New parameterization in muon decay and type of emitted neutrino

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Ref. 1) M.Doi, T. Kotani and H.N, Prog.Theor.Phys.114(2005),845-871 (hep-ph/0502136)

1. Introduction

- ν-less DBD is a best subject to determine the type of ν (the Dirac or Majorana type).
- As a complementary study, it is meaningful at present to survey the possibility of whether μ decay can be used as a tool to determine the type of ν.

$$\Downarrow$$

- We propose a new parameterization of the e^+ energy spectrum of the μ^+ decay that is suitable for investigating a deviation from the standard model and discriminating between the types of ν's.
- We propose a method in which the χ^2 value experimentally determined by assuming the Dirac type ν is compared with those determined by the Majorana type ν. This may provide a test to determine the type of ν, although it is indirect.

2. General framework

- We assume the following effective weak interaction Hamiltonian:

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}}\{j^{\dagger}_{eL\,\alpha} j^{\alpha}_{\mu L} + \lambda j^{\dagger}_{eR\,\alpha} j^{\alpha}_{\mu R} + \eta j^{\dagger}_{eR\,\alpha} j^{\alpha}_{\mu L} + \kappa j^{\dagger}_{eL\,\alpha} j^{\alpha}_{\mu R}\} + \text{H.c.} \quad (1)$$

  - This is expected from the gauge models that contain both $V-A$ and $V+A$ currents: The appearance of the coupling constant λ is due to W_R, while terms with η and κ come from the possible mixing between W_L and W_R.
  - In the $SU(2)_L \times SU(2)_R \times U(1)$ gauge model, $\kappa = \eta$

- The left- and right-handed charged weak leptonic currents:

$$j_{\ell L\,\alpha} = \sum_{j=1}^{2n} \overline{E_\ell}\gamma_\alpha(1-\gamma_5)U_{\ell j}N_j, \quad j_{\ell R\,\alpha} = \sum_{j=1}^{2n} \overline{E_\ell}\gamma_\alpha(1+\gamma_5)V_{\ell j}N_j \quad (2)$$

  for the case of the n generations. Here $U_{\ell j}$ and $V_{\ell j}$ are, respectively, the left- and right-handed lepton mixing matrices. E_ℓ is the mass eigenstate of charged leptons and N_j is of neutrinos with mass m_j.

3.1 Differential decay rate for normal $\mu^{\pm}$ decay

- The $\mu^{\pm}$ decay takes place as

$$\mu^{\pm} \to e^{\pm} \; + \; N_j \; + \; \overline{N_k}, \tag{3}$$

  - $\overline{N_k}$ represents an antineutrino for the Dirac ν case, but it should be understood as N_k for the Majorana ν case.

- If the radiative corrections are not included, the differential decay rate for emitted $e^{\pm}$ in the rest frame of polarized $\mu^{\pm}$ is expressed as

$$\frac{d^2\Gamma(\mu^{\pm} \to e^{\pm}\nu\overline{\nu})}{dx\, d\cos\theta} = \left(\frac{m_\mu\, G_F^2\, W^4}{6 \cdot 4\,(\pi)^3}\right) A\,\sqrt{x^2 - x_0^2}\, D(x,\, \theta) \tag{4}$$

where

$$x = \frac{E}{W},\; x_0 = \frac{m_e}{W} = 9.65 \times 10^{-3},\;\; W = \frac{m_\mu^2 + m_e^2}{2\, m_\mu} = 52.8 \text{ MeV}. \tag{5}$$

  - The angle θ is the direction of the emitted $e^{\pm}$ with respect to the muon polarization vector $\vec{P_\mu}$ at the instant of the $\mu^{\pm}$ decay.
  - The allowed range of x is limited kinematically as

$$x_0 \leqq x \leqq x_{\max} = (1 - r_{jk}^2) \simeq 1 \quad \text{with} \quad r_{jk}^2 = \frac{(m_j + m_k)^2}{2 m_\mu W}. \tag{6}$$

3.2 Energy spectrum of e^+ in the μ^+ decay

- The e^+ energy spectrum part is expressed as

$$xD(x,\,\theta) = x[\,N(x) + P_\mu\,\cos\theta\,P(x)\,] \tag{7}$$

where $P_\mu = |\vec{P_\mu}|$ is the rate of muon polarization, and the isotropic part $N(x)$ and anisotropic part $P(x)$ are

$$N(x) = \frac{1}{A}\left[a_+(3x - 2x^2) + 12(\,k_{+\,c} + \varepsilon_m\,k_{+\,m}\,)\,x\,(1-x)\right] \tag{8}$$

$$P(x) = \frac{1}{A}\left[a_-(-x + 2\,x^2) + 12\,(\,k_{-\,c} + \varepsilon_m k_{-\,m}\,)\,x\,(1-x)\right] \tag{9}$$

- We ignore some small terms proprotional to m_e and neutrino masses.
- The decay formulae for the Dirac and Majorana ν's are obtained by setting $\varepsilon_m = 0$ and $\varepsilon_m = 1$, respectively.
- The first terms with setting $A = a_+ = a_- = 1$ in these $N(x)$ and $P(x)$ correspond to the theoritical predictions from the standard model.
- The well-known Michel parameterization is obtained if we choose the normalization factor $A = A_{1\,0} = a_+ + 2\,k_{+\,c}$.

3.3 Coefficients

- In the Dirac ν case:

$$a_{\pm} = \left(1 \pm \lambda^2\right) \quad , \quad k_{\pm\,c} = \left(\kappa^2 \pm \eta^2\right)/2 \tag{10}$$

  - It is assumed that all ν's can be emitted in the μ decay. Then we have $\Sigma_j |U_{\ell j}|^2 = \Sigma_j |V_{\ell j}|^2 = 1$.

- In the Majorana ν case:

$$\begin{aligned} a_{\pm} &= \left[\left(1 - \overline{u_e}^{\,2}\right)\left(1 - \overline{u_\mu}^{\,2}\right) \pm \lambda^2\, \overline{v_e}^{\,2}\, \overline{v_\mu}^{\,2}\right] \\ k_{\pm\,c} &= \left[\kappa^2 \left(1 - \overline{u_e}^{\,2}\right) \overline{v_\mu}^{\,2} \pm \eta^2\, \overline{v_e}^{\,2} \left(1 - \overline{u_\mu}^{\,2}\right)\right]/2 \\ k_{\pm\,m} &= \left[\,\kappa^2\, |\,\overline{w_{e\mu}}\,|^2 \pm \eta^2\, |\,\overline{w_{e\mu\,h}}\,|^2\,\right]/2 \end{aligned} \tag{11}$$

  - $\overline{u_\ell}^{\,2}$, $\overline{v_\ell}^{\,2}$, $\overline{w_{e\,\mu}}$ and $\overline{w_{e\,\mu\,h}}$ are small quantities: We assume the existence of heavy Majorana ν's which are not emitted in the $\mu^{\pm}$ decay. Then, we have $\Sigma_j' |U_{\ell j}|^2 = 1 - \overline{u_\ell}^{\,2}$ and $\Sigma_j' |V_{\ell j}|^2 = \overline{v_\ell}^{\,2}$ where the primed sums are taken over only the light ν's. In addition, the following products of U and V appear: $\overline{w_{e\mu}} \equiv \Sigma_j'\, U_{ej}\, V_{\mu j}$ and $\overline{w_{e\mu\,h}} \equiv \Sigma_k'\, V_{ek}\, U_{\mu k}$.

3.4 New parameterization of energy spectrum

- We have proposed a new parameterization that directly represents deviations from the standard model. Here we present it taking a choice of normarization factor $A_{1\,0} = a_+ + 2\,k_{+\,c}$. In this case, the well-known Michel parameterization is obtained.
- If we assume the $SU(2)_L \times SU(2)_R \times U(1)$ model, our expression for the energy spectrum of e^+ is;

$$\begin{aligned} xD(x,\theta) = x^2(3-2x) &+ 2\rho_c x^2(3-4x) + 12\varepsilon_m \rho_m x^2(1-x) \\ &+ P_\mu \xi \cos\theta x^2(-1+2x) \end{aligned} \tag{12}$$

where

$$\rho_c = \frac{k_{+\,c}}{A_{1\,0}} > 0, \quad \rho_m = \frac{k_{+\,m}}{A_{1\,0}} > 0, \quad \text{and} \quad \xi = \frac{a_- + 2k_{-\,c}}{A_{1\,0}}. \tag{13}$$

$$\xi = \frac{1-\lambda^2}{1+\lambda^2+2\,\eta^2} \quad \text{for the Dirac } \nu \text{ case,} \tag{14}$$

$$\xi \simeq 1 \quad \text{for the Majorana } \nu \text{ case,} \tag{15}$$

$$\xi = 1 \quad \text{for the standard model.} \tag{16}$$

4.1 Analysis by the TWIST group (Dirac ν case)

The experimental data have been analyzed by the TWIST group using expressions based on the Michel parameter ρ_M in which the deviation of ρ_M from 0.75 represents the deviation from the standard model.

$$xD(x,\theta) = 6x^2[(1-x) + \frac{2}{9}\rho_M(4x-3)] + P_\mu \xi \cos\theta\, x^2(-1+2x) \qquad (17)$$

- We propose to use the deviation of the spectrum from the SM spectrum $x^2(3-2x)$. i.e. our parameterization is

$$\boxed{xD(x,\theta) = x^2(3-2x) + \rho_c x^2(6-8x) + P_\mu \xi \cos\theta\, x^2(-1+2x)}$$

  - The ρ_M is related to our ρ_c as $\rho_M = 0.75 - \frac{3}{2}\rho_c$.

- By using the χ^2-fitting, the TWIST group reported,

$$\rho_M = 0.75080 \pm 0.00032(\mathrm{stat}) \pm 0.00097(\mathrm{syst}) \pm 0.00023, \qquad (18)$$

  In term of our parameter, $\rho_c = -\frac{2}{3}(8.0 \pm 3.2 \pm 9.7 \pm 2.3) \times 10^{-4}$

- The mean value is $\rho_c = -\frac{2}{3}\,8.0 \times 10^{-4}$, although $\rho_c > 0$ is satisfied within experimental uncertainty. Theoretically we have $\rho_c \simeq (\kappa^2 + \eta^2)/2 > 0$.

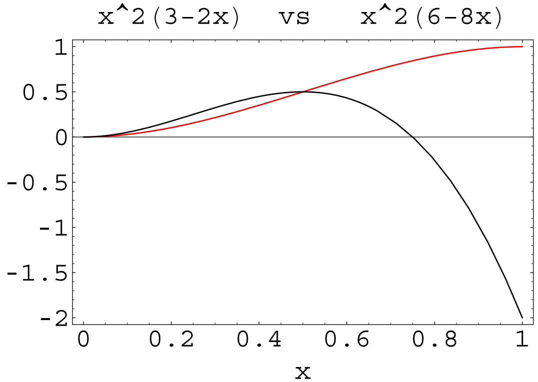
4.2 $A = A_{1,0}$ case vs $A = A_{0,0}$ case

- Analysis by TWIST group: use $A = A_{1,0}$ for the normarization factor

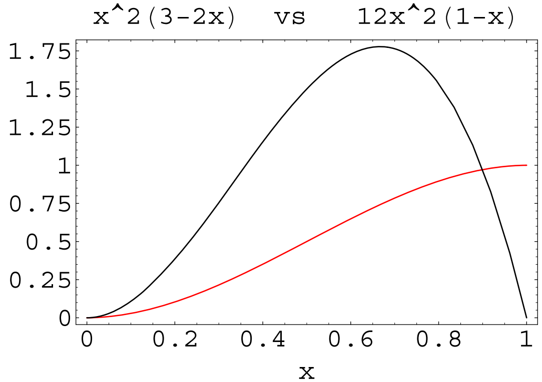
$$xD(x, \tfrac{\pi}{2}) = x^2(3-2x) + \rho_c x^2(6-8x) \qquad \text{for} \quad A = A_{1,0} = a_+ + 2k_{+c}$$

- If we use $A = A_{0,0}$,

$$xD(x, \tfrac{\pi}{2}) = x^2(3-2x) + \rho_c 12x^2(1-x) \qquad \text{for} \quad A = A_{0,0} = a_+$$



$A = A_{1,0}$ case



$A = A_{0,0}$ case

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- In the case of $A = A_{0,0}$, we will hardly obtain a negative value for ρ_c experimentally.
- Why was the negative value for ρ_c obtained experimentally in the case of $A = A_{1,0}$?

5. How to determine the type of neutrino

Our basic idea : χ^2 - fitting of energy spectrum $xD(x, = \frac{\pi}{2})$ through theoretical expressions.

$\Rightarrow$ compare χ^2 value for the Dirac ν (χ^2_d) with that for the Majorana ν (χ^2_m)

- If χ^2_m is much smaller than χ^2_d, there is a higher probability that ν's are of the Majorana type.
- The normalizaion factor A can include artificial constants n, ℓ and p:

$$A = A_{n,\ell,p} = a_+ + 2nk_{+c} + 2\ell\varepsilon_m k_{+m} + 2p\varepsilon_m \lambda d_r$$

- We parameterize the energy srectrum in order to see a deviation from the SM. Here we keep the x_0 terms, ($x_0 = \frac{m_e}{W} = 9.65 \times 10^{-3}$):

$$x_p D(x, \frac{\pi}{2}) = x_p(3x - 2x^2 - x_0^2) + H(x), \qquad x_p \equiv \sqrt{x^2 - x_0^2} \tag{19}$$

- Dirac ν case:

$$\boxed{H(x) = \rho_c x_p \{(12-6n)x - (12-4n)x^2 + 2nx_0^2\}}$$

$$\rho_c = \tfrac{1}{2}(\kappa^2 + \eta^2) > 0$$

- Majorana ν case:

$$\boxed{H(x) = (\rho_c \ + \ \rho_m) x_p \{(12-6q-6x_0\gamma)x - (12-4q)x^2 + 2qx_0^2 + 6x_0\gamma\}}$$

$$\rho_c \simeq \tfrac{1}{2}(\kappa^2\,\overline{v_\mu}^{\,2} + \eta^2\,\overline{v_e}^{\,2}) > 0, \quad \rho_m \simeq \tfrac{1}{2}(\kappa^2\,|\overline{w_{e\mu}}|^2 + \eta^2\,|\overline{w_{e\mu\,h}}|^2) > 0,$$

$q \equiv \dfrac{n\rho_c + \ell\rho_m + p\eta_m}{\rho_c + \rho_m}$ is also a artificial constant,

and $\gamma \equiv \dfrac{\eta_m}{\rho_c + \rho_m} \simeq O(1)$ with $\eta_m \simeq \lambda d_r = \lambda \frac{1}{2}\mathrm{Re}\{(\overline{w_{e\mu}})^*(\overline{w_{e\mu\,h}})\}$

$\Rightarrow$ Some differences appear only in the $x_0\gamma$ terms, although they may be difficult to be directly detected experimentally.

Summary

We have surveyed the possibility of whether μ^+ decay can be used as a tool to determine the type of ν.

- Muon decay takes place irrespective of the type of neutrino.
- There is a difference in the energy spectra of emitted e^+ between the Dirac and Majorana ν cases.

$$\Downarrow$$

- We propose a new parameterization of the e^+ energy spectrum of the μ^+ decay that is suitable for investigating a deviation from the standard model and discriminating between the types of ν's.
- We propose a method of discriminating the type of ν by using χ^2 -fitting of the e^+ energy spectrum: We propose a method in which the χ^2 value experimentally determined by assuming the Dirac type ν is compared with those determined by the Majorana type ν. This may provide a test to determine the type of ν, although it is indirect. However it may be difficult at present, because some differences appear only in the small $x_0\gamma$ terms.

Note 1. Normalization factor A

There are various possibilities for the choice of A, when experimental data are analyzed, although these choices differ only by rearrangements of the terms in the theoretical expression for the e^+ energy spectrum.
e.g.

$$f(x) + w\,g(x) = (1 + nw)\{f(x) + \tfrac{w}{1+nw}[g(x) - nf(x)]\}$$

- $A = 1$ case $\iff$ $f(x) + w\,g(x)$ is used:

  Use leading term $f(x)$ and small deviation term $wg(x)$ with weight w, then the unknown constant w is determined experimentally by minimizing the χ^2-value defined by $\chi^2 = \Sigma_k \left[\frac{ad(x_k)-y_k}{a\sigma_k}\right]^2$.
  - $y_k = f(x_k) + w\,g(x_k)$ is theoretical value. $d(x_k)$ and σ_k are, respectively, the corresponding experimental value of spectrum and its experimental error. Here a is introduced as a adjustable constant.
- $A = (1 + nw)$ case $\iff$ $f(x) + \frac{w}{1+nw}[g(x) - nf(x)]$ is used:

  We can determine the evalue of the $\frac{w}{1+nw}$ experimentally. Here n is introduced as a artificial constant.

Note 2. Analysis by the TWIST group

- The experimental data have been analyzed by the TWIST group using expressions based on the Michel parameters:

$$N(x) = 6\left[x(1-x) + \frac{2}{9}\rho_M\left(4x^2 - 3x - x_0^2\right) + \eta_M x_0(1-x)\right], \quad (20)$$

$$P(x) = 2\xi_M\sqrt{x^2 - x_0^2}\left[(1-x) + \frac{2}{3}\delta_M\left(4x - 3 - r_0^2\right)\right], \quad (21)$$

where $r_0^2 \equiv \frac{m_e^2}{m_\mu W} = \left(1 - \sqrt{1 - x_0^2}\right) = 4.66 \cdot 10^{-5}$.

- These expressions are presented for the Dirac neutrino case with $m_\nu = 0$. In the standard model, these parameters take definite values: $\rho_M = \delta_M = 0.75$, $\xi_M = 1$, and $\eta_M = 0$.
- In our model, there is no η_M term, even in the massive Dirac neutrino case, if we ignore terms proportional to m_ν. The reason why the $\eta_M \neq 0$ term appears in the Michel parameterization by Fetscher and Gerber is that it comes from the interference between the $(V \pm A)$ and $(S \pm P)$ (or T) interactions.
- The χ_d^2 value was also reported as 1804,1814,1951,1965,1993 for 5 data sets with 1887 freedom.

Note 3. $A = A_{1,0}$ *case, with ignoring* x_0 *terms*

- Dirac ν case:

$$\boxed{xD(x, \tfrac{\pi}{2}) = x^2(3-2x) + \rho_c x^2(6-8x)}$$

$$\rho_c = \tfrac{1}{2}(\kappa^2 + \eta^2) > 0$$

- Majorana ν case:

$$\boxed{xD(x, \tfrac{\pi}{2}) = x^2(3-2x) + \rho_c x^2(6-8x) + \rho_m 12x^2(1-x)}$$

$$\rho_c \simeq \tfrac{1}{2}\left(\kappa^2\, \overline{v_\mu}^{\,2} + \eta^2\, \overline{v_e}^{\,2}\right) > 0, \quad \rho_m \simeq \tfrac{1}{2}\left(\kappa^2\, |\overline{w_{e\mu}}|^2 + \eta^2\, |\overline{w_{e\mu\, h}}|^2\right) > 0$$

Note 4. $A = A_{n,\ell}$ *case, with ignoring* x_0 *terms*

We present a analysis using a normalizaion factor which includes artificial constants n and ℓ as given by

$$A = A_{n,\ell} = a_+ + 2nk_{+c} + 2\ell\varepsilon_m k_{+m} \tag{22}$$

- Dirac ν case:

$$\boxed{xD(x, \tfrac{\pi}{2}) = x^2(3-2x) + \rho_c x^2\{(12-6n) - (12-4n)x\}}$$

- Majorana ν case:

$$\boxed{xD(x, \tfrac{\pi}{2}) = x^2(3-2x) + (\rho_c + \rho_m)x^2\{(12-6q) - (12-4q)x\}}$$

  Here $q \equiv \dfrac{n\rho_c + \ell\rho_m}{\rho_c + \rho_m}$ is also a artificial constant.

- We can't determine the type of ν because the x dependences are the same.

$\Rightarrow$ Some different x-dependences appear in x_0 terms which was neglected in the present analysis.