# Nuclear Shell-Model calculations for the LSP scattering cross sections of nuclei

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# Motivation: Constituents of the Dark Matter

Dark Matter =

**Baryonic matter**

MACHOs
brown dwarfs
jupiters
stellar black-hole remnants
white dwarfs
neutron stars

**Nonbaryonic matter**

HDM
light $\nu$'s
CDM

axions
WIMPs
- heavy $\nu$'s
- neutralinos
- sneutrinos

# Motivation: CDM Detector

**Process**

LSP-nucleon neutral-current scattering in the detector

$\Downarrow$

**Measure**

Nuclear recoil signature

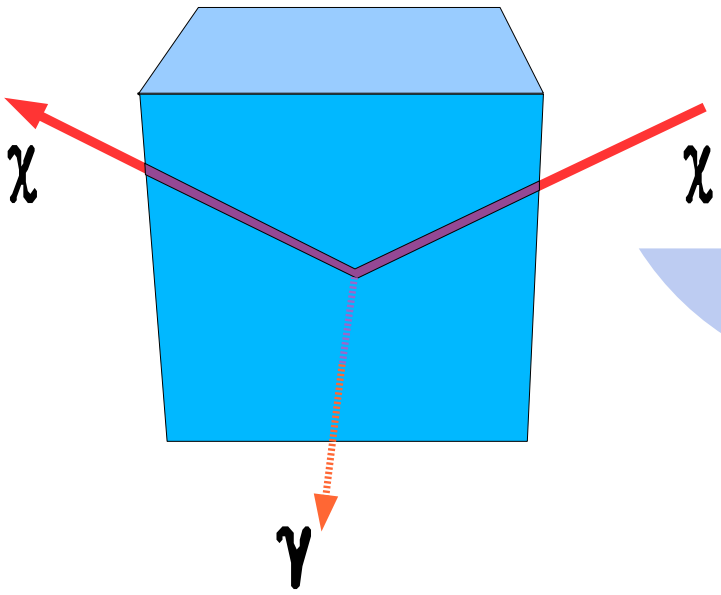
$\Updownarrow$

**Signature**

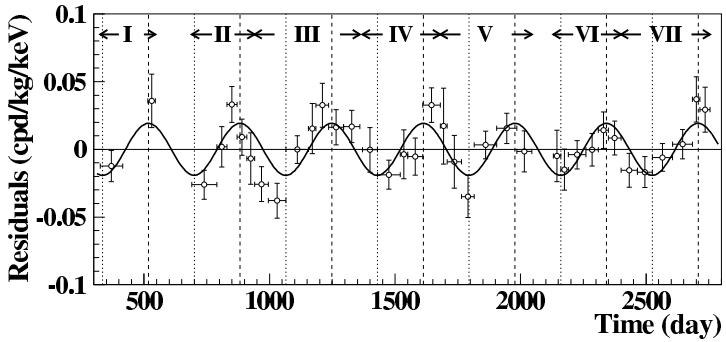
photon, phonon, ionisation

**Detector nuclei**

$^{73}$Ge, $^{71}$Ga, $^{129}$Xe, $^{131}$Xe etc.



# Motivation: the DAMA experiment vs. the Others



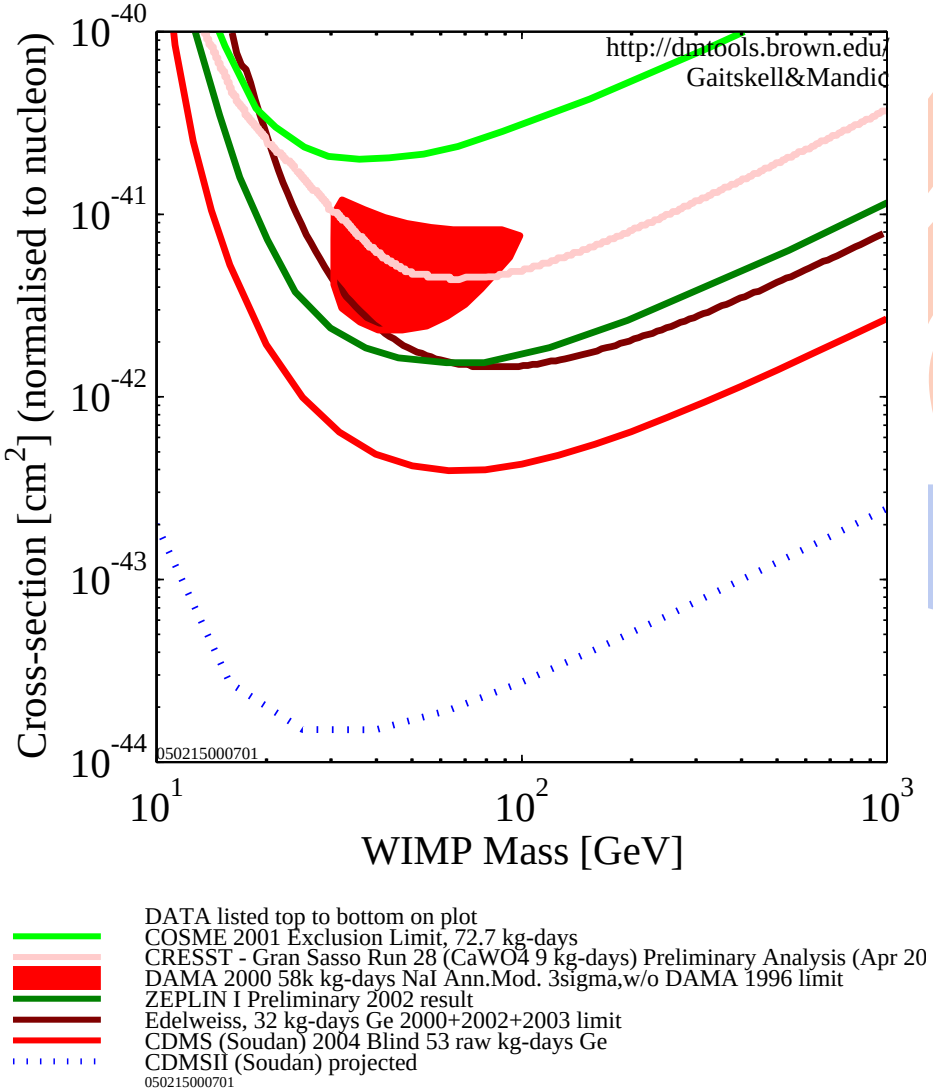

**Our Motivation**: Contradiction between the DAMA and the rest

$\Updownarrow$

NUCLEAR-STRUCTURE EFFECT?

*ASSUME*: WIMP=LSP=Lightest Supersymmetric Particle$\equiv \chi$

$$\chi = \alpha\tilde{\mathrm{B}} + \beta\tilde{\mathrm{W}}_3 + \gamma\tilde{\mathrm{H}}_1 + \delta\tilde{\mathrm{H}}_2$$



# LSP Detection Rate

## Folding with Maxwellian distribution

$$R = \frac{dN}{dt} = \frac{\rho(0)}{m_\chi} \frac{m_{\rm det}}{A m_{\rm p}} \int f(\mathbf{v} + \mathbf{v}_{\rm E})\, v\sigma(v) d^3 v,$$

$\mathbf{v}_{\rm E}$ = Earth's velocity with respect to the galactic center
$\mathbf{v} + \mathbf{v}_{\rm E}$ = LSP's velocity with respect to the galactic center

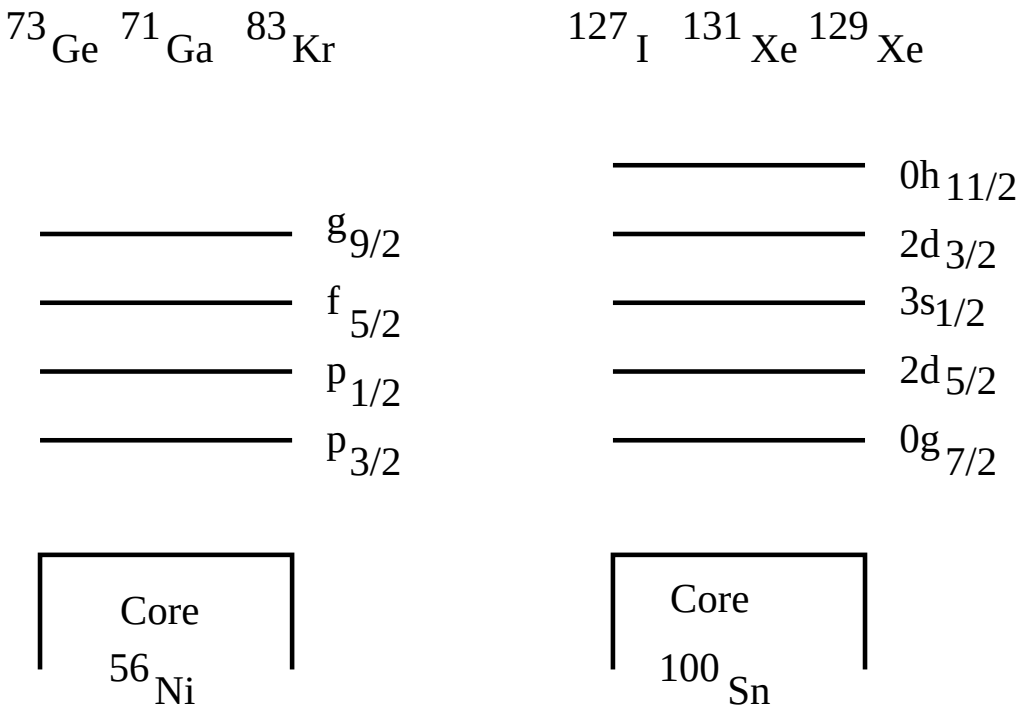
$\Downarrow$ Some calculation ...

$$R = R_0 \left[ (A0)^2 1 + 2A1A02 + (A1)^2 3 + A^2 \left( S0 - S1 \frac{A - 2Z}{A} \right)^2 4 \right],$$

$$R_0 = \frac{8.90 \times 10^8}{A m_\chi [\mathrm{GeV}] (m_{\rm p} b)^2} \mathrm{y}^{-1} \mathrm{kg}^{-1}$$

$D_i = D_i(m_\chi, Q_{\rm thr})$ Integrated nuclear factors

# Single-particle spaces for LSP detector nuclei




Effective interactions from M. Hjorth-Jensen

# Configuration restrictions

**$^{127}$I**

At least six neutrons on $0h_{11/2}$, protons not restricted.

**$^{71}$Ga, $^{73}$Ge**

Configuration centroid based restrictions.

**Dimensions**

Max. $2 \cdot 10^6$ practical (with non-parallelised version)

# Shell-Model Calculations (using EICODE 1.0)

Studied nuclei:
$^{23}$Na, $^{71}$Ga,
$^{73}$Ge, $^{127}$I

| Nucleus | $< \mathbf{S_n} >$ | $< \mathbf{S_p} >$ | $< \mathbf{L_n} >$ | $< \mathbf{L_p} >$ | $\mu_{\text{exp.}}$ | $\mu_{\text{SM}}$ | $\mu_{\text{s.p.}}$ |
|---|---|---|---|---|---|---|---|
| $^{73}$Ge | 0.4067 | 0.0048 | 3.7537 | 0.3348 | -0.879 | -1.194 | -1.913 |
| $^{71}$Ga | 0.0382 | 0.3360 | 0.2810 | 0.8687 | +2.562 | +2.599 | +3.793 |
| $^{23}$Na | 0.0199 | 0.2477 | 0.3207 | 0.9115 | +2.218 | +2.219 | +3.793 |
| $^{127}$I | 0.0382 | 0.3299 | 0.7018 | 1.4301 | +2.813 | +3.127 | +4.793 |

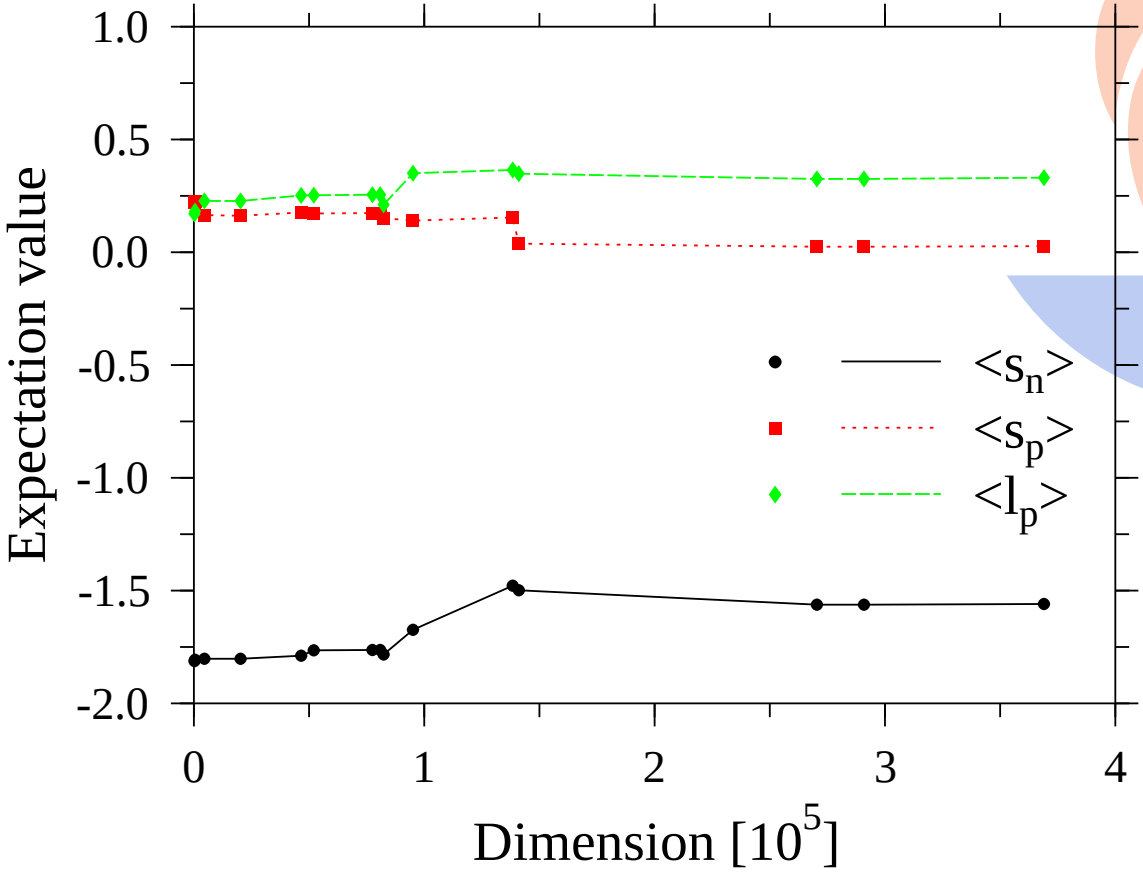
## MQPM

Detection rates in $^{71}$Ga, $^{73}$Ge and $^{127}$I (Phys. Lett. B 584 (2004) 31-39)

## LARGE-Scale EICODE calculations

(Phys. Lett. B632 (2006) 226),

$\mu_{\text{th}} = -3.826\langle \mathbf{S}_{\text{n}} \rangle + 5.586\langle \mathbf{S}_{\text{p}} \rangle + \langle \mathbf{L}_{\text{p}} \rangle$



# Theory results - elastic scattering

DAMA

- Even nuclei only sensitive to the coherent channel part of LSP scttering (nuclear form factor)
- Odd nuclei allow the detection of incoherent LSP scattering channel (spin matrix elements) $\Leftarrow$ Nuclear structure effects
- SUSY parametrisation that favors extreme spin dependence may explain the fact that DAMA sees CDM and other experiments not.

# Improving the nuclear structure part of LSP cross sections

## Shell-Model code EICODE 1.0

Uses the same methods as code NATHAN of E. Caurier et al. (PRC 59, 2033 (1999))

$$\Downarrow$$

$$H_{IJ} = A^{o}_{ij} B^{o'}_{\alpha\beta} V(o + o')\,, \quad I = i + \alpha\,, \quad J = j + \beta\,. \tag{1}$$

$$\Uparrow$$

## Density matrix element lists read from disk

| $i$ | $j$ | $A_{ij}$ |
|---|---|---|
| 0 | 0 | 1.0 |
| 1 | 0 | −0.5 |
| 1 | 1 | 0.5 |

| $\alpha$ | $\beta$ | $B_{\alpha\beta}$ |
|---|---|---|
| 1 | 3 | 1.0 |
| 3 | 1 | −0.15 |
| 3 | 3 | −0.5 |

Works well with m-scheme ... not so well with J-scheme!

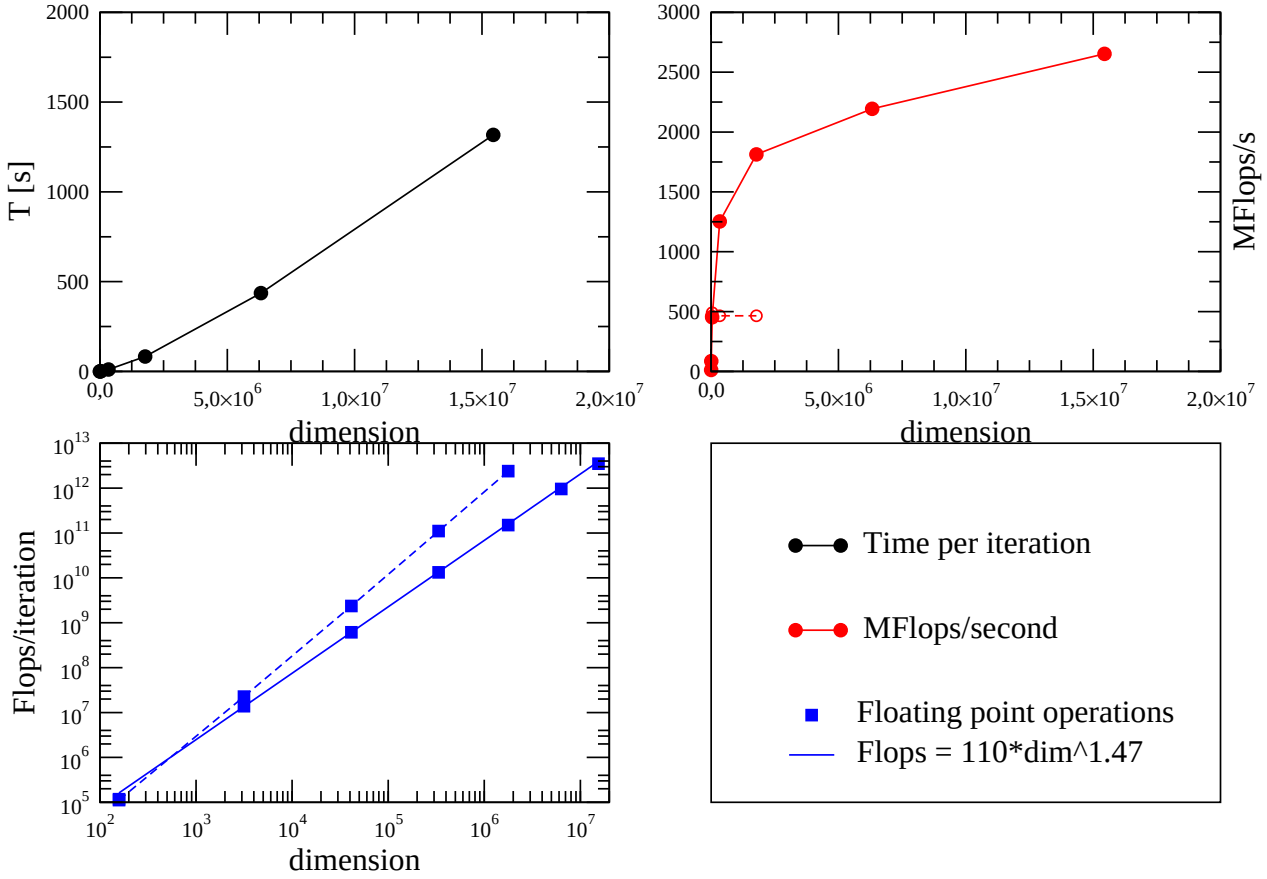
# Improving the nuclear structure part of LSP cross sections (or DBD calculations)

## Shell-Model code EICODE 2.0

- Replace element-by-element method with a more efficient method (published ... soon)
- Less floating point operations per matrix-vector product
- Lot of code optimisation, using standardised fast libraries
- More efficient CPU usage

# EICODE 2.0 Performance in pf-shell

## Scaling and efficiency



# Near future

## Nuclear structure calculations

- Parallel SM program EICODE can solve full unrestricted calculations for heavy Iodine and Xenon isotopes (and others) in $sdg_{7/2}h_{11/2}$-shell.

$\Downarrow$

## Interpreting the DAMA result vs. others

- Unrestricted description of $^{127}$I ground and excited states, spectroscopy, for accurate nuclear structure

## Inelastic LSP scattering

- Important for nuclei like $^{83}$Kr ($E_2 = 9.4$ keV), $^{127}$I ($E_2 = 58$ keV).

## Xenon detectors

- Unrestricted wavefunctions for $^{129}$Xe and $^{131}$Xe ground and excited states.

## Limits of J-scheme Shell-Model

- J-scheme dimension $10^9$ possible (200-500 CPUs)

# Conclusion

## Shell-Model EICODE at Jyväskylä

- The code is a maturing general purpose J-scheme SM code
- Algorithmic improvements $\Rightarrow$ speed increase 35-100 fold $\Rightarrow$ huge J-scheme calculations!

$$\Downarrow$$

## Application: LSP scattering, elastic/inelastic

- We can calculate the nuclear structure parts (spin matrix elements) more accurately as we did before

## Another application: $0\nu$ double beta decay

- $0 - 2\hbar\omega$ calculations for $^{48}$Ca etc.