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SHELL MODEL CALCULATIONS OF THE NEUTRINOLESS DOUBLE BETA DECAY

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The three pillars of the shell model

The Effective Interaction

The Valence Space

The Algorithms and their Codes

E. Caurier, G. Martínez-Pinedo, F. Nowacki,
A. Poves and A. P. Zuker.
"The Shell Model as a Unified View of Nuclear Structure"
Reviews of Modern Physics, 77 (2005) 427-488

The Effective Interaction: Key aspects

The correct evolution of the spherical mean field in the valence space has to be incorporated (fitted) in the realistic interactions (G-matrices)

The multipole hamiltonian does not seem to demand major changes with respect to the one derived from the realistic nucleon-nucleon potentials

The "crisis" of the calculations of the 0ν, $\beta\beta$ nuclear matrix elements

The QRPA "explosion"

g_{pp}, the miraculous factor

Does a good 2ν m.e. guarantee a good 0ν m.e.?

To quench or not to quench ...

The quest for better wave functions

Quality indicators

- Good spectroscopy for parent, daughter and grand-daughter, even better if its extend to a full mass region
- GT-strengths and strength functions, 2ν matrix elements, etc.

Large scale shell model calculations (LSSM) vs QRPA, the pros and cons

- Interaction
- Valence space
- Pairing
- Deformation

The Valence Space(s)

An ideal valence space should incorporate the most relevant degrees of freedom AND be computationally tractable

Classical $0\hbar\omega$ valence spaces are provided by the major oscillator shells p, sd and pf shells

Other physically sound and computationally accessible valence spaces are proposed below. In red, those relevant for the description of the double beta emitters

(note: in a major HO shell of principal quantum number **p** the orbit j=p+1/2 is called $intruder$ and the remaining ones are denoted by r_p)

- r_2-pf: intruders around N and/or Z=20
- r_3-$g_{9/2}$: ^{76}Ge, ^{82}Se

The Valence Space(s)

- r_3-$g_{9/2}$, $d_{5/2}$: the region around ^{80}Zr
- r_3-$g_{9/2}(d_{5/2})$ for neutrons and pf for protons: neutron rich Cr, Fe, Ni, Zn
- r_4-$h_{11/2}$ for neutrons and $p_{1/2} - g_{9/2}$-r_4 for protons: ^{96}Zr, ^{100}Mo,^{116}Cd
- r_4-$h_{11/2}$ for neutrons and protons: ^{124}Sn, $^{128-130}$Te, ^{136}Xe

The Strasbourg-Madrid codes can deal with problems involving basis of 10^{11} Slater determinants, using relatively modest computational resources

Update of the 0ν results

In the valence spaces r$_3$-$g_{9/2}$ (^{76}Ge, ^{82}Se) and r$_4$-$h_{11/2}$ (^{124}Sn, $^{128-130}$Te, ^{136}Xe) we have obtained high quality effective interactions by carrying out multi-parametrical fits whose starting point is given by realistic G-matrices. In the valence space proposed for ^{96}Zr, ^{100}Mo and ^{116}Cd, the results are still subject to further improvement

m_ν for $T_{\frac{1}{2}} = 10^{25}$ y.		$M_{0\nu}^{GT}$	1-χ_F
^{48}Ca	0.85	0.67	1.14
^{76}Ge	0.90	2.35	1.10
^{82}Se	0.42	2.26	1.10
^{128}Sn	0.45	2.11	1.13
^{128}Te	1.92	2.36	1.13
^{130}Te	0.35	2.13	1.13
^{136}Xe	0.41	1.77	1.13

Dependence on the effective interaction

The results depend only weakly on the effective interactions provided they are compatible with the spectroscopy of the region.

For the lower pf shell we have three interactions that work properly, KB3, FPD6 and GXPF1. Their predictions for the 2ν and the neutrinoless modes are quite close to each other

	KB3	FPD6	GXPF1
$M_{GT}(2\nu)$	0.083	0.104	0.107
$M_{GT}(0\nu)$	0.667	0.726	0.621

Similarly, in the $r3g$ and $r4h$ spaces, the variations among the predictions of spectroscopically tested interactions is small (10-20%)

Learning from the ^{48}Ca $\rightarrow^{48}$Ti and the (fictitious) ^{48}Ti $\rightarrow^{48}$Cr decays

The influence of deformation

Changing adequately the effective interaction we can increase or decrease the deformation of parent, grand-daughter or both, and so gauge its effect on the decays. A mismatch of deformation can reduce the $\beta\beta$ matrix elements by factors 2-3. In fact the fictitious decay Ti-Cr, using the same energetics that in Ca-Ti, has matrix elements more than twice larger. If we increase the deformation in both Ti and Cr nothing happens. On the contrary, if we reduce the deformation of Ti, the matrix elements are severely quenched. The effect of deformation is therefore quite important and cannot be overlooked

	^{48}Ca $\rightarrow^{48}$Ti	^{48}Ti $\rightarrow^{48}$Cr
$M_{GT}(2\nu)$	0.083	0.213
$M_{GT}(0\nu)$	0.667	1.298

The influence of the spin-orbit partner

Similarly, we can increase artificially the excitation energy of the spin-orbit partner of the intruder orbit. Surprisingly enough, this affects very little the values of the matrix elements, particularly in the neutrinoless case. Even removing the spin-orbit partner completely produces minor changes

	^{48}Ca $\rightarrow ^{48}$Ti	^{48}Ti $\rightarrow ^{48}$Cr
$M_{GT}(2\nu)$	0.083	0.213
$M_{GT}(0\nu)$	0.667	1.298

Without spin-orbit partner

	^{48}Ca $\rightarrow ^{48}$Ti	^{48}Ti $\rightarrow ^{48}$Cr
$M_{GT}(2\nu)$	0.049	0.274
$M_{GT}(0\nu)$	0.518	1.386

The multipole structure of the 0ν matrix element

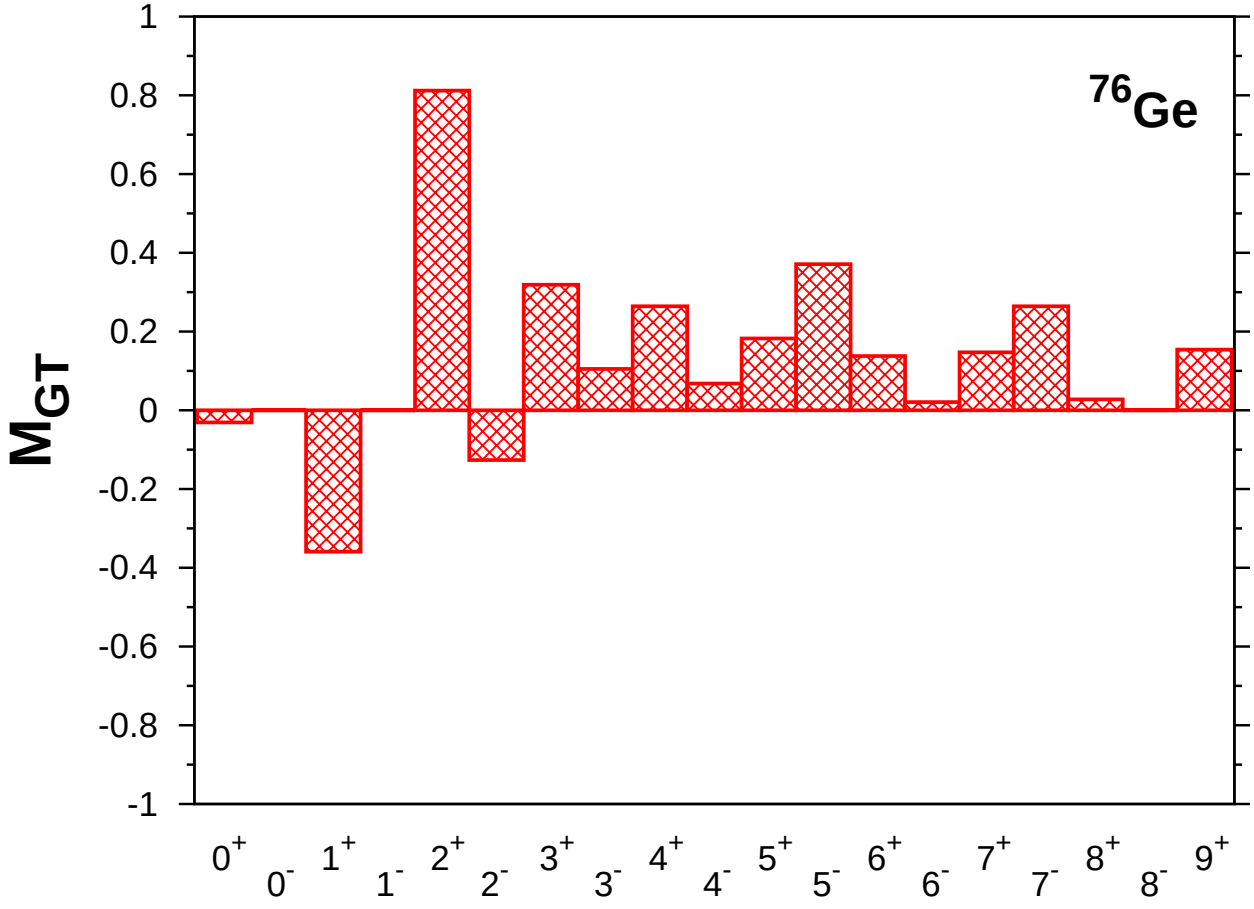
The transformation of a two body interaction from the **p-p** to the **p-h** form is highly non-unique

$[(a_1^\dagger a_2^\dagger)^J \cdot (a_1 a_2)^J]^0$ can go to
$[(a_1^\dagger a_1)^\lambda \cdot (a_2^\dagger a_2)^\lambda]^0$ or to
$[(a_1^\dagger a_2)^\gamma \cdot (a_2^\dagger a_1)^\gamma]^0$

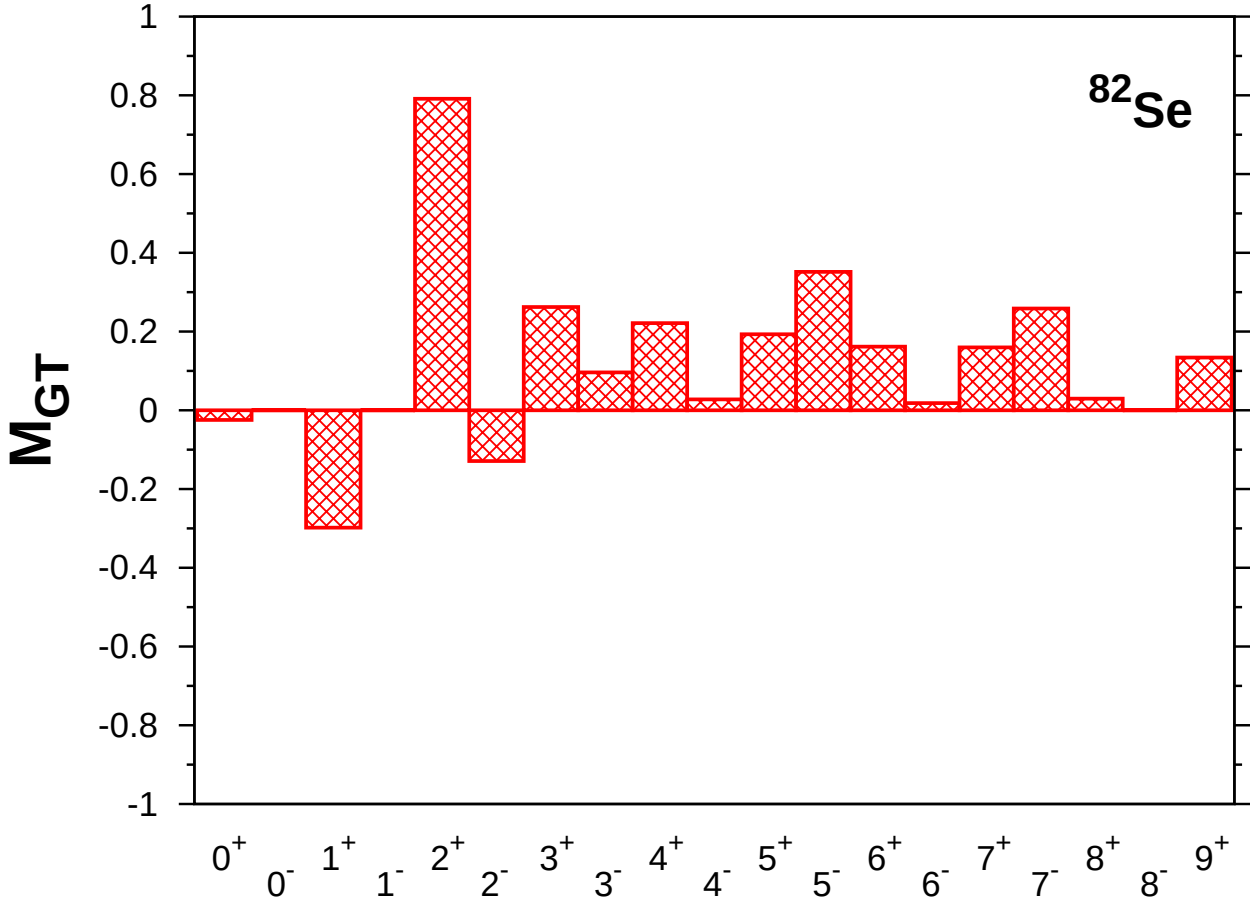
We make the choice of keeping all the orderings of the matrix elements even if they are redundant. Notice however that other choices may lead to different decompositions, that have the same physical content

Our results differ markedly of those of the QRPA calculations

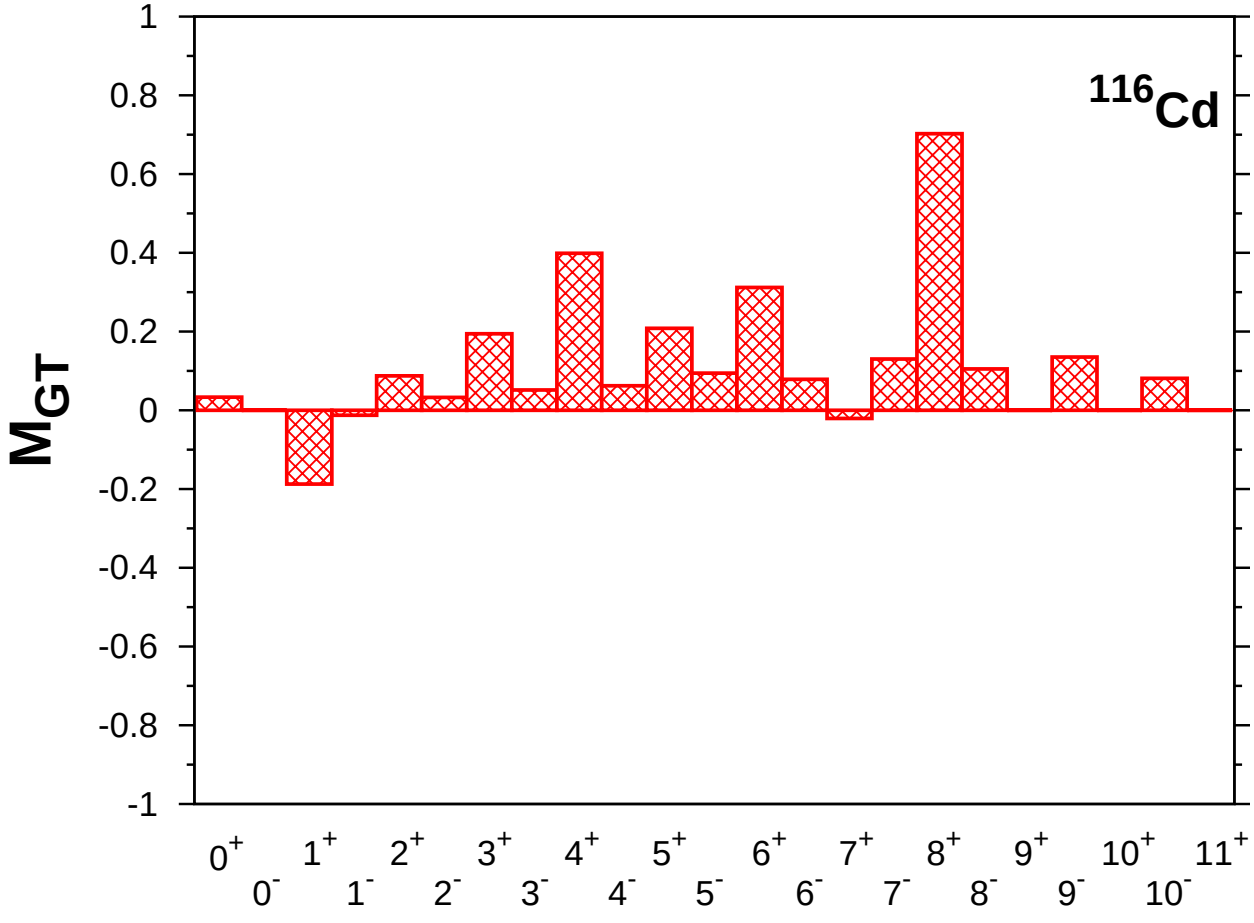
The multipole structure of the 0ν matrix element: A=76



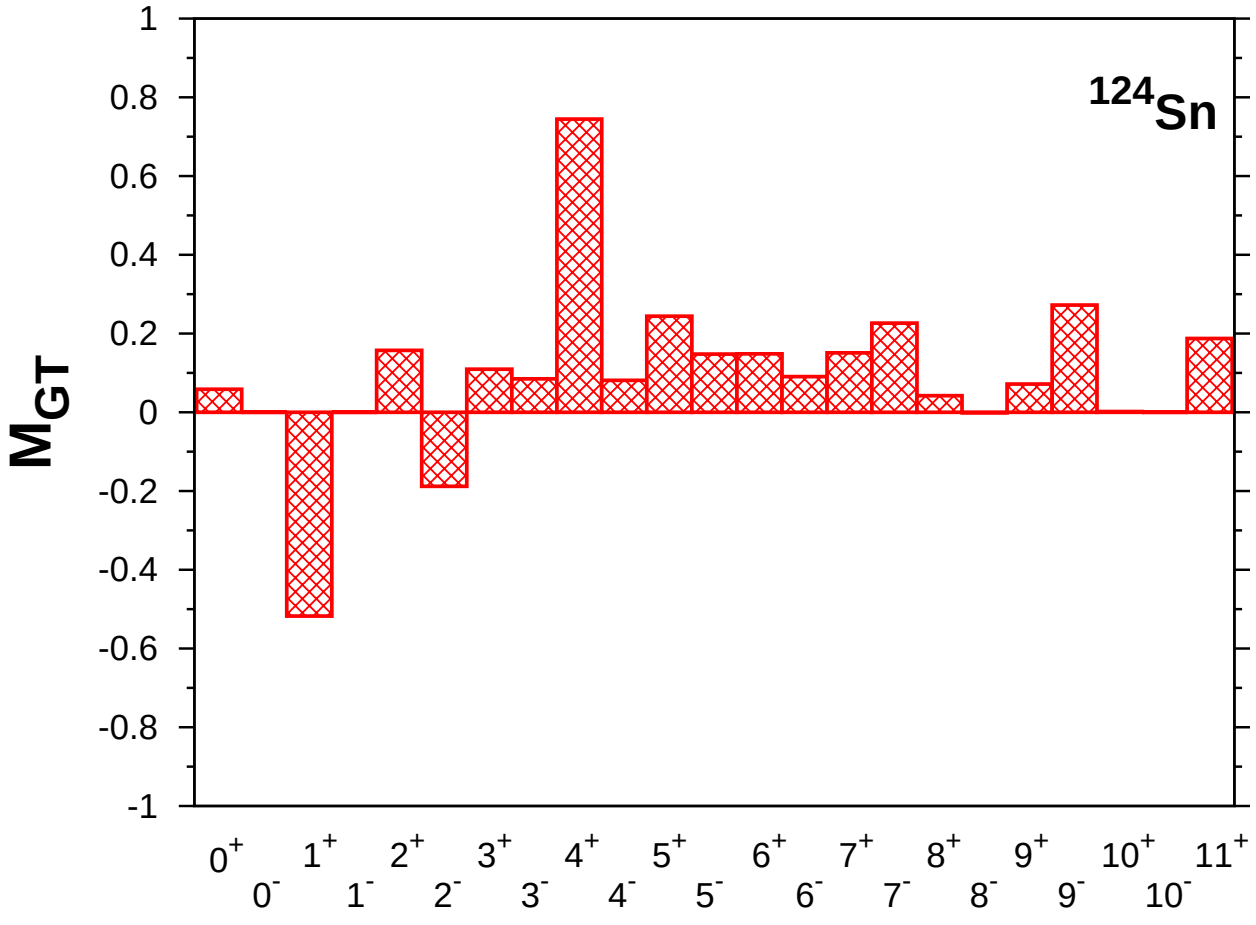
The multipole structure of the 0ν matrix element: A=82



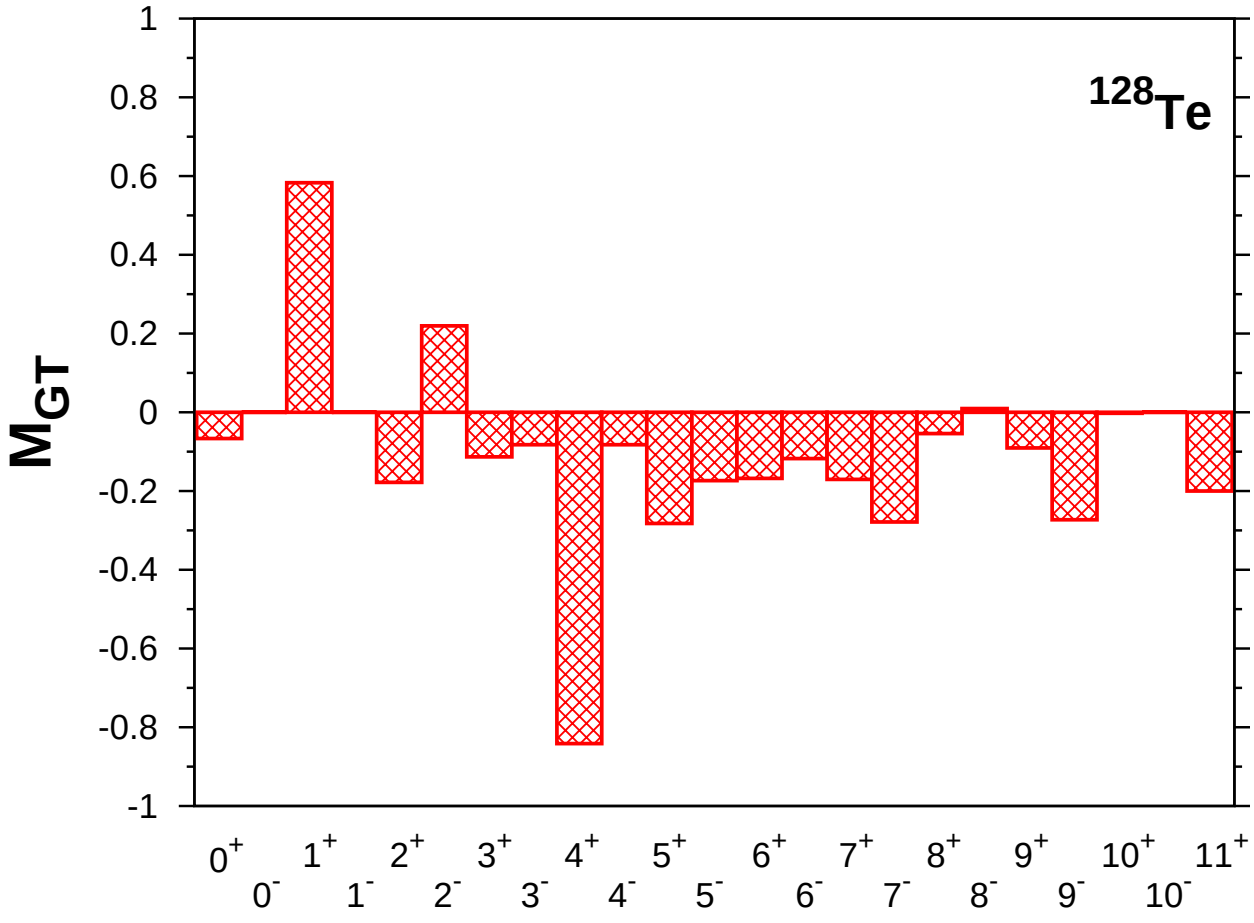
The multipole structure of the 0ν matrix element: A=116



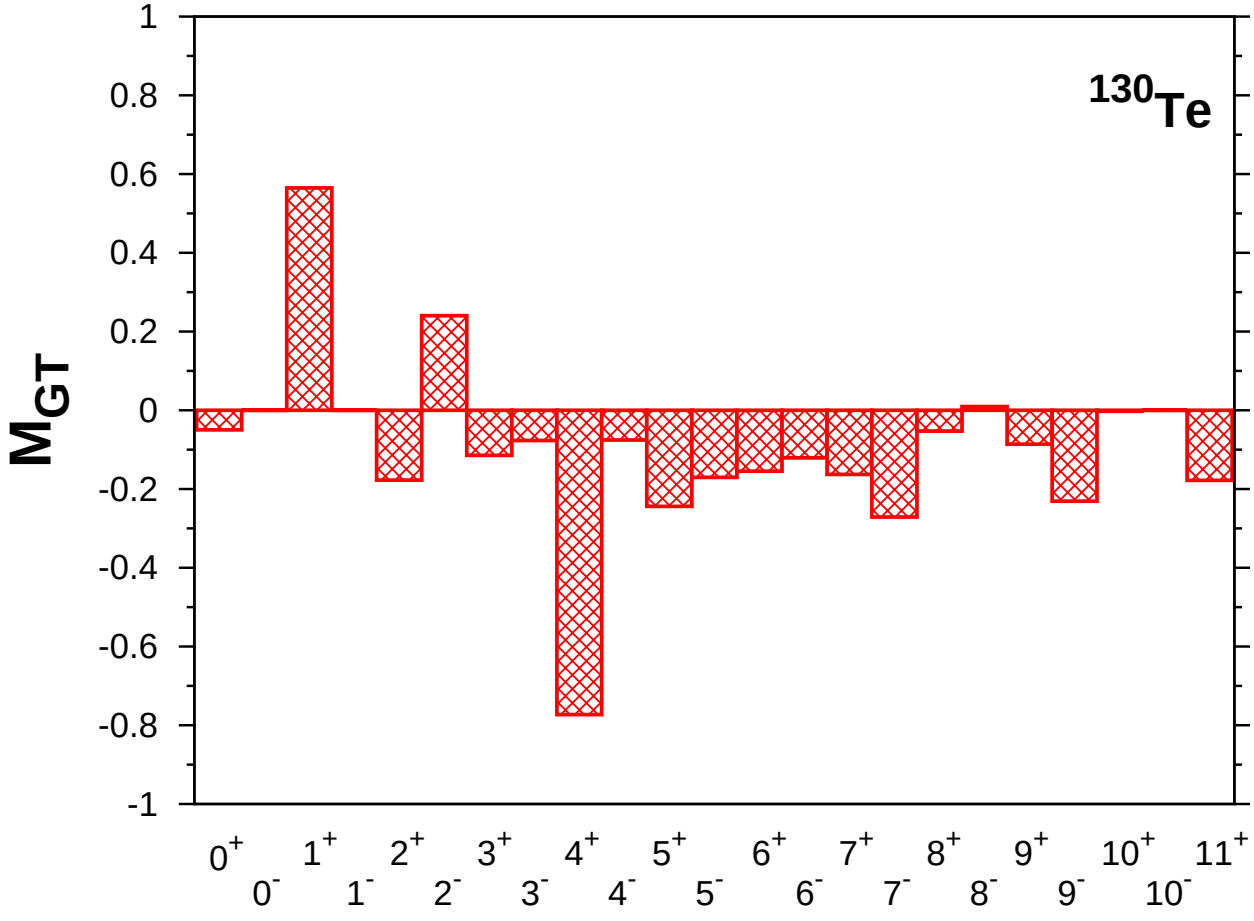
The multipole structure of the 0ν matrix element: A=124



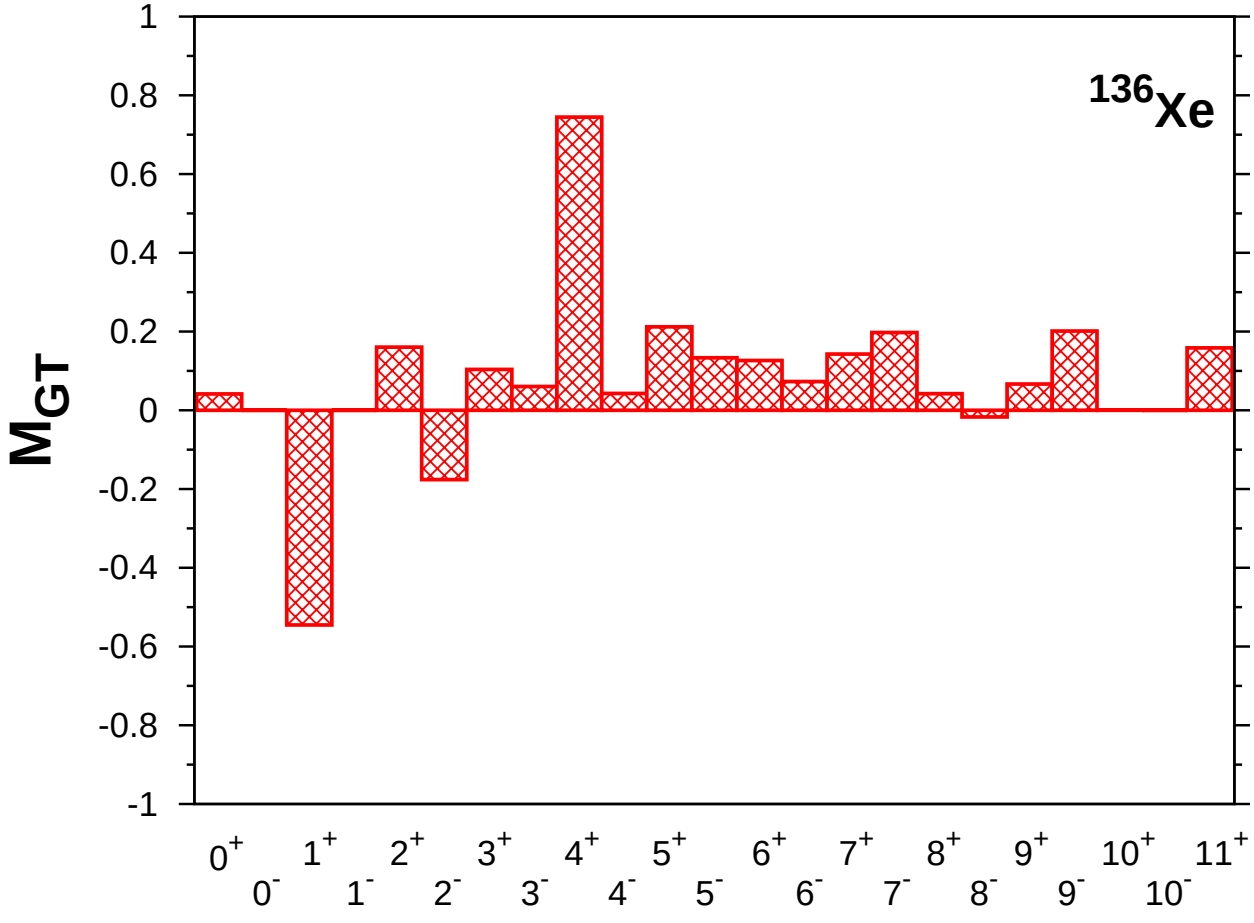
The multipole structure of the 0ν matrix element: A=128



The multipole structure of the 0ν matrix element: A=130



The multipole structure of the 0ν matrix element: A=136



An exploration of the influence of $2\hbar\omega$ correlations in the ^{48}Ca $\rightarrow^{48}$Ti decay

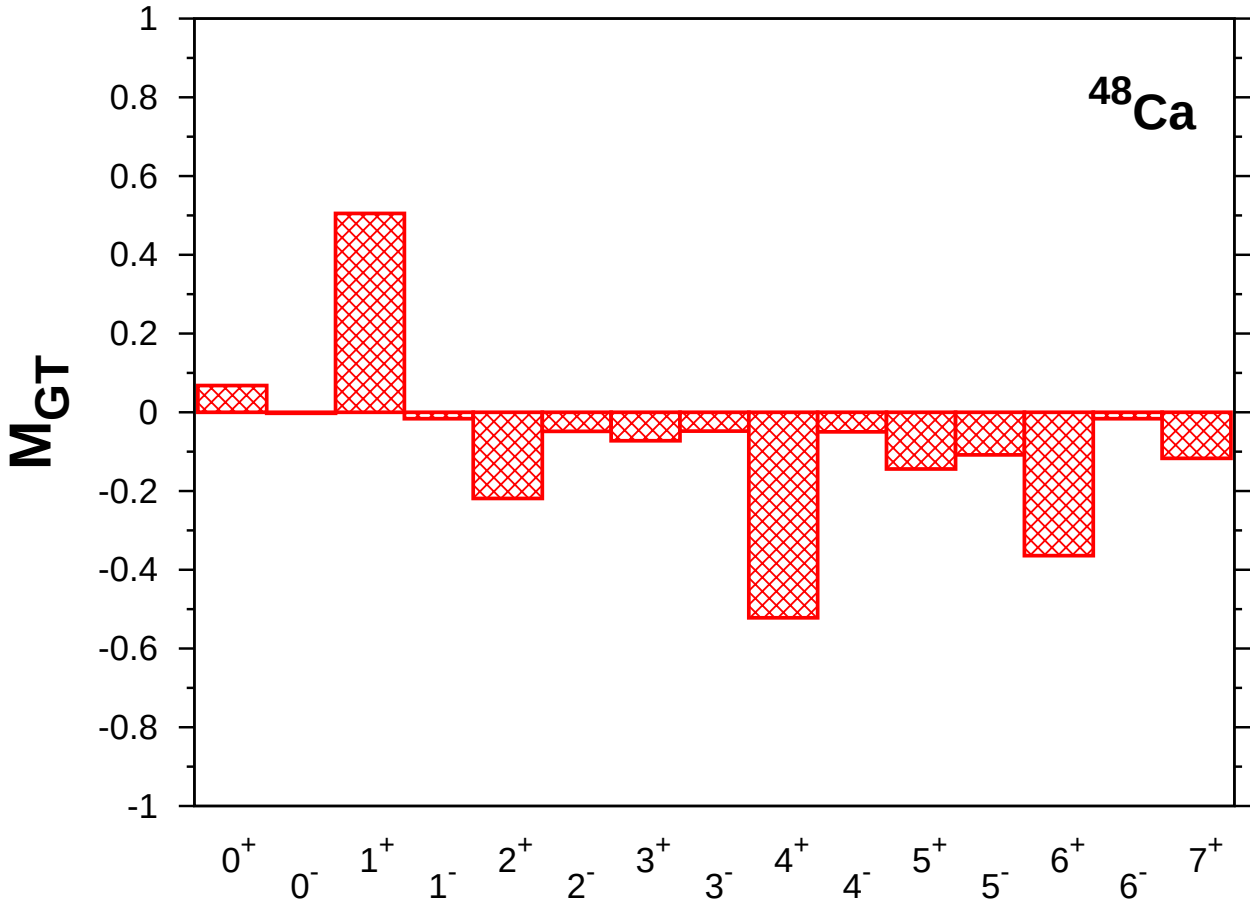
We have started studying the effect of 2p-2h excitations from the sd shell to the pf shell (core excitations) and from the pf shell to the $0g_{9/2}$ orbit. Some preliminary results are:

Excitations to the $0g_{9/2}$ orbit have no appreciable effect in the 0ν matrix element. Even for unrealistically large occupations, it barely increases by 10%

On the contrary, proton excitations from the sd shell have large effects. The 0ν matrix element increases a 25% already for a mixing as low as 2%. For reasonable amounts of core excited components in the wave functions of parent and grand-daughter (5% to 10%) the 0ν matrix element increases by 40% to 50%. The effect is less pronounced in the 2ν decay with 20% to 30% increases.

We are analyzing this effect, trying to establish if this behavior is peculiar to the case of ^{48}Ca, the only doubly magic double beta emitter, or a more general property

The multipole structure of the 0ν matrix element: A=48



CONCLUSIONS

- Large scale shell model calculations with high quality effective interactions are available or will be in the immediate future for all but one of the neutrinoless double beta emitters

- The theoretical spread of the values of the nuclear matrix elements entering in the lifetime calculations is greatly reduced if the ingredients of each calculation are examined critically and only those fulfilling a set of quality criteria are retained

- A concerted effort of benchmarking between LSSM and QRPA practitioners would be of utmost importance to increase the reliability and precision of the nuclear structure input for the double beta decay processes