Physique au LHC: théorie

Ecole doctorale Orsay

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- **Le Modèle Standard**

1. Le Modèle Standard de la physique des particules

2. Statut du MS

3. Contraintes sur M_H dans le MS

4. Problèmes et insuffisances du MS

- **Tests du MS au LHC**
- **Le Higgs au LHC**
- **SUSY au LHC**
- **Nouvelle physique au LHC**

1. The Standard Model

The SM is based on a local gauge symmetry: invariance under

$$\mathrm{G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y}$$

- The group $\mathrm{SU(3)_C}$ describes the strong force:

– interaction between q, q , q which are SU(3) triplets

– mediated by 8 gluons, G_μ^a corresponding to 8 generators of $\mathbf{SU(3)_C}$

Gell-Man 3×3 matrices: $[\mathbf{T^a}, \mathbf{T^b}] = \mathbf{if^{abc}T_c}$ with $\mathrm{Tr}[\mathbf{T^a T^b}] = \frac{1}{2}\delta_{\mathbf{ab}}$

– asymptotic freedom: interaction "weak" at high energy, $\alpha_{\mathbf{s}} = \frac{\mathbf{g_s^2}}{4\pi} \ll 1$

The Lagrangian of the theory is given by:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}\mathbf{G^a_{\mu\nu} G^{\mu\nu}_a} + \mathbf{i}\sum_{\mathbf{i}} \mathbf{\bar{q}_i}(\partial_\mu - \mathbf{i g_s T_a G^a_\mu})\gamma^\mu \mathbf{q_i}(-\sum_{\mathbf{i}} \mathbf{m_i \bar{q}_i q_i})$$

with $\mathbf{G^a_{\mu\nu}} = \partial_\mu \mathbf{G^a_\nu} - \partial_\nu \mathbf{G^a_\mu} + \mathbf{g_s}\,\mathbf{f^{abc} G^b_\mu G^c_\nu}$

– fermion gauge boson couplings : $-g_i \overline{\psi} V_\mu \gamma^\mu \psi$

– triple gauge boson couplings : $i g_i \,\mathrm{Tr}(\partial_\nu V_\mu - \partial_\mu V_\nu)[V_\mu, V_\nu]$

– quartic gauge boson couplings : $\frac{1}{2} g_i^2 \,\mathrm{Tr}[V_\mu, V_\nu]^2$

1. The SM: brief introduction

• $\mathrm{SU(2)_L \times U(1)_Y}$ **describes the electroweak interaction:**

– between the three familles of quarks and leptons

$$\mathbf{I_f^{3L,3R}} = \pm\frac{1}{2}, 0 \quad \Rightarrow \mathbf{L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_{\mathbf{L}}, \ \mathbf{R} = \mathbf{e_R^-}, \ \mathbf{Q} = \begin{pmatrix} u \\ d \end{pmatrix}_{\mathbf{L}}, \ \mathbf{u_R}, \ \mathbf{d_R}$$

$$\mathbf{Y_f = 2Q_f - 2I_f^3} \Rightarrow \mathbf{Y_L} = -1, \mathbf{Y_R} = -2, \mathbf{Y_Q} = \frac{1}{3}, \mathbf{Y_{u_R}} = \frac{4}{3}, \mathbf{Y_{d_R}} = -\frac{2}{3}$$

Same holds for the two other generations: $\mu, \nu_\mu, \mathrm{c}, \mathrm{s};\ \tau, \nu_\tau, \mathrm{t}, \mathrm{b}$.

There is no $\nu_{\mathbf{R}}$ **(and neutrinos are and stay exactly massless)!**

– mediated by the $\tilde{\mathrm{W}}_\mu$ **(isospin) and** B_μ **(hypercharge) gauge bosons**

the gauge bosons, corresp. to generators, are exactly massless

$$\mathbf{T^a} = \frac{1}{2}\tau^{\mathbf{a}}\,; \quad [\mathbf{T^a}, \mathbf{T^b}] = \mathbf{i}\epsilon^{\mathbf{abc}}\mathbf{T_c} \ \text{ and } \ [\mathbf{Y}, \mathbf{Y}] = \mathbf{0}$$

Lagrangian simple: with fields strenghs and covariant derivatives

$$\mathbf{W^a_{\mu\nu}} = \partial_\mu \mathbf{W^a_\nu} - \partial_\nu \mathbf{W^a_\mu} + \mathbf{g_2}\epsilon^{\mathbf{abc}}\mathbf{W^b_\mu W^c_\nu}, \ \mathbf{B}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$$

$$\mathbf{D}_\mu \psi = \left(\partial_\mu - \mathbf{igT_a W^a_\mu} - \mathbf{ig'}\frac{\mathbf{Y}}{\mathbf{2}}\mathbf{B}_\mu\right)\psi\,, \quad \mathbf{T^a} = \frac{1}{2}\tau^{\mathbf{a}}$$

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4}\mathbf{W^a_{\mu\nu} W_a^{\mu\nu}} - \frac{1}{4}\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu} + \bar{\mathbf{F}}_{\mathbf{Li}}\,\mathbf{iD}_\mu\gamma^\mu\,\mathbf{F_{Li}} + \bar{\mathbf{f}}_{\mathbf{Ri}}\,\mathbf{iD}_\mu\gamma^\mu\,\mathbf{f_{R_i}}$$

1. The Higgs in the SM: the potential

But if gauge boson and fermion masses are put by hand in $\mathcal{L}_{\rm SM}$

$\frac{1}{2}{\rm M}_{\rm V}^2 {\rm V}^\mu {\rm V}_\mu$ **and/or** ${\rm m_f \bar{f}_L f_R}$ **terms: breaking of gauge symmetry.**

We need a less “brutal” way to generate particle masses in the SM.

In the SM, for the mechanism of spontaneous EW symmetry breaking,

$\Rightarrow$ **introduce a doublet of complex scalar fields**

$$\mathbf{\Phi} = \begin{pmatrix} \phi^+ \\ \phi^{\mathbf{0}} \end{pmatrix} \ \ {\rm with} \ \ \mathbf{Y_\Phi = +1}$$

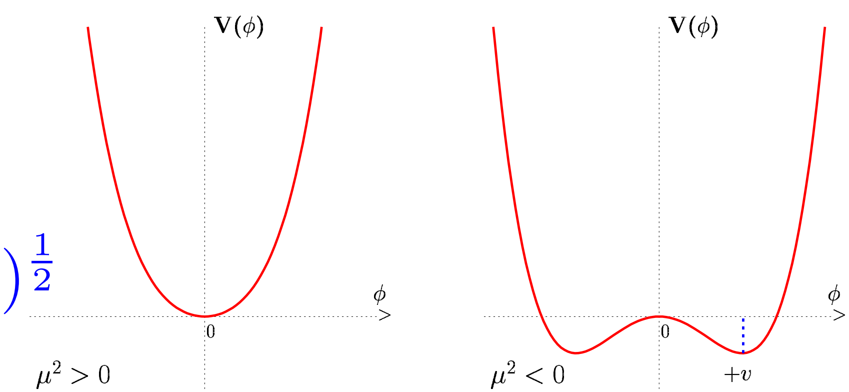
with a Lagrangian that is invariant under $\mathbf{SU(2)_L \times U(1)_Y}$

$$\mathcal{L}_{\mathbf{S}} = (\mathbf{D}^\mu \mathbf{\Phi})^\dagger (\mathbf{D}_\mu \mathbf{\Phi}) - \mu^{\mathbf{2}} \mathbf{\Phi}^\dagger \mathbf{\Phi} - \lambda (\mathbf{\Phi}^\dagger \mathbf{\Phi})^{\mathbf{2}}$$

$\mu^2 > 0$**: 4 scalar particles.**

$\mu^2 < 0$**:** Φ **develops a vev:**

$$\langle 0 | \Phi | 0 \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \ v = \ \left(-\frac{\mu^2}{\lambda}\right)^{\frac{1}{2}}$$



1. The Higgs in the SM: the physical fields

To obtain the physical states, write $\mathcal{L}_S$ with the true vacuum:

- **Write Φ in terms of four fields $\theta_{1,2,3}(x)$ and H(x) at 1st order:**

$$\Phi(x) = e^{i\theta_a(x)\tau^a(x)/v} \frac{1}{\sqrt{2}} \binom{0}{v+H(x)} \simeq \frac{1}{\sqrt{2}} \binom{\theta_2 + i\theta_1}{v+H-i\theta_3}$$

- **Make a gauge transformation on Φ to go to the unitary gauge:**

$$\Phi(x) \to e^{-i\theta_a(x)\tau^a(x)}\, \Phi(x) = \frac{1}{\sqrt{2}} \binom{0}{v+H(x)}$$

- **Then fully develop the term $|D_\mu \Phi)|^2$ of the Lagrangian $\mathcal{L}_S$:**

$$|D_\mu \Phi)|^2 = \left| \left(\partial_\mu - ig_1 \frac{\tau_a}{2} W_\mu^a - i\frac{g_2}{2} B_\mu \right) \Phi \right|^2$$

$$= \frac{1}{2} \left| \begin{pmatrix} \partial_\mu - \frac{i}{2}(g_2 W_\mu^3 + g_1 B_\mu) & -\frac{ig_2}{2}(W_\mu^1 - iW_\mu^2) \\ -\frac{ig_2}{2}(W_\mu^1 + iW_\mu^2) & \partial_\mu + \frac{i}{2}(g_2 W_\mu^3 - g_1 B_\mu) \end{pmatrix} \binom{0}{v+H} \right|^2$$

$$= \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{8} g_2^2 (v+H)^2 |W_\mu^1 + iW_\mu^2|^2 + \frac{1}{8}(v+H)^2 |g_2 W_\mu^3 - g_1 B_\mu|^2$$

- **Define the new fields $W_\mu^\pm$ and Z_μ [A_μ is the orthogonal of Z_μ]:**

$$W^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)\ ,\ Z_\mu = \frac{g_2 W_\mu^3 - g_1 B_\mu}{\sqrt{g_2^2 + g_1^2}}\ ,\ A_\mu = \frac{g_2 W_\mu^3 + g_1 B_\mu}{\sqrt{g_2^2 + g_1^2}}$$

$$\sin^2 \theta_W \equiv g_2 / \sqrt{g_2^2 + g_1^2} = e/g_2$$

1. The Higgs in the SM: the masses

- **And pick up the terms which are bilinear in the fields $W^{\pm}, Z, A$:**

$$M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{1}{2} M_A^2 A_\mu A^\mu$$

$\Rightarrow$ **3 degrees of freedom for $W_L^{\pm}, Z_L$ and thus $M_{W^\pm}, M_Z$:**

$$M_W = \frac{1}{2} v g_2 \,, \; M_Z = \frac{1}{2} v \sqrt{g_2^2 + g_1^2} \,, \; M_A = 0 \,,$$

with the value of the vev given by: $v = 1/(\sqrt{2} G_F)^{1/2} \sim 246 \, \mathrm{GeV}$.

$\Rightarrow$ **The photon stays massless, $U(1)_{\rm QED}$ is preserved.**

- **For fermion masses, use <u>same</u> doublet field Φ and its <u>conjugate</u> field $\tilde{\Phi} = i\tau_2 \Phi^*$ and introduce $\mathcal{L}_{\rm Yuk}$ which is invariant under SU(2)xU(1):**

$$\mathcal{L}_{\rm Yuk} = -f_e (\bar{e}, \bar{\nu})_L \Phi e_R - f_d (\bar{u}, \bar{d})_L \Phi d_R - f_u (\bar{u}, \bar{d})_L \tilde{\Phi} u_R + \cdots$$

$$= -\frac{1}{\sqrt{2}} f_e (\bar{\nu}_e, \bar{e}_L) \binom{0}{v+H} e_R \cdots = -\frac{1}{\sqrt{2}} (v+H) \bar{e}_L e_R \cdots$$

$$\Rightarrow m_e = \frac{f_e \, v}{\sqrt{2}} \,, \; m_u = \frac{f_u \, v}{\sqrt{2}} \,, \; m_d = \frac{f_d \, v}{\sqrt{2}}$$

With same Φ, we have generated gauge boson and fermion masses, while preserving SU(2)xU(1) gauge symmetry (which is now hidden)!

What about the residual degree of freedom?

1. The Higgs in the SM: the Higgs boson

It will correspond to the physical spin–zero scalar Higgs particle, H.

The kinetic part of H field, $\frac{1}{2}(\partial_\mu H)^2$, comes from $|D_\mu \Phi)|^2$ term.

Mass and self-interaction part from $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$:

$$V = \frac{\mu^2}{2}(0, v+H)\binom{0}{v+H} + \frac{\lambda}{2}|(0, v+H)\binom{0}{v+H}|^2$$

Doing the exercise you find that the Lagrangian containing H is,

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - V = \frac{1}{2}(\partial^\mu H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$

The Higgs boson mass is given by: $M_H^2 = 2\lambda v^2 = -2\mu^2$.

The Higgs triple and quartic self–interaction vertices are:

$$g_{H^3} = 3i\, M_H^2/v\ ,\ g_{H^4} = 3i M_H^2/v^2$$

What about the Higgs boson couplings to gauge bosons and fermions?

They were almost derived previously, when we calculated the masses:

$$\mathcal{L}_{M_V} \sim M_V^2(1+H/v)^2\ ,\ \mathcal{L}_{m_f} \sim -m_f(1+H/v)$$

$$\Rightarrow g_{Hff} = i m_f/v\ ,\ g_{HVV} = -2i M_V^2/v\ ,\ g_{HHVV} = -2i M_V^2/v^2$$

Since v is known, the only free parameter in the SM is M_H or λ.

1. SM physics: $W/Z/H$ at high energies

Propagators of gauge and Goldstone bosons in a general ζ gauge:

$\longrightarrow q$

$$\frac{-i}{q^2-M_V^2+i\epsilon}\left[g_{\mu\nu}+(\zeta-1)\frac{q_\mu q_\mu}{q^2-\zeta M_V^2}\right]$$

$\zeta=\infty$: **Landau gauge**

$\zeta=1$: **'t Hooft-Feynman**

$\omega^\pm, \omega^0$:

$\longrightarrow q$

$$\frac{-i}{q^2-\zeta M_V^2+i\epsilon}$$

- **In unitary gauge, Goldstones do not propagate and gauge bosons have usual propagators of massive spin–1 particles (old IVB theory).**
- **At very high energies, $\mathrm{s} \gg \mathrm{M_V^2}$, an approximation is $\mathrm{M_V} \sim 0$. The $\mathrm{V_L}$ components of V can be replaced by the Goldstones, $\mathrm{V_L} \to \omega$.**
- **In fact, the electroweak equivalence theorem tells that at high energies, massive vector bosons are equivalent to Goldstones. In VV scattering e.g.**

$$\mathbf{A(V_L^1 \cdots V_L^n \to V_L^1 \cdots V_L^{n'}) = (i)^n(-i)^{n'} A(w^1 \cdots w^n \to w^1 \cdots w^{n'}}$$

Thus, we simply replace V by w in the scalar potential and use w:

$$\mathbf{V = \frac{M_H^2}{2v}(H^2+w_0^2+2w^+w^-)H + \frac{M_H^2}{8v^2}(H^2+w_0^2+2w^+w^-)^2}$$

2. Status of the SM

The parameters of the SM at tree—level:

In the SM, there are 18 free parameters ($+\theta_{\rm QCD} + \nu$ sector):

- **9 fermions masses, 4 CKM parameters (see below for details).**
- **3 coupling g_s, g_2, g_1 and 2 parameters from scalar potential μ, λ**

More precise inputs, $\alpha_s, \alpha(M_Z^2), G_F, M_Z$ and M_H (unknown)

Weak interactions of fermions with gauge bosons

$$\mathcal{L}_{\rm NC} = e J_\mu^{\rm A} A^\mu + \frac{g_2}{\cos\theta_W} J_\mu^Z Z^\mu \ , \ \mathcal{L}_{\rm CC} = \frac{g_2}{\sqrt{2}} (J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})$$

$$J_\mu^A = Q_f \bar{f} \gamma_\mu f \ , \ J_\mu^Z = \frac{1}{4} \bar{f} \gamma_\mu [\hat{v}_f - \gamma_5 \hat{a}_f] f \ , \ J_\mu^+ = \frac{1}{2} \bar{f}_u \gamma_\mu (1-\gamma_5) f_d$$

with $v_f = \frac{\hat{v}_f}{4 s_W c_W} = \frac{2I_f^3 - 4 Q_f s_W^2}{4 s_W c_W} \ , \ a_f = \frac{\hat{a}_f}{4 s_W c_W} = \frac{2 I_f^3}{4 s_W c_W}$

3-families: complication in CC as current eigenstates $\neq$ mass eigenstates

connected by a unitary transformation: $(d', s', b') = V_{\rm CKM} (d, s, b)$

$V_{\rm CKM} \equiv 3 \times 3$ **unitarity matrix; NC are diagonal in both bases (GIM).**

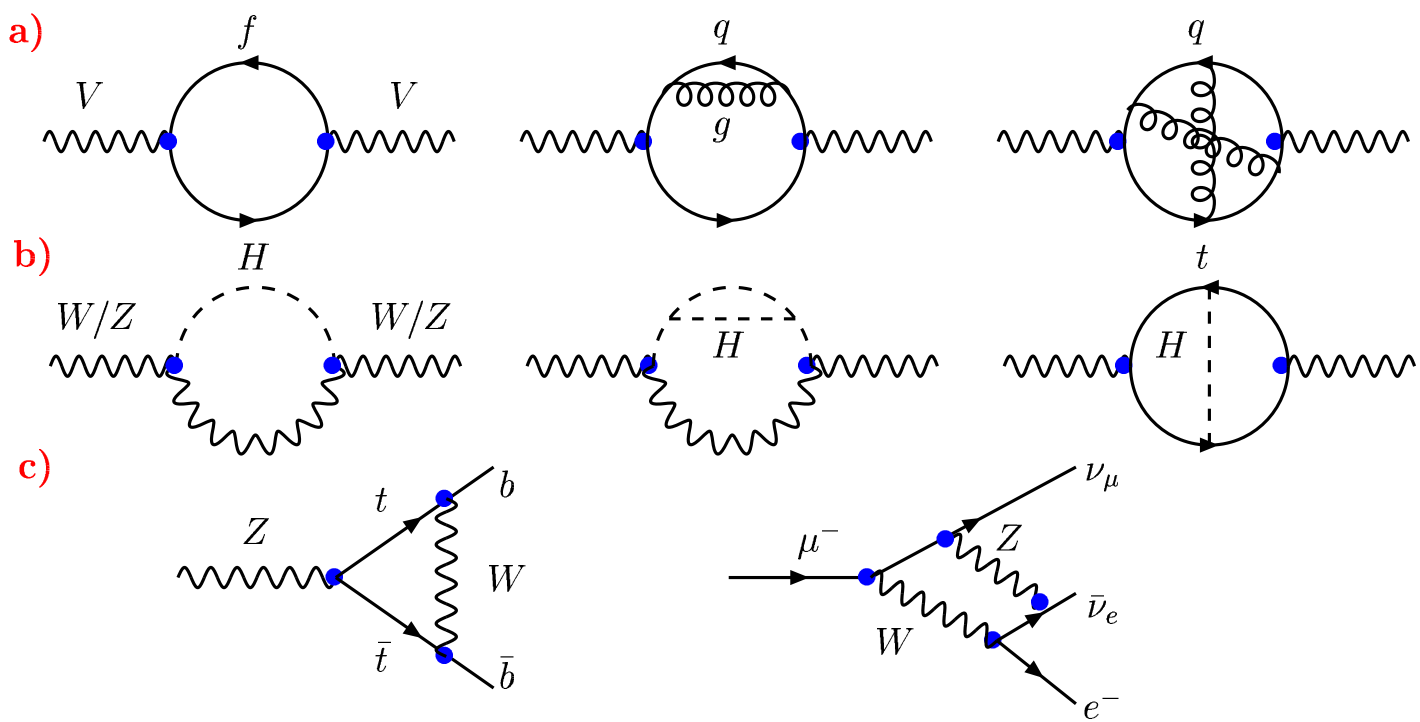
Parametrized by 3 angles and 1 CPV phase: tests at B–factories.

2. Status of the SM: precision tests

M_W **and** $\sin^2\theta_W$ **predicted:** $\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha(M_Z^2)}{2M_W^2(1-M_W^2/M_Z^2)}$; $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$

In fact, they are related by $\rho = \frac{M_W^2}{c_W^2 M_Z^2} \equiv 1$ **at tree–level in the SM**

To have very precise predictions, include the radiative corrections:



The dominant correction is, besides $\Delta\alpha$**, the one to the** ρ **parameter**

$$\rho = \frac{1}{1-\Delta\rho}\ , \ \Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} = \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2} - \frac{G_\mu M_W^2}{8\sqrt{2}\pi^2}\log\frac{M_H^2}{M_W^2} + \cdots$$

2. Status of the SM: high–precision data

- **Z boson lineshape parameters at LEP1 ($\sqrt{\rm s} \sim \rm M_Z$):**

$$M_Z, \Gamma_Z, \sigma(e^+e^- \to \textbf{hadrons})$$

- **Partial decay widths and asymmetries in Z decays at LEP1:**

$$\Gamma(Z \to f\bar{f}) = \frac{2\alpha}{3} N_c M_Z (v_f^2 + a_f^2)\ ,\ A_{FB}^f = \frac{3}{4} \frac{2a_e v_e}{v_e^2+a_e^2} \frac{2a_f v_f}{v_f^2+a_f^2}$$

- **Left–right polarized asymmetries in Z decays at SLC:**

$$A_{LR} = \frac{2a_e v_e}{v_e^2+a_e^2}\ ,\ A_{LR/FB}^f = \frac{3}{4} \frac{2a_f v_f}{v_f^2+a_f^2}$$

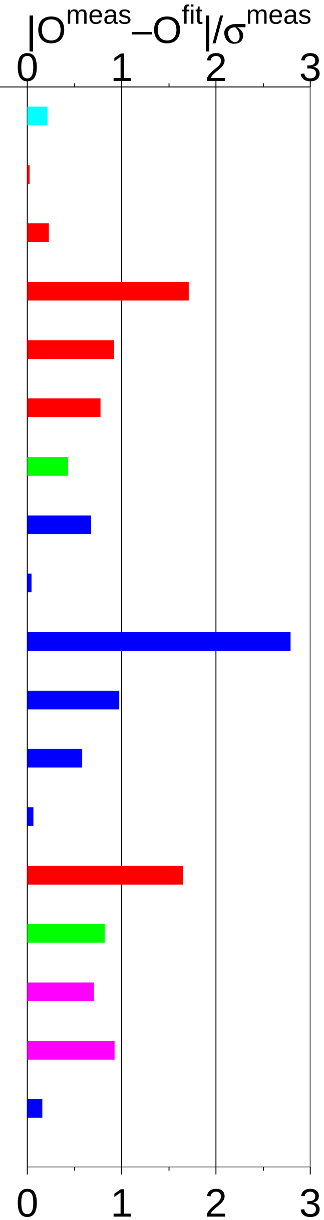
- W **boson parameters:** M_W **and** Γ_W **at LEP2 and Tevatron.**
- **Other observables at low–energy:** ν_e **DIS, PV in Cs and Th ...**
- **Use top quark mass value from Tevatron** $m_t = 171 \pm 2$ **GeV**
- **Use value of** α_s **from LEP and elsewhere:** $\alpha_s = 0.1172 \pm 0.002$
- **Use** $\alpha(M_Z)$ **with** $\Delta\alpha = 0.028 \pm 0.00036$ **from low–energy data**

$\Rightarrow$ **Very high precision tests of the SM at the quantum level: 1%–0.1%**

SM describes precisely (almost) all available experimental data!

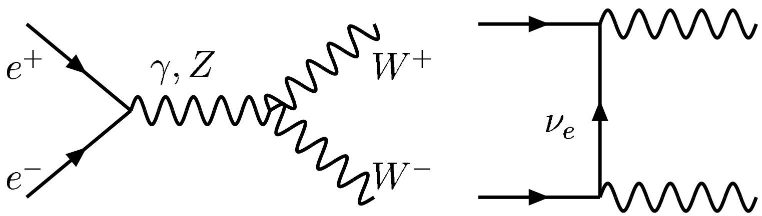
2. Status of the SM: high–precision tests

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02766
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ^0_{had} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01640
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1479
R_b	0.21629 ± 0.00066	0.21585
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0741
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1479
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.392 ± 0.029	80.371
Γ_W [GeV]	2.147 ± 0.060	2.091
m_t [GeV]	171.4 ± 2.1	171.7



2. Tests of the SM: gauge structure

WW production at at LEP2:



General CPC WWV coupling given by:

$$\mathcal{L}_{\rm eff}^{WWV} \propto g_1^V V^\mu \left(W^-_{\mu\nu} W^{+\nu} - W^+_{\mu\nu} W^{-\nu}\right) + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2} V^{\mu\nu} W_\nu^{+\rho} W^-_{\rho\mu}$$

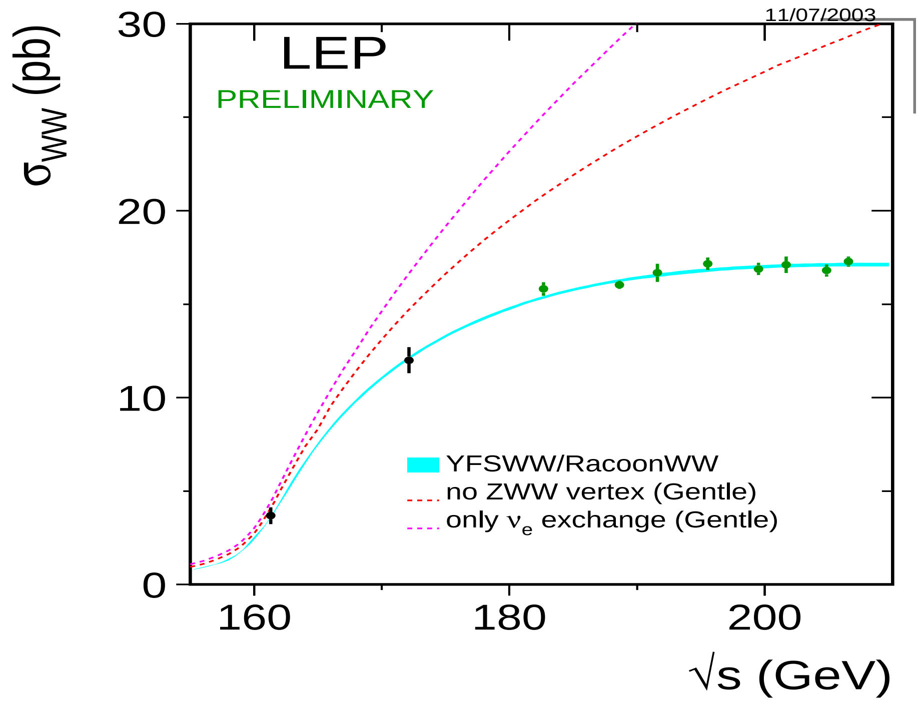
In SM: $g_1^V = 1, \kappa_V = 1, \lambda_V = 0$

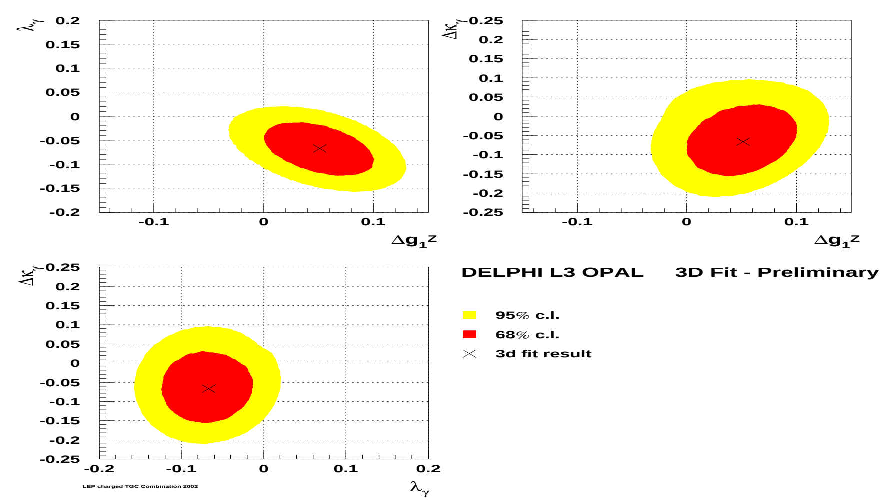
$\mathbf{SU(2)_L \times U(1)_Y}$ gauge structure checked rather precisely at LEP2

Note: QCD also very precisely tested!

– running of α_s from m_τ to LEP2.

– 3 gluon vertex determined at LEP1.

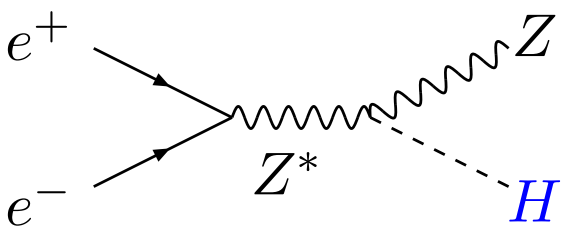




3. Constraints on M_H: experiment

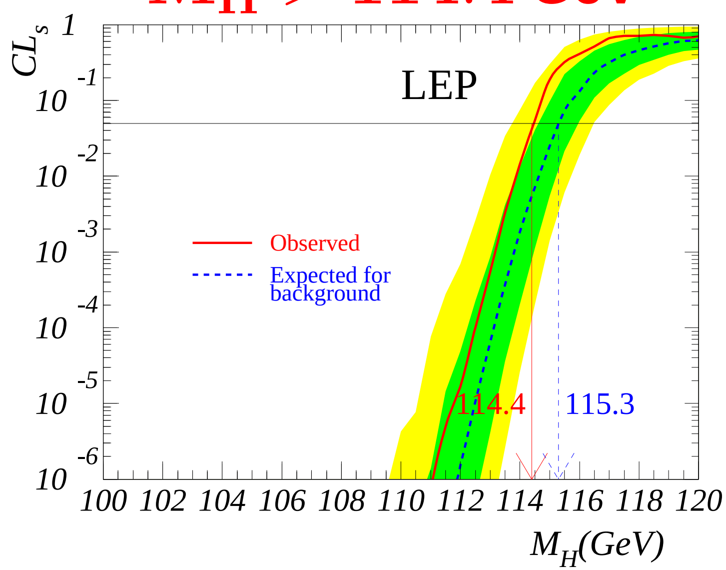
Direct searches at LEP:

H looked for in $e^+e^- \to ZH$



We have a limit at 95% CL:

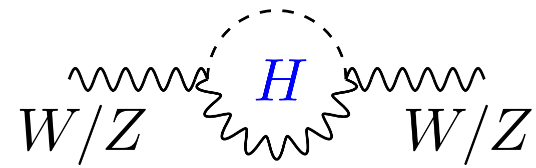
$M_H > 114.4$ **GeV**



(1.7σ excess at $M_H \sim 116$ GeV)

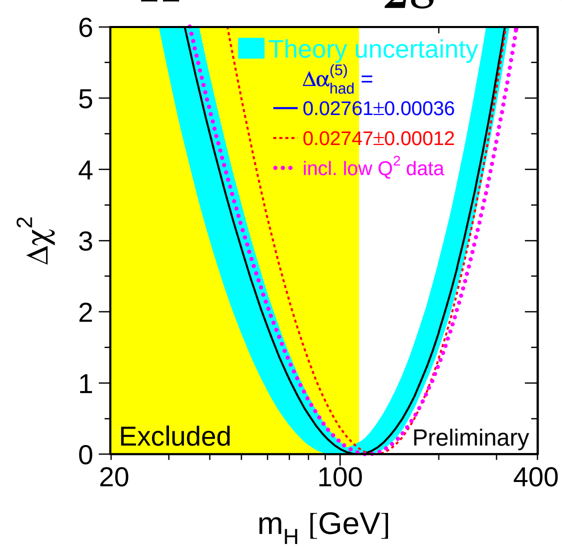
Indirect searches:

H contributes to RC to W/Z masses:



Fit the EW precision measurements:

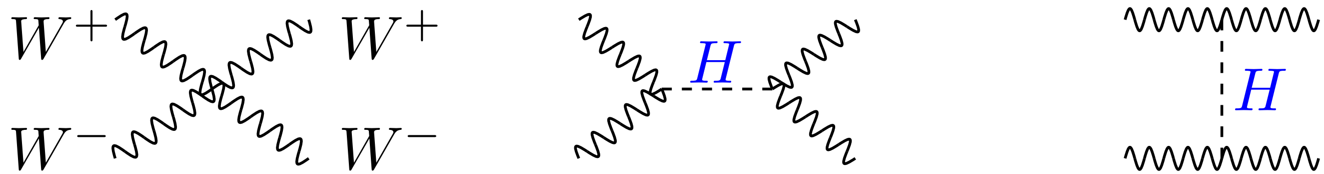
we obtain $M_H = 85^{+39}_{-28}$ GeV, or



$M_H \lesssim 160$ **GeV at 95% CL**

3. Constraints on M_H: perturbative unitarity

Scattering of massive gauge bosons $V_L V_L \to V_L V_L$ at high-energy



Because w interactions increase with energy (q^μ terms in V propagator),

$s \gg M_W^2 \Rightarrow \sigma(w^+w^- \to w^+w^-) \propto s$: $\Rightarrow$ **unitarity violation possible!**

Decomposition into partial waves and choose J=0 for $s \gg M_W^2$:

$$a_0 = -\frac{M_H^2}{8\pi v^2}\left[1 + \frac{M_H^2}{s-M_H^2} + \frac{M_H^2}{s}\log\left(1+\frac{s}{M_H^2}\right)\right]$$

For unitarity to be fullfiled, we need the condition $|\mathrm{Re}(a_0)| < 1/2$.

At high energies, $s \gg M_H, M_W$, we have: $a_0 \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$

$$\text{unitarity} \Rightarrow M_H \lesssim 870\ \text{GeV}\ \ (M_H \lesssim 710\ \text{GeV})$$

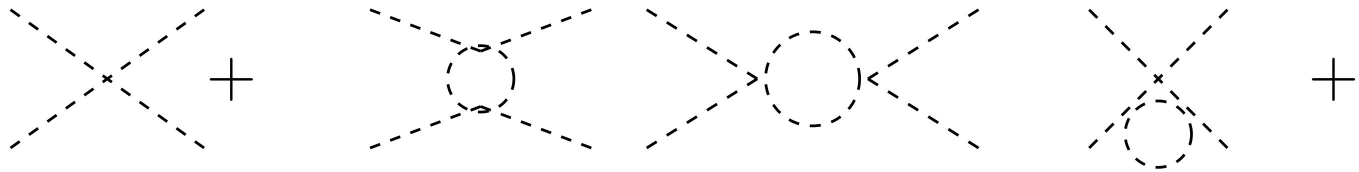
For a very heavy or no Higgs boson, we have: $a_0 \xrightarrow{s \ll M_H^2} -\frac{s}{32\pi v^2}$

$$\text{unitarity} \Rightarrow \sqrt{s} \lesssim 1.7\ \text{TeV}\ \ (\sqrt{s} \lesssim 1.2\ \text{TeV})$$

Otherwise (strong?) New Physics should appear to restore unitarity.

3. Constraints on M_H: triviality

The quartic coupling of the Higgs boson λ ($\propto M_H^2$) increases with energy.



The RGE evolution of λ with Q^2 and its solution are given by:

$$\frac{\mathrm{d}\lambda(Q^2)}{\mathrm{d}Q^2} = \frac{3}{4\pi^2}\,\lambda^2(Q^2) \Rightarrow \lambda(Q^2) = \lambda(v^2)\left[1 - \frac{3}{4\pi^2}\,\lambda(v^2)\log\frac{Q^2}{v^2}\right]^{-1}$$

- If $Q^2 \ll v^2$, $\lambda(Q^2) \to 0_+$: **the theory is said to be trivial (no int.).**
- If $Q^2 \gg v^2$, $\lambda(Q^2) \to \infty$: **Landau pole at** $Q = v\exp\left(\frac{4\pi^2 v^2}{M_H^2}\right)$.

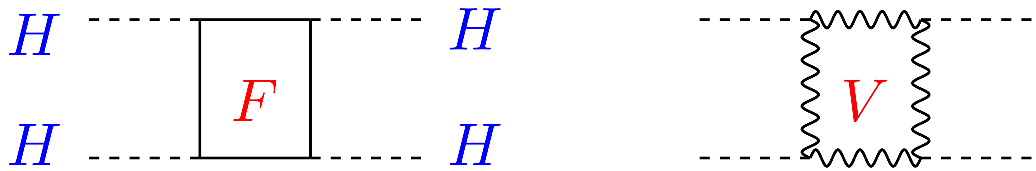
The SM is valid only at scales before λ becomes infinite:

If $\Lambda_C = M_H$, $\lambda \lesssim 4\pi \Rightarrow M_H \lesssim 650$ **GeV**

(Comparable to results obtained with simulations on the lattice!)

3. Constraints on M_H: vacuum stability

The top quark and gauge bosons also contribute to the evolution of λ.



The RGE evolution of the coupling at one–loop is given by

$$\lambda(Q^2) = \lambda(v^2) + \frac{1}{16\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right]\log\frac{Q^2}{v^2}$$

If λ is small (H is light), top loops might lead to $\lambda(0) < \lambda(v)$:

v is not the minimum of the potentiel and the EW vacuum is instable.

$\Rightarrow$ Impose that the coupling λ stays always positive:

$$\lambda(Q^2) > 0 \Rightarrow M_H^2 > \frac{v^2}{8\pi^2}\left[-12\frac{m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right]\log\frac{Q^2}{v^2}$$

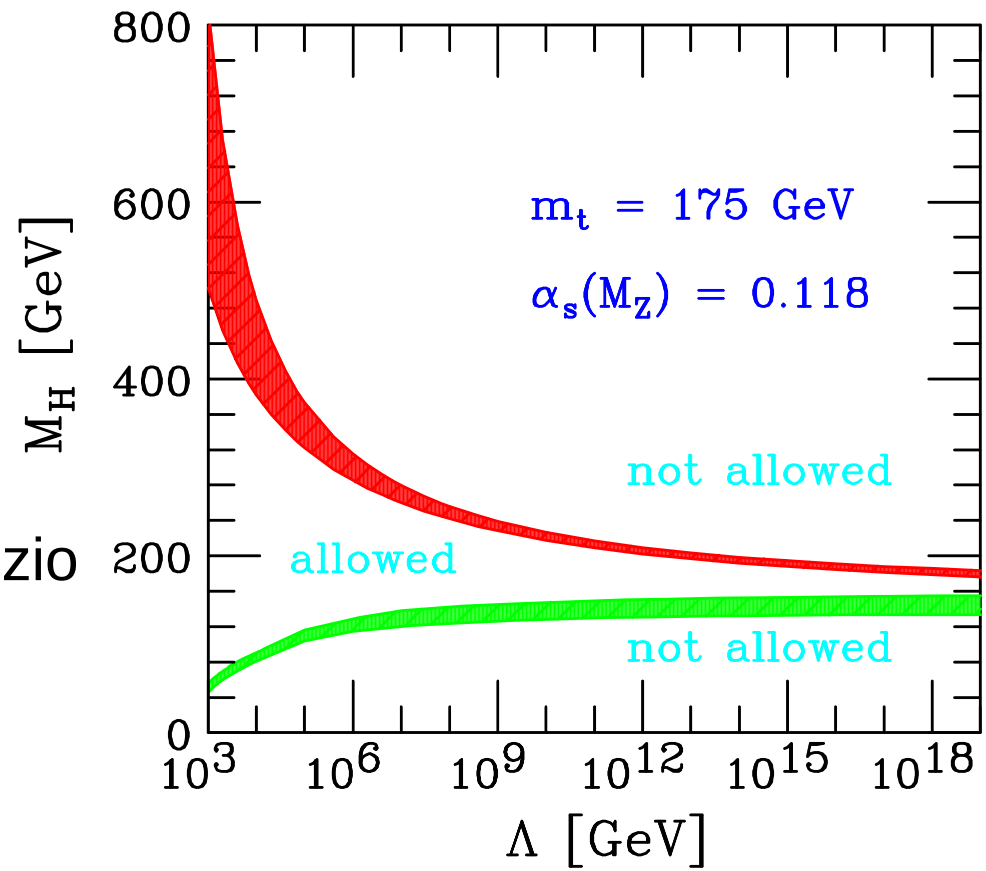
Very strong constraint: $Q = \Lambda_C \sim 1$ TeV $\Rightarrow M_H \gtrsim 70$ GeV

3. Constraints on M_H: triviality+stability

Combine the two constraints and include all possible effects:

- **corrections at two loops**
- **theoretical errors**
- **experimental errors**
- **other refinements $\cdots$**

Cabibbo, Maiani, Parisi, Petronzio
Hambye, Riesselmann



$$\Lambda_C \sim 10^3 \text{ GeV} \quad \Rightarrow \quad 70 \text{ GeV} \lesssim M_H \lesssim 700 \text{ GeV}$$

$$\Lambda_C \sim 10^{16} \text{ GeV} \quad \Rightarrow \quad 130 \text{ GeV} \lesssim M_H \lesssim 180 \text{ GeV}$$

4. Problems and shortcomings of the SM

The SM has many attractive theoretical/experimental features:

- Based on gauge principle, unitary, perturbative, renormalisable $\cdots$
- Once $\mathrm{M_H}$ fixed: everything is predictible with great accuracy.
- And has passed all experimental tests up to now.

But the model has too many shortcomings:

- Too many free parameters (19!) in the model, put by hand...
- No satisfactory explanation for $\mu^2 < 0$ (put ad hoc).
- Does not include the fourth fundamental force, gravity, ..
- Does not say anything about the masses of the neutrinos.
- Does not explain the baryon asymmetry in the universe.
- No real unification of the three gauge interactions; fast P decay.
- There is no stable, weak, massive particle for dark matter.

And above all that, there is the hierarchy or naturalness problem.

4. Problems of the SM: unification

In SM, we have 3 different gauge groups with 3 coupling constants:

$\Rightarrow$ SU(3)xSU(2)xU(1) subgroup of a bigger unifying group.

Grand Unified Theory (GUT): SU(5), SO(10), E_6 etc....

- **only one coupling constant a the GUT scale $M_{GUT} = M_U$**

Spontaneous breakdown to G_{SM} at M_U (intermdeiate scale?).

- **GUT has fundamental representation including all SM fermions.**

Ex: SO(10) has dim. 16 repr. which incorporates 15 SM fermions.

- **Space left for RH neutrinos: generation of m_ν via see–saw.**
- **Baryon asymmetry of the universe through leptogenesis**
- **Explains charge quantization (ex. in SU(5): e,d in multiplet).**
- **Can relate the masses of fermions at M_U (Yukama coupling unif.)**

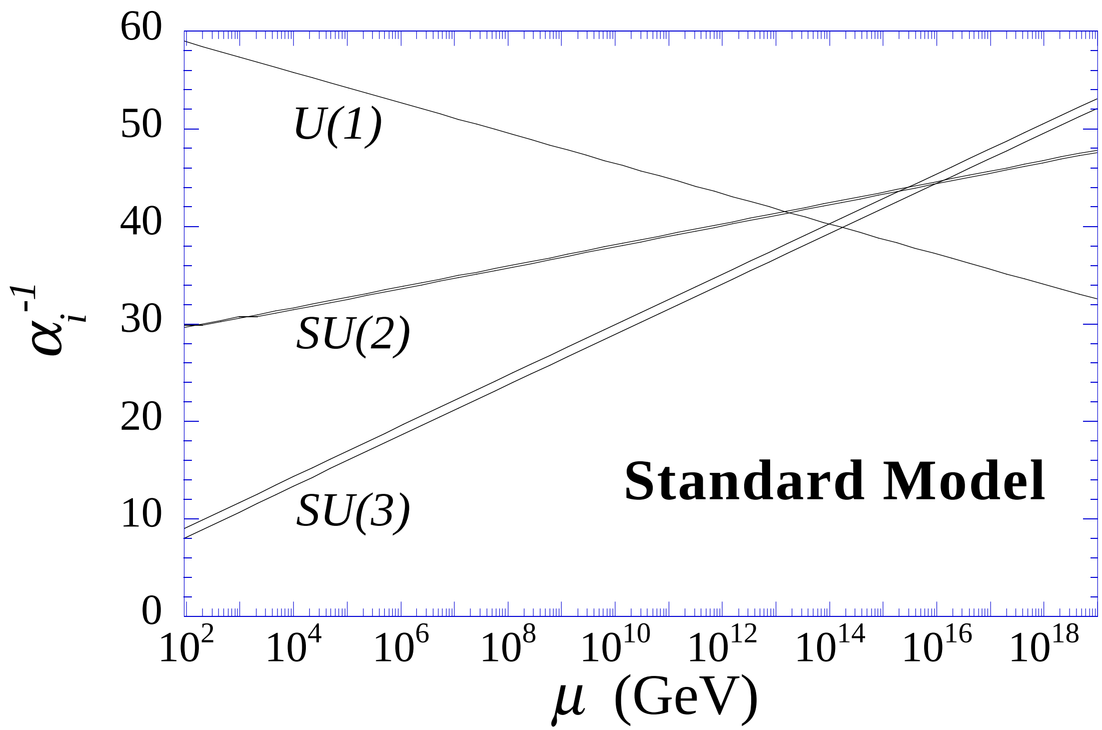
However, there is a problem in non-SUSY GUTS:

the SU(3), SU(2) and U(1) gauge couplings $\alpha_i = g_i^2/(4\pi)$ do not unify:

4. Problems of the SM: unification

The running of the coupling constants: due to radiative corrections to the interaction term in the original Lagrangian ($\gamma f \bar{f}$ in QED); equivalent to ren. of two–point functions; evolution determined by RGE which depends on relevant gauge group and particle content.

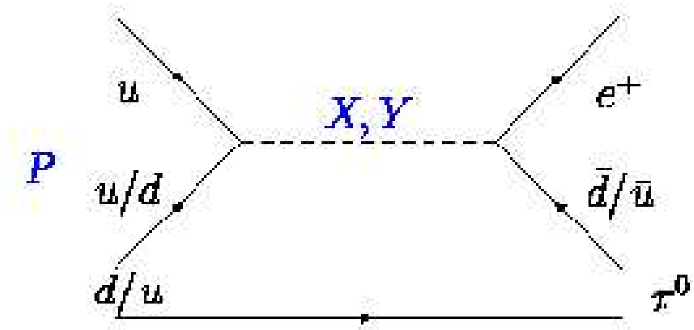
In the context of SU(5), there is no unification with SM particle content



Alternative view: couplings do not meet at a single point near M_{GUT}.

4. Problems of the SM: proton decay

P decay occurs via exchange of the heavy SU(5) gauge bosons X,Y:



$P \to e^+\pi^0, e^+\omega, e^+\rho, ...$

$P \to \nu_e \pi^+, ...$

$\tau_P = \Gamma_P \propto \alpha_U^2 \frac{m_P^5}{M_{X,Y}^4}$

– Compute the effective 4–fermion interaction (CKM... dependent).

– Run down vertices from high scale $\mathrm{M_{X,Y} \sim M_{GUT}}$ to $\mathrm{m_P}$.

– Calculate hadronic ME of the 4–fermion operator (model dep..).

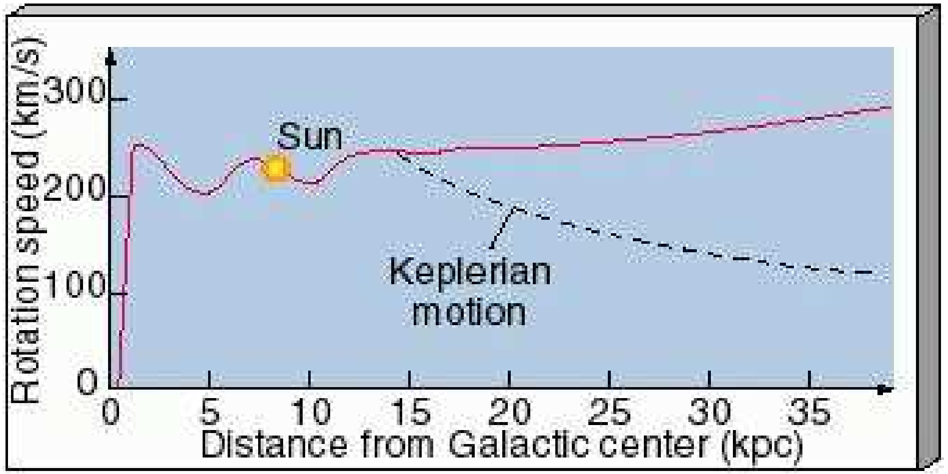
With the input GUT scale from $\mathrm{g_i's}$, $\mathrm{M_{GUT} \sim 10^{15}}$ GeV, one has:

$$\tau_{\mathbf{P}}^{\mathrm{non-SUSY\ GUT}} = \mathbf{10^{30\pm1.7}}\ \mathrm{years}$$

To be compared to $\tau_{\mathbf{P}}^{\mathrm{exp}} \gtrsim \mathbf{10^{33}}$ years: P decay is far too fast!!!

4. Problems of the SM: no cold dark matter

The experimental measurement of the galaxy rotation curve:



shows that some dark matter should be present in universe.

From large structure formation: DM should be cold (non relativistic)

The WMAP satelitte has shown that there is 25% of CDM:

$$\Omega_{\rm DM}\,h^2 \simeq 0.113 \pm 0.009 \Rightarrow 0.09 \leq \Omega_{\rm DM}\,h^2 \leq 0.14 \text{ at } 99\% \text{ CL}$$

Needs a particle that fullfils the following conditions:

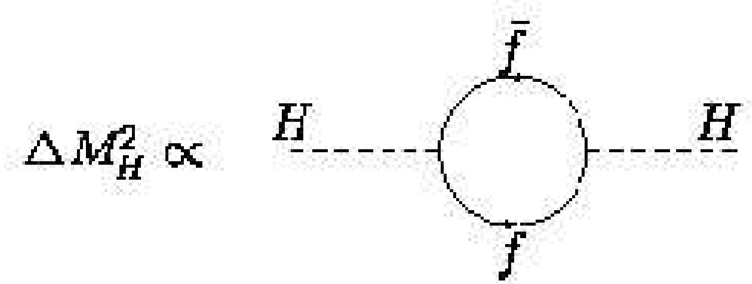
electrically neutral, weakly interacting, rather massive and stable!

There is no such a particle in the SM and also in non–SUSY GUTs!

4. Problems of the SM: the hierarchy problem

Radiative corrections to the Higgs boson mass in the SM

Let us first consider the fermion loop contribution to $\mathbf{M_H^2}$



Using a cut–off Λ (see excercises later) one obtains:

$$\mathbf{\Delta M_H^2 = N_f \frac{\lambda_f^2}{8\pi^2}\left[-\Lambda^2 + 6 m_f^2 \log\frac{\Lambda}{m_f} - 2m_f^2\right] + \mathcal{O}(1/\Lambda^2)}$$

We have thus a quadratic divergence, $\mathbf{\Delta M_H^2 \sim \Lambda^2}$.

Divergence is independent of $\mathbf{M_H}$, and does not disappear if $\mathbf{M_H = 0}$:

The choice $\mathbf{M_H = 0}$ does not increase the symmetry of $\mathcal{L}_{\rm SM}$.

If we fix the cut–off Λ to $\mathbf{M_{GUT}}$ or $\mathbf{M_P}$: $\Rightarrow \mathbf{M_H} \sim 10^{14}$ to 10^{17} GeV!

The Higgs boson mass prefers to be close to the very high scale:

This is the hierarchy problem.

4. Problems of the SM: the hierarchy problem

But we want a light Higgs ($M_H \lesssim 1$ TeV) for unitarity etc... reasons.

We need thus to make: $M_H^2|^{\rm Physical} = M_H^2|^0 + \Delta M_H^2$+ countreterm

And adjust this counterterm with a precision of 10^{-30} (30 digits)

This fine-tunning would be very unnatural...

In SM, besides fermion loops, there are also contributions to M_H

from the massive gauge bosons and from the Higgs boson itself:

$$\Rightarrow \Delta M_H^2 \propto [3(M_W^2 + M_Z^2 + M_H^2)/4 - \sum m_f^2](\Lambda^2/M_W^2)$$

We can adjust the unknown M_H so that the quadratic divergence

disappears (would be a prediction for Higgs mass, $M_H \sim 200$ GeV).

However: does not work at two–loop level or at higher orders....

Summary: the problem of the quadratic divergences to M_H is there.

Photon and fermion masses protected by gauge and chiral symmetry,

.... but here is no symmetry which protects M_H in the SM.

5. The SM at LHC

Physics at the LHC: some generalities

LHC: pp collider $\sqrt{s}=7+7=14$ TeV $\Rightarrow \sqrt{s}_{\rm eff} \sim \sqrt{s}/3 \sim 5$ TeV

$\mathcal{L} \sim 10\,{\rm fb}^{-1}$ first years and 100 ${\rm fb}^{-1}$ later

- **Huge cross sections for QCD processes.**
- **Small cross sections for EW Higgs signal.**

  S/B $\gtrsim 10^{10} \Rightarrow$ **a needle in a haystack!**
- **Need some strong selection criteria:**

Trigger: get rid of uninteresting events...

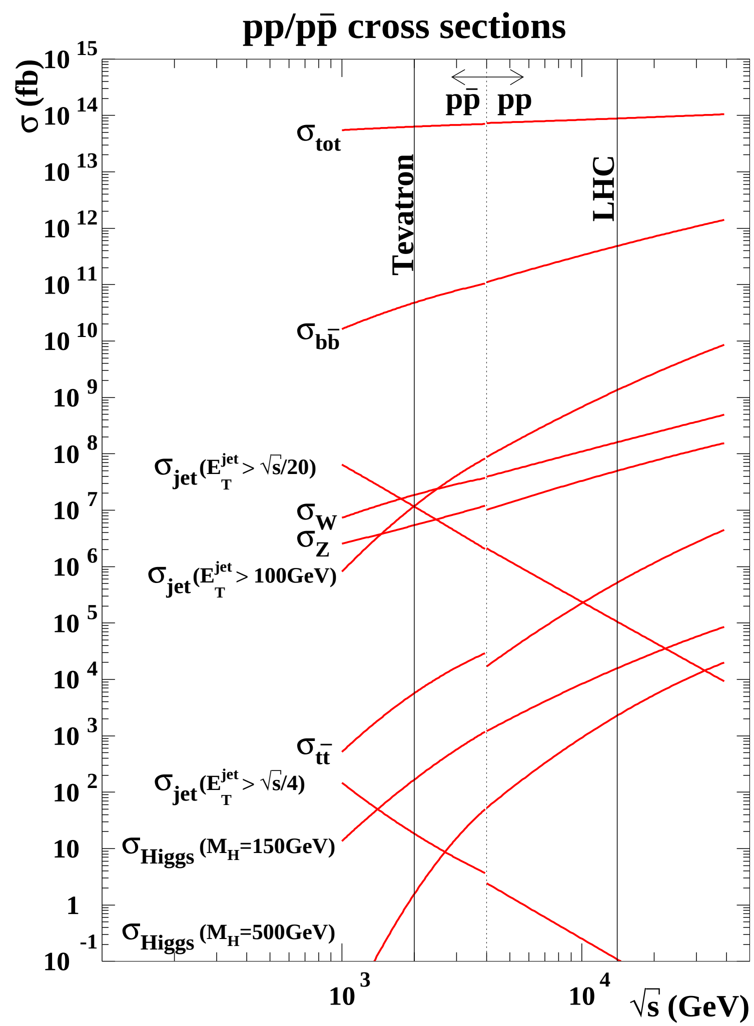
Select clean channels: $\mathrm{H} \to \gamma\gamma, \mathrm{VV} \to \ell$

Use different kinematic features for Higgs

Combine different decay/production channels

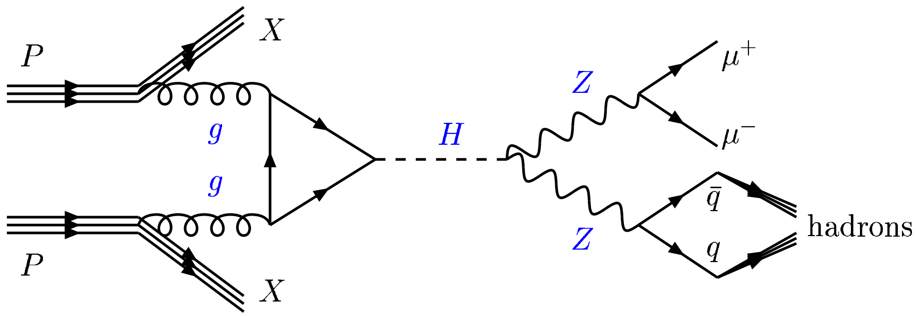
Have a precise knowledge of S and B rates.

- **Gigantic experimental (+theoretical) efforts!**



5. SM at LHC: generalities

Example of process at LHC to see how things work: $\mathrm{gg} \to \mathrm{H}$



$$\mathrm{N_{ev}} = \mathcal{L} \times \mathrm{P(g/p)} \times \hat{\sigma}(\mathrm{gg} \to \mathrm{H}) \times \mathrm{B(H \to ZZ)} \times \mathrm{B}(\mathrm{Z} \to \mu\mu) \times \mathrm{BR}(\mathrm{Z} \to \mathrm{qq})$$

For a large number of events, all these numbers should be large!

Two ingredients: hard process (σ, B) and soft process (PDF, hadr).

Factorization theorem! Here discuss production/decay process.

The partonic cross section of the subprocess, $\mathrm{gg} \to \mathrm{H}$, is:

$$\hat{\sigma}(\mathbf{gg} \to \mathbf{H}) = \int \frac{\mathbf{1}}{\mathbf{2\hat{s}}} \times \frac{\mathbf{1}}{\mathbf{2 \cdot 8}} \times \frac{\mathbf{1}}{\mathbf{2 \cdot 8}} |\mathcal{M}_{\mathbf{Hgg}}|^{\mathbf{2}} \frac{\mathrm{d}^{\mathbf{3}}\mathbf{p_H}}{(\mathbf{2}\pi)^{\mathbf{3}}\mathbf{2E_H}} (\mathbf{2}\pi^{\mathbf{4}}) \delta^{\mathbf{4}}(\mathbf{q} - \mathbf{p_H})$$

Flux factor, color/spin average, matrix element squared, phase space.

Convolute with gluon densities to obtain total hadronic cross section

$$\sigma = \int_{\mathbf{0}}^{\mathbf{1}} \mathrm{d}\mathbf{x_1} \int_{\mathbf{0}}^{\mathbf{1}} \mathrm{d}\mathbf{x_2} \frac{\pi^{\mathbf{2}}\mathbf{M_H}}{\mathbf{8\hat{s}}} \, \mathbf{\Gamma}(\mathbf{H} \to \mathbf{gg}) \mathbf{g}(\mathbf{x_1}) \mathbf{g}(\mathbf{x_2}) \delta(\hat{\mathbf{s}} - \mathbf{M_H^2})$$

5. SM at LHC: generalities

The calculation of $\sigma_{\rm born}$ is not enough in general at pp colliders:

need to include higher order radiative corrections which introduce terms of order $\alpha_s^n \log^m(Q/M_H)$ where Q is either large or small...

- **Since α_s is large, these corrections are in general very important.**
- **Choose a (natural scale) which absorbs/resums the large logs.**

Since we truncate pert. series: only NLO/NNLO corrections available.

- **The (hopefully) not known HO corrections induce a theoretical error.**
- **The scale variation is a (naive) measure of the HO: must be small.**

Also, precise knowledge of σ is not enough: need to calculate some kinematical distributions (e.g. $p_T, \eta, \frac{d\sigma}{dM}$) to distinguish S from B.

In fact, one has to do this for both the signal and background (unless directly measurable from data): the important quantity is $\sigma = \frac{N_S}{\sqrt{N_{bjg}}}$

$\Rightarrow$ a lot of theoretical work is needed!

But most complicated thing is to actually see the signal for S/B$\ll$1!

5. SM at LHC: Tests

Tests of the SM and background calibration for New Physics search:

- **High–p_T jets, γ physics and QCD:**

– allows to measure α_s, PDFs and check perturbation theory

– c, b, t production for QCD dynamics (resummation, quarkonia).

- **The physics of the bottom quarks:**

– study of QCD (as above), CKM matrix and CP violation (LHCb).

- **The physics of the top quark:**

– plays a key role origin of EWSB and clue to SM flavour problem?

– since lifetime shorter than hadronisation scale, QCD laboratory

- **The physics of W, Z bosons:**

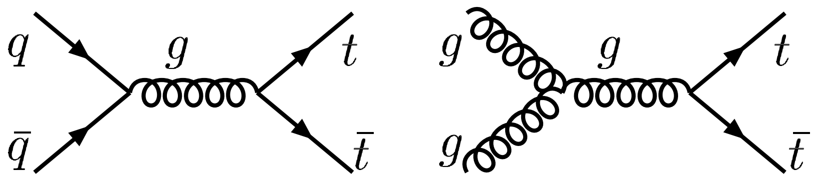
– W, Z production allow to measure precisely M_W and Γ_W

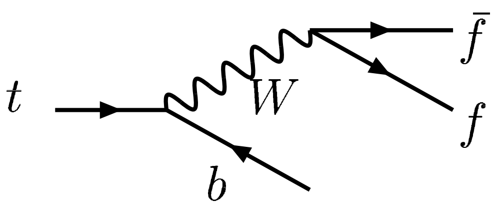
– $WW, WZ(\gamma), ZZ(\gamma)$ allow to measure the TGC.

– VV and $VV \to VV$ allow to test a strong interaction sector.

5. SM at LHC: the top quark

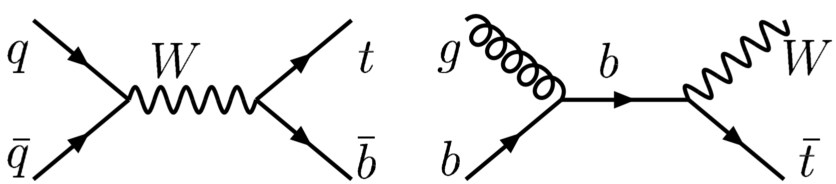
Top quark pair production: $\mathbf{pp \to t\bar{t}}$



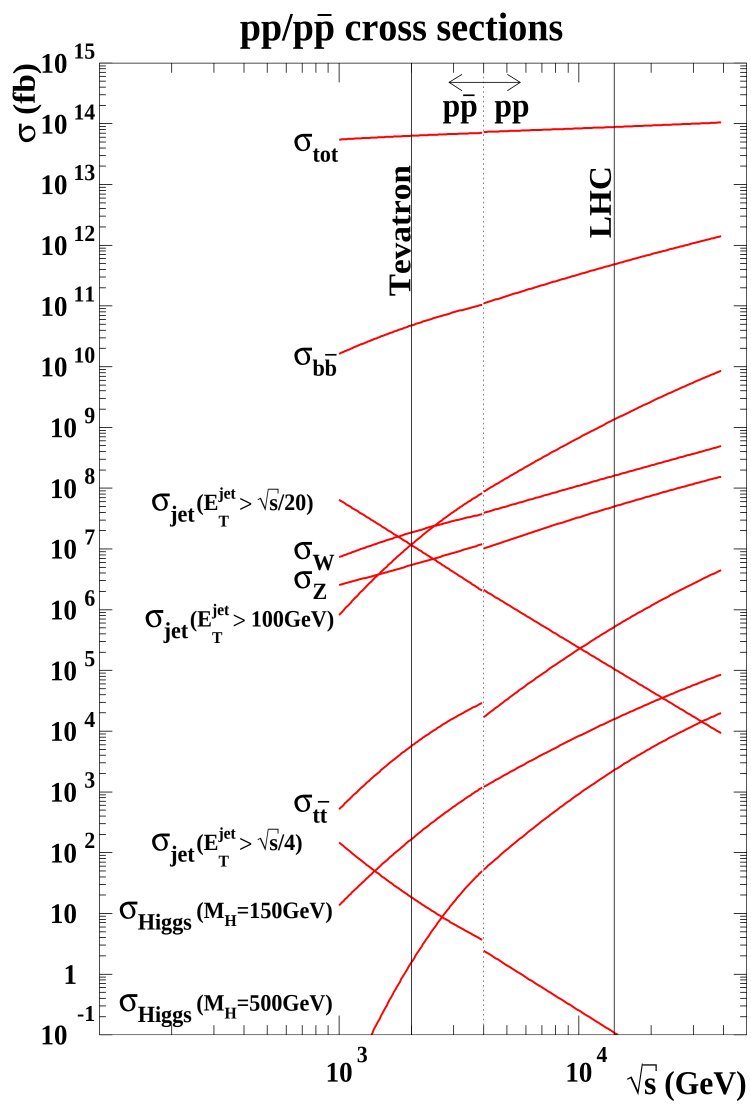


M_t measurement: $\Delta M_t \sim \pm 1$ GeV

Single top production: $\mathbf{pp \to t + X}$

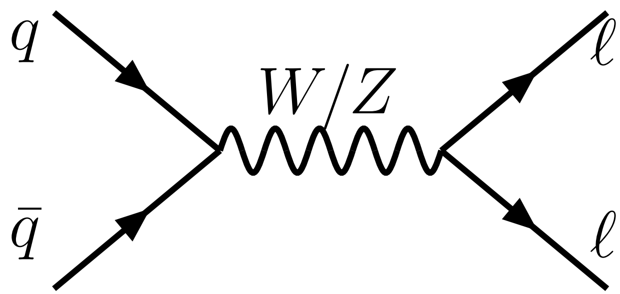


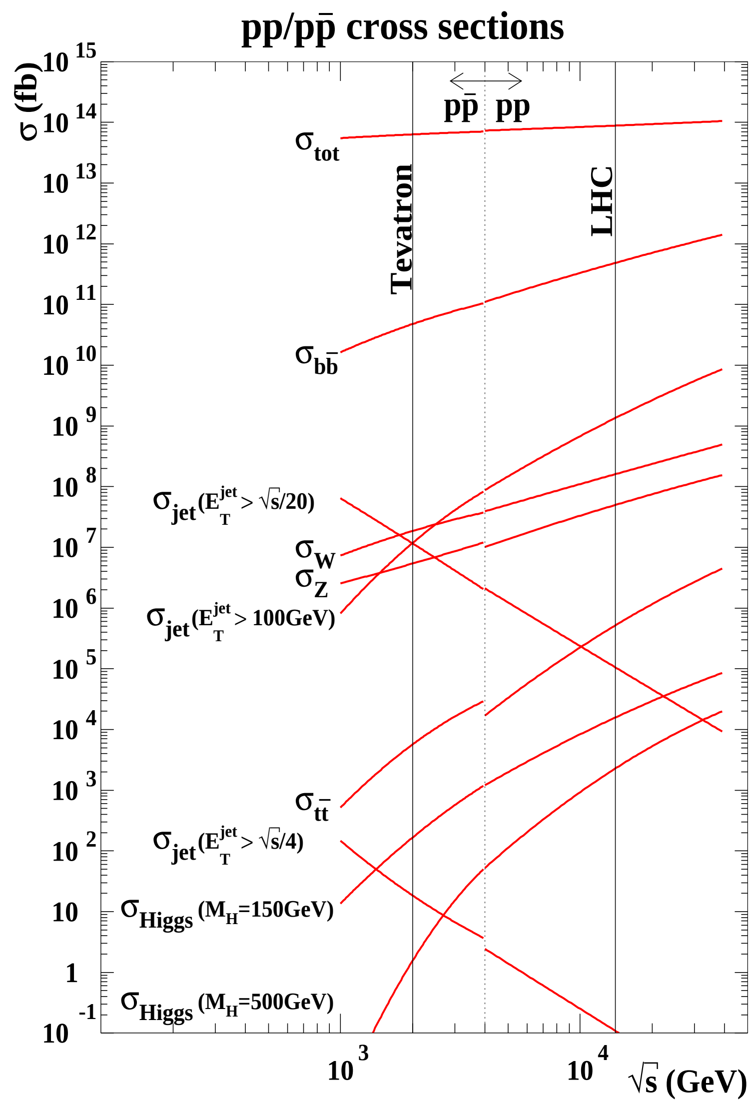
Much smaller rates by enough events
for precise $V_{bt}^{\rm CKM}$ measurement....



5. SM at LHC: the W/Z bosons

Single W/Z production: $\mathrm{pp} \to \mathrm{q\bar{q}} \to \mathbf{V}$





Very large number of events $\sim 10^9$:

Include RC: $K_V = \sigma_{\rm NNL0}/\sigma_{\rm LO} \sim 1.4$

Systematical errors ($\mathcal{L}$, PDF's, etc..)

cancell in the ratio $\sigma(\mathbf{W})/\sigma(\mathbf{Z})$

Use Z parameters from LEP1/SLC

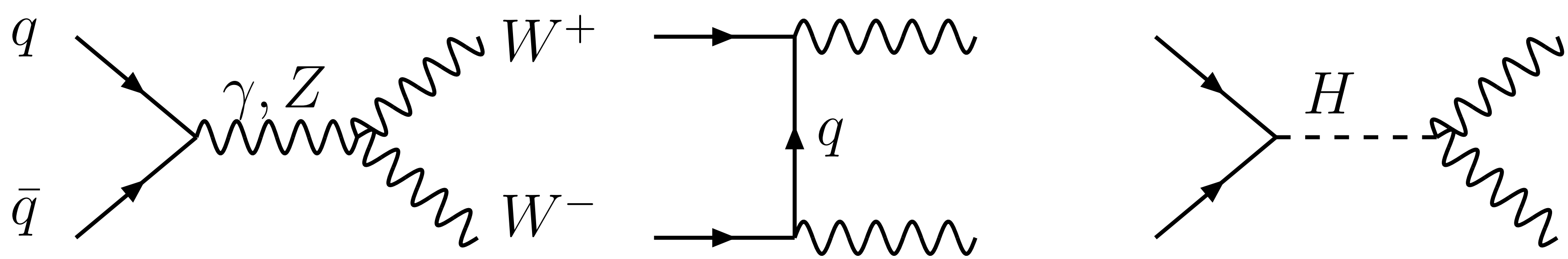
Precise measurements of W parameters:

Ex: $\Delta \mathrm{M_W} \approx 15$ **MeV (30 MeV now).....**

5. SM at LHC: W boson Physics

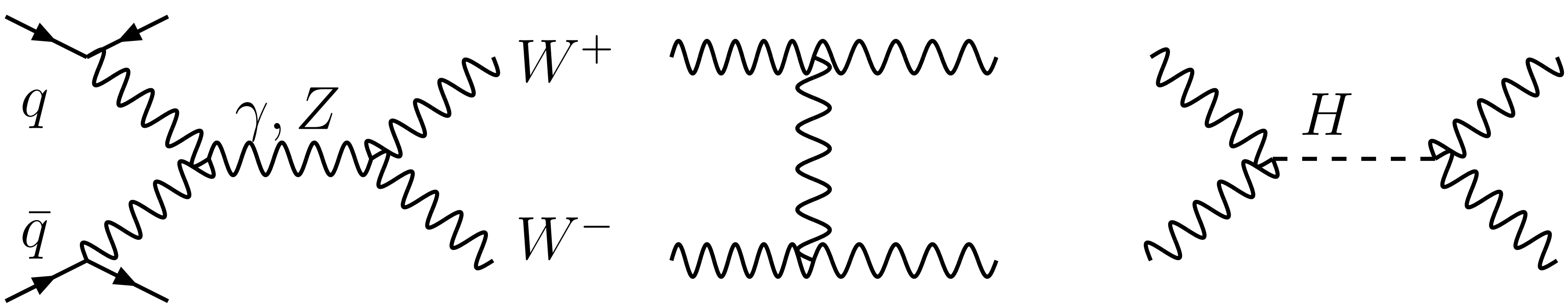
WW/ZZ/Zγ production important to check SM gauge structure

(similar than at LEP2 but with a higher precision)



WW/ZZ scattering important to check a strongly interacting sector

(in case where there is no Higgs boson found at the LHC!!)



5. SM at LHC: TGCs

General form of the trilinear gauge boson couplings:

$$\mathcal{L}_{\rm eff}^{WWV} = i g_{WWV} \Big[g_1^V V^\mu \left(W^-_{\mu\nu} W^{+\nu} - W^+_{\mu\nu} W^{-\nu} \right) + \kappa_V \, W^+_\mu W^-_\nu V^{\mu\nu}$$

$$+ \frac{\lambda_V}{M_W^2} V^{\mu\nu} W_\nu^{+\rho} W^-_{\rho\mu} + i g_5^V \varepsilon_{\mu\nu\rho\sigma} \left((\partial^\rho W^{-\mu}) W^{+\nu} - W^{-\mu} (\partial^\rho W^{+\nu}) \right) V^\sigma$$

$$+ i g_4^V W^-_\mu W^+_\nu (\partial^\mu V^\nu + \partial^\nu V^\mu) - \frac{\tilde{\kappa}_V}{2} W^-_\mu W^+_\nu \varepsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} - \frac{\tilde{\lambda}_V}{2 m_W^2} W^-_{\rho\mu} W^{+\mu}{}_\nu \varepsilon^\nu$$

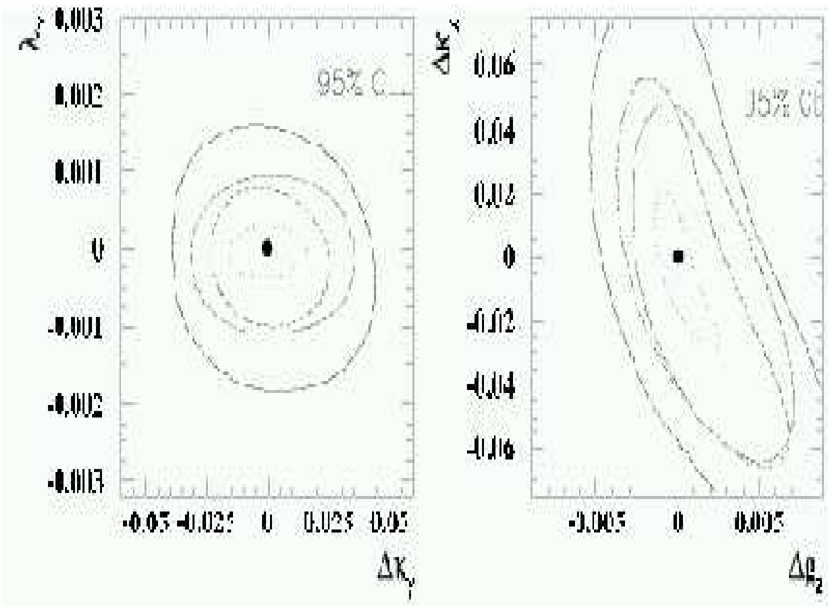
with overall couplings $g_{WW\gamma} = e$ and $g_{WWZ} = e \cot \theta_W$

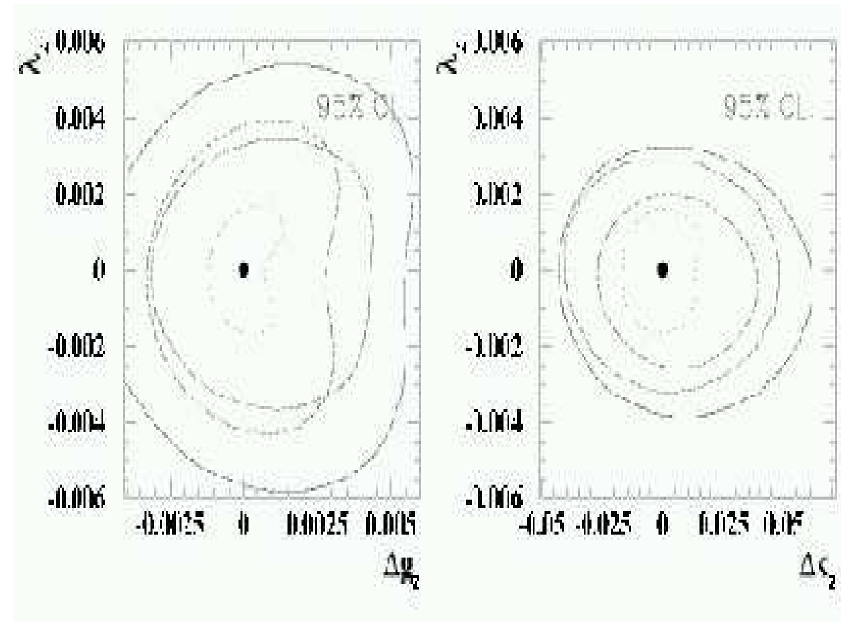
In practice: introduce deviations from their tree–level SM values

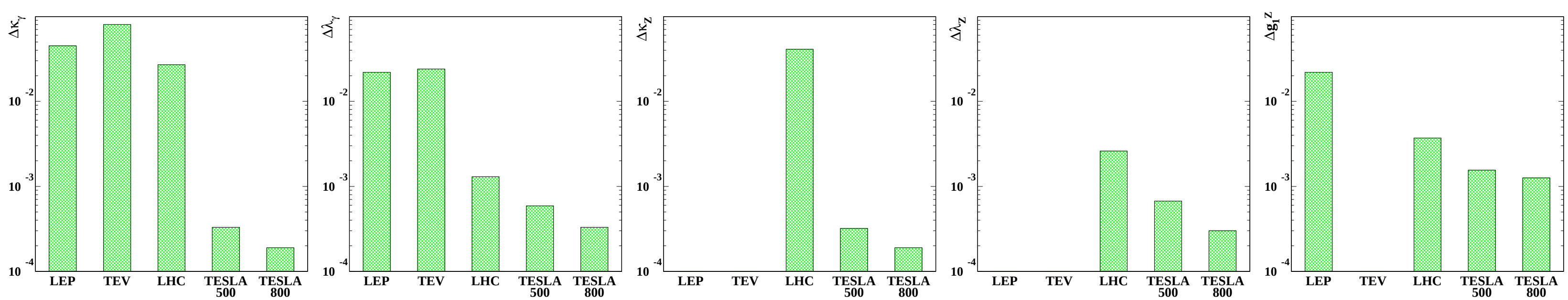
$$\Delta g_1^Z \equiv (g_1^Z - 1) \, , \;\; \Delta \kappa_\gamma \equiv (\kappa_\gamma - 1) \, , \;\; \Delta \kappa_Z \equiv (\kappa_Z - 1)$$

Use all production processes, W/Z decays, distributions and spin–correlations (many observables!) to probe Δx.

5. SM at LHC: constraints on TGCs







5. SM at LHC: strong W/Z sector

In SM with Higgs field integrated out (too heavy or absent), non–linear realisation of EWSB and $\mathcal{L}_{\rm SM}^{\rm eff}$at scale below $\Lambda_{\textsf{EWSB}} = 4\pi v \approx 3\,\mathrm{TeV}$ (and if no resonance appears) is given by

$$L_1 = \frac{\alpha_1}{16\pi^2}\frac{gg'}{2}B_{\mu\nu}\,\mathrm{tr}\Big(\sigma_3 W^{\mu\nu}\Big)\ ,\ L_2 = \frac{\alpha_2}{16\pi^2}\mathrm{i}g' B_{\mu\nu}\,\mathrm{tr}\Big(\sigma_3 V^\mu V^\nu$$

$$L_3 = \frac{\alpha_3}{16\pi^2}2\mathrm{i}g\,\mathrm{tr}\Big(W_{\mu\nu}V^\mu V^\nu\Big)\ ,\ L_4 = \frac{\alpha_4}{16\pi^2}\,\mathrm{tr}\Big(V_\mu V_\nu\Big)\,\mathrm{tr}\Big(V^\mu V^\nu$$

$$L_5 = \frac{\alpha_5}{16\pi^2}\,\mathrm{tr}\Big(V_\mu V^\mu\Big)\,\mathrm{tr}\Big(V_\nu V^\nu\Big)\ ,\ \cdots L_{6,7,8,9,10}\ (\mathrm{C/P\ non-conserving}$$

Coefficients α_i related to the NP scale Λ_i^* by $\frac{\alpha_i}{16\pi^2} = \left(\frac{v}{\Lambda_i^*}\right)^2$

- **Some operators, $L_{1,2,\ldots}$, constrained by precisison data ($\Delta\rho$, ..).**
- **$L_{1,2,3}$ contribute to TGC; probed in $qq \to WW, ZZ, Z\gamma$, ...**
- **$L_{3,4,5}$ contribute to quartic couplings and parametrize strongly interacting gauge bosons; probed in $qq \to WWqq, ZZqq$**

5. SM at LHC: strong W/Z sector