Neutrinoless Double Beta Decay and the Neutrino Mass Matrix

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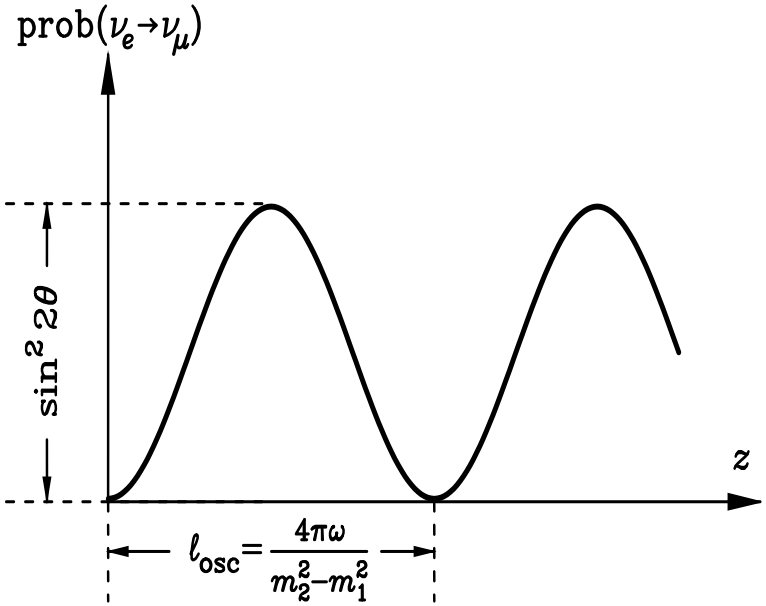
- Neutrinos and Neutrinoless Double Beta Decay
- NH vs. IH
- Neutrinoless Double Beta Decay and the Mass Matrix
- Examples for Cancellations and no Cancellation in $|m_{ee}|$

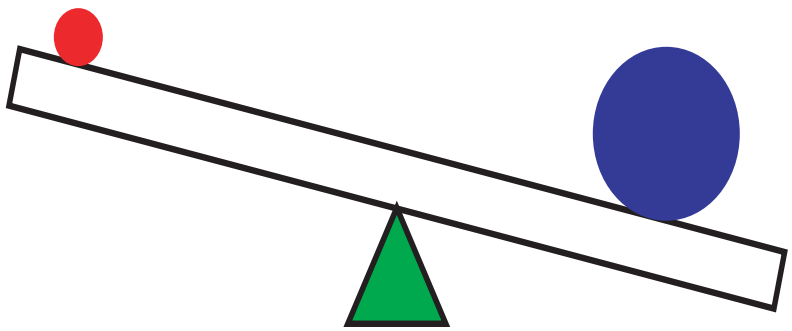
$$|m_{ee}| = \left| |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, m_2 \, e^{2i\alpha} + |U_{e3}|^2 \, m_3 \, e^{2i\beta} \right|$$

Status and goal of neutrino physics

understand form and origin of *fundamental object in low energy Lagrangian*:

$$\mathcal{L} = \frac{1}{2}\,\overline{\nu_\alpha^c}\,(m_\nu)_{\alpha\beta}\,\nu_\beta + h.c. \text{ with } m_\nu = U^*\,\mathrm{diag}(m_1, m_2, m_3)\,U^\dagger$$





$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta\,\sin^2 \frac{\Delta m^2}{4E}\,L$$

Oscillations

Masses?

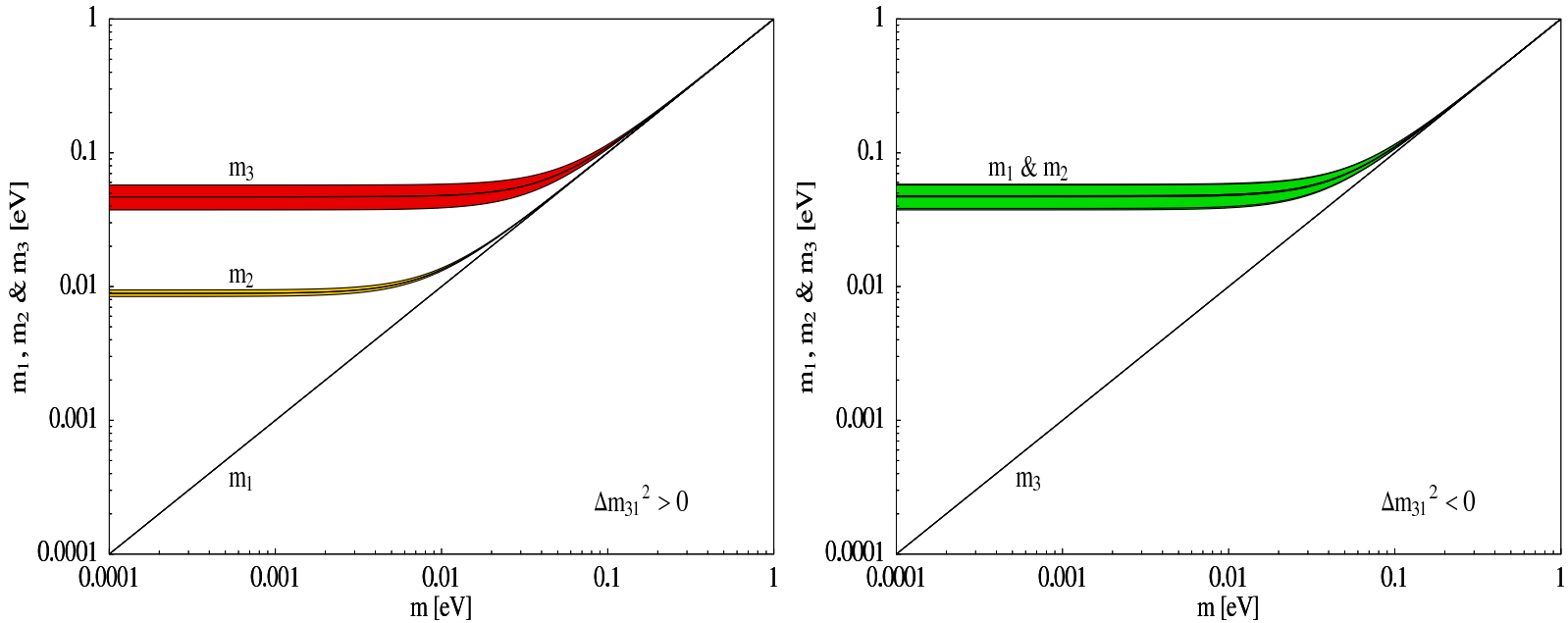
$$m_\nu = -m_D^T\,M_R^{-1}\,m_D$$

See-saw Mechanism

Lepton Number Violation!

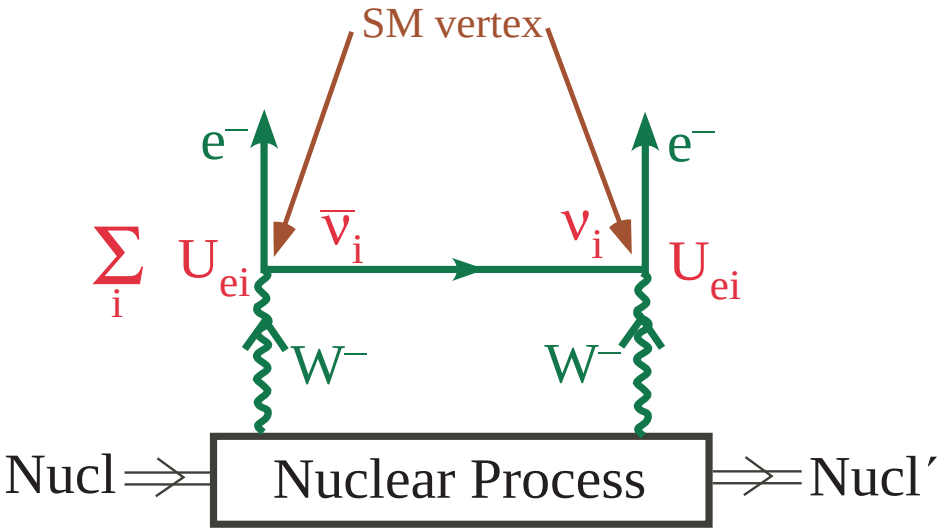
Neutrino masses

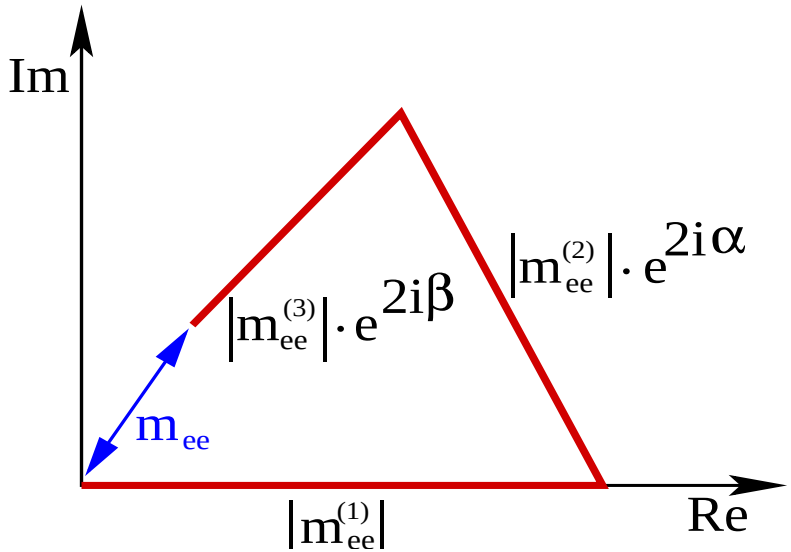
- 9 parameters in m_ν; we only know θ_{12} and θ_{23}
- neutrino masses $\leftrightarrow$ scale of their origin
- neutrino mass ordering $\leftrightarrow$ form of m_ν



- $m_3^2 \simeq \Delta m_{\rm A}^2 \gg m_2^2 \simeq \Delta m_\odot^2 \gg m_1^2$: normal hierarchy (NH)
- $m_2^2 \simeq |\Delta m_{\rm A}^2| \simeq m_1^2 \gg m_3^2$: inverted hierarchy (IH)
- $m_3 \simeq m_2 \simeq m_1 \equiv m_0 \gg \sqrt{\Delta m_{\rm A}^2}$: quasi–degeneracy (QD)

Neutrinoless Double Beta Decay



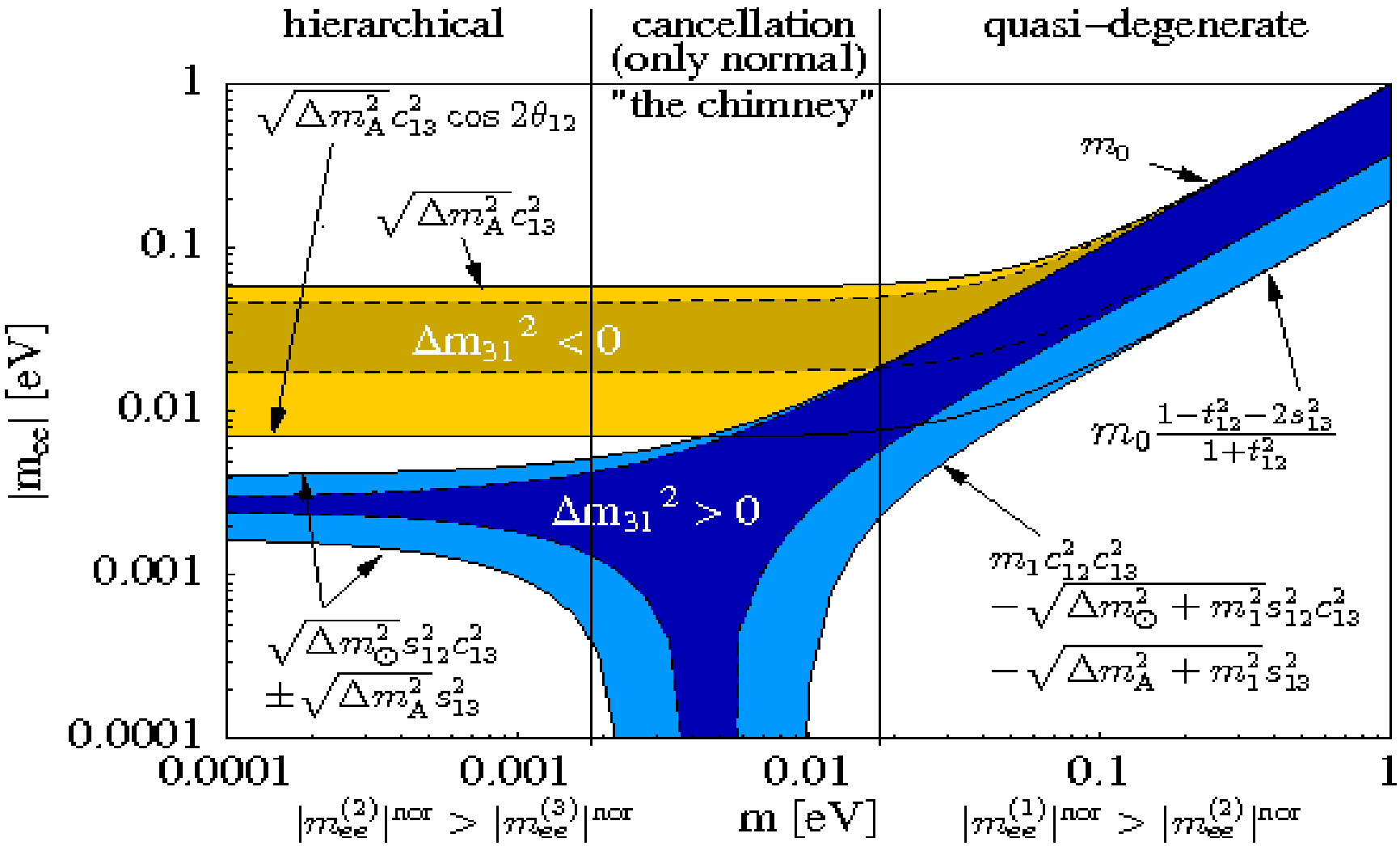


- only works when $\nu = \nu^c$ **and** $m_\nu \neq 0 \Leftrightarrow$ See–saw mechanism
- Nuclear Matrix Elements: Uncertainty $\zeta = \mathcal{O}(1)$!?

Amplitude proportional to coherent sum (“Effective mass” $|m_{ee}|$):

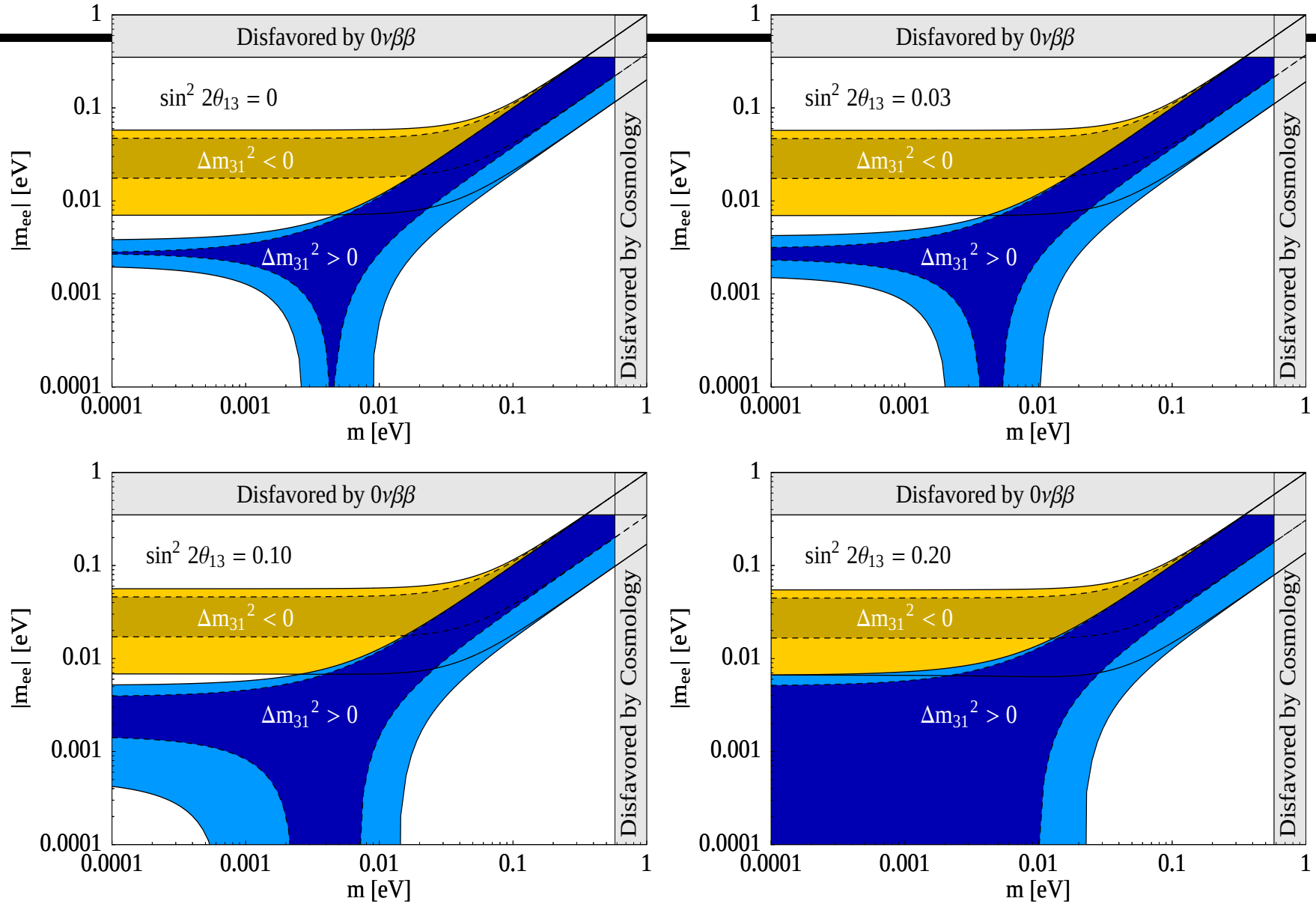
$$|m_{ee}| \equiv \left|\sum U_{ei}^2\, m_i\right| = \left|c_{12}^2\, c_{13}^2\, m_1 + s_{12}^2\, c_{13}^2\, m_2\, e^{2i\alpha} + s_{13}^2\, m_3\, e^{2i\beta}\right|$$
$$= f\left(\theta_{12}, m_i, |U_{e3}|, \mathrm{sgn}(\Delta m_{\mathrm{A}}^2), \alpha, \beta\right)$$

7 out of 9 parameters of m_ν ...



Lindner, Merle, W.R., *Phys. Rev.* D **73**, 053005 (2006)

DISTINGUISH NH FROM IH?!



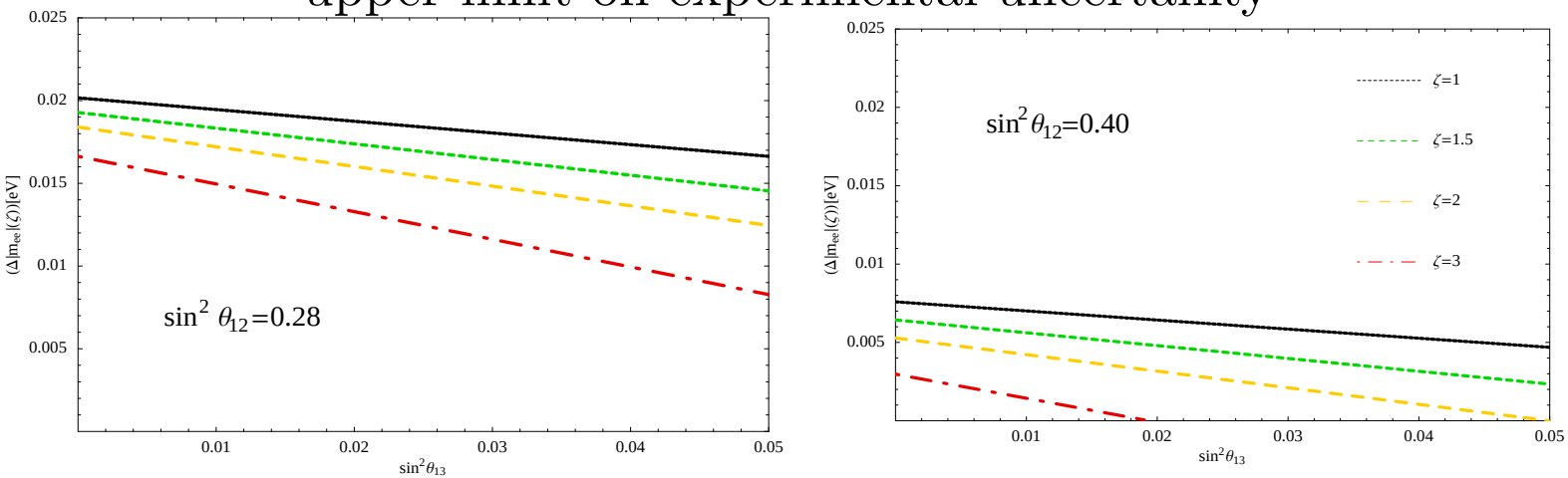
Lindner, Merle, W.R., *Phys. Rev.* D **73**, 053005 (2006)

Bilenky, Pascoli, Petcov; Klapdor, Päs, Smirnov; W.R.; Feruglio, Strumia, Vissani; Fogli *et al.*

Normal vs. Inverted Hierarchy vs. Nuclear Physics

$$\Delta|m_{ee}| \equiv |m_{ee}|^{\rm IH}_{\rm MIN} - \zeta\,|m_{ee}|^{\rm NH}_{\rm MAX} \overset{!}{>} 0$$

upper limit on experimental uncertainty



Lindner, Merle, W.R., PRD **73**, 053005 (2006);

S. Choubey, W.R.; Pascoli, Petcov, Schwetz; de Gouvea, Jenkins

- likes $\zeta \lesssim 2$
- strong dependence on θ_{12}: likes small $\sin^2\theta_{12} \lesssim 0.35$
- some dependence on U_{e3}: prefers small U_{e3} ($\leftrightarrow$ oscillations)

A simple $U(1)$ for m_ν?

With L_e, L_μ and L_τ we have only 3 allowed possibilities

L_e Normal Hierarchy Barbieri; Vissani, Buchmüller, Yanagida	$\begin{pmatrix} 0 & 0 & 0 \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix}$	$R = \frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}} \simeq \lvert U_{e3}\rvert^2$ $\tan^2\theta_{23} \simeq 1 + \lvert U_{e3}\rvert \simeq 1 + \sqrt{R}$ $\lvert m_{ee}\rvert \simeq \sqrt{\Delta m^2_{\rm A}}\,\lvert U_{e3}\rvert^2 \simeq \sqrt{\Delta m^2_\odot}$
$L_e - L_\mu - L_\tau$ Inverted Hierarchy Petcov;...	$\begin{pmatrix} 0 & a & b \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	requires U_ℓ: ideal for QLC $\tan^2\theta_{12} \simeq 1 - 4\,\lvert U_{e3}\rvert \simeq 1 - 2\sqrt{2}\sin\theta_{\rm C}$ $\lvert m_{ee}\rvert \simeq \sqrt{\Delta m^2_{\rm A}}$
$L_\mu - L_\tau$ quasi–degenerate νs Choubey, W.R.	$\begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$	only case with $\mu - \tau$ symmetry!! $\Rightarrow U_{e3} = 0$ and $\theta_{23} = \pi/4$ $\lvert m_{ee}\rvert \simeq m_0$

S. Choubey, W.R., *Phys. Rev.* D **72**, 033016 (2005)

($L_e + L_\mu + L_\tau$ means Dirac neutrinos...)

Precision Data, $|m_{ee}|$ and the Mass Matrix

Normal hierarchy: $|m_{ee}| \simeq \sqrt{\Delta m^2_\odot}$

$$m_\nu = \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & d\,\epsilon \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix} \leftrightarrow \left\{ \begin{array}{c} \mu - \tau \text{ symmetry} \\ \text{predicts } \theta_{23} = \pi/4 \text{ and } U_{e3} = 0 \end{array} \right.$$

$$\begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & b\,\epsilon \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix} \quad \text{gives} \quad \left\{ \begin{array}{c} \theta_{13} = \mathcal{O}\left(\sqrt{\dfrac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}}\right) \\ \theta_{23} - \pi/4 = \mathcal{O}\left(\dfrac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}\right) \end{array} \right.$$

$$\begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & d\,\epsilon \\ \cdot & 1+a\,\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix} \quad \text{gives} \quad \left\{ \begin{array}{c} \theta_{13} = \mathcal{O}\left(\dfrac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}\right) \\ \theta_{23} - \pi/4 = \mathcal{O}\left(\sqrt{\dfrac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}}\right) \end{array} \right.$$

Mohapatra, JHEP **0410**, 027 (2004)

Breaking μ–τ Symmetry by a phase

Mohapatra, W.R., Phys. Rev. D **72**, 053001 (2005)

Normal hierarchy

$$m_\nu = \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & d\,\epsilon \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix} \longrightarrow \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} c\,\epsilon^2 & d\,\epsilon & d\,\epsilon\, e^{i\alpha} \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$$

(is only possibility for breaking with phase)

Results:

$$\left.\begin{aligned} |U_{e3}| &\simeq \frac{d}{2}\,\epsilon\,\sqrt{1-\cos\alpha} \\ \sin\delta &\simeq -\cos\alpha/2 \\ \tan 2\theta_{12} &\simeq 2\,d\sqrt{1+\cos\alpha} \\ \theta_{23}-\pi/4 &\simeq -\frac{d}{2}\,\epsilon^2\cos\alpha/2 \end{aligned}\right\} \quad \Rightarrow \frac{|U_{e3}|}{\tan 2\theta_{12}} \simeq \frac{\epsilon}{4}\tan\alpha/2$$

maximal $|U_{e3}|$ for $\theta_{12}=0$

A neat special case

$$m_\nu = \frac{\sqrt{\Delta m^2_{\rm A}}}{2} \begin{pmatrix} d\,\epsilon^2 & a\,\epsilon\, e^{-i\alpha} & a\,\epsilon\, e^{i\alpha} \\ \cdot & 1+\epsilon & -1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$$

gives maximal CP violation!!

$$|U_{e3}| \simeq \frac{a}{\sqrt{2}}\,\epsilon \sin\alpha$$

$$\tan 2\theta_{12} \simeq 2\sqrt{2}\, a \cos\alpha$$

$$\theta_{23} - \pi/4 \simeq a^2\,\epsilon^2 \cos\alpha\, \sin\alpha$$

Harrison, Scott; Ma; Aizawa *et al.*; Mohapatra, W.R.

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & d\,\epsilon \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix}$	simple $U(1)$, broken L_e sequential dominance	$\lvert m_{ee}\rvert = c_1\sqrt{\Delta m^2_{\rm A}}\,\lvert U_{e3}\rvert^2$ $\lvert U_{e3}\rvert = c_2\sqrt{R},\ \theta_{23} = \frac{\pi}{4} - c_3\sqrt{R}$
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & d\,\epsilon \\ \cdot & 1+\epsilon & 1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$	$\mu\tau$ symmetry broken in e sector	$\lvert m_{ee}\rvert = c_1\sqrt{\Delta m^2_{\rm A}}\,\lvert U_{e3}\rvert^2$ $\lvert U_{e3}\rvert = c_2\sqrt{R},\ \theta_{23} = \frac{\pi}{4} - c_3\,R$
$\begin{pmatrix} a\,\epsilon^2 & b\,\epsilon & b\,\epsilon \\ \cdot & 1+d\epsilon & 1 \\ \cdot & \cdot & 1+\epsilon \end{pmatrix}$	$\mu\tau$ symmetry broken in $\mu\tau$ sector	$\lvert m_{ee}\rvert = c_1\sqrt{\Delta m^2_{\rm A}}\,\lvert U_{e3}\rvert$ $\lvert U_{e3}\rvert = c_2\,R,\ \theta_{23} = \frac{\pi}{4} - c_3\sqrt{R}$
$\begin{pmatrix} 0 & 0 & \epsilon \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix}$	2 zeros also $m_{ee} = m_{e\tau} = 0$	$\lvert m_{ee}\rvert = 0$ $\lvert U_{e3}\rvert = \sqrt{\frac{R}{\cos 2\theta_{12}}}\,\frac{\sin 2\theta_{12}}{2\,\tan\theta_{23}}$
$\begin{pmatrix} a^2\,\epsilon & a\sqrt{1-a^2}\,\epsilon & 0 \\ \cdot & b^2 + (1-a^2)\,\epsilon & b\sqrt{1-b^2} \\ \cdot & \cdot & 1-b^2 \end{pmatrix}$	minimal see–saw	$\lvert m_{ee}\rvert = \sqrt{\Delta m^2_\odot}\,\sin^2\theta_{12}$ $\lvert U_{e3}\rvert = \sqrt{R}/2\,\sin 2\theta_{12}\,\tan\theta_{23}$

S. Choubey, W.R., *Phys. Rev.* D **72**, 033016 (2005)

Inverted hierarchy

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} 0 & a & b \\ \cdot & \epsilon^2 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	broken $L_e - L_\mu - L_\tau$ and $U_\ell \sim V_{\rm CKM}$	$\langle m \rangle = \sqrt{\Delta m^2_{\rm A}} \left\lvert \cos 2\theta_{12} + 4i/\sin^2\theta_{23}\, J_{CP} \right\rvert$ $\tan^2\theta_{12} = 1 - 4\cos\delta\,\cot\theta_{23}\,\lvert U_{e3}\rvert$
$\begin{pmatrix} A & B & B/c \\ \cdot & D & D/c \\ \cdot & \cdot & D/c^2 \end{pmatrix}$	"Strong Scaling Ansatz" $\frac{m_{e\mu}}{m_{e\tau}} = \frac{m_{\mu\mu}}{m_{\mu\tau}} = \frac{m_{\tau\mu}}{m_{\tau\tau}} \equiv c$	$\sqrt{\Delta m^2_{\rm A}}\,\cos 2\theta_{12} \le \lvert m_{ee}\rvert \le \sqrt{\Delta m^2_{\rm A}}$ $m_3 = U_{e3} = 0,\ \tan^2\theta_{23} = 1/c^2$ no RGE running!
$\begin{pmatrix} 1 + a\,\epsilon & b\,\epsilon & d\,\epsilon \\ \cdot & \frac{1}{2} + f\,\epsilon & \frac{1}{2} + g\,\epsilon \\ \cdot & \cdot & \frac{1}{2} + h\,\epsilon \end{pmatrix}$	perturbed m^0_ν	$\langle m \rangle = \sqrt{\Delta m^2_{\rm A}}\,(1 + c_1\,\lvert U_{e3}\rvert)$ $\lvert U_{e3}\rvert = c_2\,R,\ \theta_{23} = \frac{\pi}{4} - c_3\,R$

S. Choubey, W.R., *Phys. Rev.* D **72**, 033016 (2005)

Matrix m_ν/m_0	comments	correlations
$\begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix} +$ sequential dominance	type II see–saw upgrade	$\lvert m_{ee}\rvert \simeq m_0$ $\lvert U_{e3}\rvert = c_1\sqrt{R},\ \theta_{23} = \frac{\pi}{4} - c_2\sqrt{R}$ phases shrink with m_0
$\begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & -1 \\ \cdot & \cdot & 0 \end{pmatrix}$	$L_\mu - L_\tau$ plus perturbations	$\lvert m_{ee}\rvert = m_0\ (1/\sqrt{2} + c_1\,\lvert U_{e3}\rvert)$ $\lvert U_{e3}\rvert = c_2\,\Delta m^2_{\rm A}/m_0^2 \lesssim 0.1$ $\theta_{23} = \pi/4 - c_3\,\lvert U_{e3}\rvert$
$\begin{pmatrix} a & \epsilon & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & d \end{pmatrix}$	also $m_{e\mu} = m_{\tau\tau} = 0$ and $m_{e\mu} = m_{\mu\mu} = 0$ and $m_{e\tau} = m_{\tau\tau} = 0$	$\lvert m_{ee}\rvert \simeq m_0 \simeq \sqrt{\frac{\Delta m^2_{\rm A}\ \tan^4\theta_{23}}{1-\tan^4\theta_{23}}}$ $R \simeq \frac{1+\tan^2\theta_{12}}{\tan\theta_{12}}\ \tan 2\theta_{23}\ \mathrm{Re}\ U_{e3}$ $\Rightarrow \theta_{23} \neq \pi/4$ and $\mathrm{Re}\ U_{e3} \simeq 0$
$r_\nu \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + c_\nu \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix}$	$S(3)_L \times S(3)_R$ democracy	$\lvert m_{ee}\rvert \simeq m_0$, requires $r_\nu \ll 1$ $\lvert U_{e3}\rvert \simeq \sqrt{m_e/m_\mu}$, θ_{23} large depends on $m_{e,\mu,\tau}$ and breaking
$\begin{pmatrix} a & b & d \\ \cdot & e & f \\ \cdot & \cdot & g \end{pmatrix}$	anarchy	$\lvert U_{e3}\rvert$ close to upper bound, θ_{23} close to bound extreme hierarchy unlikely

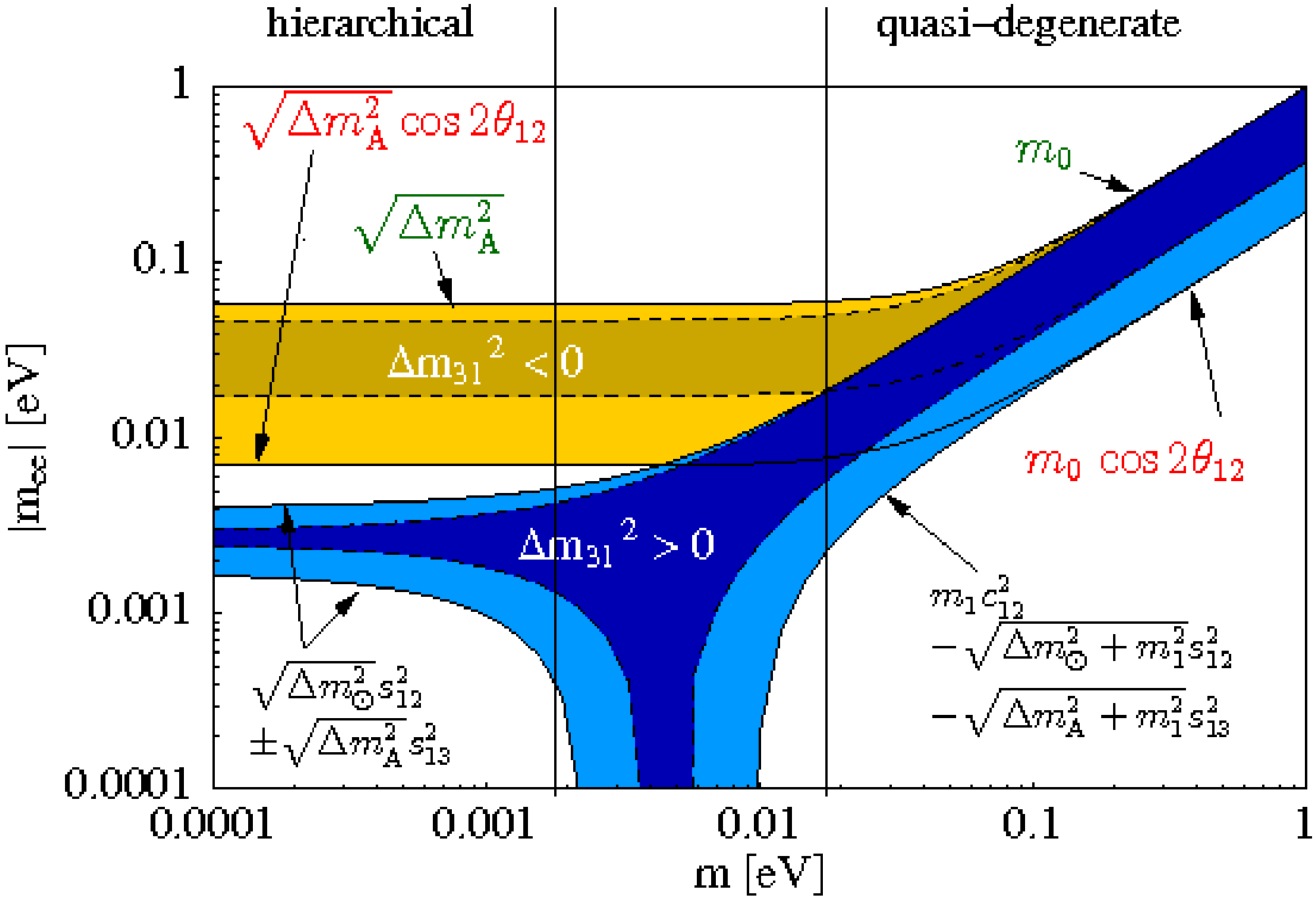
S. Choubey, W.R., *Phys. Rev.* D **72**, 033016 (2005)

Cancellation in $|m_{ee}|$

- same procedure possible for IH and QD, but little insight gained
- focused so far on μ–τ symmetry in NH: $|m_{ee}|$ hard to measure
- once you have IH or QD, the question is how much cancellation in $|m_{ee}|$?

IH:	$\sqrt{\Delta m^2_{\rm A}}\,\cos 2\theta_{12}$	$\lesssim \lvert m_{ee}\rvert \lesssim$	$\sqrt{\Delta m^2_{\rm A}}$
QD:	$m_0\,\cos 2\theta_{12}$	$\lesssim \lvert m_{ee}\rvert \lesssim$	m_0
	large cancellation		no cancellation
	$\alpha = \pi/2$		$\alpha = 0$

- Now two examples are given for large cancellation in QD and no cancellation in IH, QD



Lindner, Merle, W.R., *Phys. Rev.* D **73**, 053005 (2006)

QLC, Leptogenesis and $0\nu\beta\beta$

Parametrization (W.R., *Phys. Rev.* D **69**, 033005 (2004)):

$$U_{e2} = \sqrt{\frac{1}{2}}\,(1-\lambda) \text{ with } \lambda \simeq 0.22$$

Interpretation (Raidal, *Phys. Rev. Lett.* **93**, 161801 (2004); Minakata, Smirnov, *Phys. Rev.* D **70**, 073009 (2004)):

$$\theta_{12} + \theta_C = \frac{\pi}{4} \text{ or } |U_{e2}| + |V_{ud}| = 1/\sqrt{2}$$

"Quark-Lepton Complementarity"

Straightforward minimal implementation:

$$U = U_\ell^\dagger\, U_\nu \text{ with } U_\ell = V_{\rm CKM} \text{ and } U_\nu = \text{bimaximal}$$

- a la $SU(5)$: $m_\ell = m_{\rm down}^T \Rightarrow m_{\rm up} = \text{diag}$
- a la $SO(10)$: $m_D = m_{\rm up}$ to get see-saw $m_\nu = -m_D^T\, M_R^{-1}\, m_D = m_{\rm up}\, M_R^{-1}\, m_{\rm up}$ $\Rightarrow M_R$ gives bimaximal m_ν

Eigenvalues $M_{1,2,3}$ given by $m_{1,2,3}$ and phases $\Leftrightarrow$ Leptogenesis!

Heavy Neutrino Mass Matrix

$$m_\nu^{\rm bimax} = \begin{pmatrix} A & B\,e^{-i\phi} & -B\,e^{-i\omega} \\ \cdot & (D+\frac{A}{2})\,e^{-2i\phi} & (D-\frac{A}{2})\,e^{-i(\phi+\omega)} \\ \cdot & \cdot & (D+\frac{A}{2})\,e^{-2i\omega} \end{pmatrix} = m_D^T\,M_R^{-1}\,m_D$$

with e.g. $A = \frac{1}{2}\left(m_1 + m_2\,e^{-2i\sigma}\right)$ and $P_\nu = {\rm diag}(1, e^{i\phi}, e^{i\omega})$

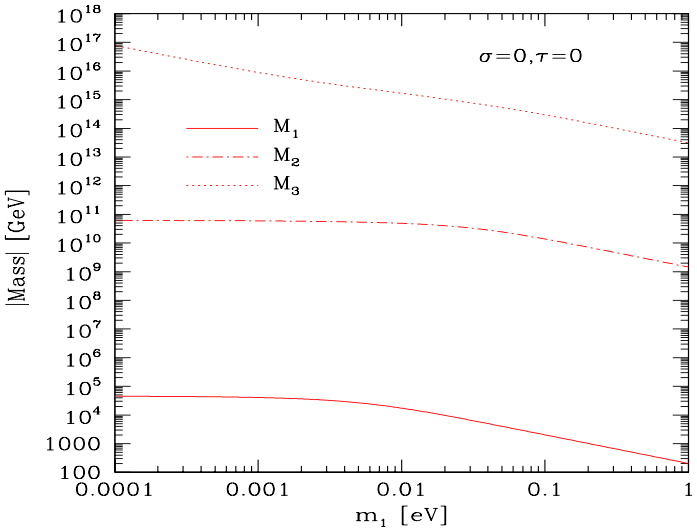
$$-M_R = m_{\rm up}\,m_\nu^{-1}\,m_{\rm up} = P_\nu \begin{pmatrix} \tilde{A}\,m_u^2 & \tilde{B}\,m_u\,m_c & -\tilde{B}\,m_u\,m_t \\ \cdot & \left(\tilde{D}+\frac{\tilde{A}}{2}\right)\,m_c^2 & \left(\tilde{D}-\frac{\tilde{A}}{2}\right)\,m_c\,m_t \\ \cdot & \cdot & \left(\tilde{D}+\frac{\tilde{A}}{2}\right)\,m_t^2 \end{pmatrix} P_\nu$$

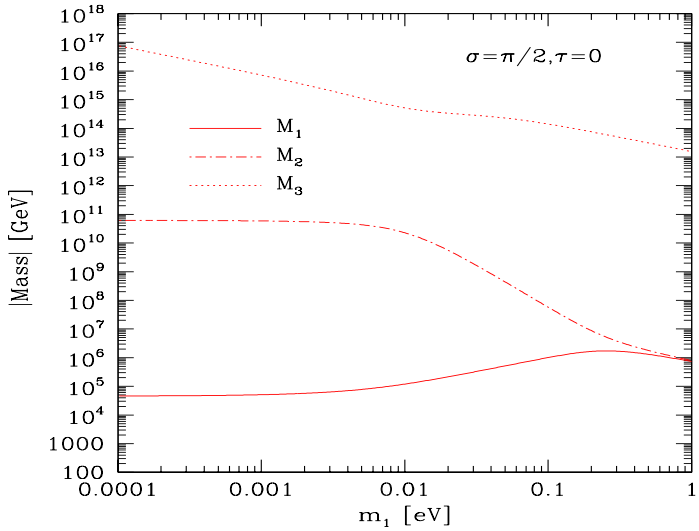
because $m_D = m_{\rm up}$ is diagonal and with e.g. $\tilde{A} = \frac{1}{2\,m_1} + \frac{e^{2i\sigma}}{2\,m_2}$

QLC, Leptogenesis and $0\nu\beta\beta$

Invert $m_\nu = -m_D^T\, M_R^{-1}\, m_D$ to get heavy Majorana masses $M_{1,2,3}$!

$M_{1,2,3}$ depend on light neutrino masses and low energy Majorana phases!!





$M_1 \lesssim 10^7$ GeV too small for successful leptogenesis $\Rightarrow$ quasi-degenerate $M_1 \simeq M_2$!

$\Rightarrow$ requires $m_1 \simeq 0.5$ eV and $\sigma \simeq \alpha \simeq \pi/2$ leading to

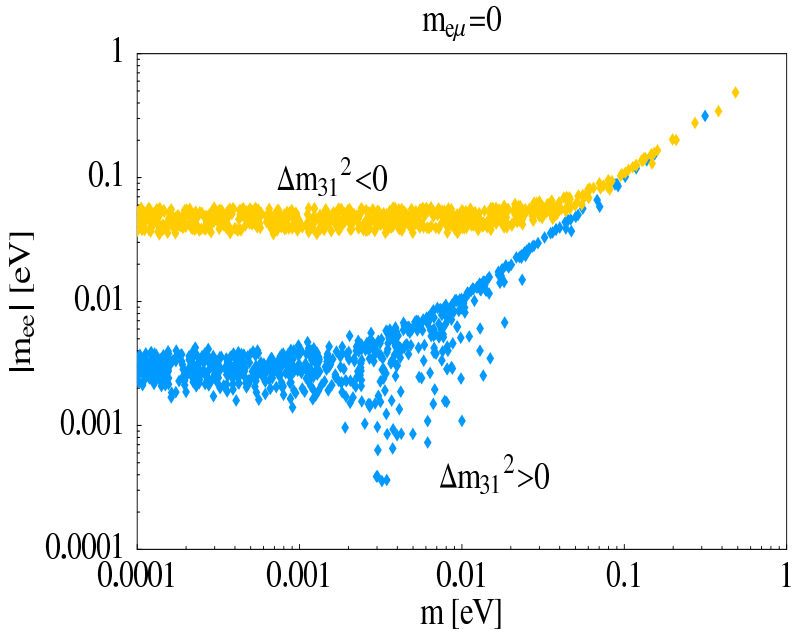
$$|m_{ee}| \simeq m_1 \cos 2\theta_{12} \simeq m_1 \sqrt{2}\,\lambda \simeq 0.16 \text{ eV}$$

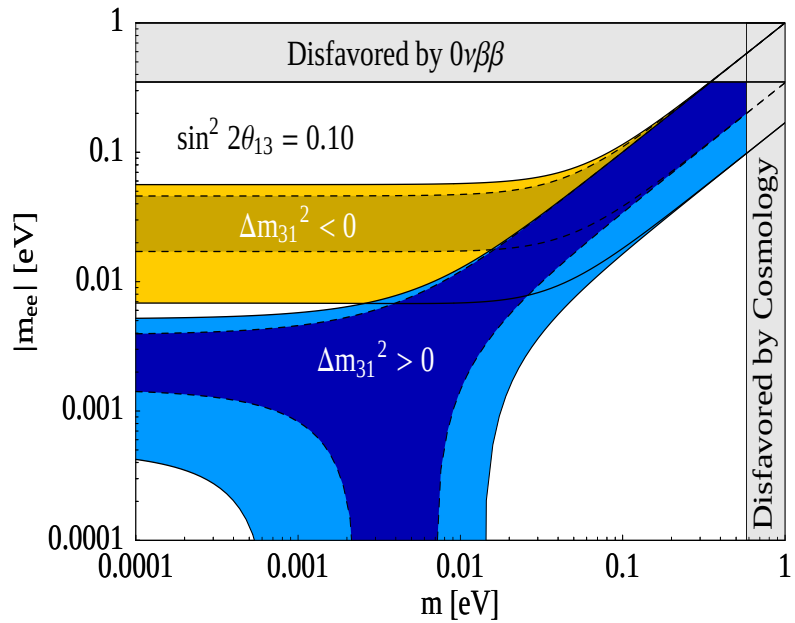
Large cancellation in $|m_{ee}|$!

K.A. Hochmuth and W.R., hep-ph/0607103

Texture Zeros in m_ν

$$|m_{e\mu}| \simeq \begin{cases} \frac{1}{2}\sqrt{\Delta m^2_\odot}\, c_{23}\, \sin 2\theta_{12} + \mathcal{O}(\theta_{13}) & \text{NH} \\ c_{23}\, \sin 2\theta_{12}\, \sqrt{\Delta m^2_{\rm A}}\, \sin\alpha + \mathcal{O}(\theta_{13}) & \text{IH} \quad \text{zero for } \alpha = 0 \Rightarrow |m_{ee}| \simeq \sqrt{\Delta m^2_{\rm A}} \\ m_0\, c_{23}\, \sin 2\theta_{12}\, \sin\alpha + \mathcal{O}(\theta_{13}) & \text{QD} \quad \text{zero for } \alpha = 0 \Rightarrow |m_{ee}| \simeq m_0 \end{cases}$$





No cancellation in $|m_{ee}|$!

(similar for $m_{\mu\mu}$, $m_{\mu\tau}$ and $m_{\tau\tau}$ in case of QD)

A. Merle and W.R., *Phys. Rev.* D **73**, 073012 (2006)

Summary

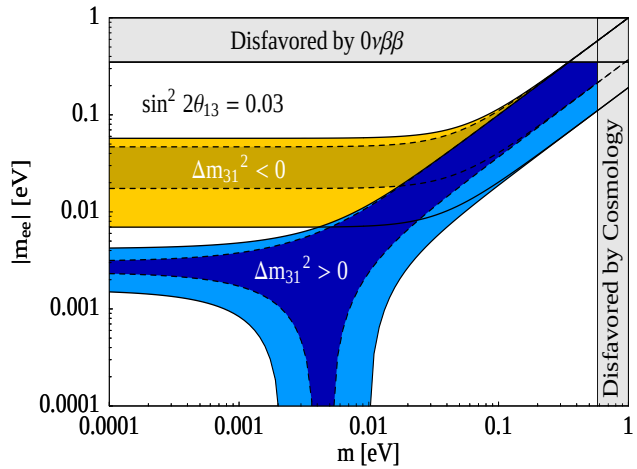
- "Neutrinos, the only -inos discovered so far"
  - oscillations $\Rightarrow m_\nu \neq 0$
  - almost all models explain $m_\nu \neq 0$ in connection with <u>Lepton Number Violation</u> $\Rightarrow 0\nu\beta\beta$
- $0\nu\beta\beta$ can
  - distinguish NH from IH
  - mass scale (consistency with KATRIN and cosmology)
  - Majorana phases (Leptogenesis?)
- Mass Matrix/Models
  - simple $U(1)$ allowed
  - breaking of μ–τ symmetry
  - cancellation or no cancellation? Examples QLC and texture zeros

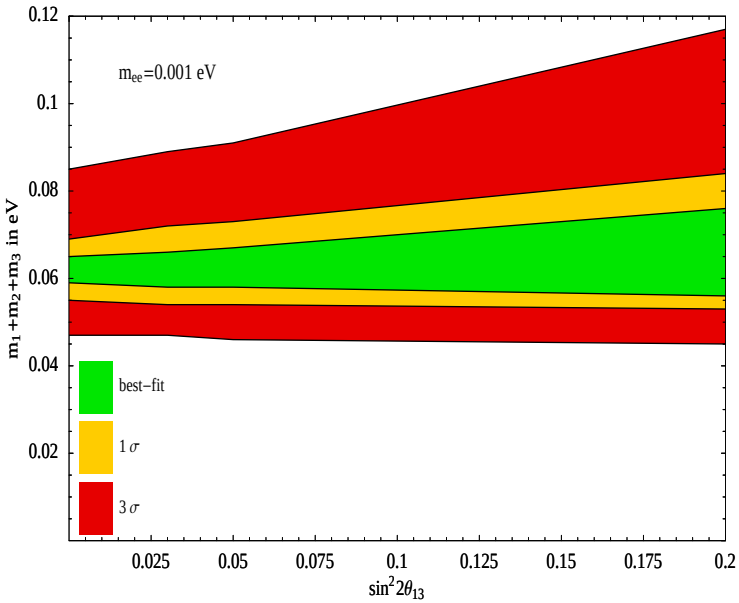
<u>$|m_{ee}|$ + precision data will help</u>

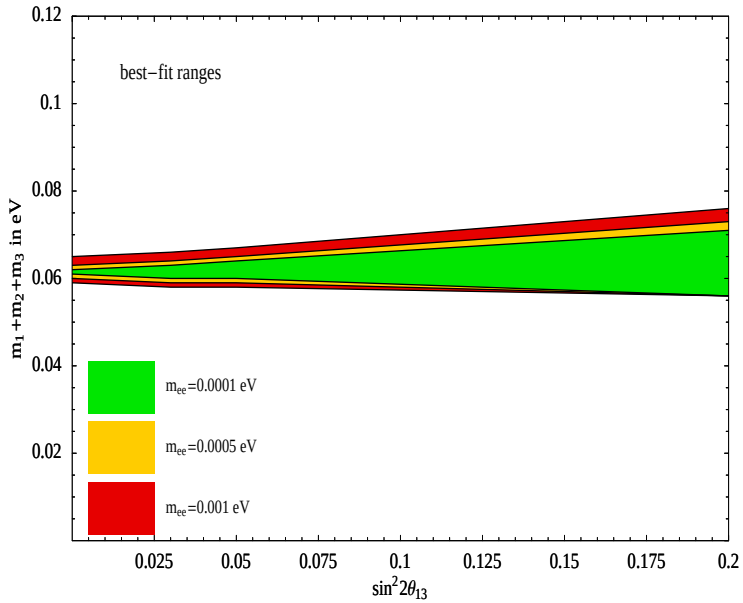
More Backup Slides than Talk Slides

(Almost) vanishing $|m_{ee}|$

means normal ordering and m_1 in "chimney"







Lindner, Merle, W.R., *Phys. Rev.* D **73**, 053005 (2006)

Scaling in the Neutrino Mass Matrix

Inverted hierarchy usually obtained by $L_e - L_\mu - L_\tau$:

$$m_\nu = \sqrt{\frac{\Delta m^2_{\rm A}}{2}} \begin{pmatrix} 0 & 1 & -1 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \text{ gives } \begin{cases} m_1 = -m_2 \ , \quad m_3 = 0 \\ U_{e3} = 0 \ , \quad \theta_{23} = \pi/4 \ , \quad \theta_{12} = \pi/4 \end{cases}$$

requires tuned and large breaking:

$$m_\nu + \sqrt{\frac{\Delta m^2_{\rm A}}{2}}\, \epsilon \begin{pmatrix} a & b & b \\ \cdot & d & e \\ \cdot & \cdot & d \end{pmatrix} \text{ gives } \begin{cases} \dfrac{\Delta m^2_\odot}{\Delta m^2_{\rm A}} \simeq \sqrt{2}\,(a+d+e)\,\epsilon \\ \sin\theta_{12} \simeq \sqrt{\frac{1}{2}} - \frac{1}{8}\,(a-d-e)\,\epsilon \end{cases}$$

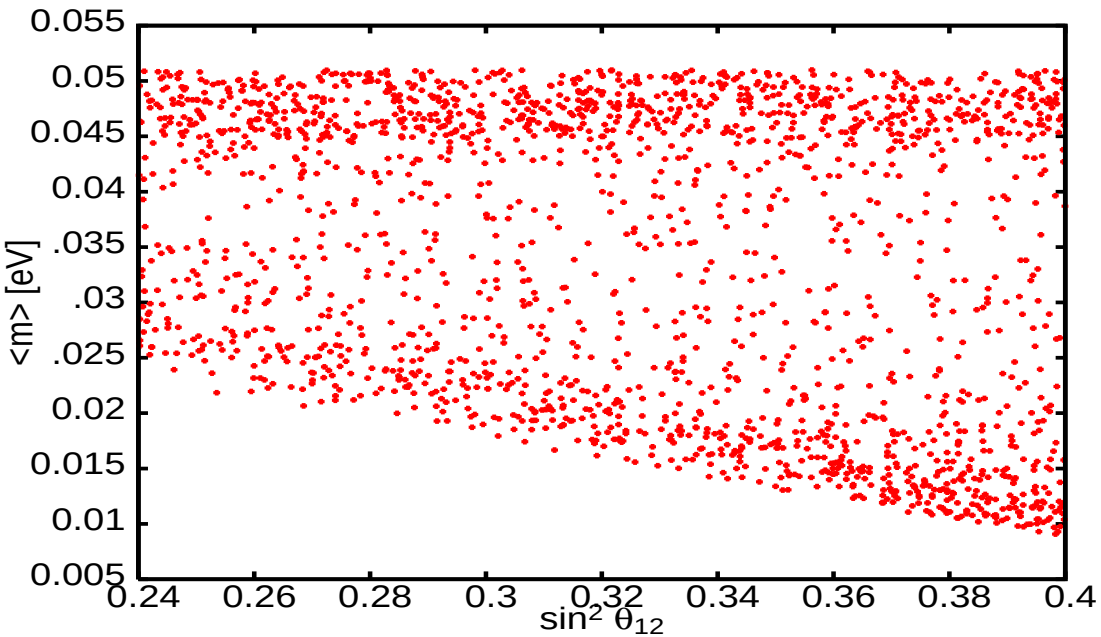
and because of neutrinoless double beta decay: $a\,\epsilon \gtrsim 0.4$

Possible Solution

"Strong Scaling Ansatz": $\dfrac{m_{\alpha\beta}}{m_{\alpha\gamma}}$ independent of flavor α

Mohapatra, W.R., hep-ph/0608111

$$m_\nu = \begin{pmatrix} A & B & B/c \\ \cdot & D & D/c \\ \cdot & \cdot & D/c^2 \end{pmatrix} \text{ gives } \begin{cases} \text{only } \beta = \mu \text{ and } \gamma = \tau \\ \text{inverted hierarchy,} \quad \tan^2\theta_{23} = 1/c^2 \\ m_3 = U_{e3} = 0 \text{ not affected by RGE!!!!} \end{cases}$$



Origin of Scaling

(1) Froggatt-Nielsen with Universality:

$m_\nu = \lambda_{\alpha\beta} \frac{v_{\rm wk}^2}{M} r^{x_\alpha + x_\beta}$ with $\lambda_{\alpha\beta} = \lambda$ (except for λ_{ee}) gives scaling with $c = r^{x_\mu - x_\tau}$

(2) Seesaw plus non-Abelian discrete symmetry group $D_4 \times Z_2$:

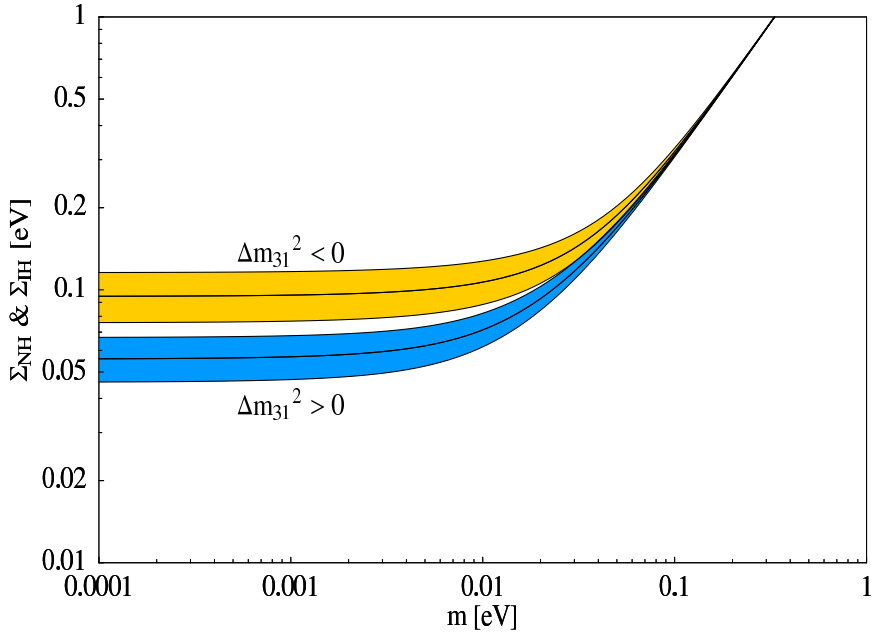
Field	$D_4 \times Z_2$ quantum number
$L_e,\ H_e,\ e_R,\ N_{e,\mu,\tau}$	1_1^+
$\begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix}, \begin{pmatrix} H_\mu \\ H_\tau \end{pmatrix}$	2^+
$\begin{pmatrix} \mu_R \\ \tau_R \end{pmatrix}$	2^-
$H_1',\ H_2'$	$1_1^-,\ 1_4^-$

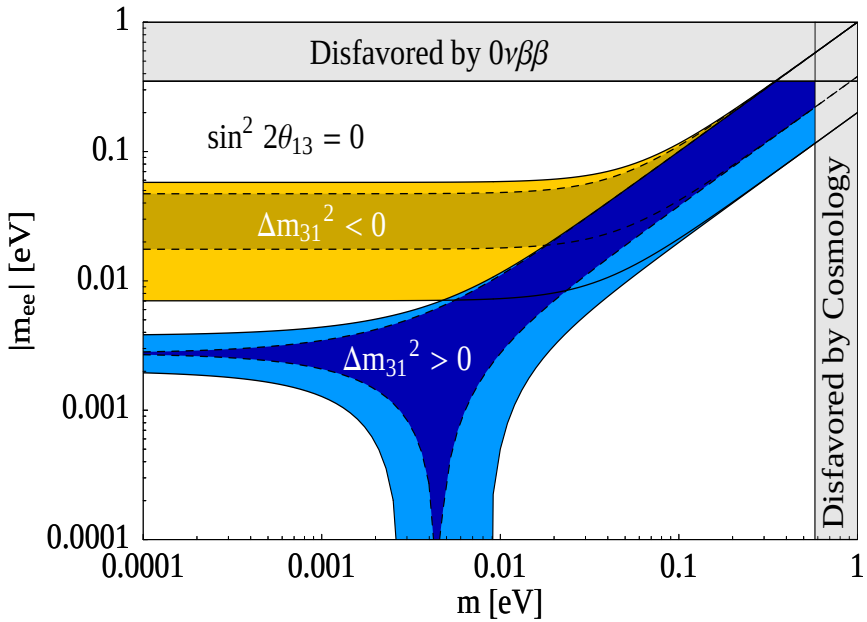
independent of form of M_R!!!

Other (non-oscillation) probes of NH vs. IH

1) Cosmology: $\Sigma = \sum m_i$

$$\Sigma^{\rm NH} \simeq \sqrt{\Delta m^2_{\rm A}} < \Sigma^{\rm IH} \simeq 2\sqrt{\Delta m^2_{\rm A}}$$



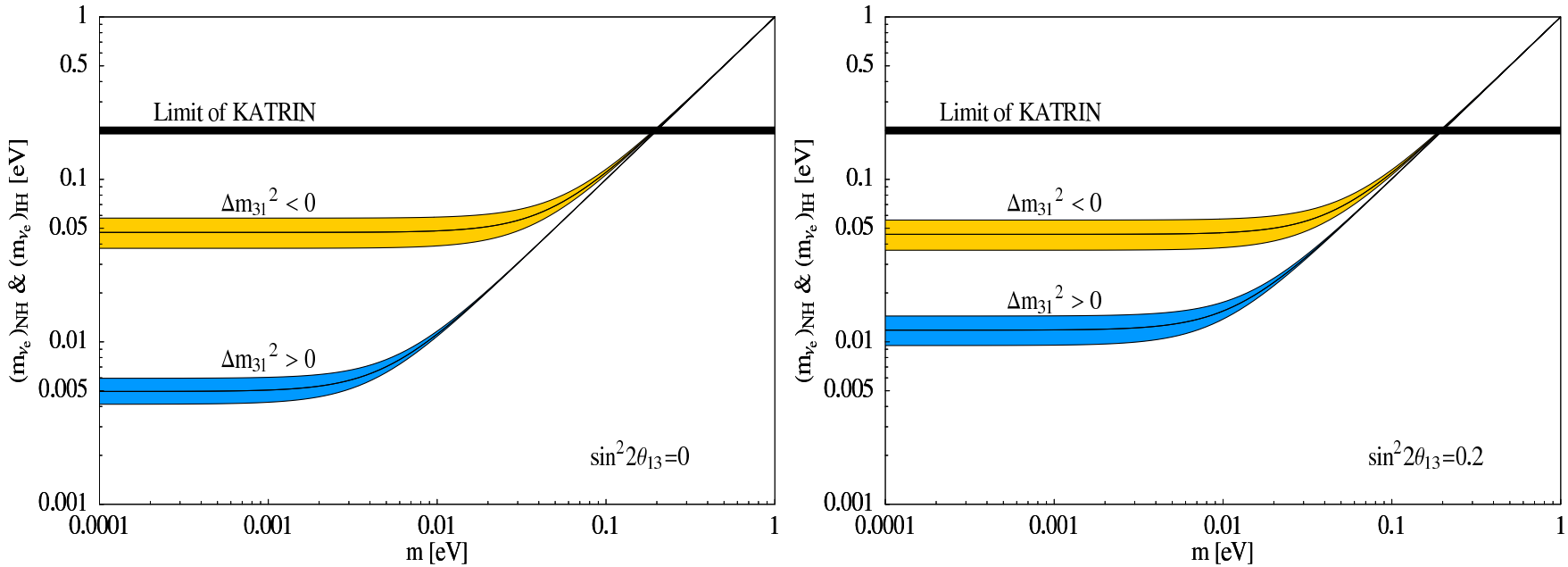


- independent on mixing angles
- systematics?
- Capability to distinguish NH from IH if $\sigma(\Sigma)$ and $\sigma(|m_{ee}|) \lesssim 0.05$ eV

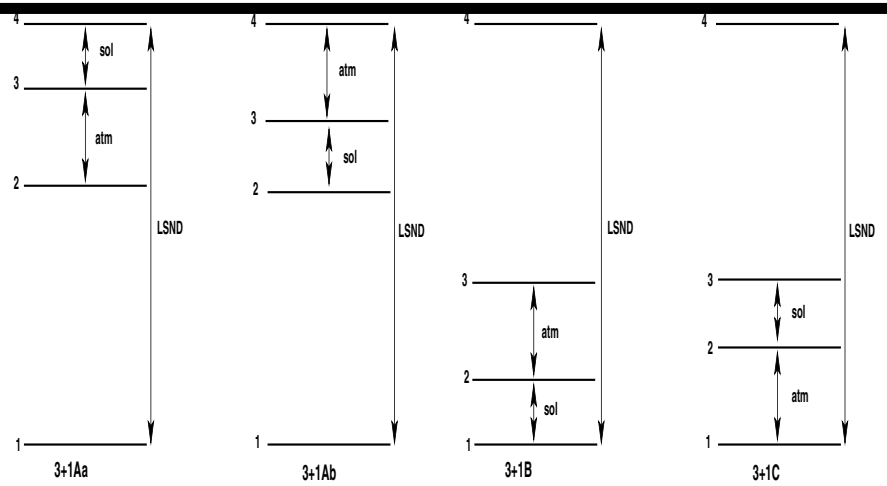
(Fogli *et al.*; Pascoli, Petcov, Schwetz; de Gouvea, Jenkins)

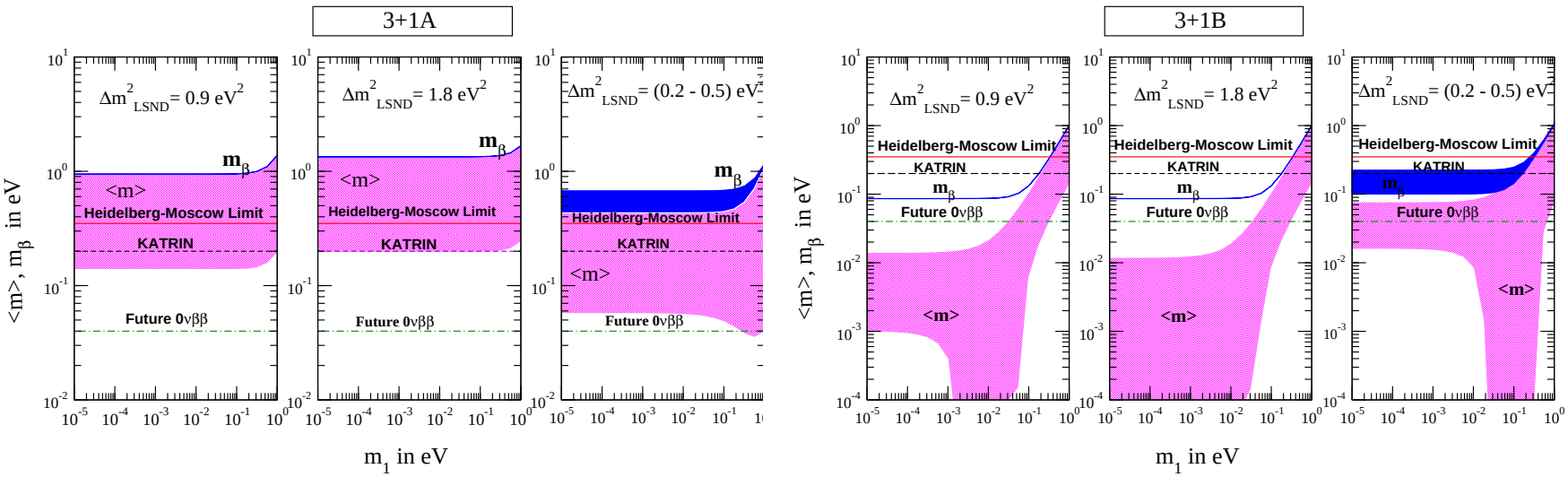
2) β-decay: $m_{\nu_e} = \sqrt{\sum |U_{ei}|^2 \, m_i^2}$

$$m_{\nu_e}^{\rm NH} \simeq \sqrt{s_{12}^2 \, c_{13}^2 \, \Delta m_\odot^2 + s_{13}^2 \, \Delta m_{\rm A}^2} \ll m_{\nu_e}^{\rm IH} \simeq \sqrt{c_{13}^2 \, \Delta m_{\rm A}^2}$$



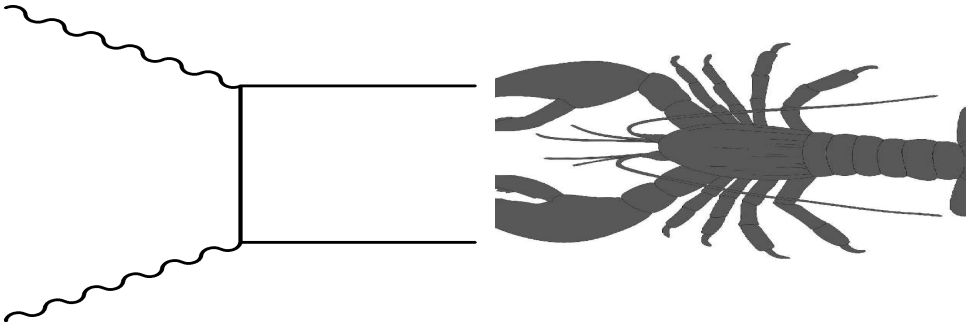
- almost independent on mixing angles
- well below KATRIN limit



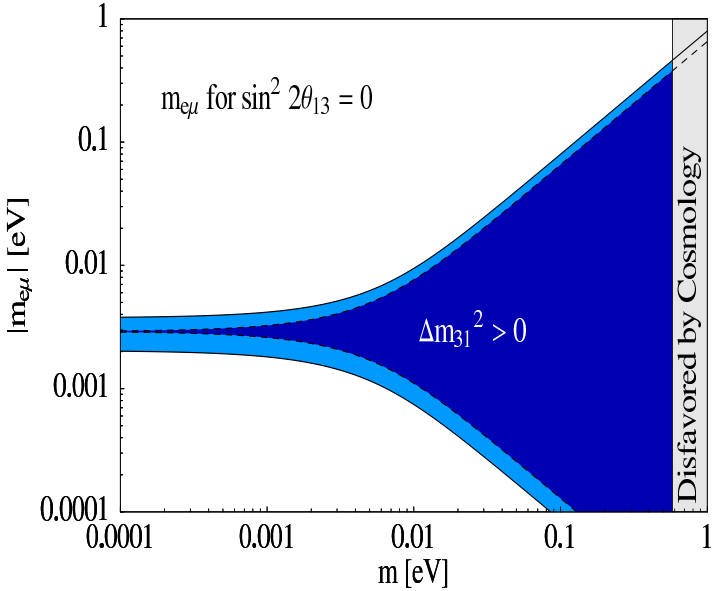


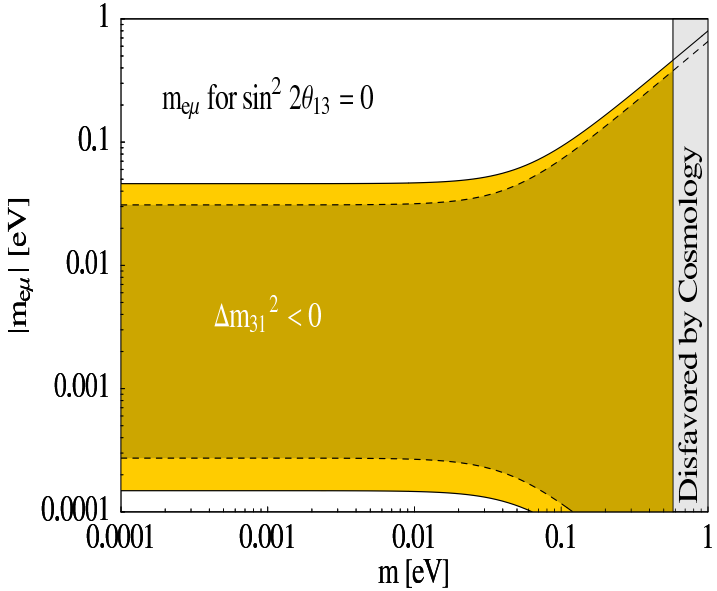
different for LSND neutrinos: Goswami, W.R., *Phys. Rev.* D **73**, 113003 (2006)

3) Other elements of m_ν: "the lobster"

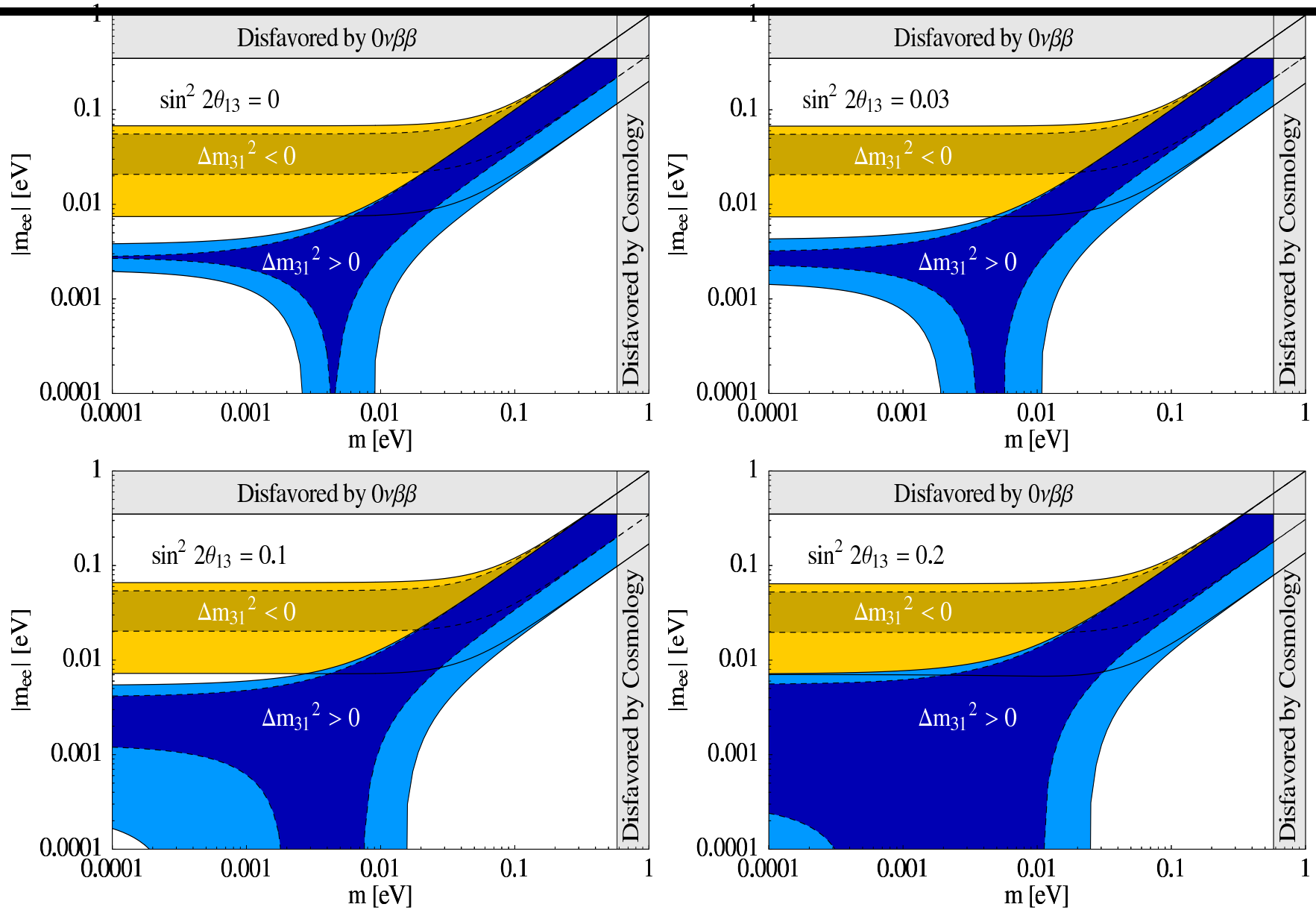


$$BR(K^+ \to \pi^- e^+ \mu^+) \propto |m_{e\mu}|^2 = \left|\sum U_{ei}\, U_{\mu i}\, m_i\right|^2 \sim 10^{-30} \left(\frac{|m_{e\mu}|}{\text{eV}}\right)^2$$





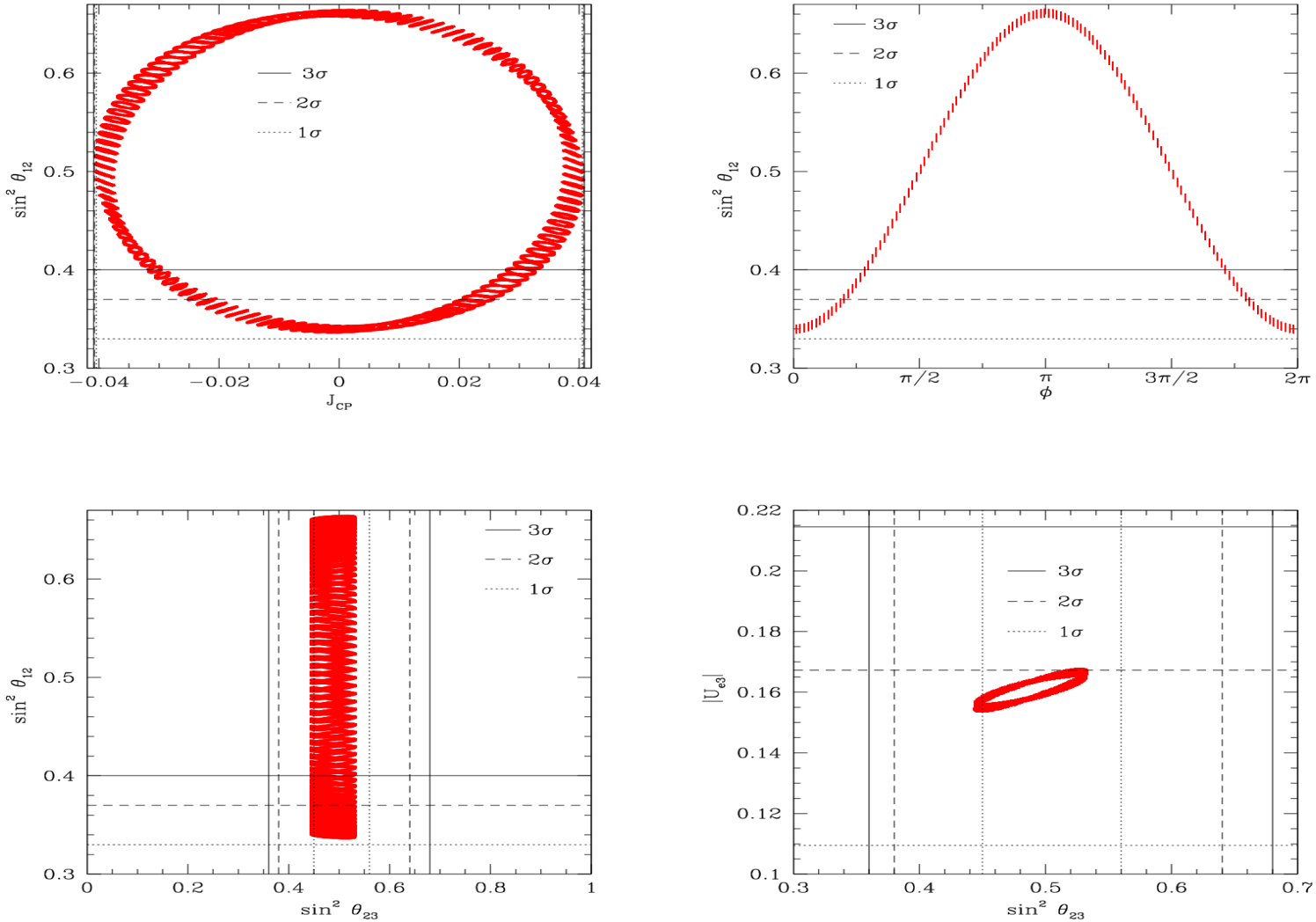
A. Merle, W.R., *Phys. Rev.* D **73**, 073012 (2006)



Larger $\Delta m^2_{\rm A}$? MINOS: $\Delta m^2_{\rm A} = (3 \pm 1) \cdot 10^{-3}$ eV^2

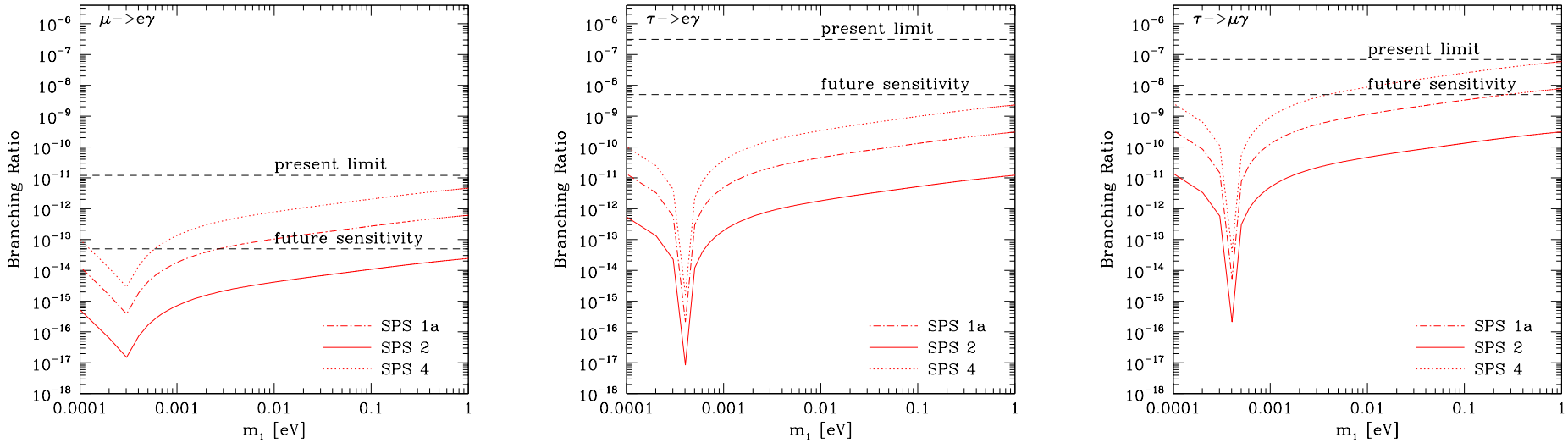
Other predictions of QLC

K.A. Hochmuth and W.R, hep-ph/0607103



Other predictions of QLC

K.A. Hochmuth and W.R, hep-ph/0607103



- $\mu \to e\gamma$ can be observable for neutrino masses above 10^{-3} eV, unless the SUSY masses approach the TeV scale;
- $\tau \to e\gamma$ is predicted to be very small:
- $\tau \to \mu\gamma$ requires rather large neutrino masses;
- $\mathrm{BR}(\mu \to e\gamma) : \mathrm{BR}(\tau \to e\gamma) : \mathrm{BR}(\tau \to \mu\gamma) \simeq \lambda^6 : \lambda^2 : 1$

What's more to $0\nu\beta\beta$?

- Mass scale: consider QD spectrum

$$m_0 \leq \frac{1+\tan^2\theta_{12}}{1-\tan^2\theta_{12}-2\,|U_{e3}|^2}\,|m_{ee}|^{\rm exp} \lesssim 5\ {\rm eV}$$

  comparable to current ^{3}H limit in the future

  S. Choubey, W.R., *Phys. Rev.* D **72**, 033016 (2005)

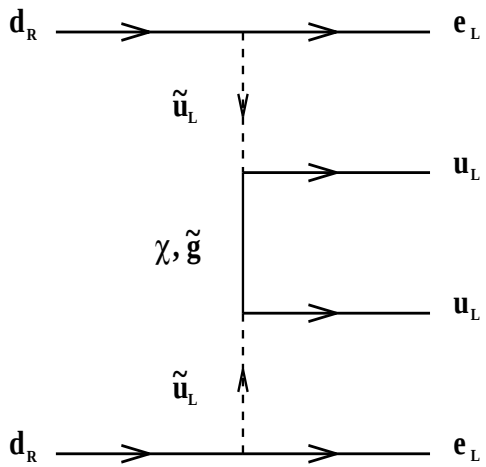
- Majorana phases: consider IH spectrum

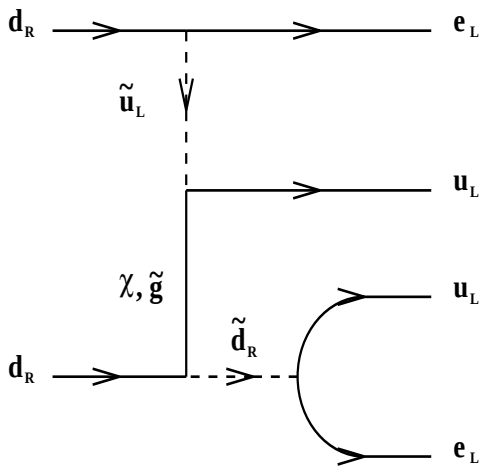
$$\sin^2\alpha = \left(1-\frac{|m_{ee}|}{\sqrt{|\Delta m^2_{\rm A}|}\,(1-|U_{e3}|^2)}\right)^2 \frac{1}{\sin^2 2\theta_{12}}$$

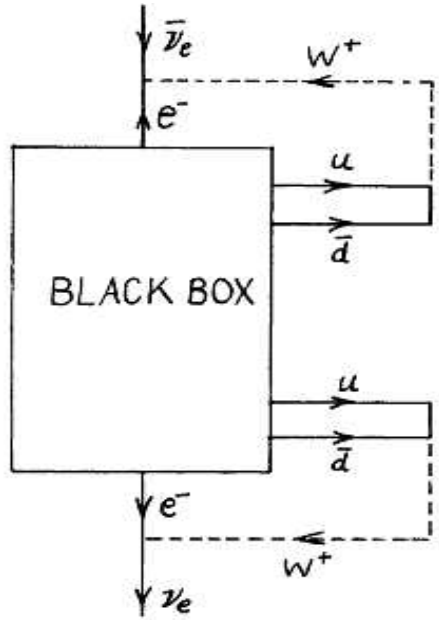
  extremely challenging unless NME uncertainty $\lesssim 1.5$ and θ_{12} rather large

  Pascoli, Petcov, W.R., *Phys. Lett.* B **549**, 177 (2002); Pascoli, Petcov, Schwetz, *Nucl. Phys.* B **734**, 24 (2006)

Other processes contributing to $0\nu\beta\beta$





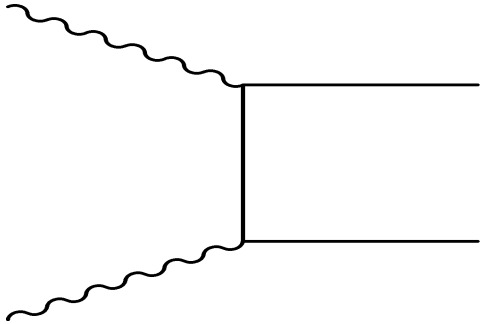


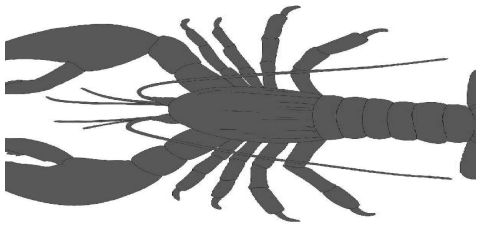
$$2n \to 2p + 2e^- \Rightarrow 2d \to 2u + 2e^- \Rightarrow 0 \to u\bar{d} + u\bar{d} + 2e^-$$

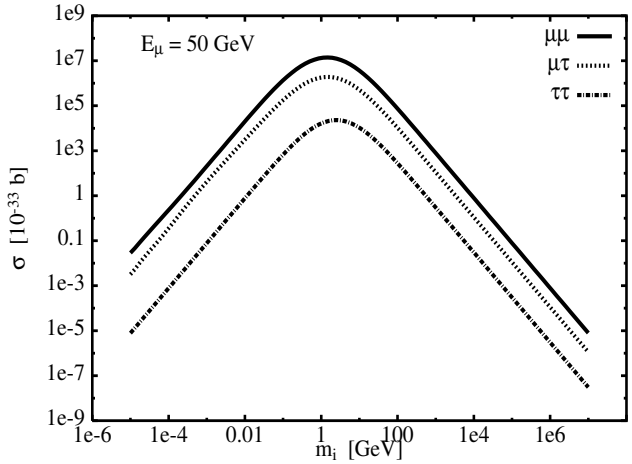
- SUSY
- Higgs triplets
- Right–handed interactions
- Majorons

$\Rightarrow$ limits on masses and couplings

Analogous processes ("The lobster")







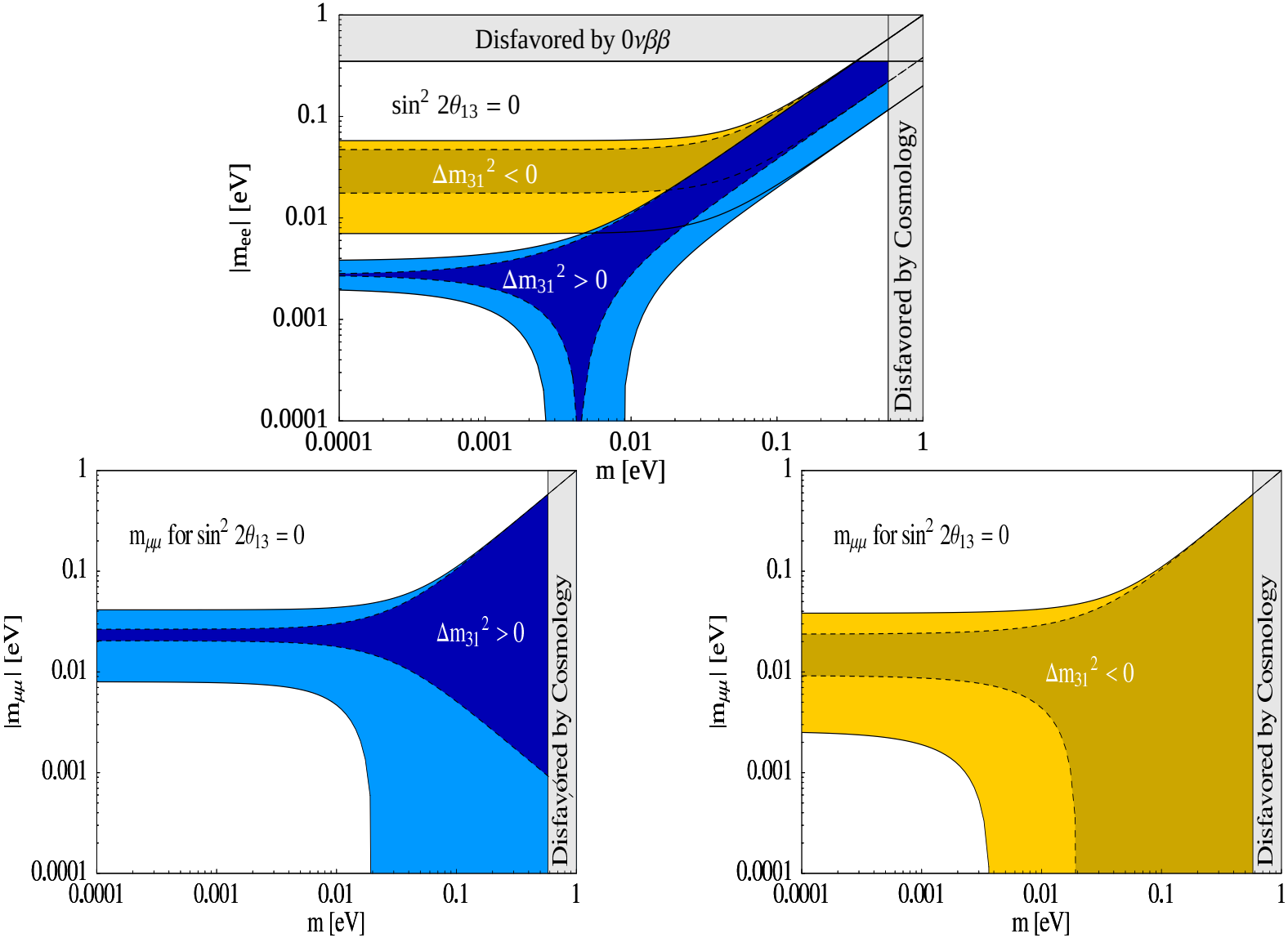
- Exotic decays, e.g.,

$$\mathrm{BR}(K^+ \to \pi^- \mu^+ \mu^+) \sim 10^{-30}\ (m_{\mu\mu}/\mathrm{eV})^2 \text{ with } m_{\mu\mu} = \left| \sum U_{\mu i}^2\, m_i \right|$$

- processes at accelerators (νN scattering, ν–fac, HERA "isolated leptons")

$$\mathrm{BR},\ \Gamma,\ \sigma \propto \frac{m^2}{(q^2 - m^2)^2} \simeq \begin{cases} m_i^2 & q^2 \gg m_i^2 \\ m_i^{-2} & q^2 \ll m_i^2 \end{cases}$$

The other elements



A. Merle, W.R., *Phys. Rev.* D **73**, 073012 (2006)

The emerging picture

$$U = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{-i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{-i\delta} & c_{23}\, c_{13} \end{pmatrix} P$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & s_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

- $\theta_{12} \simeq 33^0 \leftrightarrow$ solar + KamLAND neutrinos $\leftrightarrow \Delta m^2_\odot \simeq 8 \cdot 10^{-5}$ eV^2
- $\theta_{23} \simeq 45^0 \leftrightarrow$ atmospheric + K2K neutrinos $\leftrightarrow |\Delta m^2_{\rm A}| \simeq 2 \cdot 10^{-3}$ eV^2 $>$ or < 0?
- $\theta_{13} \lesssim 13^0 \leftrightarrow$ short baseline reactor neutrinos ("CHOOZ angle", $|U_{e3}|$)
- δ testable in (*three flavor!*) long–baseline oscillations
- Majorana phases α, β only in Lepton Number Violation $\leftrightarrow 0\nu\beta\beta$

Mass hierarchies and the effective mass

- NH: $m_3 \simeq \sqrt{\Delta m^2_{\rm A}}$, $m_2 \simeq \sqrt{\Delta m^2_{\odot}} = \sqrt{\Delta m^2_{\rm A}}\,\sqrt{R}$ and $m_1 \simeq 0$:

$$|m_{ee}|^{\rm NH} \simeq \left|\sin^2\theta_{12}\sqrt{\Delta m^2_{\odot}} + \sin^2\theta_{13}\sqrt{\Delta m^2_{\rm A}}\,e^{2i(\alpha-\beta)}\right| \lesssim 0.0066\ (0.0096)\,{\rm eV}$$

$$\text{or: } |m_{ee}|^{\rm NH} = \mathcal{O}(\sqrt{\Delta m^2_{\odot}}) \leftrightarrow \text{Next-to-next generation}$$

but can be zero!!

- IH: $m_2 \simeq m_1 \simeq \sqrt{|\Delta m^2_{\rm A}|}$ and $m_3 \simeq 0$:

$$|m_{ee}|^{\rm IH} \simeq \sqrt{|\Delta m^2_{\rm A}|}\,c^2_{13}\sqrt{1-\sin^2 2\theta_{12}\,\sin^2\alpha}$$

$$0.047\ (0.057)\,{\rm eV} \simeq \sqrt{|\Delta m^2_{\rm A}|}\,c^2_{13} \geq |m_{ee}|^{\rm IH} \geq \sqrt{|\Delta m^2_{\rm A}|}\,c^2_{13}\,\cos 2\theta_{12} \simeq 0.018\ (0.0073)\,{\rm eV}$$

$$\text{or: } |m_{ee}|^{\rm IH} = \mathcal{O}(\sqrt{|\Delta m^2_{\rm A}|}) \leftrightarrow \text{Next generation}$$

$$\Rightarrow 0 \neq |m_{ee}|^{\rm IH}_{\rm MIN} > |m_{ee}|^{\rm NH}_{\rm MAX} \Rightarrow \text{ Distinguish NH from IH!!!}$$

- QD: $m_3 \simeq m_2 \simeq m_1 \equiv m_0$:

$$m_0 \geq |m_{ee}|^{\rm QD} \geq m_0 \, \frac{1 - \tan^2\theta_{12} - 2\,\sin^2\theta_{13}}{1 + \tan^2\theta_{12}} \simeq 0.38\, m_0 \;\; (0.15\, m_0)$$

or: $|m_{ee}|^{\rm QD} = \mathcal{O}(m_0) \leftrightarrow$ Current generation

$\Rightarrow 0 \neq |m_{ee}|^{\rm QD}_{\rm MIN} > |m_{ee}|^{\rm NH,IH}_{\rm MAX} \Rightarrow$ Distinguish QD from NH and IH!!!