# Nuclear aspects of neutrino physics 

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The title is much too broad to cover in 25 minutes. However, in many applications discussed at this conference the relevant nuclear physics is just the description of the ground states of even-even nuclei. I will restrict myself to this particular issue.

The applications I have in mind are the detection of the neutralino dark matter by observing its elastic scattering on nuclei, search for the Lepton Flavor Violation by the coherent muon conversion in the field of a nucleus, and naturally and most importantly, double beta decay.

We know a lot about ground states, certainly for the stable even-even nuclei that we have in mind. We know the binding energy, the charge density, perhaps a little less but still enough about the neutron density, the quantum numbers and energies of the single-particle states near the Fermi level, the pairing energy .....

Is that enough? It turns out that it is enough for the first two applications (neutralino scattering and muon conversion), but not quite enough for $\beta \beta$ decay.

Why is that? Basically, the first two applications are truly elastic from the nuclear point of view. They involve coherent interaction with the whole nucleus (in the limit of vanishing momentum transfer), with a rather simple extension to the finite momentum transfer.

The neutralino scattering on even-even nuclei (spin-independent) is described by the formula $d \sigma / d q^{2}=\left(G_{F}{ }^{2} / v^{2}\right)\left(c_{0}{ }^{2} / 4 \pi\right) A^{2} F^{2}(q), q=\sqrt{2 M_{A} T}$, where $T$ is the nuclear recoil kinetic energy.

Here $c_{0}$ is a small parameter containing all information about the particle-physics aspect of the problem, $A$ is the nuclear mass number, and $F(q)$ is the Fourier transform of the spherical nuclear ground state density distribution. So, $F(q)$ is all the nuclear information we need to know. As simple as that.


Similarly, in the coherent muon to electron conversion, a test of Lepton Flavor Violation
$\mu^{-}+(Z, A) \longrightarrow e^{-+}(Z, A)$,
the transition amplitude again depends (to a good approximation) on the Fourier transforms of the proton and neutron densities
$F_{Z, N}(q)=1 /(Z, N) \quad d^{3} r \rho_{p, r}(r) e^{-i q r}$ for $|q|=m_{\mu}-\varepsilon_{b}$
(I consider only the coherent process, transitions to the excited states are essentially unobservable)

Thus in both these applications to the elastic processes all we have to know is the density.

## Double beta decay, for both modes, is quite different:

Now we are dealing with two nuclei, $Z, N$ and $Z+2, N-2$. To see the issue involved lets consider a hypothetical case, a $\beta \beta$ decay of ${ }^{208 \mathrm{~Pb}}$ to ${ }^{208 \mathrm{P}}$.

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Single particle states

## In reality the situation is much more complicated:

1) In real nuclei there is no sharp Fermi surface. The states above $E_{F}$ are not totally empty, and the states below $E_{F}$ are not totally occupied.
2) In real nuclei there is configuration mixing:

In the preceding example the final nucleus would have the final neutron holes distributed among $3 p_{1 / 2}, 3 p_{3 / 2}$ and $1 i_{13 / 2}$ and the final protons will be in either $1 h_{9 / 2}$ or $2 f_{7 / 2}$, with smaller but nonvanishing components in the further single-particle states.

Thus the evaluation of the $\beta \beta$ decay nuclear matrix element is not easy.

Simplest attempt: We know that even-even nuclei all have $I^{\pi}=0^{+}$ground states. So lets first of all include the nuclear force responsible for that, namely the like particle pairing. We shall assume that both nuclei contain only $\mathrm{O}^{+}$pairs, distributed around $\mathrm{E}_{\mathrm{F}}$ according to the rules of the BCS theory.

The closure (a good approximation) $M^{0 v}$ is then
$\begin{aligned} M^{0 v}= & \Sigma_{(p, n)}<\left(j_{p} j_{p}\right)^{I=0}\|O\|\left(j_{n} j_{n}\right) I=0>u_{p} v_{n} v_{p}{ }^{\prime} u_{n}{ }^{\prime} \\ & {\left[\left(2 j_{p}+1\right)\left(2 j_{n}+1\right)\right]^{1 / 2}\left\langle\Psi_{i n} \mid \Psi_{f i}\right\rangle, \quad(1) }\end{aligned}$
where the overlap factor signifies the difference between the initial and final nuclei and $u_{p} v_{n}$ refer to the initial, and $v_{p}{ }^{\prime} u_{n}$ to the final nucleus.

It turns out that this formula significantly overestimates the value of $M^{0 v}$.

If the "pairing only", eq.(1), overestimates the $M^{0 v}$ we should ask, why?, and what is missing?
The reason is that the true ground states contain besides the $0^{+}$Cooper pairs of like nucleons admixtures of "broken pair" or "higher seniority" states.

They are there because the residual force contains other components, besides the pairing force. In particular, the neutron-proton force must be included.
The neutron-proton force will cause, among other things, admixtures of states of the type
$\left[\left(j_{n} j_{n}{ }^{\prime}\right)^{I}\left(j_{p}{ }^{-1} j_{p}{ }^{\prime-1}\right)^{I}\right]^{0}$ (two particle, two hole) in the initial state, and analogous ones in the final state.

In QRPA such admixtures are governed by the strengths of the effective $p-n$ force, usually called $g_{p h}$ and $g_{p p}$. They reduce the magnitude of $M^{0 v}$ significantly.


Here is what happens when we begin adding in QRPA the contributions of the "broken pair" admixtures with the angular momentum I.
The leftmost entry is the $I=0$ i.e. the "pairing only". As we add more and more I states, the matrix element is drastically reduced.
This is from Rodin et al. for what we consider realistic values of $g_{p p}$. In SM a similar, perhaps rest drastic, reduction appears. This is presumably related to fewer s.p. states included.

If we wish to check whether the theoretical evaluation of $M^{0 v}$ makes sense, we might want to test the "pairing" and "broken pairs" parts separately.

The "pairing" part is independent of the neutron-proton interaction (and hence of $g_{\mathrm{pp}}$ and $g_{\mathrm{ph}}$ ) but depends on the degree of smearing of the Fermi level which in QRPA is characterized by the magnitude of the pairing gap $\Delta$.

The $0 v$ matrix elements depend on the degree of smearing of the Fermi level, parametrized through the magnitude of the pairing gap $\Delta$


Thus, when calculating $M^{0 v}$ for semimagic nuclei, we must be careful.

To proceed further, we need a bit of QRPA formalism:
The basic equations to solve are
where $A$ and $B$ are matrices that depend on the hamiltonian.

$$
\left(\begin{array}{cc}
A & B \\
-B & -A
\end{array}\right]\left[\begin{array}{l}
X \\
Y
\end{array}\right]=\omega\left(\begin{array}{l}
X \\
Y
\end{array}\right],
$$

In particular it depends on the interaction strengths $g_{p h}$ and $g_{p p}$. We obtain the "pairing only" part for $g_{p h}$ and $g_{p p}=0 ; Y=0$ in that case.

Once $g_{\text {ph }}$ and $g_{\text {pp }}$ are 0 , the vector $Y$ is also 0 and the


The dimension of the matrices $A$ and $B$ is given by the number of possible $p, n$ combinations included. There is no obvious prescription how many of these combinations one should include. Different people take different number of them. Does it matter?
The answer is yes and no.


Here the contribution of the $1+$ multipole, and of all other ones is plotted against the parameter $g_{p p}$ that signifies the strength of the neutron-proton interaction. The nominal value is $g_{p p}=1.0$. The dots denote the adjusted value that reproduces the $2 v$ decay rate in each nucleus. Note the steep slope of the $1^{+}$curve and the relatively gentle slope of the dashed lines.
This suggests that adjusting properly the contribution of the $1^{+}$is important.

In QRPA the $0 v$ matrix element depends on the number of s.p. states included. However, that dependence is drastically reduced if we adjust the coupling strength $g_{p p}$ accordingly (from the $2 v$ decay here).


This is a more general conclusion, adjusting $g_{\text {pp }}$ to $2 v$ decay rate makes the $\mathrm{M}^{\circ v}$ essentially independent of adjustable input.

V.Rodin, A. Faessler, F. Simkovic, and P. Vogel, PRC68,044302(2003),NPA766,107(2006).

Additional results in blue, selfconsistent renormalized QRPA (SRQRPA)


So far we did not specify the form of the operator $O$ that changes two neutrons into two protons.

In momentum representation the operator $O$ consists of three parts:
$O=\left[-1 / g_{A}{ }^{2}(\right.$ Fermi $)+\sigma_{1} \sigma_{2} f_{G T}\left(q^{2}\right)(G T)-S_{12} f_{T}\left(q^{2}\right)($ Tensor $\left.)\right] / q(q+E) \tau_{1}{ }^{+} \tau_{2}{ }^{+}$
$S_{12}=3\left(\sigma_{1} q\right)\left(\sigma_{2} q\right)-\sigma_{1} \sigma_{2}$ is the tensor operator and
$f_{G T}=1-f_{T}$, and $f_{T}=2 q^{2} / 3\left(q^{2}+m_{\pi}^{2}\right)-\left[2 q^{2} / 3\left(q^{2}+m_{\pi}^{2}\right)\right]^{2}$ is related
to the induced pseudoscalar current and is not included by everybody.

The inclusion (or not) of the induced pseudoscalar current is one of the reasons for the differences between calculated Ov matrix elements ( $\sim 30 \%$ reduction when included).

Once the operator $O$ is specified, we have to evaluate the two-body matrix element of the type $\left.\left\langle\left(j_{p} j_{p}\right)^{I}\right| O \mid\left(j_{n} j_{n}\right)^{I}\right)^{\text {. }}$. The states $j_{p}$ and $j_{n}$ describe nucleons moving in the mean field (harmonic oscillator or Woods-Saxon). Only the angular momenta are coupled, so there is nothing to prevent the two nucleons in $\left|\left(j_{p} j_{p}\right)^{I}\right\rangle$ to be close to each other.

However, the nucleon-nucleon force is highly repulsive at short distances, and the form $\left|\left(j_{p} j_{j}\right)^{\text {I }}\right\rangle$ does not reflect that.

Two-nucleon probability distribution, with and without correlations, MC with realistic interaction. O. Benhar et al. RMP65.817(1993)


This is not important for many operators. But the operator $O$ of $0 v$ decay prefers small $r_{12}$ so the role of the short range repulsion is enhanced.
Ideally, we should use an effective operator $O_{\text {eff }}$ that respects the fact that two nucleons should not be close to each other. But the exact form of $O_{\text {eff }}$ has not been worked out.

Instead, it is customary to use fOf instead of $O$ where $f=1-\exp \left(-\gamma_{1} r^{2}\right)\left[1-\gamma_{2} r^{2}\right], \gamma_{1} \sim 1.1 \mathrm{fm}^{2}, \gamma_{2} \sim 0.68 \mathrm{fm}^{2}$ in order to prevent short internucleon distance to contribute to the 0v matrix element.

So inclusion (or not) of the Jastrow function $f$ is another (and probably main) reason for the differences between the calculated $0 v$ matrix elements.

The effect of short range repulsion is contraintuitive. It is enhanced due to the partial cancellation between the "pairing, J=0" part, which is reduced by $\sim 30 \%$ and the "broken pairs" part which is reduced only by ~10\%.
The difference is then reduced by a factor of $\sim$ two.
(The internucleon spacing in Cooper pairs is on average less than in the "broken pair" with J O. Hence the factor $f^{2}$ affects the pairing part more than the "proken pairs" part.)

The integrand of $M^{0 v}, M^{0 v}=\int P(r) d r$ based on a semirealistic, exactly solvable model, see J. Engel and P.V., PRC69,034304 (2004).



## Comparison of $M^{0 v}$ of Rodin et al. (RQRPA) and the shell model results reported by A. Poves at NDM06

Nucleus
${ }^{76} \mathrm{Ge}$
${ }^{82} \mathrm{Se}$
${ }^{96} \mathrm{Zr}$
${ }^{100 \mathrm{Mo}}$
${ }^{116} \mathrm{Cd}$
${ }^{130} \mathrm{Te}$
${ }^{136} \mathrm{Xe}$

| RQRPA | Poves |
| :--- | ---: |
| $2.3-2.4$ | 2.35 |
| $1.9-2.1$ | 2.26 |
| $0.3-0.4$ |  |
| $1.1-1.2$ |  |
| $1.2-1.4$ | 2.13 |
| 1.3 | 1.77 |

Note that the SM calculations include the reduction caused by the s.r.c. but not by the induced currents (about $30 \%$ reduction). Also note that the previous (tentative and preliminary) results as privately communicated by F. Nowacki in 2004 included a rather small values for ${ }^{100} \mathrm{Mo}$ and ${ }^{96} \mathrm{Zr}$, similar to the 'hole' for ${ }^{96} \mathrm{Zr}$ in QRPA. It remains to be seen whether this feature persists.

## Summary:

- I tried to explain why proper evaluation of the $M^{0 v}$ matrix elements is difficult, and why different people can obtain different results.
- I stressed that quite generally there are two opposing tendencies. Pairing (i.e. existence of $0^{+}$like-nucleon pairs) leads to smearing of the Fermi level and increases $M^{0 v}$. On the other hand the neutron-proton force is responsible for presence of "broken pairs" (or higher seniority) states that decrease $M^{0 v}$.
- To check these aspects of the problem, one can perhaps use the two-nucleon transfer reactions to determine the population of s.p. states ( $u$ and $v$ factors), and one particle transfer reactions to study the multipole strengths in the intermediate odd-odd nuclei to test the opposing tendency (broken pair contribution).


## Summary continuation:

- A different part of the problem is the proper choice of the interaction hamiltonian (in particular of $g_{p p}$ ) and of the transition operator $O$. In other words, a consistent and as rigorous as possible determination of the effective hamiltonian and the effective $0 v \beta \beta$ transition operator.
-These issues are the main reason for the variation between the calculated $M^{00}$ by different authors. In practice the issues to resolve are whether to include (or not) and how the short range nucleon-nucleon repulsion, whether to include (or not) the induced nucleon currents, and how to properly choose the magnitude of $g_{\mathrm{pp}}$.
-These are not problems of nuclear structure per se. They canno $\dagger$ (at least I do not think that they can) be resolved by performing nuclear experiments.
- In any case, these issues must be resolved somehow if we wish to make within QRPA and its generalizations a realistic estimate of the proper value, and associated uncertainty, of $M^{0 v}$.

Spares:


Another illustration of the cancellation between the "pairing" and "broken pairs" parts. They are here expressed in terms of the virtual multipole states $J^{\pi}$ of the intermediate odd-odd nucleus.
This is for ${ }^{100}$ Mo, 13 s.p. states, $g_{\text {pp }}$ is adjusted to the $2 v \beta \beta$ rate. Short-range correlations are included.

Spares:


Many multipoles contribute in each case. Most of them, with the exception of $1^{+}$, have the same sign.
This is from Rodin et al, other calculations get a similar pattern.

## Spares:

from Civitarese \& Suhonen, Phys. Lett. B626,80(2005), (same Figs. in Nucl.Phys. A761,313(2005)




