Neutrinos and Dark Matter in Nuclear Physics
NDM06, 3-9 September 2006, Paris

Collective Oscillations of Supernova Neutrinos

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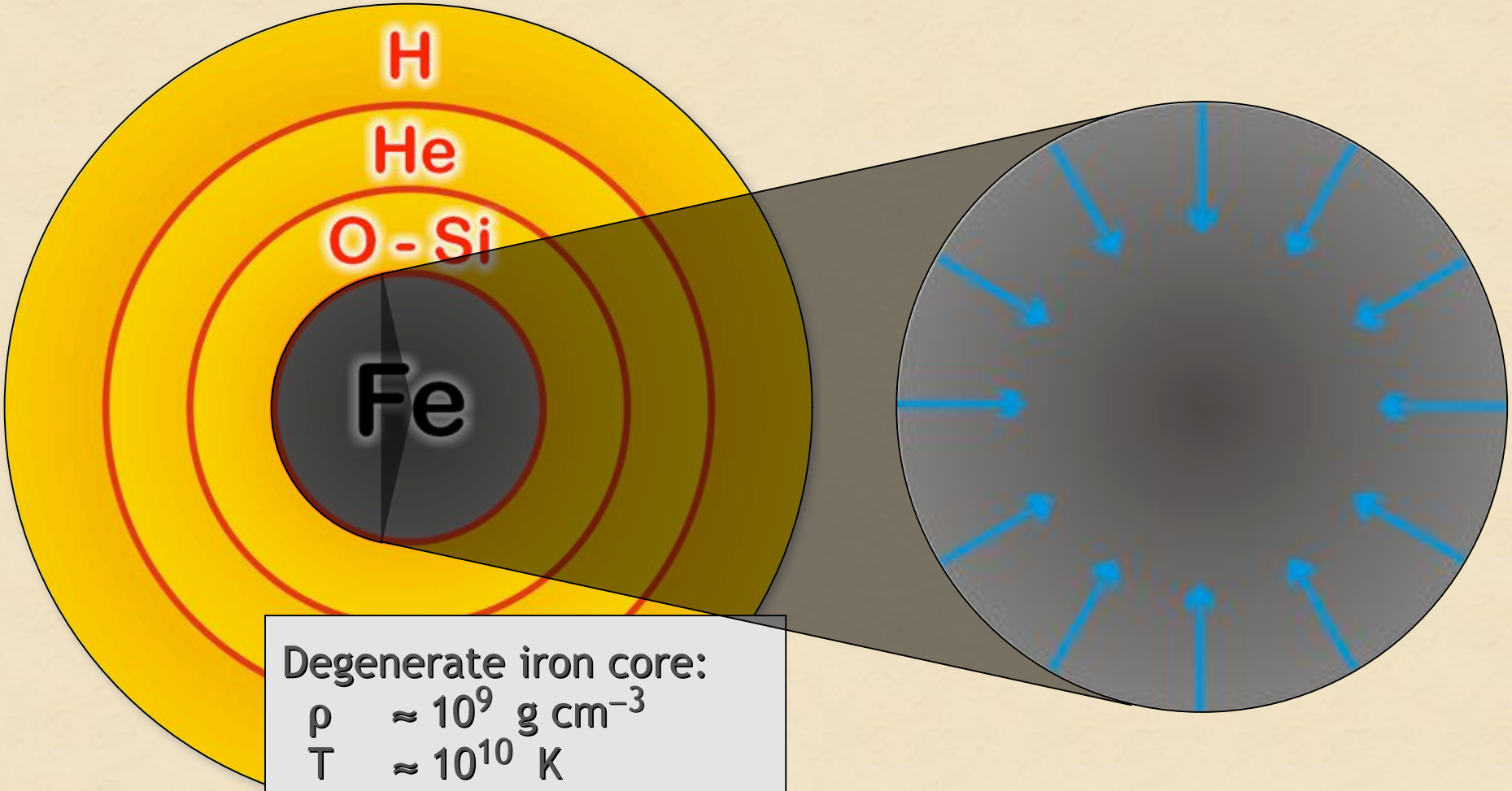
Supernova 1987A
23 February 1987

Stellar Collapse and Supernova Explosion

Onion structure

Collapse (implosion)



Degenerate iron core:

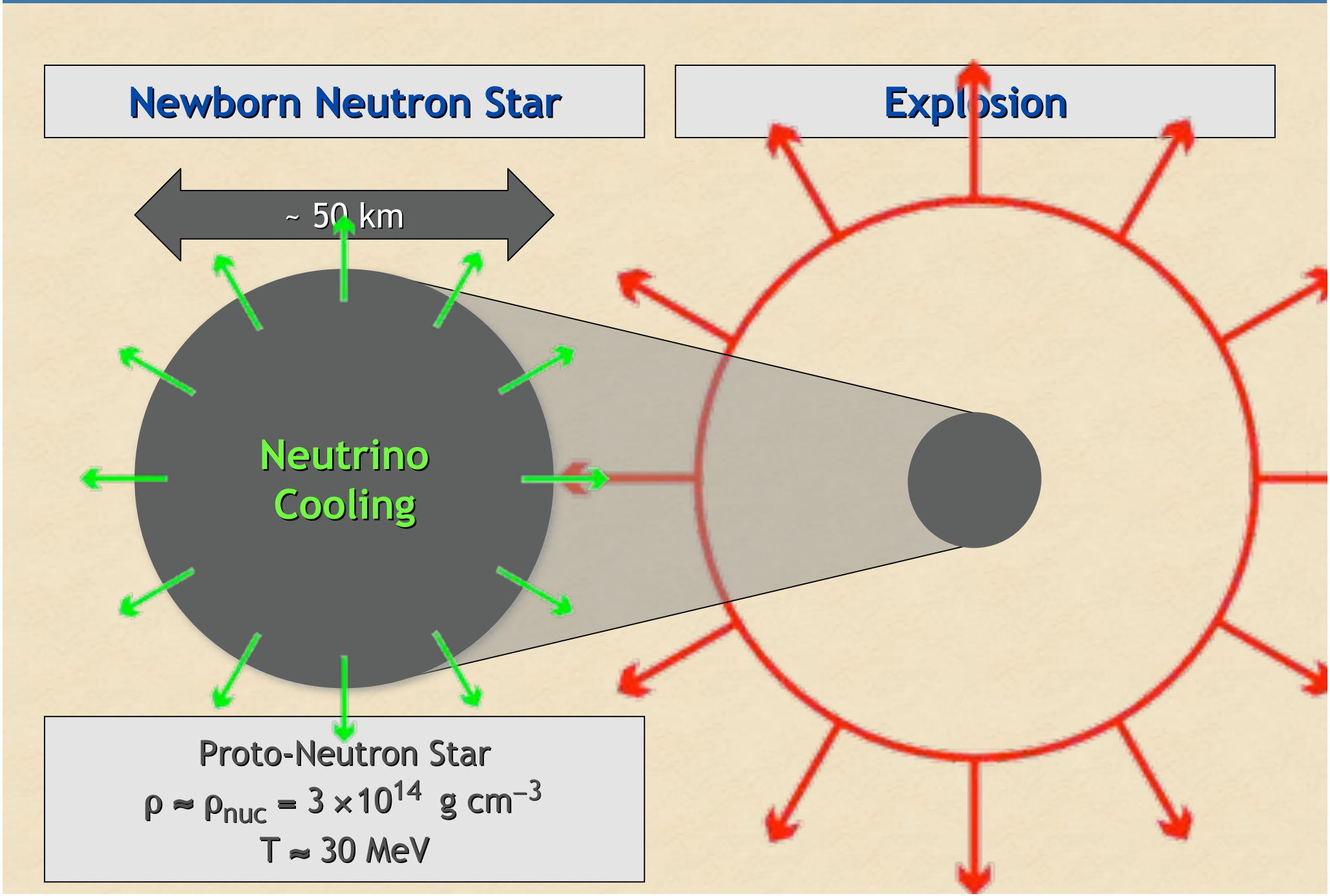
$\rho \approx 10^9$ g cm^{-3}

$T \approx 10^{10}$ K

$M_{Fe} \approx 1.5\ M_{sun}$

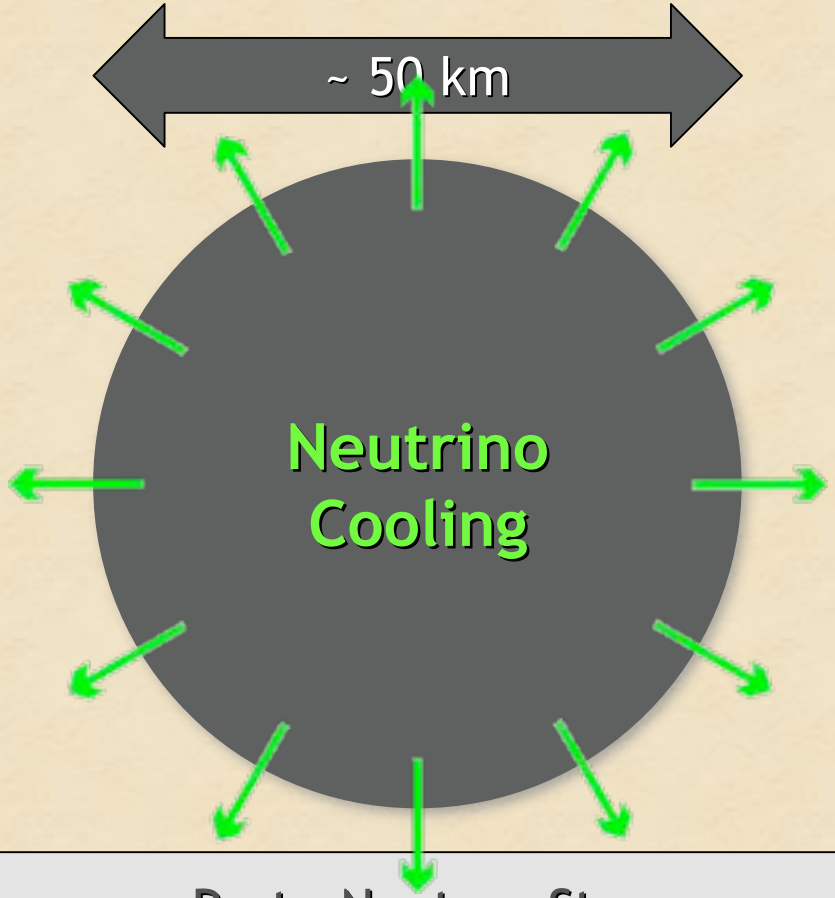
$R_{Fe} \approx 8000$ km

Stellar Collapse and Supernova Explosion



Stellar Collapse and Supernova Explosion

Newborn Neutron Star



Proto-Neutron Star

$\rho \approx \rho_{nuc} = 3 \times 10^{14}$ g cm^{-3}

T ≈ 30 MeV

Gravitational binding energy

$E_b \approx 3 \times 10^{53}$ erg $\approx$ 17% $M_{SUN}\, c^2$

This shows up as

99% Neutrinos

1% Kinetic energy of explosion (1% of this into cosmic rays)

0.01% Photons, outshine host galaxy

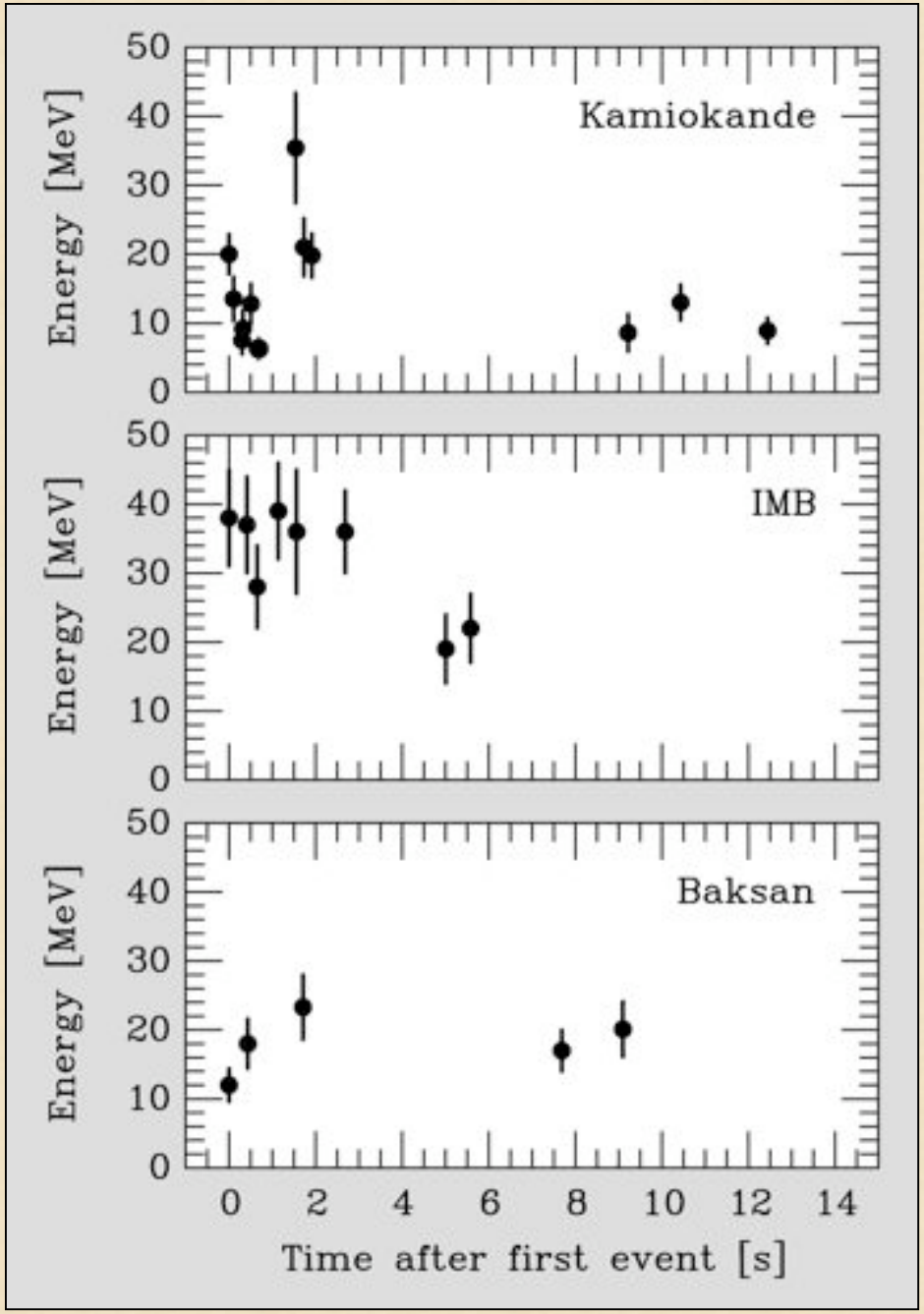
Neutrino luminosity

$L_\nu \approx 3 \times 10^{53}$ erg / 3 sec

$\approx 3 \times 10^{19}\, L_{SUN}$

While it lasts, outshines the entire visible universe

Neutrino Signal of Supernova 1987A



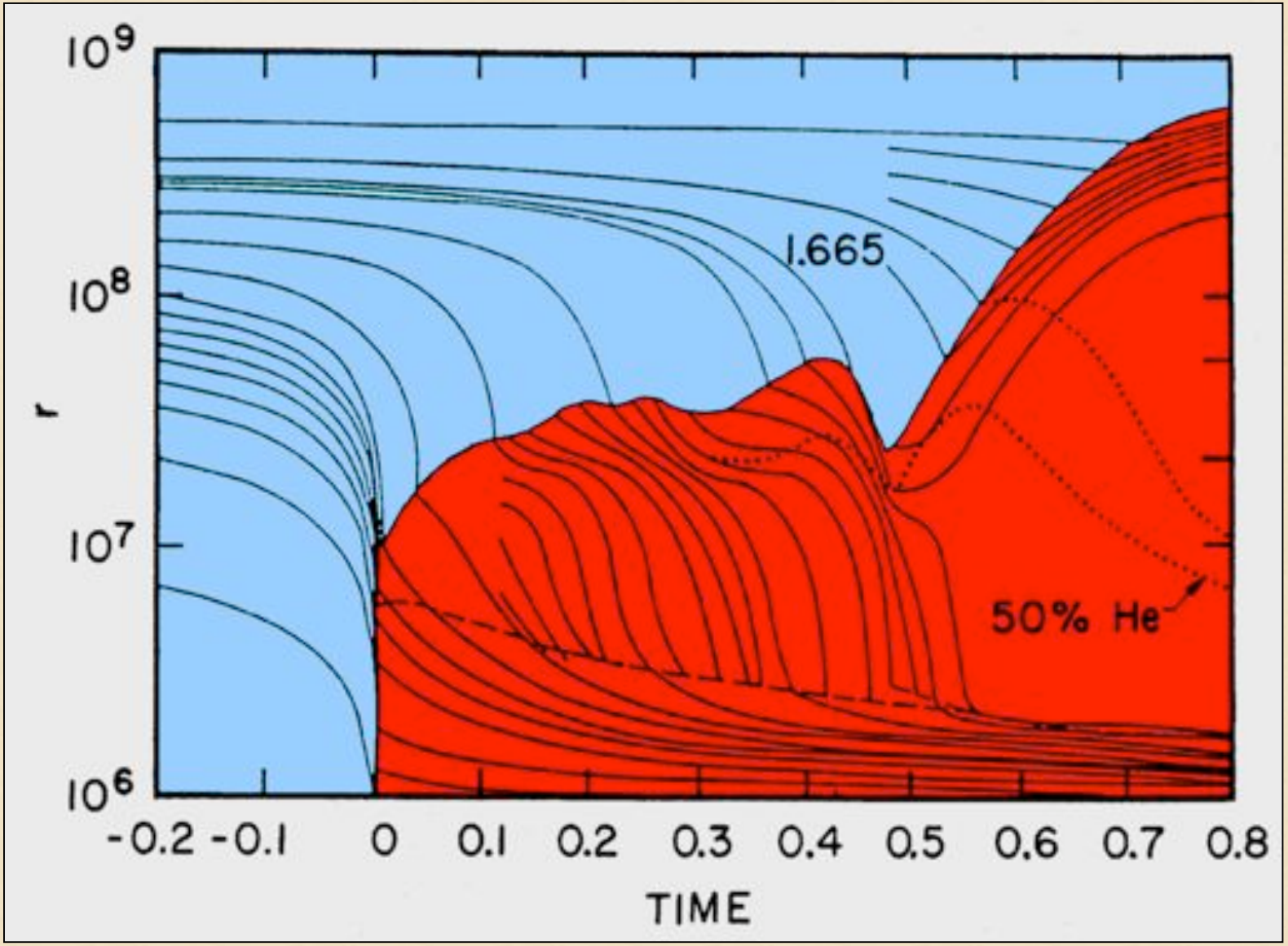
Kamiokande-II (Japan)
Water Cherenkov detector
2140 tons
Clock uncertainty ±1 min

Irvine-Michigan-Brookhaven (US)
Water Cherenkov detector
6800 tons
Clock uncertainty ±50 ms

Baksan Scintillator Telescope (Soviet Union), 200 tons
Random event cluster ~ 0.7/day
Clock uncertainty +2/-54 s

Within clock uncertainties, signals are contemporaneous

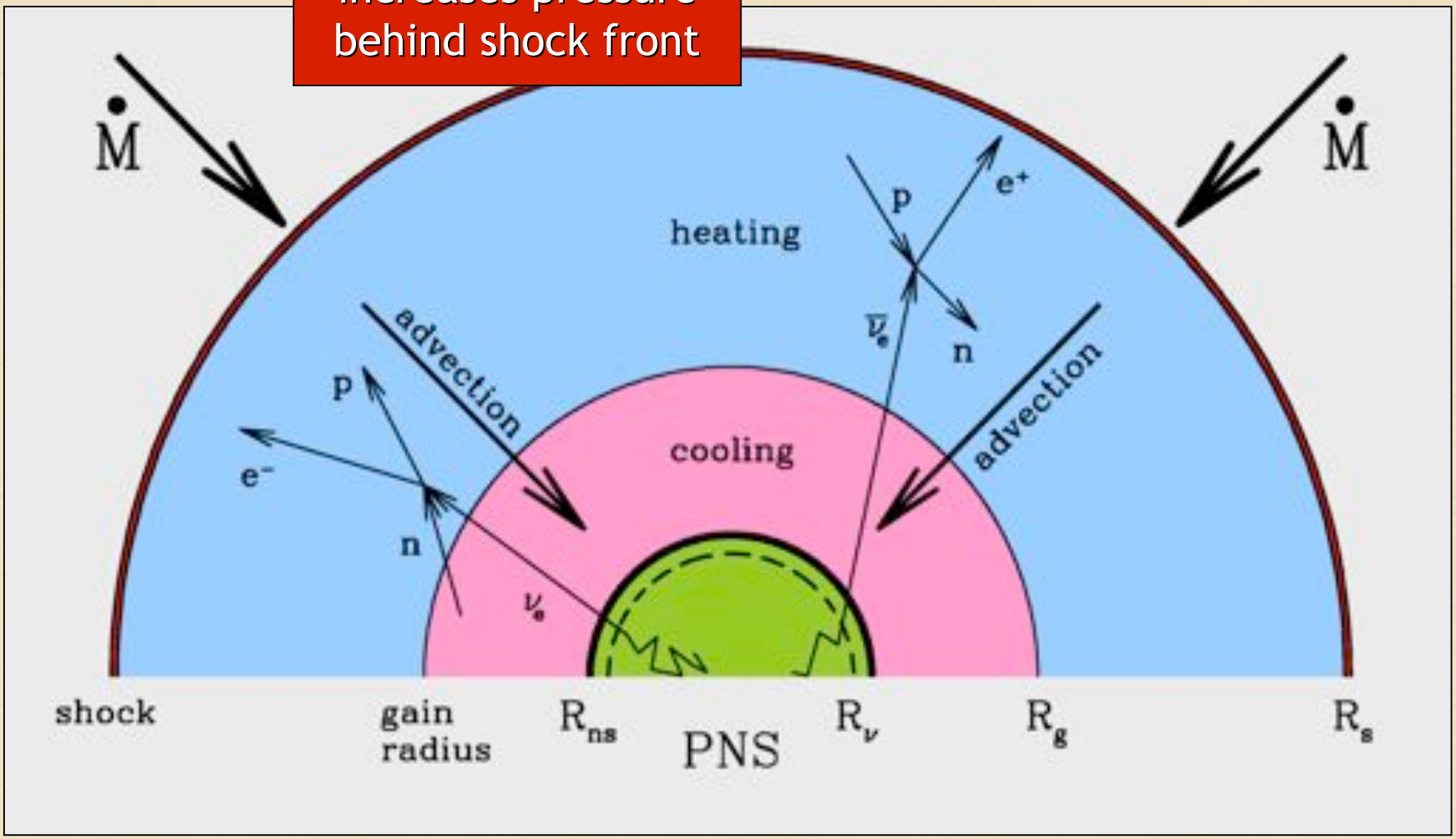
Delayed Explosion



Wilson, Proc. Univ. Illinois Meeting on Num. Astrophys.(1982)
Bethe & Wilson, ApJ 295 (1985) 14

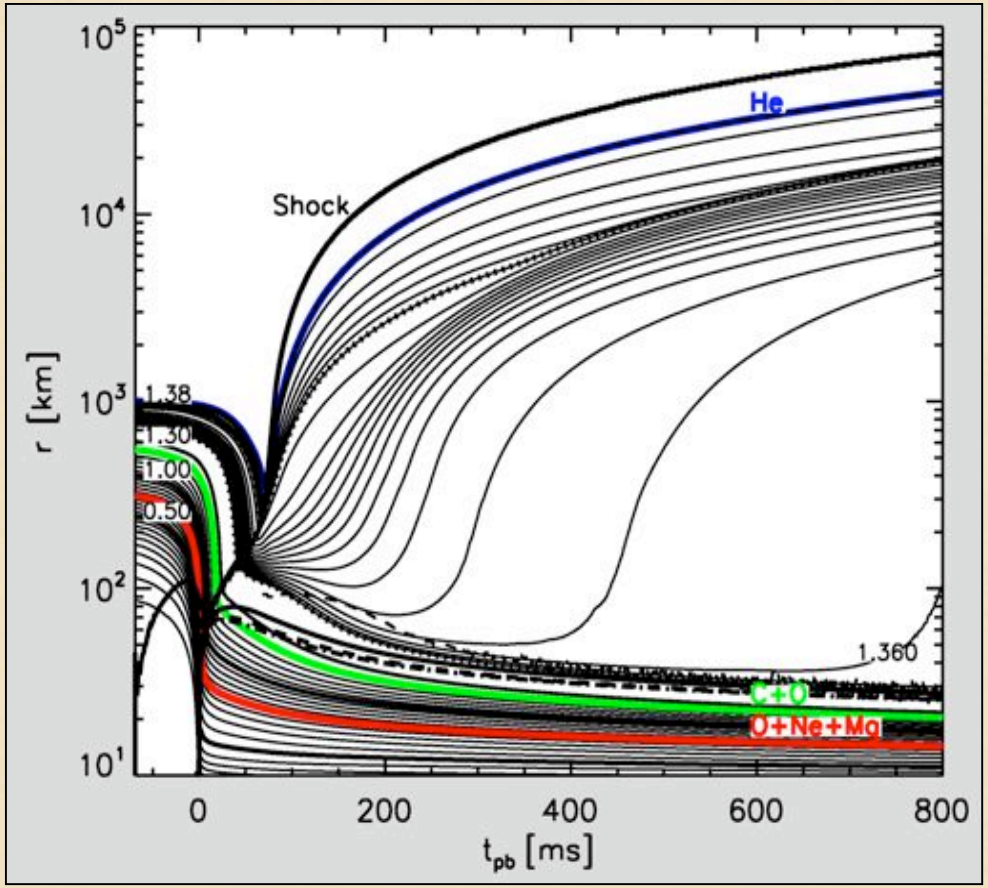
Neutrinos to the Rescue

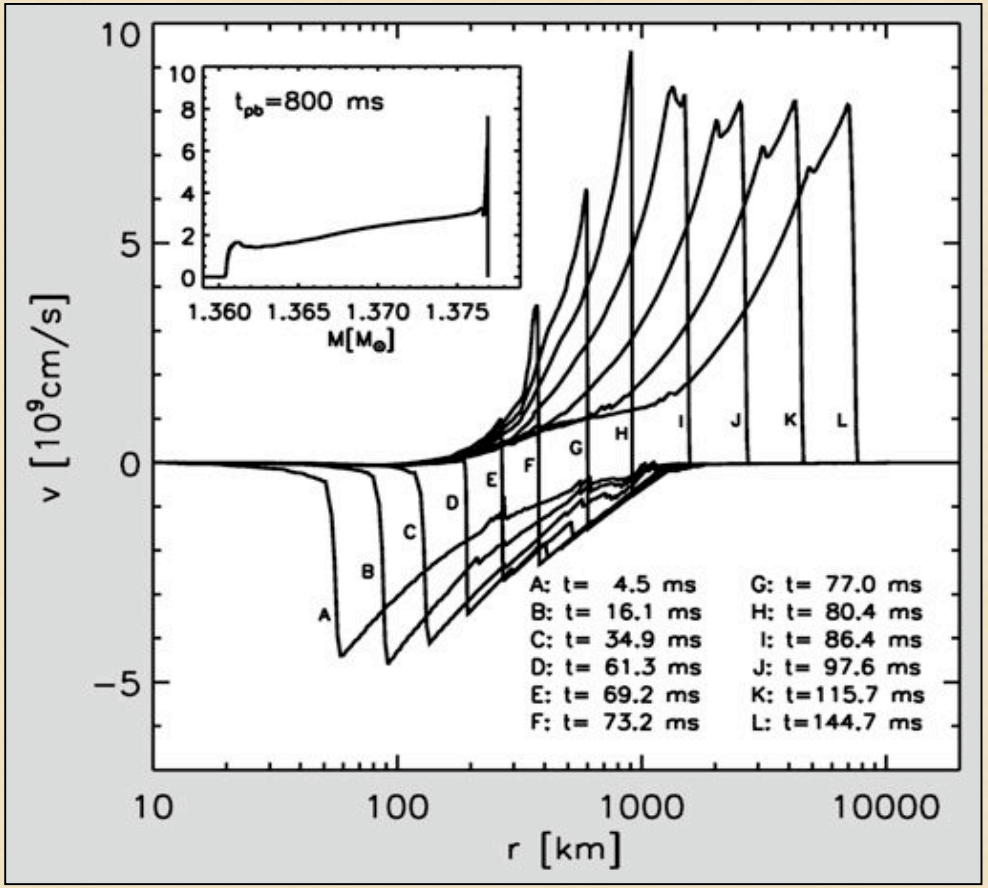
Neutrino heating increases pressure behind shock front



Picture adapted from Janka, astro-ph/0008432

Exploding Models (8-10 Solar Masses) with O-Ne-Cores

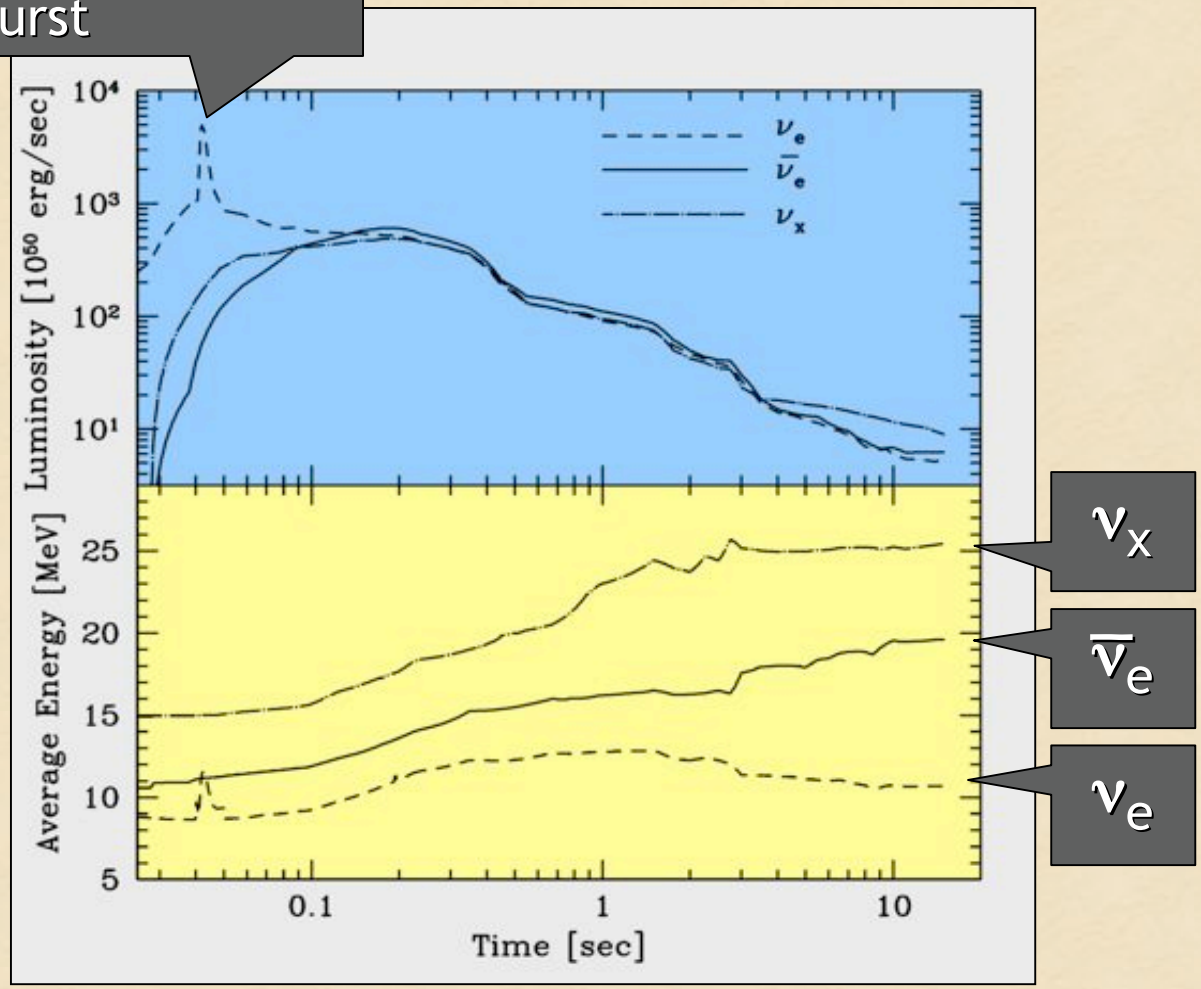




Kitaura, Janka & Hillebrandt: “Explosions of O-Ne-Mg cores, the Crab supernova, and subluminous type II-P supernovae”, astro-ph/0512065

Flavor-Dependent Fluxes and Spectra

Prompt ν_e deleptonization burst



Livermore numerical model
ApJ 496 (1998) 216

Broad characteristics

- Duration a few seconds
- $\langle E_\nu \rangle$ ~ 10–20 MeV
- $\langle E_\nu \rangle$ increases with time
- Hierarchy of energies

$$\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle$$

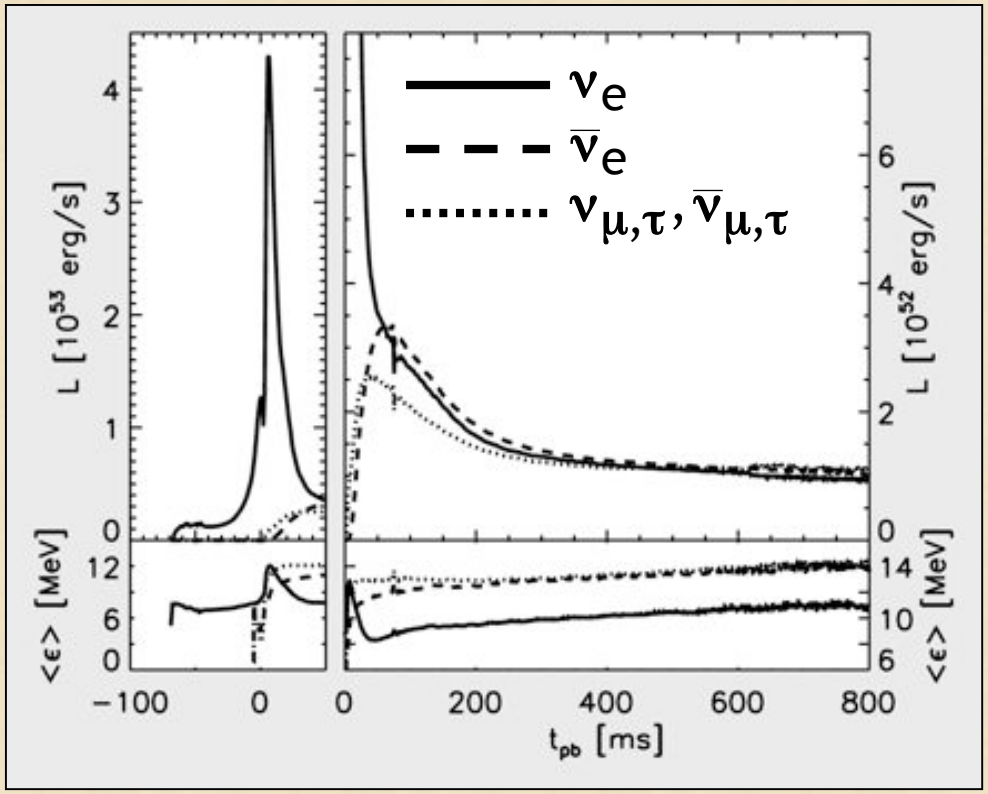
- Approximate equipartition of energy between flavors

However, in traditional simulations transport of ν_μ and ν_τ schematic

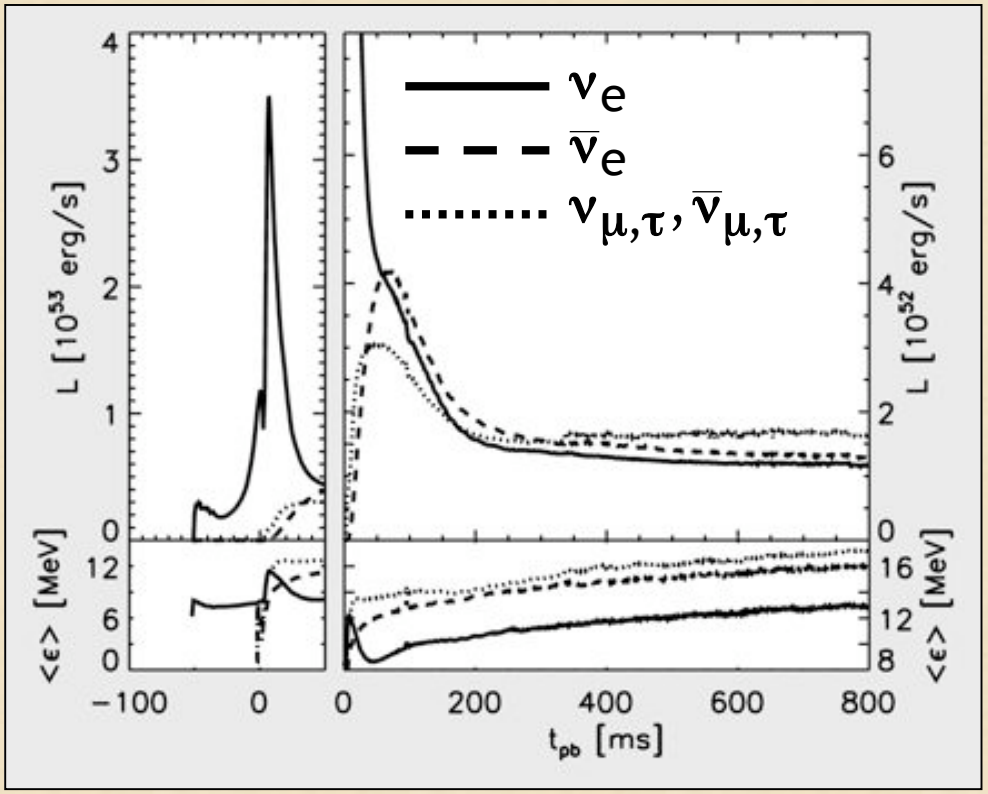
- Incomplete microphysics
- Crude numerics to couple neutrino transport with hydro code

Flavor-Dependent Neutrino Fluxes vs. Equation of State

Wolff & Hillebrandt nuclear EoS (stiff)

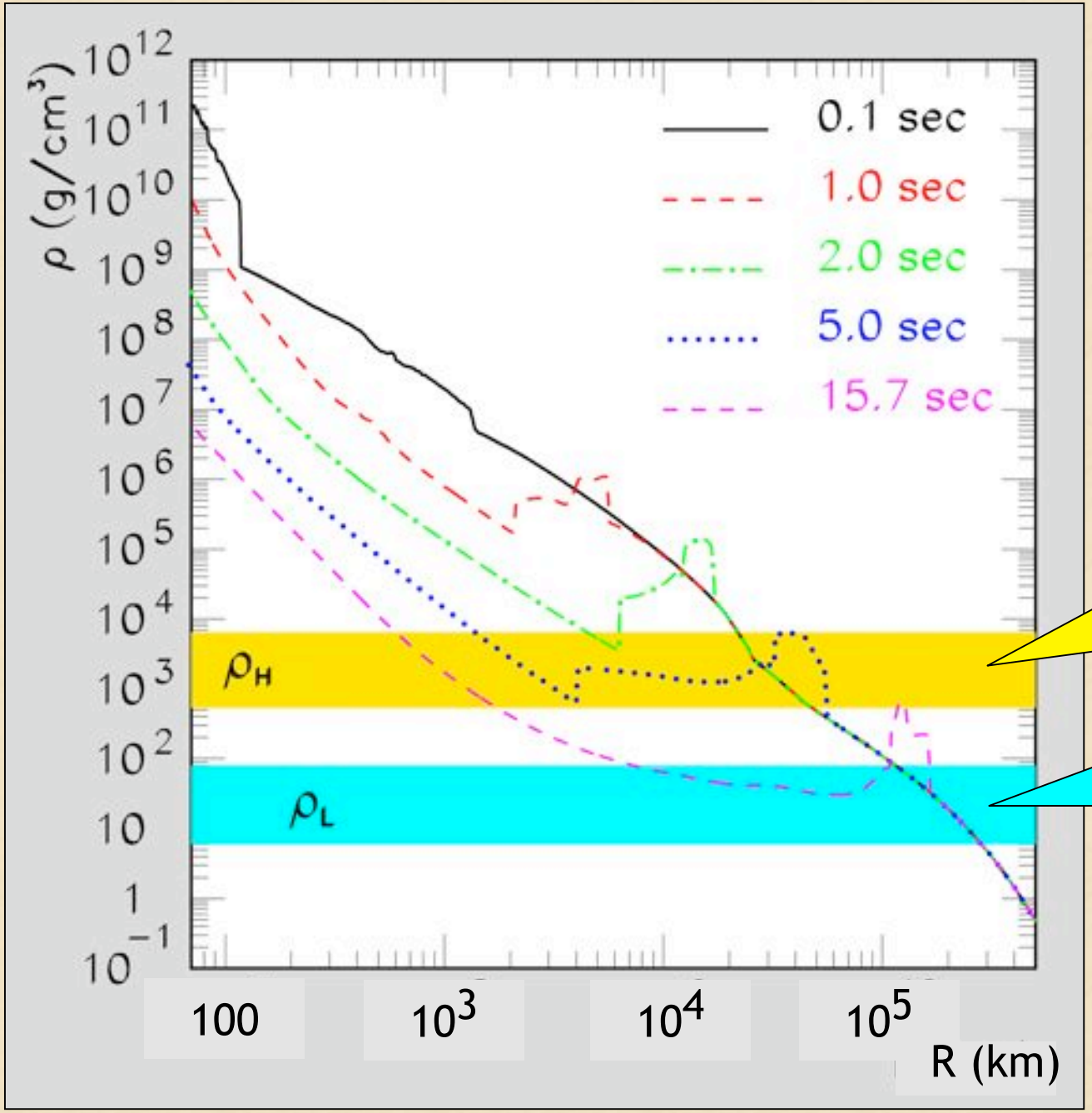


Lattimer & Swesty nuclear EoS (soft)



Kitaura, Janka & Hillebrandt, "Explosions of O-Ne-Mg cores, the Crab supernova, and subluminous Type II-P supernovae", astro-ph/0512065

H- and L-Resonance for MSW Oscillations

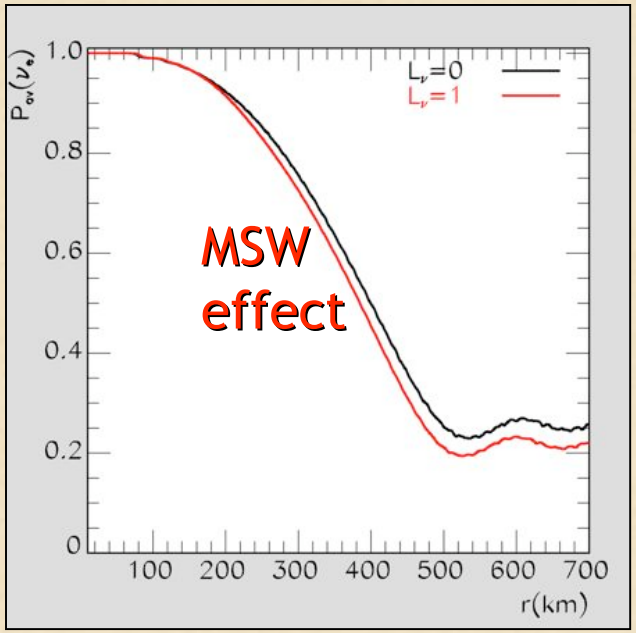
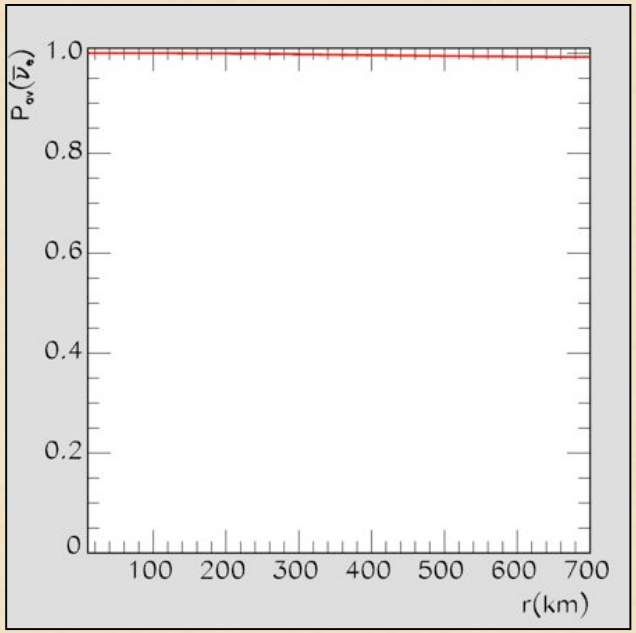
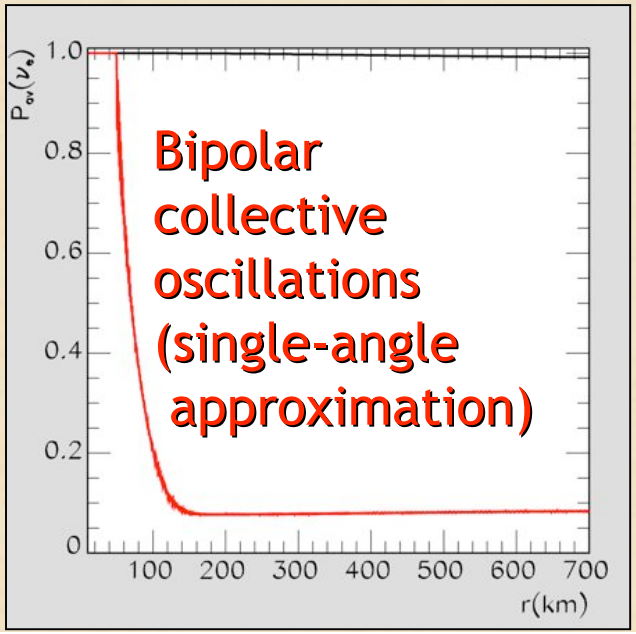
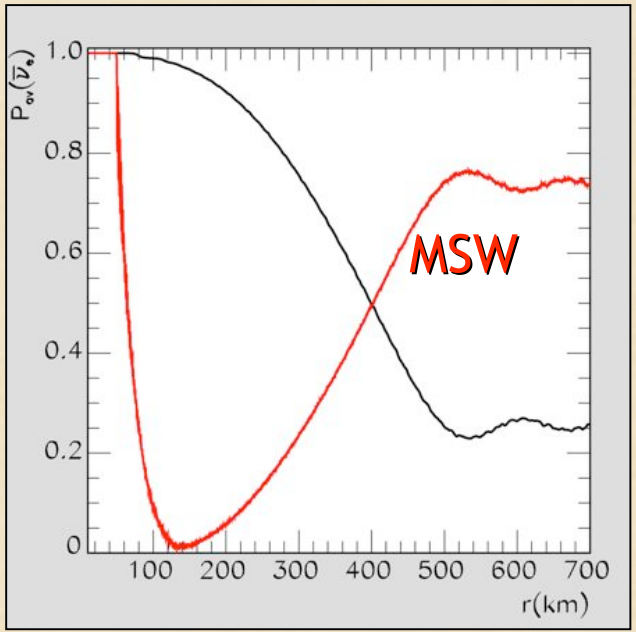


R. Tomàs, M. Kachelriess, G. Raffelt, A. Dighe, H.-T. Janka & L. Scheck: Neutrino signatures of supernova forward and reverse shock propagation [astro-ph/0407132]

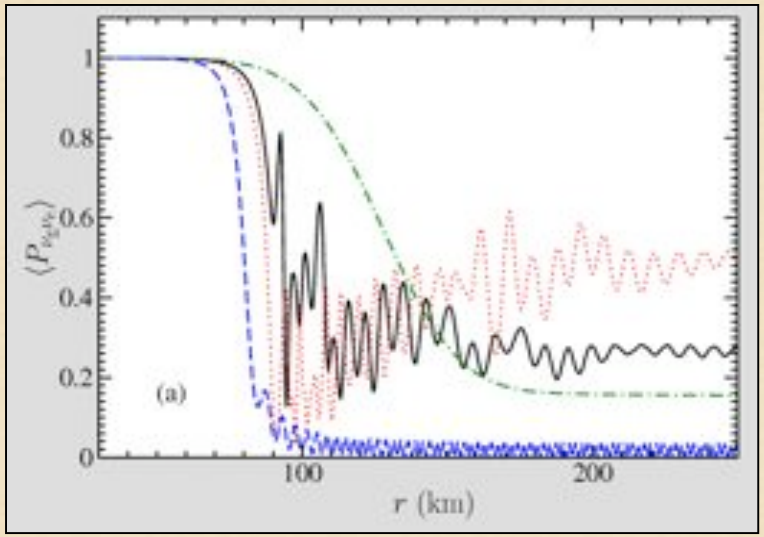
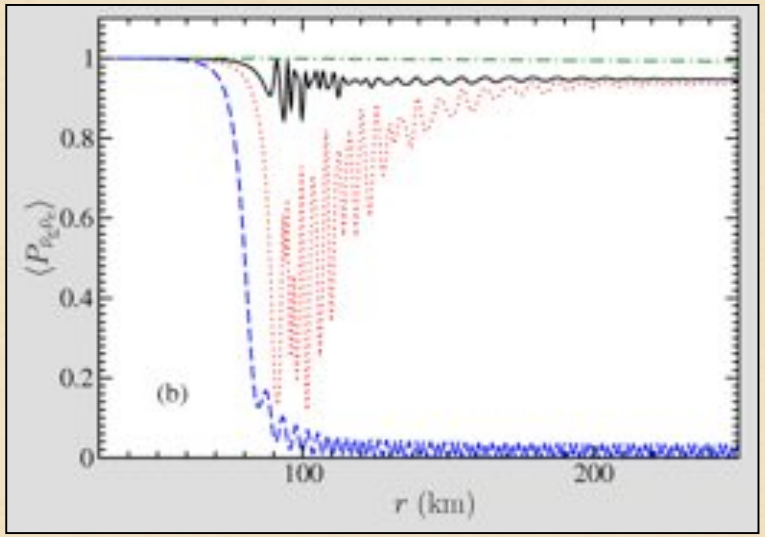
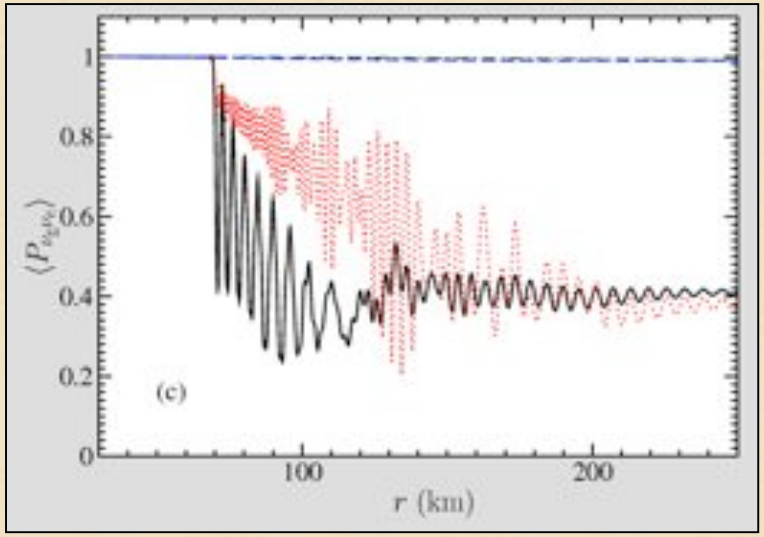
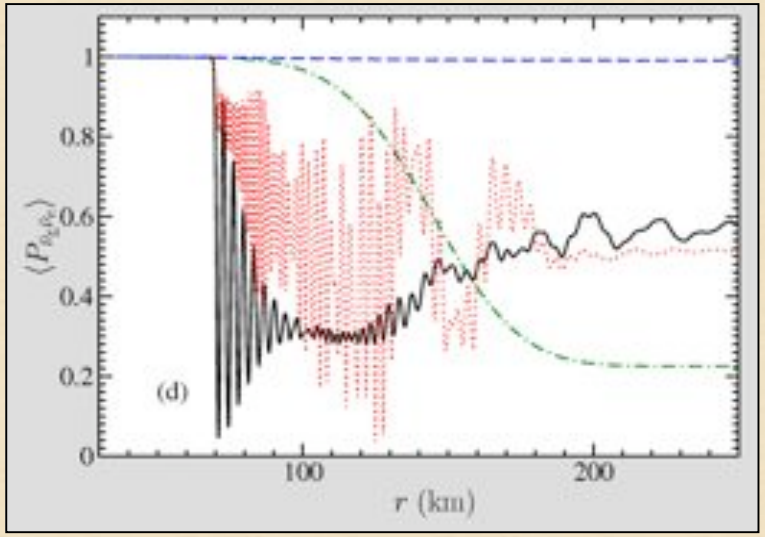
Resonance density for Δ

Resonance density for Δ

Self-Induced Flavor Oscillations of SN Neutrinos

	Survival probability ν_e	Survival probability $\bar{\nu}_e$	
Normal Hierarchy atm Δm^2 Θ_{13} close to Chooz limit			Realistic nu-nu effect No nu-nu effect
Inverted Hierarchy			Realistic nu-nu effect No nu-nu effect

Nonlinear Neutrino Conversion in Supernovae

	Survival probability ν_e	Survival probability $\bar{\nu}_e$
Normal Hierarchy atm Δm^2 Θ_{13} close to Chooz limit		
Inverted Hierarchy		

Duan, Fuller, Carlson, Qian: "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories", astro-ph/0606616. See also: astro-ph/0608050

Bipolar Oscillations of Neutrinos in a Box

Survival probability of both $\nu_{\blacksquare}$ and $\bar{\nu}_{\blacksquare}$

$\blacksquare\blacksquare\blacksquare\blacksquare|\nu_{\blacksquare} \rightarrow \nu_{\blacksquare}| = \frac{|}{\blacksquare}|\blacksquare_{\blacksquare} + \blacksquare|$

Dense gas of $\nu_{\blacksquare}$ and $\bar{\nu}_{\blacksquare}$

Take equal densities with

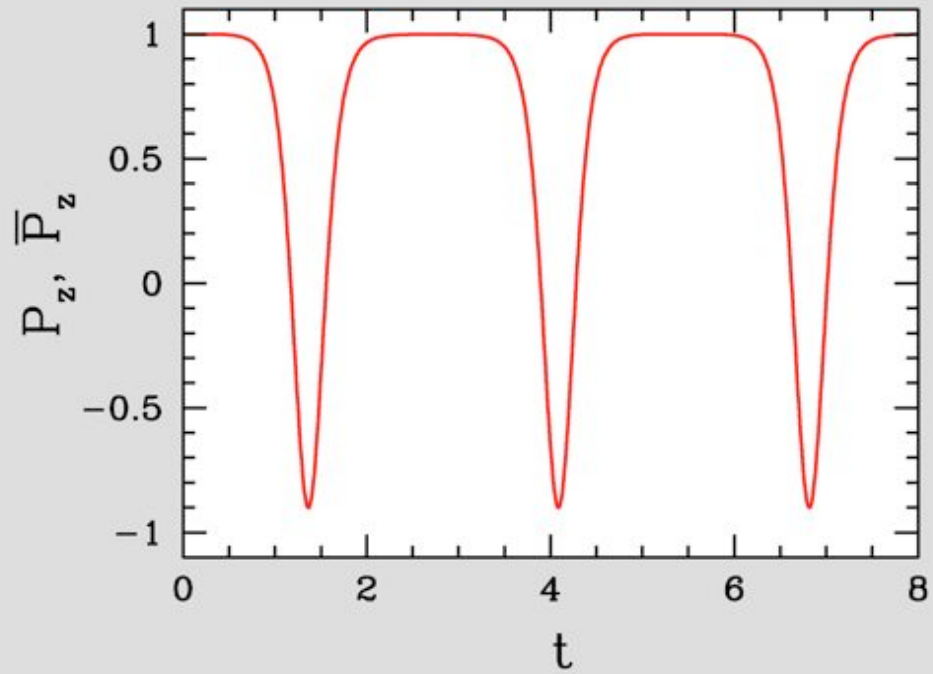
$\mu = \sqrt{\blacksquare}\,\blacksquare_{\blacksquare}\blacksquare_{\nu_{\blacksquare}} = \sqrt{\blacksquare}\,\blacksquare_{\blacksquare}\blacksquare_{\bar{\nu}_{\blacksquare}}$

Assume only one energy with

$\omega = \Delta\blacksquare^{\blacksquare}/\blacksquare\blacksquare = \frac{|}{\blacksquare\blacksquare}\,\mu$

Small mixing angle, inverted hierarchy

$\theta = \blacksquare.|$



- Time scale set by $\kappa = \sqrt{\omega\mu}$
- "Plateau phases" mean exponential growth, increase by -log(θ)
- Reduced mixing angle by ordinary matter unimportant

Note:
This is no real "flavor conversion",
Rather a "coherent pair conversion"

$\nu_{\blacksquare}\bar{\nu}_{\blacksquare} \leftrightarrow \nu_{\mu}\bar{\nu}_{\mu}$

Time scale proportional to $\sqrt{\blacksquare_{\blacksquare}}$

Neutrino Density Matrices in Flavor Space

Neutrino quantum field	$\Psi(t,\vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^3} \left[a(t,\vec{p})\, u_{\vec{p}} + b^\dagger(t,-\vec{p})\, v_{-\vec{p}} \right] e^{i\vec{p}\cdot\vec{x}}$
Spinors in flavor space	$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{pmatrix} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ Destruction operators for (anti)neutrinos

Variables for discussing neutrino flavor oscillations

Quantum states (amplitudes)	"Density matrices" (analogous to occupation numbers)
$\left\lvert \begin{matrix} \nu_1(t,\vec{p}) \\ \nu_2(t,\vec{p}) \\ \nu_3(t,\vec{p}) \end{matrix} \right\rangle = \begin{pmatrix} a_1(t,\vec{p}) \\ a_2(t,\vec{p}) \\ a_3(t,\vec{p}) \end{pmatrix}^\dagger \lvert 0 \rangle$	$\rho_{ij}(t,\vec{p}) = \langle a_i^\dagger(t,\vec{p})\, a_j(t,\vec{p}) \rangle$ Neutrinos $\bar{\rho}_{ij}(t,\vec{p}) = \langle b_i^\dagger(t,\vec{p})\, b_j(t,\vec{p}) \rangle$ Anti-neutrinos
Sufficient for "beam experiments," but confusing "wave packet debates" for quantifying decoherence effects	"Quadratic" quantities, required for dealing with decoherence, collisions, Pauli-blocking, nu-nu-refraction, etc.

General Equations of Motion

ν

$$i\partial_t \rho_p = +\left[\frac{M^2}{2p}, \rho_p\right] + \sqrt{2}G_F\left[N, \rho_p\right] + \sqrt{2}G_F\left[\int \frac{d^3q}{(2\pi)^3}\left(1 - \cos\theta_{pq}\right)\left(\rho_q - \bar{\rho}_q\right), \rho_p\right]$$

$\bar{\nu}$

$$i\partial_t \bar{\rho}_p = -\left[\frac{M^2}{2p}, \bar{\rho}_p\right] + \sqrt{2}G_F\left[N, \bar{\rho}_p\right] + \sqrt{2}G_F\left[\int \frac{d^3q}{(2\pi)^3}\left(1 - \cos\theta_{pq}\right)\left(\rho_q - \bar{\rho}_q\right), \bar{\rho}_p\right]$$

- Vacuum oscillations
  M neutrino mass matrix
- Note opposite sign between neutrinos and antineutrinos

Usual matter effect with

$$N = \begin{pmatrix} n_e - n_{\bar{e}} & 0 & 0 \\ 0 & n_\mu - n_{\bar{\mu}} & 0 \\ 0 & 0 & n_\tau - n_{\bar{\tau}} \end{pmatrix}$$

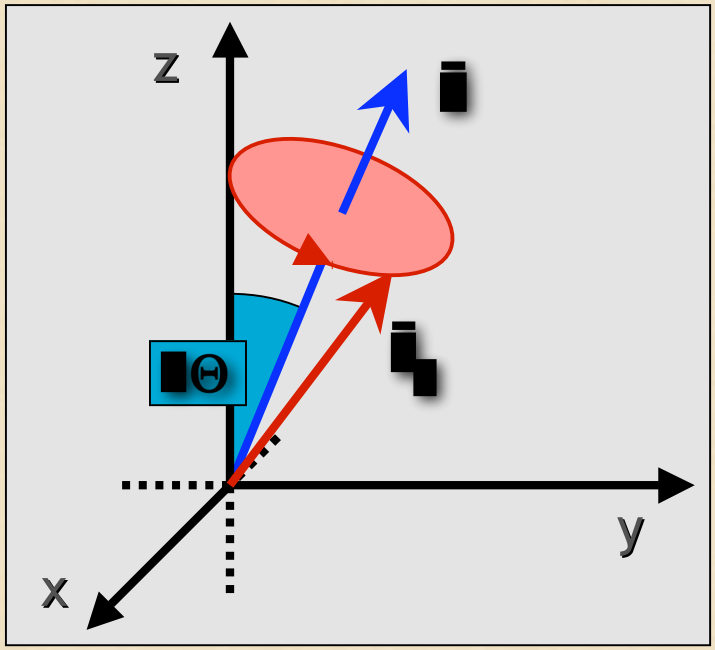
Nonlinear nu-nu effects are important when nu-nu interaction energy exceeds typical vacuum oscillation frequency (Do not compare with matter effect!)

$$\omega_{pq} = \frac{\Delta m^2}{2p} < \mu = \sqrt{2}\, G_F n_\nu \langle 1 - \cos\theta \rangle$$

Two-Flavor Neutrino Oscillations in Vacuum

Polarization vector	$\rho = \frac{1 + \bar{\sigma}\cdot\bar{P}}{2}$ or different normalization $\rho = P\,\frac{1 + \bar{\sigma}\cdot\bar{P}}{2}$ σ_i Pauli matrices
"Magnetic field" in flavor space	$\Omega \approx 1 + \frac{\Delta m^2}{2p} + \frac{\Delta m^2}{2p}\,\frac{\bar{\sigma}\cdot\bar{\Omega}}{2}$ $\quad \bar{\Omega} = \begin{pmatrix} \sin 2\theta \\ 0 \\ \cos 2\theta \end{pmatrix}$

Neutrino flavor oscillation as a spin precession



Neutrinos	Anti-neutrinos
$\partial \bar{P} = +\frac{\Delta m^2}{2p}\,\bar{\Omega}\times\bar{P}$	$\partial \bar{P} = -\frac{\Delta m^2}{2p}\,\bar{\Omega}\times\bar{P}$
Spin 1/2 ↑ ↑ Magnetic moment $+\Delta m^2/2p$	Spin 1/2 ↑ ↓ Magnetic moment $-\Delta m^2/2p$

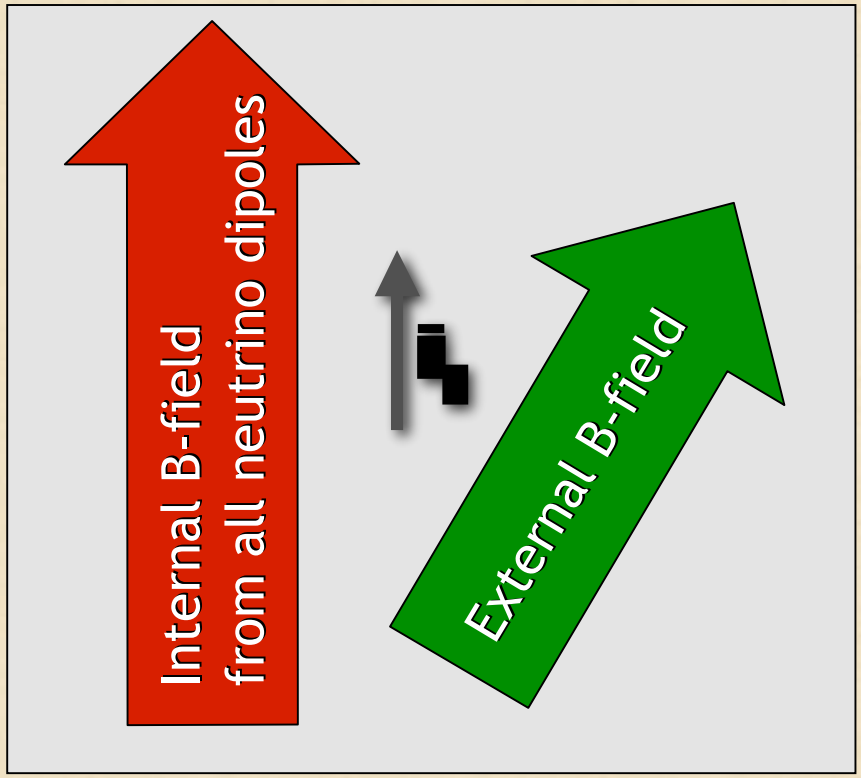
Synchronized Oscillations by Self-Interactions

$$\partial \vec{P} = \frac{\Delta m^2}{2E} \vec{B} \times \vec{P} + \sqrt{2} G_F n_\nu \vec{I} \times \vec{P} \qquad \vec{I} = \int \frac{d^3p}{(2\pi)^3} \vec{P}$$

Integrated polarization vector

Neutrinos precess in external magnetic field B (in flavor space)

The ensemble of neutrino magnetic moments creates an "internal magnetic field" that is felt by each neutrino

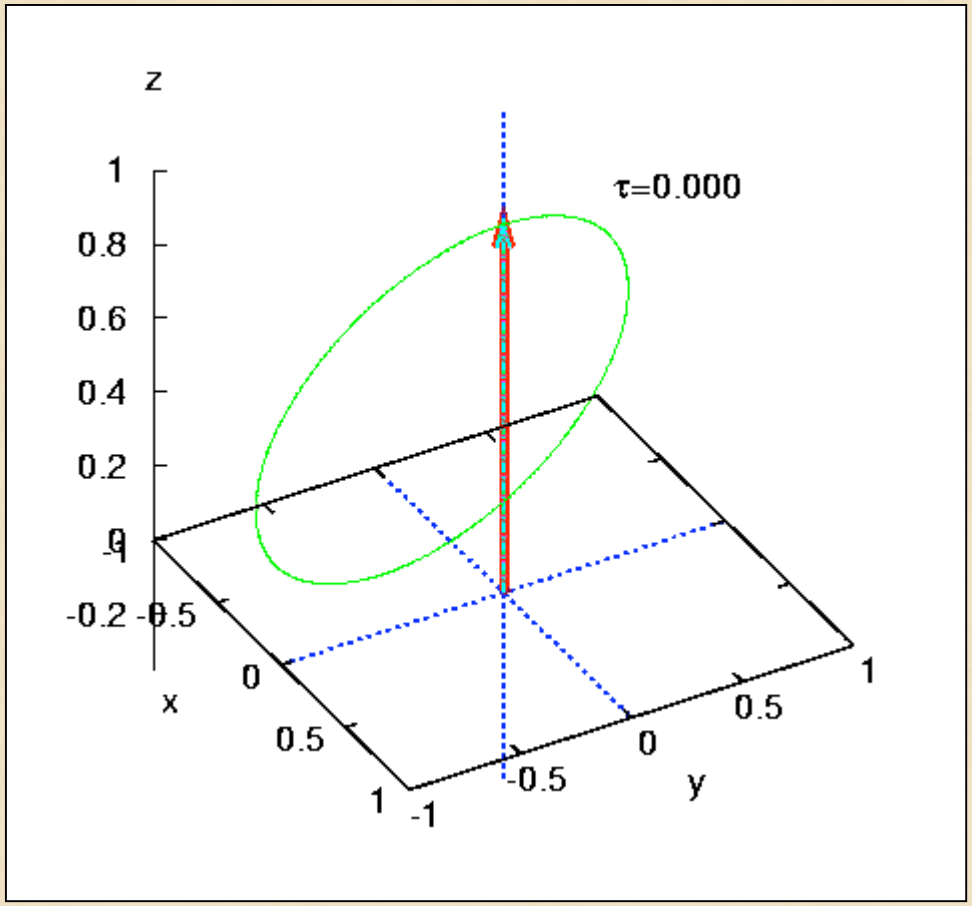


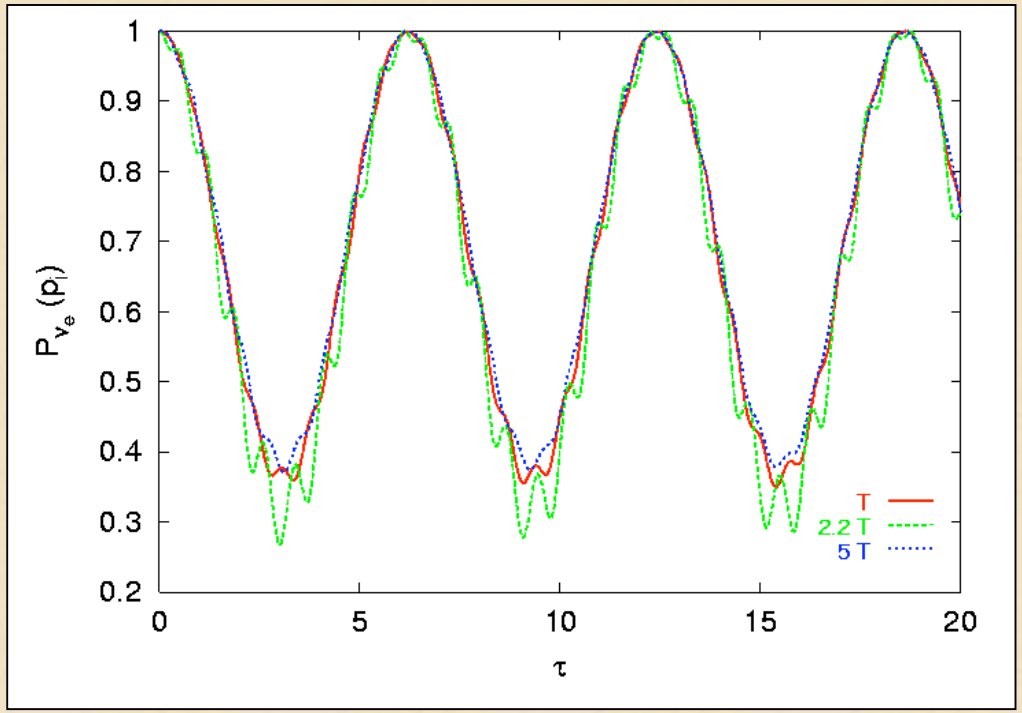
<u>Internal field ≫ external B:</u>
All modes are locked to each other and spin-precess together in analogy to spin-orbit coupling in atoms, causing the anomalous Zeeman effect.

Synchronized oscillation frequency

$$\omega_{\text{synch}} = \left\langle \frac{\Delta m^2}{2E} \right\rangle$$

Synchronized Oscillations by Self-Interactions





Individual modes precess around large common dipole moment

Pastor, Raffelt & Semikoz, PRD 65 (2002) 053011

Equations of Motion for Two-Flavor Case

ν	$\partial_t \vec{P} = +\frac{\Delta m^2}{2E}\vec{B}\times\vec{P} + \sqrt{2}G_F n_e \vec{v}\times\vec{P} + \sqrt{2}G_F \int \frac{d^3p}{(2\pi)^3}(1-\cos\theta_{\nu\nu})(\vec{P}-\vec{\bar{P}})\times\vec{P}$
$\bar{\nu}$	$\partial_t \vec{\bar{P}} = -\frac{\Delta m^2}{2E}\vec{B}\times\vec{\bar{P}} + \sqrt{2}G_F n_e \vec{v}\times\vec{\bar{P}} + \sqrt{2}G_F \int \frac{d^3p}{(2\pi)^3}(1-\cos\theta_{\nu\nu})(\vec{P}-\vec{\bar{P}})\times\vec{\bar{P}}$

ν	$\partial_t \vec{P} = +\omega\vec{B}\times\vec{P} + \mu(\vec{P}-\vec{\bar{P}})\times\vec{P}$	Isotropic neutrinos, ignore matter, only one neutrino energy $\omega = -\frac{\Delta m^2}{2E}$ $\mu = \sqrt{2}G_F n_\nu$
$\bar{\nu}$	$\partial_t \vec{\bar{P}} = -\omega\vec{B}\times\vec{\bar{P}} + \mu(\vec{P}-\vec{\bar{P}})\times\vec{\bar{P}}$	
Sum	$\partial_t \vec{S} = +\omega\vec{B}\times\vec{D} + \mu\vec{D}\times\vec{S}$	Neglect first term for $\omega \ll \mu$ $\partial_t \vec{D} = \omega\mu(\vec{B}\times\vec{D})\times\vec{S}$ Length S = 2 approximately conserved
Diff.	$\partial_t \vec{D} = \omega\vec{B}\times\vec{D}$	

Tilt angle of S relative to B-direction	$\ddot{\varphi} = \omega\mu\sin\varphi\cos\varphi$	Pendulum in flavour space • Inverted (unstable) for inverted hierarchy • Stable (harmonic oscillator) otherwise

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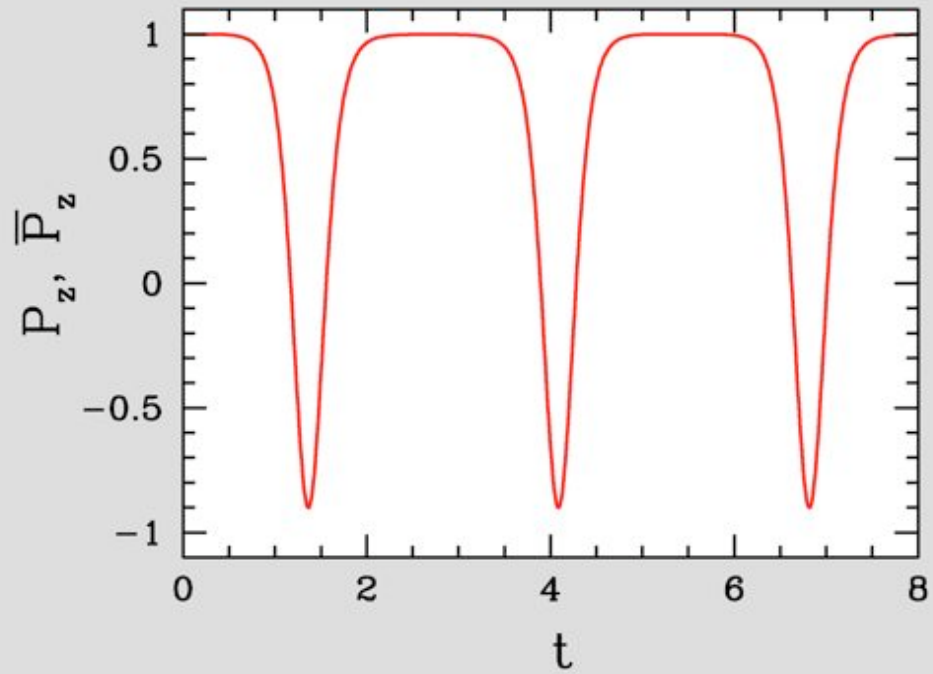

- Time scale set by $\kappa = \sqrt{\omega\mu}$
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This is no real "flavor conversion",
Rather a "coherent pair conversion"

$\nu_{\blacksquare}\bar{\nu}_{\blacksquare} \leftrightarrow \nu_{\mu}\bar{\nu}_{\mu}$

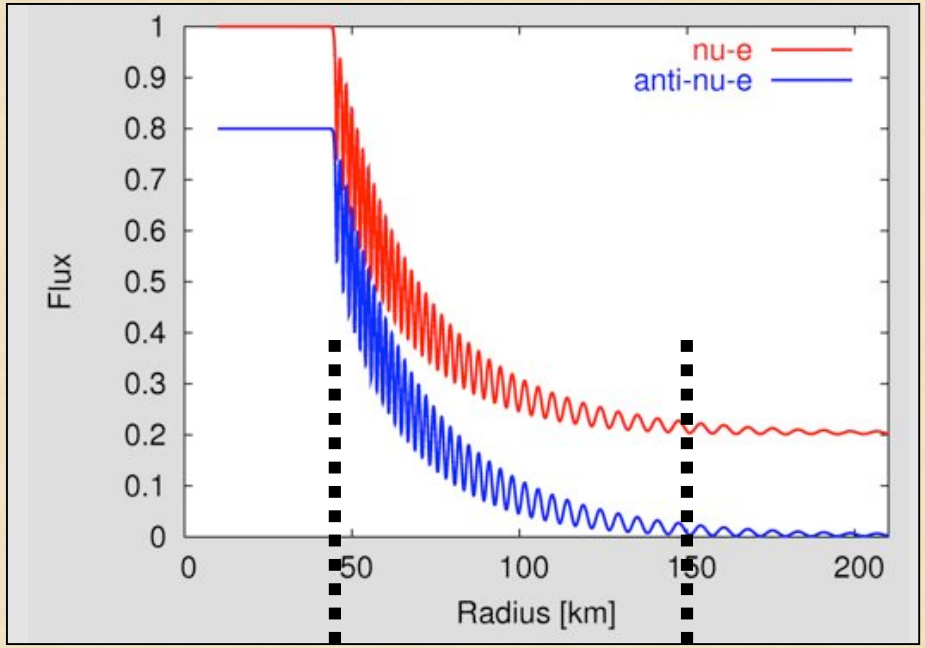
Time scale proportional to $\sqrt{\blacksquare_{\blacksquare}}$

Transition Between Different Oscillation Modes

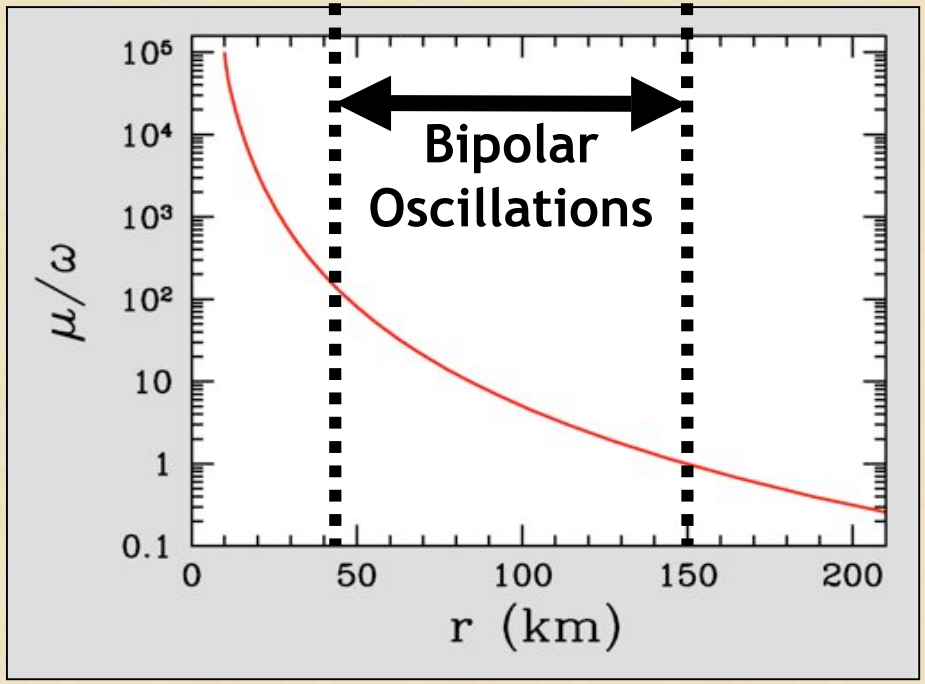
$$\partial \bar{P} = + \frac{\Delta m^2}{2E} \bar{B} \times \bar{P} + \sqrt{2} G_F n_\nu \bar{B} \times \bar{P} + \sqrt{2} G_F \int \frac{d^3 p}{(2\pi)^3} (1 - \cos\theta_{\nu\nu}) (\bar{P} - \bar{P}) \times \bar{P}$$

No collective effects for small neutrino densities	Nu-nu term never important relative to vacuum term $\sqrt{2} G_F (n_\nu + n_{\bar{\nu}}) < \frac{\Delta m^2}{2E}$
Bipolar oscillations for ***intermediate*** nu densities (Duan, Fuller & Qian astro-ph/0511275)	Nu-nu term important, depending on P's $\sqrt{2} G_F \frac{(n_\nu - n_{\bar{\nu}})^2}{2(n_\nu + n_{\bar{\nu}})} < \frac{\Delta m^2}{2E}$
Synchronised oscillations for large nu densities & sufficient asymmetry	Nu-nu term always important $\frac{\Delta m^2}{2E} < \sqrt{2} G_F \frac{(n_\nu - n_{\bar{\nu}})^2}{2(n_\nu + n_{\bar{\nu}})}$

Toy Supernova in "Single-Angle" Approximation



- Assume 80% anti-neutrinos
- Vacuum oscillation frequency $\omega = 0.3\ \mathrm{km}^{-1}$
- Neutrino-neutrino interaction energy at nu sphere (r = 10 km) $\mu = 0.3\times10^{5}\ \mathrm{km}^{-1}$
- Falls off approximately as r^{-4} (geometric flux dilution and nus become more co-linear)



Decline of oscillation amplitude explained in pendulum analogy by inreasing moment of inertia (Hannestad, Raffelt, Sigl & Wong astro-ph/0608695)

Sources of Decoherence

$$\partial \bar{\mathbf{P}} = + \frac{\Delta m^2}{2E} \bar{\mathbf{B}} \times \bar{\mathbf{P}} + \sqrt{2} G_F n_e \bar{\mathbf{v}} \times \bar{\mathbf{P}} + \sqrt{2} G_F \int \frac{d^3 p'}{(2\pi)^3} (1 - \cos\theta_{pp'}) \bar{\mathbf{P}}' - \bar{\mathbf{P}}' \times \bar{\mathbf{P}}$$

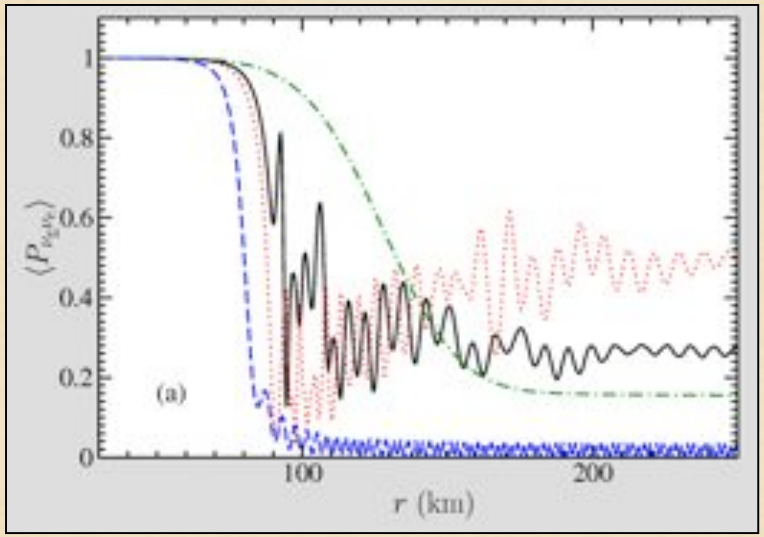
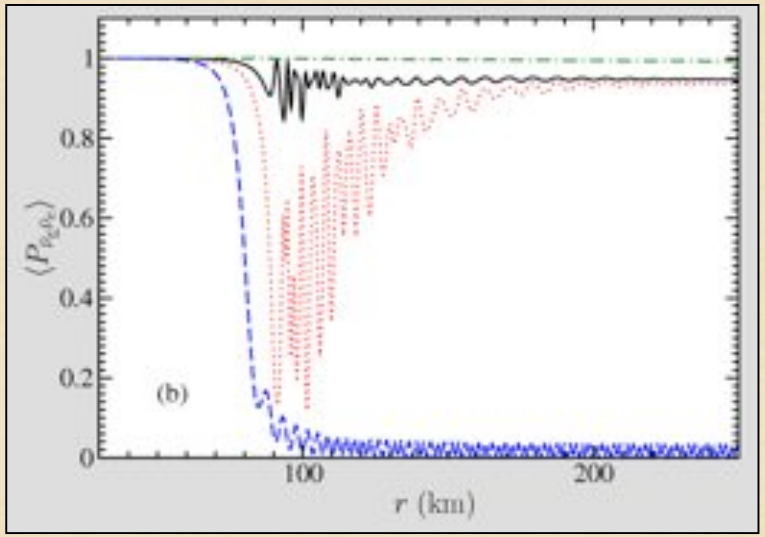
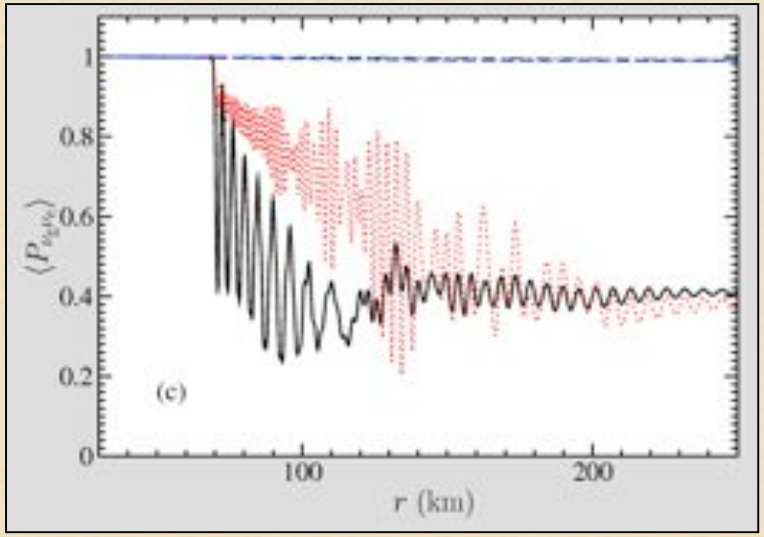
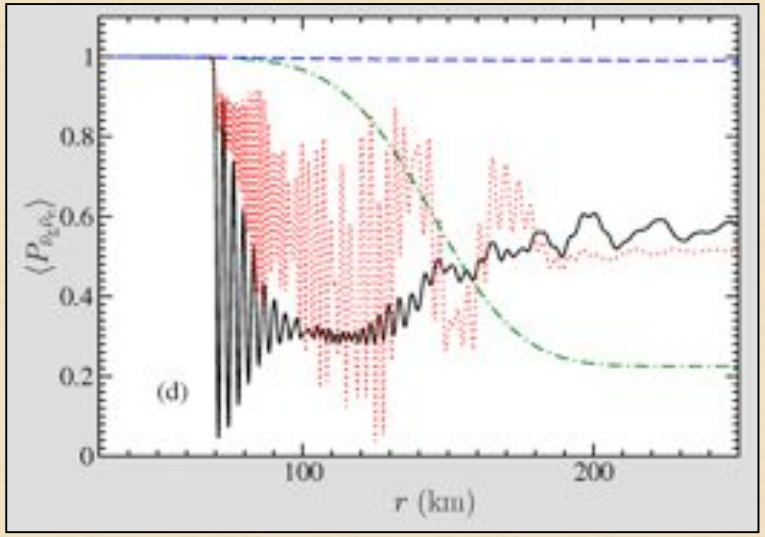
- Different oscillation frequencies do not lead to decoherence
- Evolution still governed by a single flavour variable with

  (in complete contrast to ordinary oscillations with energy dispersion, but similar to synchronised case)

(Hannestad, Raffelt, Sigl, Wong, astro-ph/0608695)

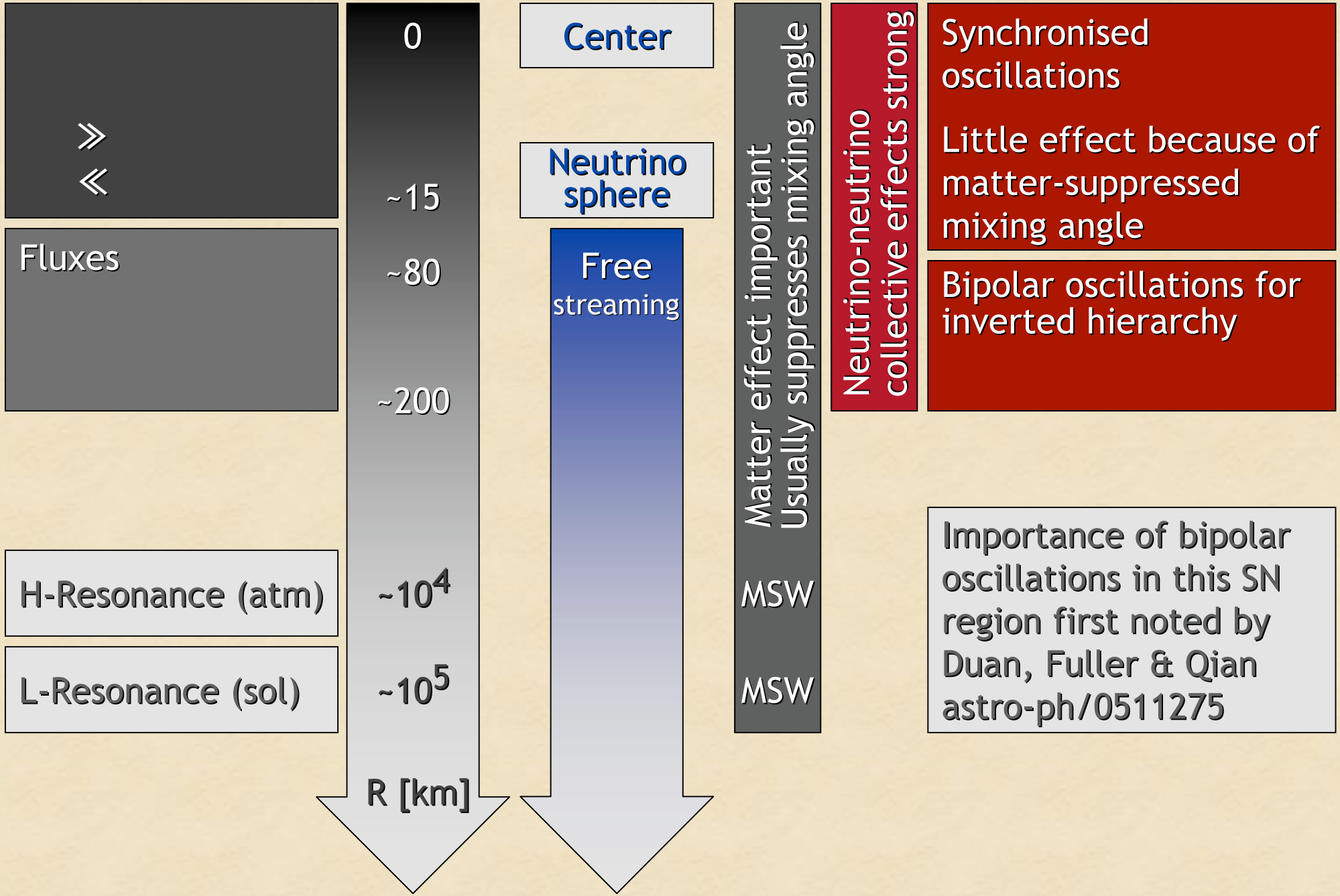
- Different coupling strengths do not average in a non-isotropic medium, e.g. nus streaming off a SN core ("multi-angle" case as numerically studied by Duan, Fuller, Carlson & Qian, astro-ph/0606616)
- Must lead to decoherence. But how much?
- Isotropic treatment a reasonable proxy for multi-angle case? (as suggested in astro-ph/0606616) If so, why?

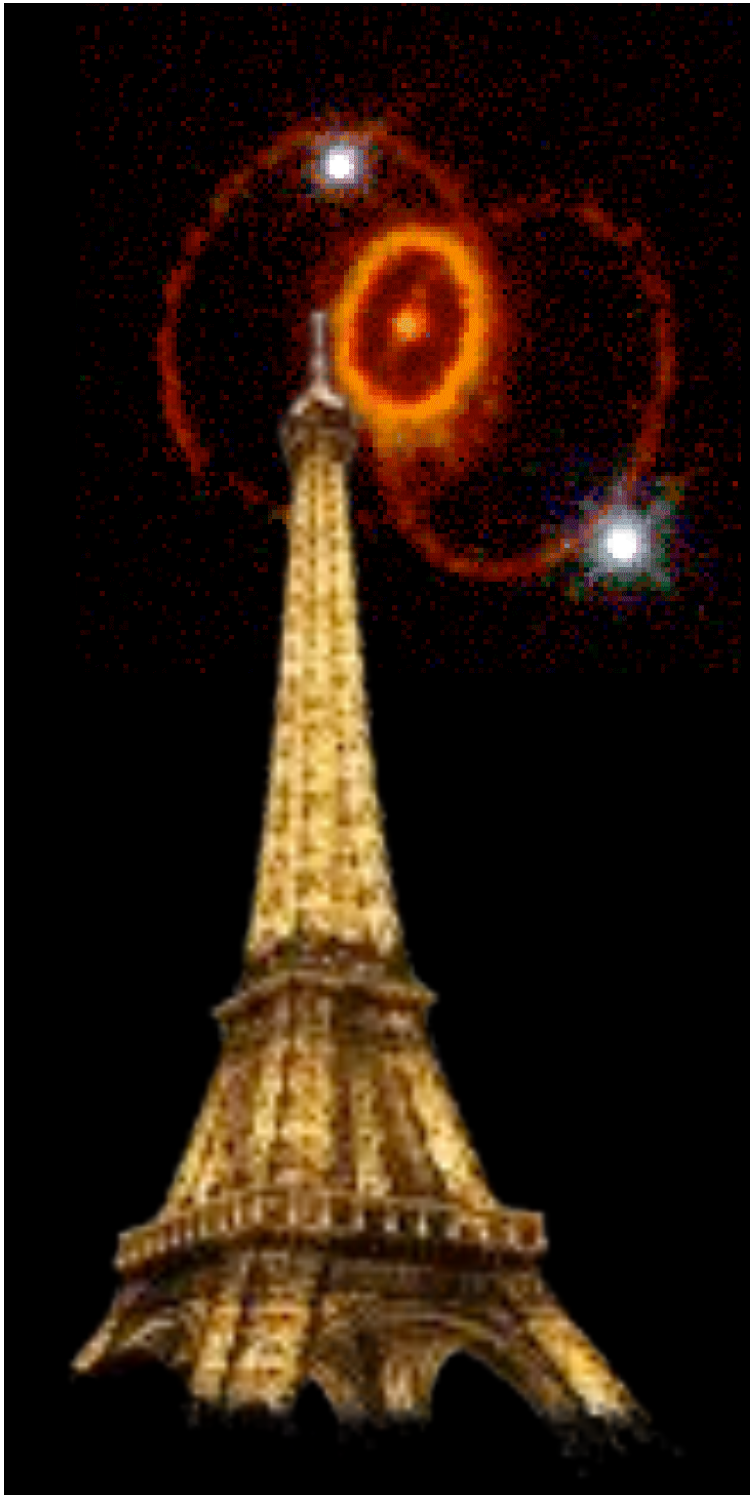
Nonlinear Neutrino Conversion in Supernovae

	Survival probability ν_e	Survival probability $\bar{\nu}_e$
Normal Hierarchy atm Δm^2 Θ_{13} close to Chooz limit		
Inverted Hierarchy		

Duan, Fuller, Carlson, Qian: "Simulation of Coherent Non-Linear Neutrino Flavor Transformation in the Supernova Environment. 1. Correlated Neutrino Trajectories", astro-ph/0606616. See also: astro-ph/0608050

Different Oscillation Modes in Supernovae





Conclusions

Simultaneous ν and $\bar{\nu}$ flavor conversion possible by bipolar collective oscillation mode at few 10 to few 100 km above neutrino sphere

Depending on primary neutrino flux spectra, may
- Modify energy transfer to shock wave
- Modify neutrino-driven nucleosynthesis
- Modify observable signatures of SN neutrino oscillations

- In a non-isotropic medium (as for neutrino streaming off a SN core), both collective conversion and kinematical decoherence possible
- Which form is more generic in the SN context?

- Large-scale numerical simulations crucial
- Reduction to theoretically tractable “toy cases” equally important for this nonlinear system

Selected Literature on Bipolar Oscillations

Samuel, PRD 48 (1993) 1462, hep-ph/9604341 Kostelecký & Samuel, hep-ph/9506262	• Numerical discovery of collective phenomena in dense neutrino gases (synchronised and bipolar) • Analytic solutions for basic cases
Pastor & Raffelt astro-ph/0207281	Strong conversion effects in SN hot bubble region numerically observed even for small Δm^2, but connection to bipolar mode not recognised
Duan, Fuller & Qian, astro-ph/0511275	• Identify bipolar oscillation mode as crucial in SN region up to a few 100 km above nu-sphere • Probably not (much) affected by ordinary matter
Duan, Fuller, Carlson & Qian, astro-ph/0606616 astro-ph/0608050	• Large-scale numerical simulations of "multi-angle effect" (including variation of nu-nu interaction for different modes in non-isotropic medium relevant for SN nus)
Hannestad, Raffelt, Sigl & Wong, astro-ph/0608695	• Identify collective motion as a "pendulum in flavor space", explains many puzzling details • Bipolar conversion is the instability of an inverted harmonic oscillator • Matter indeed causes unimportant log delay