Double beta decay within Continuum-QRPA

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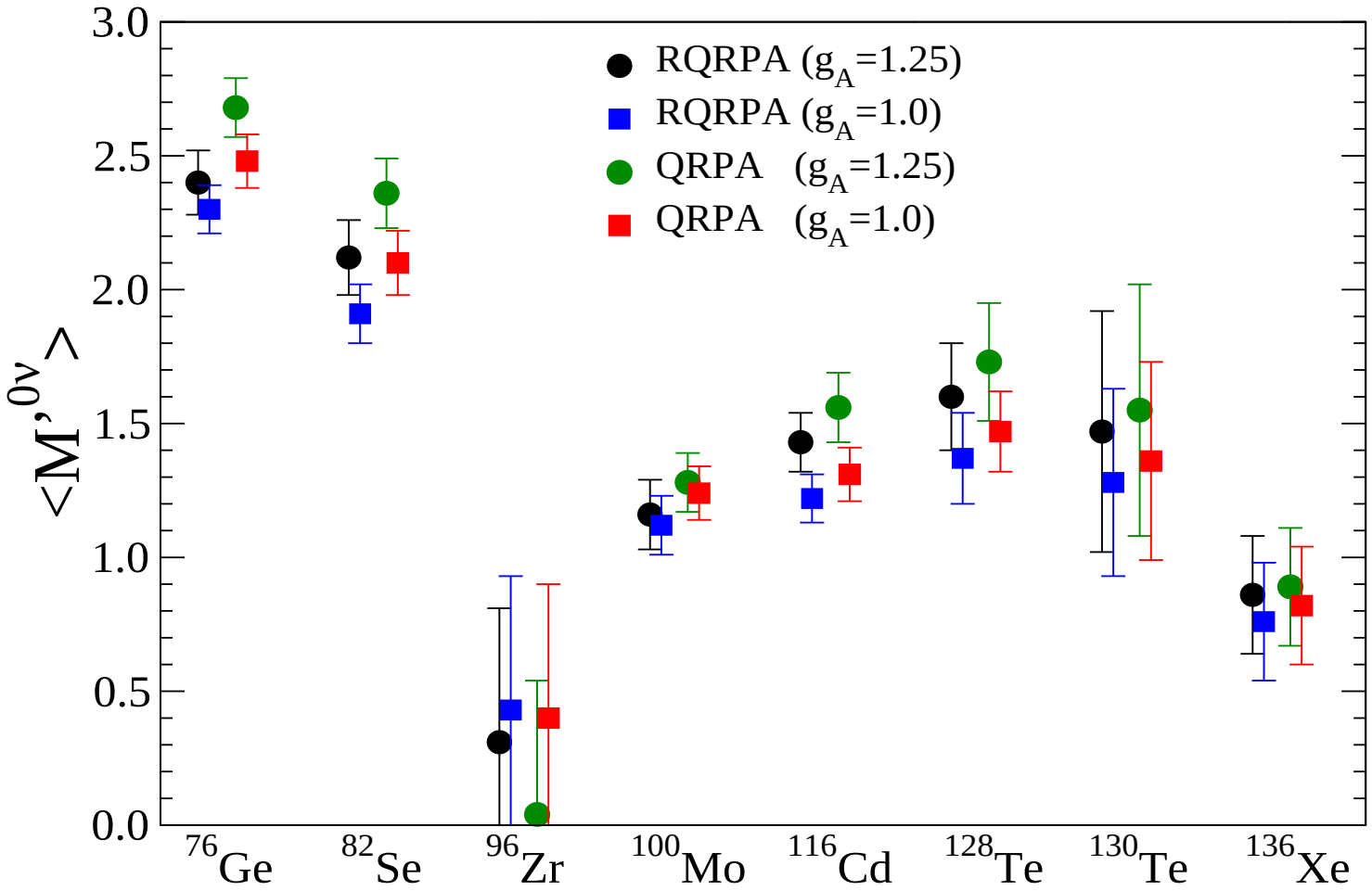
Introduction

Light Majorana Neutrino Exchange Mechanism



V. R., A. Faessler, F. Simkovic, P. Vogel, PRC **68** (2003); NPA **766** (2006)

the most thorough QRPA analysis of the $0\nu\beta\beta$-decay up-to-date

Introduction

How large is the systematic error of the QRPA
(due to neglecting many-particle configurations)?

$M^{0\nu}$ and $M^{2\nu}$ are integral quantities (sums over all intermediate states)

challenge for experimental verification, but favors QRPA description

if we knew the exact nuclear Hamiltonian to use within QRPA...

One fixes parameters of the nuclear Hamiltonian describing some observables within the QRPA

Introduction

How accurate is the "standard QRPA description" itself?

In particular, what is the best basis choice?

a priory: the larger basis - the better

Motivation for continuum-QRPA

$\Rightarrow$ to perform ultimate QRPA calculations

- To include entire single-particle basis
  no more question about the dependence of QRPA results on the s.p.-basis size
- To use realistic s.p. wave functions in continuum
  no more need for oscillator wave functions
- To get a new insight into results
  via alternative formulation of the QRPA

Basics of continuum-QRPA

$$|JM\rangle = Q^{\dagger}_{JM}|0^{+}_{RPA}\rangle \qquad Q^{\dagger}_{JM} = \sum_{12}\left[X_{12}A^{\dagger}_{12} - Y_{12}\tilde{A}_{12}\right]$$

It is not possible to handle the infinite number of the QRPA amplitudes X_s, Y_s if one wants to include the single-particle continuum.

Instead, the QRPA is reformulated in terms of four transition densities $\varrho_i^{J^\pi s}$ $(i = 1, \ldots 4)$ of the excited states in the coordinate space

Basics of continuum-QRPA

Elements $\varrho_i(r)$in terms of the pn-QRPA amplitudes $X_{\pi\nu}$ and $Y_{\pi\nu}$

$$\varrho_i(r) = \sum_{\pi\nu} \chi_\pi(r)\chi_\nu(r)R_i^{\pi\nu},$$

$$\begin{pmatrix} R^{\pi\nu}_{p-h} \\ R^{\pi\nu}_{h-p} \\ R^{\pi\nu}_{p-p} \\ R^{\pi\nu}_{h-h} \end{pmatrix} = \begin{pmatrix} u_\pi v_\nu X_{\pi\nu} + v_\pi u_\nu Y_{\pi\nu} \\ u_\pi v_\nu Y_{\pi\nu} + v_\pi u_\nu X_{\pi\nu} \\ u_\pi u_\nu X_{\pi\nu} - v_\pi v_\nu Y_{\pi\nu} \\ u_\pi u_\nu Y_{\pi\nu} - v_\pi v_\nu X_{\pi\nu} \end{pmatrix}$$

$u,\ v$ - coefficients of Bogolyubov transformation

The pn-QRPA system of equations for ϱ_i

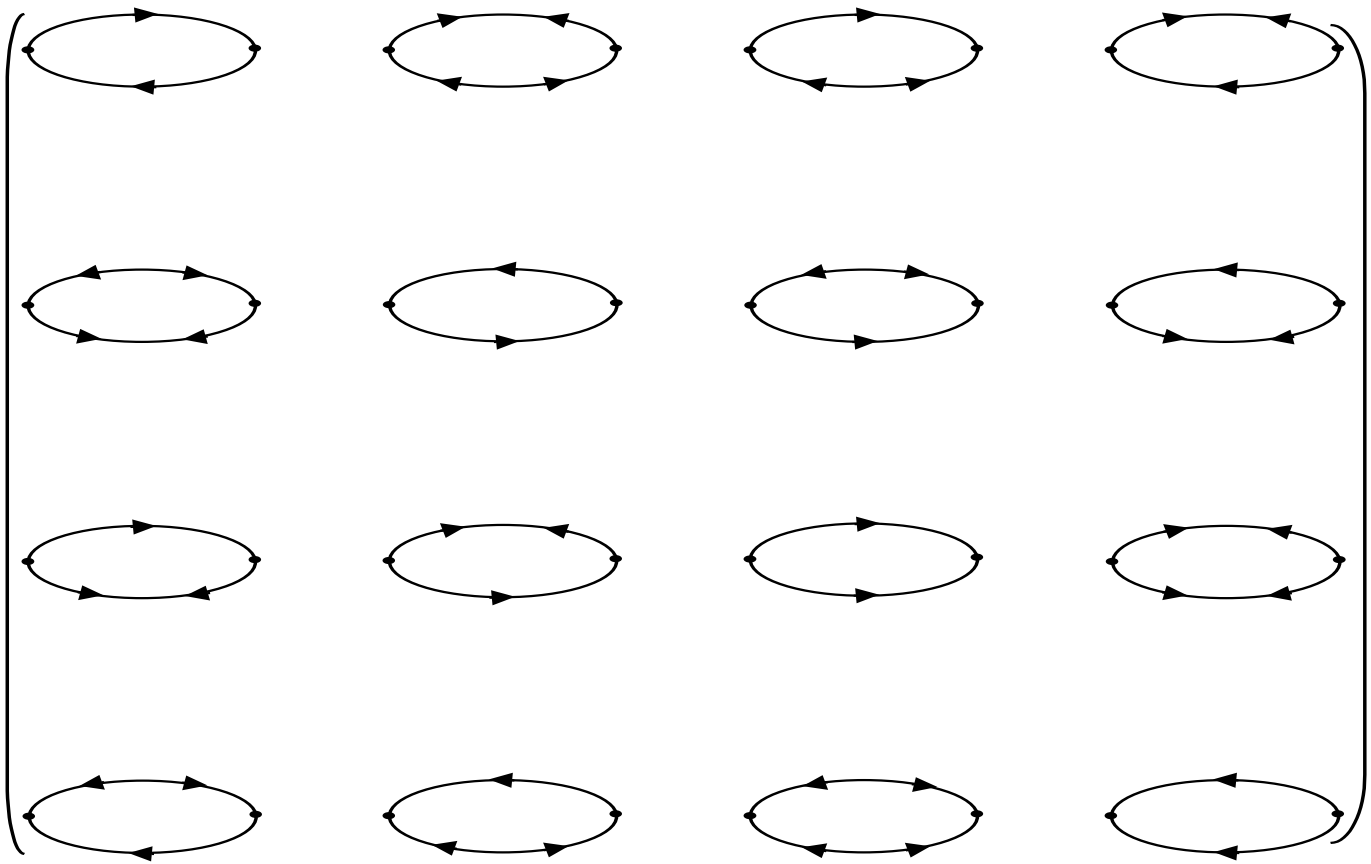
$$\varrho = \{AF\varrho\}$$

$$\varrho_i^{J^\pi s}(r) = \sum_k \int A_{ik}^{J^\pi}(rr', \omega = \omega_s)\, F_k^{J^\pi}(r'r'')\, \varrho_k^{J^\pi s}(r'')\, dr' dr'',$$

$F_k^{J^\pi}(r_1 r_2)$ — residual interaction in k-channel

"Free" two-quasiparticle propagator (response function) A_{ik}

in terms of normal and anomalous s.p. Green's functions for Fermi-systems with nucleon pairing



Basics of continuum-QRPA

$$A^J_{ik}(r_1 r_2, \omega) = \sum_{\pi\nu} \chi_\pi(r_1)\chi_\nu(r_1)\,\chi_\pi(r_2)\chi_\nu(r_2)\,A^{\pi\nu}_{ik}(\omega)$$

$$A^{\pi\nu}_{11} = \frac{u_\pi^2 v_\nu^2}{\omega - E_\pi - E_\nu} - \frac{u_\nu^2 v_\pi^2}{\omega + E_\pi + E_\nu}, \quad A^{\pi\nu}_{12} = \frac{u_\pi v_\pi v_\nu u_\nu}{\omega - E_\pi - E_\nu} - \frac{u_\pi v_\pi v_\nu u_\nu}{\omega + E_\pi + E_\nu}$$

The way to explicitly take the s.p. continuum into consideration:

1. To put for highly-excited s.p. states: $v = 0,\ u = 1,\ E = |\varepsilon - \lambda|\ (\gg \Delta)$ (accuracy $\left(\frac{\Delta}{|\varepsilon-\lambda|}\right)^2$)

2. To use the s.p. Green's function: $g(r_1 r_2, \varepsilon) = \sum_\pi \frac{\chi_\pi(r_1)\chi_\pi(r_2)}{\varepsilon - \varepsilon_\pi}$ to perform summation over the s.p. states in continuum

Total response function $\hat{A}$

including QRPA iterations of p-h and p-p interactions
$\Rightarrow$ a Bethe-Salpeter-type integral equation:

$$\hat{A} = A + \{AF\hat{A}\}$$

Spectral decomposition of $\hat{A}$

$$\hat{A}^{J^\pi}_{11}(r_1 r_2, \omega) = \sum_s \frac{\varrho_1^{J^\pi s}(r_1)\varrho_1^{J^\pi s}(r_2)}{\omega - \omega_s + i\delta} - \sum_s \frac{\varrho_2^{J^\pi s}(r_1)\varrho_2^{J^\pi s}(r_2)}{\omega + \omega_s - i\delta}$$

$$\hat{A}^{J^\pi}_{22}(r_1 r_2, \omega) = \hat{A}^{J^\pi}_{11}(r_1 r_2, -\omega)$$

$$\hat{A}^{J^\pi}_{12}(r_1 r_2, \omega) = \sum_s \frac{\varrho_1^{J^\pi s}(r_1)\varrho_2^{J^\pi s}(r_2)}{\omega - \omega_s + i\delta} - \sum_s \frac{\varrho_2^{J^\pi s}(r_1)\varrho_1^{J^\pi s}(r_2)}{\omega + \omega_s - i\delta}$$

$\beta\beta$-decay within continuum-QRPA

Strength functions for a s.p. probing operator $\hat{V}^{(\mp)}_{J\mu}$

$$S^{(\mp)}(\omega) = \sum_s \left|\langle s|\hat{V}^{(\mp)}_{J\mu}|0\rangle\right|^2 \delta(\omega - \omega_s)$$

can be calculated in term of Im $\hat{A}$:

$$S^{(-)}(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\{V_J \hat{A}^J_{11}(\omega) V_J\}$$

$$S^{(+)}(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\{V_J \hat{A}^J_{22}(\omega) V_J\}$$

$\beta\beta$-decay within continuum-QRPA

$2\nu\beta\beta$

Non-diagonal strength function

$$S^{(--)}(\omega) = \sum_s \langle 0'|\hat{V}^{(-)}_{J\bar{\mu}}|s\rangle\langle s|\hat{V}^{(-)}_{J\mu}|0\rangle\delta(\omega - \omega_s)$$

Identifying BCS vacuum $|0'\rangle$ with $|0\rangle$

$$S^{(--)}(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\{V_J\hat{A}^J_{12}(\omega)V_J\}$$

$$M^{2\nu}_{GT} = -\frac{3}{2}\{1\cdot\hat{A}^{1^+}_{12}(\omega=0)\cdot 1\} \quad + \quad \delta M^{2\nu}_{GT}$$

$$\delta M^{2\nu}_{GT} = -\frac{1}{\pi}\int d\omega\left(\frac{1}{\omega+\delta E} - \frac{1}{\omega}\right)\{1\cdot\hat{A}^J_{12}(\omega)\cdot 1\}$$

$\beta\beta$-decay within continuum-QRPA

$0\nu\beta\beta$

matrix element of a 2-body operator within the continuum-QRPA

$$\hat{W}^{(--)} = \sum_{ab} \sum_{JLS} W_J(r_a, r_b) T_{JLS\mu}(n_a) T^*_{JLS\mu}(n_b) \tau_a^{(-)} \tau_b^{(-)}$$

between the ground states $|0\rangle$ and $|0'\rangle$

$$\langle 0'|\hat{W}^{(--)}|0\rangle = \sum_J -\frac{1}{\pi} \int d\omega \, \{W_J \hat{A}^J_{12}(\omega)\}$$

Results

Model

Landau-Migdal zero-range pairing, p-h and p-p forces

Input parameters

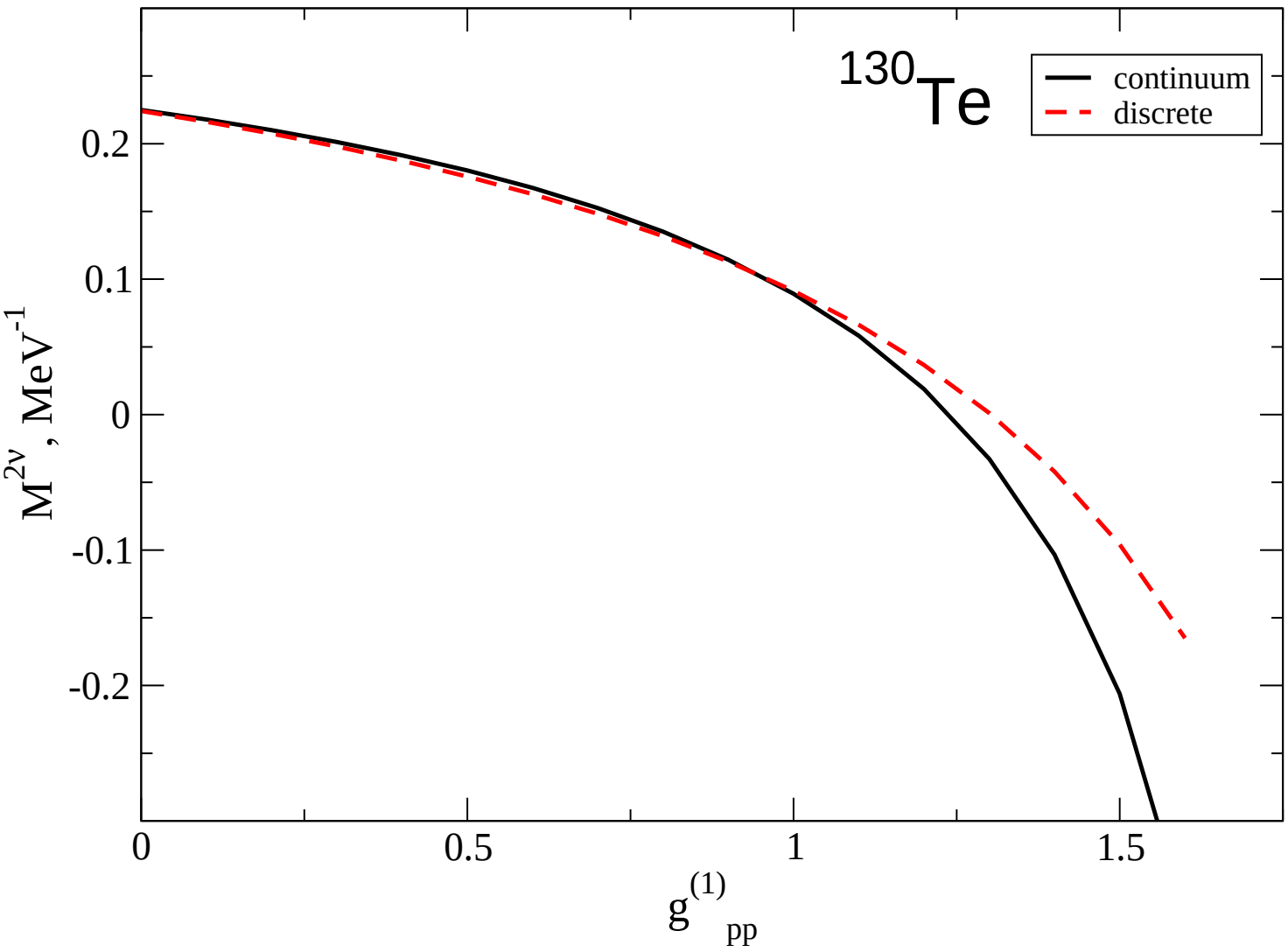
pairing strengths g_n^{pair}, g_p^{pair}

p-h strengths: isovector f_{ph}^0 and spin-isovector f_{ph}^1

p-p strengths g_{pp}^0, g_{pp}^1

Fixing input parameters

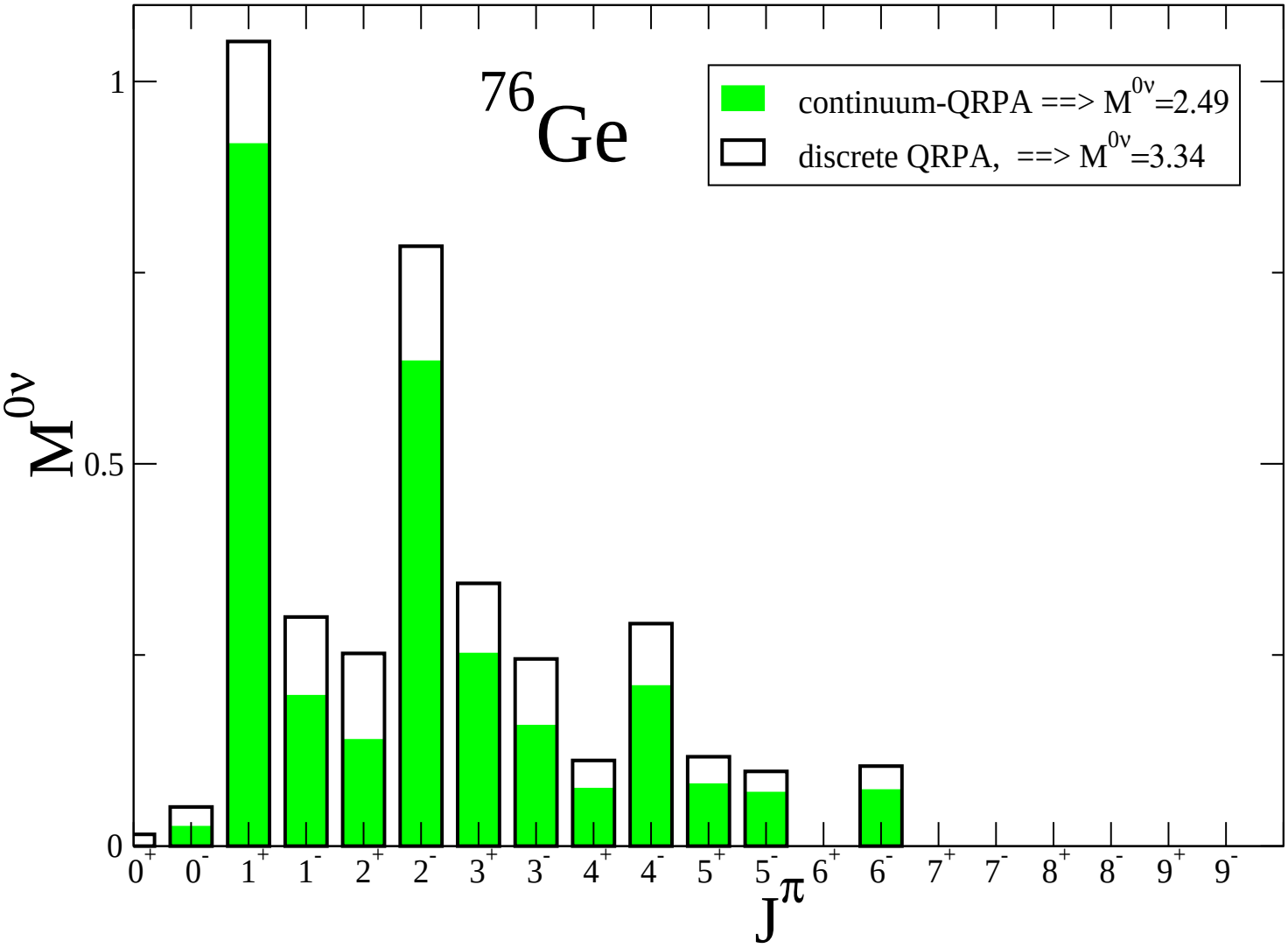
- g^{pair} – to reproduce exp. pairing energies
- $f^0_{ph} = 1.0$ – isospin-selfconsistency of p-h interaction and mean field
- f^1_{ph} – to reproduce the exp. energy of the GTR
- $g^0_{pp} = (g^{pair}_n + g^{pair}_p)/2$ – isospin-selfconsistency of p-p interaction
- g^1_{pp} – to reproduce exp. $M^{2\nu}$

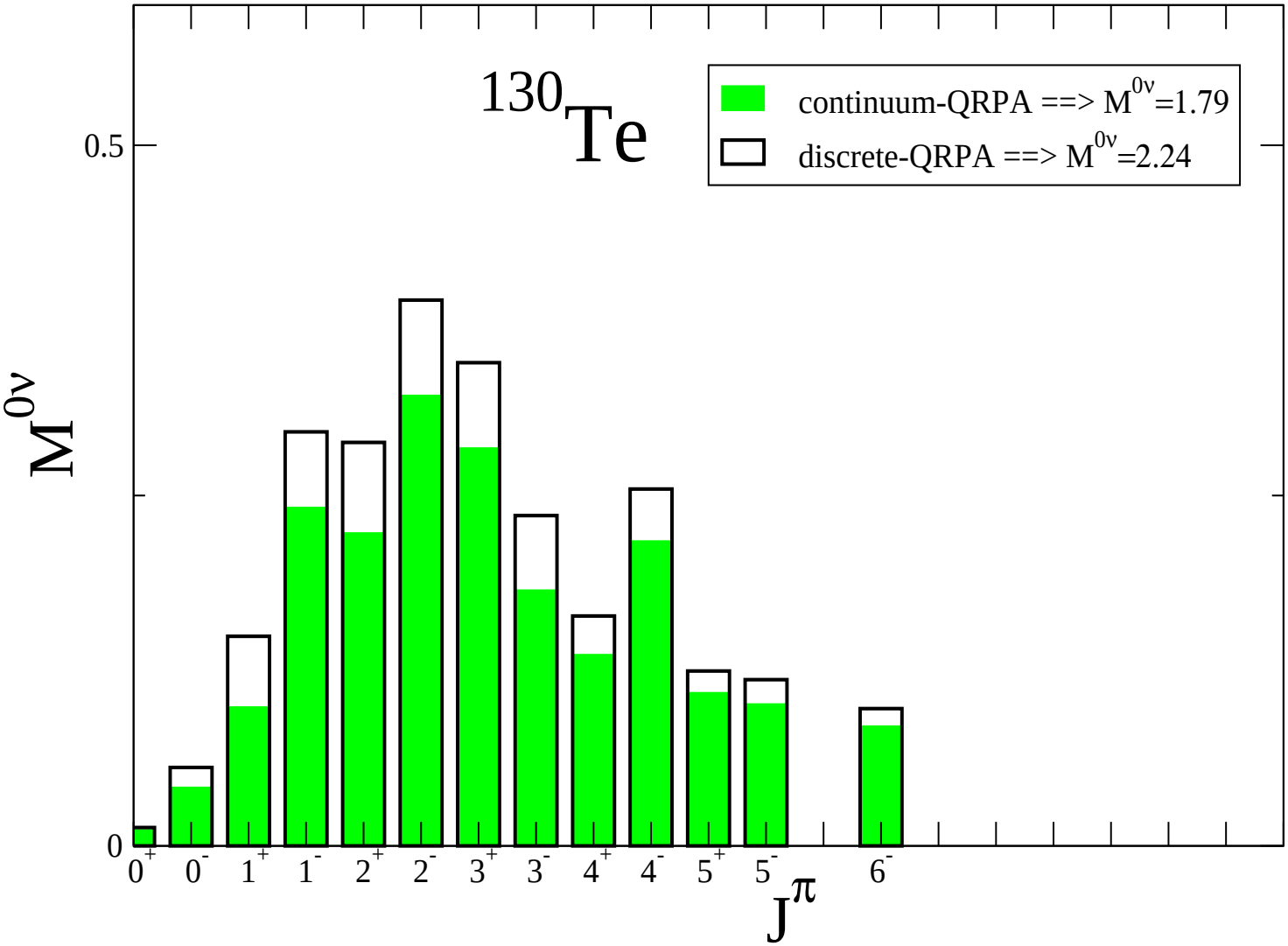

^{130}Te
continuum
discrete
0.2
0.1
0
-0.1
-0.2
$M^{2\nu}$, MeV^{-1}
0
0.5
1
1.5
$g^{(1)}_{pp}$

Results

Present calculation of $M^{0\nu}$

- two-nucleon short-range correlations are included
- the higher order terms of the nucleon current are missing ($M^{0\nu}$ can get reduced by up to 30%)
- For multipoles with $L \geq 5$ the QRPA is barely suitable — short-range physics dominates


^{76}Ge
continuum-QRPA ==> $M^{0\nu}$=2.49
discrete QRPA, ==> $M^{0\nu}$=3.34
$M^{0\nu}$
1
0.5
0
0^+ 0^- 1^+ 1^- 2^+ 2^- 3^+ 3^- 4^+ 4^- 5^+ 5^- 6^+ 6^- 7^+ 7^- 8^+ 8^- 9^+ 9^-
J^π


130Te
continuum-QRPA ==> M0ν=1.79
discrete-QRPA ==> M0ν=2.24
0.5
0
M0ν
0+ 0- 1+ 1- 2+ 2- 3+ 3- 4+ 4- 5+ 5- 6-
Jπ

Conclusions

- Continuum-QRPA approach to calculation of DBD amplitudes has been formulated

- Total $M^{0\nu}$ gets suppressed by about 20-30%

- Perspective: effective transition operators for the Shell Model ?