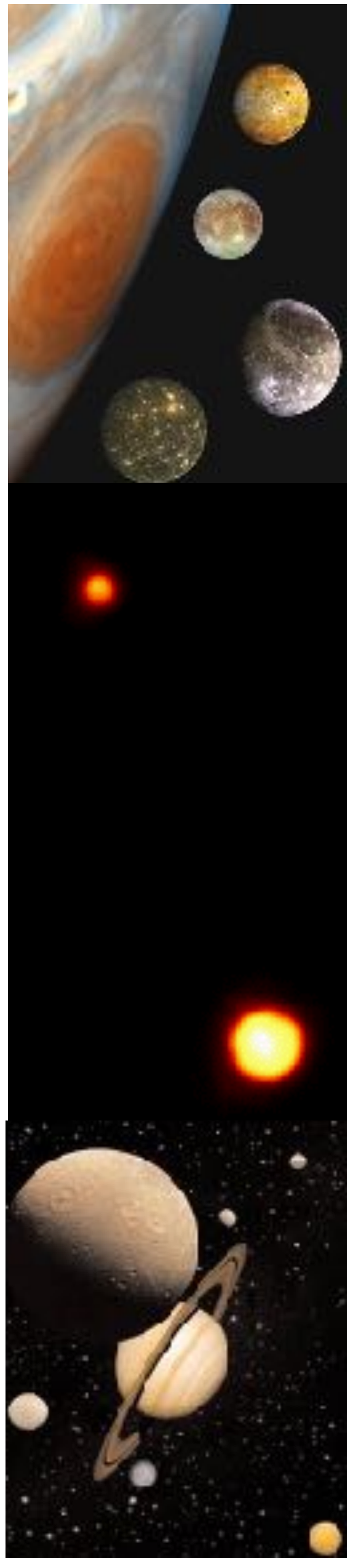


Minor bodies :

Ganymede	35
Titan	32
Io	30
Callisto	28
Pluto	7
Charon	4
Titania	3
Ceres	1







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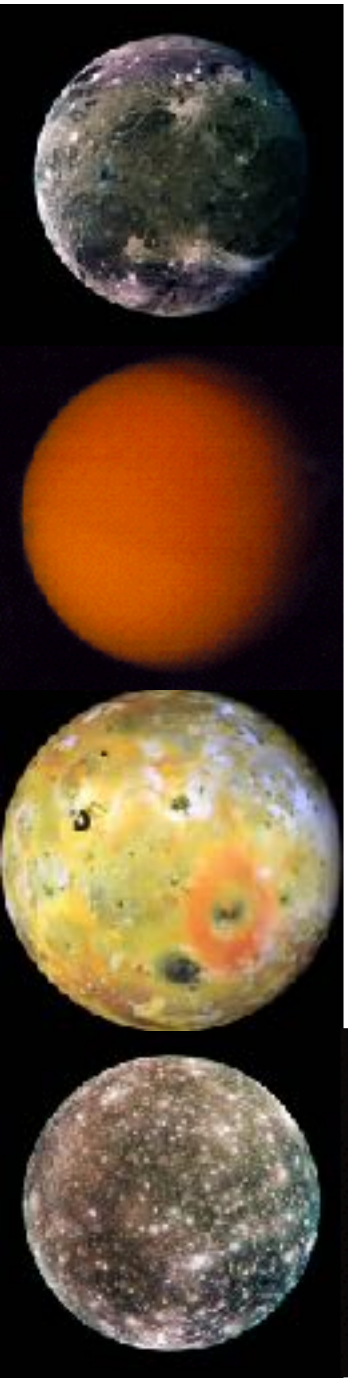
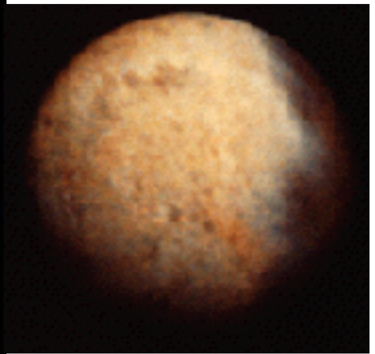
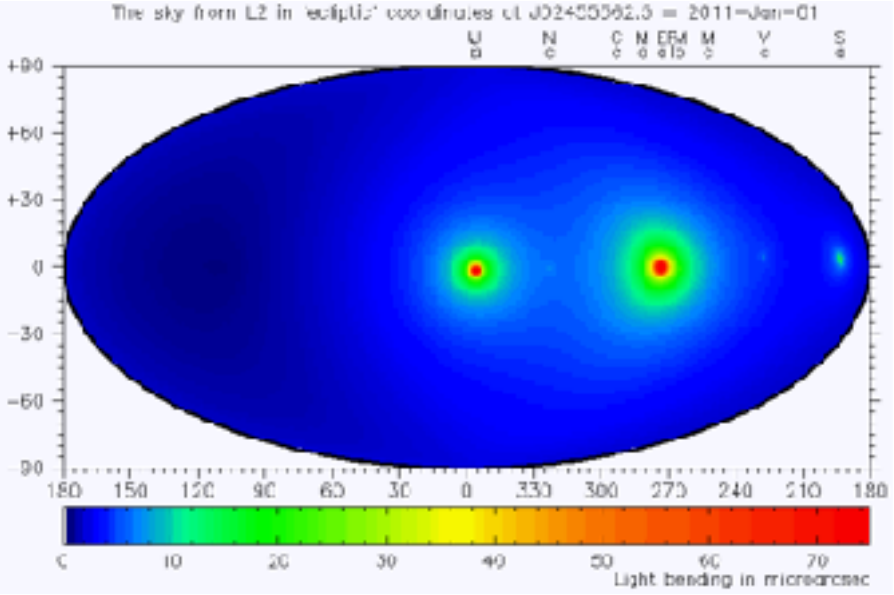




Order of magnitude for monopole light deflection.



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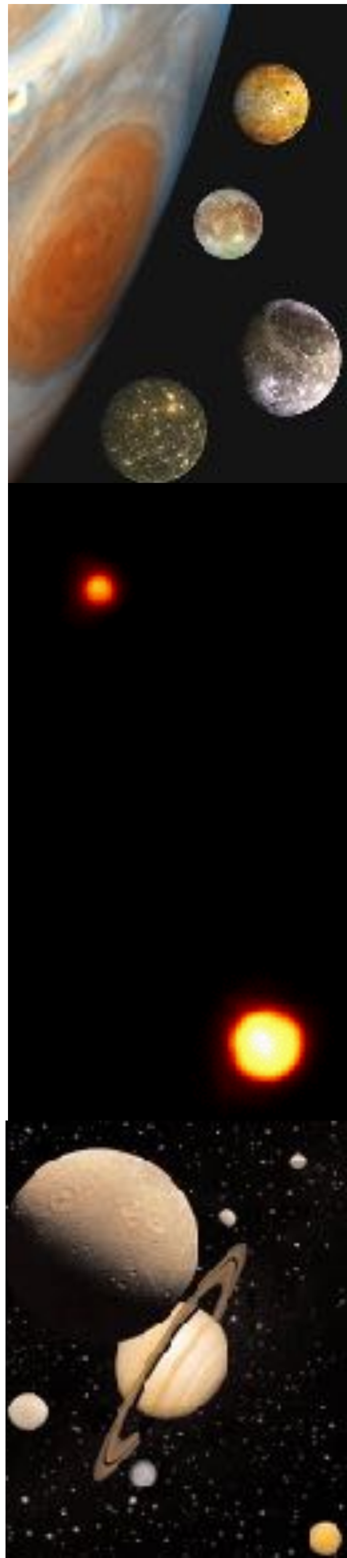
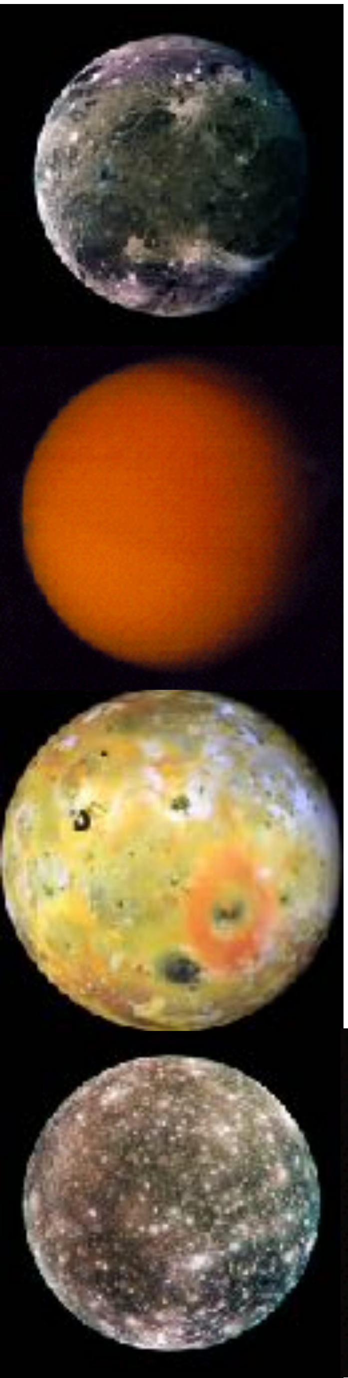
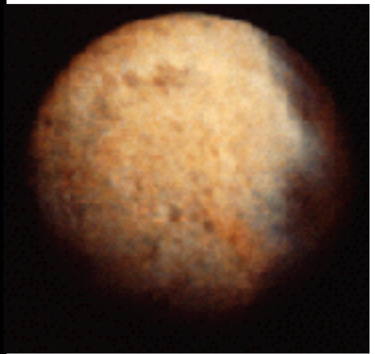


Light deflection : how much ?





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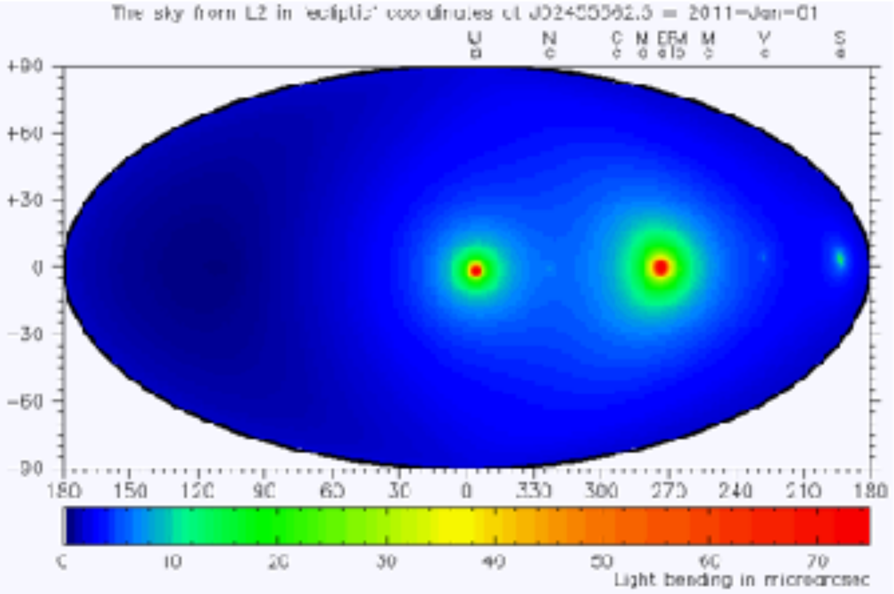
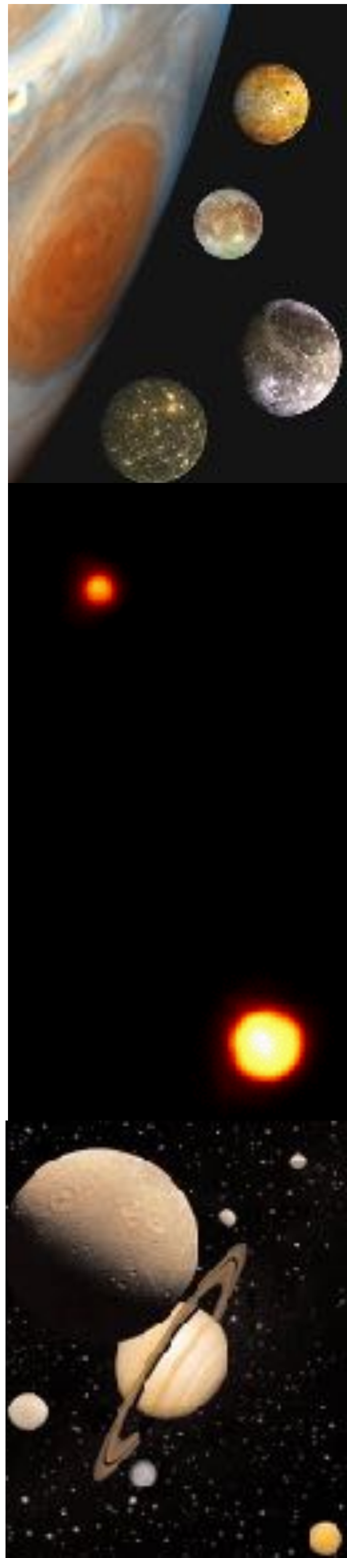
Earth	574	125°
Moon	26	5°
Mars	116	25°
Jupiter	16270	90°
Saturn	5780	17°
Uranus	2080	71°
Neptune	2533	51°

Order of magnitude for monopole light deflection.



Minor bodies :

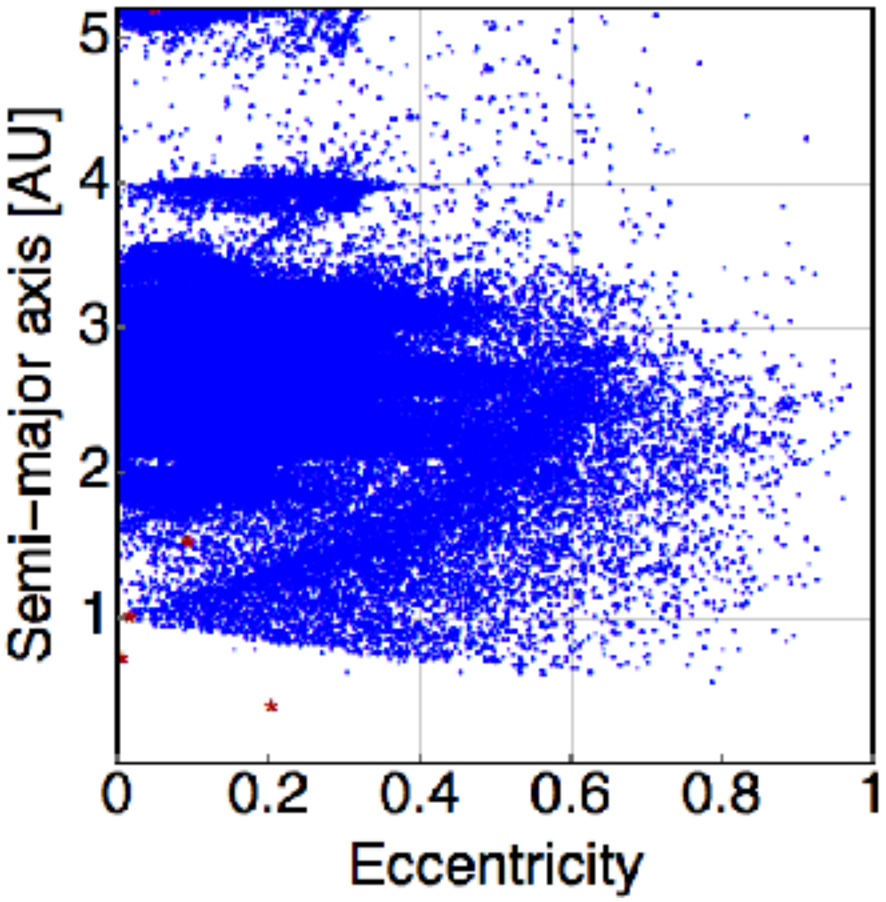
Ganymede	35
Titan	32
Io	30
Callisto	28
Pluto	7
Charon	4
Titania	3
Ceres	1

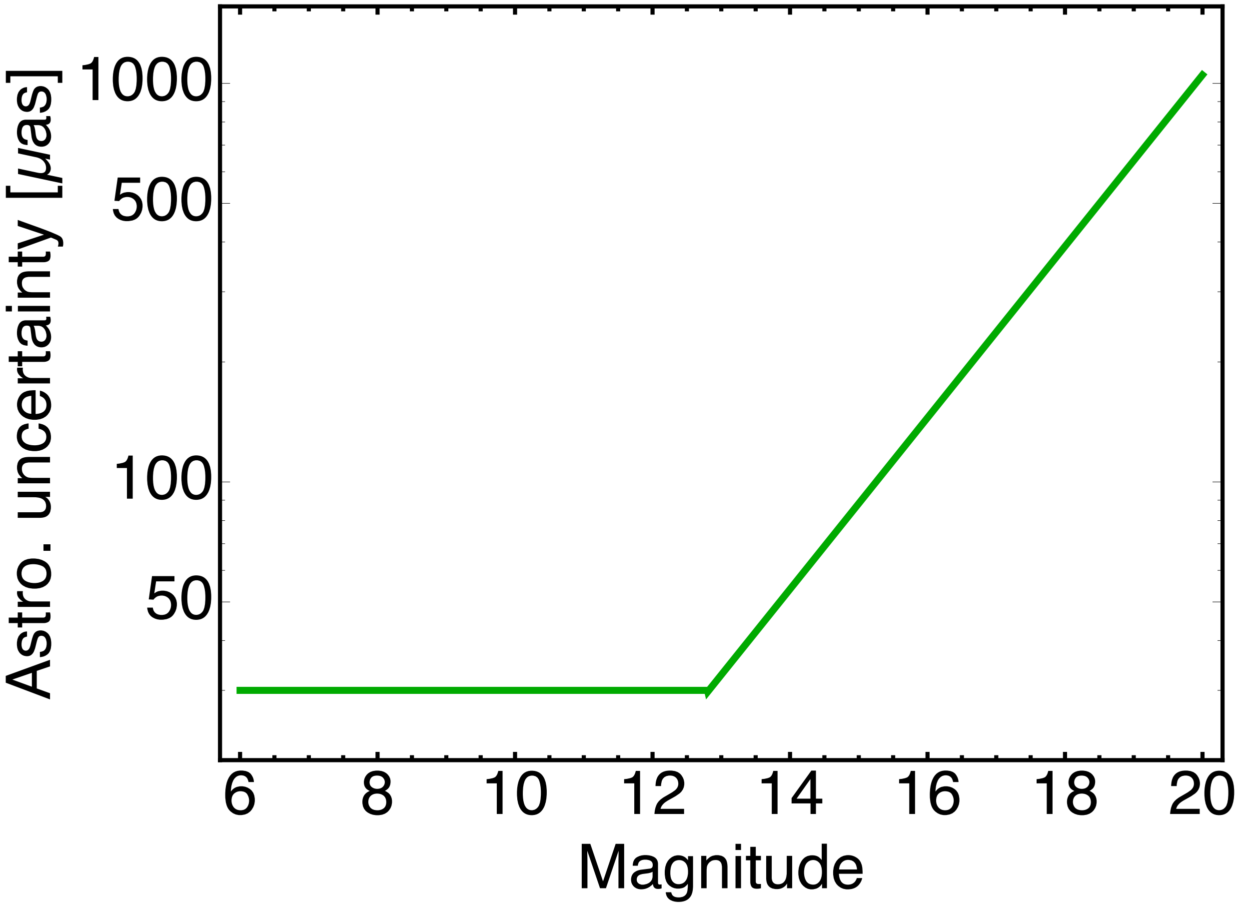


Gaia will observe ~400 000 SSO's

- Use GAIA SSO's observations to test GR: advantage of a large samples of different orbital parameters
  - decorrelation of parameters
  - complementary to planetary ephemerides (different bodies, different type of observations, different method to analyze the data)

- Uncertainty used in simulations depends on magnitude

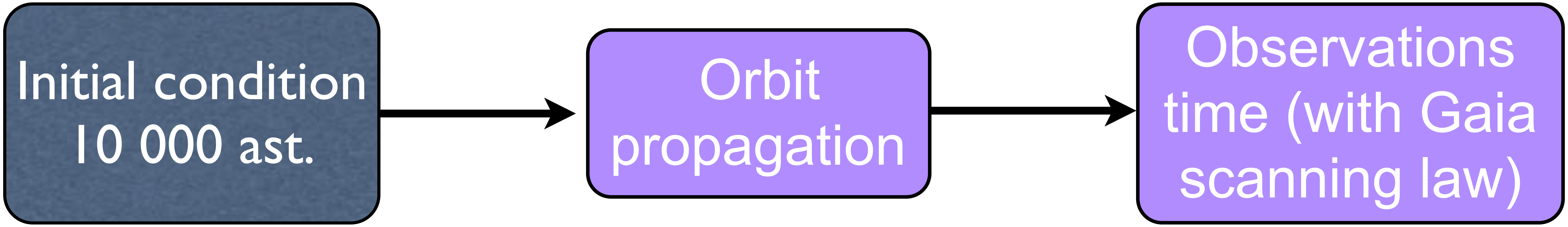




- Simulations done for 5 years and also for 10 years (extended mission)

Simulations of Gaia observations

done by Gaia WP DU460



Initial condition
10 000 ast.

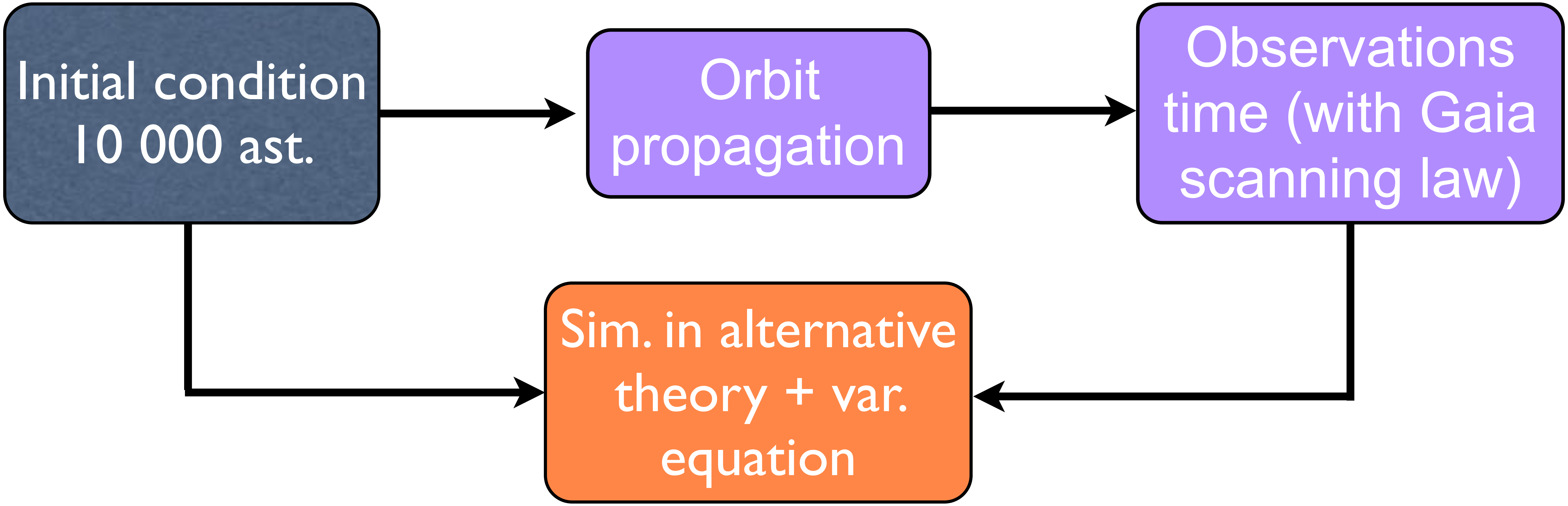
Orbit
propagation

Observations
time (with Gaia
scanning law)

Similar to what is done in Mouret, PRD, 2011

Simulations of Gaia observations

done by Gaia WP DU460



Initial condition
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Orbit propagation

Observations time (with Gaia scanning law)

Sim. in alternative theory + var. equation

Similar to what is done in Mouret, PRD, 2011

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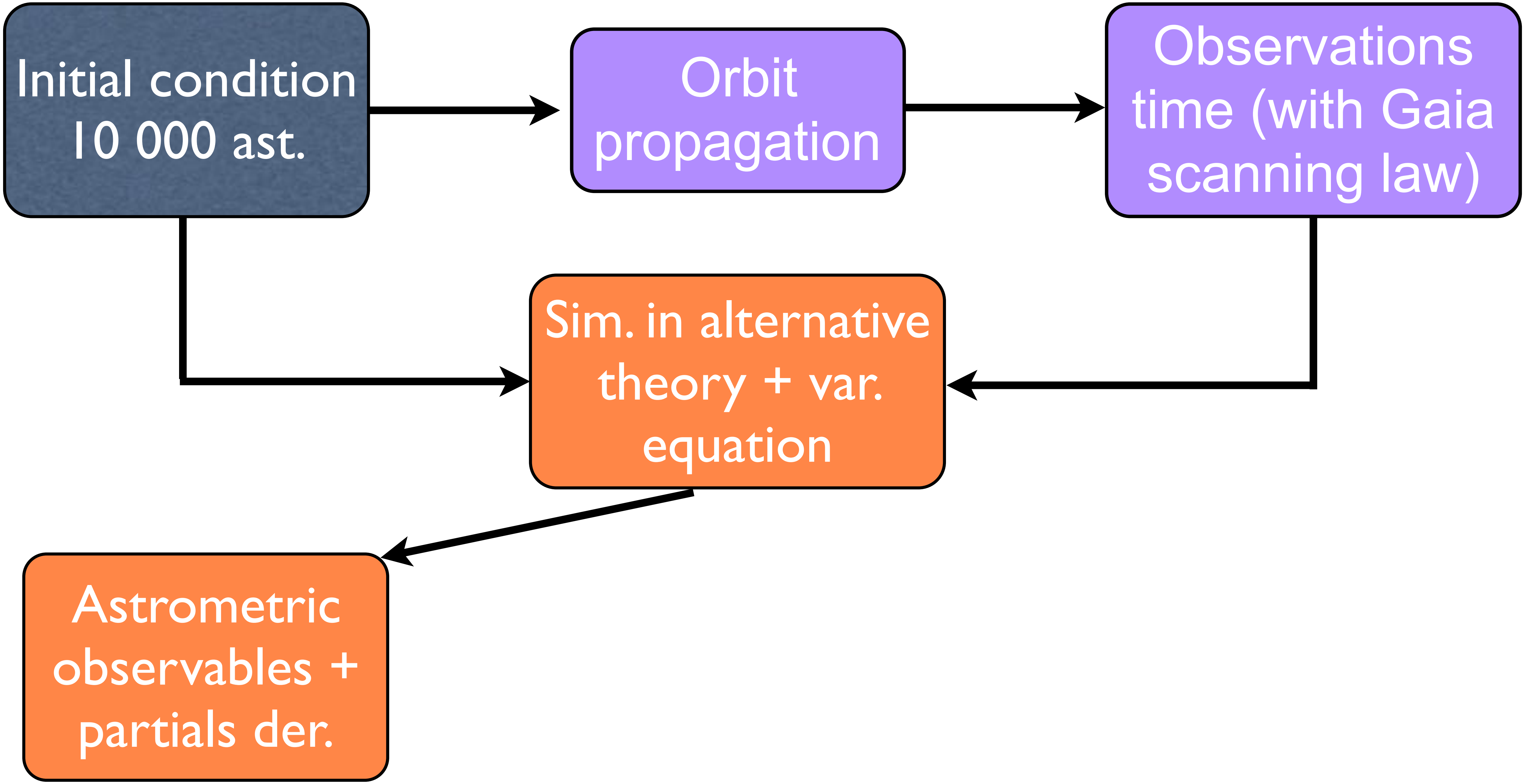

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Orbit propagation

Observations time (with Gaia scanning law)

Sim. in alternative theory + var. equation

Astrometric observables + partials der.

Similar to what is done in Mouret, PRD, 2011

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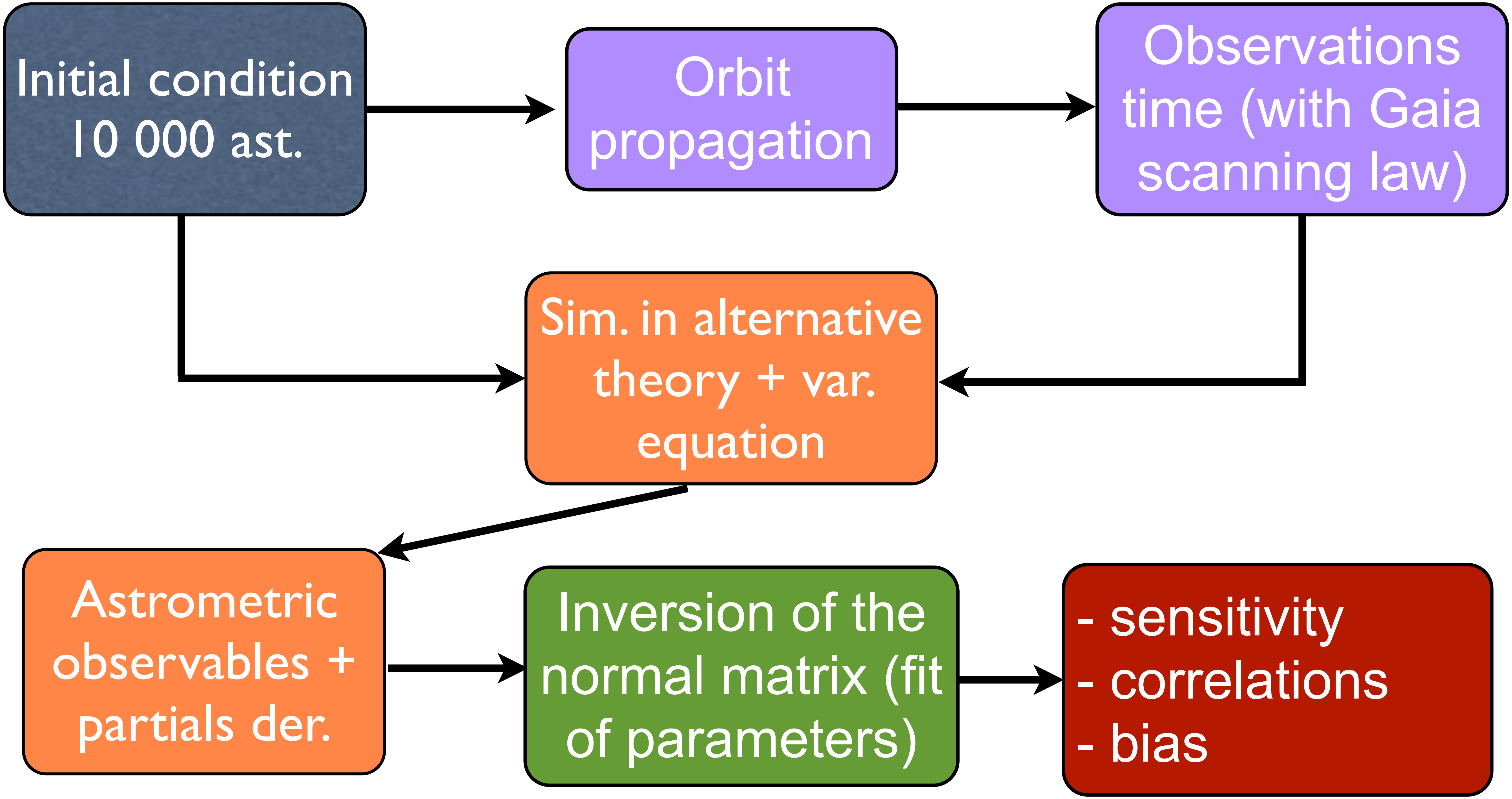

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Orbit
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Observations
time (with Gaia
scanning law)

Sim. in alternative
theory + var.
equation

Astrometric
observables +
partials der.

Inversion of the
normal matrix (fit
of parameters)

- sensitivity
- correlations
- bias

- local parameters (IC)
- global parameters (grav. theory, J_2, …)

Similar to what is done in Mouret, PRD, 2011

PPN formalism and Sun J$_2$

- highly correlated parameters: secular effect on orbital dynamics

$$\left\langle \frac{d\omega}{dt} \right\rangle = (2+2\gamma-\beta)n\frac{GM}{c^2a(1-e^2)} + \frac{3}{2}n\frac{J_2R^2}{a^2(1-e^2)^2}$$

- various asteroids orbital parameters help to decorrelate

- sensitivity:

	J_2	β
GAIA [5yr]	$\sigma_{J_2} \sim 5\times10^{-8}$	$\sigma_\beta \sim 4\times10^{-4}$
GAIA [10yr]	$\sigma_{J_2} \sim 1.5\times10^{-8}$	$\sigma_\beta \sim 10^{-4}$
INPOP	$(2.22\pm0.13)\times10^{-7}$	$(0.0\pm6.9)\times10^{-5}$

- correlation ~ 0.4

INPOP results from A. Fienga et al, Cel. Mech. Dyn. Astro. 2015

- complementary to planetary ephemerides: different analysis, not the same systematics

- Interesting: combined fit Gaia + planets

Test of the SEP can help to decorrelate β and J_2

- SEP: Universality of free fall violated for self-gravitating body

see K. Nordtvedt, Phys. Rev., 169, 1014, 1968

$$m_p = m_i + \eta \frac{E_{\mathrm{grav}}}{c^2} \qquad m_i \vec{a} = m_p \vec{\nabla} U$$

- Gaia can constrain η at 3×10^{-4} [3×10^{-5} if extended mission] while the current best constraint from LLR is $\eta = (4.4 \pm 4.5) \times 10^{-4}$

see J. Williams et al, IJMPD, 18, 1129, 2009

- In the PPN formalism $\eta = 4\beta - \gamma - 3$ helps to estimate β

	J_2	β
GAIA [5yr]	$\sigma_{J_2} \sim 4 \times 10^{-8}$	$\sigma_\beta \sim 8 \times 10^{-5}$
GAIA [10yr]	$\sigma_{J_2} \sim 1.3 \times 10^{-8}$	$\sigma_\beta \sim 8 \times 10^{-6}$
INPOP	$(2.22 \pm 0.13) \times 10^{-7}$	$(0.0 \pm 6.9) \times 10^{-5}$

no correlation remaining

INPOP results from A. Fienga et al, Cel. Mech. Dyn. Astro. 2015

- Considering a violation of the SEP reduces σ_β by a factor 5

A fifth force is a well motivated phenomenology

- deviation from Newtonian gravity characterized by a Yukawa potential

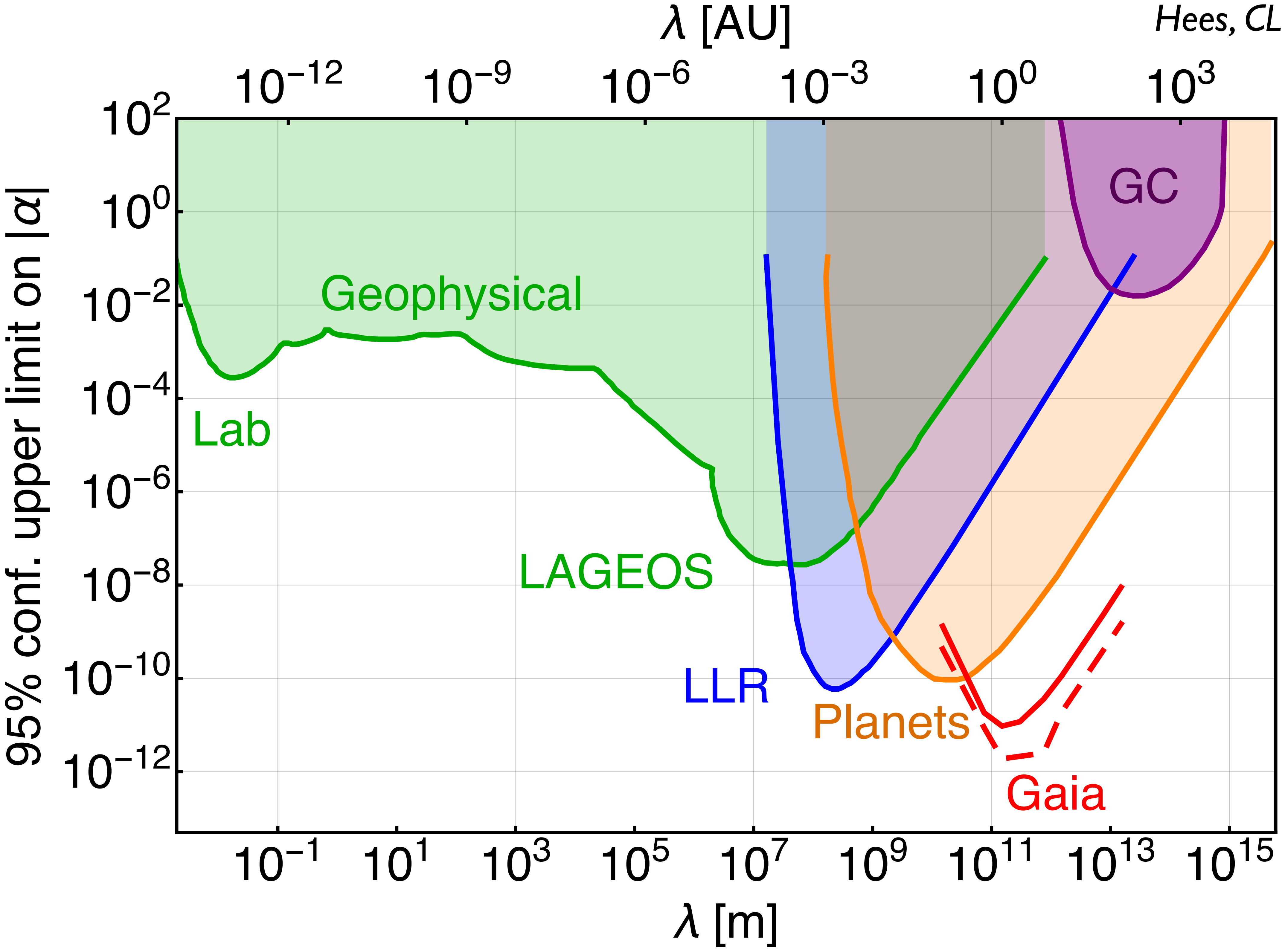
$$\phi(r) = \frac{GM}{c^2 r}\left(1 + \alpha e^{-r/\lambda}\right)$$

See E.G. Adelberger, Progress in Part. and Nucl. Phys., 62/102, 2009
“The Search for Non-Newtonian gravity”, E. Fischbach, C. Talmadge, 1998

- Phenomenology motivated by
  - new interaction with a massive gauge boson Fischbach and Talmadge, Nature, 1989
  - high dimension theories Krause and Fischbach, arXiv: hep-ph/9912276
  - Braneworld scenarios Arkani-Hamed, et al, PRD, 1999
  - massive gravity Will, PRD, 1998
  - massive tensor-scalar theory Alsing, et al, PRD, 2012
  - …

A fifth force can be tested with Gaia

Hees, CLPL, 2017



correlation with the Sun GM needs to be assessed carefully

Lense-Thirring effect due to the Sun

- Relativistic frame dragging effect produced by the rotation of a body (due to the Spin S)
- impossible to estimate the Sun Lense-Thirring with planetary ephemerides: completely correlated with J_2 see W. Folkner et al, IPN, 2014
- Asteroids can decorrelate but Gaia not powerful enough

$$\frac{\sigma_S}{S} \sim 6.5 \quad [1.7 \text{ for } 10\text{yr}]$$

- Combination with radar observations to be considered
- But… not including the LT in the modeling leads to bias:

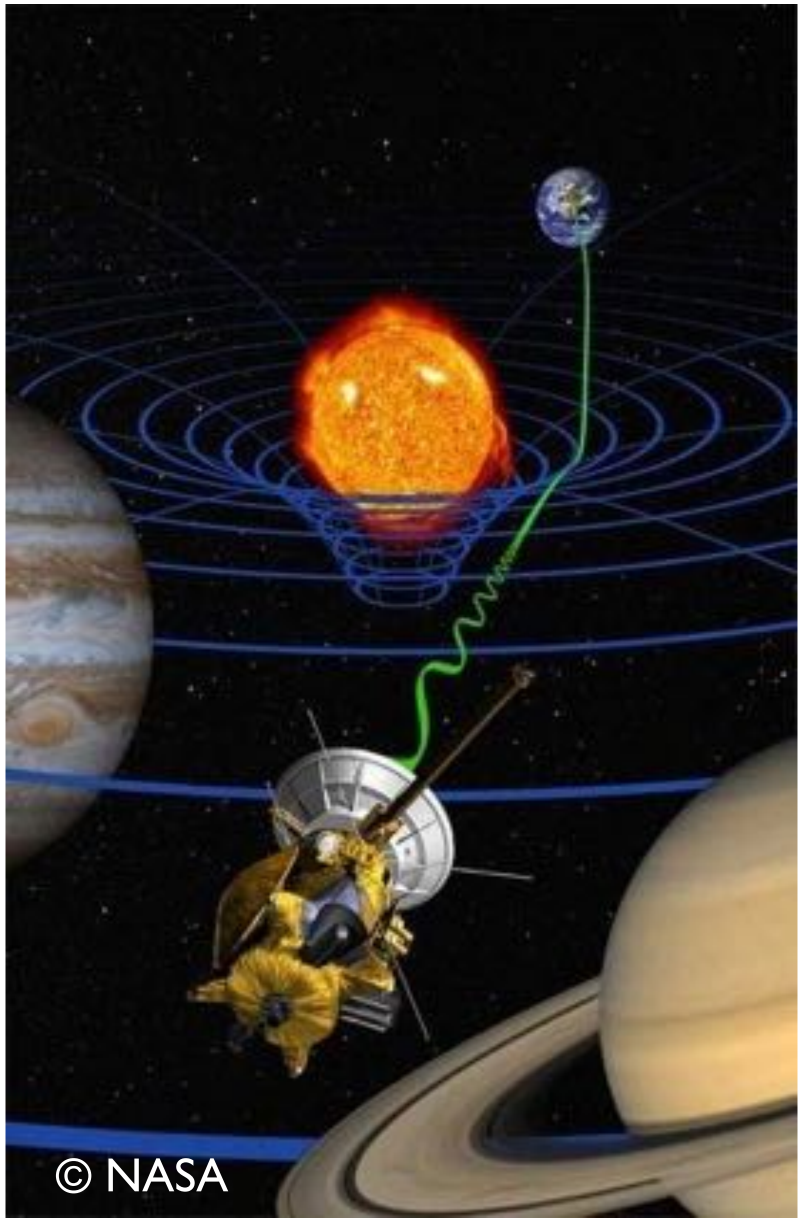
- 10^{-8} on the J_2 (i.e. 10% of its value)

- $5\text{x}10^{-5}$ on the β PPN

Constraints on PPN parameters

- Measurement of the Shapiro time delay with Cassini[1]

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$



© NASA

- Planetary ephemerides INPOP[2]

$$\beta - 1 = (0.2 \pm 2.5) \times 10^{-5}$$

- Dynamic of the orbit of the Moon with LLR[3]

$$\beta - 1 = (2.1 \pm 1.1) \times 10^{-4}$$

© NASA

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[1] B. Bertotti, L. Iess, P. Tortora, Nature, 425/374, 2003

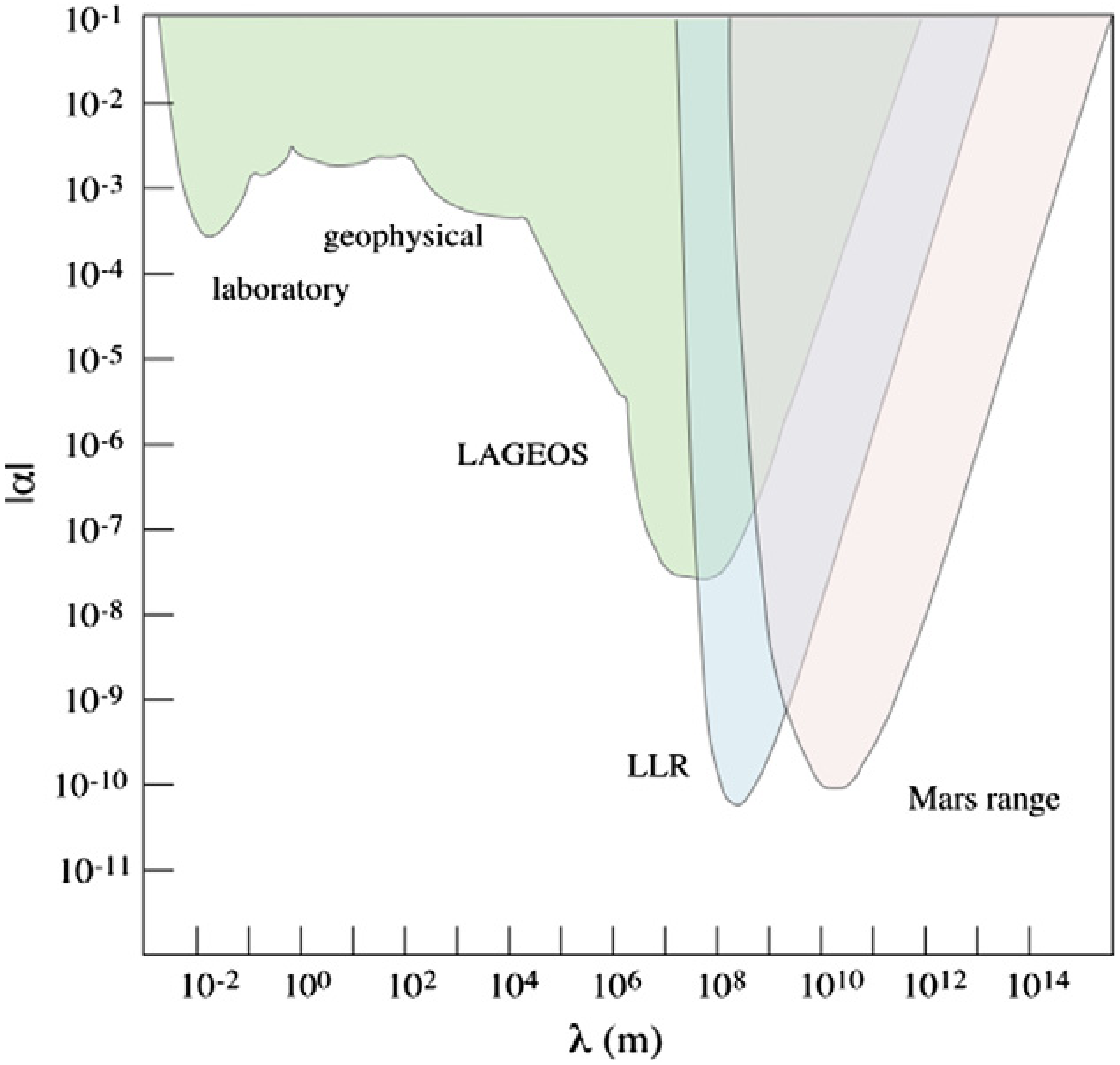
[2] A. Verma et al, A & A, 561, A115, 2014

[3] J. Williams, S. Turyshev, D. Boggs, IJMP D, 18/1129, 2009

Fifth force formalism

- Search for a deviation of the Newton potential of the form of a Yukawa potential[1] $\phi(r) = \frac{GM}{c^2 r}\left(1 + \alpha e^{-r/\lambda}\right)$

Excluded area for fifth force



- Very good constraints in this formalism except at small and large distances

from A.Konopliv et al, Icarus, 211/401, 2011

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[1] E.G. Adelberger, Progress in Part. and Nucl. Phys., 62/102, 2009
"The Search for Non-Newtonian gravity", E. Fischbach, C. Talmadge, 1998

Is it enough ?

- Still strong motivations to improve the current tests:
  - tensor-scalar theories “naturally” converging towards GR[1]
  - screening theories: modification of GR “hidden” in certain region of space-time: chameleons[2], symmetron[3], Vainshtein mechanism[4]
  - tensor-scalar theories with a decoupling of the scalar field[5]

We have strong motivations to pursue this kind of tests!

[1] T. Damour, K. Nordtvedt, PRD 48/3436 and PRL 70/2217, 1993

[2] J. Khoury, A. Weltman, PRD 69/044026 and PRL 93/171104, 2004

[3] K. Hinterbichler, et al, PRD84/103521 and PRL104/231301, 2010

[4] A. Vainshtein, Phys. Let. B, 39/393, 1972

[5] T. Damour, A. Polyakov, Nucl. Phys. B, 1994
O. Minazzoli, A. Hees, PRD 88/1504, 2013

Is it necessary to go beyond ?

Post Einsteinian Grav.

- phenomenology
- non local field equation: quantization ?

$$G_{\mu\nu}[k] = \chi_{\mu\nu}{}^{\alpha\beta}[k] T_{\alpha\beta}[k]$$

- metric: parametrized by 2 arbitrary functions

M.T. Jaekel, S. Reynaud, CQG, 2005

SME

- phenomenology
- violation of Lorentz symmetry coming from a fundamental level
- action parametrized by a tensor $\bar{s}^{\mu\nu}$

Q. Bailey, A. Kostelecky, PRD, 2006

Fab Four

- General 2nd order tensor-scalar theory
- developed in cosmology: Dark Energy
- weak-field metric: parametrized by 4 parameters

J.P. Bruneton et al, Adv. in Astr., 2012

MOND

- phenomenology
- developed for galactic observations: Dark Matter (galactic rotation curves)
- main effect in the Solar System: External Field Effect $U = \frac{GM}{r} + \frac{Q_2}{2} x^i x^j \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$

L. Blanchet, J. Novak, MNRAS, 2011

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~~PPN formalism : $\gamma, \boldsymbol{\beta}, \ldots$~~

~~5th force formalism: α, λ~~

Is it necessary to go beyond ?

Post Einsteinian Grav.

SME

Fab Four

Currently: lack of constraints from Solar System for these theories !

Interesting to consider them and to constrain them using Solar System observations

MOND

- phenomenology
- developed for galactic observations: Dark Matter (galactic rotation curves)
- main effect in the Solar System: External Field Effect

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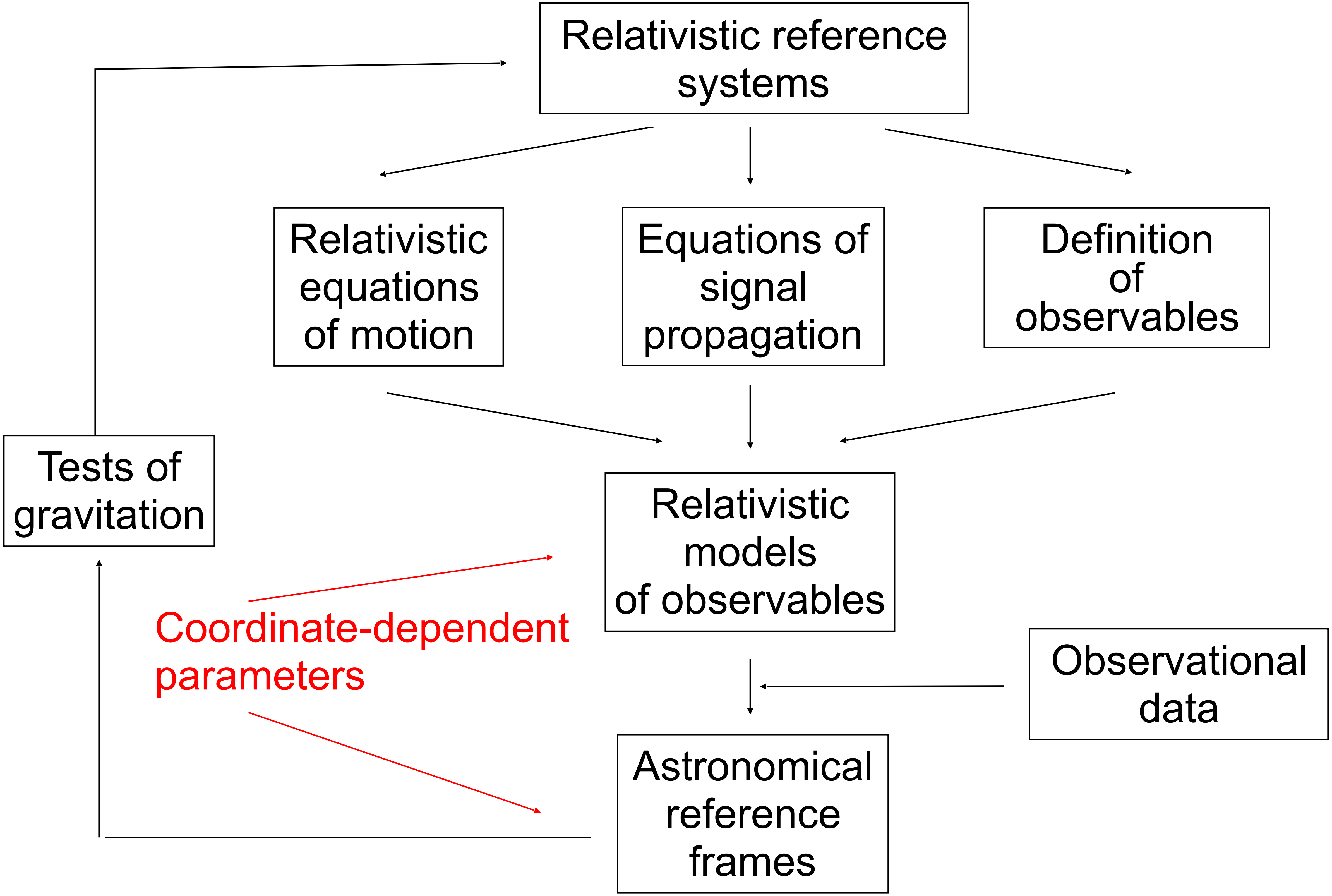
Relativistic Astronomy : some basics



Relativistic reference systems

Relativistic equations of motion

Equations of signal propagation

Definition of observables

Tests of gravitation

Relativistic models of observables

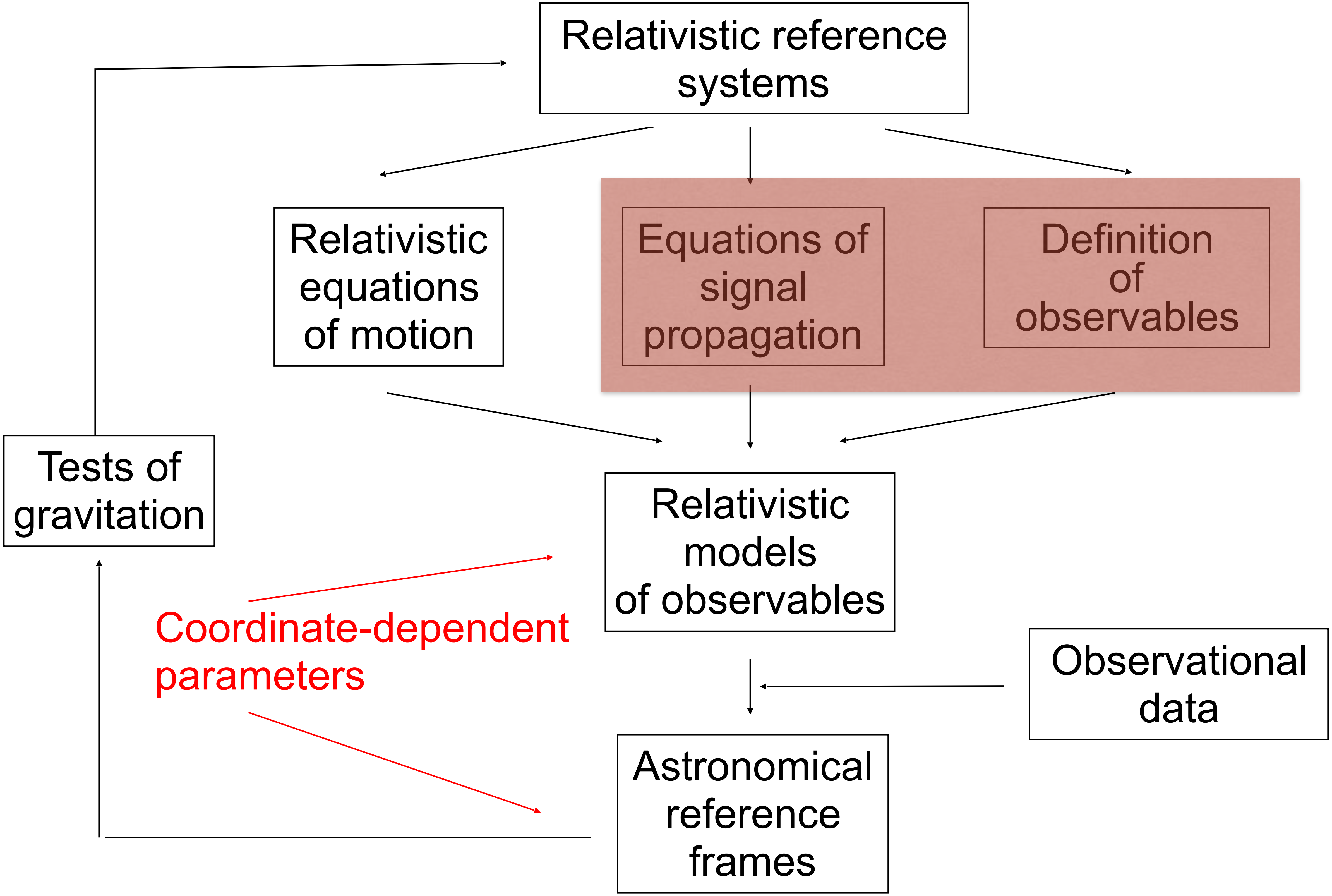
Coordinate-dependent parameters

Observational data

Astronomical reference frames

Klioner, 2003

Relativistic Astronomy : some basics



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Light propagation is crucial in the modeling of Sol. Sys. observations

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1) Range observable

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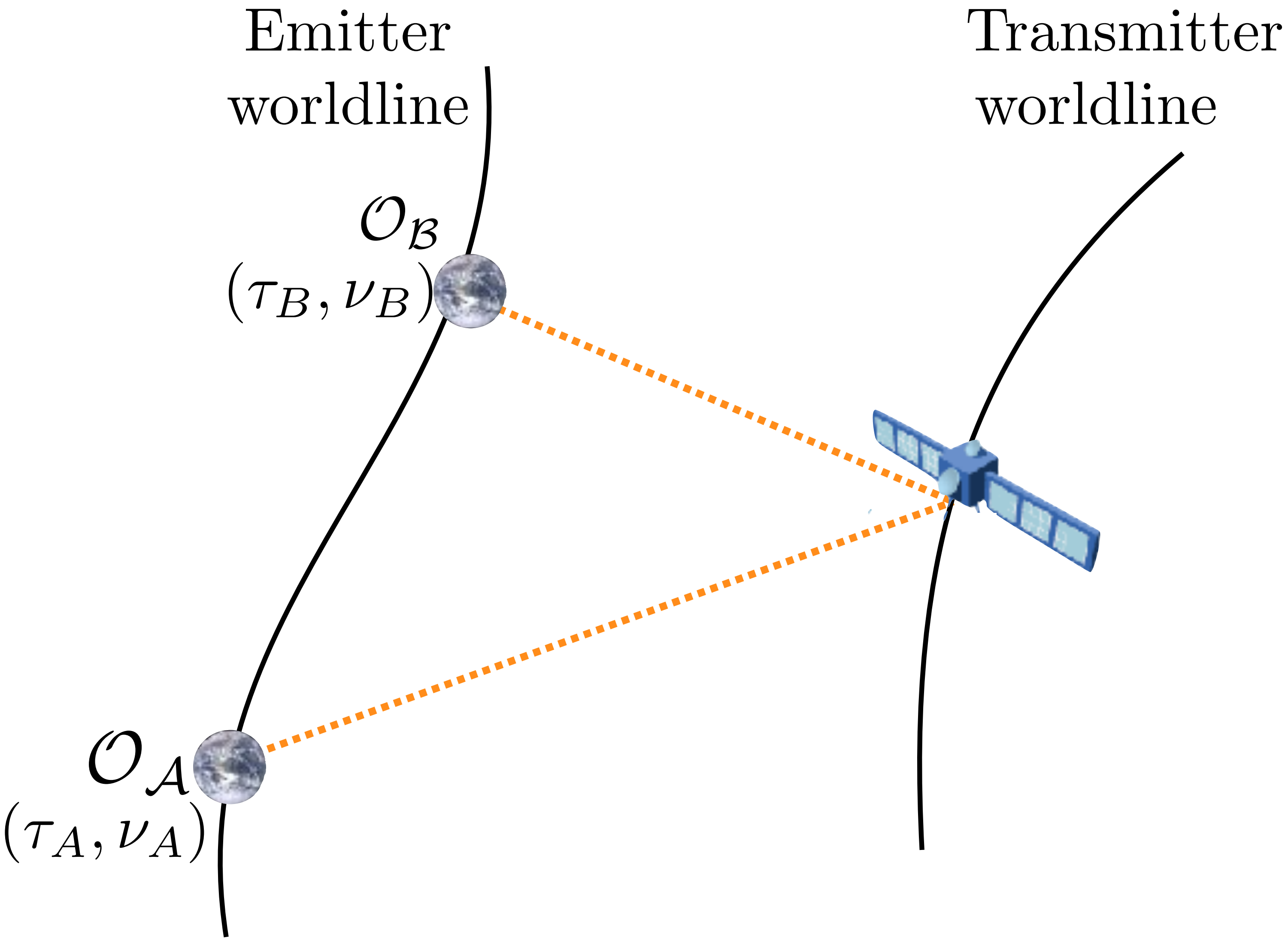
- Difference in proper time

$$\text{Range} = c(\tau_B - \tau_A)$$



Light propagation is crucial in the modeling of Sol. Sys. observations

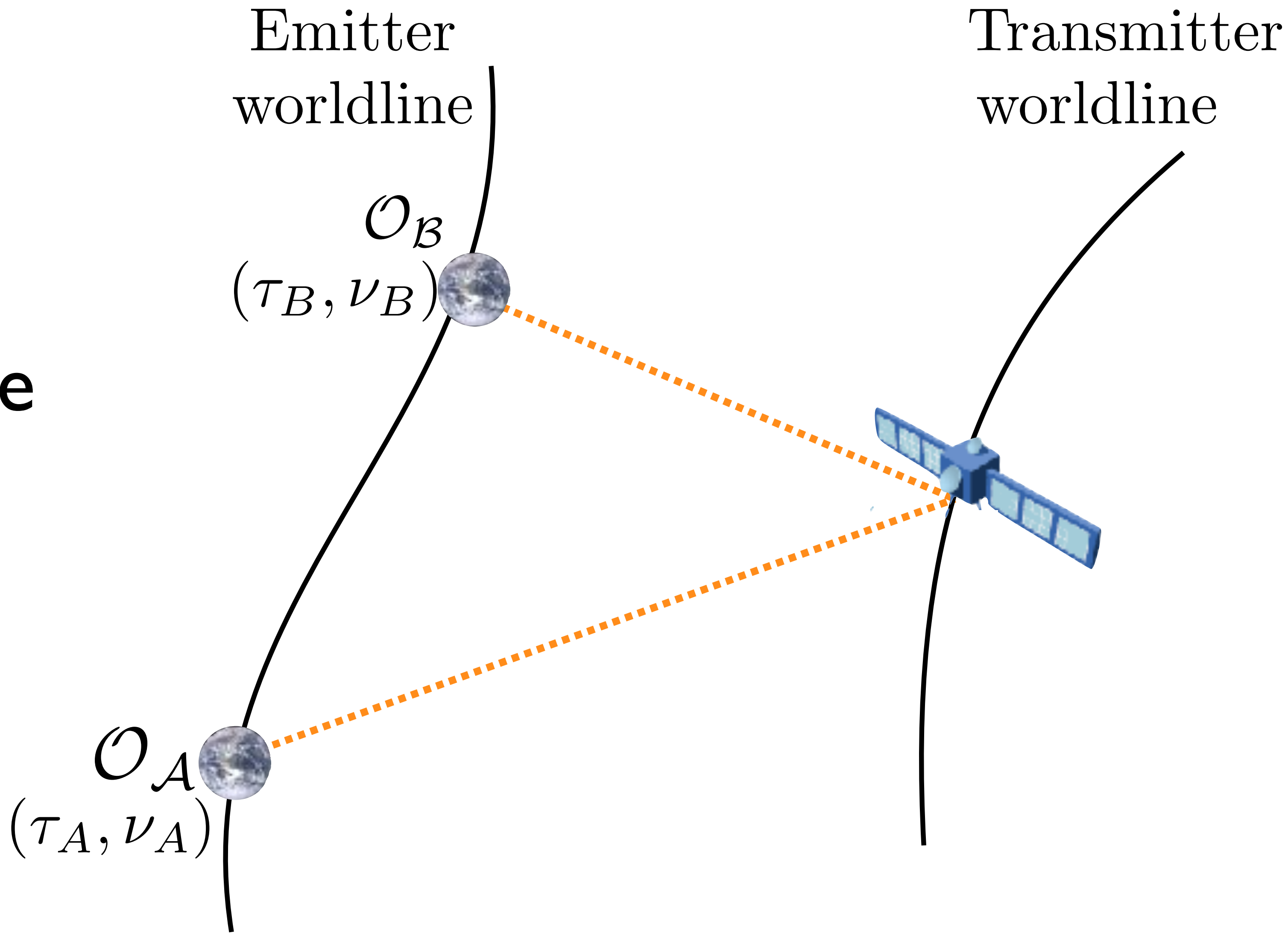
1) Range observable

- Difference in proper time

$$\text{Range} = c(\tau_B - \tau_A)$$

- Depends on the difference in coord. time (amongst other parameters)

$$t_B - t_A$$



Light propagation is crucial in the modeling of Sol. Sys. observations

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2) Doppler observable

Light propagation is crucial in the modeling of Sol. Sys. observations

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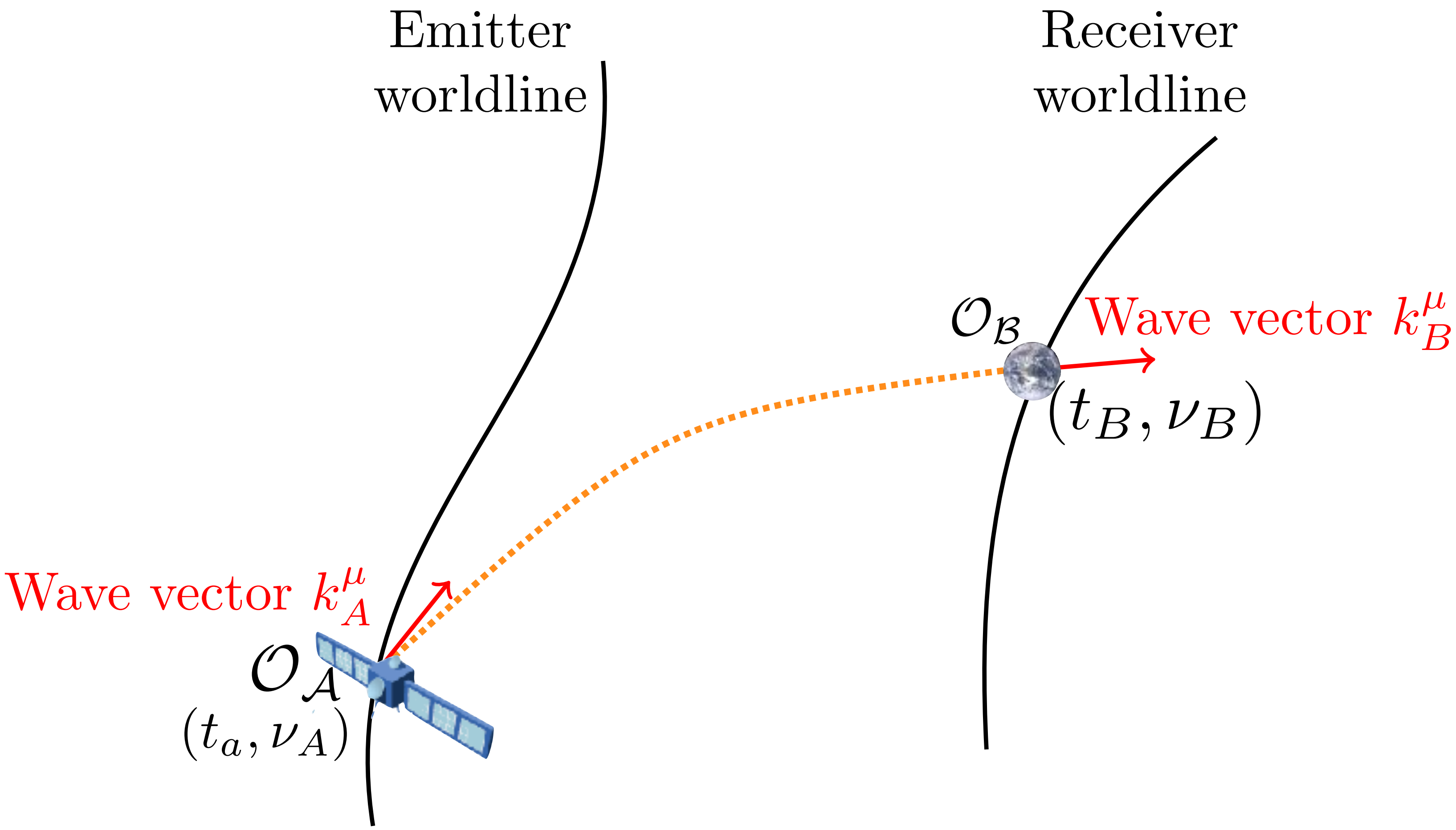
- Ratio of proper frequency $D = \frac{\nu_B}{\nu_A} = \left(\frac{d\tau}{dt}\right)_A \left(\frac{d\tau}{dt}\right)_B^{-1} \frac{k_0^B}{k_0^A} \frac{1+\beta_B^i \hat{k}_i^B}{1+\beta_A^i \hat{k}_i^A}$

with $\beta^i = v^i/c$ and

$$\hat{k}_i = \frac{k_i}{k_0}$$



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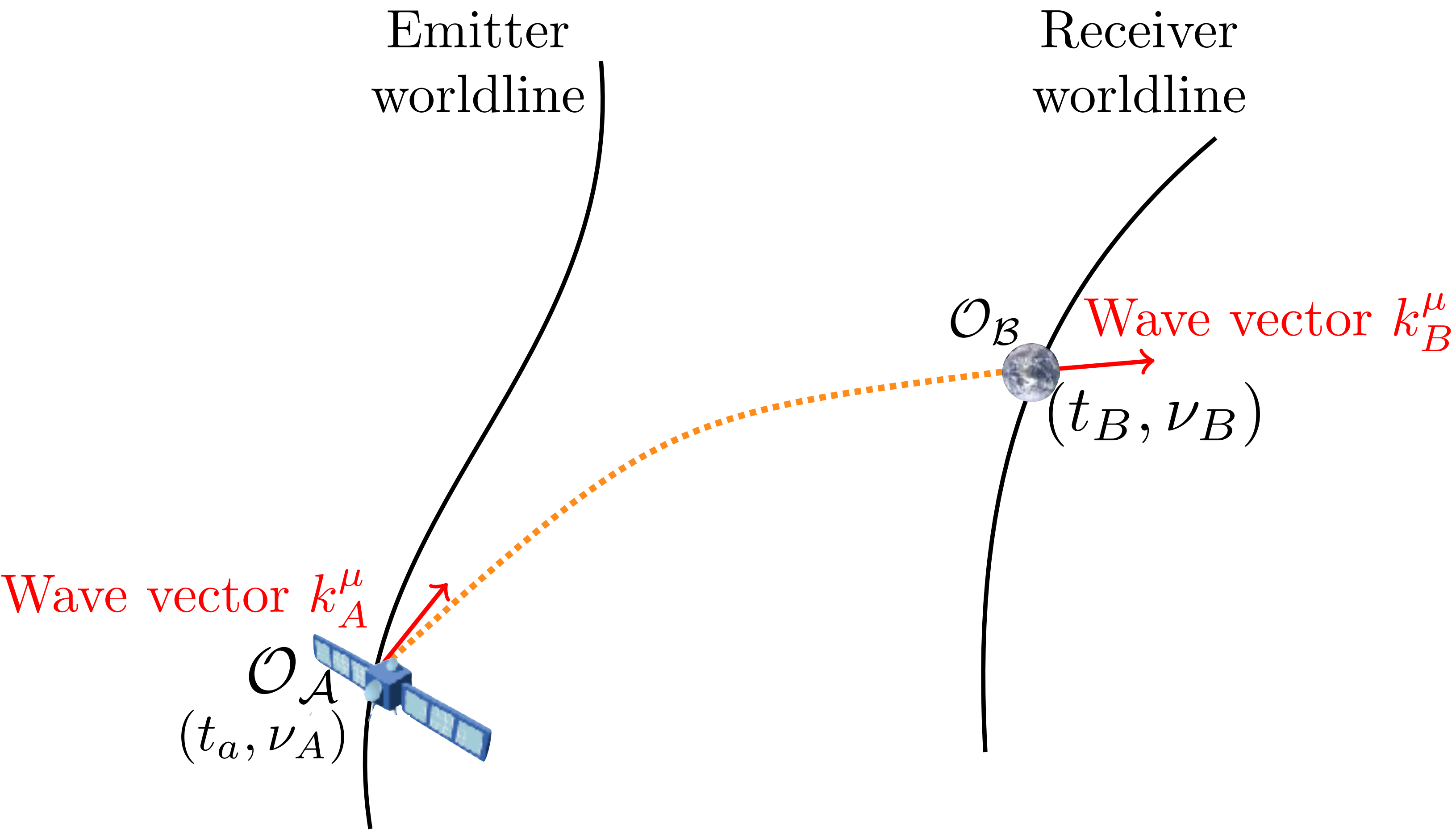
$$\boxed{\hat{k}_i = \frac{k_i}{k_0}}$$

- Wave vector at emission and reception needed



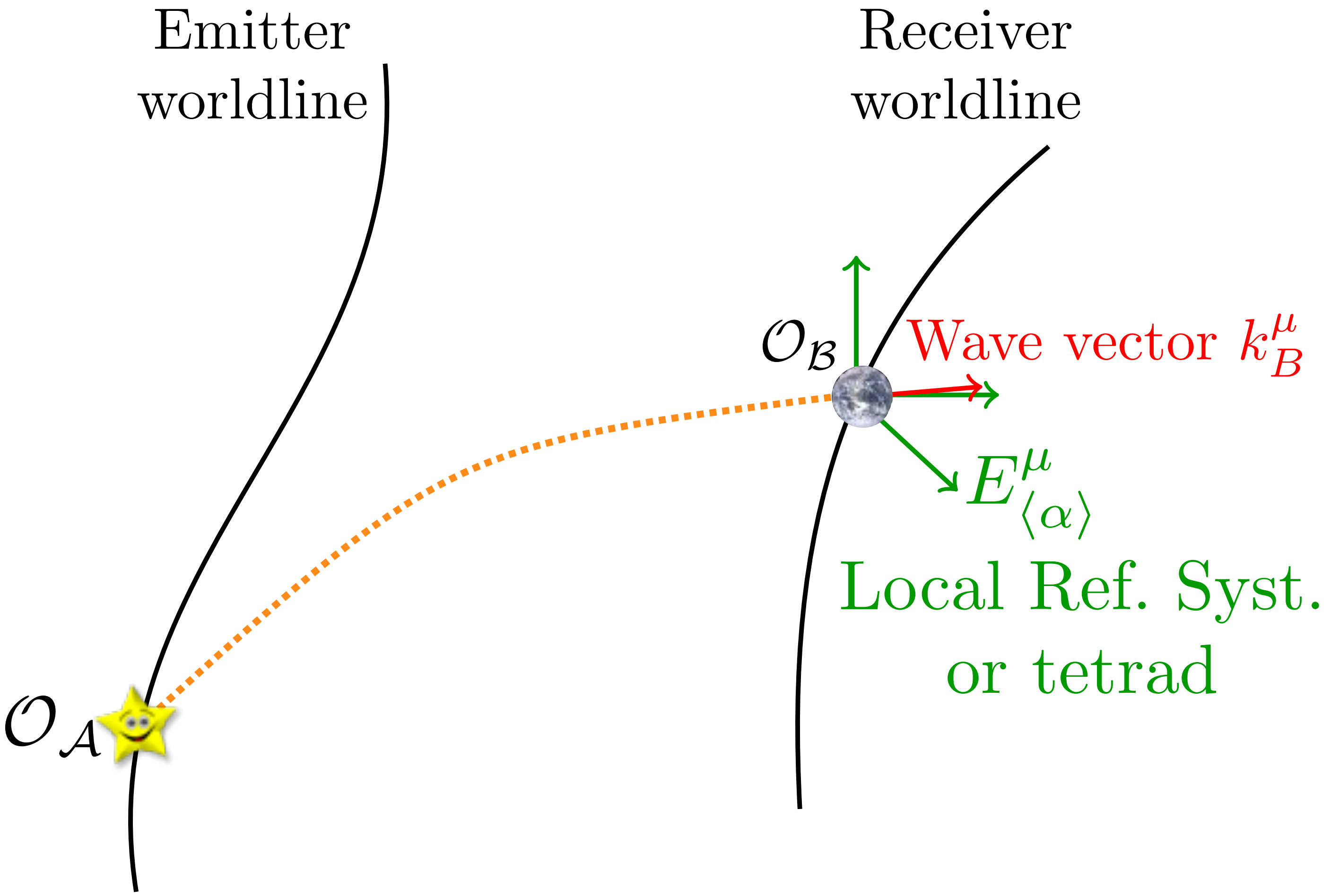
Light propagation is crucial in the modeling of Sol. Sys. observations

Light propagation is crucial in the modeling of Sol. Sys. observations

3) Astrometric observables

Direction of observation of the light ray in a local reference system (or tetrad)

$$n^{\langle i\rangle}=-\frac{E^0_{\langle i\rangle}+E^j_{\langle i\rangle}\hat{k}^B_j}{E^0_{\langle 0\rangle}+E^j_{\langle 0\rangle}\hat{k}^B_j}$$



Light propagation is crucial in the modeling of Sol. Sys. observations

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Direction of observation of the light ray in a local reference system (or tetrad)

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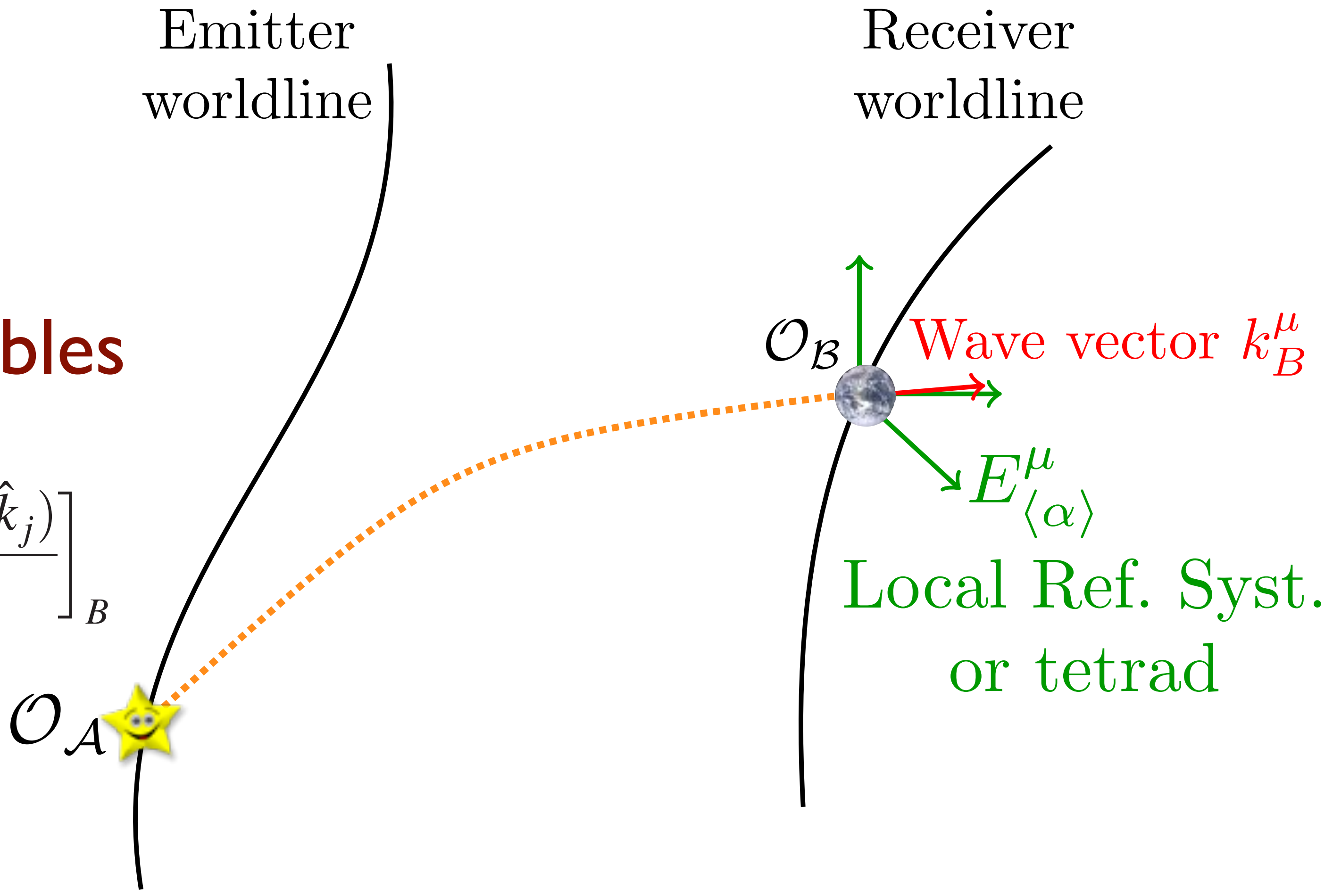
4) Differential astrometric observables

$$\sin^2\frac{\phi}{2} = -\frac{1}{4}\left[\frac{(g_{00}+2g_{0k}\beta^k+g_{kl}\beta^k\beta^l)g^{ij}(\hat{k}'_i-\hat{k}_i)(\hat{k}'_j-\hat{k}_j)}{(1+\beta^m\hat{k}_m)(1+\beta^l\hat{k}_l{}')}\right]_B$$

Angle between 2 incoming light rays



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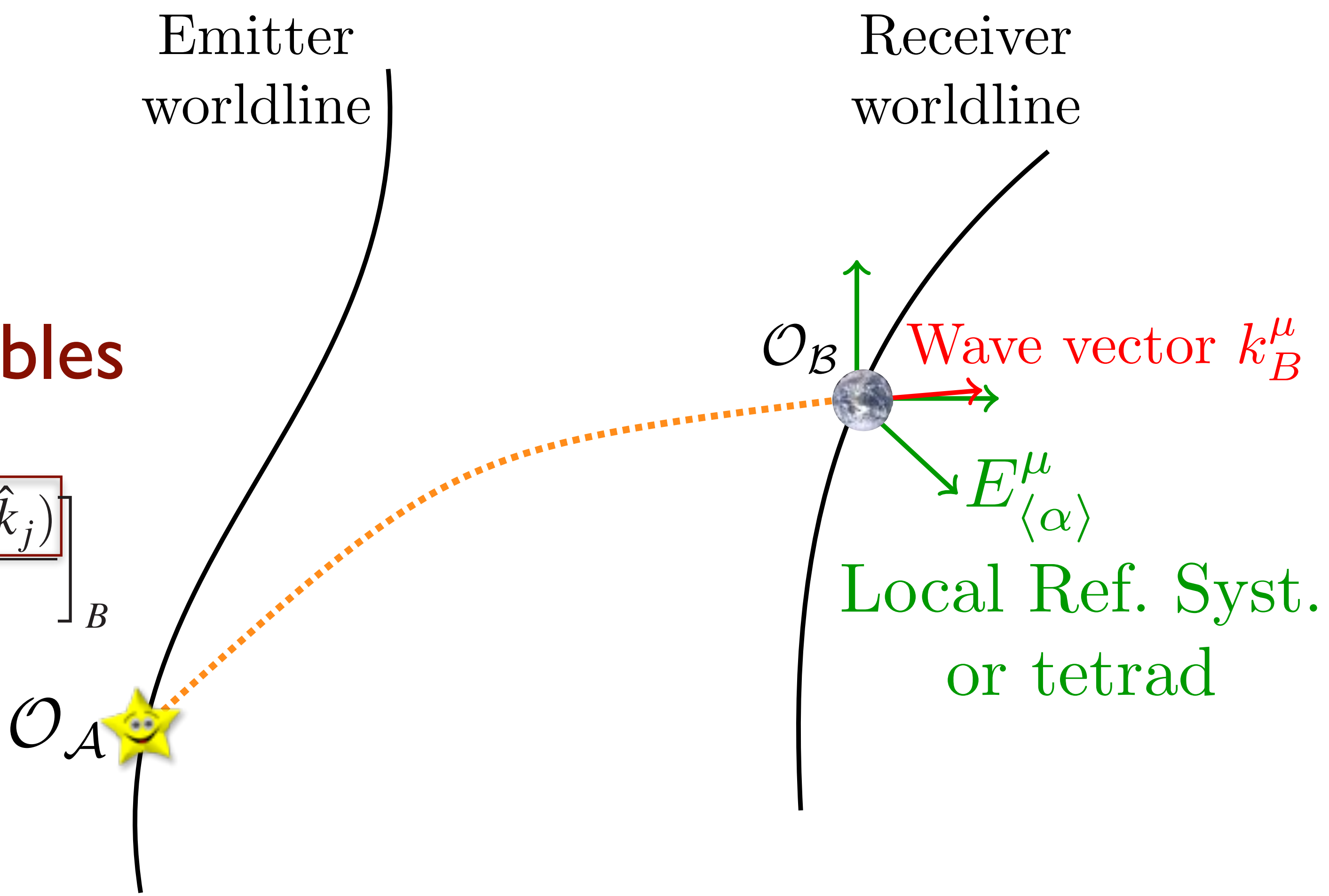

Wave vector at reception needed only

How to determine the light propagation ?

- At the geometric optics approximation: photons follow null geodesics

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0$$

$$k^\mu k_\mu = 0$$

an Initial Value Problem

with $k^\mu = \dfrac{dx^\mu}{d\lambda}$ the tangent vector

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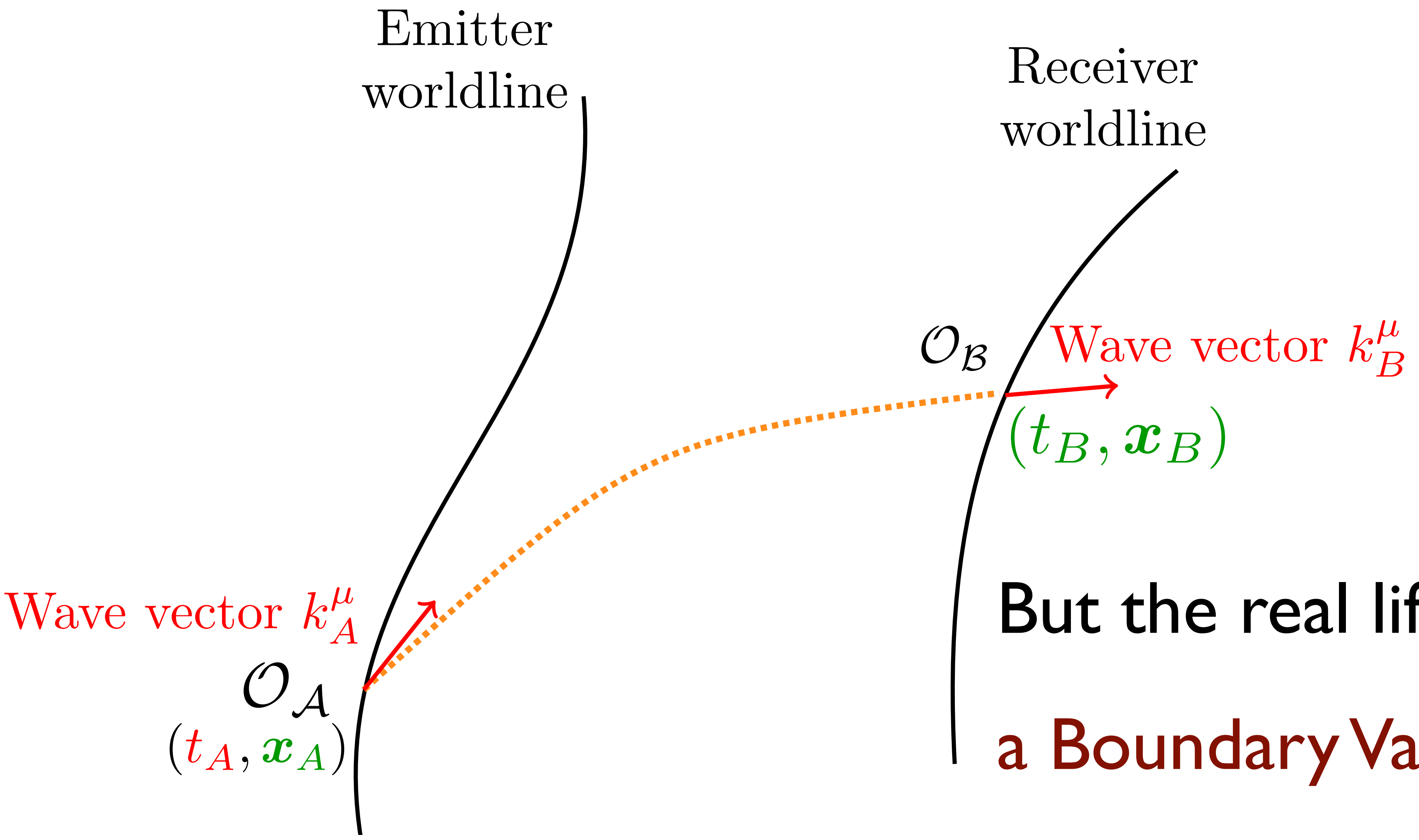

But the real life :

a Boundary Value Problem

Methods to solve the null geodesic eqs.

Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method

see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007
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- Analytical solutions for weak gravitational field:
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  - static extended bodies with multipolar expansion at 1pM
  - 2 pM Schwarzschild metric

see E. Shapiro, PRL 13, 26, 789, 1964

see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
S. Klioner, A & A, 404, 783, 2003
De Felice et al., ApJ, 653, 2, 1552-1565, 2006

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- Use of the eikonal equation:
  - perturbative solution for spherically symmetric space-time

see for example N. Ashby, B. Bertotti, CQG 27, 145013, 2010

… and the Time Transfer Functions

see CLPL, et al, CQG 21, 4463, 2004
P. Teyssandier and CLPL, CQG 25, 145020, 2008

… and the Time Transfer Functions

see CLPL, et al, CQG 21, 4463, 2004
P. Teyssandier and CLPL, CQG 25, 145020, 2008

- The Time Transfer Functions - TTF - are defined by

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) \qquad t_B - t_A = \mathcal{T}_e(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B)$$

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- The derivatives of the TTF are of crucial interest since

$$\hat{k}_i^A = c\frac{\partial \mathcal{T}_r}{\partial x_A^i} \qquad \hat{k}_i^B = -c\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1} \qquad \frac{k_0^B}{k_0^A} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B}$$

Range, Doppler, astrometric observables can be written in terms of the TTF and its derivatives

Fundamental properties of TTF's

see CLPL, et al, CQG 21, 4463, 2004
P. Teyssandier and CLPL, CQG 25, 145020, 2008

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$$g^{00}(x_A^0 + c\mathcal{T}_e, \boldsymbol{x}_B) - 2c\,g^{0i}(x_A^0 + c\mathcal{T}_e, \boldsymbol{x}_B)\frac{\partial \mathcal{T}_e}{\partial x_B^i} + c^2\,g^{ij}(x_A^0 + c\mathcal{T}_e, \boldsymbol{x}_B)\frac{\partial \mathcal{T}_e}{\partial x_B^i}\frac{\partial \mathcal{T}_e}{\partial x_B^j} = 0.$$

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TTF is a dedicated World Function to light ray.

General Post-Minkowskian expansions are possible

Post-Minkowskian expansion of the TTF

see P. Teyssandier and CLPL, CQG 25, 145020, 2008

- A pM expansion of the TTF: $\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \frac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}_r^{(n)}$

- Computation with an iterative procedure involving integrations over a straight line between the emitter and the spatial position of the receiver !

- Example at 1 pM: $\mathcal{T}_r^{(1)} = \frac{R_{AB}}{2c} \int_0^1 \left[g_{(1)}^{00} - 2N_{AB}^i g_{(1)}^{0i} + N_{AB}^i N_{AB}^j g_{(1)}^{ij} \right]_{z^\alpha(\lambda)} d\lambda$

  with $z^\alpha(\lambda)$ the straight Mink. null path between em. and rec.

- Main advantages:

  - analytical computations relatively easy

  - very well adapted to numerical evaluation

Analytical results in Schwarzschild space-time

- A pM expansion of the TTF: $\mathcal{T} = \frac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}^{(n)}$

$$\mathcal{T}^{(1)} = \frac{(1+\gamma)m}{c} \ln \frac{r_A + r_B + |\boldsymbol{x}_B - \boldsymbol{x}_A|}{r_A + r_B - |\boldsymbol{x}_B - \boldsymbol{x}_A|}$$

see E. Shapiro, PRL 13, 26, 789, 1964

$$\mathcal{T}^{(2)} = \frac{m^2}{r_A r_B} \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c} \left[\kappa \frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} - \frac{(1+\gamma)^2}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B} \right]$$

see CLPL, et al, CQG 21, 4463, 2004
S. Klioner, S. Zschocke, CQG 27, 075015, 2010

$$\mathcal{T}^{(3)} = \frac{m^3}{r_A r_B} \left(\frac{1}{r_A} + \frac{1}{r_B} \right) \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c(1 + \boldsymbol{n}_A.\boldsymbol{n}_B)} \left[\kappa_3 - (1+\gamma)\kappa \frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} + \frac{(1+\gamma)^3}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B} \right]$$

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014

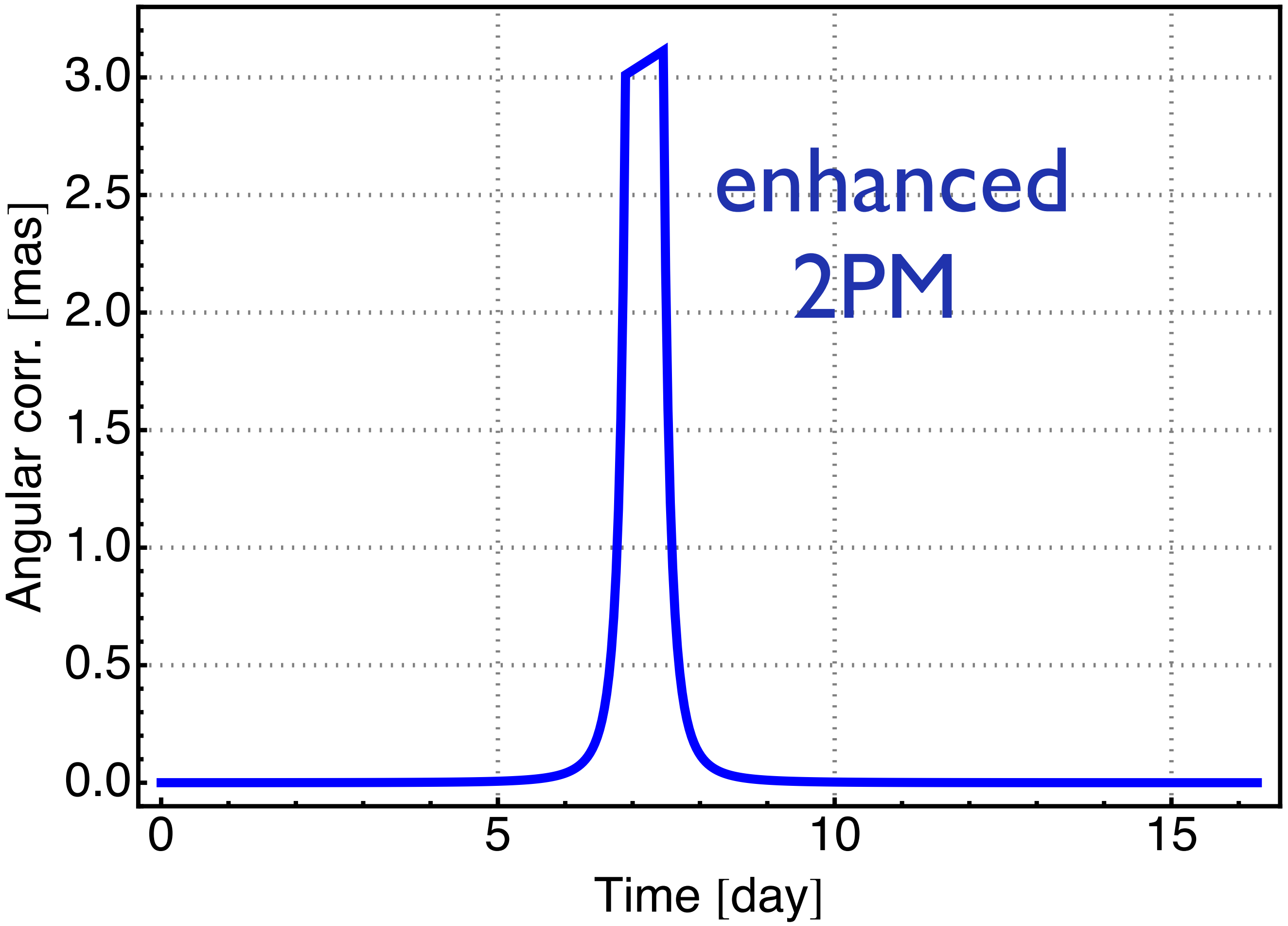
with $\kappa = 2 + 2\gamma - \beta + \frac{3}{4}\epsilon$

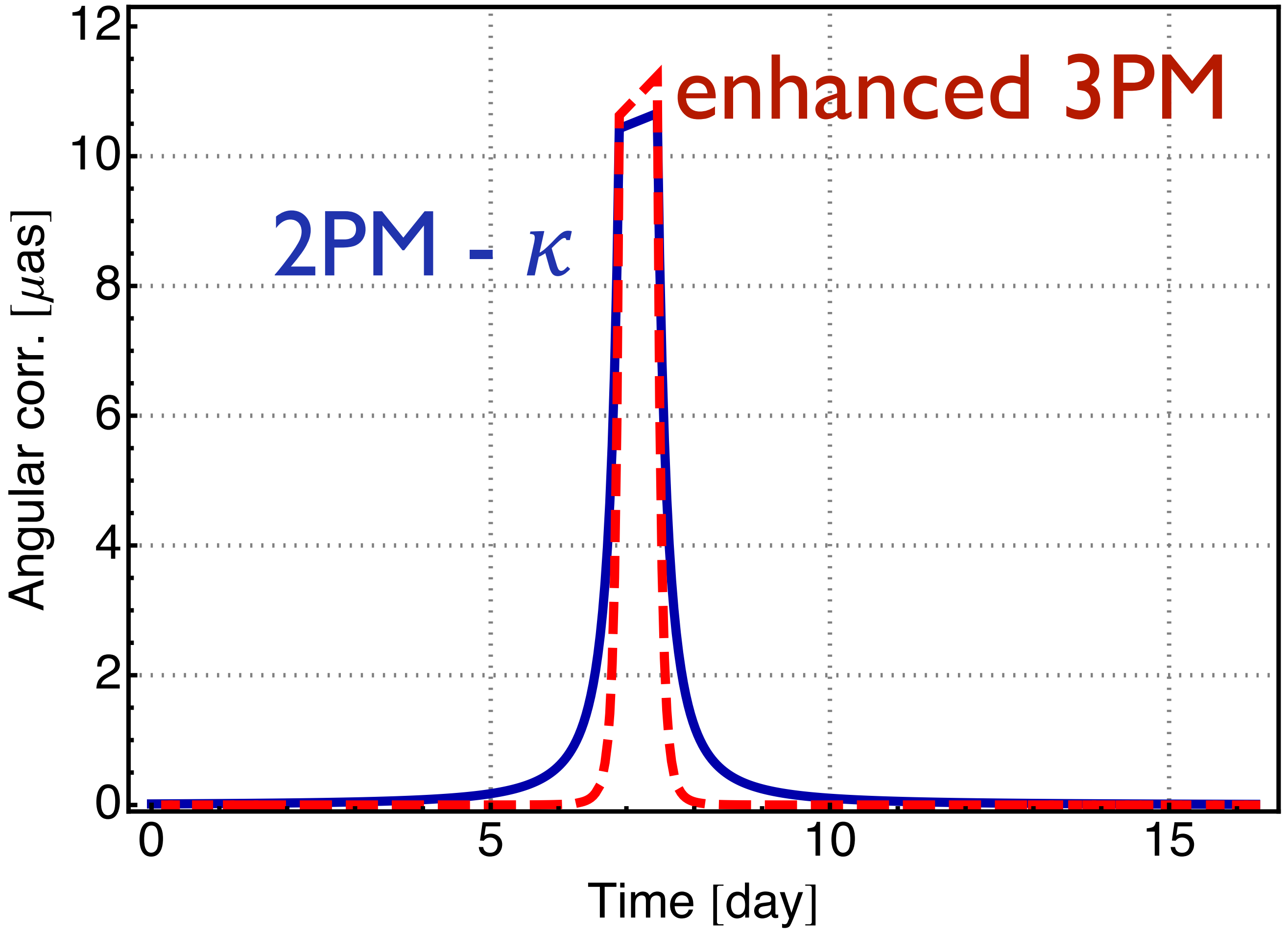
$\kappa_3 = 2\kappa - 2\beta(1+\gamma) + \frac{1}{4}(3\beta_3 + \gamma_3)$

and $\boldsymbol{n}_{A/B} = \frac{\boldsymbol{x}_{A/B}}{r_{A/B}}$

Is it necessary to go to the 3rd order?

- In a conjunction geometry, at each order n, there are enhanced terms proportional to $(1+\gamma)^n$
- Ex. with light deflection for Sun grazing rays: AGP space mission (old GAME). Expected accuracy: μas
  $\Rightarrow$ 3pM term needed





see A. Hees, S. Bertone, CLPL, PRD 89, 064045, 2014
P. Teyssandier, B. Linet, proceedings of JSR 2013, arXiv:1312.3510

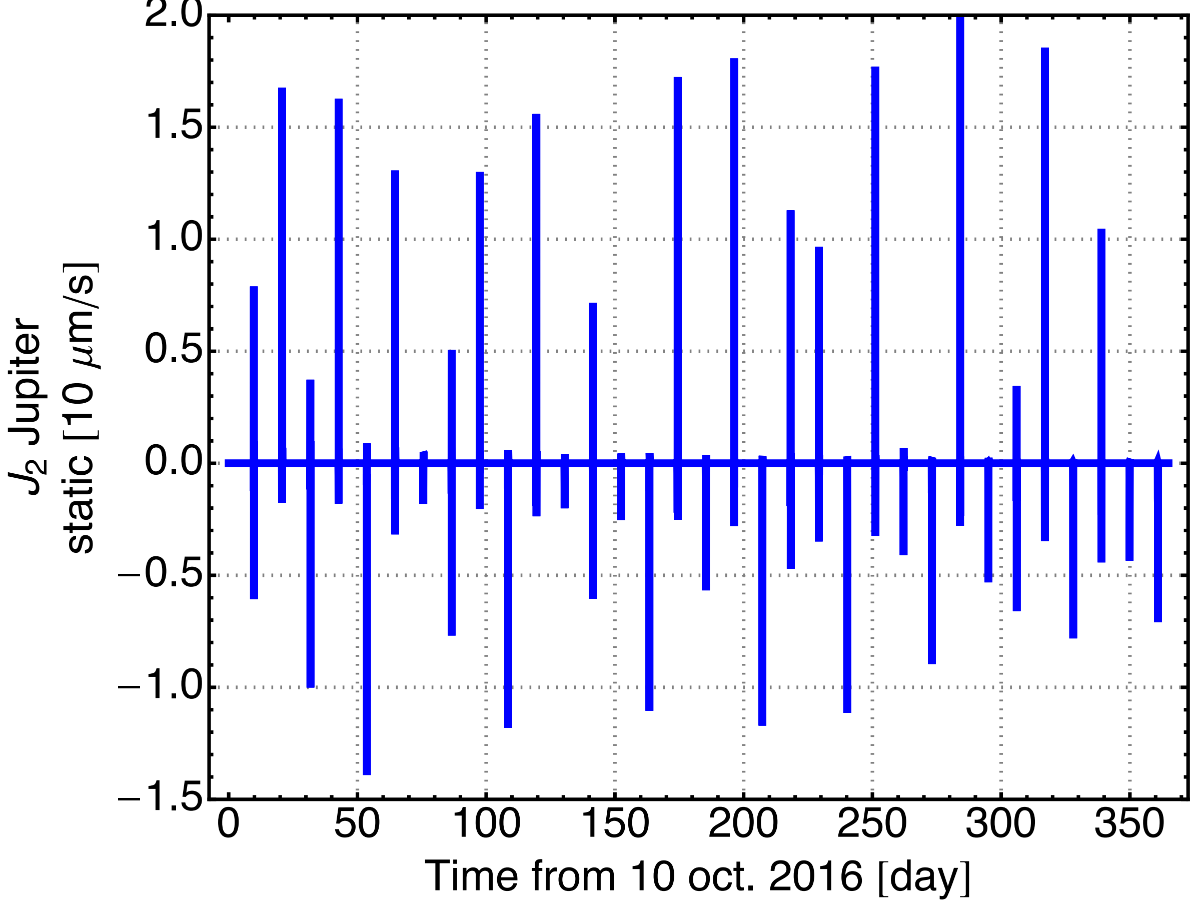
Analytical result around axisymmetric bodies

- Influence of all the multipole moments J_n from the grav. potential

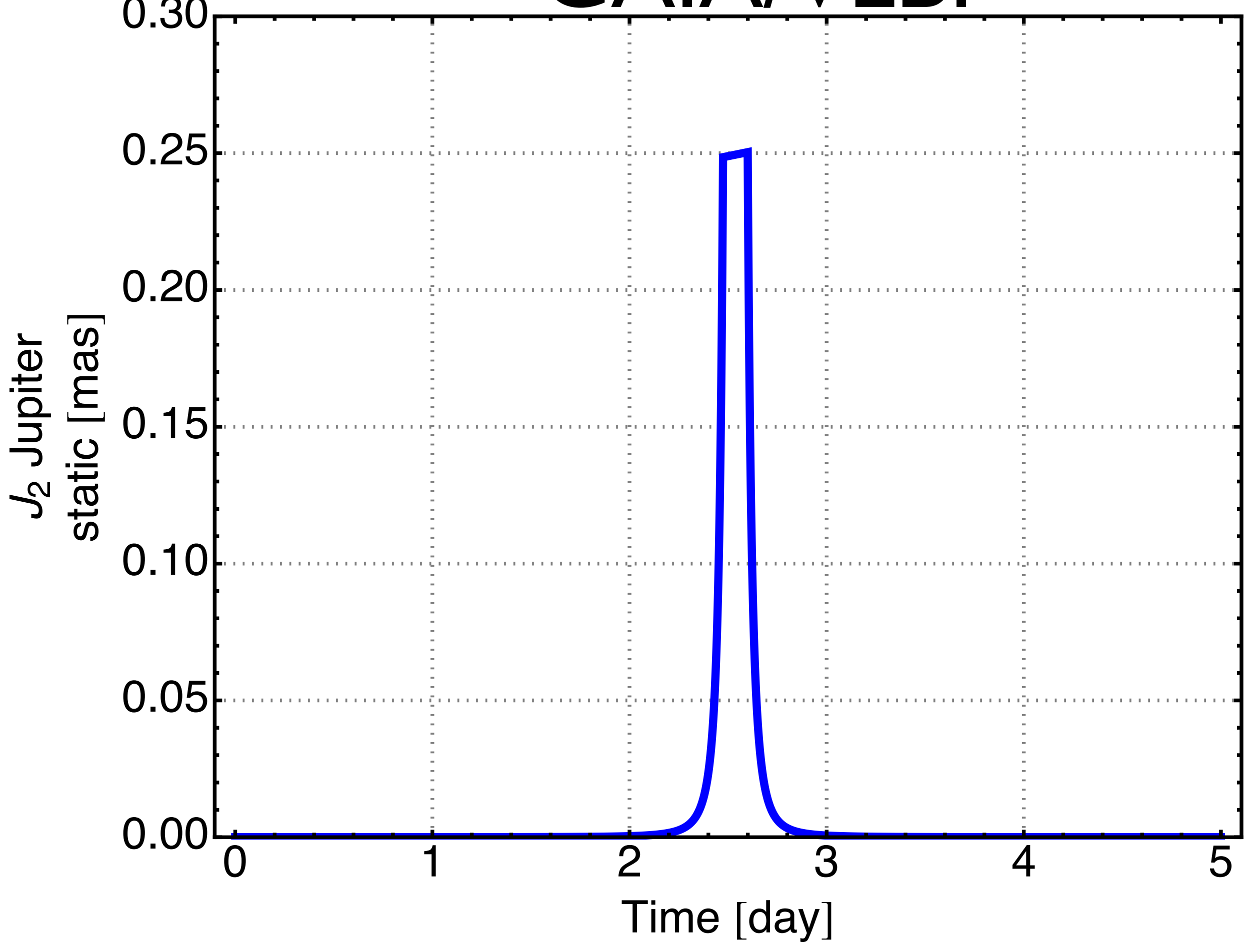
see CLPL, P. Teyssandier, PRD 77, 044029, 2008 for a computation with the TTF
or S. Kopeikin, J. of Math. Physics 38, 2587, 1997 for another approach

- Influence of Jupiter J_2 on the JUNO Doppler (1 μm/s accuracy) and for GAIA (10 μas acc.)

JUNO



GAIA/VLBI



- terms important for the data analysis for these missions

see Hees, Bertone, CLPL, PRD 90, 084020, 2014

What happens if the body is moving ?

see Hees, Bertone, CLPL, PRD 90, 084020, 2014

- At first pM order, the TTF for uniformly moving bodies can easily be derived from the TTF generated by a static body

$$\Delta(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \gamma(1 - \boldsymbol{N}_{AB}.\boldsymbol{\beta})\tilde{\Delta}(\boldsymbol{R}_A + \gamma\boldsymbol{\beta}R_{AB}, \boldsymbol{R}_B)$$

TTF in the moving case

static TTF

with $\boldsymbol{\beta} = \boldsymbol{v}/c, \quad \gamma = (1 - \beta^2)^{-1/2}$

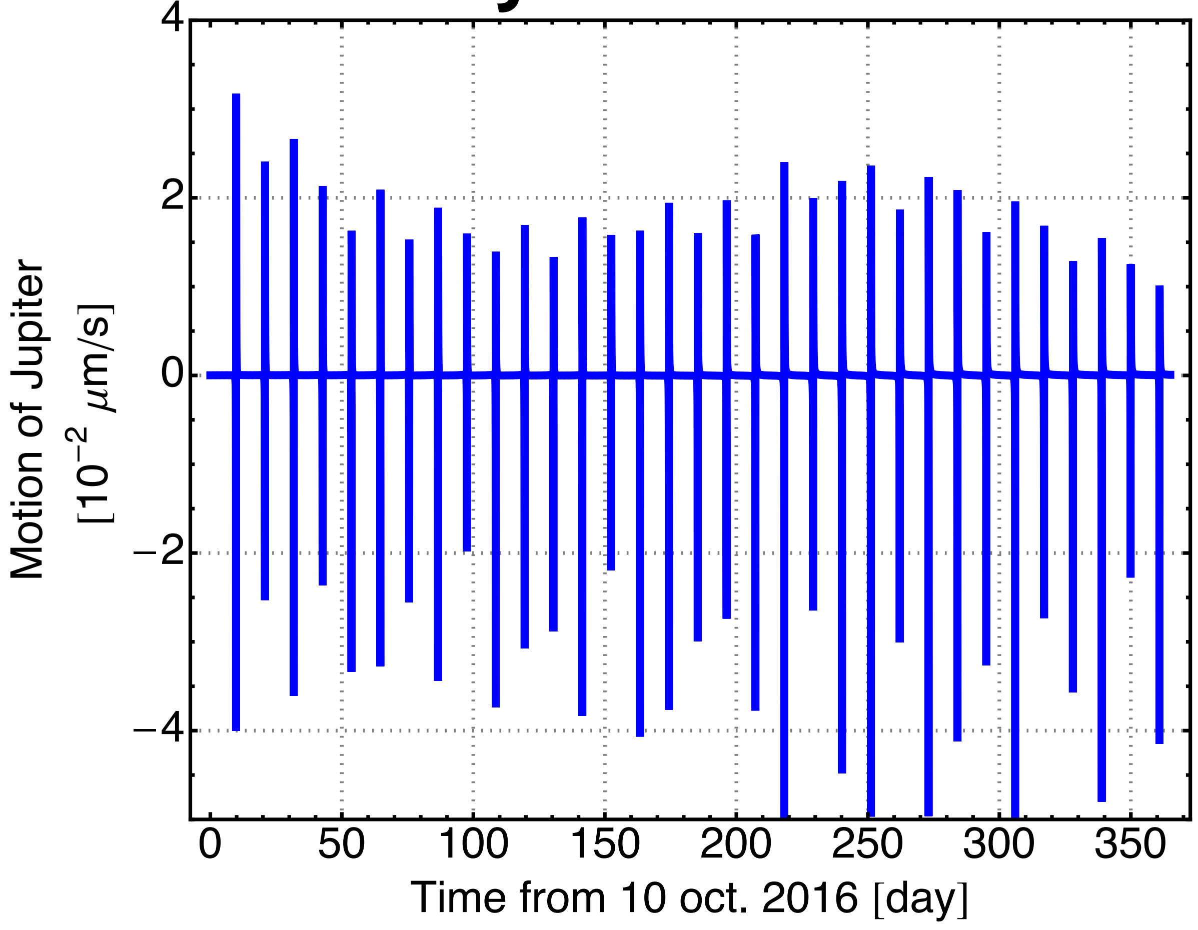
and $\boldsymbol{R}_X$ depends on $\boldsymbol{x}_X, \boldsymbol{\beta}$

- All the analytical results computed for a static source can be extended in the case of a uniformly moving source

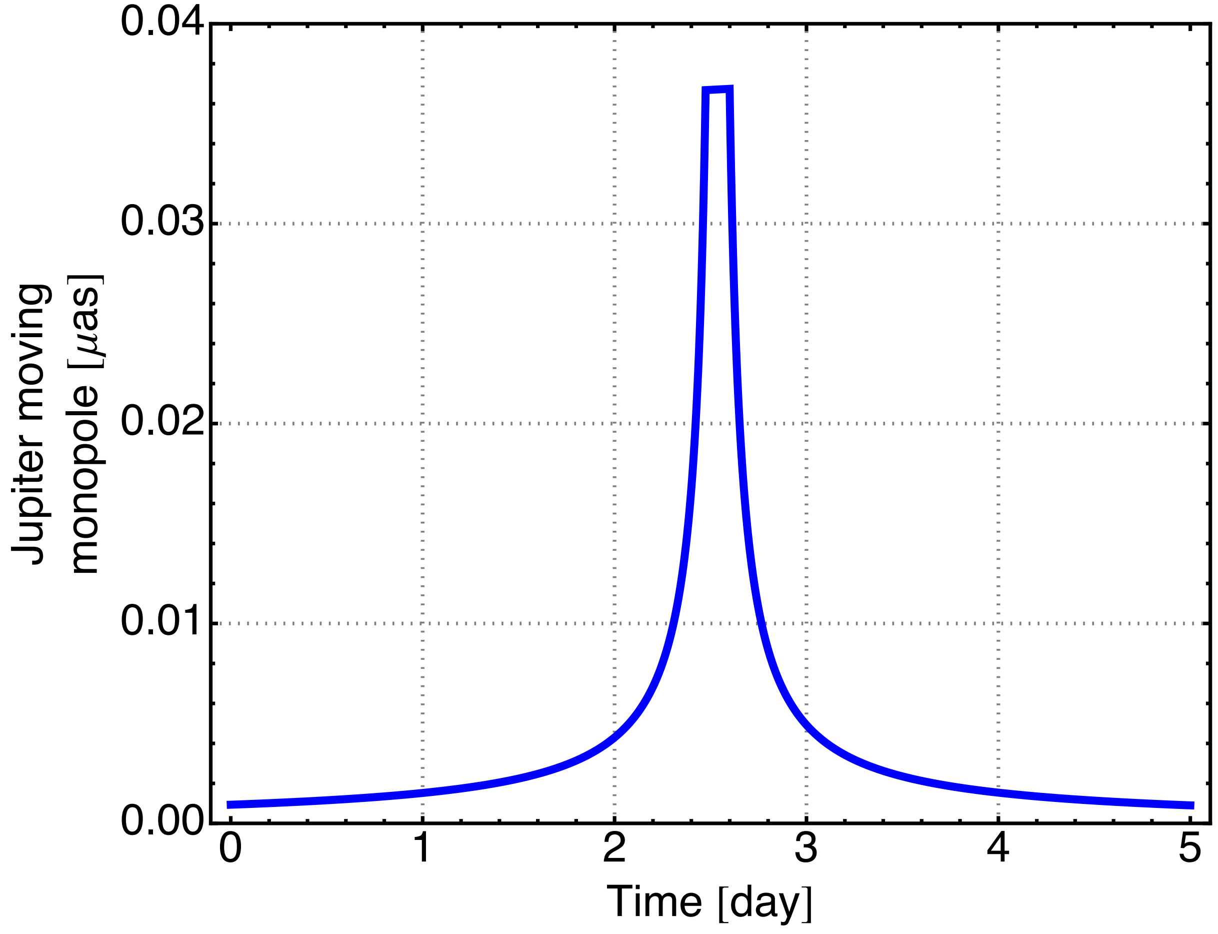
Ex.: motion of Jupiter

- Influence of Jupiter velocity on the JUNO Doppler (1μm/s accuracy) and for GAIA (10 μas acc.)

JUNO



GAIA/VLBI



- depend highly on the orbit geometry: conjunction and $\beta.N_{AB}$
- In particular: should be reassessed for JUICE orbit

see Hees, Bertone, CLPL, PRD 90, 084020, 2014

Numerical evaluation of the TTF

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$$\frac{\partial \mathcal{T}^{(1)}}{\partial x^i_{A/B}} = \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

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- At 2pM order: a double integral to evaluate

$$\mathcal{T}^{(2)} = \int_0^1 \int_0^1 n\left[z^\alpha(\mu\lambda);\; g^{(2)}_{\alpha\beta},\; g^{(1)}_{\alpha\beta},\; g^{(1)}_{\alpha\beta,\gamma},\; \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

$$\frac{\partial \mathcal{T}^{(2)}}{\partial x^i_{A/B}} = \int_0^1 \int_0^1 n_{A/B}\left[z^\alpha(\mu\lambda);\; g^{(2)}_{\alpha\beta},\; g^{(2)}_{\alpha\beta,\gamma},\; g^{(1)}_{\alpha\beta},\; g^{(1)}_{\alpha\beta,\gamma},\; g^{(1)}_{\alpha\beta,\gamma\delta},\; \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

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- Numerically efficient ; useful when no analytical solution can be found

see Hees, Bertone, CLPL, PRD 89, 064045, 2014

Some fundamental unified theories break Lorentz symmetry

- like e.g.: strings, noncommutative space-time, loop quan. theory
- General framework to study Lorentz violation:

  Standard-Model Extension (SME)

developped by Kostelecky and collaborators in the 90ies

- SME is an effective field theory developed from a Lagrangian

$$\mathcal{L} = \underset{\substack{\nearrow \\ \text{standard} \\ \text{model}}}{\mathcal{L}_{\mathrm{SM}}} + \underset{\substack{\uparrow \\ \text{General} \\ \text{Relativity}}}{\mathcal{L}_{\mathrm{GR}}} + \boxed{\mathcal{L}_{\mathrm{LV}}}$$

All possible Lorentz violating terms constructed from SM & GR fields and background coefficients

Credit to Q. Bailey

The gravity sector of the minimal SME introduces 9 coefficients

- minimal SME = linearized gravity limit $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\mathcal{L}_{\mathrm{LV}} = \frac{c^3}{32\pi G} h^{\mu\nu} \boxed{\bar{s}^{\alpha\beta}} \mathcal{G}_{\alpha\mu\nu\beta} + \boxed{\dots}$$

STF tensor: 9 Lorentz-violating coefficients

non-minimal higher order terms

See Kostelecky, PRD, 04 - Bailey and Kostelecky, PRD, 06 - Kostelecky and Mewes, PLB, 16

- this modifies the equations of motion

$$\left[\frac{d^2x^j}{dt^2}\right]_{\mathrm{SME}} = \frac{GM}{r^3}\left[\boxed{\bar{s}^{jk}} r^k - \frac{3}{2}\boxed{\bar{s}^{kl}}\frac{r^k r^l}{r^2} r^j + 2\boxed{\bar{s}^{0k}}\frac{v^k}{c} r^j - 2\boxed{\bar{s}^{0j}}\frac{v^k}{c} r^k + \dots\right]$$

- Can be constrained by: LLR, pulsars, VLBI, atom interferometry, etc …

Bourgoin et al., PRL, 2016

Le Poncin-Lafitte at al., PRD, 2016

for a review of the tests of SME in gravity sector, see Hees et al, Universe, 2016

SME Post-fit analysis of experiments

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Several papers on Lunar Laser Ranging, Gravity Probe B and binary pulsars :

SME Post-fit analysis of experiments

Several papers on Lunar Laser Ranging, Gravity Probe B and binary pulsars :

$$|\bar{s}^{TT}| < 1.6 \times 10^{-5} \quad (95\% \text{ C.L.}),$$

Shao PRD 2014

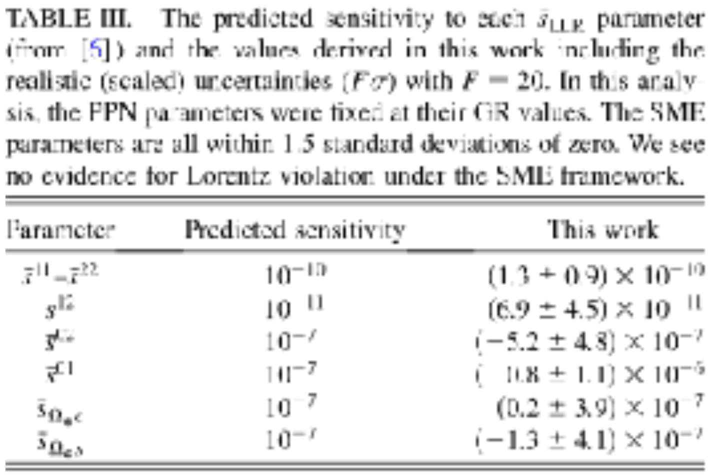

TABLE III. The predicted sensitivity to each $\bar{s}_{\rm LLR}$ parameter (from [5]) and the values derived in this work including the realistic (scaled) uncertainties ($F\sigma$) with $F = 20$. In this analysis, the PPN parameters were fixed at their GR values. The SME parameters are all within 1.5 standard deviations of zero. We see no evidence for Lorentz violation under the SME framework.

Parameter	Predicted sensitivity	This work
$\bar{s}^{11}-\bar{s}^{22}$	10^{-10}	$(1.3 \pm 0.9) \times 10^{-10}$
$\bar{s}^{12}$	10^{-11}	$(6.9 \pm 4.5) \times 10^{-11}$
$\bar{s}^{02}$	10^{-7}	$(-5.2 \pm 4.8) \times 10^{-7}$
$\bar{s}^{01}$	10^{-7}	$(-0.8 \pm 1.1) \times 10^{-8}$
$\bar{s}_{\Omega_\oplus c}$	10^{-7}	$(0.2 \pm 3.9) \times 10^{-7}$
$\bar{s}_{\Omega_\oplus s}$	10^{-7}	$(-1.3 \pm 4.1) \times 10^{-7}$

Battat et al. PRL 2007

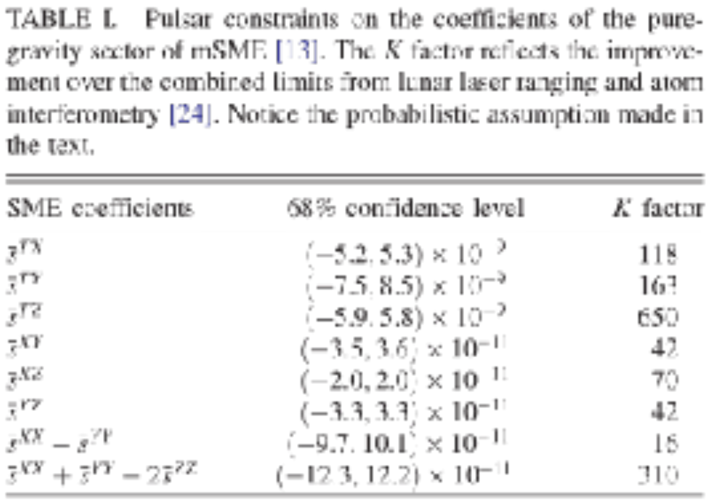

TABLE I. Pulsar constraints on the coefficients of the pure-gravity sector of mSME [13]. The K factor reflects the improvement over the combined limits from lunar laser ranging and atom interferometry [24]. Notice the probabilistic assumption made in the text.

SME coefficients	68% confidence level	K factor
$\bar{s}^{TX}$	$(-5.2, 5.3) \times 10^{-9}$	118
$\bar{s}^{TY}$	$(-7.5, 8.5) \times 10^{-9}$	163
$\bar{s}^{TZ}$	$(-5.9, 5.8) \times 10^{-9}$	650
$\bar{s}^{XY}$	$(-3.5, 3.6) \times 10^{-11}$	42
$\bar{s}^{XZ}$	$(-2.0, 2.0) \times 10^{-11}$	70
$\bar{s}^{YZ}$	$(-3.3, 3.3) \times 10^{-11}$	42
$\bar{s}^{XX}-\bar{s}^{YY}$	$(-9.7, 10.1) \times 10^{-11}$	15
$\bar{s}^{XX}+\bar{s}^{YY}-2\bar{s}^{ZZ}$	$(-12.3, 12.2) \times 10^{-11}$	310

Shao PRL 2014

SME Post-fit analysis of experiments

Several papers on Lunar Laser Ranging, Gravity Probe B and binary pulsars :

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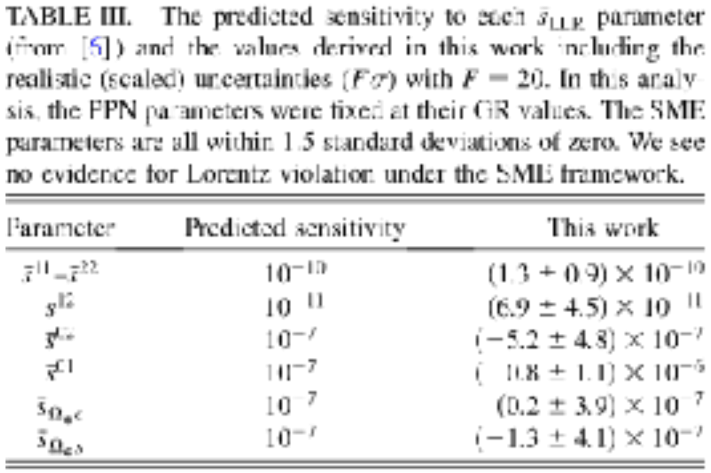
Shao PRD 2014

But can we speak about *Constraints* ?



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$\bar{s}^{12}$	10^{-11}	$(6.9 \pm 4.5) \times 10^{-11}$
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$\bar{s}^{01}$	10^{-7}	$(-0.8 \pm 1.1) \times 10^{-6}$
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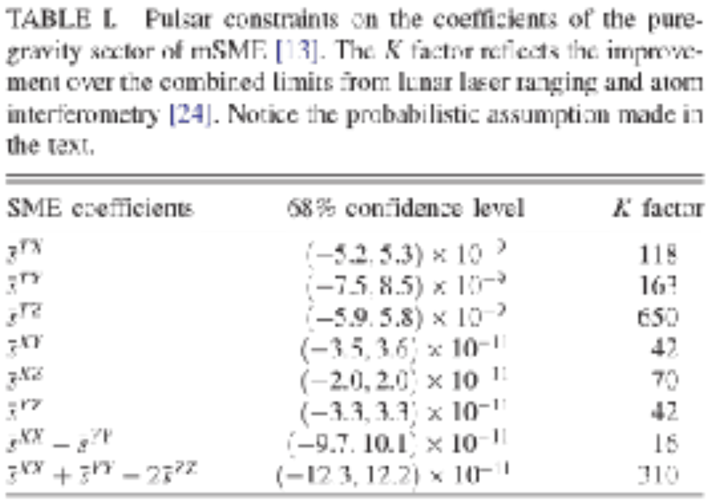
Battat et al. PRL 2007



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Shao PRL 2014

SME Post-fit analysis of experiments

Several papers on Lunar Laser Ranging, Gravity Probe B and binary pulsars :

$$|\bar{s}^{TT}| < 1.6 \times 10^{-5} \quad (95\% \text{ C.L.}),$$

Shao PRD 2014

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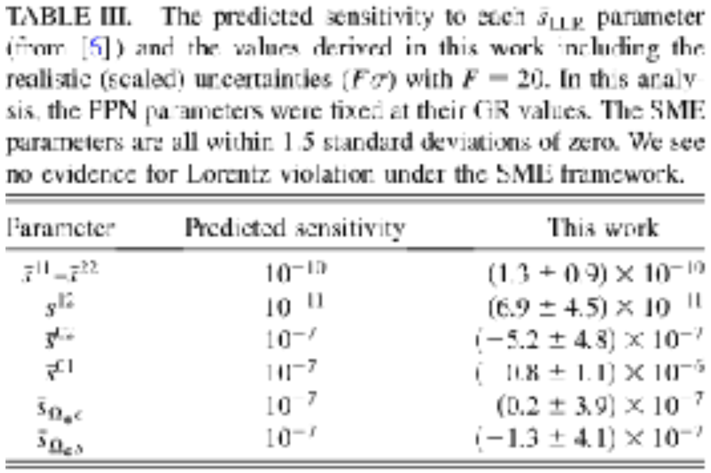

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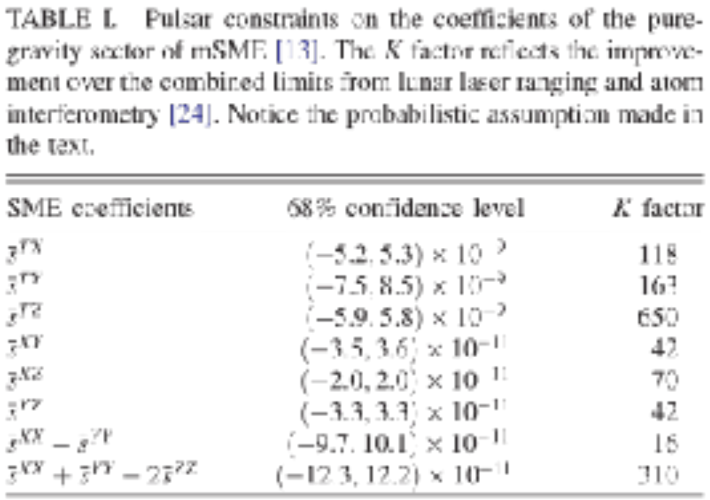
Battat et al. PRL 2007



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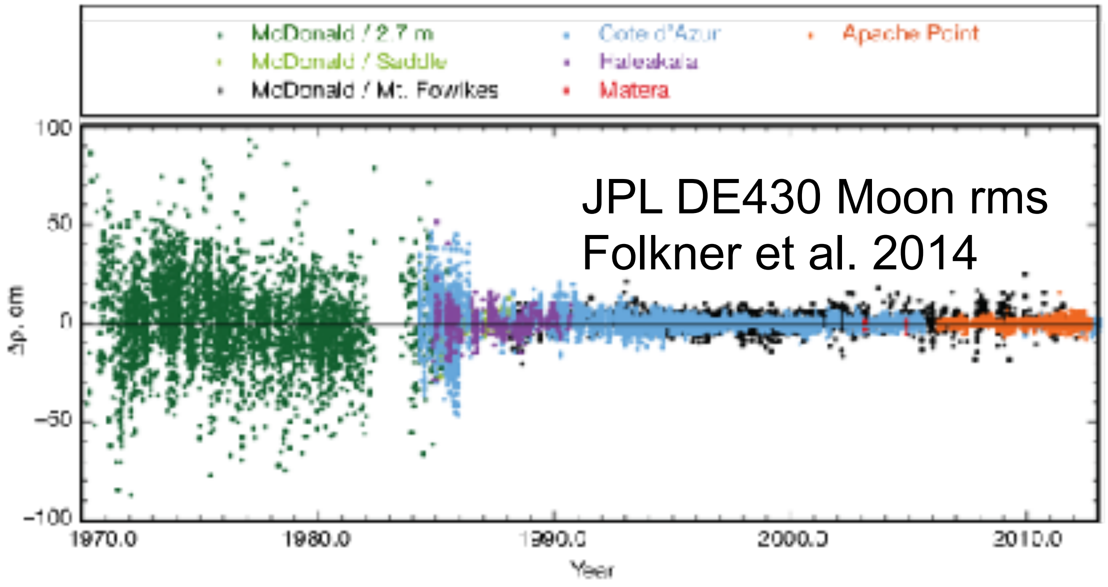
Shao PRL 2014



JPL DE430 Moon rms
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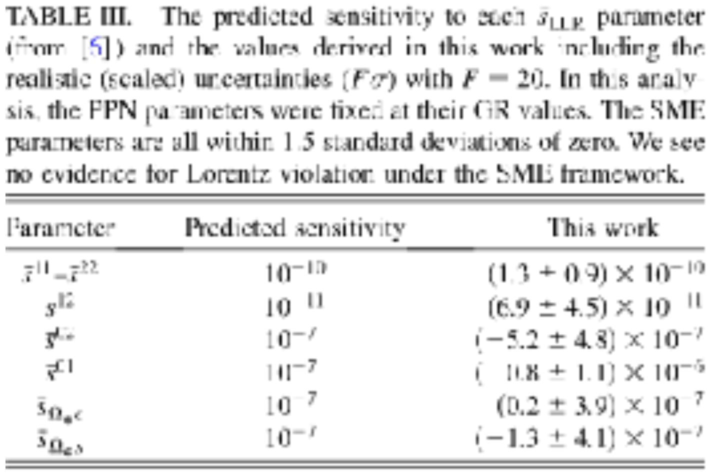

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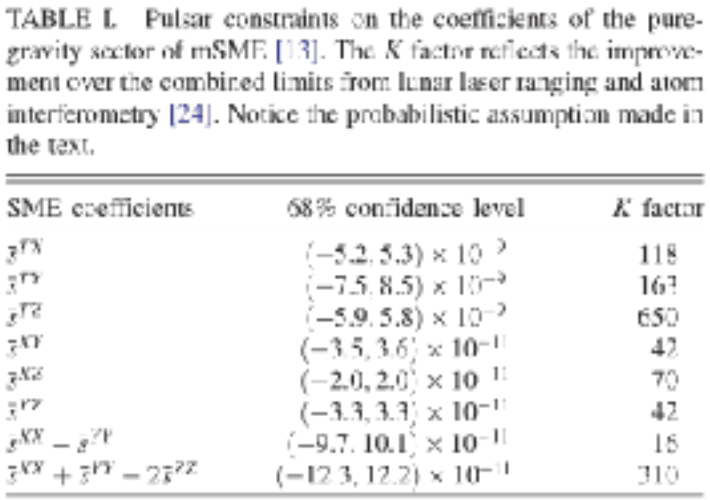
Battat et al. PRL 2007



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Shao PRL 2014

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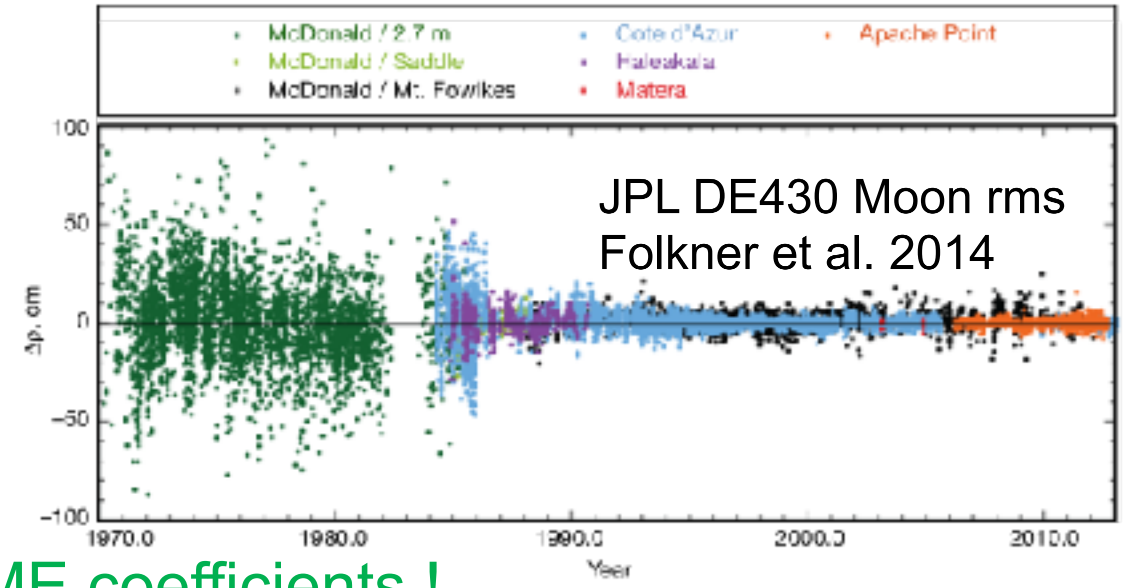

JPL DE430 Moon rms
Folkner et al. 2014

→ Over-estimation of SME coefficients !

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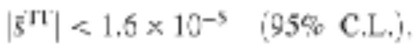

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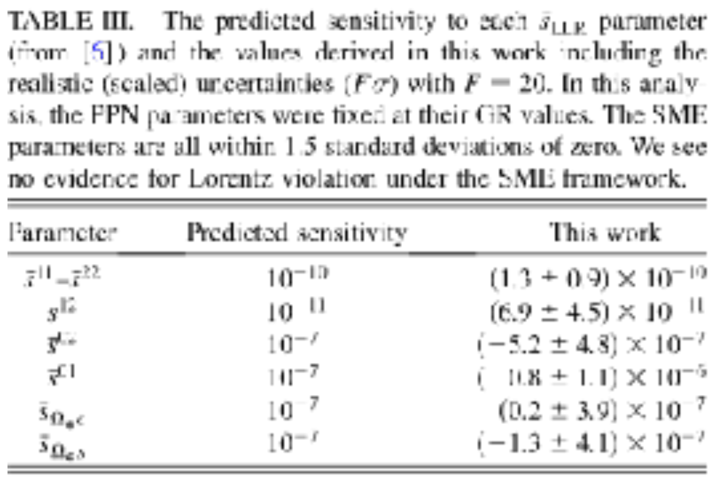

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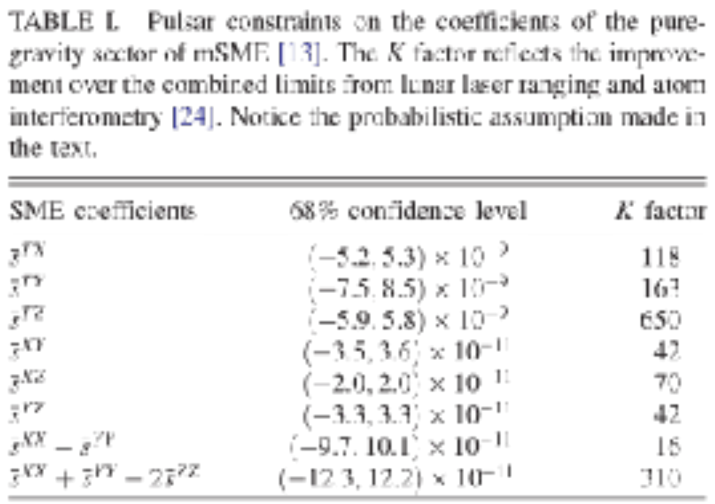

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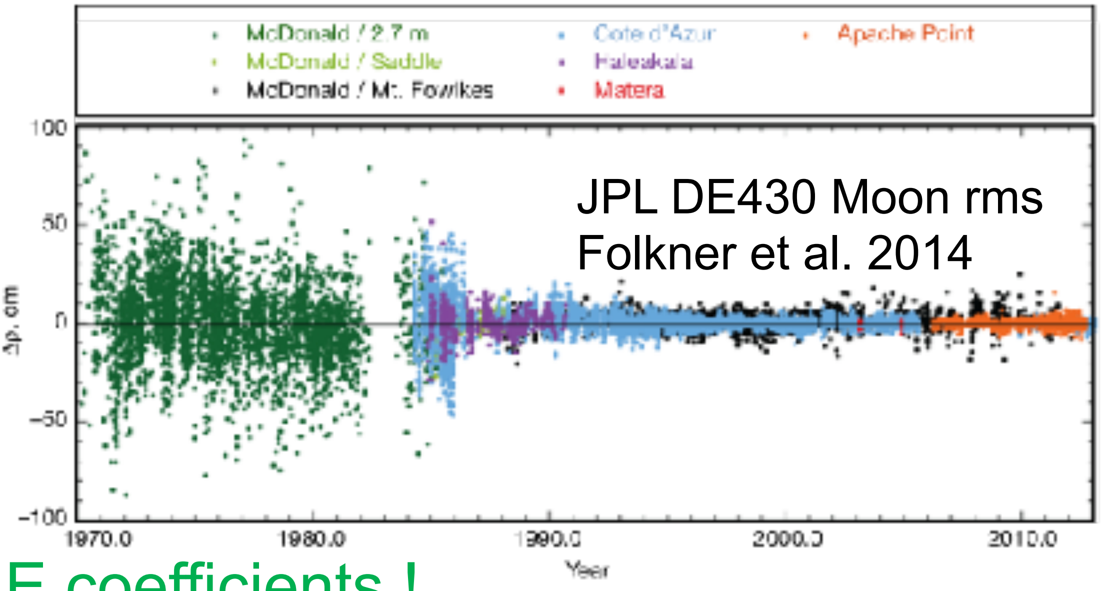

JPL DE430 Moon rms
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Up to now :

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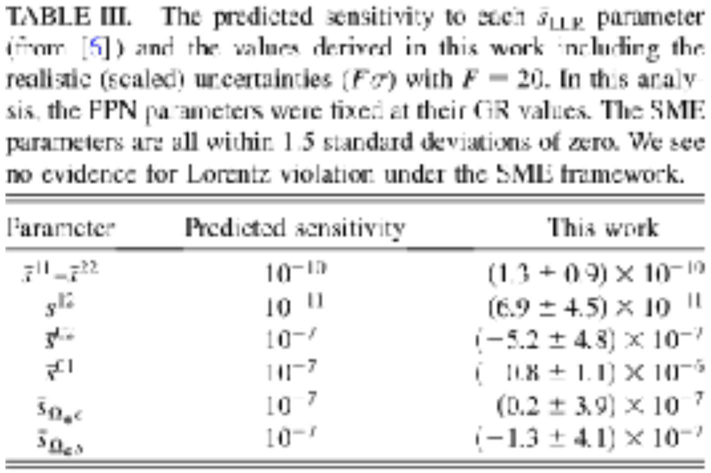

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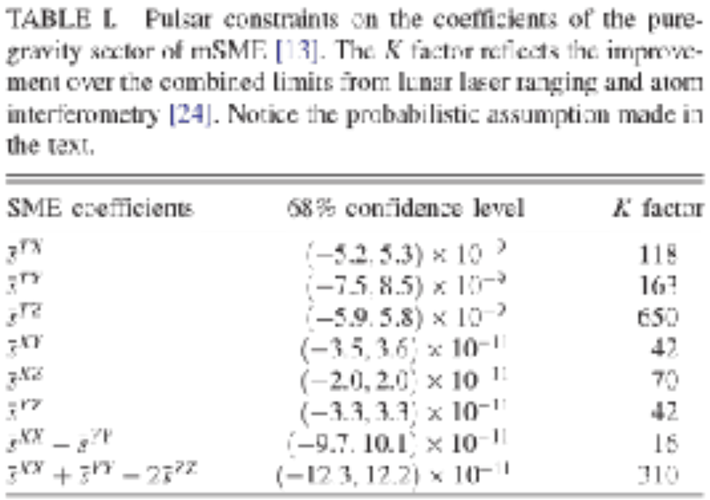

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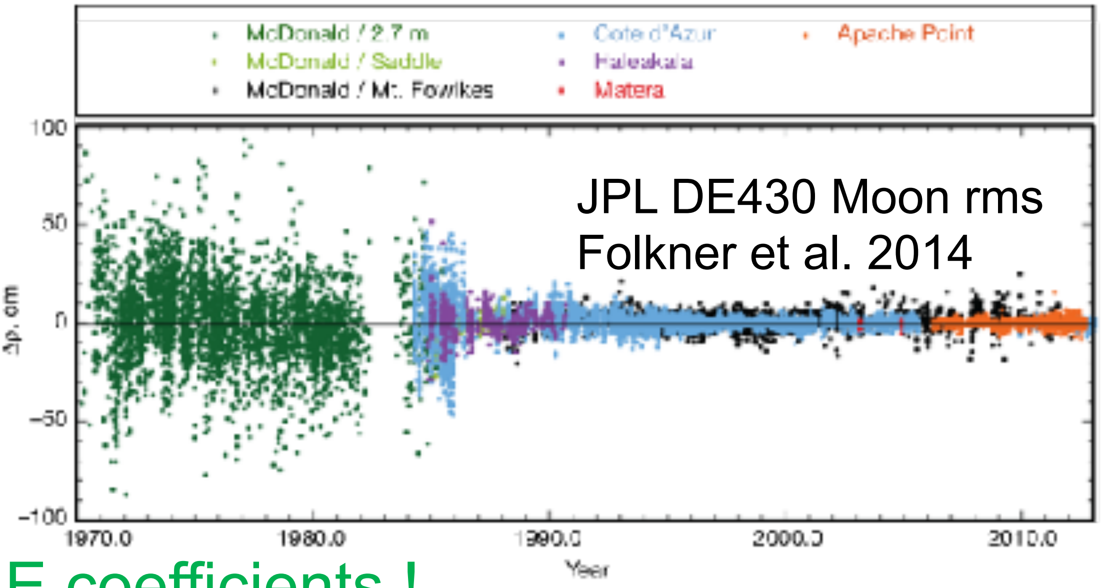

JPL DE430 Moon rms
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→ Over-estimation of SME coefficients !

Up to now :
- NO constraint on gravitational sector of SME
- but ONLY Upper Limit

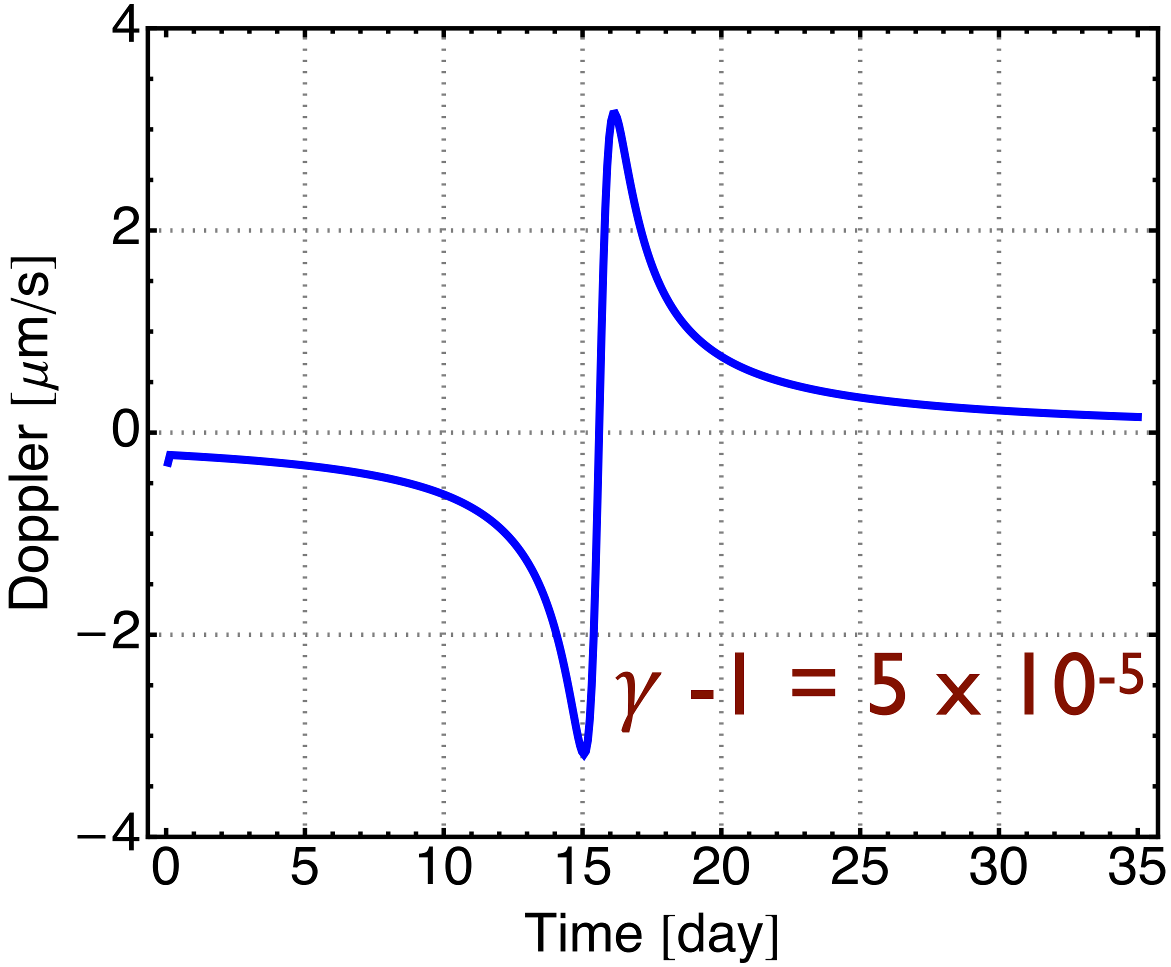
→ Pre-Process data, dynamical modeling & fit in a complete SME framework

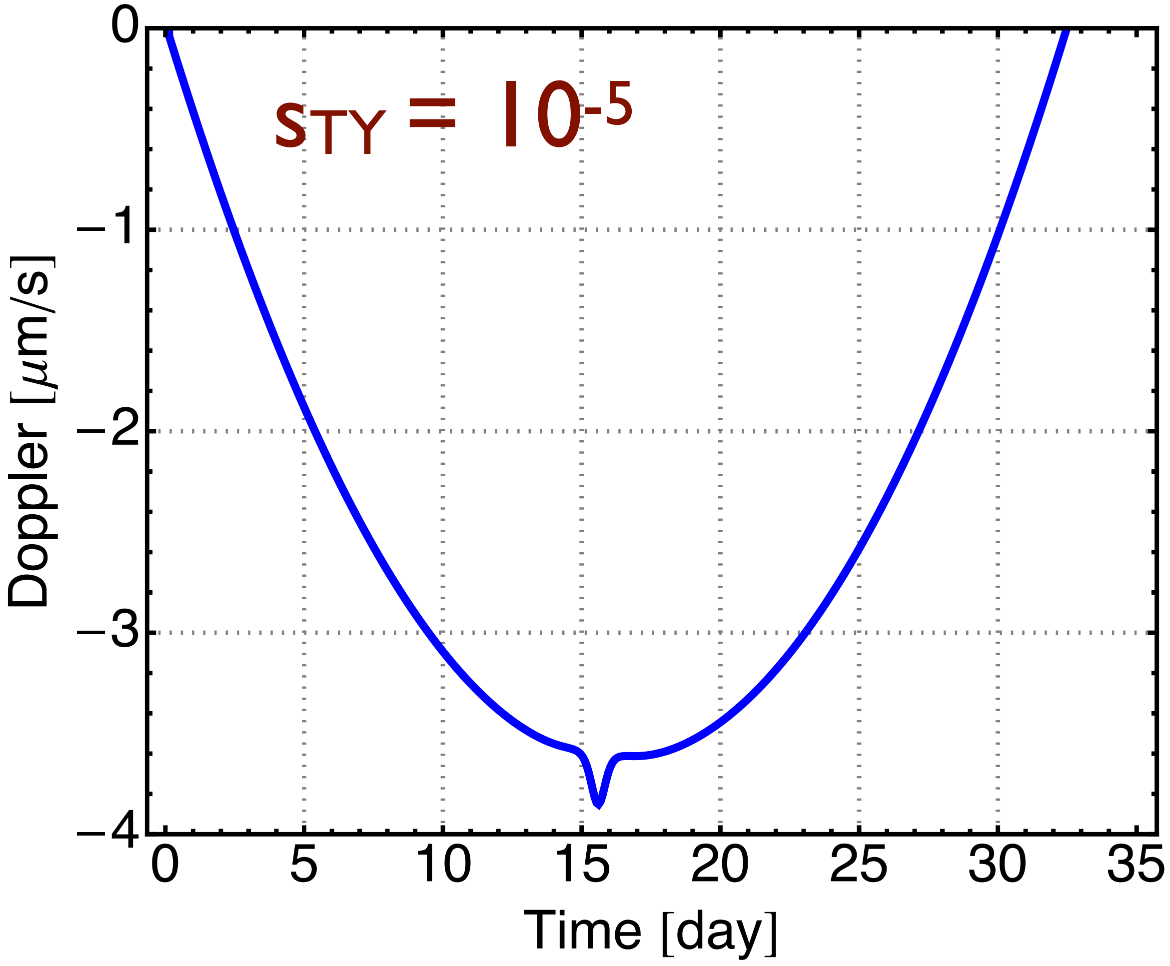
In 3 words : Our Ultimate Goal !

Numerical evaluation of the TTF in SME

- Numerical evaluation appropriate to evaluate effects due to alternative theories of gravitation
- Example: Doppler for 30 days of Cassini tracking between Jupiter and Saturn ("γ experiment")
- Effect of the γ PPN and of Standard Model Extension s_{TY} on Cassini Doppler

for SME, see Q. Bailey and A. Kostelecky, PRD 74, 045001, 2006





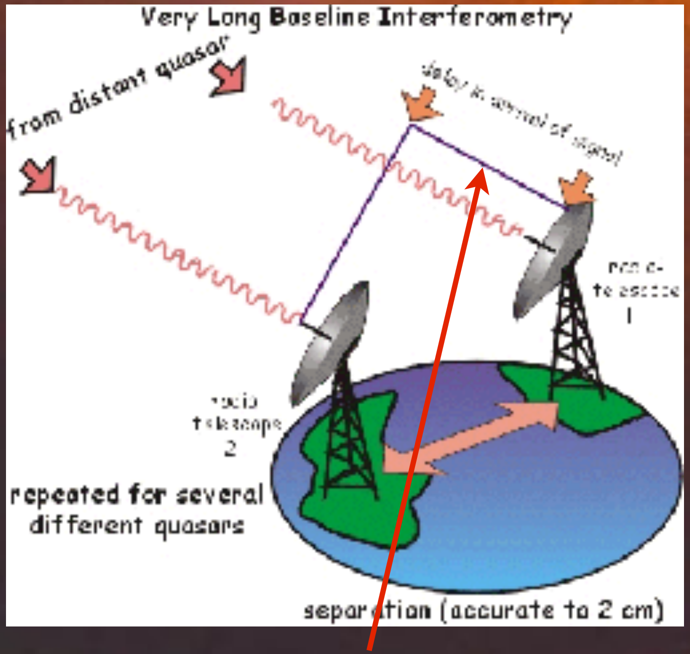
see A. Hees, et al, CQG 29, 235027, 2012 ; Hees et al., CLS 2014

Towards VLBI test of SME

Observations of quasars

1. Statically not moving
2. With 2 radiotelescopes, one are able to fix the Earth orientation with respect to quasars

Position kinematical position of the Earth in space



observable :
time delay between the reception of signal at the radiotelescopes

Modeling SME-VLBI delay & fit

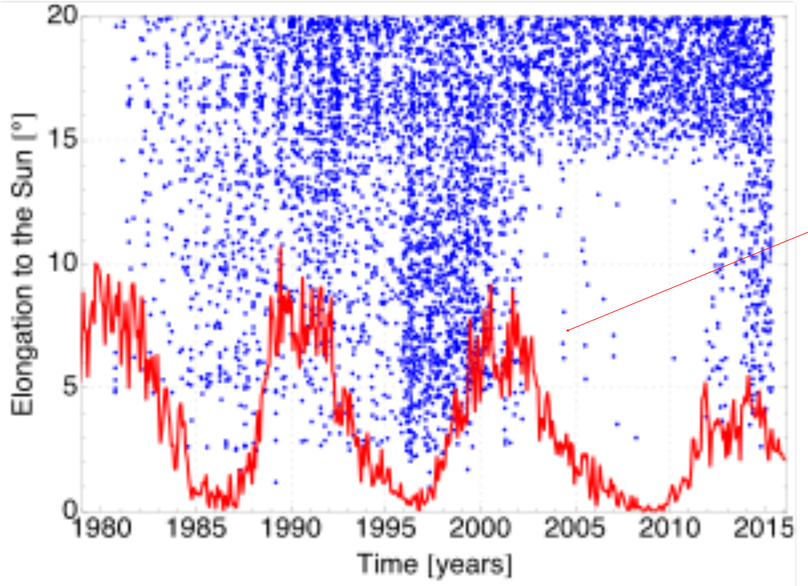
Lambert & CLPL, 2009 and 2011 : determination of PPN Gamma at the level of 10^{-4},
1 order of mag below Cassini **but strong statistics & robustess**

First, we derive the VLBI delay in SME from Bailey (2009) :

$$\Delta\tau_{(\text{grav})} = 2\frac{\widetilde{GM}}{c^3}(1-\frac{2}{3}\bar{s}^{TT})\ln\frac{r_1+\boldsymbol{k}.\boldsymbol{x}_1}{r_2+\boldsymbol{k}.\boldsymbol{x}_2}+\frac{2}{3}\frac{\widetilde{GM}}{c^3}\bar{s}^{TT}\,(\boldsymbol{n}_2.\boldsymbol{k}-\boldsymbol{n}_1.\boldsymbol{k})\,.$$

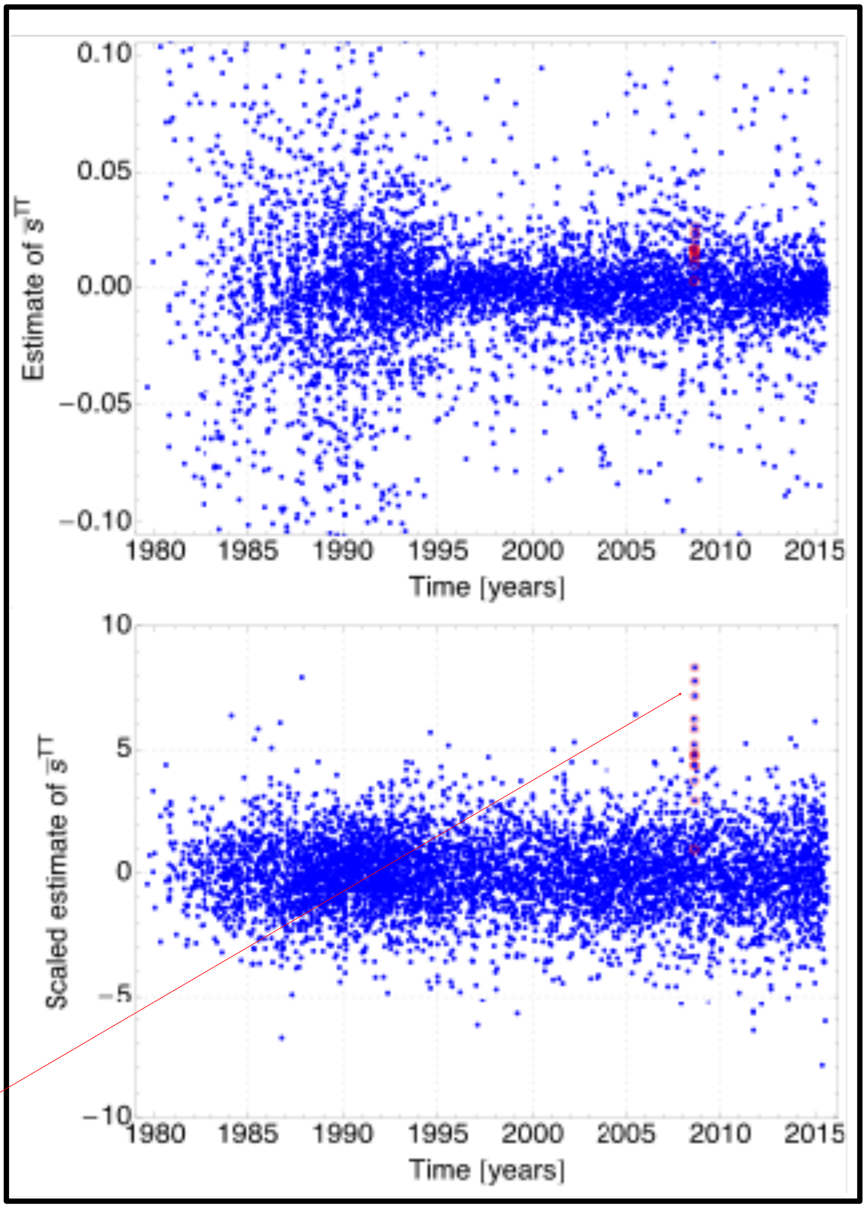
with $\boldsymbol{x}_{1/2}$ positions of stations and $r_{1/2}=|\boldsymbol{x}_{1/2}$ $\boldsymbol{n}_{1/2}=\frac{\boldsymbol{x}_{1/2}}{r_{1/2}}$

and $\boldsymbol{k}$ is the direction of the source.



- Modification of CALC with module USERPART. Test with post-fit analysis :

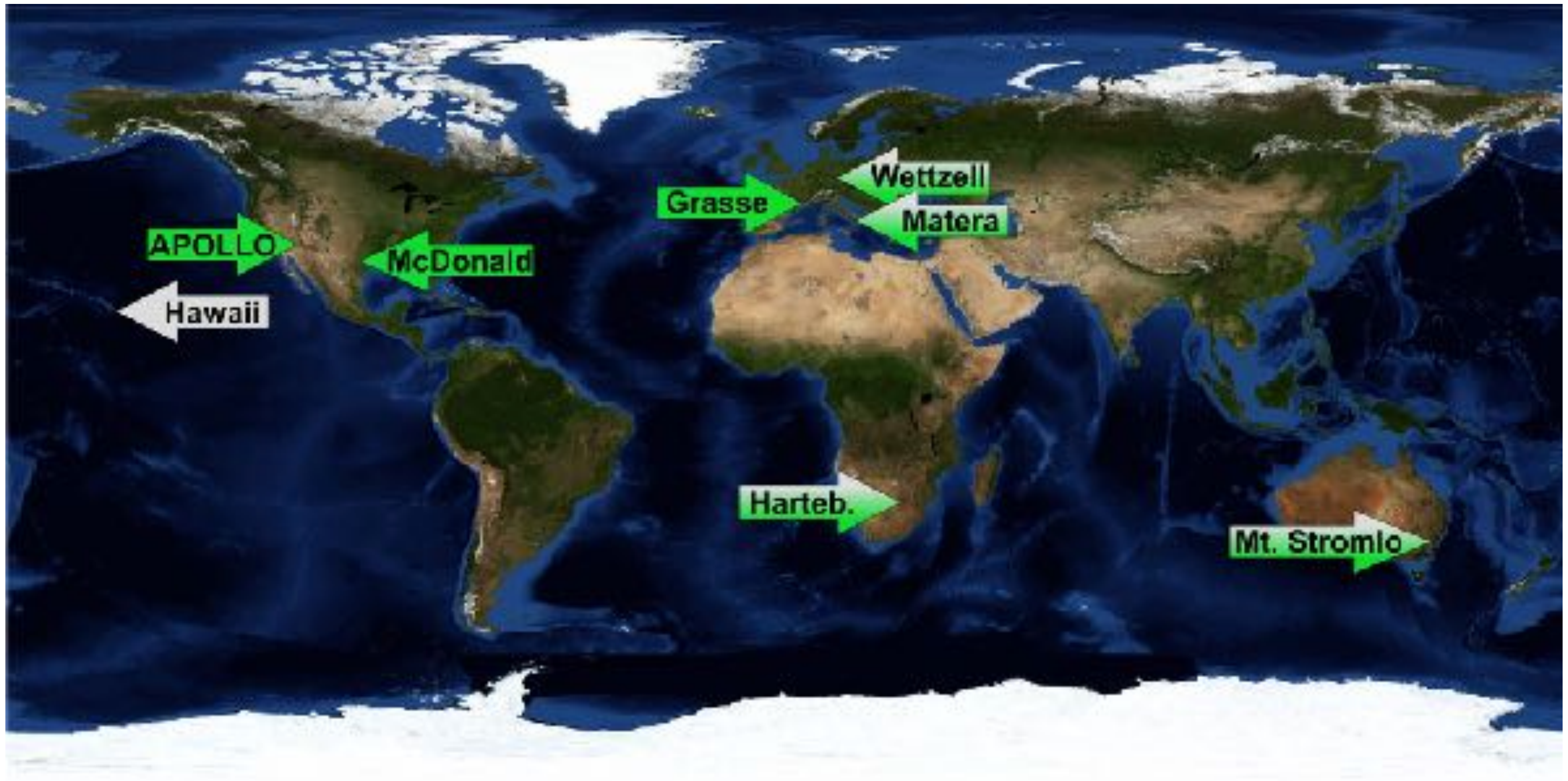
  $\bar{s}^{TT}=(-0.6\pm 2.1)\times 10^{-8}$
- 2 & 8 Ghz for solar activity
- 8 Ghz for SME analysis
- Systematics on CONT08 data but we kept them.



CLPL, Hees & Lambert, PRD 2016
arXiv:1604.01663

$\bar{s}^{TT}=(-5\pm 8)\times 10^{-5}$

International Laser Ranging Service (ILRS)



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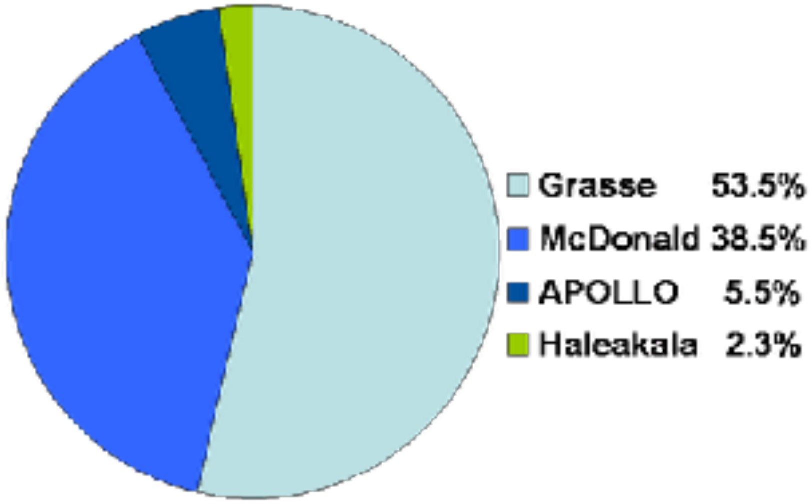


polac

POLAC @ SYRTE/obs. de Paris
Lead : S. Bouquillon

Official ILRS LLR data center

- 20721 normal points 08/1969 to 12/2013
- Prediction tool for LLR station
- Validation of LLR observations
- Analytical theory of Moon motion : ELP

SME and planetary motions

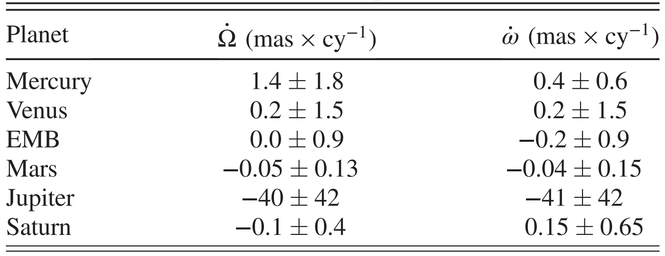
LLR + gravimetry

PHYSICAL REVIEW D **92,** 064049 (2015)

Testing Lorentz symmetry with planetary orbital dynamics

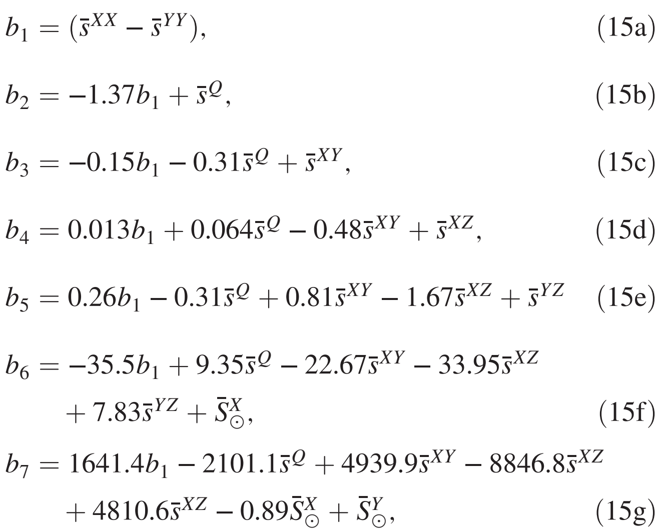
A. Hees,[1,*] Q. G. Bailey,[2] C. Le Poncin-Lafitte,[3] A. Bourgoin,[3] A. Rivoldini,[4] B. Lamine,[5]
F. Meynadier,[3] C. Guerlin,[6,3] and P. Wolf[3]

- Main advantage: decorrelation of the SME parameters
- Use of perihelion precession



Planet	$\dot{\Omega}$ (mas × cy^{-1})	$\dot{\omega}$ (mas × cy^{-1})
Mercury	1.4 ± 1.8	0.4 ± 0.6
Venus	0.2 ± 1.5	0.2 ± 1.5
EMB	0.0 ± 0.9	-0.2 ± 0.9
Mars	-0.05 ± 0.13	-0.04 ± 0.15
Jupiter	-40 ± 42	-41 ± 42
Saturn	-0.1 ± 0.4	0.15 ± 0.65

- 7 linear combinations determined



$$b_1 = (\bar{s}^{XX} - \bar{s}^{YY}), \qquad (15a)$$

$$b_2 = -1.37 b_1 + \bar{s}^{Q}, \qquad (15b)$$

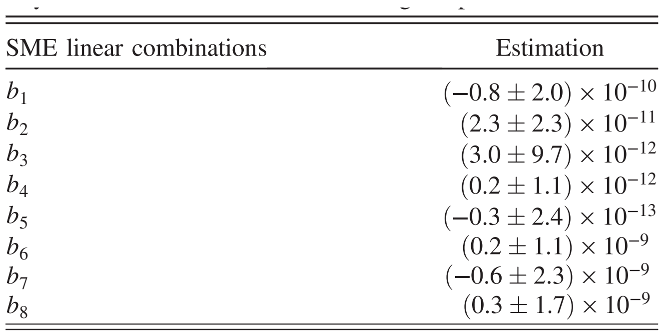
$$b_3 = -0.15 b_1 - 0.31 \bar{s}^{Q} + \bar{s}^{XY}, \qquad (15c)$$

$$b_4 = 0.013 b_1 + 0.064 \bar{s}^{Q} - 0.48 \bar{s}^{XY} + \bar{s}^{XZ}, \qquad (15d)$$

$$b_5 = 0.26 b_1 - 0.31 \bar{s}^{Q} + 0.81 \bar{s}^{XY} - 1.67 \bar{s}^{XZ} + \bar{s}^{YZ} \qquad (15e)$$

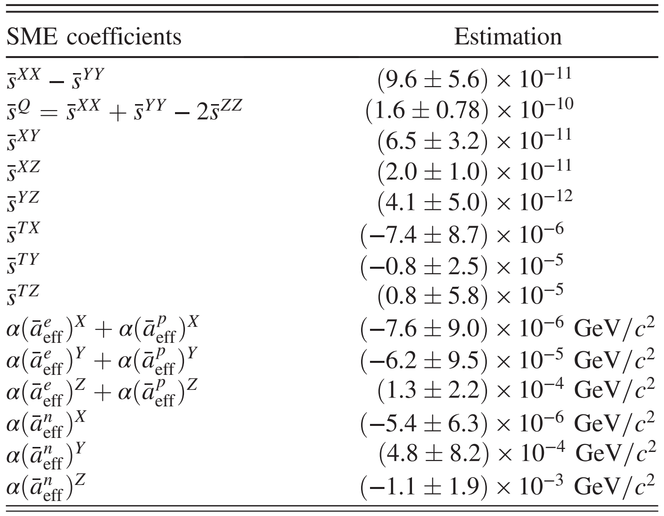
$$b_6 = -35.5 b_1 + 9.35 \bar{s}^{Q} - 22.67 \bar{s}^{XY} - 33.95 \bar{s}^{XZ} + 7.83 \bar{s}^{YZ} + \bar{S}^{X}_{\odot}, \qquad (15f)$$

$$b_7 = 1641.4 b_1 - 2101.1 \bar{s}^{Q} + 4939.9 \bar{s}^{XY} - 8846.8 \bar{s}^{XZ} + 4810.6 \bar{s}^{XZ} - 0.89 \bar{S}^{X}_{\odot} + \bar{S}^{Y}_{\odot}, \qquad (15g)$$



SME linear combinations	Estimation
b_1	$(-0.8 \pm 2.0) \times 10^{-10}$
b_2	$(2.3 \pm 2.3) \times 10^{-11}$
b_3	$(3.0 \pm 9.7) \times 10^{-12}$
b_4	$(0.2 \pm 1.1) \times 10^{-12}$
b_5	$(-0.3 \pm 2.4) \times 10^{-13}$
b_6	$(0.2 \pm 1.1) \times 10^{-9}$
b_7	$(-0.6 \pm 2.3) \times 10^{-9}$
b_8	$(0.3 \pm 1.7) \times 10^{-9}$

- Use of Lunar Laser Ranging and gravimetry



SME coefficients	Estimation
$\bar{s}^{XX} - \bar{s}^{YY}$	$(9.6 \pm 5.6) \times 10^{-11}$
$\bar{s}^{Q} = \bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	$(1.6 \pm 0.78) \times 10^{-10}$
$\bar{s}^{XY}$	$(6.5 \pm 3.2) \times 10^{-11}$
$\bar{s}^{XZ}$	$(2.0 \pm 1.0) \times 10^{-11}$
$\bar{s}^{YZ}$	$(4.1 \pm 5.0) \times 10^{-12}$
$\bar{s}^{TX}$	$(-7.4 \pm 8.7) \times 10^{-6}$
$\bar{s}^{TY}$	$(-0.8 \pm 2.5) \times 10^{-5}$
$\bar{s}^{TZ}$	$(0.8 \pm 5.8) \times 10^{-5}$
$\alpha(\bar{a}^{e}_{\mathrm{eff}})^{X} + \alpha(\bar{a}^{p}_{\mathrm{eff}})^{X}$	$(-7.6 \pm 9.0) \times 10^{-6}\ \mathrm{GeV}/c^2$
$\alpha(\bar{a}^{e}_{\mathrm{eff}})^{Y} + \alpha(\bar{a}^{p}_{\mathrm{eff}})^{Y}$	$(-6.2 \pm 9.5) \times 10^{-5}\ \mathrm{GeV}/c^2$
$\alpha(\bar{a}^{e}_{\mathrm{eff}})^{Z} + \alpha(\bar{a}^{p}_{\mathrm{eff}})^{Z}$	$(1.3 \pm 2.2) \times 10^{-4}\ \mathrm{GeV}/c^2$
$\alpha(\bar{a}^{n}_{\mathrm{eff}})^{X}$	$(-5.4 \pm 6.3) \times 10^{-6}\ \mathrm{GeV}/c^2$
$\alpha(\bar{a}^{n}_{\mathrm{eff}})^{Y}$	$(4.8 \pm 8.2) \times 10^{-4}\ \mathrm{GeV}/c^2$
$\alpha(\bar{a}^{n}_{\mathrm{eff}})^{Z}$	$(-1.1 \pm 1.9) \times 10^{-3}\ \mathrm{GeV}/c^2$

Fitted in residuals.
Different experiments = different linear combination
All together = find constraint on single coefficient…

Gaia is very powerful to constrain SME

- Main advantage: decorrelation of the 9 SME parameters because of the variety of orbital parameters (not feasible with planets)

SME Parameter	1σ - 5yr	1σ - 10yr
$\bar{s}^{XX} - \bar{s}^{YY}$	3.7×10^{-12}	6.5×10^{-12}
$\bar{s}^{XX} + \bar{s}^{YY} - \bar{s}^{ZZ}$	6.4×10^{-12}	2.1×10^{-12}
$\bar{s}^{XY}$	1.6×10^{-12}	7.0×10^{-13}
$\bar{s}^{XZ}$	9.2×10^{-13}	3.7×10^{-13}
$\bar{s}^{YZ}$	1.7×10^{-12}	5.8×10^{-13}
$\bar{s}^{TX}$	5.6×10^{-9}	1.1×10^{-9}
$\bar{s}^{TY}$	8.8×10^{-9}	2.0×10^{-9}
$\bar{s}^{TZ}$	1.6×10^{-8}	4.0×10^{-9}

1 order of magnitude improvement wrt current constraints

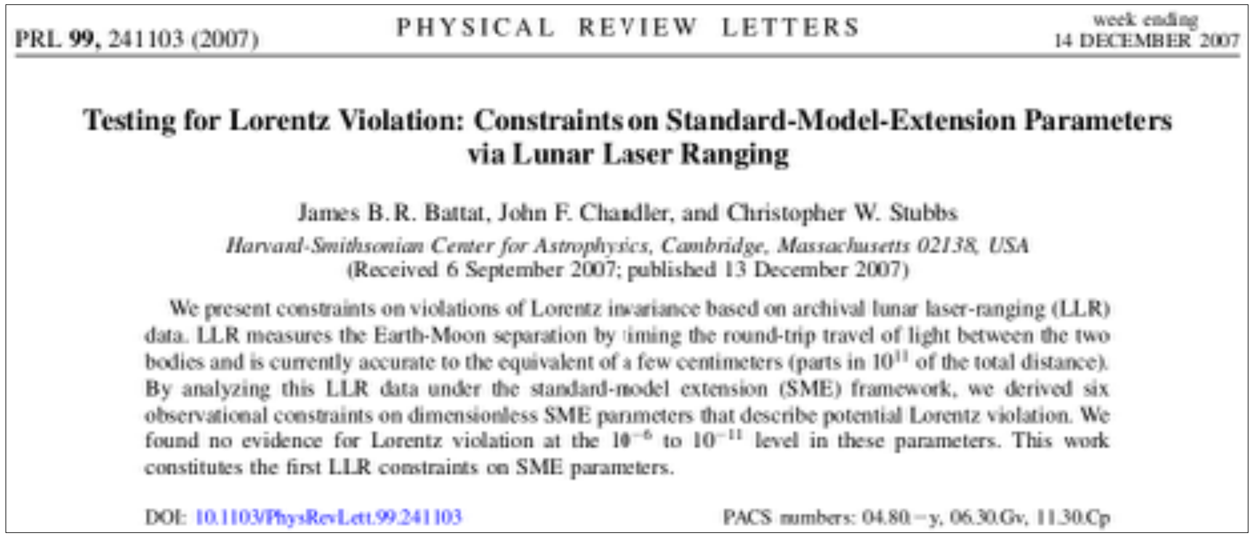
- To be extended to take into account violations of the Einstein Equivalence Principle (gravity-matter Lorentz violating terms)

Extremely promising results !

see Hees et al, Universe, 2016

LLR and SME

State of the art



PRL **99,** 241103 (2007) PHYSICAL REVIEW LETTERS week ending 14 DECEMBER 2007

Testing for Lorentz Violation: Constraints on Standard-Model-Extension Parameters via Lunar Laser Ranging

James B. R. Battat, John F. Chandler, and Christopher W. Stubbs

Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA

(Received 6 September 2007; published 13 December 2007)

We present constraints on violations of Lorentz invariance based on archival lunar laser-ranging (LLR) data. LLR measures the Earth-Moon separation by timing the round-trip travel of light between the two bodies and is currently accurate to the equivalent of a few centimeters (parts in 10^{11} of the total distance). By analyzing this LLR data under the standard-model extension (SME) framework, we derived six observational constraints on dimensionless SME parameters that describe potential Lorentz violation. We found no evidence for Lorentz violation at the 10^{-6} to 10^{-11} level in these parameters. This work constitutes the first LLR constraints on SME parameters.

DOI: 10.1103/PhysRevLett.99.241103 PACS numbers: 04.80.−y, 06.30.Gv, 11.30.Cp

14401 normal points spanning over 09/1969 to 12/2003.

Post-fit LLR analysis:

- Looked for analytical signals derived in Bailey *et. al.*, 2006.
- Planetary Ephemeris Program (PEP) developed at MIT to re-iterate => cross-correlation possible

Provide 6 *SME coefficient estimates* combinations at the level 10^{-6} and 10^{-11}.
Realistic error $\sigma_r = F\sigma$ with $F = 20$, from PPN analysis.

$\alpha = cste$ and $\beta = cste$. From observation $T = 2\pi/\dot{\alpha} = 2\pi/\dot{\beta} = 18, 6\ y$.
Analytic solution only accounting for short periodic terms. Only available for few years time-span.

Least-square fit, estimating only SME coefficients. No correlations taken into account with others global parameters, only cross-correlation.

$$\delta r_{SME}(t) = A_{SME}\cos\left(2\omega t + 2\theta\right) + B_{SME}\sin\left(2\omega t + 2\theta\right)$$

SME oscillating signatures at the same frequencies than natural frequencies :

Lunar potential
2d degree !

$$\delta r_{2\omega,2\theta}(t) = \left[A_{20} + A_{22}\right]\cos\left(2\omega t + 2\theta\right) + B_{22}\sin\left(2\omega t + 2\theta\right)$$

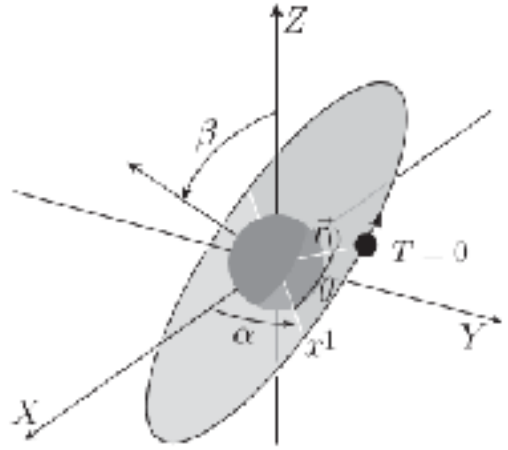


FIG. 2. Lunar orbital parameters: here the Earth is shown translated to the center of the Sun-centered coordinate system. The lunar orbit is described by r_0, the mean distance between the Earth and Moon, e (not labeled) the eccentricity of the lunar orbit, α, the longitude of the ascending node, β, the angle between the normal to the lunar orbital plane and the normal to the Earth's equatorial plane, and θ, the angle, along the Lunar orbit, subtended by the ascending node line and the position of the Moon at $t = 0$. Reprinted figure with permission from [6]. Copyright 2006 by the American Physical Society.

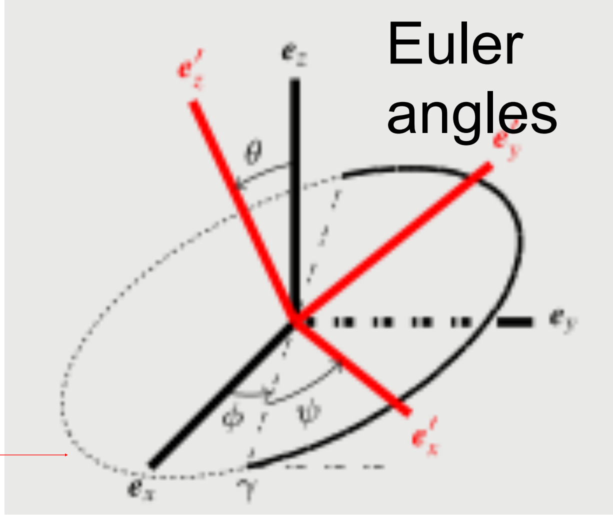
ELPN, SME lunar ephemeride

Dynamical modeling from scratch :

- Integrate the motion of Solar System bodies
  - Newtonian point-mass interactions.
  - Figure potential of bodies :
    - Orientation of bodies,
    - J_2 of the Sun,
    - J_2, J_3, J_4 and J_5 of the Earth,
    - Degree 2, 3, 4 and 5 of the Moon.
  - Tidal and spin effects :
    - Dissipation inside anelastic bodies,
    - Time-lag of degree 2 with RK4.
  - Relativistic point-mass interactions :
    - Solar system barycentre,
    - Integrate the time scale correction (in pure GR).
    - SME correction for Earth-Moon system only
  - Lunar librations :
    - Momentums due to punctual (5*th* degree) and extended (2*th* degree) bodies,
    - Geodetic precession effect,
    - Fluid lunar core,
    - Friction between solid mantle and fluid core.
- Integrate partials at the same time than forces and momentums.

ODE system of 6000 equations

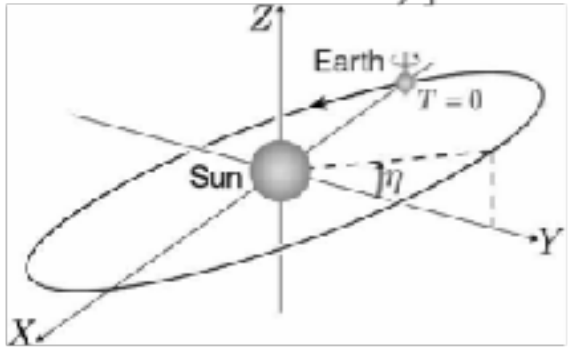
ODEX, Everheart
quadruple precision = 10^{-34}
6 CPU days of a solution !



Pre-processing of LLR normal points :
Update of the CAROLL software

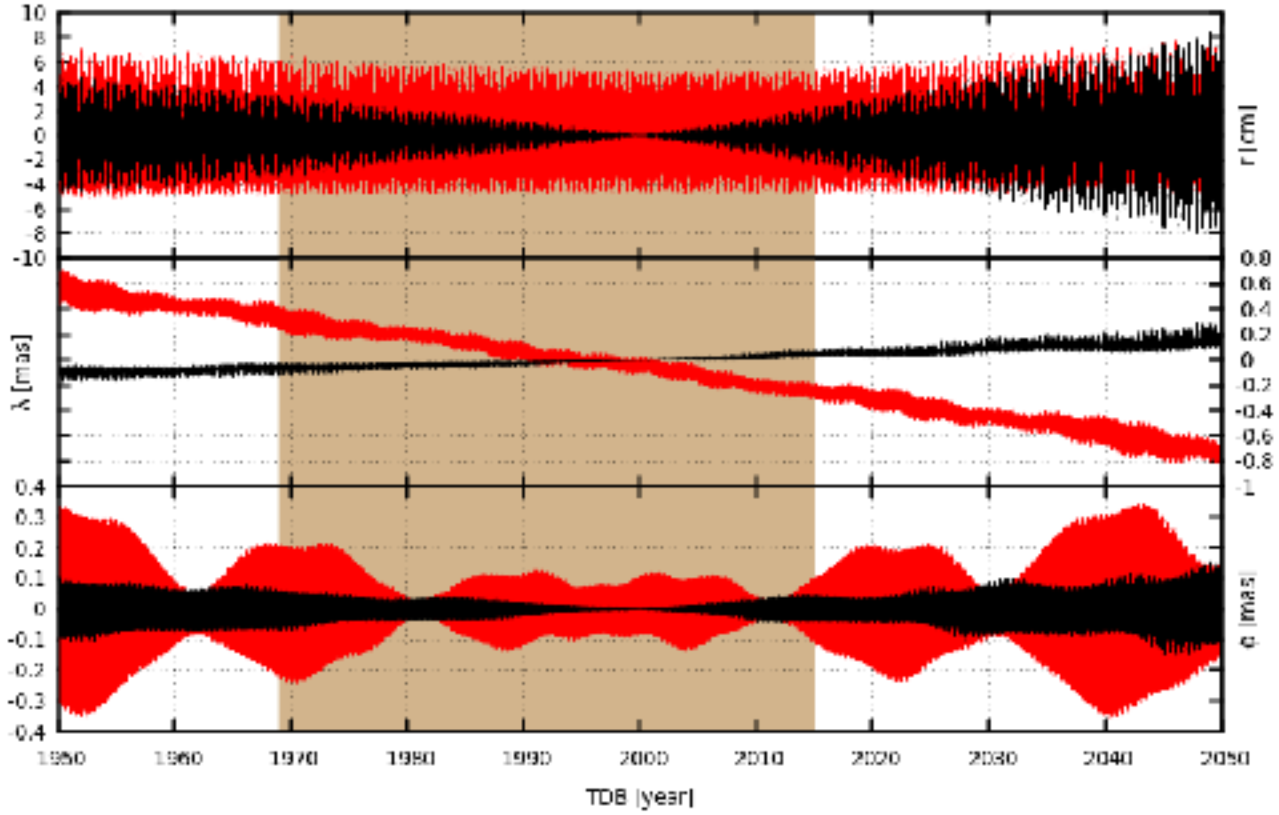
- Tchebychev polynomials of the solution and partials.
- LLR analysis in SME framework by chi-square fitting

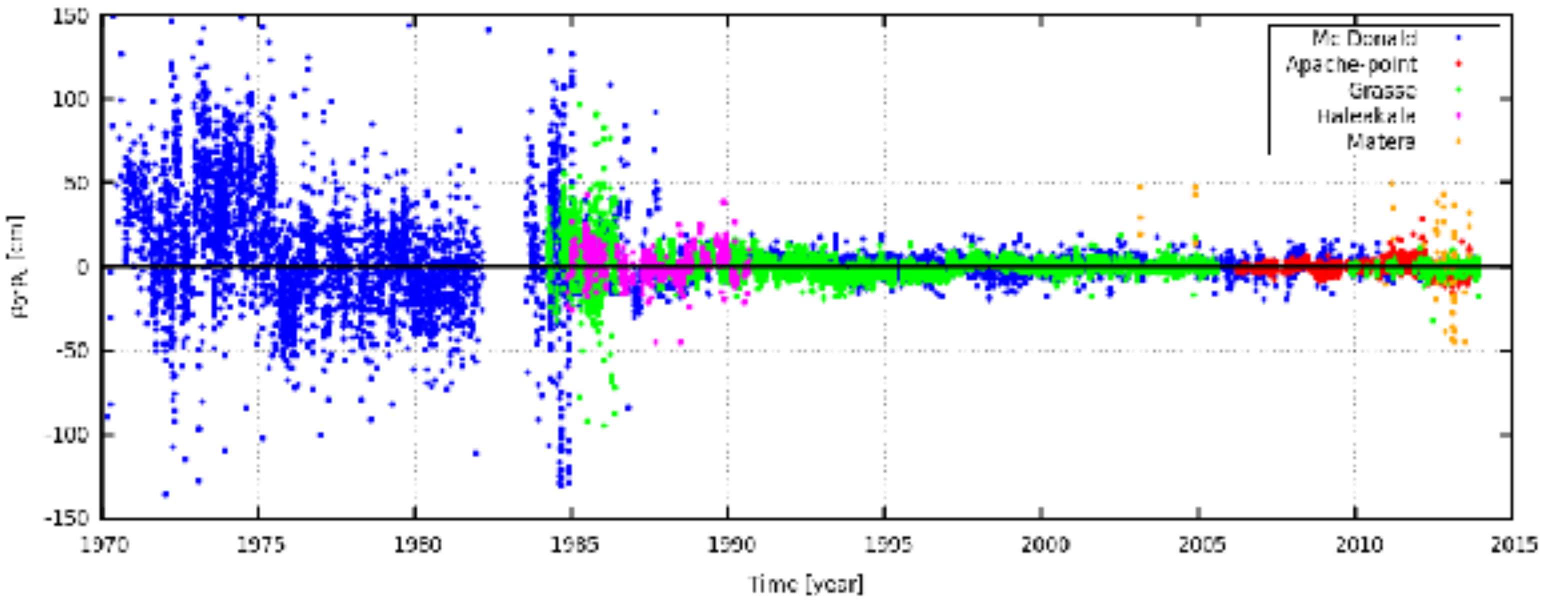
$$a^J_{\rm LV} = \frac{\bar{G}M}{r^3}\Big[\bar{s}^{JK}_t r^K - \frac{3}{2}\bar{s}^{KL}_t \hat{r}^K \hat{r}^L r^J + 3\bar{s}^{TK}\hat{V}^K r^J - \bar{s}^{TJ}\hat{V}^K r^K - \bar{s}^{TK}\hat{V}^J r^K + 3\bar{s}^{TL}\hat{V}^K \hat{r}^K \hat{r}^L r^J + 2\frac{\delta m}{M}\left(\bar{s}^{TK}\hat{v}^K r^J - \bar{s}^{TJ}\hat{v}^K r^K\right)\Big],$$



ELPN vs JPL-DE430

We compute difference pre (*black curve*) and post-fit (*red curve*) between DE430 and ELPN over the orbit of the Moon.





LLR Stations	Years	NP	*wrms* (*cm*)
Mc Donald 270*cm*	1969-1985	3480	29.3
Mc Donald MLRS1	1983-1988	584	43.6
Mc Donald MLRS2	1988-2014	3100	4.5
Grasse (Rubis)	1984-1986	1154	14.7
Grasse (Yag)	1987-2005	9294	3.4
Grasse (MeO)	2009-2014	966	1.2
Haleakala	1984-1990	735	7.4
Matera	2003-2014	79	9.3
Apache Point	2006-2014	1787	3.5

Our SME analysis

see Bourgoin et *al.*, PRL, 2016

SME parameters considered :

$$\begin{aligned} \bar{s}^A &= \bar{s}^{XX} - \bar{s}^{YY} & \bar{s}^B &= \bar{s}^{XX} + \bar{s}^{YY} - 2\ \bar{s}^{ZZ} \\ \bar{s}^C &= \bar{s}^{TY} + 0.43\ \bar{s}^{TZ} & \bar{s}^D &= \bar{s}^B - 0.045 \bar{s}^{YZ}. \\ & \bar{s}^{TX}, \bar{s}^{XY}, \bar{s}^{XZ}. \end{aligned}$$

$\bar{s}^A$ and $\bar{s}^B$

Enforce the traceless condition

Our SME analysis

see Bourgoin et *al.*, PRL, 2016

SME parameters considered :

$$\bar{s}^A = \bar{s}^{XX} - \bar{s}^{YY} \qquad \bar{s}^B = \bar{s}^{XX} + \bar{s}^{YY} - 2\,\bar{s}^{ZZ}$$

$$\bar{s}^C = \bar{s}^{TY} + 0.43\,\bar{s}^{TZ} \qquad \bar{s}^D = \bar{s}^B - 0.045\bar{s}^{YZ}.$$

$$\bar{s}^{TX}, \bar{s}^{XY}, \bar{s}^{XZ}.$$

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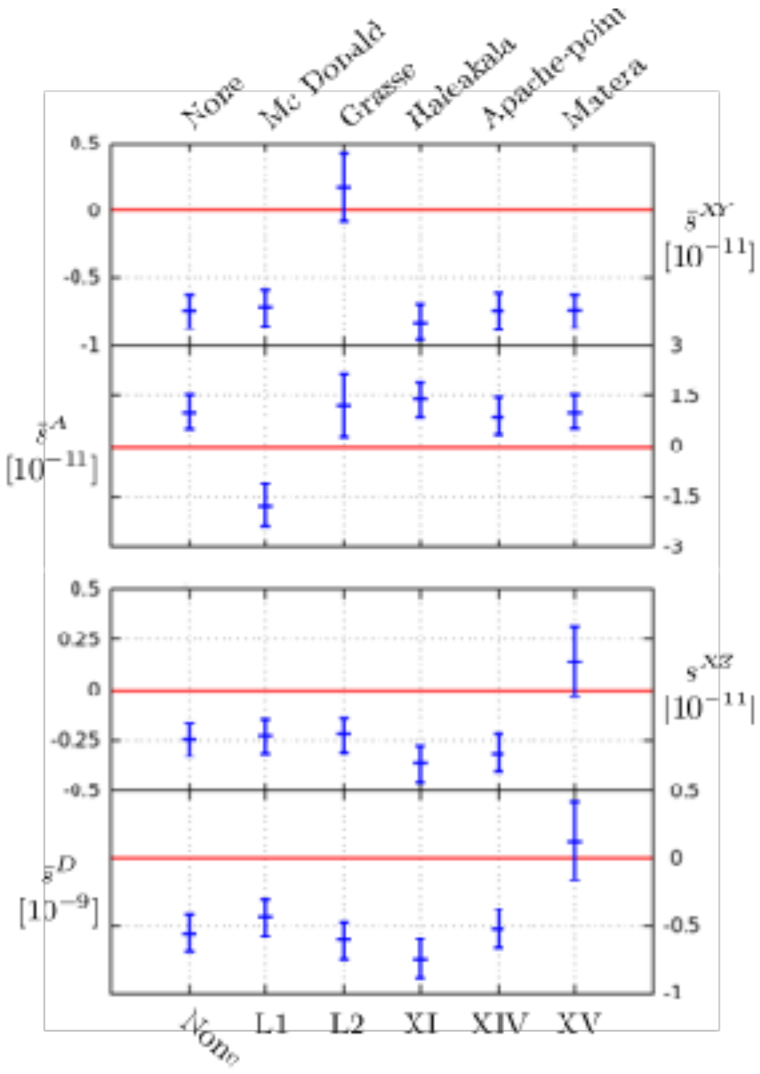

Study of systematics, *i.e.* split data set !

Our SME analysis

see Bourgoin et *al.*, PRL, 2016

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$$\bar{s}^A = \bar{s}^{XX} - \bar{s}^{YY} \qquad \bar{s}^B = \bar{s}^{XX} + \bar{s}^{YY} - 2\,\bar{s}^{ZZ}$$

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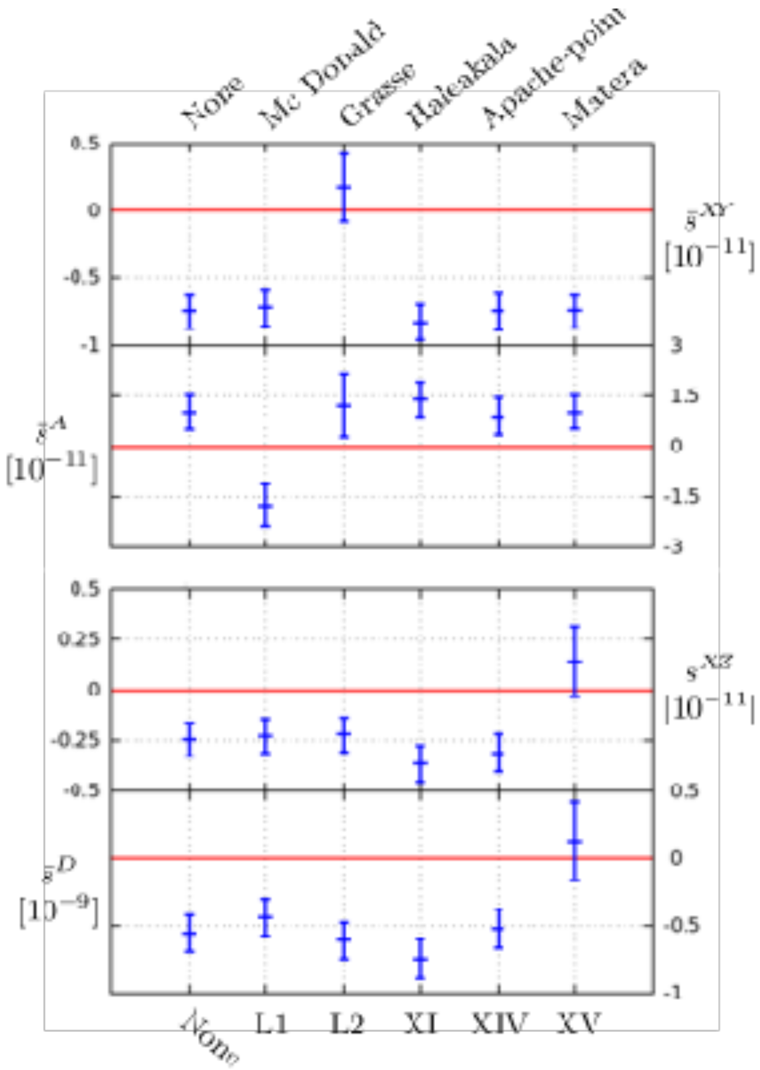

Variation of SME sigma with data-set

Sigma over-estimated. Need to find a realistic scale factor F (not from PPN !)

Study of systematics, *i.e.* split data set !

Our SME analysis

see Bourgoin et *al.*, PRL, 2016

SME parameters considered :

$$\bar{s}^A = \bar{s}^{XX} - \bar{s}^{YY} \qquad \bar{s}^B = \bar{s}^{XX} + \bar{s}^{YY} - 2\ \bar{s}^{ZZ}$$
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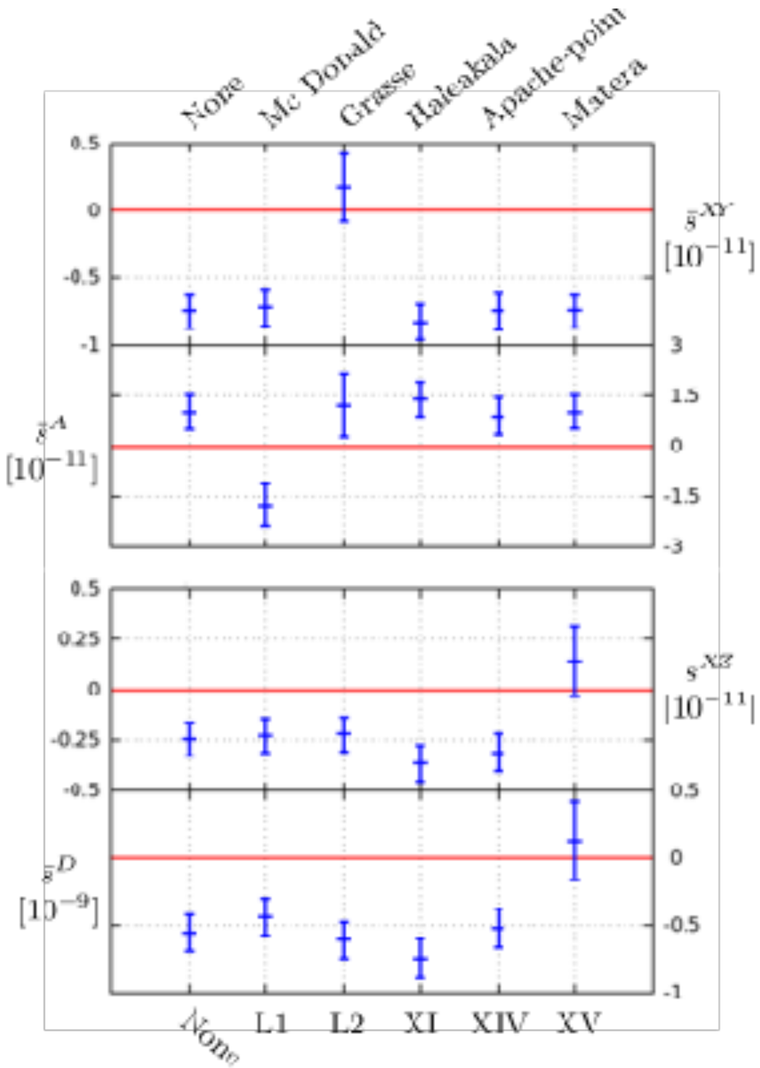

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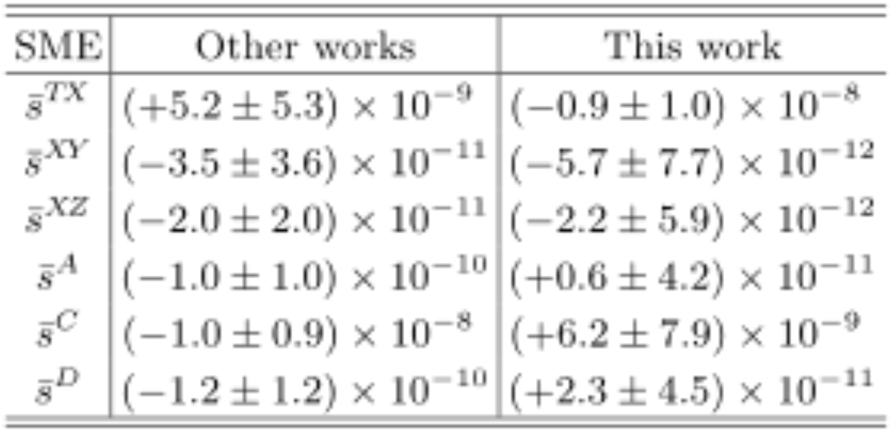
Jackknife resampling method allows to estimate systematics uncertainties :



SME	Other works	This work
$\bar{s}^{TX}$	$(+5.2 \pm 5.3) \times 10^{-9}$	$(-0.9 \pm 1.0) \times 10^{-8}$
$\bar{s}^{XY}$	$(-3.5 \pm 3.6) \times 10^{-11}$	$(-5.7 \pm 7.7) \times 10^{-12}$
$\bar{s}^{XZ}$	$(-2.0 \pm 2.0) \times 10^{-11}$	$(-2.2 \pm 5.9) \times 10^{-12}$
$\bar{s}^A$	$(-1.0 \pm 1.0) \times 10^{-10}$	$(+0.6 \pm 4.2) \times 10^{-11}$
$\bar{s}^C$	$(-1.0 \pm 0.9) \times 10^{-8}$	$(+6.2 \pm 7.9) \times 10^{-9}$
$\bar{s}^D$	$(-1.2 \pm 1.2) \times 10^{-10}$	$(+2.3 \pm 4.5) \times 10^{-11}$

Our SME analysis

see Bourgoin et *al.*, PRL, 2016

SME parameters considered :

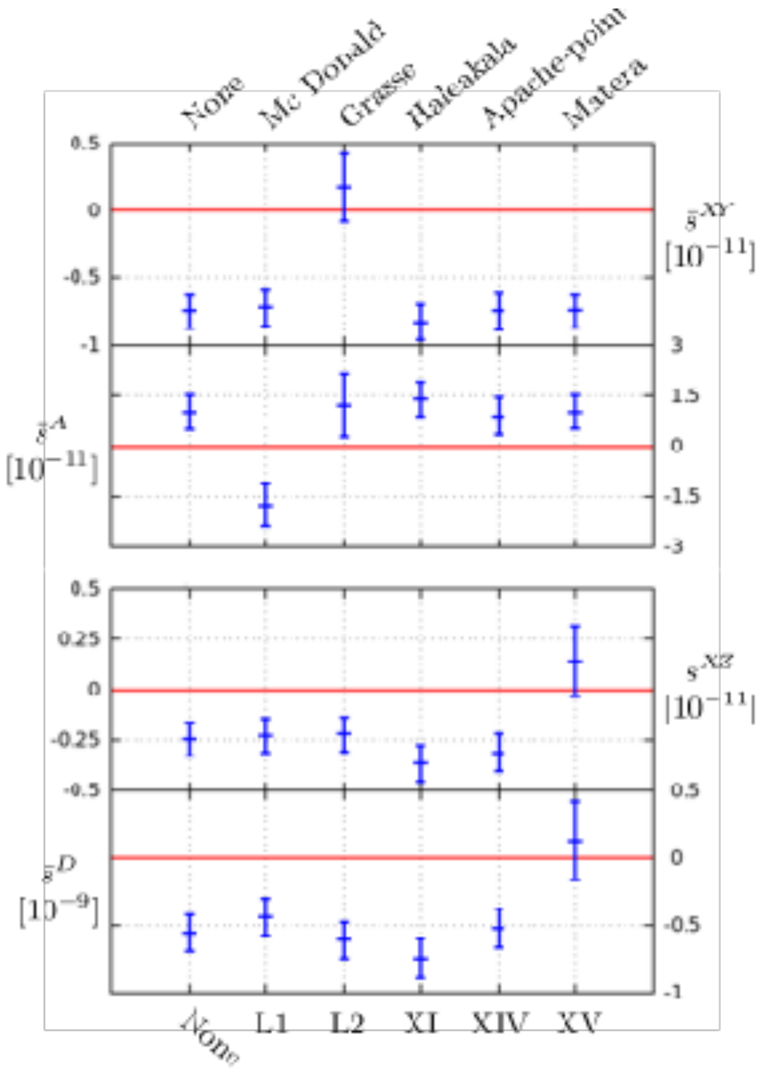
$$\bar{s}^A = \bar{s}^{XX} - \bar{s}^{YY} \qquad \bar{s}^B = \bar{s}^{XX} + \bar{s}^{YY} - 2\ \bar{s}^{ZZ}$$

$$\bar{s}^C = \bar{s}^{TY} + 0.43\ \bar{s}^{TZ} \qquad \bar{s}^D = \bar{s}^B - 0.045\bar{s}^{YZ}.$$

$$\bar{s}^{TX}, \bar{s}^{XY}, \bar{s}^{XZ}.$$

$\bar{s}^A$ and $\bar{s}^B$

Enforce the traceless condition



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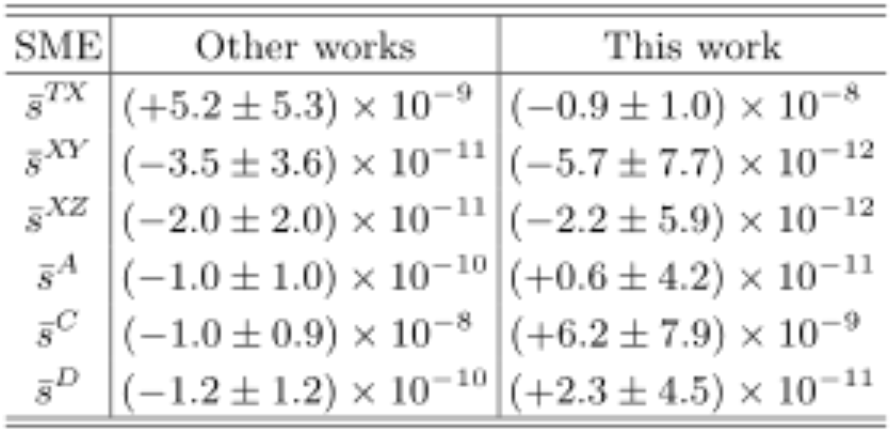

SME	Other works	This work
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$\bar{s}^D$	$(-1.2 \pm 1.2) \times 10^{-10}$	$(+2.3 \pm 4.5) \times 10^{-11}$

Study of systematics, *i.e.* split data set !

We improve :

- by a factor 30 to 800 results from Battat *et al.* 2007 (different combinaison)
- by a factor 5 post-fit analysis of 1 coefficient from binary pulsars

LLR & GRAIL

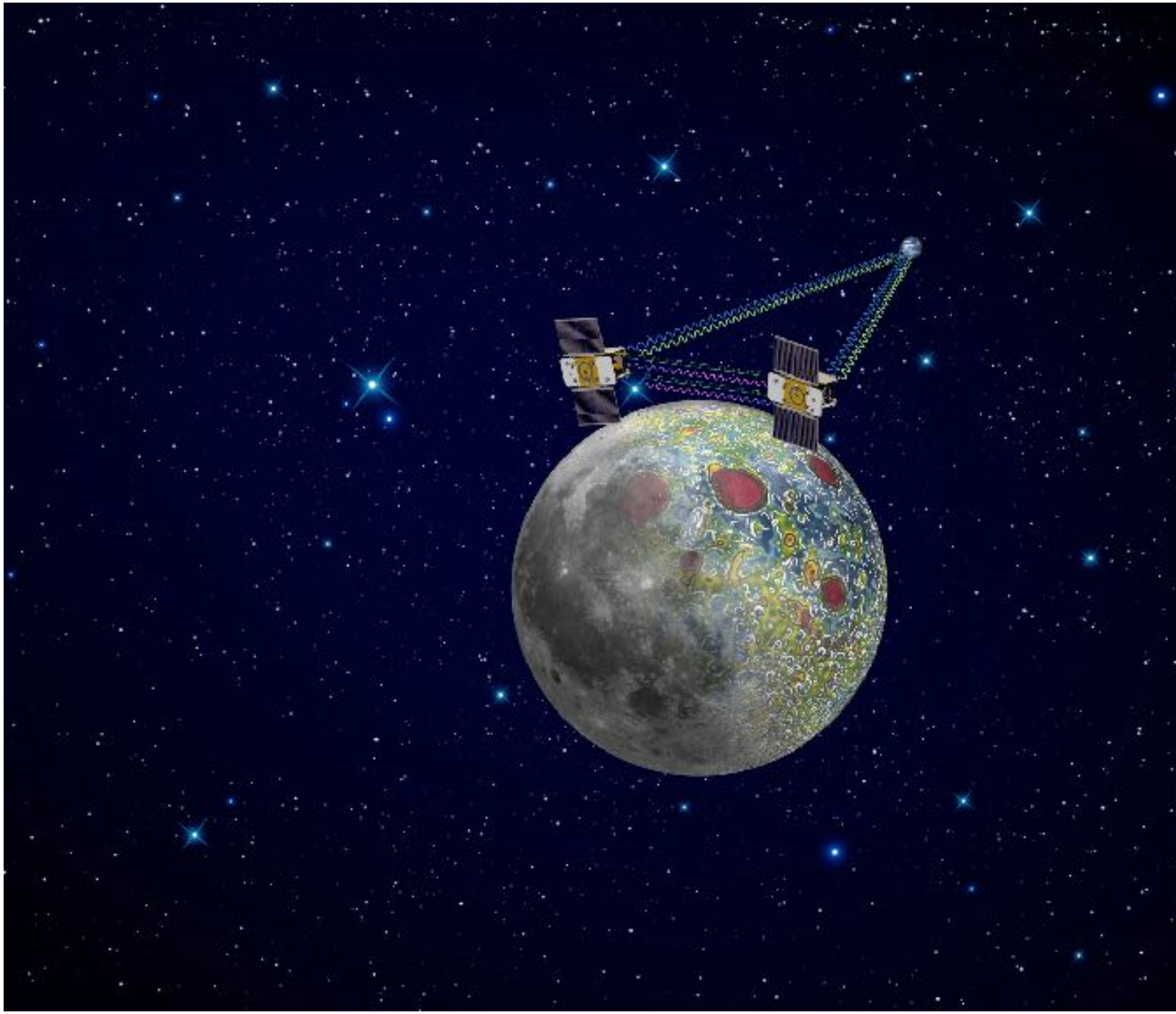
Launched : 09 / 2011
Science : 03-05 /2012

Improved knowledge of Moon's gravity field by a factor 100.

Our idea :
Combine LLR & GRAIL data together.

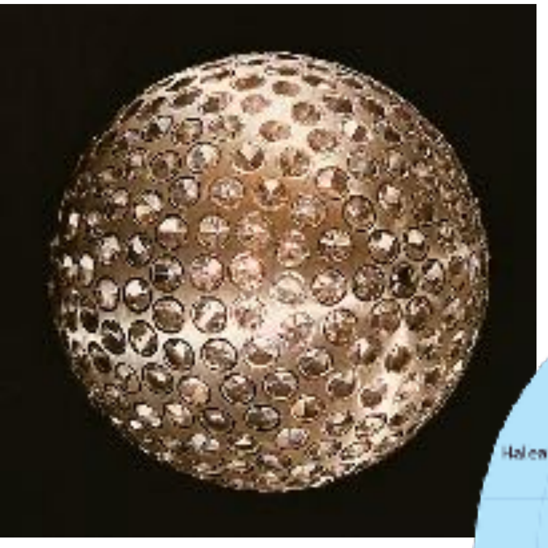
Why ?
Traceless condition is slighty correlated with effects from lunar potential (at same frequency)

Project ongoing @ SYRTE (CLPL, Bourgoin) + UNI Bern (Bertone)



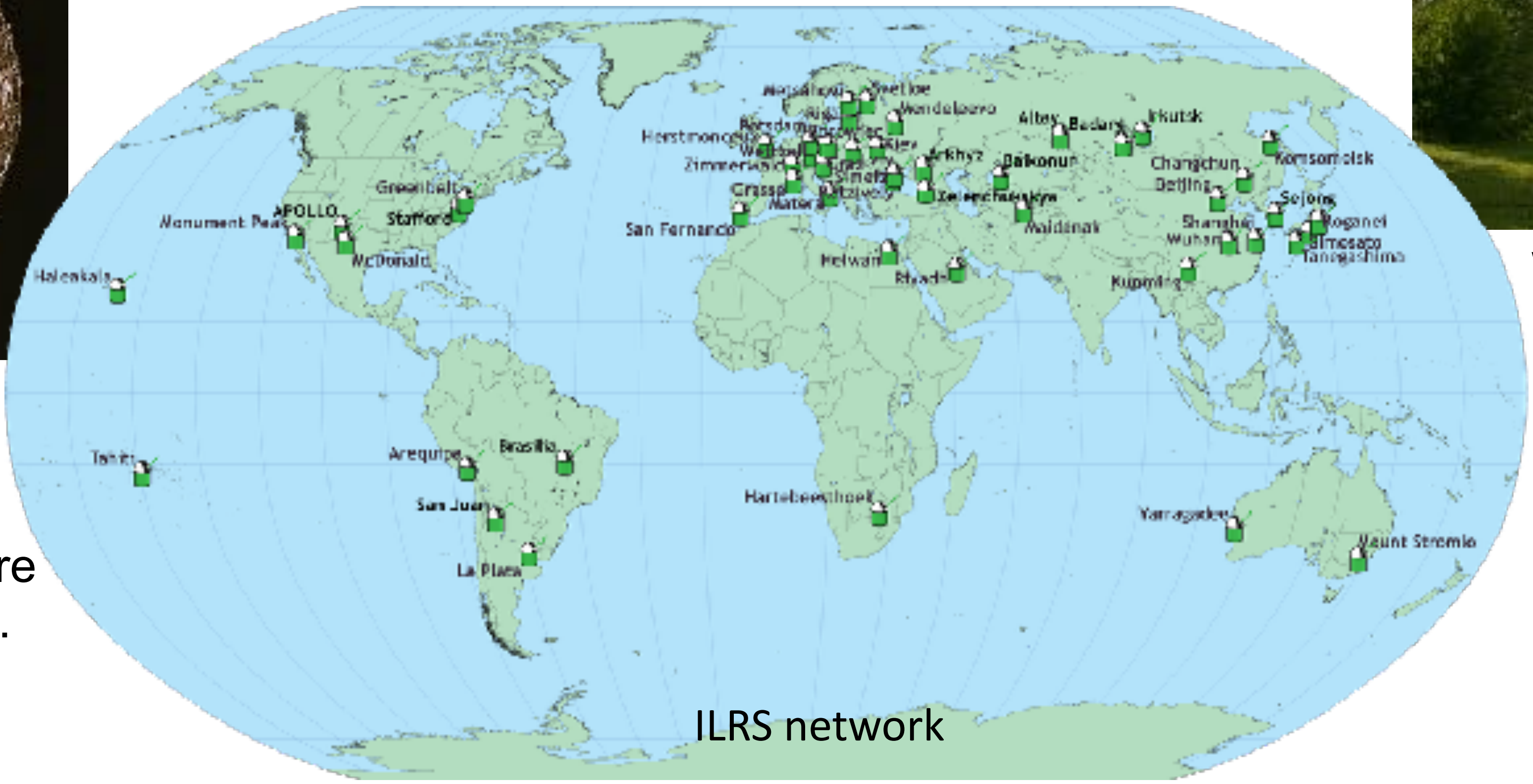
What’s next ?

Satellite Laser Ranging (SLR)



LAGEOS
1976 – today !

But a lot of probes are tracked every day…



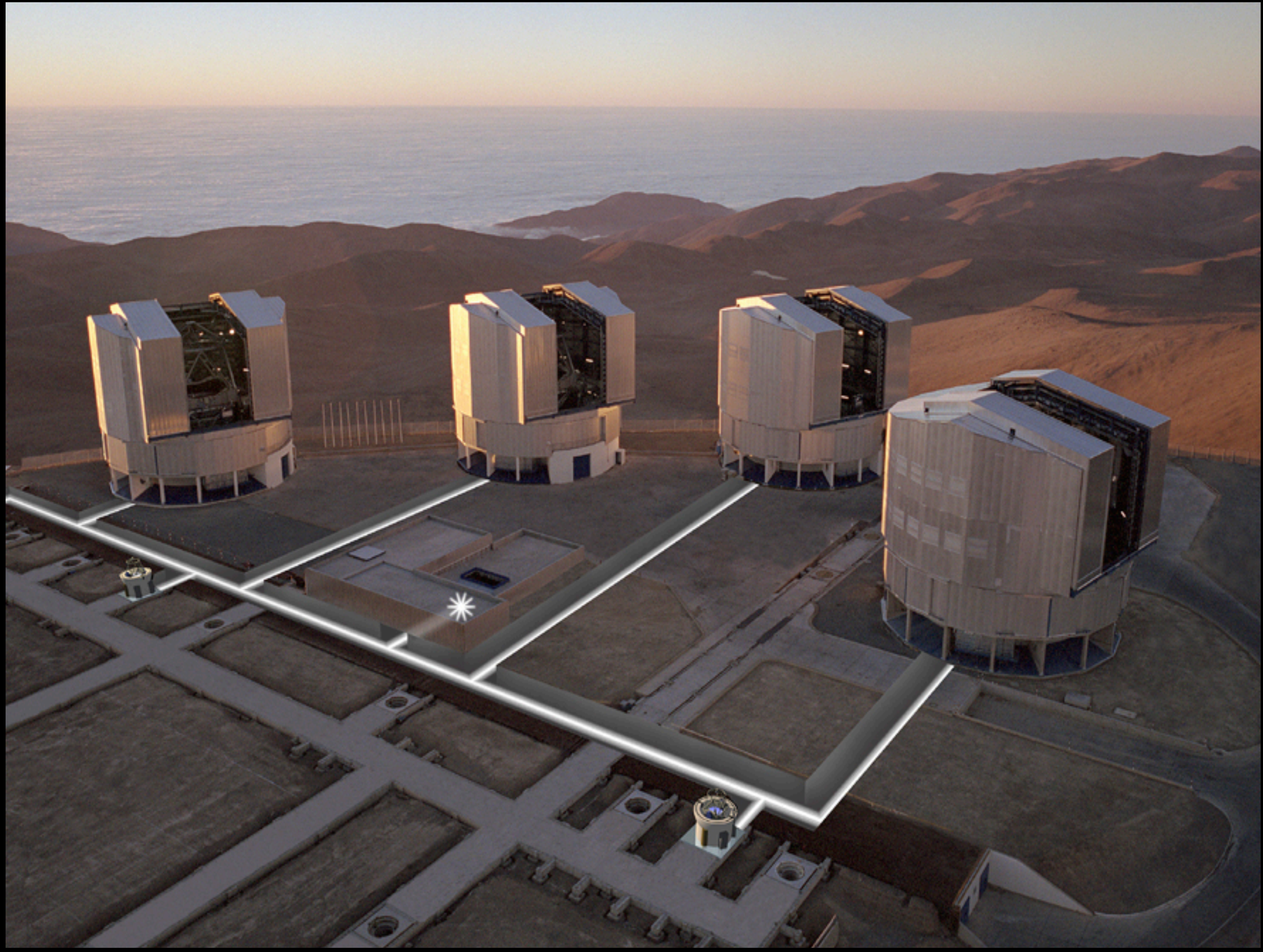
ILRS network

Wetzell Geodetic Obs., Germany

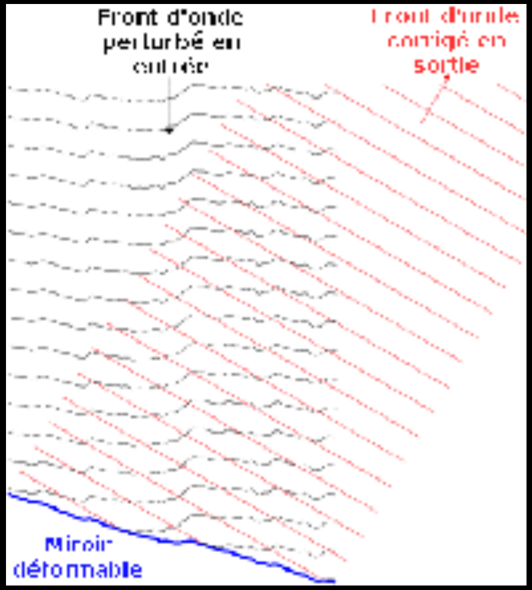
Modification of CNES/GINS software (similar to OD-ODP, Orbit14 & GeoDyn) to be able to perform orbit determination of probe in SME framework :

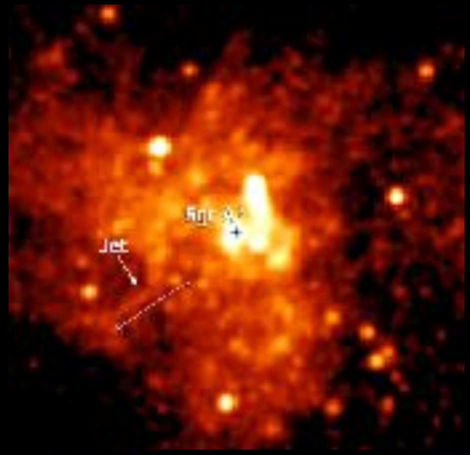
- apply to LAGEOS, LARES.
- Consider also the case of interplanetary probe as Cassini, JUNO or JUICE

Les tests en champ fort: expérience GRAVITY

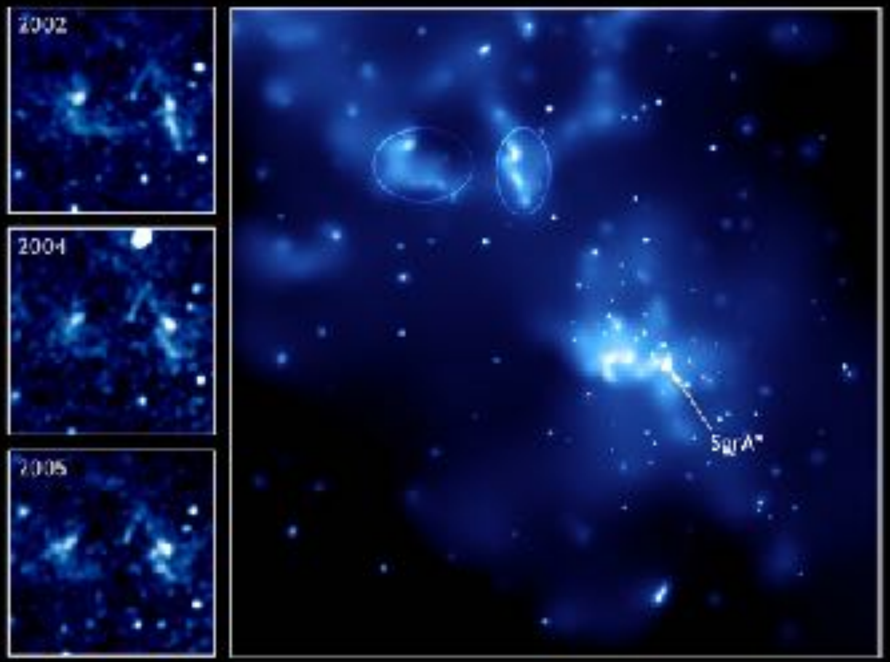


Mesure d'interférométrie
longueur d'onde infrarouge
Utilisation de l'optique adaptative

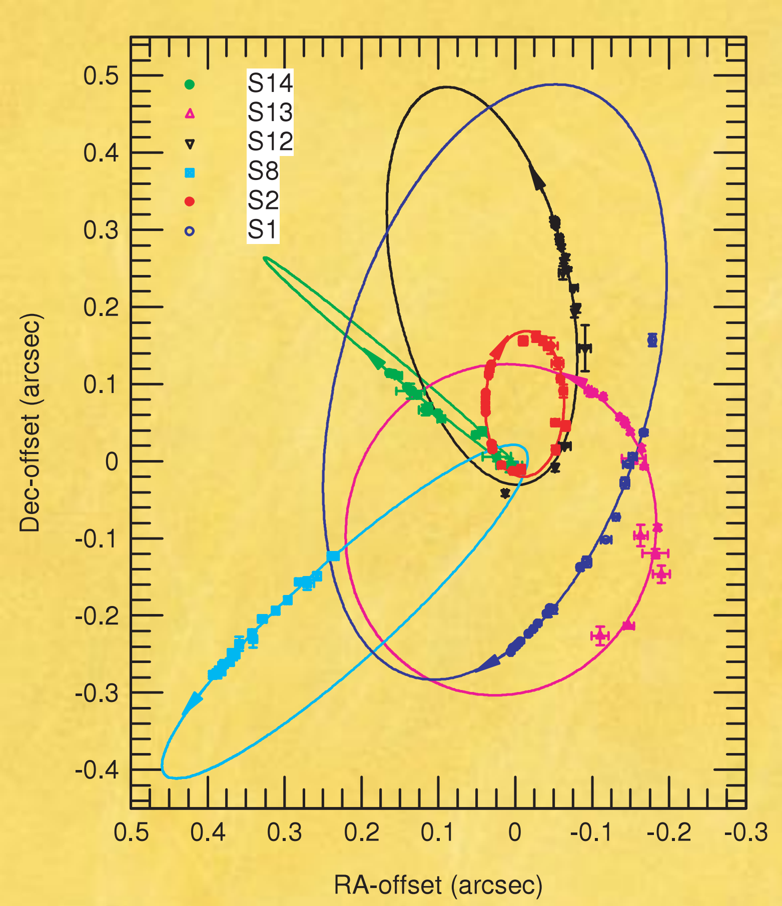




On observera Sagitarus A (près du centre galactique) près du trou noir central.



observations de Sgr A en X du satellite Chandra



Eisenhauer et al. 2005

Mais alors... Einstein a toujours raison aujourd'hui?

Mais alors... Einstein a toujours raison aujourd'hui?

Deux petits exemples d'anomalies
qui font (ou "ont fait") jaser 😉

L'anomalie Pioneer

Révélée par le JPL en 2002

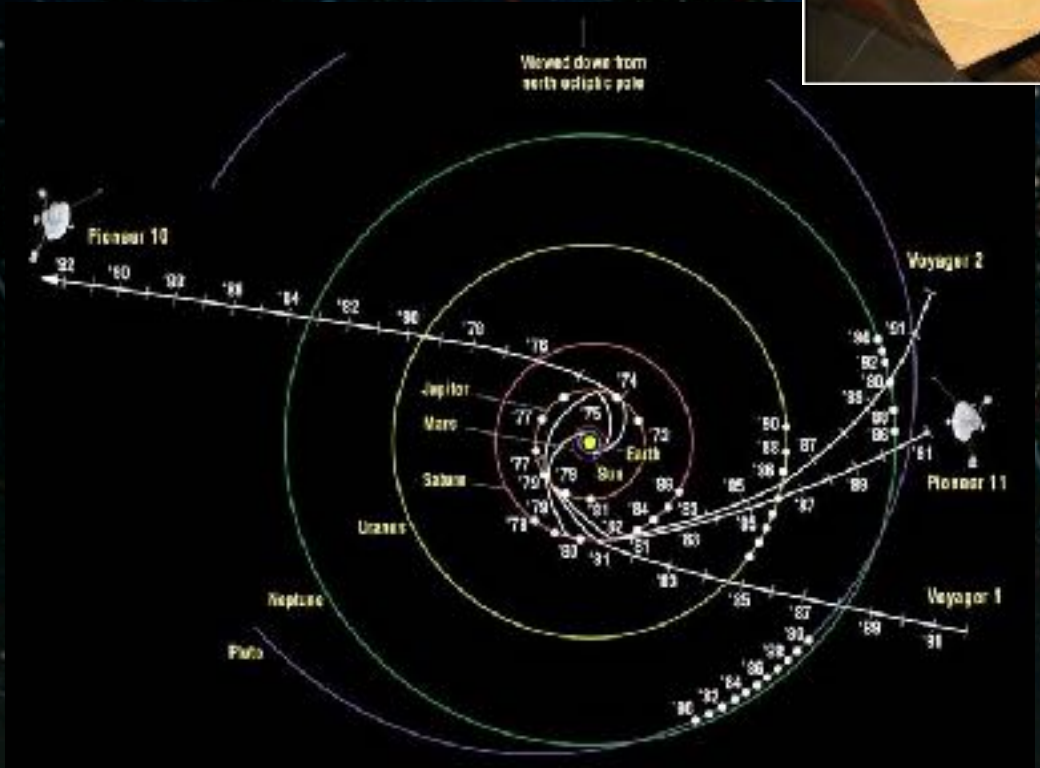
Les théoriciens se sont amusés à imaginer les théories les plus exotiques possible : des centaines de publications !

Officieusement a sombré *corps et biens* depuis quelques semaines.

A ce jour, on attend encore la publication de la part du JPL...

Ces sondes ne sortaient pas suffisamment vite du système solaire.

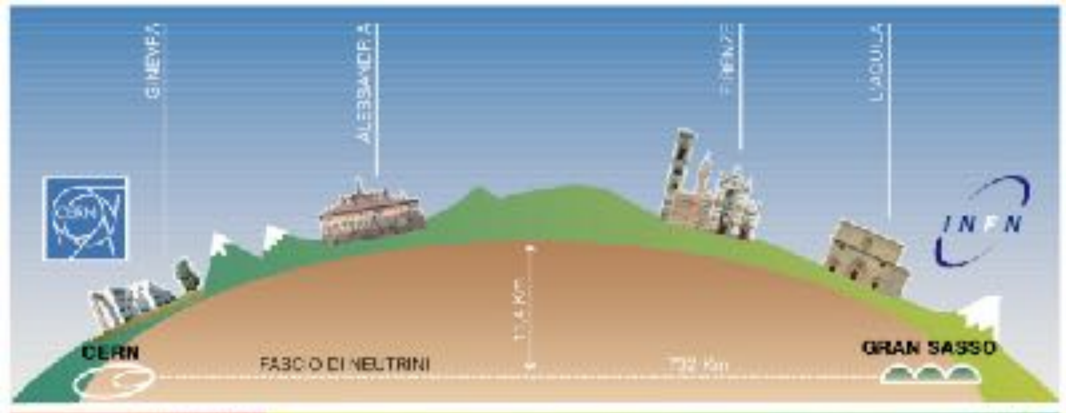
Une longue analyse a eu lieu pour déceler l'effet thermique manquant



Un neutrino fou pris en excès de vitesse : -3 points

Sept. 2011, pour 60 ns de trop, un neutrino dépasse la limitation de vitesse. Le gendarme en charge de l'affaire : la collaboration OPERA.

Centre de "*traitement des amendes*" de Gran Sasso.



Labo. d'OPERA à Gran Sasso

Mars 2012 : une collaboration concurrente, ICARUS, démontre que le résultat n'est pas fiable.

Le parquet a décidé de classer l'affaire sans suite...

cause probable de l'erreur: un câble défectueux du récepteur GPS du "*radar*"

Conclusions

- At moment, GR is tested via PPN formalism and fifth force
- Existing VLBI, LLR, SLR and probe navigation data are existing, growing with time and with better accuracy
- New space missions are ongoing (Gaia, JUNO, Aces, JUICE) with unprecedented accuracy
- Since 5 years, people try to extend gravitational tests to more alternative theories
    - SME, since it happens even at Newtonian Level, is a good first candidate
    - But… we fit in the residuals.
- New generation of SME patch for real data analysis code is ongoing for Gaia and VLBI. Soon for LLR and in the future SLR.
- The resulting methodology will be next applied to other theories and objects (binary pulsars)